Chapter 1
INTRODUCTION AND OVERVIEW

Thermodynamics, Heat Transfer, and Fluid Mechanics

1-1C Classical thermodynamics is based on experimental observations whereas statistical thermodynamics is based on the average behavior of large groups of particles.

1-2C On a downhill road the potential energy of the bicyclist is being converted to kinetic energy, and thus the bicyclist picks up speed. There is no creation of energy, and thus no violation of the conservation of energy principle.

1-3C There is no truth to his claim. It violates the second law of thermodynamics.

1-4C Thermodynamics deals with the amount of heat transfer as a system undergoes a process from one equilibrium state to another. Heat transfer, on the other hand, deals with the rate of heat transfer as well as the temperature distribution within the system at a specified time.

1-5C The driving force is (*a*) the temperature difference for heat transfer, (*b*) the electric potential difference for electric current flow (voltage), and (*c*) the pressure difference for fluid flow.

1-6C Heat transfer is a non-equilibrium phenomena since in a system that is in equilibrium there can be no temperature differences and thus no heat flow.

1-7C The caloric theory is based on the assumption that heat is a fluid-like substance called the "caloric" which is a massless, colorless, and odorless substance. It was abandoned in the middle of the nineteenth century after it was shown that there is no such thing as the caloric.

1-8C No, there cannot be any heat transfer between two bodies that are at the same temperature (regardless of pressure) since the driving force for heat transfer is temperature difference.

1-9C **Stress** is defined as force per unit area, and is determined by dividing the force by the area upon which it acts. The normal component of a force acting on a surface per unit area is called the **normal stress**, and the tangential component of a force acting on a surface per unit area is called **shear stress**. In a fluid, the normal stress is called **pressure**.

1-10C A certain region of a flow is called the **flow field**. The fluid particles that follow the same path form a line, which is called the **streamline**. The streamlines are tangent to the velocity vector and thus the direction of flow at every point in the flow field, and no fluid particle can cross a streamline.

Mass, Force, and Acceleration

1-11C Pound-mass lbm is the mass unit in English system whereas pound-force lbf is the force unit. One pound-force is the force required to accelerate a mass of 32.174 lbm by 1 ft/s^2. In other words, the weight of a 1-lbm mass at sea level is 1 lbf.

1-12C Kg is the mass unit in the SI system whereas kg-force is a force unit. 1-kg-force is the force required to accelerate a 1-kg mass by 9.807 m/s^2. In other words, the weight of 1-kg mass at sea level is 1 kg-force.

1-13C There is no acceleration, thus the net force is zero in both cases.

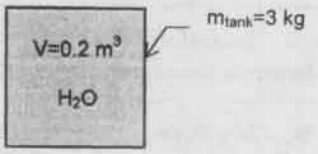
1-14 A plastic tank is filled with water. The weight of the combined system is to be determined.

Assumptions The density of water is constant throughout.

Properties The density of water is given to be $\rho = 1000\ \text{kg/m}^3$.

Analysis The mass of the water in the tank and the total mass are

$$m_w = \rho V = (1000\ \text{kg/m}^3)(0.2\ \text{m}^3) = 200\ \text{kg}$$
$$m_{\text{total}} = m_w + m_{tank} = 200 + 3 = 203\ \text{kg}$$



Thus,

$$W = mg = (203\ \text{kg})(9.807\text{m/s}^2)\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{1991N}$$

1-15 The interior dimensions of a room are given. The mass and weight of the air in the room are to be determined.

Assumptions The density of air is constant throughout the room.

Properties The density of air is given to be $\rho = 1.6\ \text{kg/m}^3$.

Analysis The mass of the air in the room is

$$m = \rho V = (1.16\ \text{kg/m}^3)(6\times 6\times 8\ \text{m}^3) = \mathbf{334.1\ kg}$$



Thus,

$$W = mg = (334.1\text{kg})(9.807\text{m/s}^2)\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{3277N}$$

1-16 The variation of gravitational acceleration above the sea level is given as a function of altitude. The height at which the weight of a body will decrease by 1% is to be determined.

Analysis The weight of a body at the elevation z can be expressed as

$$W = mg = m(9.807 - 3.32 \times 10^{-6} z)$$

In our case,

$$W = 0.99W_s = 0.99mg_s = 0.99(m)(9.807)$$

Substituting,

$$0.99(9.807) = (9.807 - 3.32 \times 10^{-6} z) \longrightarrow z = \mathbf{29{,}539m}$$



1-17E An astronaut took his scales with him to space. It is to be determined how much he will weigh on the spring and beam scales in space.

Analysis (*a*) A spring scale measures weight, which is the local gravitational force applied on a body:

$$W = mg = (150\text{lbm})(5.48\text{ft/s}^2)\left(\frac{1\text{lbf}}{32.174\text{lbm} \cdot \text{ft/s}^2}\right) = \mathbf{25.5lbf}$$

(*b*) A beam scale compares masses and thus is not affected by the variations in gravitational acceleration. The beam scale will read what it reads on earth,

$$W = \mathbf{150\ lbf}$$

1-18 The acceleration of an aircraft is given in g's. The net upward force acting on a man in the aircraft is to be determined.

Analysis From the Newton's second law, the force applied is

$$F = ma = m(6\,\text{g}) = (90\text{ kg})(6 \times 9.807\text{m/s}^2)\left(\frac{1\text{N}}{1\,\text{kg} \cdot \text{m/s}^2}\right) = \mathbf{5292\ N}$$

1-19 A rock is thrown upward with a specified force. The acceleration of the rock is to be determined.

Analysis The weight of the rock is

$$W = mg = (5\text{kg})(9.79\text{m/s}^2)\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{48.95N}$$

Then the net force that acts on the rock is

$$F_{net} = F_{up} - F_{down} = 150 - 48.95 = 101.05\ N$$

From the Newton's second law, the acceleration of the rock becomes

$$a = \frac{F}{m} = \frac{101.05\text{N}}{5\text{kg}}\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right) = \mathbf{20.2m/s^2}$$



1-20 Gravitational acceleration g and thus the weight of bodies decreases with increasing elevation. The percent reduction in the weight of an airplane cruising at 13,000 m is to be determined. √

Properties The gravitational acceleration g is given to be 9.807 m/s^2 at sea level and 9.767 m/s^2 at an altitude of 13,000 m.

Analysis Weigh is proportional to the gravitational acceleration g, and thus the percent reduction in weight is equivalent to the percent reduction in the gravitational acceleration, which is determined from

$$\%\text{Reduction in weight} = \%\text{Reduction in } g = \frac{\Delta g}{g}\times 100 = \frac{9.807 - 9.767}{9.807}\times 100 = \mathbf{0.41\%}$$

Therefore, the airplane and the people in it will weight 0.41% less at 13,000 m altitude.

Discussion Note that the weight loss at cruising altitudes is negligible.



Systems and Properties

1-21C The radiator should be analyzed as an open system since mass is crossing the boundaries of the system.

1-22C A can of soft drink should be analyzed as a closed system since no mass is crossing the boundaries of the system.

1-23C Intensive properties do not depend on the size (extent) of the system but extensive properties do.

Solving Engineering Problems

1-24C The *rating* problems deal with the determination of the *heat transfer rate* for an existing system at a specified temperature difference. The *sizing* problems deal with the determination of the *size* of a system in order to transfer heat at a *specified rate* for a *specified temperature difference.*

1-25C The experimental approach (testing and taking measurements) has the advantage of dealing with the actual physical system, and getting a physical value within the limits of experimental error. However, this approach is expensive, time consuming, and often impractical. The analytical approach (analysis or calculations) has the advantage that it is fast and inexpensive, but the results obtained are subject to the accuracy of the assumptions and idealizations made in the analysis.

1-26C Modeling makes it possible to predict the course of an event before it actually occurs, or to study various aspects of an event mathematically without actually running expensive and time-consuming experiments. When preparing a mathematical model, all the variables that affect the phenomena are identified, reasonable assumptions and approximations are made, and the interdependence of these variables are studied. The relevant physical laws and principles are invoked, and the problem is formulated mathematically. Finally, the problem is solved using an appropriate approach, and the results are interpreted.

1-27C The right choice between a crude and complex model is usually the *simplest* model which yields *adequate* results. Preparing very accurate but complex models is not necessarily a better choice since such models are not much use to an analyst if they are very difficult and time consuming to solve. At the minimum, the model should reflect the essential features of the physical problem it represents.

1-28C The description of most scientific problems involve relations that relate the *changes* in some key variables to each other, and the smaller the increment chosen in the changing variables, the more accurate the description. In the limiting case of infinitesimal changes in variables, we obtain *differential equations*, which provide precise mathematical formulations for the physical principles and laws by representing the rates of changes as *derivatives*.

1-29C Despite the convenience and capability the engineering software packages offer, they are still just tools, and they will not replace the traditional engineering courses. They will simply cause a shift in emphasis in the course material from mathematics to physics. They are of great value in engineering practice, however, as engineers today rely on software packages for solving large and complex problems in a short time, and perform optimization studies efficiently.

Conservation of Mass

1-30C Mass flow rate is the amount of mass flowing through a cross-section per unit time whereas the volume flow rate is the amount of volume flowing through a cross-section per unit time.

1-31C The amount of mass or energy entering a control volume does not have to be equal to the amount of mass or energy leaving during an unsteady-flow process.

1-32C Flow through a control volume is steady when it involves no changes with time at any specified position.

1-33C No, a flow with the same volume flow rate at the inlet and the exit is not necessarily steady (unless the density is constant). To be steady, the mass flow rate through the device must remain constant.

1-34E A garden hose is used to fill a water bucket. The volume and mass flow rates of water, the filling time, and the discharge velocity are to be determined.

Assumptions **1** Water is an incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis (*a*) The volume and mass flow rates of water are

$$\dot{V} = A\mathbf{V} = (\pi D^2/4)\mathbf{V} = [\pi(1/12\text{ ft})^2/4](8\text{ ft/s}) = \mathbf{0.04363\ ft^3/s}$$

$$\dot{m} = \rho\dot{V} = (62.4\text{ lbm/ft}^3)(0.04363\text{ ft}^3/\text{s}) = \mathbf{2.72\ lbm/s}$$



(*b*) The time it takes to fill a 20-gallon bucket is

$$\Delta t = \frac{V}{\dot{V}} = \frac{20\text{ gal}}{0.04363\text{ ft}^3/\text{s}}\left(\frac{1\text{ ft}^3}{7.4804\text{ gal}}\right) = \mathbf{61.3\ s}$$

(*c*) The average discharge velocity of water at the nozzle exit is

$$\mathbf{V}_e = \frac{\dot{V}}{A_e} = \frac{\dot{V}}{\pi D_e^2/4} = \frac{0.04363\text{ ft}^3/\text{s}}{[\pi(0.5/12\text{ ft})^2/4]} = \mathbf{32\ ft/s}$$

Discussion Note that for a given flow rate, the average velocity is inversely proportional to the square of the velocity. Therefore, when the diameter is reduced by half, the velocity quadruples.

1-35 Air is accelerated in a nozzle. The mass flow rate, the exit temperature, and the exit area of the nozzle are to be determined.

Assumptions Flow through the nozzle is steady.

Properties The density of air is given to be 2.21 kg/m^3 at the inlet, and 0.762 kg/m^3 at the exit.

Analysis (a) The mass flow rate of air is determined from the inlet conditions to be

$$\dot{m} = \rho_1 A_1 V_1 = (2.21\,\text{kg/m}^3)(0.008\,\text{m}^2)(30\,\text{m/s}) = \mathbf{0.530\ kg/s}$$



(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.
Then the exit area of the nozzle is determined to be

$$\dot{m} = \rho_2 A_2 \mathbf{V}_2 \longrightarrow A_2 = \frac{\dot{m}}{\rho_2 \mathbf{V}_2} = \frac{0.530\,\text{kg/s}}{(0.762\,\text{kg/m}^3)(180\,\text{m/s})} = 0.00387\,\text{m}^2 = \mathbf{38.7\ cm^2}$$

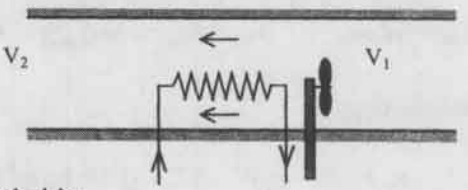
1-36 Air is expanded and is accelerated as it is heated by a hair dryer of constant diameter. The percent increase in the velocity of air as it flows through the drier is to be determined.

Assumptions Flow through the nozzle is steady.

Properties The density of air is given to be 1.20 kg/m^3 at the inlet, and 1.05 kg/m^3 at the exit.

Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Then,

$$\dot{m}_1 = \dot{m}_2$$
$$\rho_1 A \mathbf{V}_1 = \rho_2 A \mathbf{V}_2$$
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\rho_1}{\rho_2} = \frac{1.20\,\text{kg/m}^3}{1.05\,\text{kg/m}^3} = 1.14 \quad \text{(or, and increase of } \mathbf{14\%}\text{)}$$



Therefore, the air velocity increases 14% as it flows through the hair drier.

1-37E The ducts of an air-conditioning system pass through an open area. The inlet velocity and the mass flow rate of air are to be determined.

Assumptions Flow through the air conditioning duct is steady.

Properties The density of air is given to be 0.078 lbm/ft^3 at the inlet.

Analysis The inlet velocity of air and the mass flow rate through the duct are

$$\mathbf{V}_1 = \frac{\dot{V}_1}{A_1} = \frac{\dot{V}_1}{\pi D^2/4} = \frac{450 \text{ ft}^3/\text{min}}{\pi(10/12 \text{ ft})^2/4} = \mathbf{825 \text{ ft/min} = 13.8 \text{ ft/s}}$$



$$\dot{m} = \rho_1 \dot{V}_1 = (0.078 \text{ lbm/ft}^3)(450 \text{ ft}^3/\text{min}) = 35.1 \text{ lbm/min} = \mathbf{0.585 lbm/s}$$

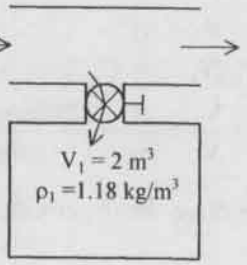
1-38 A rigid tank initially contains air at atmospheric conditions. The tank is connected to a supply line, and air is allowed to enter the tank until the density rises to a specified level. The mass of air that entered the tank is to be determined.

Assumptions Flow through the nozzle is steady.

Properties The density of air is given to be 1.18 kg/m^3 at the beginning, and 7.20 kg/m^3 at the end.

Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. The mass balance for this system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2 - m_1 = \rho_2 V - \rho_1 V$



Substituting,

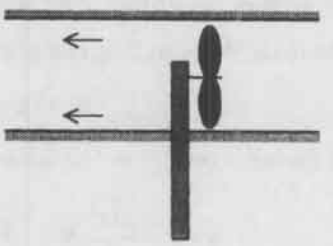
$$m_i = (\rho_2 - \rho_1)V = [(7.20 - 1.18) \text{ kg/m}^3](2 \text{ m}^3) = 9.58 \text{ kg}$$

Therefore, 9.58 kg of mass entered the tank.

1-39 The ventilating fan of the bathroom of a building runs continuously. The mass of air "vented out" per day is to be determined.

Assumptions Flow through the fan is steady.

Properties The density of air in the building is given to be 1.20 kg/m^3.



Analysis The mass flow rate of air vented out is

$$\dot{m}_{air} = \rho \dot{V}_{air} = (1.20\ \text{kg/m}^3)(0.030\ \text{m}^3/\text{s}) = 0.036\ \text{kg/s}$$

Then the mass of air vented out in 24 h becomes

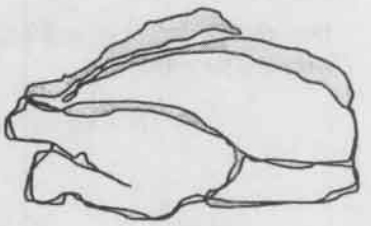
$$m = \dot{m}_{air}\,\Delta t = (0.036\ \text{kg/s})(24 \times 3600\ \text{s}) = \mathbf{3110\ kg}$$

Discussion Note that more than 3 tons of air is vented out by a bathroom fan in one day.

1-40E Chickens are to be cooled by chilled water in an immersion chiller. The mass flow rate of chicken through the chiller is to be determined.

Assumptions Chickens are dropped into the chiller steadily.

Properties The average mass of a chicken is 4.5 lbm.



Analysis Chickens are dropped into the chiller at a rate of 500 per hour. Therefore, chickens can be considered to flow steadily through the chiller at a mass flow rate of

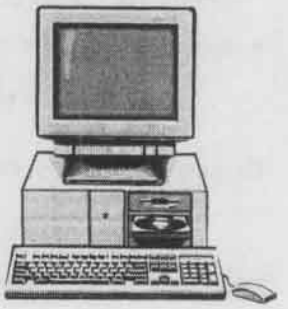
$$\dot{m}_{chicken} = (500\ \text{chicken/h})(4.5\ \text{lbm/chicken}) = 2250\ \text{kg/h} = \mathbf{0.625\ lbm/s}$$

Note that chicken can be treated conveniently as a "flowing fluid" in calculations.

1-41 A desktop computer is to be cooled by a fan at a high elevation where the air density is low. The mass flow rate of air through the fan and the diameter of the casing for a given velocity are to be determined.

Assumptions Flow through the fan is steady.

Properties The density of air at a high elevation is given to be 0.7 kg/m^3.



Analysis The mass flow rate of air is

$$\dot{m}_{air} = \rho \dot{V}_{air} = (0.7\,\text{kg/m}^3)(0.34\,\text{m}^3/\text{min}) = 0.238\,\text{kg/min} = 0.0040\,\text{kg/s}$$

If the mean velocity is 110 m/min, the diameter of the casing is

$$\dot{V} = A\mathbf{V} = \frac{\pi D^2}{4}\mathbf{V} \quad \rightarrow \quad D = \sqrt{\frac{4\dot{V}}{\pi \mathbf{V}}} = \sqrt{\frac{4(0.34\,\text{m}^3/\text{min})}{\pi(110\,\text{m/min})}} = \mathbf{0.063\,m}$$

Therefore, the diameter of the casing must be at least 0.063 cm to ensure that the mean velocity does not exceed 110 m/min.

Discussion This problem shows that engineering systems are sized to satisfy certain constraints imposed by certain considerations.

Review Problems

1-42 The gravitational acceleration changes with altitude. Accounting for this variation, the weights of a body at different locations are to be determined.

Analysis The weight of an 80-kg man at various locations is obtained by substituting the altitude z (values in m) into the relation

$$W = mg = (80\text{kg})(9.807 - 3.32\times10^{-6}\,z\,\text{m/s}^2)\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)$$

Sea level: (z = 0 m): W = 80×(9.807-3.32x10^{-6}×0) = 80×9.807 = **784.6 N**
Denver: (z = 1610 m): W = 80×(9.807-3.32x10^{-6}×1610) = 80×9.802 = **784.2 N**
Mt. Ev.: (z = 8848 m): W = 80×(9.807-3.32x10^{-6}×8848) = 80×9.778 = **782.2 N**

1-43 A water tank open to the atmosphere is initially filled with water. The tank discharges to the atmosphere through a long pipe connected to a valve. The initial discharge velocity from the tank and the time required to empty the tank are to be determined. √

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The tank is considered to be empty when the water level drops to the center of the valve.

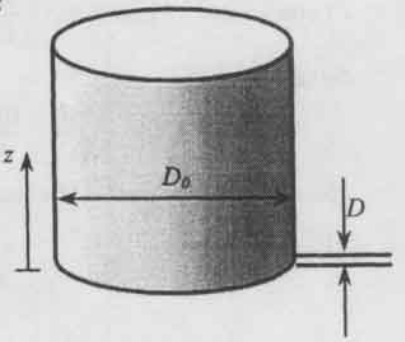
Analysis (*a*) Substituting the known quantities, the discharge velocity can be expressed as

$$\mathbf{V} = \sqrt{\frac{2gz}{1.5 + fL/D}} = \sqrt{\frac{2gz}{1.5 + 0.0015(100\,\text{m})/(0.10\,\text{m})}} = \sqrt{0.1212gz}$$

Then the initial discharge velocity becomes

$$\mathbf{V}_1 = \sqrt{0.1212gz_1} = \sqrt{0.1212(9.81\,\text{m/s}^2)(2\,\text{m})} = 1.54\,\text{m/s}$$

where z is the water height relative to the center of the orifice at that time.



(*b*) The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{V} = A_{\text{pipe}}\mathbf{V}_2 = \frac{\pi D^2}{4}\sqrt{0.1212gz}$$

Then the amount of water that flows through the pipe during a differential time interval dt is

$$dV = \dot{V}dt = \frac{\pi D^2}{4}\sqrt{0.1212gz}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV = A_{\text{tank}}(-dz) = -\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{0.1212gz}\,dt = -\frac{\pi D_0^2}{4}dz \rightarrow dt = -\frac{D_0^2}{D^2}\frac{dz}{\sqrt{0.1212gz}} = -\frac{D_0^2}{D^2\sqrt{0.1212g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2\sqrt{0.1212g}}\int_{z=z_1}^{0} z^{-1/2}dz \rightarrow t_f = -\frac{D_0^2}{D^2\sqrt{0.1212g}}\left|\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2\sqrt{0.1212g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{z_1}{0.1212g}} = \frac{(10\,\text{m})^2}{(0.1\,\text{m})^2}\sqrt{\frac{2\,\text{m}}{0.1212(9.81\,\text{m/s}^2)}} = 25{,}940\,\text{s} = \mathbf{7.21\,h}$$

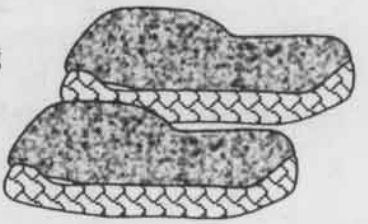
Discussion The draining time can be shortened considerably by installing a pump in the pipe.

1-44 A man is considering buying a 12-oz steak for $3.15, or a 320-g steak for $2.80. The steak that is a better buy is to be determined.

Assumptions The steaks are of identical quality.

Analysis To make a comparison possible, we need to express the cost of each steak on a common basis. Let us choose 1 kg as the basis for comparison. Using proper conversion factors, the unit cost of each steak is determined to be

12 ounce steak: $$\text{Unit Cost} = \left(\frac{\$3.15}{12\text{ oz}}\right)\left(\frac{16\text{ oz}}{1\text{ lbm}}\right)\left(\frac{1\text{ lbm}}{0.45359\text{ kg}}\right) = \mathbf{\$9.26/kg}$$



320 gram steak:

$$\text{Unit Cost} = \left(\frac{\$2.80}{320\text{ g}}\right)\left(\frac{1000\text{ g}}{1\text{ kg}}\right) = \mathbf{\$8.75/kg}$$

Therefore, the steak at the international market is a better buy.

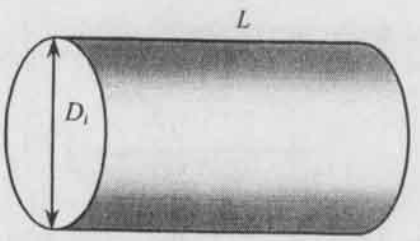
1-45 Milk is transported from Texas to California in a cylindrical tank. The amount of milk in the tank is to be determined.

Assumptions Milk is mostly water, and thus the properties of water can be used for milk.

Properties The density of milk is the same as that of water, $\rho = 1000\text{ kg/m}^3$.

Analysis Noting that the thickness of insulation is 0.05 m on all sides, the volume and mass of the milk in a full tank is determined to be

$$V_{\text{milk}} = (\pi D_i^2/4)L_i = [\pi(1.9\text{ m})^2/4](6.9\text{ m}) = 19.56\text{ m}^3$$
$$m_{\text{milk}} = \rho V_{\text{milk}} = (1000\text{ kg/m}^3)(19.56\text{ m}^3) = 19{,}560\text{ kg}$$



The volume of the milk in gallons is

$$V_{\text{milk}} = (19.56\text{ m}^3)\left(\frac{264.17\text{ gal}}{1\text{ m}^3}\right) = \mathbf{5167\text{ gal}}$$

1-46 The rate of accumulation of water in a pool and the rate of discharge are given. The rate supply of water to the pool is to be determined.

Assumptions **1** Water is supplied and discharged steadily. **2** The rate of evaporation of water is negligible. **3** No water is supplied or removed through other means.

Analysis The conservation of mass principle applied to the pool requires that the rate of increase in the amount of water in the pool be equal to the difference between the rate of supply of water and the rate of discharge. That is,

$$\frac{dm_{\text{pool}}}{dt} = \dot{m}_i - \dot{m}_e \quad \rightarrow \quad \dot{m}_i = \frac{dm_{\text{pool}}}{dt} + \dot{m}_e \quad \rightarrow \quad \dot{V}_i = \frac{dV_{\text{pool}}}{dt} + \dot{V}_e$$

since the density of water is constant and thus the conservation of mass is equivalent to conservation of volume. The rate of discharge of water is

$$\dot{V}_e = A_e \mathbf{V}_e = (\pi D^2/4)\mathbf{V}_e = [\pi(0.05\,\text{m})^2/4](5\,\text{m/s}) = 0.00982\,\text{m}^3/\text{s}$$

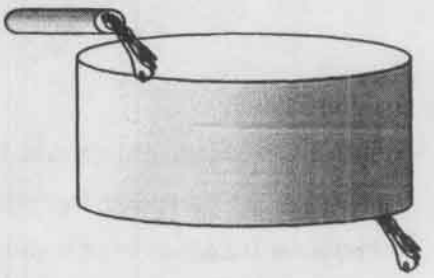
The rate of accumulation of water in the pool is equal to the cross-section of the pool times the rate at which the water level rises,

$$\frac{dV_{\text{pool}}}{dt} = A_{\text{cross-section}}\,\mathbf{V}_{\text{level}} = (3\,\text{m} \times 4\,\text{m})(0.015\,\text{m/min}) = 0.18\,\text{m}^3/\text{min} = 0.00300\,\text{m}^3/\text{s}$$

Substituting, the rate at which water is supplied to the pool is determined to be

$$\dot{V}_i = \frac{dV_{\text{pool}}}{dt} + \dot{V}_e = 0.003 + 0.00982 = \mathbf{0.01282\,m^3/s}$$

Therefore, water is supplied at a rate of 0.01282 m^3/s = 12.82 L/s.



1-47 The thrust developed by the jet engine of a Boeing 777 is given to be 85,000 pounds. This thrust is to be expressed in N and kgf.

Analysis Noting that 1 lbf = 4.448 N and 1 kgf = 9.807 N, the thrust developed can be expressed in two other units as

Thrust in N: $$\text{Thrust} = (85{,}000\,\text{lbf})\left(\frac{4.448\,\text{N}}{1\,\text{lbf}}\right) = \mathbf{378{,}080\,N}$$

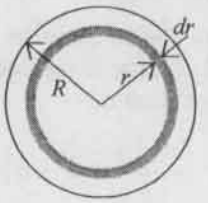
Thrust in kgf: $$\text{Thrust} = (378{,}080\,\text{N})\left(\frac{1\,\text{kgf}}{9.807\,\text{N}}\right) = \mathbf{38{,}552\,kgf}$$



1-48 A fluid is flowing in a circular pipe. A relation is to be obtained for the average fluid velocity in therms of $V(r)$, R, and r.

Analysis Choosing a circular ring of area dA = 2πrdr as our differential area, the mass flow rate through a cross-sectional area can be expressed as

$$\dot{m} = \int_A \rho \mathbf{V}(r)\, dA = \int_0^R \rho \mathbf{V}(r) 2\pi r\, dr$$



Setting this equal to and solving for V_{av},

$$\mathbf{V}_{av} = \frac{2}{R^2} \int_0^R \mathbf{V}(r) r\, dr$$

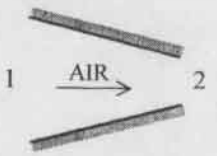
1-49 Air is accelerated in a nozzle. The density of air at the nozzle exit is to be determined.

Assumptions Flow through the nozzle is steady.

Properties The density of air is given to be 4.18 kg/m^3 at the inlet.

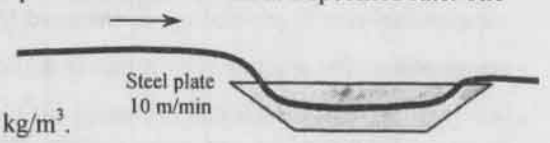
Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Then,

$$\dot{m}_1 = \dot{m}_2$$
$$\rho_1 A_1 \mathbf{V}_1 = \rho_2 A_2 \mathbf{V}_2$$
$$\rho_2 = \frac{A_1}{A_2} \frac{\mathbf{V}_1}{\mathbf{V}2} \rho_1 = 2 \frac{120 \text{ m/s}}{380 \text{ m/s}} (4.18 \text{ kg/m}^3) = 2.64 \text{ kg/m}^3$$



Discussion Note that the density of air decreases considerably despite a decrease in the cross-sectional area of the nozzle.

1-50 A long roll of large 1-Mn manganese steel plate is to be quenched in an oil bath at a specified rate. The mass flow rate of the plate is to be determined.



Assumptions The plate moves through the bath steadily.

Properties The density of steel plate is given to be $\rho = 7854\ \text{kg/m}^3$.

Analysis The mass flow rate of the sheet metal through the oil bath is

$$\dot{m} = \rho\dot{V} = \rho wtV = (7854\ \text{kg/m}^3)(1\ \text{m})(0.005\ \text{m})(10\ \text{m/min}) = 393\ \text{kg/min} = \mathbf{6.55\ kg/s}$$

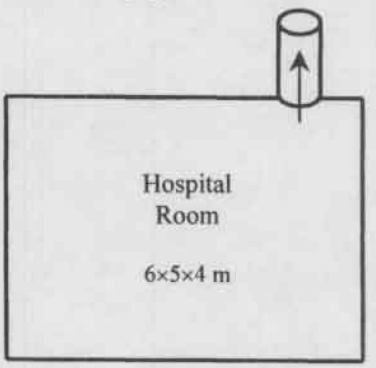
Therefore, steel plate can be treated conveniently as a "flowing fluid" in calculations.

1-51 The air in a hospital room is to be replaced every 20 minutes. The minimum diameter of the duct is to be determined if the air velocity is not to exceed a certain value.

Assumptions **1** The volume occupied by the furniture etc in the room is negligible. **2** The incoming conditioned air does not mix with the air in the room.

Analysis The volume of the room is

$$V = (6\ \text{m})(5\ \text{m})(4\ \text{m}) = 120\ \text{m}^3$$



To empty this air in 20 min, the volume flow rate must be

$$\dot{V} = \frac{V}{\Delta t} = \frac{120\ \text{m}^3}{20 \times 60\ \text{s}} = 0.10\ \text{m}^3/\text{s}$$

If the mean velocity is 5 m/s, the diameter of the duct is

$$\dot{V} = A\mathbf{V} = \frac{\pi D^2}{4}\mathbf{V} \quad \rightarrow \quad D = \sqrt{\frac{4\dot{V}}{\pi \mathbf{V}}} = \sqrt{\frac{4(0.10\ \text{m}^3/\text{s})}{\pi(5\ \text{m/s})}} = \mathbf{0.16\ m}$$

Therefore, the diameter of the duct must be at least 0.16 m to ensure that the air in the room is exchanged completely within 20 min while the mean velocity does not exceed 5 m/s.

Discussion This problem shows that engineering systems are sized to satisfy certain constraints imposed by certain considerations.

1-52E A study quantifies the cost and benefits of enhancing IAQ by increasing the building ventilation. The net monetary benefit of installing an enhanced IAQ system to the employer per year is to be determined.

Assumptions The analysis in the report is applicable to this work place.

Analysis The report sates that enhancing IAQ increases the productivity of a person by $90 per year, and decreases the cost of the respiratory illnesses by $39 a year while increasing the annual energy consumption by $6 and the equipment cost by about $4 a year. The net monetary benefit of installing an enhanced IAQ system to the employer per year is determined by adding the benefits and subtracting the costs to be

$$\text{Net benefit} = \text{Total benefits} - \text{total cost} = (90+39) - (6+4) = \$119/\text{year} \quad \text{(per person)}$$

The total benefit is determined by multiplying the benefit per person by the number of employees,

$$\text{Total net benefit} = \text{No. of employees} \times \text{Net benefit per person} = 120 \times \$119/\text{year} = \mathbf{\$14{,}280/year}$$

Discussion Note that the unseen savings in productivity and reduced illnesses can be very significant when they are properly quantified.

Chapter 2
BASIC CONCEPTS OF THERMODYNAMICS

State, Process, Forms of Energy

2-1C In electric heaters, electrical energy is converted to sensible internal energy.

2-2C The forms of energy involved are electrical energy and sensible internal energy. Electrical energy is converted to sensible internal energy, which is transferred to the water as heat.

2-3C The *macroscopic* forms of energy are those a system possesses as a whole with respect to some outside reference frame. The *microscopic* forms of energy, on the other hand, are those related to the molecular structure of a system and the degree of the molecular activity, and are independent of outside reference frames.

2-4C The sum of all forms of the energy a system possesses is called *total energy*. In the absence of magnetic, electrical and surface tension effects, the total energy of a system consists of the kinetic, potential, and internal energies.

2-5C The internal energy of a system is made up of sensible, latent, chemical and nuclear energies. The sensible internal energy is due to translational, rotational, and vibrational effects.

2-6C Thermal energy is the sensible and latent forms of internal energy, and it is referred to as heat in daily life.

2-7C For a system to be in thermodynamic equilibrium, the temperature has to be the same throughout but the pressure does not. However, there should be no unbalanced pressure forces present. The increasing pressure with depth in a fluid, for example, should be balanced by increasing weight.

2-8C A process during which a system remains almost in equilibrium at all times is called a quasi-equilibrium process. Many engineering processes can be approximated as being quasi-equilibrium. The work output of a device is maximum and the work input to a device is minimum when quasi-equilibrium processes are used instead of nonquasi-equilibrium processes.

2-9C A process during which the temperature remains constant is called isothermal; a process during which the pressure remains constant is called isobaric; and a process during which the volume remains constant is called isochoric.

2-10C The state of a simple compressible system is completely specified by two independent, intensive properties.

2-11C Yes, because temperature and pressure are two independent properties and the air in an isolated room is a simple compressible system.

2-12 A 1000-MW power plant is powered by nuclear fuel. The amount of nuclear fuel consumed per year is to be determined

Assumptions **1** The power plant operates continuously. **2** The conversion efficiency of the power plant remains constant. **3** The nuclear fuel is uranium. **4** The uranium undergoes complete fission in the plant (this is not the case in practice).

Properties The complete fission of 1 kg of uranium-235 releases 6.73×10^{10} kJ/kg of heat (given in text).

Analysis Noting that the conversion efficiency is 30%, the amount of energy consumed by the power plant is

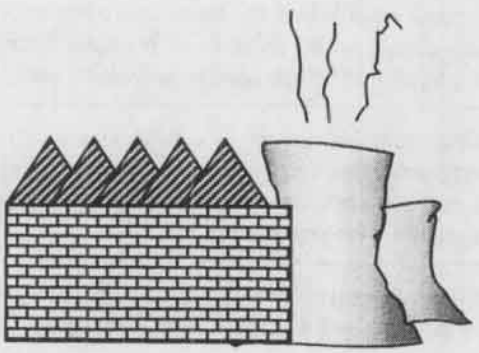
$$\text{Energy consumption rate} = \text{Power production/Efficiency}$$
$$= (1000\text{ MW})/0.3 = 3333\text{ MW} = 3.333\times10^{6}\text{ kJ/s}$$

$$\text{Annual energy consumption} = (\text{Energy consumtion rate})(1\text{ year})$$
$$= (3.333\times10^{6}\text{ kJ/s})(365\times24\times3600\text{ s/year})$$
$$= 1.051\times10^{14}\text{ kJ/year}$$

Noting that the complete fission of uranium-235 releases 6.73×10^{10} kJ/kg of heat, the amount of uranium that needs to be supplied to the power plant per year is

$$\text{Annual fuel consumption} = \frac{\text{Annual energy consumption}}{\text{Heating value of fuel}} = \frac{1.051\times10^{14}\text{ kJ/year}}{6.73\times10^{10}\text{ kJ/kg}}$$
$$= \mathbf{1562\text{ kg/year}}$$

Therefore, this power plant will consume about one and a half tons of nuclear fuel per year.



2-13 A 1000-MW power plant is powered by burning coal. The amount of coal consumed per year is to be determined

Assumptions **1** The power plant operates continuously. **2** The conversion efficiency of the power plant remains constant.

Properties The heating value of the coal is given to be 28,000 kJ/kg.

Analysis Noting that the conversion efficiency is 30%, the amount of chemical energy consumed by the power plant is

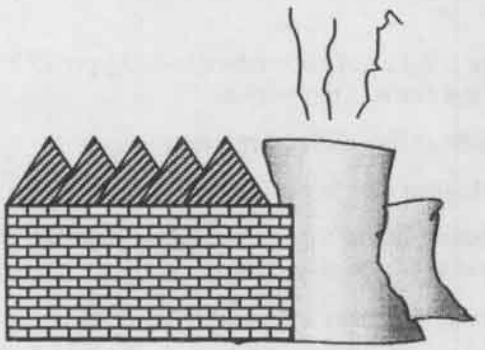
$$\text{Energy consumption rate} = \text{Power production/Efficiency}$$
$$= (1000\,\text{MW})/0.3 = 3333\,\text{MW} = 3.333\times10^{6}\ \text{kJ/s}$$

$$\text{Annual energy consumption} = (\text{Energy consumtion rate})(1\,\text{year})$$
$$= (3.333\times10^{6}\ \text{kJ/s})(365\times24\times3600\,\text{s/year})$$
$$= 1.051\times10^{14}\ \text{kJ/year}$$

Noting that the heating value of the coal is 28,000 kJ/kg, the amount of coal that needs to be supplied to the power plant per year is

$$\text{Annual fuel consumption} = \frac{\text{Annual energy consumption}}{\text{Heating value of fuel}} = \frac{1.051\times10^{14}\ \text{kJ/year}}{28{,}000\,\text{kJ/kg}}$$
$$= 3.754\times10^{9}\ \text{kg/year}$$
$$= \mathbf{3{,}754{,}000\ tons/year}$$

Therefore, this power plant will consume almost 4 millions tons of coal per year.



2-14 A power plant that burns natural gas produces 0.59 kg of carbon dioxide (CO_2) per kWh. The amount of CO_2 production that is due to the refrigerators in a city is to be determined.

Assumptions The city uses electricity produced by a natural gas power plant.

Properties 0.59 kg of CO_2 is produced per kWh of electricity generated (given).

Analysis Noting that there are 200,000 households in the city and each household consumes 700 kWh of electricity for refrigeration, the total amount of CO_2 produced is

$$\begin{aligned}\text{Amount of } CO_2 \text{ produced} &= (\text{Amount of electricity consumed})(\text{Amount of } CO_2 \text{ per kWh}) \\ &= (200{,}000 \text{ household})(700 \text{ kWh/household})(0.59 \text{ kg/kWh}) \\ &= 8.26\times10^7 \ CO_2 \text{ kg/year} \\ &= \mathbf{82{,}600\ CO_2\ ton/year}\end{aligned}$$

Therefore, the refrigerators in this city are responsible for the production of 82,600 tons of CO_2.

2-15 A power plant that burns coal, produces 1.1 kg of carbon dioxide (CO_2) per kWh. The amount of CO_2 production that is due to the refrigerators in a city is to be determined.

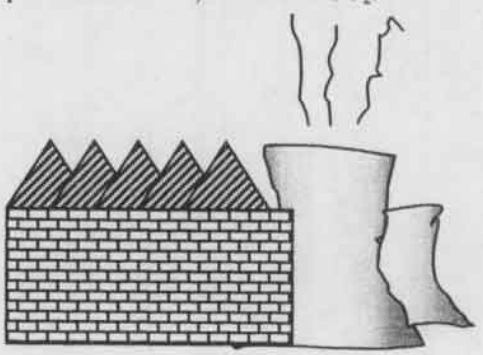
Assumptions The city uses electricity produced by a coal power plant.

Properties 1.1 kg of CO_2 is produced per kWh of electricity generated (given).

Analysis Noting that there are 200,000 households in the city and each household consumes 700 kWh of electricity for refrigeration, the total amount of CO_2 produced is

$$\begin{aligned}\text{Amount of } CO_2 \text{ produced} &= (\text{Amount of electricity consumed})(\text{Amount of } CO_2 \text{ per kWh}) \\ &= (200{,}000 \text{ household})(700 \text{ kWh/household})(1.1 \text{ kg/kWh}) \\ &= 15.4\times10^7 \ CO_2 \text{ kg/year} \\ &= \mathbf{154{,}000\ CO_2\ ton/year}\end{aligned}$$

Therefore, the refrigerators in this city are responsible for the production of 154,000 tons of CO_2.



2-16E A household uses fuel oil for heating, and electricity for other energy needs. Now the household reduces its energy use by 20%. The reduction in the CO_2 production this household is responsible for is to be determined.

Properties The amount of CO_2 produced is 1.54 lbm per kWh and 26.4 lbm per gallon of fuel oil (given).

Analysis Noting that this household consumes 8000 kWh of electricity and 1500 gallons of fuel oil per year, the amount of CO_2 production this household is responsible for is

$$\begin{aligned}\text{Amount of } CO_2 \text{ produced} &= (\text{Amount of electricity consumed})(\text{Amount of } CO_2 \text{ per kWh}) \\ &\quad + (\text{Amount of fuel oil consumed})(\text{Amount of } CO_2 \text{ per gallon}) \\ &= (8000\,\text{kWh/yr})(1.54\,\text{lbm/kWh}) + (1500\,\text{gal/yr})(26.4\ \text{lbm/gal}) \\ &= 51{,}920\ CO_2\ \text{lbm/year}\end{aligned}$$

Then reducing the electricity and fuel oil usage by 20% will reduce the annual amount of CO_2 production by this household by

$$\begin{aligned}\text{Reduction in } CO_2 \text{ produced} &= (0.20)(\text{Current amount of } CO_2 \text{ production}) \\ &= (0.20)(51{,}920\ CO_2\ \text{kg/year}) \\ &= \mathbf{10{,}384\ CO_2\ lbm/year}\end{aligned}$$

Therefore, any measure that saves energy also reduces the amount of pollution emitted to the environment.



2-17 A household has 2 cars, a natural gas furnace for heating, and uses electricity for other energy needs. The annual amount of NO_x emission to the atmosphere this household is responsible for is to be determined.

Properties The amount of NO_x produced is 7.1 g per kWh, 4.3 g per therm of natural gas, and 11 kg per car (given).

Analysis Noting that this household has 2 cars, consumes 1200 therms of natural gas, and 9,000 kWh of electricity per year, the amount of NO_x production this household is responsible for is

$$\begin{aligned}\text{Amount of } NO_x \text{ produced} &= (\text{No. of cars})(\text{Amount of } NO_x \text{ produced per car}) \\ &\quad + (\text{Amount of electricity consumed})(\text{Amount of } NO_x \text{ per kWh}) \\ &\quad + (\text{Amount of gas consumed})(\text{Amount of } NO_x \text{ per gallon}) \\ &= (2\,\text{cars})(11\,\text{kg/car}) + (9000\,\text{kWh/yr})(0.0071\,\text{kg/kWh}) \\ &\qquad + (1200\,\text{therms/yr})(0.0043\,\text{kg/therm}) \\ &= \mathbf{91.06\ NO_x\ kg/year}\end{aligned}$$

Discussion Any measure that saves energy will also reduce the amount of pollution emitted to the atmosphere.



Temperature

2-18C The zeroth law of thermodynamics states that two bodies are in thermal equilibrium if both have the same temperature reading, even if they are not in contact.

2-19C They are Celsius(°C) and Kelvin (K) in the SI, and Fahrenheit (°F) and Rankine (R) in the English system.

2-20C Probably, but not necessarily. The operation of these two thermometers is based on the thermal expansion of a fluid. If the thermal expansion coefficients of both fluids vary linearly with temperature, then both fluids will expand at the same rate with temperature, and both thermometers will always give identical readings. Otherwise, the two readings may deviate.

2-21 A temperature is given in °C. It is to be expressed in K.

Analysis The Kelvin scale is related to Celsius scale by

$$T(\text{K}) = T(^\circ\text{C}) + 273$$

Thus,

$$T(\text{K}) = 37^\circ\text{C} + 273 = \mathbf{310\ K}$$

2-22E A temperature is given in °C. It is to be expressed in °F, K, and R.

Analysis Using the conversion relations between the various temperature scales,

$$T(\text{K}) = T(^\circ\text{C}) + 273 = 18^\circ\text{C} + 273 = \mathbf{291\ K}$$
$$T(^\circ\text{F}) = 1.8T(^\circ\text{C}) + 32 = (1.8)(18) + 32 = \mathbf{64.4^\circ F}$$
$$T(\text{R}) = T(^\circ\text{F}) + 460 = 64.4 + 460 = \mathbf{524.4\ R}$$

2-23 A temperature change is given in °C. It is to be expressed in K.

Analysis This problem deals with temperature changes, which are identical in Kelvin and Celsius scales. Thus,

$$\Delta T(\text{K}) = \Delta T(^\circ\text{C}) = \mathbf{30\ K}$$

2-24E A temperature change is given in °F. It is to be expressed in °C, K, and R.

Analysis This problem deals with temperature changes, which are identical in Rankine and Fahrenheit scales. Thus,

$$\Delta T(\text{R}) = \Delta T(^\circ\text{F}) = \mathbf{27\ R}$$

The temperature changes in Celsius and Kelvin scales are also identical, and are related to the changes in Fahrenheit and Rankine scales by

$$\Delta T(\text{K}) = \Delta T(\text{R})/1.8 = 27/1.8 = \mathbf{15\ K}$$

and

$$\Delta T(^\circ\text{C}) = \Delta T(\text{K}) = \mathbf{15^\circ C}$$

2-25 Two systems having different temperatures and energy contents are brought in contact. The direction of heat transfer is to be determined.

Analysis Heat transfer occurs from warmer to cooler objects. Therefore, heat will be transferred from system B to system A until both systems reach the same temperature.

Pressure, Manometer, and Barometer

2-26C The pressure relative to the atmospheric pressure is called the *gage pressure*, and the pressure relative to an absolute vacuum is called *absolute pressure*.

2-27C The atmospheric air pressure which is the external pressure exerted on the skin decreases with increasing elevation. Therefore, the pressure is lower at higher elevations. As a result, the difference between the blood pressure in the veins and the air pressure outside increases. This pressure imbalance may cause some thin-walled veins such as the ones in the nose to burst, causing bleeding. The shortness of breath is caused by the lower air density at higher elevations, and thus lower amount of oxygen per unit volume.

2-28C No, the absolute pressure in a liquid of constant density does not double when the depth is doubled. It is the *gage pressure* that doubles when the depth is doubled.

2-29C If the lengths of the sides of the tiny cube suspended in water by a string are very small, the magnitudes of the pressures on all sides of the cube will be the same.

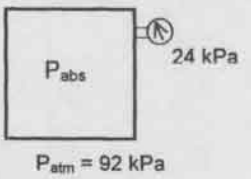
2-30C *Pascal's principle* states that *the pressure applied to a confined fluid increases the pressure throughout by the same amount*. This is a consequence of the pressure in a fluid remaining constant in the horizontal direction. An example of Pascal's principle is the operation of the hydraulic car jack.

2-31C The density of air at sea level is higher than the density of air on top of a high mountain. Therefore, the volume flow rates of the two fans running at identical speeds will be the same, but the mass flow rate of the fan at sea level will be higher.

2-32 The pressure in a vacuum chamber is measured by a vacuum gage. The absolute pressure in the chamber is to be determined.

Analysis The absolute pressure in the chamber is determined from

$$P_{abs} = P_{atm} - P_{vac} = 92 - 24 = \mathbf{68\ kPa}$$



2-33E The pressure in a tank is measured with a manometer by measuring the differential height of the manometer fluid. The absolute pressure in the tank is to be determined for the cases of the manometer arm with the higher and lower fluid level being attached to the tank .

Assumptions The fluid in the manometer is incompressible.

Properties The specific gravity of the fluid is given to be $\rho_s = 1.25$. The density of water at 32°F is 62.4 lbm/ft^3 (Table A-3E).

Analysis The density of the fluid is obtained by multiplying its specific gravity by the density of water,

$$\rho = (\rho_s)(\rho_{H_2O}) = (1.25)(62.4\,\text{lbm/ft}^3) = 78.0\,\text{lbm/ft}^3$$

The pressure difference corresponding to a differential height of 28 in between the two arms of the manometer is

$$\Delta P = \rho g h = (78\,\text{lbm/ft}^3)(32.174\,\text{ft/s}^2)(28/12\,\text{ft})\left(\frac{1\,\text{lbf}}{32.174\,\text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right) = 1.26\,\text{psia}$$

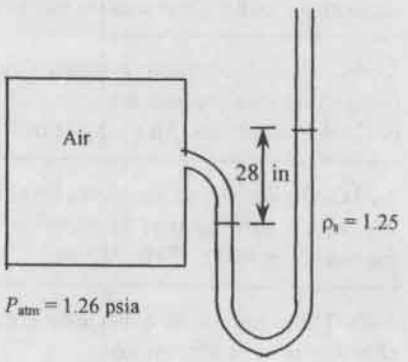
Then the absolute pressures in the tank for the two cases become:

(*a*) The fluid level in the arm attached to the tank is higher (vacuum):

$$P_{abs} = P_{atm} - P_{vac} = 12.7 - 1.26 = \mathbf{11.44\ psia}$$

(*b*) The fluid level in the arm attached to the tank is lower:

$$P_{abs} = P_{gage} + P_{atm} = 12.7 + 1.26 = \mathbf{13.96\ psia}$$



Discussion Note that we can determine whether the pressure in a tank is above or below atmospheric pressure by simply observing the side of the manometer arm with the higher fluid level.

2-34 The pressure in a pressurized water tank is measured by a multi-fluid manometer. The gage pressure of air in the tank is to be determined.

Assumptions The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air-water interface.

Properties The densities of mercury, water, and oil are given to be 13,600, 1000, and 850 kg/m^3, respectively.

Analysis Starting with the pressure at point 1 at the air-water interface, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach point 2, and setting the result equal to P_{atm} since the tube is open to the atmosphere gives

$$P_1 + \rho_{water} g h_1 + \rho_{oil} g h_2 - \rho_{mercury} g h_3 = P_{atm}$$

Solving for P_1,

$$P_1 = P_{atm} - \rho_{water} g h_1 - \rho_{oil} g h_2 + \rho_{mercury} g h_3$$

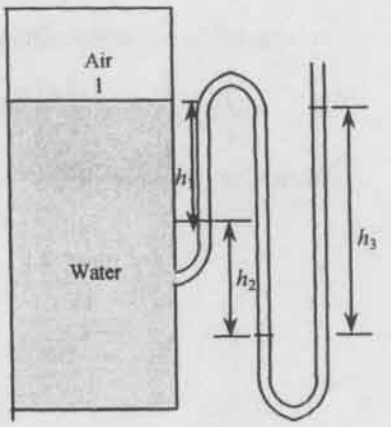
or,

$$P_1 - P_{atm} = g(\rho_{mercury} h_3 - \rho_{water} h_1 - \rho_{oil} h_2)$$

Noting that $P_{1,gage} = P_1 - P_{atm}$ and substituting,

$$P_{1,gage} = (9.81\,\text{m/s}^2)[(13{,}600\,\text{kg/m}^3)(0.46\,\text{m}) - (1000\,\text{kg/m}^3)(0.2\,\text{m}) - (850\,\text{kg/m}^3)(0.3\,\text{m}) +]\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1000\,\text{N/m}^2}\right)$$

$$= \mathbf{56.9\,kPa}$$



Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.

2-35 The barometric reading at a location is given in height of mercury column. The atmospheric pressure is to be determined.

Properties The density of mercury is given to be 13,600 kg/m^3.

Analysis The atmospheric pressure is determined directly from

$$P_{atm} = \rho g h$$

$$= (13{,}600\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.750\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1000\,\text{N/m}^2}\right)$$

$$= \mathbf{100.1\,kPa}$$

2-36 The gage pressure in a liquid at a certain depth is given. The gage pressure in the same liquid at a different depth is to be determined.

Assumptions The variation of the density of the liquid with depth is negligible.

Analysis The gage pressure at two different depths of a liquid can be expressed as

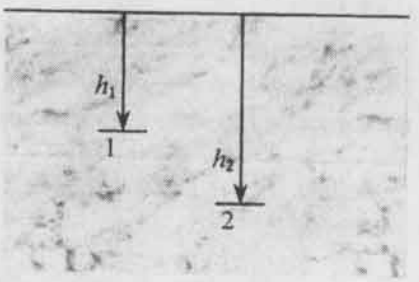
$$P_1 = \rho g h_1 \qquad \text{and} \qquad P_2 = \rho g h_2$$

Taking their ratio,

$$\frac{P_2}{P_1} = \frac{\rho g h_2}{\rho g h_1} = \frac{h_2}{h_1}$$

Solving for P_2 and substituting gives

$$P_2 = \frac{h_2}{h_1} P_1 = \frac{12\ \text{m}}{3\ \text{m}}(28\ \text{kPa}) = 112\ \text{kPa}$$



Discussion Note that the gage pressure in a given fluid is proportional to depth.

2-37 The absolute pressure in water at a specified depth is given. The local atmospheric pressure and the absolute pressure at the same depth in a different liquid are to be determined.

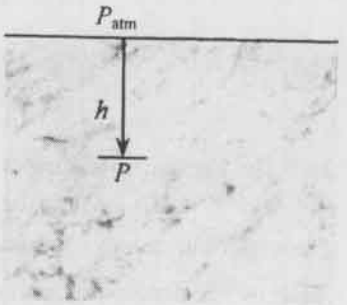
Assumptions The liquid and water are incompressible.

Properties The specific gravity of the fluid is given to be $\rho_s = 0.85$. We take the density of water to be 1000 kg/m^3. Then density of the liquid is obtained by multiplying its specific gravity by the density of water,

$$\rho = (\rho_s)(\rho_{H_2O}) = (0.85)(1000\ \text{kg/m}^3) = 850\ \text{kg/m}^3$$

Analysis (*a*) Knowing the absolute pressure, the atmospheric pressure can be determined from

$$\begin{aligned} P_{atm} &= P - \rho g h \\ &= (145\ \text{kPa}) + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^3}\right) \\ &= \mathbf{96.0\ kPa} \end{aligned}$$



(*b*) The absolute pressure at a depth of 5 m in the other liquid is

$$\begin{aligned} P &= P_{atm} + \rho g h \\ &= (96.0\ \text{kPa}) + (850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})\left(\frac{1\text{kPa}}{1000\text{N/m}^3}\right) \\ &= 137.7\ \mathbf{kPa} \end{aligned}$$

Discussion Note that at a given depth, the pressure in the lighter fluid is lower, as expected.

2-38E It is to be shown that 1 kgf/cm^2 = 14.223 psi .

Analysis Noting that 1 kgf = 9.80665 N, 1 N = 0.22481 lbf, and 1 in = 2.54 cm, we have

$$1\text{ kgf} = 9.80665\text{ N} = (9.80665\text{ N})\left(\frac{0.22481\text{ lbf}}{1\text{ N}}\right) = 2.20468\text{ lbf}$$

and

$$1\text{ kgf/cm}^2 = 2.20468\text{ lbf/cm}^2 = (2.20468\text{ lbf/cm}^2)\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)^2 = 14.223\text{ lbf/in}^2 = \mathbf{14.223\text{ psi}}$$

2-39E The weight and the foot imprint area of a person are given. The pressures this man exerts on the ground when he stands on one and on both feet are to be determined.

Assumptions The weight of the person is distributed uniformly on foot imprint area.

Analysis The weight of the man is given to be 200 lbf. Noting that pressure is force per unit area, the pressure this man exerts on the ground is



(*a*) On one foot: $$P = \frac{W}{A} = \frac{200\text{ lbf}}{36\text{ in}^2} = 5.56\text{ lbf/in}^2 = \mathbf{5.56\text{ psi}}$$

(*a*) On both feet: $$P = \frac{W}{2A} = \frac{200\text{ lbf}}{2\times 36\text{ in}^2} = 2.78\text{ lbf/in}^2 = \mathbf{2.78\text{ psi}}$$

Discussion Note that the pressure exerted on the ground (and on the feet) is reduced by half when the person stands on both feet.

2-40 The mass of a woman is given. The minimum imprint area per shoe needed to enable her to walk on the snow without sinking is to be determined.

Assumptions **1** The weight of the person is distributed uniformly on the imprint area of the shoes. **2** One foot carries the entire weight of a person during walking, and the shoe is sized for walking conditions (rather than standing). **3** The weight of the shoes is negligible.

Analysis The mass of the woman is given to be 70 kg. For a pressure of 0.5 kPa on the snow, the imprint area of one shoe must be

$$A = \frac{W}{P} = \frac{mg}{P} = \frac{(70\text{ kg})(9.81\text{ m/s}^2)}{0.5\text{ kPa}}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \left(\frac{1\text{ kPa}}{1000\text{ N/m}^2}\right) = \mathbf{1.37\text{ m}^2}$$

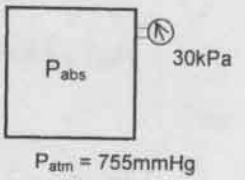


Discussion This is a very large area for a shoe, and such shoes would be impractical to use. Therefore, some sinking of the snow should be allowed to have shoes of reasonable size.

2-41 The vacuum pressure reading of a tank is given. The absolute pressure in the tank is to be determined.
√

Properties The density of mercury is given to be $\rho = 13{,}590\ \text{kg/m}^3$.

Analysis The atmospheric (or barometric) pressure can be expressed as



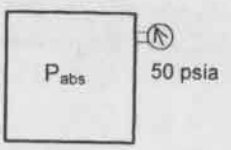
$$\begin{aligned} P_{atm} &= \rho g h \\ &= (13{,}590\text{kg/m}^3)(9.807\text{m/s}^2)(0.755\text{m})\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{kPa}}{1000\text{N/m}^2}\right) \\ &= 100.6\ \text{kPa} \end{aligned}$$

Then the absolute pressure in the tank becomes

$$P_{abs} = P_{atm} - P_{vac} = 100.6 - 30 = \mathbf{70.6kPa}$$

2-42E A pressure gage connected to a tank reads 50 psi. The absolute pressure in the tank is to be determined.

Properties The density of mercury is given to be $\rho = 848.4\ \text{lbm/ft}^3$.



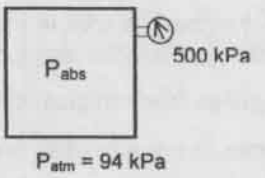
Analysis The atmospheric (or barometric) pressure can be expressed as

$$\begin{aligned} P_{atm} &= \rho g h \\ &= (848.4\text{lbm/ft}^3)(32.174\text{ft/s}^2)(29.1/12\text{ft})\left(\frac{1\text{lbf}}{32.174\text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ft}^2}{144\text{in}^2}\right) \\ &= 14.29\text{psia} \end{aligned}$$

Then the absolute pressure in the tank is

$$P_{abs} = P_{gage} + P_{atm} = 50 + 14.29 = \mathbf{64.29\ psia}$$

2-43 A pressure gage connected to a tank reads 500 kPa. The absolute pressure in the tank is to be determined.



Analysis The absolute pressure in the tank is determined from

$$P_{abs} = P_{gage} + P_{atm} = 500 + 94 = \mathbf{594 kPa}$$

2-44 A mountain hiker records the barometric reading before and after a hiking trip. The vertical distance climbed is to be determined.

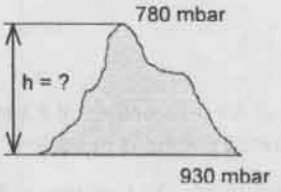
Assumptions The variation of air density and the gravitational acceleration with altitude is negligible.

Properties The density of air is given to be $\rho = 1.20$ kg/m^3.

Analysis Taking an air column between the top and the bottom of the mountain and writing a force balance per unit base area, we obtain

$$W_{air} / A = P_{bottom} - P_{top}$$

$$(\rho g h)_{air} = P_{bottom} - P_{top}$$



$$(1.20 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h)\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ bar}}{100{,}000 \text{ N/m}^2}\right) = (0.930 - 0.780) \text{ bar}$$

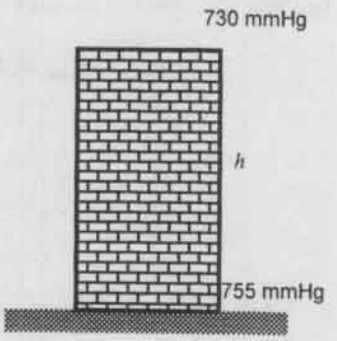
It yields $h = \mathbf{1274 \text{ m}}$

which is also the distance climbed.

2-45 A barometer is used to measure the height of a building by recording reading at the bottom and at the top of the building. The height of the building is to be determined.

Assumptions The variation of air density with altitude is negligible.

Properties The density of air is given to be $\rho = 1.18$ kg/m^3.



Analysis Atmospheric pressures at the top and at the bottom of the building are

$$\begin{aligned} P_{top} &= (\rho g h)_{top} \\ &= (1.18\ \text{kg/m}^3)(9.807\ \text{m/s}^2)(0.730\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= 97.36\ \text{kPa} \end{aligned}$$

$$\begin{aligned} P_{bottom} &= (\rho g h)_{bottom} \\ &= (1.18\ \text{kg/m}^3)(9.807\ \text{m/s}^2)(0.755\ \text{m})\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= 100.70\ \text{kPa} \end{aligned}$$

Taking an air column between the top and the bottom of the building and writing a force balance per unit base area, we obtain

$$W_{air} / A = P_{bottom} - P_{top}$$

$$(\rho g h)_{air} = P_{bottom} - P_{top}$$

$$(1.18\ \text{kg/m}^3)(9.807\ \text{m/s}^2)(h)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) = (100.70 - 97.36)\ \text{kPa}$$

It yields $h = \mathbf{288.6\ m}$

which is also the height of the building.

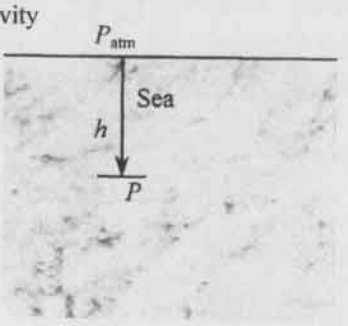
2-46 A diver is moving at a specified depth from the water surface. The pressure exerted on the surface of the diver by water is to be determined.

Assumptions The variation of the density of water with depth is negligible.

Properties The specific gravity of sea water is given to be $\rho_s = 1.03$. We take the density of water to be 1000 kg/m^3.

Analysis The density of the sea water is obtained by multiplying its specific gravity by the density of water which is taken to be 1000 kg/m^3:

$$\rho = (\rho_s)(\rho_{H_2O}) = (1.03)(1000\ \text{kg/m}^3) = 1030\ kg/m^3$$



The pressure exerted on a diver at 30 m below the free surface of the sea is the absolute pressure at that location:

$$\begin{aligned} P &= P_{atm} + \rho g h \\ &= (101\text{kPa}) + (1030\text{kg/m}^3)(9.807\text{m/s}^2)(30\text{m})\left(\frac{1\text{kPa}}{1000\text{N/m}^3}\right) \\ &= \mathbf{404.0kPa} \end{aligned}$$

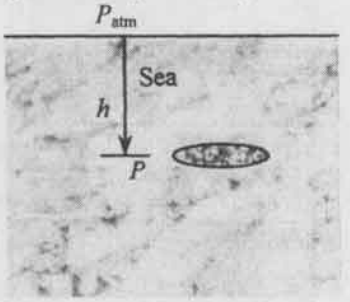
2-47E A submarine is cruising at a specified depth from the water surface. The pressure exerted on the surface of the submarine by water is to be determined.

Assumptions The variation of the density of water with depth is negligible.

Properties The specific gravity of sea water is given to be $\rho_s = 1.03$. The density of water at 32°F is 62.4 lbm/ft^3 (Table A-3E).

Analysis The density of the sea water is obtained by multiplying its specific gravity by the density of water ,

$$\rho = (\rho_s)(\rho_{H_2O}) = (1.03)(62.4 \text{ lbm / ft}^3) = 64.27 \text{ lbm / ft}^3$$



The pressure exerted on the surface of the submarine cruising 300 ft below the free surface of the sea is the absolute pressure at that location:

$$\begin{aligned} P &= P_{atm} + \rho g h \\ &= (14.7\text{psia}) + (64.27\text{lbm/ft}^3)(32.174\text{ft/s}^2)(300\text{ft})\left(\frac{1\text{lbf}}{32.174\text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ft}^2}{144\text{in}^2}\right) \\ &= \mathbf{148.6psia} \end{aligned}$$

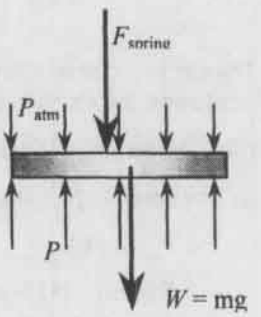
2-48 A gas contained in a vertical piston-cylinder device is pressurized by a spring and by the weight of the piston. The pressure of the gas is to be determined.

Analysis Drawing the free body diagram of the piston and balancing the vertical forces yield

$$PA = P_{atm}A + W + F_{spring}$$

Thus,

$$\begin{aligned} P &= P_{\text{atm}} + \frac{mg + F_{\text{spring}}}{A} \\ &= (95\text{ kPa}) + \frac{(4\text{ kg})(9.807\text{ m/s}^2) + 60\text{ N}}{35\times10^{-4}\text{ m}^2}\left(\frac{1\text{ kPa}}{1000\text{ N/m}^2}\right) \\ &= \mathbf{123.4kPa} \end{aligned}$$



2-49 Both a gage and a manometer are attached to a gas to measure its pressure. For a specified reading of gage pressure, the difference between the fluid levels of the two arms of the manometer is to be determined for mercury and water.

Properties The densities of water and mercury are given to be $\rho_{water} = 1000$ kg/m^3 and be $\rho_{Hg} = 13{,}600$ kg/m^3.

Analysis The gage pressure is related to the vertical distance h between the two fluid levels by

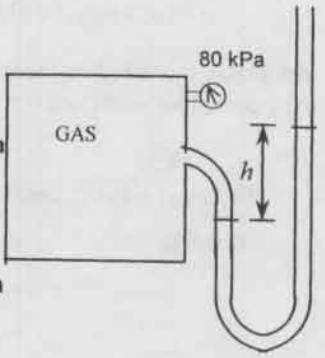
$$P_{gage} = \rho g h \longrightarrow h = \frac{P_{gage}}{\rho g}$$

(*a*) For mercury,

$$h = \frac{P_{gage}}{\rho_{Hg} g} = \frac{80 \text{ kPa}}{(13600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)}\left(\frac{1 \text{ kN/m}^2}{1 \text{ kPa}}\right)\left(\frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kN}}\right) = \mathbf{0.60\ m}$$

(*b*) For water,

$$h = \frac{P_{gage}}{\rho_{H_2O} g} = \frac{80 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.807 \text{m/s}^2)}\left(\frac{1 \text{ kN/m}^2}{1 \text{ kPa}}\right)\left(\frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kN}}\right) = \mathbf{8.16\ m}$$

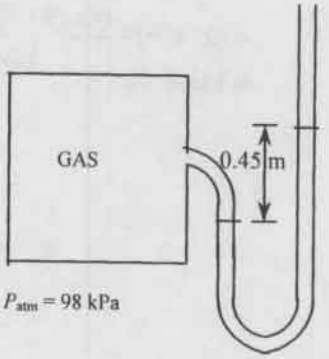


2-50 The gas pressure in a tank is measured by an oil manometer. For a given oil-level difference between the two columns, the absolute pressure in the tank is to be determined.

Properties The density of oil is given to be $\rho = 850$ kg/m^3.

Analysis The absolute pressure in the tank is determined from

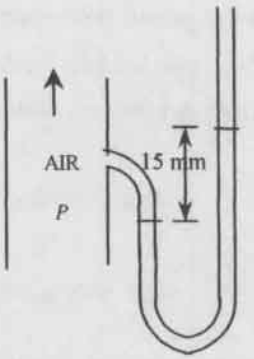
$$\begin{aligned} P &= P_{atm} + \rho g h \\ &= (98\text{kPa}) + (850\text{kg/m}^3)(9.807\text{m/s}^2)(0.45\text{m})\left(\frac{1\text{kPa}}{1000\text{N/m}^2}\right) \\ &= \mathbf{101.75kPa} \end{aligned}$$



2-51 The air pressure in a duct is measured by a mercury manometer. For a given mercury-level difference between the two columns, the absolute pressure in the duct is to be determined.

Properties The density of mercury is given to be $\rho = 13{,}600\ \text{kg/m}^3$.

Analysis (a) The pressure in the duct is above atmospheric pressure since the fluid column on the duct side is at a lower level.



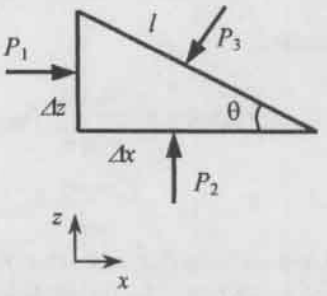
(*b*) The absolute pressure in the duct is determined from

$$\begin{aligned} P &= P_{atm} + \rho g h \\ &= (100\text{kPa}) + (13{,}600\text{kg/m}^3)(9.807\text{m/s}^2)(0.015\text{m})\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{kPa}}{1000\,\text{N/m}^2}\right) \\ &= \mathbf{102.0kPa} \end{aligned}$$

2-52 By considering a wedge-shaped fluid element in the *xy*-plane (with unit depth in the *z*-direction), it is to be shown that the pressure in an accelerating fluid is the same in all directions.

Assumptions The shear forces are negligible.

Analysis We consider a small wedge-shaped fluid element of unit dept (into the paper). The mean pressures at the three surfaces are P_1, P_2, and P_3, and the force acting on a surface is the product of mean pressure and the surface area. From Newton's 2nd law, a force balance in the *x*- and *z*-directions gives



$$\sum F_x = ma_x: \qquad P_1\Delta z - P_3 l\sin\theta = \frac{1}{2}\rho\Delta x\Delta z a_x$$

$$\sum F_z = ma_z: \qquad P_2\Delta x - P_3 l\cos\theta - \frac{1}{2}\rho\Delta x\Delta z a_z$$

where ρ is the density, $m = \rho\Delta x\Delta z/2$ is the mass, and $W = mg = \rho g\Delta x\Delta z/2$ is the weight of the fluid element. Noting that the wedge is a right triangle, we have $\Delta x = l\cos\theta$ and $\Delta z = l\sin\theta$. Substituting these geometric relations and dividing the first relation by Δz and the second relation by Δx gives

$$P_1 - P_3 = \frac{1}{2}\rho\Delta x a_x$$

$$P_2 - P_3 - \frac{1}{2}\rho g\Delta z = \frac{1}{2}\rho\Delta z a_x$$

The terms that involve Δx and Δz in the above relations drop out as $\Delta x \to 0$ and $\Delta z \to 0$ and the wedge becomes infinitesimal, and thus the fluid element shrinks to a point. Then combining the results of these two relations gives

$$P_1 = P_2 = P_3 = P$$

regardless of the angle θ. We can repeat the analysis for an element in the *xz*-plane, and obtain a similar result. Thus we conclude that *the pressure at a point in a fluid has the same magnitude in all directions* for fluids at rest and fluids in motion as well as in the absence of shear forces.

2-53 The systolic and diastolic pressures of a healthy person are given in mmHg. These pressures are to be expressed in kPa, psi, and meter water column.

Assumptions Both mercury and water are incompressible substances.

Properties We take the densities of water and mercury to be 1000 kg/m^3 and 13,600 kg/m^3, respectively.

Analysis Using the relation $P = \rho gh$ for gage pressure, the high and low pressures are expressed as

$$P_{\text{high}} = \rho g h_{\text{high}} = (13{,}600\text{kg/m}^3)(9.81\text{ m/s}^2)(0.12\text{ m})\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{kPa}}{1000\text{N/m}^2}\right) = \mathbf{16.0\ kPa}$$

$$P_{\text{low}} = \rho g h_{\text{low}} = (13{,}600\text{kg/m}^3)(9.81\text{m/s}^2)(0.08\text{ m})\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{kPa}}{1000\text{N/m}^2}\right) = \mathbf{10.7\ kPa}$$

Noting that 1 psi = 6.895 kPa,

$$P_{\text{high}} = (16.0\text{ Pa})\left(\frac{1\text{ psi}}{6.895\text{kPa}}\right) = \mathbf{2.32\ psi} \qquad \text{and} \qquad P_{\text{low}} = (10.7\text{Pa})\left(\frac{1\text{ psi}}{6.895\text{kPa}}\right) = \mathbf{1.55\ psi}$$

For a given pressure, the relation $P = \rho gh$ can be expressed for mercury and water as $P = \rho_{\text{water}} g h_{\text{water}}$ and $P = \rho_{\text{mercury}} g h_{\text{mercury}}$. Setting these two relations equal to each other and solving for water height gives

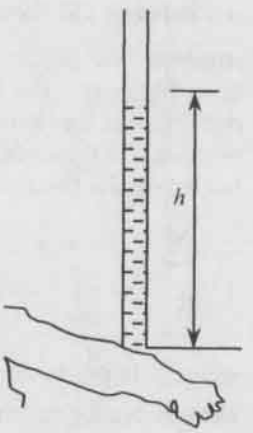
$$P = \rho_{\text{water}} g h_{\text{water}} = \rho_{\text{water}} g h_{\text{water}} \quad \rightarrow \quad h_{\text{water}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury}}$$

Therefore,

$$h_{\text{water, high}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury, high}} = \frac{13{,}600\text{ kg/m}^3}{1000\text{ kg/m}^3}(0.12\text{ m}) = \mathbf{1.63\ m}$$

$$h_{\text{water, low}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury, low}} = \frac{13{,}600\text{ kg/m}^3}{1000\text{ kg/m}^3}(0.08\text{ m}) = \mathbf{1.09\ m}$$

Discussion Note that measuring blood pressure with a "water" monometer would involve differential fluid heights higher than the person, and thus it is impractical. This problem shows why mercury is a suitable fluid for blood pressure measurement devices.



2-54 A vertical tube open to the atmosphere is connected to the vein in the arm of a person. The height that the blood will rise in the tube is to be determined.

Assumptions **1** The density of blood is constant. **2** The gage pressure of blood is 120 mmHg.

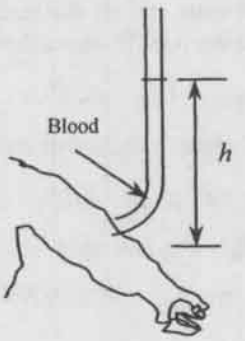
Properties The density of blood is given to be $\rho = 1050\ \text{kg/m}^3$.

Analysis For a given gage pressure, the relation $P = \rho gh$ can be expressed for mercury and blood as $P = \rho_{\text{blood}} g h_{\text{blood}}$ and $P = \rho_{\text{mercury}} g h_{\text{mercury}}$. Setting these two relations equal to each other we get

$$P = \rho_{\text{blood}} g h_{\text{blood}} = \rho_{\text{mercury}} g h_{\text{mercury}}$$

Solving for blood height and substituting gives

$$h_{\text{blood}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{blood}}} h_{\text{mercury}} = \frac{13{,}600\ \text{kg/m}^3}{1050\ \text{kg/m}^3}(0.12\ \text{m}) = \mathbf{1.55\ m}$$



Discussion Note that the blood can rise about one and a half meters in a tube connected to the vein. This explains why IV tubes must be placed high to force a fluid into the vein of a patient.

2-55 A man is standing in water vertically while being completely submerged. The difference between the pressures acting on the head and on the toes is to be determined.

Assumptions Water is an incompressible substances, and thus the density does not change with depth.

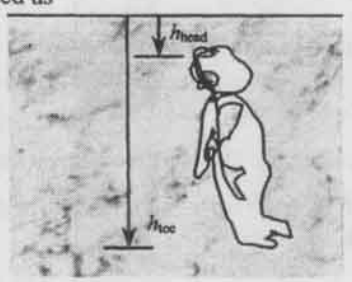
Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

Analysis The pressures at the head and toes of the person can be expressed as

$$P_{\text{head}} = P_{\text{atm}} + \rho g h_{\text{head}} \quad \text{and} \quad P_{\text{toe}} = P_{\text{atm}} + \rho g h_{\text{toe}}$$

where h is the vertical distance of the location in water from the free surface. The pressure difference between the toes and the head is determined by subtracting the first relation above from the second,

$$P_{\text{toe}} - P_{\text{head}} = \rho g h_{\text{toe}} - \rho g h_{\text{head}} = \rho g (h_{\text{toe}} - h_{\text{head}})$$



Substituting,

$$P_{\text{toe}} - P_{\text{head}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.80\ \text{m} - 0)\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{kPa}}{1000\text{N/m}^2}\right) = \mathbf{17.7\ kPa}$$

Discussion This problem can also be solved by noting that the atmospheric pressure (1 atm = 101.325 kPa) is equivalent to 10.3-m of water height, and finding the pressure that corresponds to a water height of 1.8 m.

2-56 Water is poured into the U-tube from one arm and ethyl alcohol from the other arm. The water column height in one arm and the ratio of the heights of the two fluids in the other arm are given. The height of each fluid in that arm is to be determined.

Assumptions Both water and ethyl alcohol are incompressible substances.

Properties The density of ethyl alcohol is given to be $\rho = 790$ kg/m^3. We take the density of water to be $\rho = 1000$ kg/m^3.

Analysis The height of water column in the left arm of the monometer is given to be $h_{w1} = 0.70$ m. We let the height of water and alcohol in the right arm to be h_{w2} and h_a, respectively. Then, $h_a = 6h_{w2}$. Noting that both arms are open to the atmosphere, the pressure at the bottom of the U-tube can be expressed as

$$P_{\text{bottom}} = P_{\text{atm}} + \rho_w g h_{w1} \quad \text{and} \quad P_{\text{bottom}} = P_{\text{atm}} + \rho_w g h_{w2} + \rho_a g h_a$$

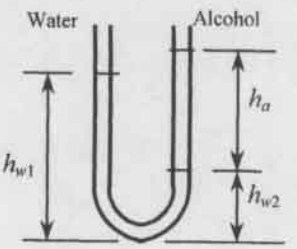
Setting them equal to each other and simplifying,

$$\rho_w g h_{w1} = \rho_w g h_{w2} + \rho_a g h_a \quad \rightarrow \quad \rho_w h_{w1} = \rho_w h_{w2} + \rho_a h_a \quad \rightarrow \quad h_{w1} = h_{w2} + (\rho_a / \rho_w) h_a$$

Noting that $h_a = 6h_{w2}$, the water and alcohol column heights in the second arm are determined to be

$$0.7 \text{ m} = h_{w2} + (790/1000)6h_{w2} \quad \rightarrow \quad h_{w2} = \mathbf{0.122 \text{ m}}$$

$$0.7 \text{ m} = 0.122 \text{ m} + (790/1000)h_a \quad \rightarrow \quad h_a = \mathbf{0.732 \text{ m}}$$



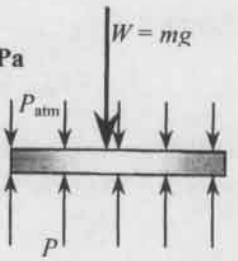
Discussion Note that the fluid height in the arm that contains alcohol is higher. This is expected since alcohol is lighter than water.

2-57 The hydraulic lift in a car repair shop is to lift cars. The fluid gage pressure that must be maintained in the reservoir is to be determined.

Assumptions The weight of the piston of the lift is negligible.

Analysis Pressure is force per unit area, and thus the gage pressure required is simply the ratio of the weight of the car to the area of the lift,

$$P_{\text{gage}} = \frac{W}{A} = \frac{mg}{\pi D^2/4} = \frac{(2000 \text{ kg})(9.81 \text{ m/s}^2)}{\pi (0.30 \text{ m})^2/4} \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 277 \text{ kN/m}^2 = \mathbf{277 \text{ kPa}}$$



Discussion Note that the pressure level in the reservoir can be reduced by using a piston with a larger area.

2-58 Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible.

Properties The density of seawater is given to be $\rho_{sea} = 1035$ kg/m^3. We take the density of water to be $\rho_w = 1000$ kg/m^3.

Analysis Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the sea water pipe (point 2), and setting the result equal to P_2 gives

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{air} g h_{air} + \rho_{sea} g h_{sea} = P_2$$

Rearranging and neglecting the effect of air column on pressure,

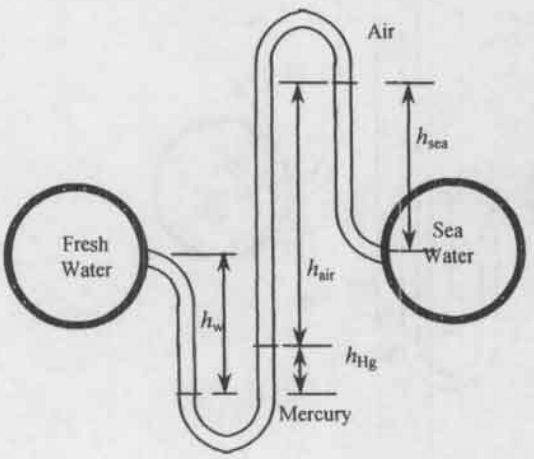
$$P_1 - P_2 = -\rho_w g h_w + \rho_{Hg} g h_{Hg} - \rho_{sea} g h_{sea} = g(\rho_{Hg} h_{Hg} - \rho_w h_w - \rho_{sea} h_{sea})$$

Substituting,

$$\begin{aligned} P_1 - P_2 &= (9.81\,\text{m/s}^2)[(13600\,\text{kg/m}^3)(0.1\,\text{m}) \\ &\quad -(1000\,\text{kg/m}^3)(0.6\,\text{m}) - (1035\,\text{kg/m}^3)(0.4\,\text{m})]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) \\ &= 3.39\,\text{kN/m}^2 = \mathbf{3.39\,kPa} \end{aligned}$$

Therefore, the pressure in the fresh water pipe is 3.39 kPa higher than the pressure in the sea water pipe.

Discussion A 0.70-m high air column with a density of 1.2 kg/m^3 corresponds to a pressure difference of 0.008 kPa. Therefore, its effect on the pressure difference between the two pipes is negligible.



2-59 Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

Assumptions All the liquids are incompressible.

Properties The density of seawater is given to be $\rho_{sea} = 1035$ kg/m^3. We take the density of water to be $\rho_w = 1000$ kg/m^3. The specific gravity of oil is given to be 0.72, and thus its density is 720 kg/m^3.

Analysis Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the sea water pipe (point 2), and setting the result equal to P_2 gives

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} + \rho_{sea} g h_{sea} = P_2$$

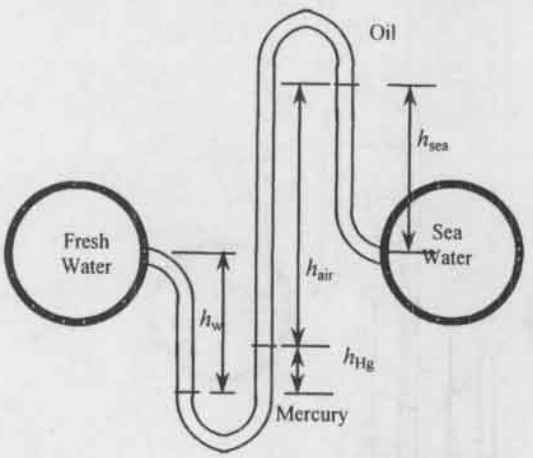
Rearranging,

$$\begin{aligned} P_1 - P_2 &= -\rho_w g h_w + \rho_{Hg} g h_{Hg} + \rho_{oil} g h_{oil} - \rho_{sea} g h_{sea} \\ &= g(\rho_{Hg} h_{Hg} + \rho_{oil} h_{oil} - \rho_w h_w - \rho_{sea} h_{sea}) \end{aligned}$$

Substituting,

$$\begin{aligned} P_1 - P_2 &= (9.81\,\text{m/s}^2)[(13600\,\text{kg/m}^3)(0.1\,\text{m}) + (720\,\text{kg/m}^3)(0.7\,\text{m}) - (1000\,\text{kg/m}^3)(0.6\,\text{m}) \\ &\quad - (1035\,\text{kg/m}^3)(0.4\,\text{m})]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) \\ &= 8.34\,\text{kN/m}^2 = \mathbf{8.34\,kPa} \end{aligned}$$

Therefore, the pressure in the fresh water pipe is 8.34 kPa higher than the pressure in the sea water pipe.



2-60E The pressure in a natural gas pipeline is measured by a double U-tube manometer with one of the arms open to the atmosphere. The absolute pressure in the pipeline is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible. **3** The pressure throughout the natural gas (including the tube) is uniform since its density is low.

Properties We take the density of water to be $\rho_w = 62.4$ lbm/ft^3. The specific gravity of mercury is given to be 13.6, and thus its density is $\rho_{Hg} = 13.6 \times 62.4 = 848.6$ lbm/ft^3.

Analysis Starting with the pressure at point 1 in the natural gas pipeline, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives

$$P_1 - \rho_{Hg} g h_{Hg} - \rho_{water} g h_{water} = P_{atm}$$

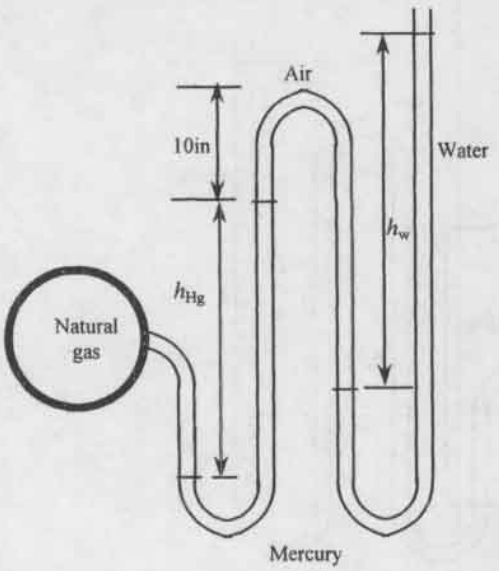
Solving for P_1,

$$P_1 = P_{atm} + \rho_{Hg} g h_{Hg} + \rho_{water} g h_1$$

Substituting,

$$P = 14.2\,\text{psia} + (32.2\,\text{ft/s}^2)[(848.6\,\text{lbm/ft}^3)(6/12\,\text{ft}) + (62.4\,\text{lbm/ft}^3)(27/12\,\text{ft})]\left(\frac{1\,\text{lbf}}{32.2\,\text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right)$$
$$= \mathbf{18.1\,psia}$$

Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly. Also, it can be shown that the 15-in high air column with a density of 0.075 lbm/ft^3 corresponds to a pressure difference of 0.00065 psi. Therefore, its effect on the pressure difference between the two pipes is negligible.



2-61E The pressure in a natural gas pipeline is measured by a double U-tube manometer with one of the arms open to the atmosphere. The absolute pressure in the pipeline is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The pressure throughout the natural gas (including the tube) is uniform since its density is low.

Properties We take the density of water to be $\rho_w = 62.4$ lbm/ft^3. The specific gravity of mercury is given to be 13.6, and thus its density is $\rho_{Hg} = 13.6 \times 62.4 = 848.6$ lbm/ft^3. The specific gravity of oil is given to be 0.69, and thus its density is $\rho_{oil} = 0.69 \times 62.4 = 43.1$ lbm/ft^3.

Analysis Starting with the pressure at point 1 in the natural gas pipeline, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives

$$P_1 - \rho_{Hg} g h_{Hg} + \rho_{oil} g h_{oil} - \rho_{water} g h_{water} = P_{atm}$$

Solving for P_1.

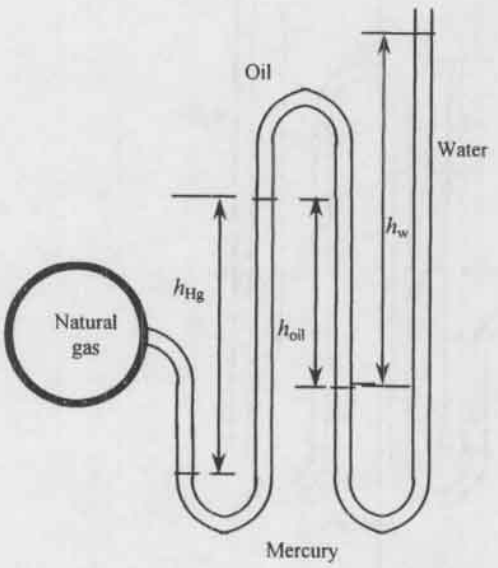
$$P_1 = P_{atm} + \rho_{Hg} g h_{Hg} + \rho_{water} g h_1 - \rho_{oil} g h_{oil}$$

Substituting,

$$P_1 = 14.2\,\text{psia} + (32.2\,\text{ft/s}^2)[(848.6\,\text{lbm/ft}^3)(6/12\,\text{ft}) + (62.4\,\text{lbm/ft}^3)(27/12\,\text{ft}) - (43.1\,\text{lbm/ft}^3)(15/12\,\text{ft})]\left(\frac{1\,\text{lbf}}{32.2\,\text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right)$$

$$= \mathbf{17.7\,psia}$$

Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.

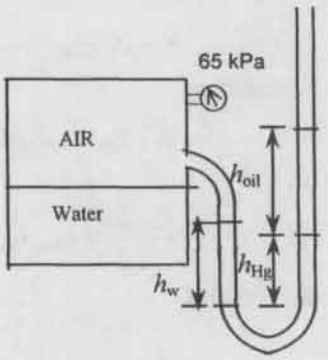


2-62 The gage pressure of air in a pressurized water tank is measured simultaneously by both a pressure gage and a manometer. The differential height h of the mercury column is to be determined.

Assumptions The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

Properties We take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$. The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.

Analysis Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives



$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} = P_{atm}$$

Rearranging,

$$P_1 - P_{atm} = \rho_{oil} g h_{oil} + \rho_{Hg} g h_{Hg} - \rho_w g h_w$$

or,

$$\frac{P_{1,gage}}{\rho_w g} = \rho_{s,oil} h_{oil} + \rho_{s,Hg} h_{Hg} - h_w$$

Substituting,

$$\frac{65\ \text{kPa}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kPa}\cdot\text{m}^2}\right) = 0.72\times(0.75\ \text{m}) + 13.6\times h_{Hg} - 0.3\ \text{m}$$

Solving for h_{Hg} gives $h_{Hg} = \mathbf{0.47\ m}$. Therefore, the differential height of the mercury column must be 47 cm.

Discussion Double instrumentation like this allows one to verify the measurement of one of the instruments by the measurement of another instrument.

2-63 The gage pressure of air in a pressurized water tank is measured simultaneously by both a pressure gage and a manometer. The differential height h of the mercury column is to be determined.

Assumptions The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

Properties We take the density of water to be ρ_w=1000 kg/m^3. The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.

Analysis Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} = P_{atm}$$

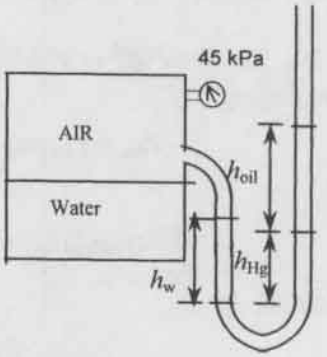
Rearranging,

$$P_1 - P_{atm} = \rho_{oil} g h_{oil} + \rho_{Hg} g h_{Hg} - \rho_w g h_w$$

or,

$$\frac{P_{1,gage}}{\rho_w g} = \rho_{s,oil} h_{oil} + \rho_{s,Hg} h_{Hg} - h_w$$

Substituting,



$$\frac{45\,\text{kPa}}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}\left[\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\,\text{kPa}\cdot\text{m}^2}\right] = 0.72\times(0.75\,\text{m}) + 13.6\times h_{Hg} - 0.3\,\text{m}$$

Solving for h_{Hg} gives h_{Hg} = **0.32 m**. Therefore, the differential height of the mercury column must be 32 cm.

Discussion Double instrumentation like this allows one to verify the measurement of one of the instruments by the measurement of another instrument.

2-64 The top part of a water tank is divided into two compartments, and a fluid with an unknown density is poured into one side. The levels of the water and the liquid are measured. The density of the fluid is to be determined.

Assumptions **1** Both water and the added liquid are incompressible substances. **2** The added liquid does not mix with water.

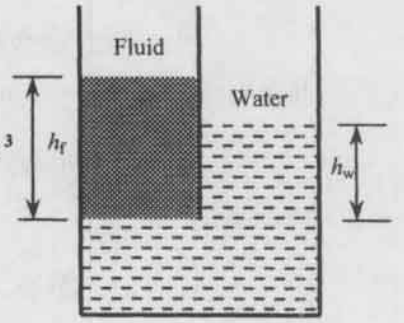
Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

Analysis Both fluids are open to the atmosphere. Noting that the pressure of both water and the added fluid is the same at the contact surface, the pressure at this surface can be expressed as

$$P_{\text{contact}} = P_{\text{atm}} + \rho_{\text{f}} g h_{\text{f}} = P_{\text{atm}} + \rho_{\text{w}} g h_{\text{w}}$$

Simplifying and solving for ρ_{f} gives

$$\rho_{\text{f}} g h_{\text{f}} = \rho_{\text{w}} g h_{\text{w}} \quad \rightarrow \quad \rho_f = \frac{h_w}{h_f}\rho_w = \frac{45\ \text{cm}}{80\ \text{cm}}(1000\ \text{kg/m}^3) = \mathbf{562.5\ kg/m^3}$$



Discussion Note that the added fluid is lighter than water as expected (a heavier fluid would sink in water).

2-65 A load on a hydraulic lift is to be raised by pouring oil from a thin tube. The height of oil in the tube required in order to raise that weight is to be determined.

Assumptions **1** The cylinders of the lift are vertical. **2** There are no leaks. **3** Atmospheric pressure act on both sides, and thus it can be disregarded.

Properties The density of oil is given to be $\rho = 780\ \text{kg/m}^3$.

Analysis Noting that pressure is force per unit area, the gage pressure in the fluid under the load is simply the ratio of the weight to the area of the lift,

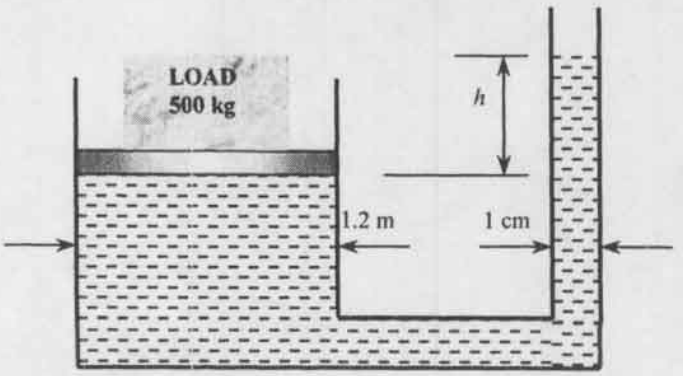
$$P_{\text{gage}} = \frac{W}{A} = \frac{mg}{\pi D^2/4} = \frac{(500\ \text{kg})(9.81\ \text{m/s}^2)}{\pi(1.20\ \text{m})^2/4}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 4.34\ \text{kN/m}^2 = 4.34\ \text{kPa}$$

The required oil height that will cause 4.34 kPa of pressure rise is

$$P_{\text{gage}} = \rho g h \quad \rightarrow \quad h = \frac{P_{\text{gage}}}{\rho g} = \frac{4.34\ \text{kN/m}^2}{(780\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\text{kN/m}^2}\right) = \mathbf{0.567\ m}$$

Therefore, a 500 kg load can be raised by this hydraulic lift by simply raising the oil level in the tube by 56.7 cm.

Discussion Note that large weights can be raised by little effort in hydraulic lift by making use of Pascal's principle.



2-66E Two oil tanks are connected to each other through a mercury manometer. For a given differential height, the pressure difference between the two tanks is to be determined.

Assumptions **1** Both the oil and mercury are incompressible fluids. **2** The oils in both tanks have the same density.

Properties The densities of oil and mercury are given to be $\rho_{oil} = 45$ lbm/ft^3 and $\rho_{Hg} = 848$ lbm/ft^3.

Analysis Starting with the pressure at the bottom of tank 1 (where pressure is P_1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the bottom of tank 2 (where pressure is P_2) gives

$$P_1 + \rho_{oil} g(h_1 + h_2) - \rho_{Hg} g h_2 - \rho_{oil} g h_1 = P_2$$

where $h_1 = 10$ in and $h_2 = 32$ in. Rearranging and simplifying,

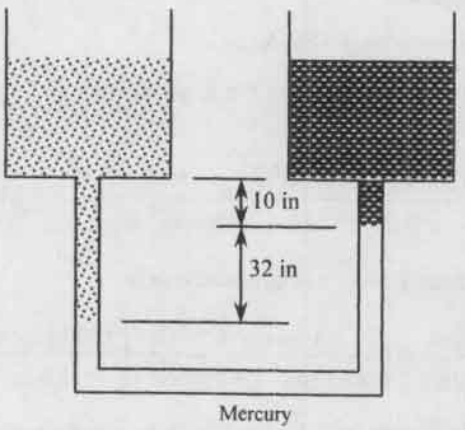
$$P_1 - P_2 = \rho_{Hg} g h_2 - \rho_{oil} g h_2 = (\rho_{Hg} - \rho_{oil}) g h_2$$

Substituting,

$$\Delta P = P_1 - P_2 = (848 - 45 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(32/12 \text{ ft})\left(\frac{1 \text{lbf}}{32.2 \text{lbm} \cdot \text{ft/s}^2}\right)\left(\frac{1 \text{ft}^2}{144 \text{in}^2}\right) = \mathbf{14.9 psia}$$

Therefore, the pressure in the left oil tank is 15.7 psia higher than the pressure in the right oil tank.

Discussion Note that large pressure differences can be measured conveniently by mercury manometers. If a water manometer were used in this case, the differential height would be over 30 ft.



Review Problems

2-67E The efficiency of a refrigerator increases by 3% per °C rise in the minimum temperature. This increase is to be expressed per °F, K, and R rise in the minimum temperature.

Analysis The magnitudes of 1 K and 1°C are identical, so are the magnitudes of 1 R and 1°F. Also, a change of 1 K or 1°C in temperature corresponds to a change of 1.8 R or 1.8°F. Therefore, the increase in efficiency is

(a) **3%** for each K rise in temperature, and
(b), (c) 3/1.8 = **1.67%** for each R or °F rise in temperature.

2-68E The boiling temperature of water decreases by 3°C for each 1000 m rise in altitude. This decrease in temperature is to be expressed in °F, K, and R.

Analysis The magnitudes of 1 K and 1°C are identical, so are the magnitudes of 1 R and 1°F. Also, a change of 1 K or 1°C in temperature corresponds to a change of 1.8 R or 1.8°F. Therefore, the decrease in the boiling temperature is

(a) **3 K** for each 1000 m rise in altitude, and
(b), (c) 3×1.8 = **5.4°F** for each 1000 m rise in altitude.

2-69E The average body temperature of a person rises by about 2°C during strenuous exercise. This increase in temperature is to be expressed in °F, K, and R.

Analysis The magnitudes of 1 K and 1°C are identical, so are the magnitudes of 1 R and 1°F. Also, a change of 1 K or 1°C in temperature corresponds to a change of 1.8 R or 1.8°F. Therefore, the rise in the body temperature during strenuous exercise is

(a) **2 K**
(b) 2×1.8 = **3.6°F**
(c) 2×1.8 = **3.6 R**

2-70E Hyperthermia of 5°C is considered fatal. This fatal level temperature change of body temperature is to be expressed in °F, K, and R.

Analysis The magnitudes of 1 K and 1°C are identical, so are the magnitudes of 1 R and 1°F. Also, a change of 1 K or 1°C in temperature corresponds to a change of 1.8 R or 1.8°F. Therefore, the fatal level of hypothermia is

(a) **5 K**
(b) 5×1.8 = **9°F**
(c) 5×1.8 = **9 R**

2-71E A house is losing heat at a rate of 3000 kJ/h per °C temperature difference between the indoor and the outdoor temperatures. The rate of heat loss is to be expressed per °F, K, and R of temperature difference between the indoor and the outdoor temperatures.

Analysis The magnitudes of 1 K and 1°C are identical, so are the magnitudes of 1 R and 1°F. Also, a change of 1 K or 1°C in temperature corresponds to a change of 1.8 R or 1.8°F. Therefore, the rate of heat loss from the house is

(a) **3000 kJ/h** per K difference in temperature, and
(b), (c) 3000/1.8 = **1667 kJ/h** per R or °F rise in temperature.

2-72 The average temperature of the atmosphere is expressed as $T_{atm} = 288.15 - 6.5z$ where z is altitude in km. The temperature outside an airplane cruising at 12,000 m is to be determined.

Analysis Using the relation given, the average temperature of the atmosphere at an altitude of 12,000 m is determined to be

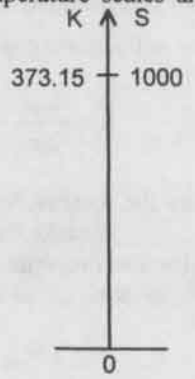
$$T_{atm} = 288.15 - 6.5z$$
$$= 288.15 - 6.5\times12$$
$$= \mathbf{210.15\ K = -63°C}$$

Discussion This is the "average" temperature. The actual temperature at different times can be different.

2-73 A new "Smith" absolute temperature scale is proposed, and a value of 1000 S is assigned to the boiling point of water. The ice point on this scale, and its relation to the Kelvin scale are to be determined.

Analysis All linear absolute temperature scales read zero at absolute zero pressure, and are constant multiples of each other. For example, T(R) = 1.8 T(K). That is, multiplying a temperature value in K by 1.8 will give the same temperature in R.

The proposed temperature scale is an acceptable absolute temperature scale since it differs from the other absolute temperature scales by a constant only. The boiling temperature of water in the Kelvin and the Smith scales are 315.15 K and 1000 K, respectively. Therefore, these two temperature scales are related to each other by



$$T(S) = \frac{1000}{373.15} T(K) = \mathbf{2.6799\ T(K)}$$

The ice point of water on the Smith scale is

$$T(S)_{ice} = 2.6799\ T(K)_{ice} = 2.6799 \times 273.15 = \mathbf{732.0\ S}$$

2-74 An expression for the equivalent wind chill temperature is given in English units. It is to be converted to SI units.

Analysis The required conversion relations are 1 mph = 1.609 km/h and $T(°F) = 1.8T(°C) + 32$. The first thought that comes to mind is to replace $T(°F)$ in the equation by its equivalent $1.8T(°C) + 32$, and V in mph by 1.609 km/h, which is the "regular" way of converting units. However, the equation we have is not a regular dimensionally homogeneous equation, and thus the regular rules do not apply. The V in the equation is a constant whose value is equal to the numerical value of the velocity in mph. Therefore, if V is given in km/h, we should divide it by 1.609 to convert it to the desired unit of mph. That is,

$$T_{equiv}(°F) = 91.4 - [91.4 - T_{ambient}(°F)][0.475 - 0.0203(V/1.609) + 0.304\sqrt{V/1.609}]$$

or

$$T_{equiv}(°F) = 91.4 - [91.4 - T_{ambient}(°F)][0.475 - 0.0126V + 0.240\sqrt{V}]$$

where V is in km/h. Now the problem reduces to converting a temperature in °F to a temperature in °C, using the proper convection relation:

$$1.8T_{equiv}(°C) + 32 = 91.4 - [91.4 - (1.8T_{ambient}(°C) + 32)][0.475 - 0.0126V + 0.240\sqrt{V}]$$

which simplifies to

$$T_{equiv}(°C) = 33.0 - (33.0 - T_{ambient})(0.475 - 0.0126V + 0.240\sqrt{V})$$

where the ambient air temperature is in °C.

2-75 A water tank open to the atmosphere is initially filled with water. The plug near the bottom of a water tank is pulled out. The time it will take for the gage pressure at the bottom of the tank to drop to a specified value is to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The distance between the bottom of the tank and the center of the hole is negligible compared to the total water height.

Properties The density of water is taken to be $\rho = 1000\ \text{kg/m}^3$.

Analysis Noting that the gage pressure in a liquid is expressed as $P_{\text{gage}} = \rho g h$, the water height in the tank that will cause a gage pressure of 15.7 kPa is determined to be

$$h = \frac{P_{\text{gage}}}{\rho g} = \frac{15.7\ \text{kPa}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\text{kN}}\right)\left(\frac{1\ \text{kN/m}^2}{1\text{kPa}}\right) = 1.60\ \text{m}$$

Now the question reduces to finding the time it takes for water level in the tank to drop to 1.6 m from 3.0 m.

We take the interior volume of the tank as the control volume. This is an unsteady flow problem since the properties (such as the amount of mass) within the control volume change with time. The conservation of mass relation for any system undergoing any process is given in the rate form as

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{system}}}{dt} \qquad (1)$$

During this process no mass enters the control volume ($\dot{m}_{\text{in}} = 0$), and the mass flow rate of discharged water can be expressed as

$$\dot{m}_{\text{out}} = (\rho V_{ave} A)_{\text{out}} = \rho\sqrt{2gh}A_{\text{jet}} \qquad (2)$$

where $A_{\text{jet}} = \pi D_{\text{jet}}^2/4$ is the cross-sectional area of the jet, which is constant. Noting that the density of water is constant, the mass of water in the tank at any time is

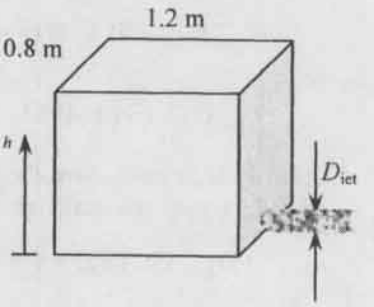
$$m_{\text{system}} = \rho V = \rho A_{\text{tank}} h \qquad (3)$$

where A_{tank} is the base area of the tank. Substituting Eqs. (2) and (3) into (1) gives

$$-\rho\sqrt{2gh}A_{\text{jet}} = \frac{d(\rho A_{\text{tank}} h)}{dt} \quad \rightarrow \quad -\rho\sqrt{2gh}A_{\text{jet}} = \frac{\rho A_{\text{tank}}\, dh}{dt}$$

Canceling the densities and separating the variables give

$$dt = -\frac{A_{\text{tank}}}{A_{\text{jet}}}\frac{dh}{\sqrt{2gh}}$$



Integrating from $t = 0$ at which $h = h_0$ to $t = t$ at which $h = h_2$ gives

$$\int_0^t dt = -\frac{A_{\text{tank}}}{A_{\text{jet}}\sqrt{2g}}\int_{h_0}^{h_2}\frac{dh}{\sqrt{h}} \quad \rightarrow \quad t = \frac{A_{\text{tank}}}{A_{\text{jet}}}\frac{\sqrt{h_0}-\sqrt{h_2}}{\sqrt{g/2}}$$

Substituting, the time of discharge is determined to be

$$t = \frac{\sqrt{3\ \text{m}}-\sqrt{1.6\ \text{m}}}{\sqrt{(9.81\ \text{m/s}^2)/2}}\left(\frac{0.8\ \text{m}\times 1.2\ \text{m}}{\pi(0.012\ \text{m})^2/4}\right) = 1790\ \text{s} = \mathbf{29.8\ min}$$

Therefore, it will take about half an hour for the gage pressure at the bottom of the tank to drop to 15.7 kPa after the discharge hole is unplugged.

Discussion It can be shown that it will take 111 min for the tank to empty completely and the gage pressure at the bottom of the tank to drop to zero.

2-76 One section of the duct of an air-conditioning system is laid underwater. The upward force the water will exert on the duct is to be determined.

Assumptions **1** The diameter given is the outer diameter of the duct (or, the thickness of the duct material is negligible). **2** The weight of the duct and the air in is negligible.

Properties The density of air is given to be $\rho = 1.30$ kg/m^3. We take the density of water to be 1000 kg/m^3.

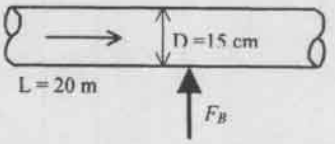
Analysis Noting that the weight of the duct and the air in it is negligible, the net upward force acting on the duct is the buoyancy force exerted by water. The volume of the underground section of the duct is

$$V = AL = (\pi D^2/4)L = [\pi(0.15\,\text{m})^2/4](20\,\text{m}) = 216\,\text{kg/min} = 0.353\ \text{m}^3$$

Then the buoyancy force becomes

$$F_B = \rho g V = (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.353\,\text{m}^3)\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{3.46\,kN}$$

Discussion The upward force exerted by water on the duct is 3.46 kN, which is equivalent to the weight of a mass of 353 kg. Therefore, this force must be treated seriously.



2-77 A helium balloon tied to the ground carries 2 people. The acceleration of the balloon when it is first released is to be determined.

Assumptions The weight of the cage and the ropes of the balloon is negligible.

Properties The density of air is given to be $\rho = 1.16$ kg/m^3. The density of helium gas is 1/7th of this.

Analysis The buoyancy force acting on the balloon is

$$V_{balloon} = 4\pi r^3/3 = 4\pi(5m)^3/3 = 523.6m^3$$

$$F_B = \rho_{air} g V_{balloon}$$
$$= (1.16\text{kg/m}^3)(9.807\text{m/s}^2)(523.6\text{m}^3)\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right) = 5956.5\text{N}$$



The total mass is

$$m_{He} = \rho_{He} V = \left(\frac{1.16}{7}\text{kg/m}^3\right)(523.6\text{m}^3) = 86.8\text{kg}$$

$$m_{total} = m_{He} + m_{people} = 86.8 + 2\times 70 = 226.8\text{kg}$$

The total weight is

$$W = m_{total} g = (226.8\text{kg})(9.807\text{m/s}^2)\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right) = 2224.2\text{N}$$

Thus the net force acting on the balloon is

$$F_{net} = F_B - W = 5956.5 - 2224.2 = 3732.3\ \text{N}$$

Then the acceleration becomes

$$a = \frac{F_{net}}{m_{total}} = \frac{3732.2\text{N}}{226.8\text{kg}}\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right) = \mathbf{16.5\,m/s^2}$$

2-78 A balloon is filled with helium gas. The maximum amount of load the balloon can carry is to be determined.

Assumptions The weight of the cage and the ropes of the balloon is negligible.

Properties The density of air is given to be $\rho = 1.16$ kg/m^3. The density of helium gas is 1/7th of this.

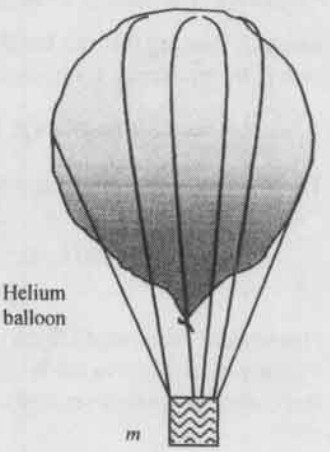
Analysis In the limiting case, the net force acting on the balloon will be zero. That is, the buoyancy force and the weight will balance each other:

$$W = mg = F_B$$

$$m_{total} = \frac{F_B}{g} = \frac{5956.5 \text{ N}}{9.807 \text{ m/s}^2} = 607.4 \text{ kg}$$

Thus,

$$m_{people} = m_{total} - m_{He} = 607.4 - 86.8 = \mathbf{520.6\ kg}$$



2-79E The pressure in a steam boiler is given in kgf/cm^2. It is to be expressed in psi, kPa, atm, and bars.

Analysis We note that 1 atm = 1.03323 kgf/cm^2, 1 atm = 14.696 psi, 1 atm = 101.325 kPa, and 1 atm = 1.01325 bar = 100 kPa (inner cover page of text). Then the desired conversions become:

In atm:
$$P = (75 \text{ kgf/cm}^2)\left(\frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2}\right) = 72.6 \text{ atm}$$

In psi:
$$P = (75 \text{ kgf/cm}^2)\left(\frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2}\right)\left(\frac{14.696 \text{ psi}}{1 \text{ atm}}\right) = 1067 \text{ psi}$$

In kPa:
$$P = (75 \text{ kgf/cm}^2)\left(\frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2}\right)\left(\frac{101.325 \text{ kPa}}{1 \text{ atm}}\right) = 7574 \text{ } kPa$$

In bars:
$$P = (75 \text{ kgf/cm}^2)\left(\frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2}\right)\left(\frac{1.01325 \text{ bar}}{1 \text{ atm}}\right) = 75.74 \text{ } bar$$

Discussion Note that the units atm, kgf/cm^2, and bar are almost identical to each other.

2-80 A barometer is used to measure the altitude of a plane relative to the ground. The barometric reading at the ground and in the plane are given. The altitude of the plane is to be determined.

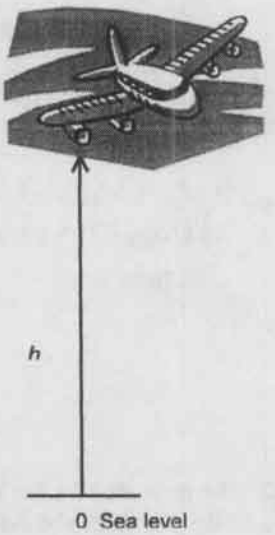
Assumptions The variation of air density with altitude is negligible.

Properties The density of air is given to be $\rho = 1.20\ \text{kg/m}^3$.

Analysis Atmospheric pressures at the location of the plane and the ground level are

$$P_{\text{plane}} = (\rho g h)_{\text{plane}}$$
$$= (13{,}600\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.690\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$
$$= 91.96\ \text{kPa}$$

$$P_{\text{ground}} = (\rho g h)_{\text{ground}}$$
$$= (13{,}600\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.753\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$
$$= 100.36\ \text{kPa}$$



Taking an air column between the airplane and the ground and writing a force balance per unit base area, we obtain

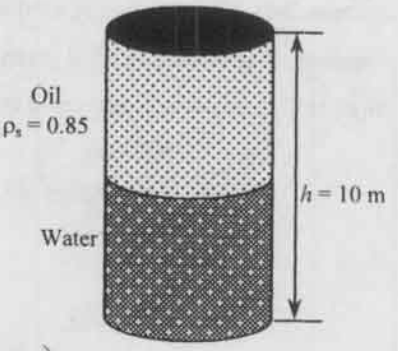
$$W_{\text{air}} / A = P_{\text{ground}} - P_{\text{plane}}$$
$$(\rho g h)_{\text{air}} = P_{\text{ground}} - P_{\text{plane}}$$
$$(1.20\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(h)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) = (100.36 - 91.96)\ \text{kPa}$$

It yields $h = \mathbf{714\ m}$

which is also the altitude of the airplane.

2-81 A 10-m high cylindrical container is filled with equal volumes of water and oil. The pressure difference between the top and the bottom of the container is to be determined.

Properties The density of water is given to be ρ = 1000 kg/m^3. The specific gravity of oil is given to be 0.85.



Analysis The density of the oil is obtained by multiplying its specific gravity by the density of water,

$$\rho = (\rho_s)(\rho_{H_2O}) = (0.85)(1000\ \text{kg/m}^3) = 850\ \text{kg/m}^3$$

The pressure difference between the top and the bottom of the cylinder is the sum of the pressure differences across the two fluids,

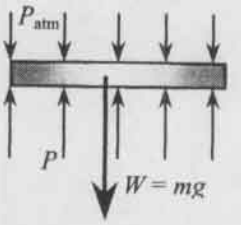
$$\begin{aligned}\Delta P_{total} &= \Delta P_{oil} + \Delta P_{water} = (\rho g h)_{oil} + (\rho g h)_{water} \\ &= \left[(850\text{kg/m}^3)(9.807\text{m/s}^2)(5\text{m}) + (1000\text{kg/m}^3)(9.807\text{m/s}^2)(5\text{m})\right]\left(\frac{1\text{kPa}}{1000\text{N/m}^2}\right) \\ &= \mathbf{90.7kPa}\end{aligned}$$

2-82 The pressure of a gas contained in a vertical piston-cylinder device is measured to be 500 kPa. The mass of the piston is to be determined.

Assumptions There is no friction between the piston and the cylinder.

Analysis Drawing the free body diagram of the piston and balancing the vertical forces yield

$$\begin{aligned} W &= PA - P_{atm}A \\ mg &= (P - P_{atm})A \\ (m)(9.807\text{m/s}^2) &= (500 - 100\text{kPa})(30\times10^{-4}\text{m}^2)\left(\frac{1000\text{kg/m}\cdot\text{s}^2}{1\text{kPa}}\right)\end{aligned}$$



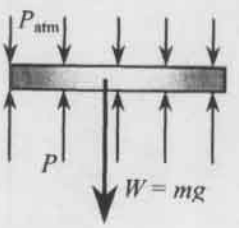
It yields m = **122.4 kg**

2-83 The gage pressure in a pressure cooker is maintained constant at 100 kPa by a petcock. The mass of the petcock is to be determined.

Assumptions There is no blockage of the pressure release valve.

Analysis Atmospheric pressure is acting on all surfaces of the petcock, which balances itself out. Therefore, it can be disregarded in calculations if we use the gage pressure as the cooker pressure. A force balance on the petcock ($\Sigma F_y = 0$) yields

$$\begin{aligned} W &= P_{gage}A \\ m &= \frac{P_{gage}A}{g} = \frac{(100kPa)(4\times10^{-6}m^2)}{9.807m/s^2}\left(\frac{1000kg/m\cdot s^2}{1kPa}\right) \\ &= \mathbf{0.0408kg}\end{aligned}$$



2-84 A glass tube open to the atmosphere is attached to a water pipe, and the pressure at the bottom of the tube is measured. It is to be determined how high the water will rise in the tube.

Properties The density of water is given to be $\rho = 1000$ kg/m^3.

Analysis The pressure at the bottom of the tube can be expressed as

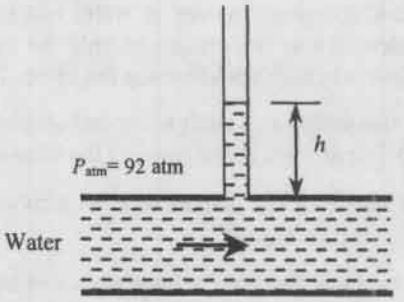
$$P = P_{atm} + (\rho g h)_{tube}$$

Solving for h,

$$h = \frac{P - P_{atm}}{\rho g}$$

$$= \frac{(115-92)\text{ kPa}}{(1000\text{ kg/m}^3)(9.8\text{ m/s}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)$$

$$= \mathbf{2.35\ m}$$



2-85 The average atmospheric pressure is given as $P_{atm} = 101.325(1 - 0.02256z)^{5.256}$ where z is the altitude in km. The atmospheric pressures at various locations are to be determined.

Analysis The atmospheric pressures at various locations are obtained by substituting the altitude z values in km into the relation

$$P_{atm} = 101.325(1 - 0.02256z)^{5.256}$$

Atlanta:	(z = 0.306 km): $P_{atm} = 101.325(1 - 0.02256\times0.306)^{5.256} = \mathbf{97.7\ kPa}$
Denver:	(z = 1.610 km): $P_{atm} = 101.325(1 - 0.02256\times1.610)^{5.256} = \mathbf{83.4\ kPa}$
M. City:	(z = 2.309 km): $P_{atm} = 101.325(1 - 0.02256\times2.309)^{5.256} = \mathbf{76.5\ kPa}$
Mt. Ev.:	(z = 8.848 km): $P_{atm} = 101.325(1 - 0.02256\times8.848)^{5.256} = \mathbf{31.4\ kPa}$

2-86 The air pressure in a duct is measured by an inclined manometer. For a given vertical level difference, the gage pressure in the duct and the length of the differential fluid column are to be determined.

Assumptions The manometer fluid is an incompressible substance.

Properties The density of the liquid is given to be $\rho = 0.81$ kg/L = 810 kg/m^3.

Analysis The gage pressure in the duct is determined from

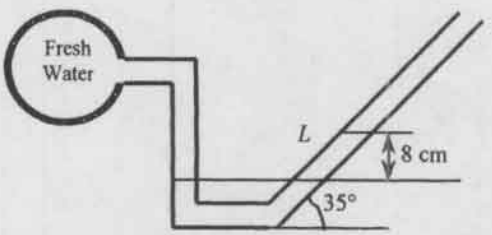
$$P_{gage} = P_{abs} - P_{atm} = \rho g h$$

$$= (810\text{kg/m}^3)(9.81\text{m/s}^2)(0.08\text{m})\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{Pa}}{1\text{N/m}^2}\right)$$

$$= \mathbf{635\ Pa}$$

The length of the differential fluid column is

$$L = h/\cos\theta = (8\text{cm})/\sin 35° = \mathbf{13.9\ cm}$$

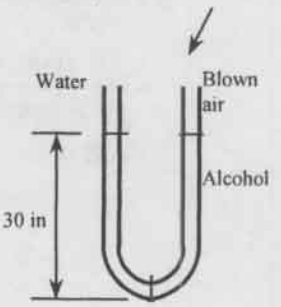


Discussion Note that the length of the differential fluid column is extended considerably by inclining the manometer arm for better readability.

2-87E Equal volumes of water and ethyl alcohol are poured into a U-tube from different arms, and the alcohol side is pressurized until the contact surface of the two fluids moves to the bottom and the liquid levels in both arms become the same. The excess pressure applied on the alcohol side is to be determined.

Assumptions **1** Both water and alcohol are incompressible substances. **2** Alcohol does not mix with water. **3** The cross-sectional area of the U-tube is constant.

Properties The density of ethyl alcohol is given to be $\rho_a = 49.3$ lbm/ft^3. We take the density of water to be $\rho_w = 62.4$ lbm/ft^3.



Analysis Noting that the pressure of both the water and the alcohol is the same at the contact surface, the pressure at this surface can be expressed as

$$P_{\text{contact}} = P_{\text{blow}} + \rho_a g h_a = P_{\text{atm}} + \rho_w g h_w$$

Noting that $h_a = h_w$ and rearranging,

$$\begin{aligned} P_{\text{gage,blow}} &= P_{\text{blow}} - P_{\text{atm}} = (\rho_w - \rho_a) g h \\ &= (62.4 - 49.3 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(30/12 \text{ ft}) \left(\frac{1 \text{lbf}}{32.2 \text{lbm} \cdot \text{ft/s}^2} \right) \left(\frac{1 \text{ft}^2}{144 \text{in}^2} \right) \\ &= 0.227 \textbf{ psi} \end{aligned}$$

Discussion When the person stops blowing, the alcohol will rise and some water will flow into the right arm. It can be shown that when the curvature effect of the tube are disregarded, the differential height of water will be 23.7 in to balance 30-in of alcohol.

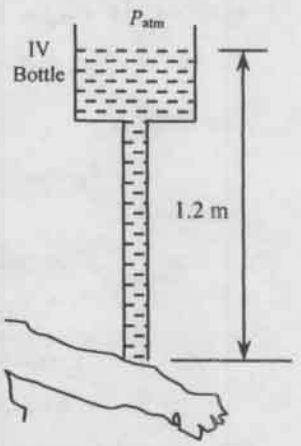
2-88 It is given that an IV fluid and the blood pressures balance each other when the bottle is at a certain height, and a certain gage pressure at the arm level is needed for sufficient flow rate. The gage pressure of the blood and elevation of the bottle required to maintain flow at the desired rate are to be determined.

Assumptions **1** The IV fluid is incompressible. **2** The IV bottle is open to the atmosphere.

Properties The density of the IV fluid is given to be $\rho = 1020\ \text{kg/m}^3$.

Analysis (*a*) Noting that the IV fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, the gage pressure of the blood in the arm is simply equal to the gage pressure of the IV fluid at a depth of 1.2 m,

$$\begin{aligned} P_{\text{gage, arm}} &= P_{\text{abs}} - P_{\text{atm}} = \rho g h_{\text{arm-bottle}} \\ &= (1020\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.20\ \text{m})\left(\frac{1\text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) \\ &= \mathbf{12.0\ Pa} \end{aligned}$$



(*b*) To provide a gage pressure of 20 kPa at the arm level, the height of the bottle from the arm level is again determined from $P_{\text{gage, arm}} = \rho g h_{\text{arm-bottle}}$ to be

$$\begin{aligned} h_{\text{arm-bottle}} &= \frac{P_{\text{gage, arm}}}{\rho g} \\ &= \frac{20\ \text{kPa}}{(1020\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right)\left(\frac{1\ \text{kN/m}^2}{1\ \text{kPa}}\right) = 2.0\ m \end{aligned}$$

Discussion Note that the height of the reservoir can be used to control flow rates in gravity driven flows. When there is flow, the pressure drop in the tube due to friction should also be considered. This will result in raising the bottle a little higher to overcome pressure drop.

2-89 A gasoline line is connected to a pressure gage through a double-U manometer. For a given reading of the pressure gage, the gage pressure of the gasoline line is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible.

Properties The specific gravities of alcohol, mercury, and gasoline are given to be 0.79, 13.6, and 0.70, respectively. We take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$.

Analysis Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the gasoline pipe, and setting the result equal to P_{gasoline} gives

$$P_{\text{gage}} - \rho_w g h_w + \rho_{\text{alcohol}} g h_{\text{alcohol}} - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{gasoline}} g h_{\text{gasoline}} = P_{\text{gasoline}}$$

Rearranging,

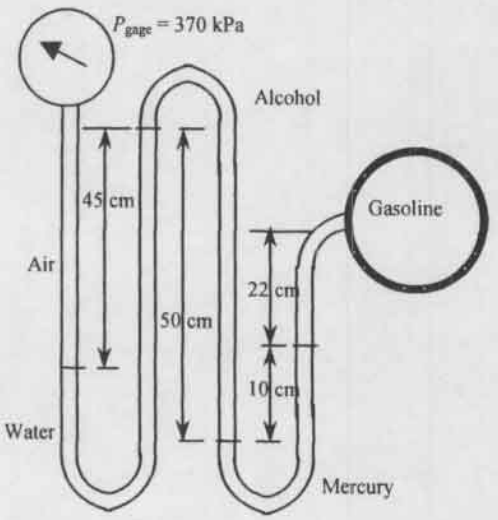
$$P_{\text{gasoline}} = P_{\text{gage}} - \rho_w g(h_w - \rho_{\text{s,alcohol}} h_{\text{alcohol}} + \rho_{\text{s,Hg}} h_{\text{Hg}} + \rho_{\text{s,gasoline}} h_{\text{gasoline}})$$

Substituting,

$$\begin{aligned} P_{\text{gasoline}} &= 370\ \text{kPa} - (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(0.45\ \text{m}) - 0.79(0.5\ \text{m}) + 13.6(0.1\ \text{m}) + 0.70(0.22\ \text{m})] \\ &\quad \times \left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) \\ &= 354.6\ \textbf{kPa} \end{aligned}$$

Therefore, the pressure in the gasoline pipe is 15.4 kPa lower than the pressure reading of the pressure gage.

Discussion Note that sometimes the use of specific gravity offers great convenience in the solution of problems that involve several fluids.



2-90 A gasoline line is connected to a pressure gage through a double-U manometer. For a given reading of the pressure gage, the gage pressure of the gasoline line is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible.

Properties The specific gravities of alcohol, mercury, and gasoline are given to be 0.79, 13.6, and 0.70, respectively. We take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$.

Analysis Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the gasoline pipe, and setting the result equal to P_{gasoline} gives

$$P_{gage} - \rho_w g h_w + \rho_{\text{alcohol}} g h_{\text{alcohol}} - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{gasoline}} g h_{\text{gasoline}} = P_{\text{gasoline}}$$

Rearranging,

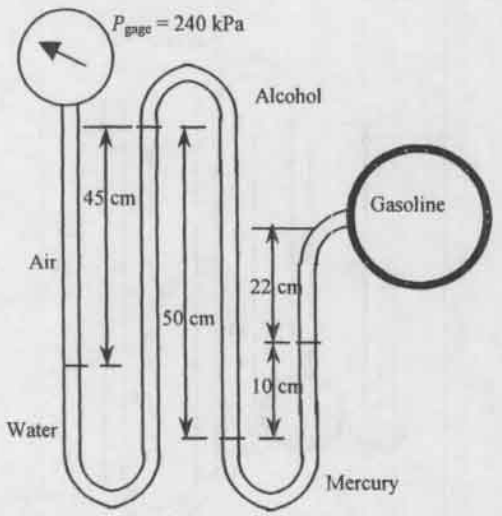
$$P_{\text{gasoline}} = P_{\text{gage}} - \rho_w g(h_w - \rho_{s,\text{alcohol}} h_{s,\text{alcohol}} + \rho_{s,\text{Hg}} h_{\text{Hg}} + \rho_{s,\text{gasoline}} h_{s,\text{gasoline}})$$

Substituting,

$$P_{\text{gasoline}} = 240\ \text{kPa} - (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(0.45\ \text{m}) - 0.79(0.5\ \text{m}) + 13.6(0.1\ \text{m}) + 0.70(0.22\ \text{m})]$$
$$\times \left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)$$
$$= 224.6\ \textbf{kPa}$$

Therefore, the pressure in the gasoline pipe is 15.4 kPa lower than the pressure reading of the pressure gage.

Discussion Note that sometimes the use of specific gravity offers great convenience in the solution of problems that involve several fluids.



2-91E A water pipe is connected to a double-U manometer whose free arm is open to the atmosphere. The absolute pressure at the center of the pipe is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The solubility of the liquids in each other is negligible.

Properties The specific gravities of alcohol, mercury, and oil are given to be 0.79, 13.6, and 0.80, respectively. We take the density of water to be $\rho_w = 62.4$ lbm/ft^3.

Analysis Starting with the pressure at the center of the water pipe, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives

$$P_{\text{water pipe}} - \rho_{\text{water}} g h_{\text{water}} + \rho_{\text{alcohol}} g h_{\text{alcohol}} - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{oil}} g h_{\text{oil}} = P_{atm}$$

Solving for $P_{\text{water pipe}}$,

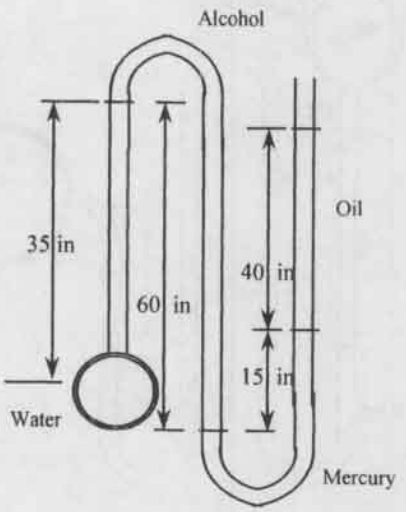
$$P_{\text{water pipe}} = P_{atm} + \rho_{\text{water}} g (h_{\text{water}} - \rho_{\text{s, alcohol}} h_{\text{alcohol}} + \rho_{\text{s, Hg}} h_{\text{Hg}} + \rho_{\text{s, oil}} h_{\text{oil}})$$

Substituting,

$$\begin{aligned} P_{\text{water pipe}} &= 14.2\,\text{psia} + (62.4\,\text{lbm/ft}^3)(32.2\,\text{ft/s}^2)[(35/12\,\text{ft}) - 0.79(60/12\,\text{ft}) + 13.6(15/12\,\text{ft}) \\ &\quad + 0.8(40/12\,\text{ft})] \times \left(\frac{1\,\text{lbf}}{32.2\,\text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right) \\ &= 22.3\,\textbf{psia} \end{aligned}$$

Therefore, the absolute pressure in the water pipe is 22.3 psia.

Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.



Chapter 3
PROPERTIES OF PURE SUBSTANCES

Pure Substances, Phase Change Processes, Phase Diagrams

3-1C Yes. Because it has the same chemical composition throughout.

3-2C A liquid which is about to vaporize is saturated liquid; otherwise it is compressed liquid.

3-3C A vapor which is about to condense is saturated vapor; otherwise it is superheated vapor.

3-4C No.

3-5C No.

3-6C Yes. The saturation temperature of a pure substance depends on pressure. The higher the pressure, the higher the saturation or boiling temperature.

3-7C The temperature will also increase since the boiling or saturation temperature of a pure substance depends on pressure.

3-8C Because one cannot be varied while holding the other constant. In other words, when one changes, so does the other one.

3-9C At critical point the saturated liquid and the saturated vapor states are identical. At triple point the three phases of a pure substance coexist in equilibrium.

3-10C Yes.

3-11C Case (c) when the pan is covered with a heavy lid. Because the heavier the lid, the greater the pressure in the pan, and thus the greater the cooking temperature.

3-12C At supercritical pressures, there is no distinct phase change process. The liquid uniformly and gradually expands into a vapor. At subcritical pressures, there is always a distinct surface between the phases.

Property Tables

3-13C A given volume of water will boil at a higher temperature in a **tall and narrow pot** since the pressure at the bottom will (and thus the corresponding saturation pressure) will be higher in that case.

3-14C A perfectly fitting pot and its lid often stick after cooking as a result of the vacuum created inside as the temperature and thus the corresponding saturation pressure inside the pan drops. An easy way of removing the lid is to reheat the food. When the temperature rises to boiling level, the pressure rises to atmospheric value and thus the lid will come right off.

3-15C The molar mass of gasoline (C_8H_{18}) is 114 kg/kmol, which is much larger than the molar mass of air which is 29 kg/kmol. Therefore, the gasoline vapor will settle down instead of rising even if it is at a much higher temperature than the surrounding air. As a result, the warm mixture of air and gasoline on top of an open gasoline can will most likely settle down instead of rising in a cooler environment.

3-16C Ice can be made by evacuating the air in a water tank. During evacuation, vapor is also thrown out, and thus the vapor pressure in the tank drops, causing a difference between the vapor pressures at the water surface and in the tank. This pressure difference is the driving force of vaporization, and forces the liquid to evaporate. But the liquid must absorb the heat of vaporization before it can vaporize, and it absorbs it from the liquid and the air in the neighborhood, causing the temperature in the tank to drop. The process continues until water starts freezing. The process can be made more efficient by insulating the tank well so that the entire heat of vaporization comes essentially from the water.

3-17C Yes. Otherwise we can create energy by alternately vaporizing and condensing a substance.

3-18C No. Because in the thermodynamic analysis we deal with the changes in properties; and the changes are independent of the selected reference state.

3-19C The term h_{fg} represents the amount of energy needed to vaporize a unit mass of saturated liquid at a specified temperature or pressure. It can be determined from $h_{fg} = h_g - h_f$.

3-20C Yes; the higher the temperature the lower the h_{fg} value.

3-21C Quality is the fraction of vapor in a saturated liquid-vapor mixture. It has no meaning in the superheated vapor region.

3-22C Completely vaporizing 1 kg of saturated liquid at 1 atm pressure since the higher the pressure, the lower the h_{fg}.

3-23C Yes. It decreases with increasing pressure and becomes zero at the critical pressure.

3-24C No. Quality is a mass ratio, and it is not identical to the volume ratio.

3-25C The compressed liquid can be approximated as a saturated liquid at the given temperature. Thus $v_{T,P} \cong v_{f@T}$.

3-26 *Complete the following table for H_2O:*

T, °C	P, kPa	v, m³ / kg	Phase description
50	***12.349***	4.16	***Saturated mixture***
120.23	200	***0.8857***	Saturated vapor
250	400	***0.5951***	***Superheated vapor***
110	600	***0.001052***	***Compressed liquid***

3-27E *Complete the following table for H_2O:*

T, °F	P, psia	u, Btu / lbm	Phase description
250	***29.82***	851	***Saturated mixture***
227.96	20	***196.19***	Saturated liquid
500	120	***1174.2***	***Superheated vapor***
400	400	***374.27***	***Compressed liquid***

3-28 *Complete the following table for H_2O:*

T, °C	P, kPa	*h*, kJ / kg	*x*	Phase description
136.30	325	*1435.57*	0.4	*Saturated mixture*
160	*617.8*	1682	*0.483*	*Saturated mixture*
177.69	950	*753.02*	0.0	*Saturated liquid*
80	500	*334.91*	---	*Compressed liquid*
350	800	3161.7	---	*Superheated vapor*

3-29 *Complete the following table for Refrigerant-134a:*

T, °C	P, kPa	*v*, m^3 / kg	Phase description
-12	600	*0.0007498*	*Compressed liquid*
20	*571.60*	0.022	*Saturated mixture*
2.48	320	*0.0632*	Saturated vapor
100	600	*0.04790*	*Superheated vapor*

3-30 *Complete the following table for Refrigerant-134a:*

T, °C	P, kPa	*u*, kJ / kg	Phase description
30	*770.06*	120	*Saturated mixture*
-8	*217.04*	*39.38*	Saturated liquid
86.88	400	300	*Superheated vapor*
8	600	*60.43*	*Compressed liquid*

3-31E *Complete the following table for Refrigerant-134a:*

T, °F	P, psia	*h*, Btu / lbm	*x*	Phase description
58.35	70	64	*0.426*	*Saturated mixture*
20	*33.137*	*78.549*	0.7	*Saturated mixture*
10	70	*14.66*	---	*Compressed liquid*
160	180	128.77	---	*Superheated vapor*
110	*161.04*	*115.96*	1.0	*Saturated vapor*

3-32 *Complete the following table for H_2O:*

T, °C	P, kPa	*v*, m^3 / kg	Phase description
125	*232.1*	0.53	*Saturated mixture*
179.91	1000	*0.001127*	Saturated liquid
25	750	*0.001003*	*Compressed liquid*
500	*2709*	0.130	*Superheated vapor*

3-33 *Complete the following table for H_2O:*

T, °C	P, kPa	*u*, kJ / kg	Phase description
136.30	325	2452	***Saturated mixture***
170	***791.7***	***2576.5***	Saturated vapor
190	2000	***806.19***	***Compressed liquid***
466.7	4000	3040	***Superheated vapor***

3-34E The temperature in a pressure cooker during cooking at sea level is measured to be 250°F. The absolute pressure inside the cooker and the effect of elevation on the answer are to be determined.

Assumptions Properties of pure water can be used to approximate the properties of juicy water in the cooker.

Properties The saturation pressure of water at 250°F is 29.82 psia (Table A-5E). The standard atmospheric pressure at sea level is 1 atm = 14.7 psia.

Analysis The absolute pressure in the cooker is simply the saturation pressure at the cooking temperature,

$$P_{\text{abs}} = P_{sat@250°\text{F}} = \mathbf{29.82\ psia}$$



It is equivalent to

$$P_{\text{abs}} = 29.82\text{psia}\left(\frac{1\,\text{atm}}{14.7\,\text{psia}}\right) = \mathbf{2.03\ atm}$$

The elevation has **no effect** on the absolute pressure inside when the temperature is maintained constant at 250°F.

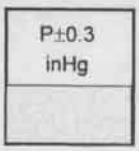
3-35E The local atmospheric pressure, and thus the boiling temperature, changes with the weather conditions. The change in the boiling temperature corresponding to a change of 0.3 in of mercury in atmospheric pressure is to be determined.

Assumptions Properties of pure water can be used to approximate the properties of juicy water in the cooker.

Properties The saturation pressures of water at 200 and 212°F are 11.529 and 14.698 psia, respectively (Table A-5E). One in. of mercury is equivalent to 1 inHg = 3.387 kPa = 0.491 psia (inner cover page).

Analysis A change of 0.3 in of mercury in atmospheric pressure corresponds to

$$\Delta P = (0.3\ \text{inHg})\left(\frac{0.491\,\text{psia}}{1\,\text{inHg}}\right) = 0.147\text{psia}$$



At about boiling temperature, the change in boiling temperature per 1 psia change in pressure is determined using data at 200 and 212°F to be

$$\frac{\Delta T}{\Delta P} = \frac{(212-200)°\,\text{F}}{(14.696-11.529)\ \text{psia}} = 3.789\ °\,\text{F/psia}$$

Then the change in saturation (boiling) temperature corresponding to a change of 0.147 psia becomes

$$\Delta T_{\text{boiling}} = (3.789\ °\,\text{F/psia})\Delta P = (3.789\ °\,\text{F/psia})(0.147\ \text{psia}) = \mathbf{0.56°\,F}$$

which is very small. Therefore, the effect of variation of atmospheric pressure on the boiling temperature is negligible.

3-36 A person cooks a meal in a pot that is covered with a well-fitting lid, and leaves the food to cool to the room temperature. It is to be determined if the lid will open or the pan will move up together with the lid when the person attempts to open the pan by lifting the lid up.

Assumptions **1** The local atmospheric pressure is 1 atm = 101.325 kPa. **2** The weight of the lid is small and thus its effect on the boiling pressure and temperature is negligible. **3** No air has leaked into the pan during cooling.

Properties The saturation pressure of water at 20°C is 2.339 kPa (Table A-4).

Analysis Noting that the weight of the lid is negligible, the reaction force F on the lid after cooling at the pan-lid interface can be determined from a force balance on the lid in the vertical direction to be

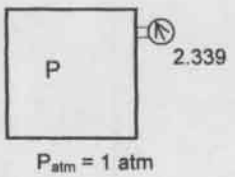
$$PA = P_{atm}A + F$$

or,

$$F = A(P_{atm} - P) = (\pi D^2/4)(P_{atm} - P)$$

$$= \frac{\pi(0.3\text{ m})^2}{4}(101{,}325 - 2339)\text{ Pa}$$

$$= 6997\text{ m}^2\text{Pa} = 6997\text{ N} \quad (\text{since } 1\text{ Pa} = 1\text{ N/m}^2)$$



The weight of the pan and its contents is

$$W = mg = (8\text{ kg})(9.8\text{ m/s}^2) = 78\text{ N}$$

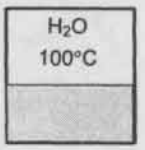
which is much less than the reaction force of 6997 N at the pan-lid interface. Therefore, the pan will **move up** together with the lid when the person attempts to open the pan by lifting the lid up. In fact, it looks like the lid will not open even if the mass of the pan and its contents is several hundred kg.

3-37 Water is boiled at sea level (1 atm pressure) in a pan placed on top of a 3-kW electric burner that transfers 60% of the heat generated to the water. The rate of evaporation of water is to be determined.

Properties The properties of water at 1 atm and thus at the saturation temperature of 100°C are h_{fg} = 2257 kJ/kg (Table A-4).

Analysis The net rate of heat transfer to the water is

$$\dot{Q} = 0.60 \times 3\text{ kW} = 1.8\text{ kW}$$



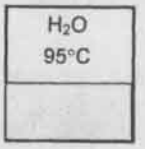
Noting that it takes 2257 kJ of energy to vaporize 1 kg of saturated liquid water, the rate of evaporation of water is determined to be

$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}}{h_{fg}} = \frac{1.8\text{ kJ/s}}{2257\text{ kJ/kg}} = \mathbf{0.80 \times 10^{-3}\ kg/s} = 2.87\text{ kg/h}$$

3-38 Water is boiled at 1500 m (84.5 kPa pressure) in a pan placed on top of a 3-kW electric burner that transfers 60% of the heat generated to the water. The rate of evaporation of water is to be determined.

Properties The properties of water at 84.5 kPa and thus at the saturation temperature of 95°C are h_{fg} = 2270.2 kJ/kg (Table A-4).

Analysis The net rate of heat transfer to the water is



$$\dot{Q} = 0.60 \times 3 \text{ kW} = 1.8 \text{ kW}$$

Noting that it takes 2270.2 kJ of energy to vaporize 1 kg of saturated liquid water, the rate of evaporation of water is determined to be

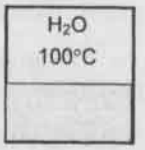
$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}}{h_{fg}} = \frac{1.8 \text{ kJ/s}}{2270.2 \text{ kJ/kg}} = \mathbf{0.79 \times 10^{-3}\ kg/s} = 2.85 \text{ kg/h}$$

3-39 Water is boiled at 1 atm pressure in a pan placed on an electric burner. The water level drops by 10 cm in 30 min during boiling. The rate of heat transfer to the water is to be determined.

Properties The properties of water at 1 atm and thus at a saturation temperature of T_{sat} = 100°C are h_{fg} = 2257 kJ/kg and v_f = 0.001044 m^3/kg (Table A-4).

Analysis The rate of evaporation of water is

$$m_{\text{evap}} = \frac{V_{\text{evap}}}{v_f} = \frac{(\pi D^2/4)L}{v_f} = \frac{[\pi(0.2 \text{ m})^2/4](0.10 \text{ m})}{0.001044} = 3.009 \text{ kg}$$



$$\dot{m}_{\text{evap}} = \frac{m_{\text{evap}}}{\Delta t} = \frac{3.009 \text{ kg}}{30 \times 60 \text{ s}} = 0.00167 \text{ kg/s}$$

Then the rate of heat transfer to water becomes

$$\dot{Q} = \dot{m}_{\text{evap}} h_{fg} = (0.00167 \text{ kg/s})(2257 \text{ kJ/kg}) = \mathbf{3.77\ kW}$$

3-40 Water is boiled at a location where the atmospheric pressure is 79.5 kPa in a pan placed on an electric burner. The water level drops by 10 cm in 30 min during boiling. The rate of heat transfer to the water is to be determined.

Properties The properties of water at 79.5 kPa are $T_{sat} = 93.2°C$, $h_{fg} = 2275$ kJ/kg and $v_f = 0.001038$ m^3/kg (Table A-5).

Analysis The rate of evaporation of water is

$$m_{evap} = \frac{V_{evap}}{v_f} = \frac{(\pi D^2/4)L}{v_f} = \frac{[\pi(0.2\text{ m})^2/4](0.10\text{ m})}{0.001038} = 3.027\text{ kg}$$



$$\dot{m}_{evap} = \frac{m_{evap}}{\Delta t} = \frac{3.027\text{ kg}}{30 \times 60\text{ s}} = 0.00168\text{ kg/s}$$

Then the rate of heat transfer to water becomes

$$\dot{Q} = \dot{m}_{evap} h_{fg} = (0.00168\text{ kg/s})(2275\text{ kJ/kg}) = \mathbf{3.82\text{ kW}}$$

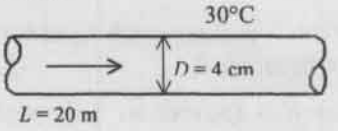
3-41 Saturated steam at $T_{sat} = 30°C$ condenses on the outer surface of a cooling tube at a rate of 45 kg/h. The rate of heat transfer from the steam to the cooling water is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The condensate leaves the condenser as a saturated liquid at 30°C.

Properties The properties of water at the saturation temperature of 30°C are $h_{fg} = 2431$ kJ/kg.

Analysis Noting that 2431 kJ of heat is released as 1 kg of saturated vapor at 30°C condenses, the rate of heat transfer from the steam to the cooling water in the tube is determined directly from

$$\dot{Q} = \dot{m}_{evap} h_{fg} = (45\text{ kg/h})(2431\text{ kJ/kg}) = 109{,}395\text{ kJ/h} = \mathbf{30.4\text{ kW}}$$



3-42 The average atmospheric pressure in Denver is 83.4 kPa. The boiling temperature of water in Denver is to be determined.

Analysis The boiling temperature of water in Denver is the saturation temperature corresponding to the atmospheric pressure in Denver which is 83.4 kPa:

$$T = T_{sat@83.4\ kPa} = \mathbf{94.4°C}$$

3-43 The boiling temperature of water in a 5-cm deep pan is given. The boiling temperature in a 40-cm deep pan is to be determined.

Assumptions Both pans are full of water.

Properties The density of liquid water is approximately $\rho = 1000\ \text{kg/m}^3$.

Analysis The pressure at the bottom of the 5-cm pan is the saturation pressure corresponding to the boiling temperature of 98°C:

$$P = P_{\text{sat@98°C}} = 94.63\ \text{kPa}$$

The pressure difference between the bottoms of two pans is

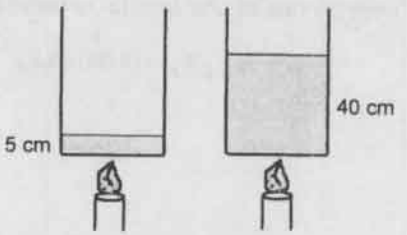
$$\Delta P = \rho g h = (1000\text{kg/m}^3)(9.8\text{m/s}^2)(0.35\text{m})\left(\frac{1\text{kPa}}{1000\text{kg/m}\cdot\text{s}^2}\right) = 3.43\text{kPa}$$

Then the pressure at the bottom of the 40-cm deep pan is

$$P = 94.63 + 3.43 = 98.06\ \text{kPa}$$

Then the boiling temperature becomes

$$T_{\text{boiling}} = T_{\text{sat@98.06kPa}} = \mathbf{99.0^\circ C}$$



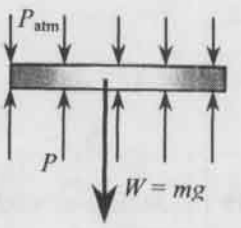
3-44 A cooking pan is filled with water and covered with a 4-kg lid. The boiling temperature of water is to be determined.

Analysis The pressure in the pan is determined from a force balance on the lid,

$$PA = P_{atm}A + W$$

or,

$$P = P_{atm} + \frac{mg}{A}$$
$$= (101\text{kPa}) + \frac{(4\text{kg})(9.807\text{m/s}^2)}{\pi(0.1\text{m})^2}\left(\frac{1\text{kPa}}{1000\text{kg/m}\cdot\text{s}^2}\right)$$
$$= 102.25\text{kPa}$$



The boiling temperature is the saturation temperature corresponding to this pressure,

$$T = T_{sat\ @\ 102.25\ kPa} = \mathbf{100.2^\circ C}$$

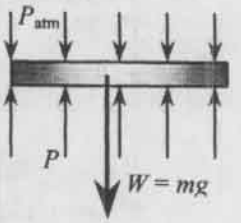
3-45 A vertical piston-cylinder device is filled with water and covered with a 20-kg piston that serves as the lid. The boiling temperature of water is to be determined. √

Analysis The pressure in the cylinder is determined from a force balance on the piston,

$$PA = P_{atm}A + W$$

or,

$$\begin{aligned} P &= P_{\text{atm}} + \frac{mg}{A} \\ &= (100\ \text{kPa}) + \frac{(20\text{kg})(9.807\text{m/s}^2)}{0.01\text{m}^2}\left(\frac{1\ \text{kPa}}{1000\text{kg/m}\cdot\text{s}^2}\right) \\ &= 119.61\ \text{kPa} \end{aligned}$$



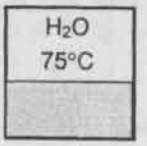
The boiling temperature is the saturation temperature corresponding to this pressure,

$$T = T_{sat\,@\,119.61\,kPa} = \mathbf{104.6°C}$$

3-46 A rigid tank that is filled with saturated liquid-vapor mixture is heated. The temperature at which the liquid in the tank is completely vaporized is to be determined, and the *T-v* diagram is to be drawn.

Analysis This is a constant volume process (v = V/m = constant), and the specific volume is determined to be

$$v = \frac{V}{m} = \frac{2.5\ \text{m}^3}{5\ \text{kg}} = 0.5\ \text{m}^3/\text{kg}$$

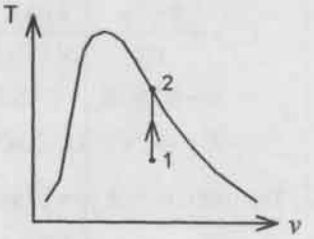


When the liquid is completely vaporized the tank will contain saturated vapor only. Thus,

$$v_2 = v_g = 0.5\ \text{m}^3/\text{kg}$$

The temperature at this point is the temperature which corresponds to this v_g value,

$$T = T_{\text{sat@}v_g=0.5\ \text{m}^3/\text{kg}} = \mathbf{140.7°C}$$



3-47 A rigid vessel is filled with refrigerant-134a. The total volume and the total internal energy is to be determined.

Properties The properties of R-134a at the given state are (Table A-13).

$$\left.\begin{array}{l} P = 900 \text{ kPa} \\ T = 80^\circ\text{C} \end{array}\right\} \begin{array}{l} u = 288.87 \text{ kJ/kg} \\ \text{v} = 0.02861 \text{ m}^3\text{/kg} \end{array}$$

R-134a
2 kg
900 kPa
80°C

Analysis The total volume and internal energy are determined from

$$V = m\text{v} = (2\text{kg})(0.02861\text{m}^3\text{/kg}) = \mathbf{0.05722m^3}$$
$$U = mu = (2\text{kg})(288.87\text{kJ/kg}) = \mathbf{577.74kJ}$$

3-48E A rigid tank contains water at a specified pressure. The temperature, total enthalpy, and the mass of each phase are to be determined.

Analysis (a) The specific volume of the water is

$$\text{v} = \frac{V}{m} = \frac{5 \text{ ft}^3}{5 \text{ lbm}} = 1.0 \text{ ft}^3\text{/lbm}$$

At 20 psia, v_f = 0.01683 ft^3/lbm and v_g = 20.09 ft^3/lbm. Thus the tank contains saturated liquid-vapor mixture since $v_f < v < v_g$, and the temperature must be the saturation temperature at the specified pressure,

$$T = T_{sat@20\text{ psia}} = \mathbf{227.96^\circ F}$$

(b) The quality of the water and its total enthalpy are determined from

$$x = \frac{v - v_f}{v_{fg}} = \frac{1.0 - 0.01683}{20.09 - 0.01683} = 0.0490$$
$$h = h_f + xh_{fg} = 196.26 + 0.049 \times 960.1 = 243.3\text{Btu/lbm}$$
$$H = mh = (5\text{lbm})(243.3\text{Btu/lbm}) = \mathbf{1216.5Btu}$$

H_2O
5 lbm
20 psia

(c) The mass of each phase is determined from

$$m_g = xm_t = 0.049 \times 5 = \mathbf{0.245 \text{ lbm}}$$
$$m_f = m_t + m_g = 5 - 0.245 = \mathbf{4.755 \text{ lbm}}$$

3-49 A rigid vessel contains R-134a at specified temperature. The pressure, total internal energy, and the volume of the liquid phase are to be determined.

Analysis (a) The specific volume of the refrigerant is

$$v = \frac{V}{m} = \frac{0.5\ \text{m}^3}{10\ \text{kg}} = 0.05\ \text{m}^3/\text{kg}$$

At -20°C, $v_f = 0.0007361\ \text{m}^3/\text{kg}$ and $v_g = 0.1464\ \text{m}^3/\text{kg}$. Thus the tank contains saturated liquid-vapor mixture since $v_f < v < v_g$, and the pressure must be the saturation pressure at the specified temperature,

$$P = P_{\text{sat@}-20^\circ\text{C}} = \mathbf{132.99\ kPa}$$

(b) The quality of the refrigerant-134a and its total internal energy are determined from

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.05 - 0.0007361}{0.1464 - 0.0007361} = 0.338$$

$$u = u_f + xu_{fg} = 24.17 + 0.338 \times (215.84 - 24.17) = 88.95\ \text{kJ/kg}$$

$$U = mu = (10\ \text{kg})(88.95\ \text{kJ/kg}) = \mathbf{889.5\ kJ}$$

R-134a
10 kg
-20°C

(c) The mass of the liquid phase and its volume are determined from

$$m_f = (1 - x)m_t = (1 - 0.338) \times 10 = \mathbf{6.62\ kg}$$

$$V_f = m_f v_f = (6.62\ \text{kg})(0.0007361\ \text{m}^3/\text{kg}) = \mathbf{0.00487\ m^3}$$

3-50 A piston-cylinder device contains a saturated liquid-vapor mixture of water at 800 kPa pressure. The mixture is heated at constant pressure until the temperature rises to 350°C. The initial temperature, the total of water, the final volume are to be determined, and the *P-v* diagram is to be drawn.

Analysis (a) Initially two phases coexist in equilibrium, thus we have a saturated liquid-vapor mixture. Then the temperature in the tank must be the saturation temperature at the specified pressure,

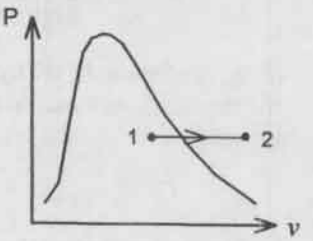
$$T = T_{\text{sat @ 800 kPa}} = \mathbf{170.43\ °C} \qquad \text{(Table A-5)}$$

(b) The total mass in this case can easily be determined by adding the mass of each phase,

$$m_f = \frac{V_f}{v_f} = \frac{0.1\ \text{m}^3}{0.001115\ \text{m}^3/\text{kg}} = 89.69\ \text{kg}$$

$$m_g = \frac{V_g}{v_g} = \frac{0.9\ \text{m}^3}{0.2404\ \text{m}^3/\text{kg}} = 3.74\ \text{kg}$$

$$m_t = m_f + m_g = 89.69 + 3.74 = \mathbf{93.43\ kg}$$



(c) At the final state water is superheated vapor, and its specific volume is

$$\left.\begin{array}{l} P_2 = 800\ \text{kPa} \\ T_2 = 350^\circ\text{C} \end{array}\right\} \; v_2 = 0.3544\ \text{m}^3/\text{kg} \qquad \text{(Table A-6)}$$

Then,

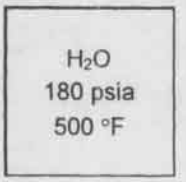
$$V_2 = m_t v_2 = (93.43\ \text{kg})(0.3544\ \text{m}^3/\text{kg}) = \mathbf{33.1\ m^3}$$

3-51E Superheated water vapor cools at constant volume until the temperature drops to 250°F. At the final state, the pressure, the quality, and the enthalpy are to be determined.

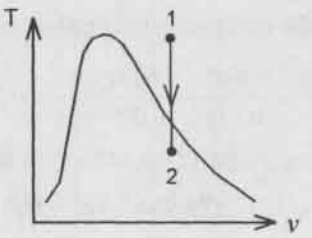
Analysis This is a constant volume process ($v = V/m$ = constant), and the initial specific volume is determined to be

$$\left.\begin{array}{l} P_1 = 180 \text{ psia} \\ T_1 = 500^\circ\text{F} \end{array}\right\} v_1 = 3.042 \text{ ft}^3/\text{lbm} \qquad \text{(Table A-6E)}$$

At 250°F, $v_f = 0.017011$ ft^3/lbm and $v_g = 13.826$ ft^3/lbm. Thus at the final state, the tank will contain saturated liquid-vapor mixture since $v_f < v < v_g$, and the final pressure must be the saturation pressure at the final temperature,



$$P = P_{sat@250^\circ\text{F}} = \mathbf{29.82 \text{ psia}}$$



(b) The quality at the final state is determined from

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{3.042 - 0.017001}{13.826 - 0.017001} = \mathbf{0.219}$$

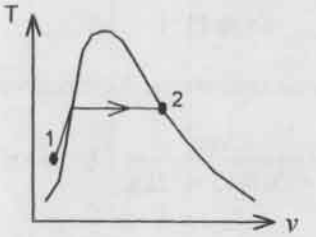
(c) The enthalpy at the final state is determined from

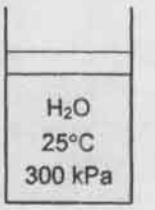
$$h = h_f + xh_{fg} = 218.59 + 0.219 \times 945.6 = \mathbf{425.7 \text{ Btu/lbm}}$$

3-52 A piston-cylinder device that is initially filled with water is heated at constant pressure until all the liquid has vaporized. The mass of water, the final temperature, and the total enthalpy change are to be determined, and the *T-v* diagram is to be drawn.

Analysis Initially the cylinder contains compressed liquid (since $P > P_{atm\ @\ 25°C}$) which can be approximated as a saturated liquid at the specified temperature,

$$v_1 \cong v_{f@25°C} = 0.001003 \text{ m}^3/\text{kg}$$
$$h_1 \cong h_{f@25°C} = 104.89 \text{ kJ/kg}$$





(*a*) The mass is determined from

$$m = \frac{V_1}{v_1} = \frac{0.050 \text{ m}^3}{0.001003 \text{ m}^3/\text{kg}} = \mathbf{49.85 \text{ kg}}$$

(*b*) At the final state, the cylinder contains saturated vapor and thus the final temperature must be the saturation temperature at the final pressure,

$$T = T_{sat@300\text{kPa}} = \mathbf{133.55^\circ\text{C}}$$

(*c*) The final enthalpy is $h_2 = h_{g\ @\ 300\ \text{kPa}} = 2725.3$ kJ/kg. Thus,

$$\Delta H = m(h_2 - h_1) = (49.85 \text{ kg})(2725.3 - 104.89) \text{ kJ/kg} = \mathbf{130{,}627 \text{ kJ}}$$

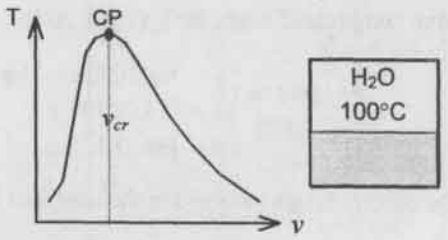
3-53 A rigid vessel that contains a saturated liquid-vapor mixture is heated until it reaches the critical state. The mass of the liquid water and the volume occupied by the liquid at the initial state are to be determined.

Analysis This is a constant volume process ($v = V/m$ = constant) to the critical state, and thus the initial specific volume will be equal to the final specific volume, which is equal to the critical specific volume of water,

$$v_1 = v_2 = v_{cr} = 0.003155 \text{ m}^3/\text{kg} \qquad \text{(last row of Table A-4)}$$

The total mass is

$$m = \frac{V}{v} = \frac{0.5 \text{ m}^3}{0.003155 \text{ m}^3/\text{kg}} = 158.48 \text{ kg}$$



At 100°C, $v_f = 0.001044$ m^3/kg and $v_g = 1.6729$ m^3/kg. Then the quality of water at the initial state is

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.003155 - 0.001044}{1.6729 - 0.001044} = 0.001263$$

Then the mass of the liquid phase and its volume at the initial state are determined from

$$m_f = (1 - x_1)m_t = (1 - 0.001263)(158.48) = \mathbf{158.28 \text{ kg}}$$

$$V_f = m_f v_f = (158.28 \text{ kg})(0.001044 \text{ m}^3/\text{kg}) = \mathbf{0.165 \text{ m}^3}$$

3-54 The properties of compressed liquid water at a specified state are to be determined using the compressed liquid tables, and also by using the saturated liquid approximation, and the results are to be compared.

Analysis Compressed liquid can be approximated as saturated liquid at the given temperature. Then from Table A-4,

$$\begin{aligned} \text{T} = 100°\text{C} \;\Rightarrow\quad & v \cong v_{\text{f@100°C}} = 0.001044\ \text{m}^3/\text{kg} && (0.76\%\ \text{error}) \\ & \text{u} \cong \text{u}_{\text{f@100°C}} = 418.94\ \text{kJ/kg} && (1.01\%\ \text{error}) \\ & \text{h} \cong \text{h}_{\text{f@100°C}} = 419.04\ \text{kJ/kg} && (2.61\%\ \text{error}) \end{aligned}$$

From compressed liquid table (Table A-7),

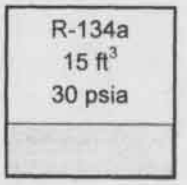
$$\left.\begin{matrix} P = 15\text{MPa} \\ T = 100°\text{C} \end{matrix}\right\} \begin{matrix} v = 0.0010361\ \text{m}^3/\text{kg} \\ u = 414.74\ \text{kJ/kg} \\ h = 430.28\ \text{kJ/kg} \end{matrix}$$

The percent errors involved in the saturated liquid approximation are listed above in parentheses.

3-55E A rigid tank contains saturated liquid-vapor mixture of R-134a. The quality and total mass of the refrigerant are to be determined.

Analysis At 30 psia, $v_f = 0.01209\ \text{ft}^3/\text{lbm}$ and $v_g = 1.5408\ \text{ft}^3/\text{lbm}$. The volume occupied by the liquid and the vapor phases are

$$V_f = 1.5\ \text{ft}^3 \qquad \text{and} \qquad V_g = 13.5\ \text{ft}^3$$



Thus the mass of each phase is

$$m_f = \frac{V_f}{\text{v}_f} = \frac{1.5\ \text{ft}^3}{0.01209\ \text{ft}^3/\text{lbm}} = 124.1\ \text{lbm}$$

$$m_g = \frac{V_g}{\text{v}_g} = \frac{13.5\ \text{ft}^3}{1.5408\ \text{ft}^3/\text{lbm}} = 8.76\ \text{lbm}$$

Then the total mass and the quality of the refrigerant are

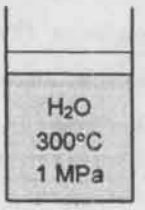
$$m_t = m_f + m_g = 124.1 + 8.76 = \mathbf{132.86\ lbm}$$

$$x = \frac{m_g}{m_t} = \frac{8.76}{132.86} = \mathbf{0.0659}$$

3-56 Superheated steam in a piston-cylinder device is cooled at constant pressure until half of the mass condenses. The final temperature and the volume change are to be determined, and the process should be shown on a *T-v* diagram.

Analysis (b) At the final state the cylinder contains saturated liquid-vapor mixture, and thus the final temperature must be the saturation temperature at the final pressure,

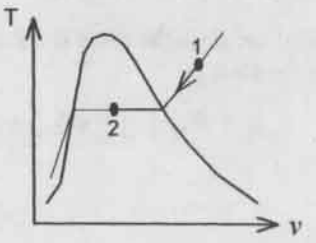
$$T = T_{\text{sat@1 MPa}} = \mathbf{179.91^\circ C}$$



(c) The quality at the final state is specified to be $x_2 = 0.5$.
The specific volumes at the initial and the final states are

$$\left.\begin{matrix} P_1 = 1.0\ \text{MPa} \\ T_1 = 300^\circ\text{C} \end{matrix}\right\} v_1 = 0.2579\ \text{m}^3/\text{kg}$$

$$\left.\begin{matrix} P_2 = 1.0\text{MPa} \\ x_2 = 0.5 \end{matrix}\right\} v_2 = v_f + x_2 v_{fg}$$
$$= 0.001127 + 0.5 \times (0.19444 - 0.001127)$$
$$= 0.0978\ \text{m}^3/\text{kg}$$



Thus,

$$\Delta V = m(v_2 - v_1) = (0.8\ \text{kg})(0.0978 - 0.2579)\ \text{m}^3/\text{kg} = \mathbf{-0.128\ m^3}$$

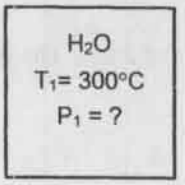
3-57 The water in a rigid tank is cooled until the vapor starts condensing. The initial pressure in the tank is to be determined.

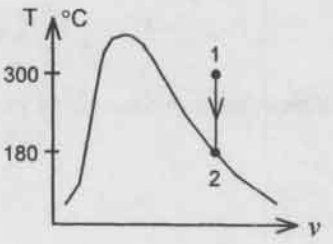
Analysis This is a constant volume process (v = V/m = constant), and the initial specific volume is equal to the final specific volume which is

$$v_1 = v_2 = v_{g@180^\circ\text{C}} = 0.19405\ \text{m}^3/\text{kg}$$

since the vapor starts condensing at 180°C.
Then from Table A-6,

$$\left.\begin{matrix} T_1 = 300^\circ\text{C} \\ v_1 = 0.194005\ \text{m}^3/\text{kg} \end{matrix}\right\} \mathbf{P_1 = 1.325MPa}$$





Vapor Pressure and Phase Equilibrium

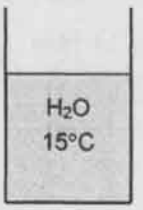
3-58 A glass of water is left in a room. The vapor pressures at the free surface of the water and in the room far from the glass are to be determined.

Assumptions The water in the glass is at a uniform temperature.

Properties The saturation pressure of water is 2.339 kPa at 20°C, and 1.703 kPa at 15°C (Table A-4).

Analysis The vapor pressure at the water surface is the saturation pressure of water at the water temperature,

$$P_{v,\text{ water surface}} = P_{\text{sat}\,@\,T_{\text{water}}} = P_{\text{sat@15°C}} = \mathbf{1.7057\ kPa}$$



Noting that the air in the room is not saturated, the vapor pressure in the room far from the glass is

$$P_{v,\text{air}} = \phi P_{\text{sat}\,@\,T_{\text{air}}} = \phi P_{\text{sat@20°C}} = (0.6)(2.339\ \text{kPa}) = \mathbf{1.4034\ kPa}$$

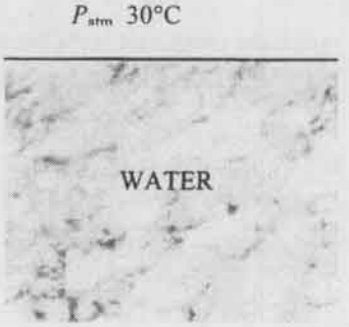
3-59 The vapor pressure in the air at the beach when the air temperature is 30°C is claimed to be 5.2 kPa. The validity of this claim is to be evaluated.

Properties The saturation pressure of water at 30°C is 4.246 kPa (Table A-4).

Analysis The maximum vapor pressure in the air is the saturation pressure of water at the given temperature, which is

$$P_{v,\text{max}} = P_{\text{sat}\,@\,T_{\text{air}}} = P_{\text{sat@30°C}} = \mathbf{4.246\ kPa}$$

which is less than the claimed value of 5.2 kPa. Therefore, the claim is **false**.



3-60 The temperature and relative humidity of air over a swimming pool are given. The water temperature of the swimming pool when phase equilibrium conditions are established is to be determined.

Assumptions The temperature and relative humidity of air over the pool remain constant.

Properties The saturation pressure of water at 20°C is 2.339 kPa (Table A-4).

Analysis The vapor pressure of air over the swimming pool is

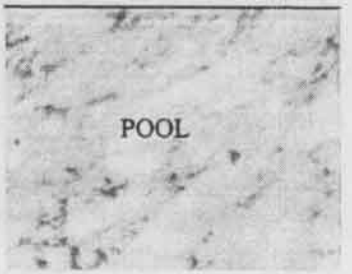
$$P_{v,\text{air}} = \phi P_{\text{sat}@T_{\text{air}}} = \phi P_{\text{sat@20°C}} = (0.4)(2.339\ \text{kPa}) = 0.9356\ \text{kPa}$$

Phase equilibrium will be established when the vapor pressure at the water surface equals the vapor pressure of air far from the surface. Therefore,

$$P_{v,\text{water surface}} = P_{v,\text{air}} = 0.9356\ \text{kPa}$$

and

$$T_{\text{water}} = T_{\text{sat}@P_v} = T_{\text{sat}@0.9356\ \text{kPa}} = \mathbf{5.8°C}$$



Discussion Note that the water temperature drops to 5.8°C in an environment at 20°C when phase equilibrium is established.

3-61 Two rooms are identical except that they are maintained at different temperatures and relative humidities. The room that contains more moisture is to be determined.

Properties The saturation pressure of water is 2.339 kPa at 20°C, and 4.246 kPa at 30°C (Table A-4).

Analysis The vapor pressures in the two rooms are

Room 1: $$P_{v1} = \phi_1 P_{\text{sat}@T_1} = \phi_1 P_{\text{sat@30°C}} = (0.4)(4.246\ \text{kPa}) = \mathbf{1.6984\ kPa}$$

Room 2: $$P_{v2} = \phi_2 P_{\text{sat}@T_2} = \phi_2 P_{\text{sat@20°C}} = (0.7)(2.339\ \text{kPa}) = \mathbf{1.6373\ kPa}$$

Therefore, room 1 at 30°C and 40% relative humidity contains more moisture.

3-62E A thermos bottle half-filled with water is left open to air in a room at a specified temperature and pressure. The temperature of water when phase equilibrium is established is to be determined.

Assumptions The temperature and relative humidity of air over the bottle remain constant.

Properties The saturation pressure of water at 70°F is 0.3632 psia (Table A-4E).

Analysis The vapor pressure of air in the room is

$$P_{v,\,\text{air}} = \phi P_{\text{sat}\,@\,T_{\text{air}}} = \phi P_{\text{sat@70°F}} = (0.35)(0.3632\,\text{psia}) = 0.1271\,\text{psia}$$

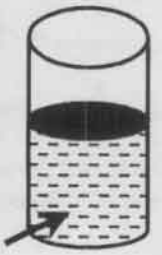
Phase equilibrium will be established when the vapor pressure at the water surface equals the vapor pressure of air far from the surface. Therefore,

$$P_{v,\,\text{water surface}} = P_{v,\,\text{air}} = 0.1271\ \text{psia}$$

and

$$T_{\text{water}} = T_{\text{sat}\,@\,P_v} = T_{\text{sat}\,@\,0.1271\,\text{psia}} = \mathbf{41°F}$$

Discussion Note that the water temperature drops to 41°F in an environment at 70°F when phase equilibrium is established.



3-63 A person buys a supposedly cold drink in a hot and humid summer day, yet no condensation occurs on the drink. The claim that the temperature of the drink is below 10°C is to be evaluated.

Properties The saturation pressure of water at 35°C is 5.628 kPa (Table A-4).

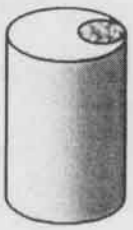
Analysis The vapor pressure of air is

$$P_{v,\,\text{air}} = \phi P_{\text{sat}\,@\,T_{\text{air}}} = \phi P_{\text{sat@35°C}} = (0.7)(5.628\,\text{kPa}) = 3.9396\,\text{kPa}$$

The saturation temperature corresponding to this pressure (called the dew-point temperature) is

$$T_{\text{sat}} = T_{\text{sat}\,@\,P_v} = T_{\text{sat@3.9396 kPa}} = 28.7°\text{C}$$

That is, the vapor in the air will condense at temperatures below 28.7°C. Noting that no condensation is observed on the can, the claim that the drink is at 10°C is **false**.



Ideal Gas

3-64C Propane (molar mass = 44.1 kg/kmol) poses a greater fire danger than methane (molar mass = 16 kg/kmol) since propane is heavier than air (molar mass = 29 kg/kmol), and it will settle near the floor. Methane, on the other hand, is lighter than air and thus it will rise and leak out.

3-65C A gas can be treated as an ideal gas when it is at a high temperature or low pressure relative to its critical temperature and pressure.

3-66C R_u is the universal gas constant which is the same for all gases whereas R is the specific gas constant which is different for different gases. These two are related to each other by $R = R_u / M$, where M is the molar mass of the gas.

3-67C Mass m is simply the amount of matter; molar mass M is the mass of one mole in grams or the mass of one kmol in kilograms. These two are related to each other by $m = NM$, where N is the number of moles.

3-68C It represent the deviation from ideal gas behavior. The further away it is from 1, the more the gas deviates from ideal gas behavior.

3-69C All gases have the same compressibility factor Z at the same reduced temperature and pressure.

3-70C Reduced pressure is the pressure normalized with respect to the critical pressure; and reduced temperature is the temperature normalized with respect to the critical temperature.

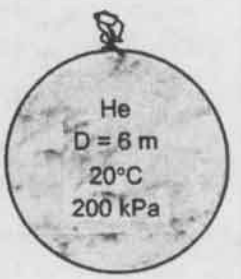
3-71 A balloon is filled with helium gas. The mole number and the mass of helium in the balloon are to be determined.

Assumptions At specified conditions, helium behaves as an ideal gas.

Properties The universal gas constant is R_u = 8.314 kPa.m^3/kmol.K. The molar mass of helium is 4.0 kg/kmol (Table A-1).

Analysis The volume of the sphere is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3\text{ m})^3 = 113.04\text{ m}^3$$



Assuming ideal gas behavior, the mole numbers of He is determined from

$$N = \frac{PV}{R_u T} = \frac{(200\text{kPa})(113.04\text{m}^3)}{(8.314\text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(293\text{K})} = \mathbf{9.28\ kmol}$$

Then the mass of He can be determined from

$$m = NM = (9.28\text{ kmol})(4.0\text{ kg/kmol}) = \mathbf{37.15\ kg}$$

3-72 An automobile tire is inflated with air. The pressure rise of air in the tire when the tire is heated and the amount of air that must be bled off to reduce the temperature to the original value are to be determined.

Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tire remains constant.

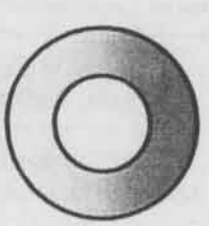
Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1).

Analysis Initially, the absolute pressure in the tire is

$$P_1 = P_g + P_{atm} = 210 + 100 = 310 \text{ kPa}$$

Treating air as an ideal gas and assuming the volume of the tire to remain constant, the final pressure in the tire can be determined from

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \longrightarrow P_2 = \frac{T_2}{T_1}P_1 = \frac{323 \text{ K}}{298 \text{ K}}(310 \text{ kPa}) = 336 \text{ kPa}$$



Thus the pressure rise is

$$\Delta P = P_2 - P_1 = 336 - 310 = \mathbf{26 \text{ kPa}}$$

The amount of air that needs to be bled off to restore pressure to its original value is

$$m_1 = \frac{P_1V}{RT_1} = \frac{(310 \text{ kPa})(0.025 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(298 \text{ K})} = 0.0906 \text{ kg}$$

$$m_2 = \frac{P_2V}{RT_2} = \frac{(310 \text{ kPa})(0.025 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(323 \text{ K})} = 0.0836 \text{ kg}$$

$$\Delta m = m_1 - m_2 = 0.0906 - 0.0836 = \mathbf{0.0070 \text{ kg}}$$

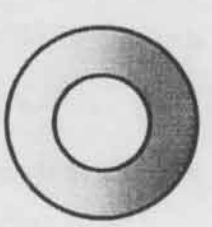
3-73E An automobile tire is under inflated with air. The amount of air that needs to be added to the tire to raise its pressure to the recommended value is to be determined.

Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tire remains constant.

Properties The gas constant of air is $R = 0.3705$ psia.ft^3/lbm.R (Table A-1E).

Analysis The initial and final absolute pressures in the tire are

$$P_1 = P_{g1} + P_{atm} = 20 + 14.6 = 34.6 \text{ psia}$$
$$P_2 = P_{g2} + P_{atm} = 30 + 14.6 = 44.6 \text{ psia}$$



Treating air as an ideal gas, the initial mass in the tire is

$$m_1 = \frac{P_1V}{RT_1} = \frac{(34.6 \text{ psia})(0.53 \text{ ft}^3)}{(0.3705 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(550 \text{ R})} = 0.0900 \text{ lbm}$$

Noting that the temperature and the volume of the tire remain constant, the final mass in the tire becomes

$$m_2 = \frac{P_2V}{RT_2} = \frac{(44.6 \text{ psia})(0.53 \text{ ft}^3)}{(0.3705 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(550 \text{ R})} = 0.1160 \text{ lbm}$$

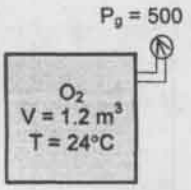
Thus the amount of air that needs to be added is

$$\Delta m = m_2 - m_1 = 0.1160 - 0.0900 = \mathbf{0.0260 \text{ lbm}}$$

3-74 The pressure and temperature of oxygen gas in a storage tank are given. The mass of oxygen in the tank is to be determined.

Assumptions At specified conditions, oxygen behaves as an ideal gas

Properties The gas constant of oxygen is $R = 0.2598$ kPa.m^3/kg.K (Table A-1).



Analysis The absolute pressure of O_2 is

$$P = P_g + P_{atm} = 500 + 97 = 597 \text{ kPa}$$

Treating O_2 as an ideal gas, the mass of O_2 in tank is determined to be

$$m = \frac{PV}{RT} = \frac{(597 \text{ kPa})(1.2 \text{ m}^3)}{(0.2598 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(297 \text{ K})} = \mathbf{9.28 \text{ kg}}$$

3-75E A rigid tank contains slightly pressurized air. The amount of air that needs to be added to the tank to raise its pressure to the recommended value is to be determined. √

Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tank remains constant.

Properties The gas constant of air is $R = 0.3704$ psia.ft^3/lbm.R (Table A-1E).

Analysis Treating air as an ideal gas, the initial volume and the final mass in the tank are determined to be

$$V = \frac{m_1 RT_1}{P_1} = \frac{(20 \text{ lbm})(0.3704 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(530 \text{ R})}{20 \text{ psia}} = 196.3 \text{ ft}^3$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(35 \text{ psia})(196.3 \text{ ft}^3)}{(0.3704 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(550 \text{ R})} = 33.73 \text{ lbm}$$

Air, 20 lbm
20 psia
70°F

Thus the amount of air added is

$$\Delta m = m_2 - m_1 = 33.73 - 20.0 = \mathbf{13.73 \text{ lbm}}$$

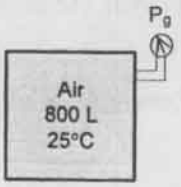
3-76 A rigid tank contains air at a specified state. The gage pressure of the gas in the tank is to be determined..

Assumptions At specified conditions, air behaves as an ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1E).

Analysis Treating air as an ideal gas, the absolute pressure in the tank is determined from

$$P_2 = \frac{mRT}{V} = \frac{(10\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{ K})}{0.8\text{ m}^3} = 1069.1\text{ kPa}$$



Thus the gage pressure is

$$P_g = P - P_{atm} = 1069.1 - 97 = \mathbf{972.1\text{ kPa}}$$

3-77 Two rigid tanks connected by a valve to each other contain air at specified conditions. The volume of tank and the final equilibrium temperature when the valve is opened are to be determined.

Assumptions At specified conditions, air behaves as an ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1E).

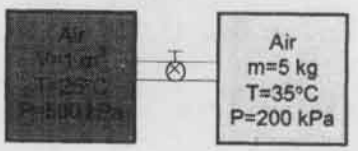
Analysis Let's call the first and the second tanks A and B. Treating air as an ideal gas, the volume of the second tank and the mass of air in the first tank are determined to be

$$V_B = \left(\frac{m_1 RT_1}{P_1}\right)_B = \frac{(5\text{kg})(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(308\text{K})}{200\text{kPa}} = \mathbf{2.21m^3}$$

$$m_A = \left(\frac{P_1 V}{RT_1}\right)_A = \frac{(500\text{kPa})(1.0\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{K})} = 5.846\text{kg}$$

Thus,

$$V = V_A + V_B = 1.0 + 2.21 = 3.21\text{ m}^3$$
$$m = m_A + m_B = 5.846 + 5.0 = 10.846\text{ kg}$$



Then the final equilibrium pressure becomes

$$P_2 = \frac{mRT_2}{V} = \frac{(10.846\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})}{3.21\text{ m}^3} = \mathbf{284.1\text{ kPa}}$$

Other Equations of State

3-78C The constant a represents the increase in pressure as a result of intermolecular forces; the constant b represents the volume occupied by the molecules. They are determined from the requirement that the critical isotherm has an inflection point at the critical point.

3-79 The pressure of nitrogen in a tank at a specified state is to be determined using the ideal gas, van der Waals, and Beattie-Bridgeman equations. The error involved in each case is to be determined.

Properties The gas constant, molar mass, critical pressure, and critical temperature of nitrogen are (Table A-1)

$R = 0.2968$ kPa·m³/kg·K, $M = 28.013$ kg/kmol, $T_{cr} = 126.2$ K, $P_{cr} = 33.90$ MPa

Analysis The specific volume of nitrogen is

$$v = \frac{V}{m} = \frac{3.27\ \text{m}^3}{100\ \text{kg}} = 0.0327\ \text{m}^3/\text{kg}$$

N_2
0.0327 m³/kg
225 K

(*a*) From the ideal gas equation of state,

$$P = \frac{RT}{v} = \frac{(0.2968\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(225\ \text{K})}{0.0327\ \text{m}^3/\text{kg}} = \mathbf{2042\ kPa} \qquad (2.1\%\ error)$$

(*b*) The van der Waals constants for nitrogen are determined from

$$a = \frac{27R^2T_{cr}^2}{64P_{cr}} = \frac{(27)(0.2968\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})^2(126.2\ \text{K})^2}{(64)(3390\ \text{kPa})} = 0.175\ \text{m}^6\cdot\text{kPa}/\text{kg}^2$$

$$b = \frac{RT_{cr}}{8P_{cr}} = \frac{(0.2968\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(126.2\ \text{K})}{8\times 3390\ \text{kPa}} = 0.00138\ \text{m}^3/\text{kg}$$

Then,

$$P = \frac{RT}{v-b} - \frac{a}{v^2} = \frac{0.2968\times 225}{0.0327-0.00138} - \frac{0.175}{(0.0327)^2} = \mathbf{1969\ kPa}\ (1.6\%\ \text{error})$$

(*c*) The constants in the Beattie-Bridgeman equation are

$$A = A_o\left(1-\frac{a}{\bar{v}}\right) = 136.2315\left(1-\frac{0.02617}{0.9160}\right) = 132.339$$

$$B = B_o\left(1-\frac{b}{\bar{v}}\right) = 0.05046\left(1-\frac{-0.00691}{0.9160}\right) = 0.05084$$

$$c = 4.2\times 10^4\ \text{m}^3\cdot\text{K}^3/\text{kmol}$$

since $\bar{v} = Mv = (28.013\ \text{kg/kmol})(0.0327\ \text{m}^3/\text{kg}) = 0.9160\ \text{m}^3/\text{kmol}$. Substituting,

$$P = \frac{R_u T}{\bar{v}^2}\left(1-\frac{c}{\bar{v}T^3}\right)(\bar{v}+B) - \frac{A}{\bar{v}^2}$$

$$= \frac{8.314\times 225}{(0.9160)^2}\left(1-\frac{4.2\times 10^4}{0.9160\times 225^3}\right)(0.9160+0.05084) - \frac{132.339}{(0.9160)^2} = \mathbf{1989\ kPa}\ (0.6\%\ \text{error})$$

3-80 The temperature of steam in a tank at a specified state is to be determined using the ideal gas relation, van der Waals equation, and the steam tables. The error involved in the first two cases is to be determined.

Properties The gas constant, critical pressure, and critical temperature of steam are (Table A-1)

$R = 0.4615$ kPa·m³/kg·K, $T_{cr} = 647.3$ K, $P_{cr} = 22.09$ MPa

Analysis The specific volume of steam is

$$v = \frac{V}{m} = \frac{1\ \text{m}^3}{2.841\ \text{kg}} = 0.3520\ \text{m}^3/\text{kg}$$

H_2O
1 m³
2.841 kg
0.6 MPa

(*a*) From the ideal gas equation of state,

$$T = \frac{Pv}{R} = \frac{(600\ \text{kPa})(0.352\ \text{m}^3/\text{kg})}{0.4615\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}} = \mathbf{457.6\ K}$$

(*b*) The van der Waals constants for steam are determined from

$$a = \frac{27R^2T_{cr}^2}{64P_{cr}} = \frac{(27)(0.4615\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})^2(647.3\ \text{K})^2}{(64)(22{,}090\ \text{kPa})} = 1.704\ \text{m}^6\cdot\text{kPa}/\text{kg}^2$$

$$b = \frac{RT_{cr}}{8P_{cr}} = \frac{(0.4615\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(647.3\ \text{K})}{8\times 22{,}090\ \text{kPa}} = 0.00169\ \text{m}^3/\text{kg}$$

Then,

$$T = \frac{1}{R}\left(P + \frac{a}{v^2}\right)(v-b) = \frac{1}{0.4615}\left(600 + \frac{1.704}{(0.3520)^2}\right)(0.352 - 0.00169) = \mathbf{465.9\ K}$$

(*c*) From the superheated steam table (Tables A-6),

$$\left.\begin{array}{l} P = 0.6\ \text{MPa} \\ v = 0.352\ \text{m}^3/\text{kg} \end{array}\right\} T = \mathbf{200^\circ C} \quad (= 473\ \text{K})$$

3-81E The temperature of R-134a in a tank at a specified state is to be determined using the ideal gas relation, the van der Waals equation, and the refrigerant tables. √

Properties The gas constant, critical pressure, and critical temperature of R-134a are (Table A-1)

$R = 0.1052$ psia·ft^3/lbm·R, $T_{cr} = 673.65$ R, $P_{cr} = 590$ psia

Analysis (a) From the ideal gas equation of state,

$$T = \frac{Pv}{R} = \frac{(100 \text{ psia})(0.5388 \text{ ft}^3/\text{lbm})}{0.1052 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R}} = \mathbf{512.2\ R}$$

(*b*) The van der Waals constants for the refrigerant are determined from

$$a = \frac{27R^2T_{cr}^2}{64P_{cr}} = \frac{(27)(0.1052 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})^2(673.65 \text{ R})^2}{(64)(590 \text{ psia})} = 3.591 \text{ ft}^6 \cdot \text{psia/lbm}^2$$

$$b = \frac{RT_{cr}}{8P_{cr}} = \frac{(0.1052 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(673.65 \text{ R})}{8 \times 590 \text{ psia}} = 0.0150 \text{ ft}^3/\text{lbm}$$

Then,

$$T = \frac{1}{R}\left(P + \frac{a}{v^2}\right)(v - b) = \frac{1}{0.1052}\left(100 + \frac{3.591}{(0.5388)^2}\right)(0.5388 - 0.0150) = \mathbf{559.5\ R}$$

(c) From the superheated refrigerant table (Table A-13E),

$$\left.\begin{array}{l} P = 100 \text{psia} \\ \text{v} = 0.5388 \text{ft}^3/\text{lbm} \end{array}\right\} T = \mathbf{120^\circ F}\,(580\text{R})$$

3-82 The pressure of nitrogen in a tank at a specified state is to be determined using the ideal gas relation and the Beattie-Bridgeman equation. The error involved in each case is to be determined.

Properties The gas constant and molar mass of nitrogen are (Table A-1)

$$R = 0.2968 \text{ kPa·m}^3\text{/kg·K} \quad \text{and} \quad M = 28.013 \text{ kg/kmol}$$

Analysis (*a*) From the ideal gas equation of state,

$$P = \frac{RT}{\text{v}} = \frac{(0.2968 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(150 \text{ K})}{0.041884 \text{ m}^3/\text{kg}} = \mathbf{1063 \text{ kPa}} \qquad (6.3\% \textit{ error})$$

(*b*) The constants in the Beattie-Bridgeman equation are

$$A = A_o\left(1-\frac{a}{\bar{\text{v}}}\right) = 136.2315\left(1-\frac{0.02617}{1.1733}\right) = 133.193$$

$$B = B_o\left(1-\frac{b}{\bar{\text{v}}}\right) = 0.05046\left(1-\frac{-0.00691}{1.1733}\right) = 0.05076$$

$$c = 4.2\times10^4 \text{ m}^3\cdot\text{K}^3\text{/kmol}$$

N_2
0.041884 m^3/kg
150 K

since $\bar{\text{v}} = M\text{v} = (28.013 \text{ kg/kmol})(0.041884 \text{ m}^3\text{/kg}) = 1.1733 \text{ m}^3\text{/kmol}$. Substituting,

$$P = \frac{R_u T}{\bar{\text{v}}^2}\left(1-\frac{c}{\bar{\text{v}}T^3}\right)(\bar{\text{v}}+B)-\frac{A}{\bar{\text{v}}^2}$$

$$= \frac{8.314\times150}{(1.1733)^2}\left(1-\frac{4.2\times10^4}{1.1733\times150^3}\right)(1.1733+0.05076)-\frac{133.193}{(1.1733)^2}$$

$$= \mathbf{1000.4 \text{ kPa}} \text{ (negligibleerror)}$$

Specific Heats, Δu and Δh of Ideal Gases

3-83C It can be used for any kind of process of an ideal gas.

3-84C It can be used for any kind of process of an ideal gas.

3-85C The desired result is obtained by multiplying the first relation by the molar mass M,

$$MC_p = MC_v + MR$$

or $$\overline{C}_p = \overline{C}_v + R_u$$

3-86C Very close, but no. Because the heat transfer during this process is $Q = mC_p\Delta T$, and C_p varies with temperature.

3-87C It can be either. The difference in temperature in both the K and °C scales is the same.

3-88C The energy required is $mC_p\Delta T$, which will be the same in both cases. This is because the C_p of an ideal gas does not vary with pressure.

3-89C The energy required is $mC_p\Delta T$, which will be the same in both cases. This is because the C_p of an ideal gas does not vary with volume.

3-90C For the constant pressure case. This is because the heat transfer to an ideal gas is $mC_p\Delta T$ at constant pressure, $mC_v\Delta T$ at constant volume, and C_p is always greater than C_v.

3-91 The enthalpy change of nitrogen gas during a heating process is to be determined using an empirical specific heat relation, constant specific heat at average temperature, and constant specific heat at room temperature.

Analysis (*a*) Using the empirical relation for $\overline{C}_p(T)$ from Table A-2c,

$$\overline{C}_p = a + bT + cT^2 + dT^3$$

where a = 28.90, b = -0.1571×10^{-2}, c = 0.8081×10^{-5}, and d = -2.873×10^{-9}. From Eq. 3-40,

$$\Delta\overline{h} = \int_1^2 \overline{C}_p(T)dT = \int_1^2 \left[a + bT + cT^2 + dT^3\right]dT$$

$$= a(T_2 - T_1) + \tfrac{1}{2}b(T_2^2 + T_1^2) + \tfrac{1}{3}c(T_2^3 - T_1^3) + \tfrac{1}{4}d(T_2^4 - T_1^4)$$

$$= 28.90(1000 - 600) - \tfrac{1}{2}(0.1571\times10^{-2})(1000^2 - 600^2)$$

$$+ \tfrac{1}{3}(0.8081\times10^{-5})(1000^3 - 600^3) - \tfrac{1}{4}(2.873\times10^{-9})(1000^4 - 600^4)$$

$$= 12{,}544\text{kJ/kmol}$$

$$\Delta h = \frac{\Delta\overline{h}}{M} = \frac{12{,}544\text{kJ/kmol}}{28.013\text{kg/kmol}} = \mathbf{447.8kJ/kg} \quad (0.2\%\text{error})$$

(*b*) Using the constant C_p value from Table A-2b at the average temperature of 800 K,

$$C_{p,ave} = C_{p@800\text{K}} = 1.121\text{kJ/kg}\cdot\text{K}$$

$$\Delta h = C_{p,ave}(T_2 - T_1) = (1.121\text{kJ/kg}\cdot\text{K})(1000 - 600)\text{K} = \mathbf{448.4kJ/kg}$$

(*c*) Using the constant C_p value from Table A-2a at room temperature,

$$C_{p,ave} = C_{p@300\text{K}} = 1.039\text{kJ/kg}\cdot\text{K}$$

$$\Delta h = C_{p,ave}(T_2 - T_1) = (1.039\text{kJ/kg}\cdot\text{K})(1000 - 600)\text{K} = \mathbf{415.6kJ/kg}$$

3-92E The enthalpy change of oxygen gas during a heating process is to be determined using an empirical specific heat relation, constant specific heat at average temperature, and constant specific heat at room temperature.

Analysis (*a*) Using the empirical relation for $\overline{C}_p(T)$ from Table A-2Ec,

$$\overline{C}_p = a + bT + cT^2 + dT^3$$

where a = 6.085, b = 0.2017×10^{-2}, c = -0.05275×10^{-5}, and d = 0.05372×10^{-9}. From Eq. 3-40,

$$\begin{aligned}\Delta\overline{h} &= \int_1^2 \overline{C}_p(T)dT = \int_1^2 \left[a + bT + cT^2 + dT^3\right]dT \\ &= a(T_2 - T_1) + \tfrac{1}{2}b(T_2^2 + T_1^2) + \tfrac{1}{3}c(T_2^3 - T_1^3) + \tfrac{1}{4}d(T_2^4 - T_1^4) \\ &= 6.085(1500 - 800) + \tfrac{1}{2}(0.2017\times10^{-2})(1500^2 - 800^2) \\ &\quad - \tfrac{1}{3}(0.05275\times10^{-5})(1500^3 - 800^3) + \tfrac{1}{4}(0.05372\times10^{-9})(1500^4 - 800^4) \\ &= 5442.3\text{Btu/lbmol}\end{aligned}$$

$$\Delta h = \frac{\Delta\overline{h}}{M} = \frac{5442.3\text{Btu/lbmol}}{31.999\text{lbm/lbmol}} = \mathbf{170.1Btu/lbm}$$

(*b*) Using the constant C_p value from Table A-2Eb at the average temperature of 1150 R,

$$C_{p,ave} = C_{p@1100\,\text{R}} = 0.255\ \text{Btu / lbm}\cdot\text{R}$$

$$\Delta h = C_{p,ave}(T_2 - T_1) = (0.255\ \text{Btu / lbm}\cdot\text{R})(1500 - 800)\text{R} = \mathbf{178.5\ Btu / lbm}$$

(*c*) Using the constant C_p value from Table A-2Ea at room temperature,

$$C_{p,ave} = C_{p@537\,\text{R}} = 0.219\ \text{Btu / lbm}\cdot\text{R}$$

$$\Delta h = C_{p,ave}(T_2 - T_1) = (0.219\ \text{Btu / lbm}\cdot\text{R})(1500 - 800)\text{R} = \mathbf{153.3\ Btu / lbm}$$

3-93 The internal energy change of hydrogen gas during a heating process is to be determined using an empirical specific heat relation, constant specific heat at average temperature, and constant specific heat at room temperature.

Analysis (*a*) Using the empirical relation for $\overline{C}_p(T)$ from Table A-2c and relating it to $\overline{C}_v(T)$,

$$\overline{C}_v(T) = \overline{C}_p - R_u = (a - R_u) + bT + cT^2 + dT^3$$

where a = 29.11, b = -0.1916×10^{-2}, c = 0.4003×10^{-5}, and d = -0.8704×10^{-9}. From Eq. 3-40,

$$\begin{aligned}
\Delta\overline{u} &= \int_1^2 \overline{C}_v(T)dT = \int_1^2 \left[(a - R_u) + bT + cT^2 + dT^3\right]dT \\
&= (a - R_u)(T_2 - T_1) + \tfrac{1}{2}b(T_2^2 + T_1^2) + \tfrac{1}{3}c(T_2^3 - T_1^3) + \tfrac{1}{4}d(T_2^4 - T_1^4) \\
&= (29.11 - 8.314)(1000 - 400) - \tfrac{1}{2}(0.1961\times10^{-2})(1000^2 - 400^2) \\
&\quad + \tfrac{1}{3}(0.4003^{-5})(1000^3 - 400^3) - \tfrac{1}{4}(0.8704\times10^{-9})(1000^4 - 400^4) \\
&= 12{,}691\text{kJ/kmol} \\
\Delta u &= \frac{\Delta\overline{u}}{M} = \frac{12{,}691\text{kJ/kmol}}{2.016\text{kg/kmol}} = \mathbf{6295.3kJ/kg}
\end{aligned}$$

(*b*) Using a constant C_p value from Table A-2b at the average temperature of 700 K,

$$\begin{aligned}
C_{v,ave} &= C_{v@700\text{K}} = 10.48\text{kJ/kg}\cdot\text{K} \\
\Delta u &= C_{v,ave}(T_2 - T_1) = (10.48\text{kJ/kg}\cdot\text{K})(1000 - 400)\text{K} = \mathbf{6288kJ/kg}
\end{aligned}$$

(*c*) Using a constant C_p value from Table A-2a at room temperature,

$$\begin{aligned}
C_{v,ave} &= C_{v@300\text{K}} = 10.183\text{kJ/kg}\cdot\text{K} \\
\Delta u &= C_{v,ave}(T_2 - T_1) = (10.183\text{kJ/kg}\cdot\text{K})(1000 - 400)\text{K} = \mathbf{6100kJ/kg}
\end{aligned}$$

Review Problems

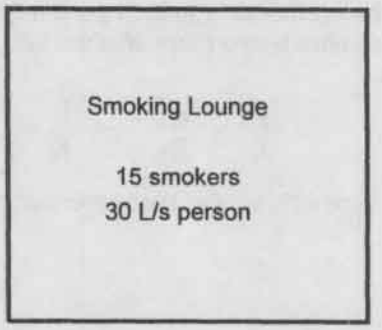
3-94 A smoking lounge that can accommodate 15 smokers is considered. The required minimum flow rate of air that needs to be supplied to the lounge and the diameter of the duct are to be determined. √

Assumptions Infiltration of air into the smoking lounge is negligible.

Properties The minimum fresh air requirements for a smoking lounge is given to be 30 L/s per person.

Analysis The required minimum flow rate of air that needs to be supplied to the lounge is determined directly from

$$\dot{V}_{\text{air}} = \dot{V}_{\text{air per person}}(\text{No. of persons})$$
$$= (30\ \text{L/s}\cdot\text{person})(15\ \text{persons}) = 450\ \text{L/s} = \mathbf{0.45\ m^3/s}$$



Solving for the diameter D and substituting,

$$D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{4(0.45\ \text{m}^3/\text{s})}{\pi(8\ \text{m/s})}} = \mathbf{0.268\ m}$$

Therefore, the diameter of the fresh air duct should be at least 26.8 cm if the velocity of air is not to exceed 8 m/s.

3-95 The minimum fresh air requirements of a residential building is specified to be 0.35 air changes per hour. The size of the fan that needs to be installed and the diameter of the duct are to be determined. √

Analysis The volume of the building and the required minimum volume flow rate of fresh air are

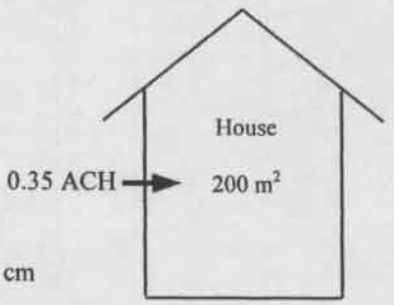
$$V_{\text{room}} = (2.7\ \text{m})(200\ \text{m}^2) = 540\ \text{m}^3$$
$$\dot{V} = V_{\text{room}} \times \text{ACH} = (540\ \text{m}^3)(0.35/\text{h}) = 189\ \text{m}^3/\text{h} = 159 = 189{,}000\ \text{L/h} = \mathbf{5050\ L/min}$$

The volume flow rate of fresh air can be expressed as

$$\dot{V} = VA = V(\pi D^2/4)$$

Solving for the diameter D and substituting,

$$D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{4(189/3600\ \text{m}^3/\text{s})}{\pi(6\ \text{m/s})}} = \mathbf{0.106\ m}$$



Therefore, the diameter of the fresh air duct should be at least 10.6 cm if the velocity of air is not to exceed 6 m/s.

3-96 The pressure in an automobile tire increases during a trip while its volume remains constant. The percent increase in the absolute temperature of the air in the tire is to be determined.

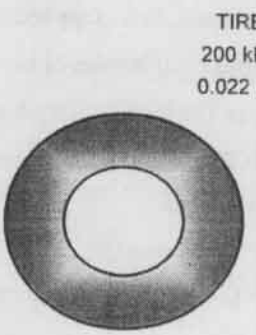
Assumptions **1** The volume of the tire remains constant. **2** Air is an ideal gas.

Properties The local atmospheric pressure is 90 kPa.

Analysis The absolute pressures in the tire before and after the trip are

$$P_1 = P_{\text{gage},1} + P_{\text{atm}} = 200 + 90 = 290 \text{ kPa}$$

$$P_2 = P_{\text{gage},2} + P_{\text{atm}} = 220 + 90 = 310 \text{ kPa}$$



Noting that air is an ideal gas and the volume is constant, the ratio of absolute temperatures after and before the trip are

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \rightarrow \quad \frac{T_2}{T_1} = \frac{P_2}{P_1} = \frac{310 \text{ kPa}}{290 \text{ kPa}} = 1.069$$

Therefore, the absolute temperature of air in the tire will increase by **6.9%** during this trip.

3-97 A hot air balloon with 3 people in its cage is hanging still in the air. The average temperature of the air in the balloon for two environment temperatures is to be determined.

Assumptions Air is an ideal gas.

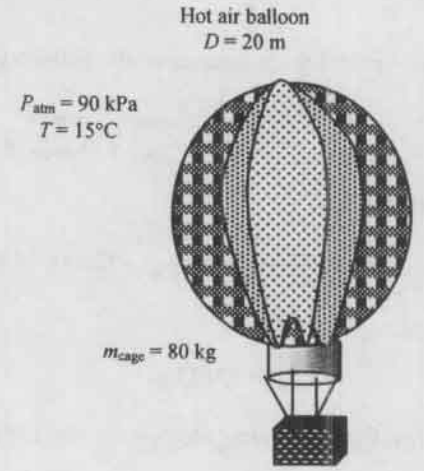
Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K.

Analysis The buoyancy force acting on the balloon is

$$V_{\text{balloon}} = 4\pi r^3/3 = 4\pi(10\text{m})^3/3 = 4189\text{m}^3$$

$$\rho_{\text{cool air}} = \frac{P}{RT} = \frac{90\text{kPa}}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(288\text{K})} = 1.089\text{kg/m}^3$$

$$F_B = \rho_{\text{cool air}} g V_{\text{balloon}}$$
$$= (1.089\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(4189\ \text{m}^3)\left(\frac{1\ \text{N}}{1\text{kg}\cdot\text{m/s}^2}\right) = 44{,}700\ \text{N}$$



The vertical force balance on the balloon gives

$$F_B = W_{\text{hot air}} + W_{\text{cage}} + W_{\text{people}}$$
$$= (m_{\text{hot air}} + m_{\text{cage}} + m_{\text{people}})g$$

Substituting,

$$44{,}700\ \text{N} = (m_{\text{hot air}} + 80\ \text{kg} + 195\ \text{kg})(9.8\ \text{m/s}^2)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

which gives

$$m_{\text{hot air}} = 4287\ \text{kg}$$

Therefore, the average temperature of the air in the balloon is

$$T = \frac{PV}{mR} = \frac{(90\ \text{kPa})(4189\ \text{m}^3)}{(4287\ \text{kg})(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})} = \mathbf{306.5\ K}$$

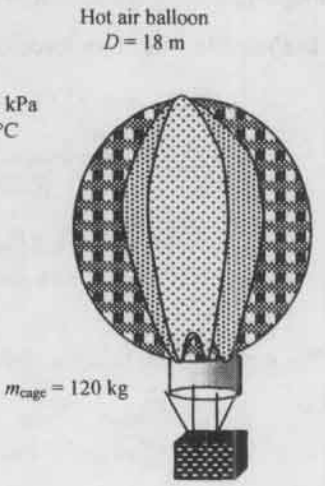
Repeating the solution above for an atmospheric air temperature of 30°C gives **323.6 K** for the average air temperature in the balloon.

3-98 A hot air balloon with 2 people in its cage is about to take off. The average temperature of the air in the balloon for two environment temperatures is to be determined.

Assumptions Air is an ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K.

Analysis The buoyancy force acting on the balloon is



$$V_{\text{balloon}} = 4\pi r^3/3 = 4\pi(9\text{ m})^3/3 = 3054\text{ m}^3$$

$$\rho_{\text{coolair}} = \frac{P}{RT} = \frac{93\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(285\text{ K})} = 1.137\text{ kg/m}^3$$

$$F_B = \rho_{\text{coolair}} g V_{\text{balloon}}$$
$$= (1.137\text{ kg/m}^3)(9.8\text{ m/s}^2)(3054\text{ m}^3)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 34{,}025\text{ N}$$

The vertical force balance on the balloon gives

$$F_B = W_{\text{hot air}} + W_{\text{cage}} + W_{\text{people}}$$
$$= (m_{\text{hot air}} + m_{\text{cage}} + m_{\text{people}})g$$

Substituting,

$$34{,}025\text{ N} = (m_{\text{hot air}} + 120\text{ kg} + 140\text{ kg})(9.8\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

which gives

$$m_{\text{hot air}} = 3212\text{ kg}$$

Therefore, the average temperature of the air in the balloon is

$$T = \frac{PV}{mR} = \frac{(93\text{ kPa})(3054\text{ m}^3)}{(3212\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})} = \mathbf{308\text{ K}}$$

Repeating the solution above for an atmospheric air temperature of 25°C gives **323 K** for the average air temperature in the balloon.

3-99E Water in a pressure cooker boils at 250°F. The absolute pressure in the pressure cooker is to be determined.

Analysis The absolute pressure in the pressure cooker is the saturation pressure that corresponds to the boiling temperature,

$$P = P_{\text{sat@250°F}} = \mathbf{29.82\ psia}$$



3-100 The refrigerant in a rigid tank is allowed to cool. The pressure at which the refrigerant starts condensing is to be determined, and the process is to be shown on a *P-v* diagram.

Analysis This is a constant volume process ($v = V/m$ = constant), and the specific volume is determined to be

$$\mathrm{v} = \frac{V}{m} = \frac{0.07\ \mathrm{m}^3}{1\ \mathrm{kg}} = 0.07\ \mathrm{m}^3/\mathrm{kg}$$

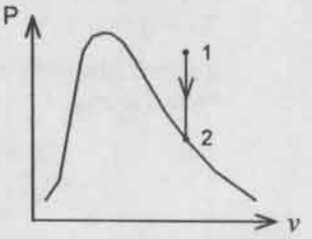


When the refrigerant starts condensing, the tank will contain saturated vapor only. Thus,

$$v_2 = v_g = 0.07\ \mathrm{m}^3/\mathrm{kg}$$

The pressure at this point is the pressure which corresponds to this v_g value,

$$P_2 = P_{\text{sat@}v_g=0.07\mathrm{m}^3/\mathrm{kg}} = \mathbf{0.29MPa}$$

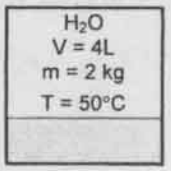


3-101 The rigid tank contains saturated liquid-vapor mixture of water. The mixture is heated until it exists in a single phase. For a given tank volume, it is to be determined if the final phase is a liquid or a vapor.

Analysis This is a constant volume process ($v = V/m$ = constant), and thus

the final specific volume will be equal to the initial specific volume,

$$v_2 = v_1$$



The critical specific volume of water is 0.003155 m^3/kg. Thus if the final specific volume is smaller than this value, the water will exist as a liquid, otherwise as a vapor.

$$V = 4\ L \longrightarrow v = \frac{V}{m} = \frac{0.004\ \text{m}^3}{2\ \text{kg}} = 0.002\ \text{m}^3/\text{kg} < v_{cr}. \quad \text{Thus, } \textbf{liquid}.$$

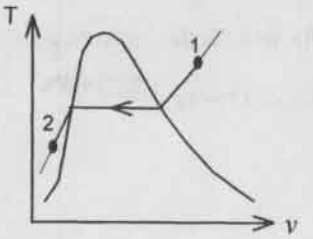
$$V = 400\ L \longrightarrow v = \frac{V}{m} = \frac{0.4\ \text{m}^3}{2\ \text{kg}} = 0.2\ \text{m}^3/\text{kg} > v_{cr}. \quad \text{Thus, } \textbf{vapor}.$$

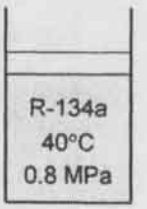
3-102 Superheated refrigerant-134a is cooled at constant pressure until it exists as a compressed liquid. The changes in total volume and internal energy are to be determined, and the process is to be shown on a T-v diagram. √

Analysis The refrigerant is a superheated vapor at the initial state and a compressed liquid at the final state. From Tables A-13 and A-11,

$$\left.\begin{matrix} P_1 = 0.8\text{MPa} \\ T_1 = 40^\circ\text{C} \end{matrix}\right\} \begin{matrix} u_1 = 252.13\text{kJ/kg} \\ v_1 = 0.02691\text{m}^3/\text{kg} \end{matrix}$$

$$\left.\begin{matrix} P_2 = 0.8\text{MPa} \\ T_2 = 20^\circ\text{C} \end{matrix}\right\} \begin{matrix} u_2 \cong u_{f@20^\circ\text{C}} = 76.80\text{kJ/kg} \\ v_2 \cong v_{f@20^\circ\text{C}} = 0.0008157\text{m}^3/\text{kg} \end{matrix}$$





Thus,

$$\Delta V = m(v_2 - v_1) = (10\ \text{kg})(0.0008157 - 0.02691)\ \text{m}^3/\text{kg} = \textbf{-0.261 m}^3$$

and

$$\Delta U = m(u_2 - u_1) = (10\ \text{kg})(76.80 - 252.13)\ \text{kJ/kg} = \textbf{-1753.3 kJ}$$

3-103 Two rigid tanks that contain hydrogen at two different states are connected to each other. Now a valve is opened, and the two gases are allowed to mix while achieving thermal equilibrium with the surroundings. The final pressure in the tanks is to be determined.

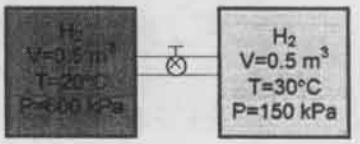
Analysis Let's call the first and the second tanks A and B. Treating H_2 as an ideal gas, the total volume and the total mass of H_2 are

$$V = V_A + V_B = 0.5 + 0.5 = 1.0\text{m}^3$$

$$m_A = \left(\frac{P_1 V}{RT_1}\right)_A = \frac{(600\text{kPa})(0.5\text{m}^3)}{(4.124\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{K})} = 0.248\ \text{kg}$$

$$m_B = \left(\frac{P_1 V}{RT_1}\right)_B = \frac{(150\text{kPa})(0.5\text{m}^3)}{(4.124\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(303\text{K})} = 0.060\ \text{kg}$$

$$m = m_A + m_B = 0.248 + 0.060 = 0.308\text{kg}$$



Then the final pressure can be determined from

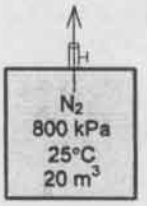
$$P = \frac{mRT_2}{V} = \frac{(0.308\ \text{kg})(4.124\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(288\ \text{K})}{1.0\ \text{m}^3} = \mathbf{365.8\ kPa}$$

3-104 A large tank contains nitrogen at a specified temperature and pressure. Now some nitrogen is allowed to escape, and the temperature and pressure of nitrogen drop to new values. The amount of nitrogen that has escaped is to be determined.

Analysis Treating N_2 as an ideal gas, the initial and the final masses in the tank are determined to be

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(800\ \text{kPa})(20\ \text{m}^3)}{(0.2968\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\ \text{K})} = 180.9\ \text{kg}$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(600\ \text{kPa})(20\ \text{m}^3)}{(0.2968\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\ \text{K})} = 138.0\ \text{kg}$$



Thus the amount of N_2 that escaped is

$$\Delta m = m_1 - m_2 = 180.9 - 138.0 = \mathbf{42.9\ kg}$$

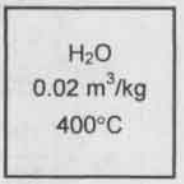
3-105 The temperature of steam in a tank at a specified state is to be determined using the ideal gas relation and the steam tables.

Properties The gas constant, the critical pressure, and the critical temperature of water are, from Table A-1,

$$R = 0.4615\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}, \quad T_{cr} = 647.3\ \text{K}, \quad P_{cr} = 22.09\ \text{MPa}$$

Analysis (a) From the ideal gas equation of state,

$$P = \frac{RT}{v} = \frac{(0.4615\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(673)}{0.02\ \text{m}^3/\text{kg}} = \mathbf{15{,}529\ kPa}$$



(*b*) From the superheated steam table (Table A-6),

$$\left.\begin{array}{l} T = 400^\circ\text{C} \\ v = 0.02\ \text{m}^3/\text{kg} \end{array}\right\} P = \mathbf{12{,}500\ kPa}$$

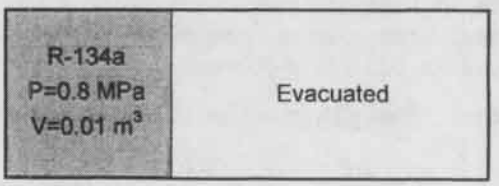
3-106 One section of a tank is filled with saturated liquid R-134a while the other side is evacuated. The partition is removed, and the temperature and pressure in the tank are measured. The volume of the tank is to be determined.

Analysis The mass of the refrigerant contained in the tank is

$$m = \frac{V_1}{v_1} = \frac{0.01\ \text{m}^3}{0.0008454\ \text{m}^3/\text{kg}} = 11.83\ \text{kg}$$

since

$$v_1 = v_{f@0.8\ \text{MPa}} = 0.0008454\ \text{m}^3/\text{kg}$$



At the final state (Table A-13),

$$\left.\begin{array}{l} P_2 = 200\ \text{kPa} \\ T_2 = 25^\circ\text{C} \end{array}\right\} v_2 = 0.11625\ \text{m}^3/\text{kg}$$

Thus,

$$V_{\text{tank}} = V_2 = mv_2 = (11.83\ \text{kg})(0.11625\ \text{m}^3/\text{kg}) = \mathbf{1.375\ m^3}$$

3-107 A propane tank contains 5 L of liquid propane at the ambient temperature. Now a leak develops at the top of the tank and propane starts to leak out. The temperature of propane when the pressure drops to 1 atm and the amount of heat transferred to the tank by the time the entire propane in the tank is vaporized are to be determined.

Properties The properties of propane at 1 atm are T_{sat} = -42.1°C, $\rho = 581\ \text{kg/m}^3$, and h_{fg} = 427.8 kJ/kg (Table A-3).

Analysis The temperature of propane when the pressure drops to 1 atm is simply the saturation pressure at that temperature,

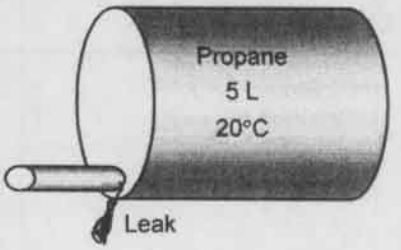
$$T = T_{sat\ @\ 1\ atm} = \mathbf{-42.1°C}$$

The initial mass of liquid propane is

$$m = \rho V = (581\ \text{kg/m}^3)(0.005\ \text{m}^3) = 2.905\ \text{kg}$$

The amount of heat absorbed is simply the total heat of vaporization,

$$Q_{absorbed} = mh_{fg} = (2.905\ \text{kg})(427.8\ \text{kJ/kg}) = \mathbf{1243\ kJ}$$



3-108 An isobutane tank contains 5 L of liquid isobutane at the ambient temperature. Now a leak develops at the top of the tank and isobutane starts to leak out. The temperature of isobutane when the pressure drops to 1 atm and the amount of heat transferred to the tank by the time the entire isobutane in the tank is vaporized are to be determined.

Properties The properties of isobutane at 1 atm are T_{sat} = -11.7°C, $\rho = 593.8\ \text{kg/m}^3$, and h_{fg} = 367.1 kJ/kg (Table A-3).

Analysis The temperature of isobutane when the pressure drops to 1 atm is simply the saturation pressure at that temperature,

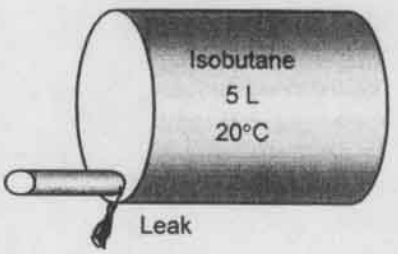
$$T = T_{sat\ @\ 1\ atm} = \mathbf{-11.7°C}$$

The initial mass of liquid isobutane is

$$m = \rho V = (593.8\ \text{kg/m}^3)(0.005\ \text{m}^3) = 2.969\ \text{kg}$$

The amount of heat absorbed is simply the total heat of vaporization,

$$Q_{absorbed} = mh_{fg} = (2.969\ \text{kg})(367.1\ \text{kJ/kg}) = \mathbf{1090\ kJ}$$



Computer, Design, and Essay Problems

3-110 A claim that fruits and vegetables are cooled by 6°C for each percentage point of weight loss as moisture during vacuum cooling is to be evaluated.

Analysis Assuming the fruits and vegetables are cooled from 30°C and 0°C, the average heat of vaporization can be taken to be 2466 kJ/kg, which is the value at 15°C, and the specific heat of products can be taken to be 4 kJ/kg.°C. Then the vaporization of 0.01 kg water will lower the temperature of 1 kg of produce by 24.66/4 = 6°C. Therefore, the vacuum cooled products will lose 1 percent moisture for each 6°C drop in temperature. Thus the claim is **reasonable**.

3-112 It is helium.



Chapter 4
ENERGY TRANSFER BY HEAT, WORK, AND MASS

Heat Transfer and Work

4-1C Energy can cross the boundaries of a closed system in two forms: heat and work.

4-2C The form of energy that crosses the boundary of a closed system because of a temperature difference is heat; all other forms are work.

4-3C An adiabatic process is a process during which there is no heat transfer. A system that does not exchange any heat with its surroundings is an adiabatic system.

4-4C It is a work interaction.

4-5C It is a work interaction since the electrons are crossing the system boundary, thus doing electrical work.

4-6C It is a heat interaction since it is due to the temperature difference between the sun and the room.

4-7C This is neither a heat nor a work interaction since no energy is crossing the system boundary. This is simply the conversion of one form of internal energy (chemical energy) to another form (sensible energy).

4-8C Point functions depend on the state only whereas the path functions depend on the path followed during a process. Properties of substances are point functions, heat and work are path functions.

Boundary Work

4-9C It represents the boundary work for quasi-equilibrium processes.

4-10C Yes.

4-11C The area under the process curve, and thus the boundary work done, is greater in the constant pressure case.

4-12C $1\,\text{kPa}\cdot\text{m}^3 = 1\,\text{k}(\text{N}/\text{m}^2)\cdot\text{m}^3 = 1\,\text{kN}\cdot\text{m} = 1\,\text{kJ}$

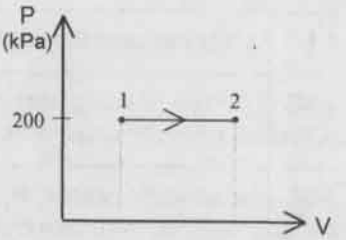
4-13 Saturated water vapor in a cylinder is heated at constant pressure until its temperature rises to a specified value. The boundary work done during this process is to be determined.

Assumptions The process is quasi-equilibrium.

Properties Noting that the pressure remains constant during this process, the specific volumes at the initial and the final states are (Table A-4 through A-6)

$$\left.\begin{array}{l} P_1 = 200\ \text{kPa} \\ \text{Sat.vapor} \end{array}\right\} v_1 = v_{g@200\text{kPa}} = 0.8857\ \text{m}^3/\text{kg}$$

$$\left.\begin{array}{l} P_2 = 200\ \text{kPa} \\ T_2 = 300^\circ\text{C} \end{array}\right\} v_2 = 1.3162\ \text{m}^3/\text{kg}$$



Analysis The boundary work is determined from its definition to be

$$W_{b,\text{out}} = \int_1^2 P\,dV = P(V_2 - V_1) = mP(v_2 - v_1)$$

$$= (5\text{kg})(200\text{kPa})(1.3162 - 0.8857)\,\text{m}^3/\text{kg}\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$

$$= \mathbf{430.5kJ}$$

Discussion The positive sign indicates that work is done by the system (work output).

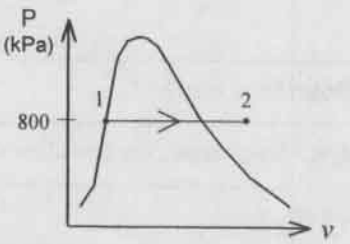
4-14 Refrigerant-134a in a cylinder is heated at constant pressure until its temperature rises to a specified value. The boundary work done during this process is to be determined.

Assumptions The process is quasi-equilibrium.

Properties Noting that the pressure remains constant during this process, the specific volumes at the initial and the final states are (Table A-11 through A-13)

$$\left.\begin{array}{l} P_1 = 800\ \text{kPa} \\ \text{Sat.liquid} \end{array}\right\} v_1 = v_{f@800\text{kPa}} = 0.0008454\text{m}^3/\text{kg}$$

$$\left.\begin{array}{l} P_2 = 800\text{kPa} \\ T_2 = 50^\circ\text{C} \end{array}\right\} v_2 = 0.02846\text{m}^3/\text{kg}$$



Analysis The boundary work is determined from its definition to be

$$m = \frac{V_1}{v_1} = \frac{0.2\ \text{m}^3}{0.0008454\ \text{m}^3/\text{kg}} = 236.6\ \text{kg}$$

and

$$W_{b,\text{out}} = \int_1^2 P\,dV = P(V_2 - V_1) = mP(v_2 - v_1)$$

$$= (236.6\text{kg})(800\text{kPa})(0.02846 - 0.0008454)\text{m}^3/\text{kg}\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$

$$= \mathbf{5227kJ}$$

Discussion The positive sign indicates that work is done by the system (work output).

4-15E Superheated water vapor in a cylinder is cooled at constant pressure until 70% of it condenses. The boundary work done during this process is to be determined.

Assumptions The process is quasi-equilibrium.

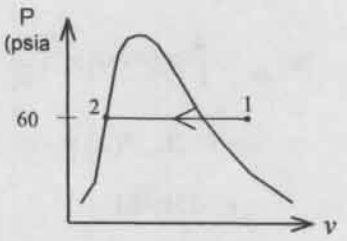
Properties Noting that the pressure remains constant during this process, the specific volumes at the initial and the final states are (Table A-4E through A-6E)

$$\left.\begin{matrix} P_1 = 60 \text{ psia} \\ T_1 = 500^\circ\text{F} \end{matrix}\right\} v_1 = 9.399 \text{ft}^3/\text{lbm}$$

$$\left.\begin{matrix} P_2 = 60 \text{ psia} \\ x_2 = 0.3 \end{matrix}\right\} v_2 = v_f + x_2 v_{fg}$$

$$= 0.017378 + 0.3(7.177 - 0.017378)$$

$$= 2.165 \text{ft}^3/\text{lbm}$$



Analysis The boundary work is determined from its definition to be

$$W_{b,\text{out}} = \int_1^2 P\,dV = P(V_2 - V_1) = mP(v_2 - v_1)$$

$$= (12 \text{ lbm})(60 \text{ psia})(2.165 - 9.399)\text{ft}^3/\text{lbm}\left(\frac{1 \text{ Btu}}{5.4039 \text{ psia} \cdot \text{ft}^3}\right)$$

$$= \mathbf{-963.8 \text{ Btu}}$$

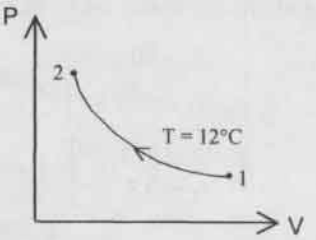
Discussion The negative sign indicates that work is done on the system (work input).

4-16 Air in a cylinder is compressed at constant temperature until its pressure rises to a specified value. The boundary work done during this process is to be determined.

Assumptions **1** The process is quasi-equilibrium. **2** Air is an ideal gas.

Properties The gas constant of air is $R = 0.287$ kJ/kg.K (Table A-1).

Analysis The boundary work is determined from its definition to be



$$W_{b,out} = \int_1^2 P\,dV = P_1V_1 \ln\frac{V_2}{V_1} = mRT\ln\frac{P_1}{P_2}$$

$$= (1.2\text{kg})(0.287\text{kJ/kg}\cdot\text{K})(285\text{K})\ln\frac{150\text{kPa}}{600\text{kPa}}$$

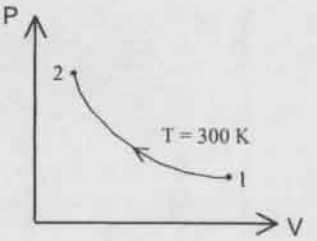
$$= \mathbf{-136.1kJ}$$

Discussion The negative sign indicates that work is done on the system (work input).

4-17 Nitrogen gas in a cylinder is compressed at constant temperature until its pressure rises to a specified value. The boundary work done during this process is to be determined.

Assumptions **1** The process is quasi-equilibrium. **2** Nitrogen is an ideal gas.

Analysis The boundary work is determined from its definition to be



$$W_{b,out} = \int_1^2 P\,dV = P_1V_1 \ln\frac{V_2}{V_1} = P_1V_1 \ln\frac{P_1}{P_2}$$

$$= (150\text{kPa})(0.2\text{m}^3)\left(\ln\frac{150\text{kPa}}{800\text{kPa}}\right)\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$

$$= \mathbf{-50.2kJ}$$

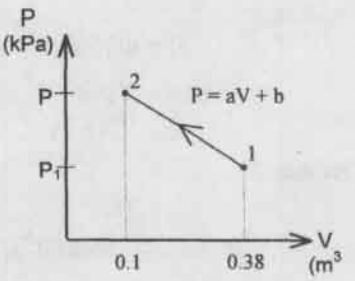
Discussion The negative sign indicates that work is done on the system (work input).

4-18 A gas in a cylinder is compressed to a specified volume in a process during which the pressure changes linearly with volume. The boundary work done during this process is to be determined by plotting the process on a P-V diagram and also by integration.

Assumptions The process is quasi-equilibrium.

Analysis (*a*) The pressure of the gas changes linearly with volume, and thus the process curve on a P-V diagram will be a straight line. The boundary work during this process is simply the area under the process curve, which is a trapezoidal. Thus,

$$P_1 = aV_1 + b = (-1200\ \text{kPa/m}^3)(0.42\ \text{m}^3) + (600\ \text{kPa}) = 96\ \text{kPa}$$
$$P_2 = aV_2 + b = (-1200\ \text{kPa/m}^3)(0.12\ \text{m}^3) + (600\ \text{kPa}) = 456\ \text{kPa}$$



and

$$W_{b,\text{out}} = \text{Area} = \frac{P_1 + P_2}{2}(V_2 - V_1)$$
$$= \frac{(96+456)\text{kPa}}{2}(0.12-0.42)\text{m}^3\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= \mathbf{-82.8kJ}$$

(*b*) The boundary work can also be determined by integration to be

$$W_{b,\text{out}} = \int_1^2 P\,dV = \int_1^2 (aV+b)\,dV = a\frac{V_2^2 - V_1^2}{2} + b(V_2 - V_1)$$
$$= (-1200\text{kPa/m}^3)\frac{(0.12^2 - 0.42^2)\text{m}^6}{2} + (600\text{kPa})(0.12-0.42)\text{m}^3$$
$$= \mathbf{-82.8kJ}$$



Discussion The negative sign indicates that work is done on the system (work input).

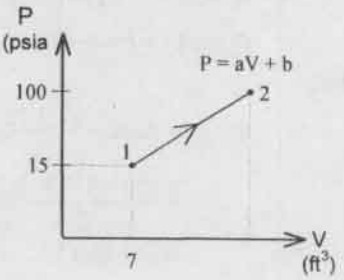
4-19E A gas in a cylinder is heated and is allowed to expand to a specified pressure in a process during which the pressure changes linearly with volume. The boundary work done during this process is to be determined.

Assumptions The process is quasi-equilibrium.

Analysis (*a*) The pressure of the gas changes linearly with volume, and thus the process curve on a P-V diagram will be a straight line. The boundary work during this process is simply the area under the process curve, which is a trapezoidal. Thus,

At state 1:

$$P_1 = aV_1 + b$$
$$15\ \text{psia} = (5\ \text{psia}/\text{ft}^3)(7\ \text{ft}^3) + b$$
$$b = -20\ \text{psia}$$



At state 2:

$$P_2 = aV_2 + b$$
$$100\ \text{psia} = (5\ \text{psia}/\text{ft}^3)V_2 + (-20\ \text{psia})$$
$$V_2 = 24\ \text{ft}^3$$

and,

$$W_{b,\text{out}} = \text{Area} = \frac{P_1 + P_2}{2}(V_2 - V_1) = \frac{(100+15)\text{psia}}{2}(24-7)\text{ft}^3\left(\frac{1\text{Btu}}{5.4039\text{psia}\cdot\text{ft}^3}\right)$$
$$= \mathbf{180.9Btu}$$

Discussion The positive sign indicates that work is done by the system (work output).

4-20 A gas in a cylinder expands polytropically to a specified volume. The boundary work done during this process is to be determined.

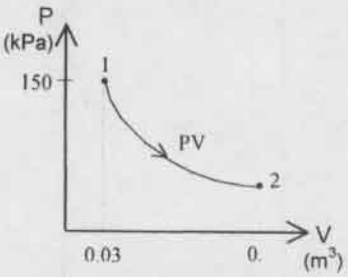
Assumptions The process is quasi-equilibrium.

Analysis The boundary work for this polytropic process can be determined directly from

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^n = (150\text{kPa})\left(\frac{0.03\text{m}^3}{0.2\text{m}^3}\right)^{1.3} = \mathbf{12.74kPa}$$

and,

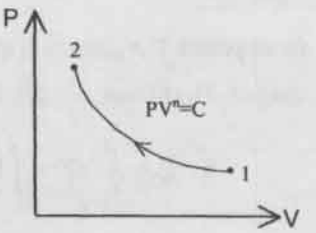
$$W_{b,out} = \int_1^2 P\,dV = \frac{P_2V_2 - P_1V_1}{1-n}$$
$$= \frac{(12.74\times0.2 - 150\times0.03)\text{kPa}\cdot\text{m}^3}{1-1.3}\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= \mathbf{6.51kJ}$$



Discussion The positive sign indicates that work is done by the system (work output).

4-21 Nitrogen gas in a cylinder is compressed polytropically until the temperature rises to a specified value. The boundary work done during this process is to be determined.

Assumptions **1** The process is quasi-equilibrium. **2** Nitrogen is an ideal gas.



Analysis The boundary work for this polytropic process can be determined from

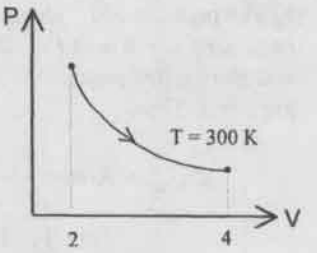
$$W_{b,\text{out}} = \int_1^2 P\,dV = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$
$$= \frac{(2\text{kg})(0.2968\text{kJ/kg}\cdot\text{K})(360-300)\text{K}}{1-1.4}$$
$$= \mathbf{-89.0\ kJ}$$

Discussion The negative sign indicates that work is done on the system (work input).

4-22 A gas whose equation of state is $\bar{v}(P+10/\bar{v}^2) = R_uT$ expands in a cylinder isothermally to a specified volume. The unit of the quantity 10 and the boundary work done during this process are to be determined.

Assumptions The process is quasi-equilibrium.

Analysis (*a*) The term $10/\bar{v}^2$ must have pressure units since it is added to P. Thus the quantity 10 must have the unit kPa·m^6/kmol^2.



(*b*) The boundary work for this process can be determined from

$$P = \frac{R_uT}{\bar{v}} - \frac{10}{\bar{v}^2} = \frac{R_uT}{V/N} - \frac{10}{(V/N)^2} = \frac{NR_uT}{V} - \frac{10N^2}{V^2}$$

and

$$W_{b,\text{out}} = \int_1^2 P\,dV = \int_1^2 \left(\frac{NR_uT}{V} - \frac{10N^2}{V^2}\right)dV = NR_uT\ln\frac{V_2}{V_1} + 10N^2\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$
$$= (0.5\text{kmol})(8.314\text{kJ/kmol}\cdot\text{K})(300\text{K})\ln\frac{4\text{m}^3}{2\text{m}^3}$$
$$+ (10\text{kPa}\cdot\text{m}^6/\text{kmol}^2)(0.5\text{kmol})^2\left(\frac{1}{4\text{m}^3} - \frac{1}{2\text{m}^3}\right)\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= \mathbf{863.8\text{kJ}}$$

Discussion The positive sign indicates that work is done by the system (work output).

4-23 CO_2 gas in a cylinder is compressed until the volume drops to a specified value. The pressure changes during the process with volume as $P = aV^{-2}$. The boundary work done during this process is to be determined.

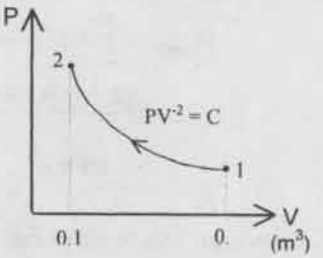
Assumptions The process is quasi-equilibrium.

Analysis The boundary work done during this process is determined from

$$W_{b,\text{out}} = \int_1^2 P\,dV = \int_1^2 \left(\frac{a}{V^2}\right) dV = -a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

$$= -(8\text{kPa}\cdot\text{m}^6)\left(\frac{1}{0.1\text{m}^3} - \frac{1}{0.3\text{m}^3}\right)\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$

$$= \mathbf{-53.3\text{kJ}}$$



Discussion The negative sign indicates that work is done on the system (work input).

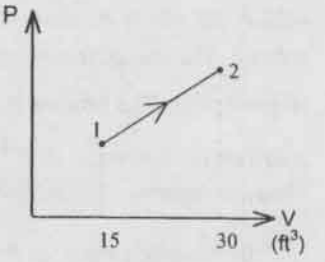
4-24E Hydrogen gas in a cylinder equipped with a spring is heated. The gas expands and compresses the spring until its volume doubles. The final pressure, the boundary work done by the gas, and the work done against the spring are to be determined, and a *P-V* diagram is to be drawn.

Assumptions **1** The process is quasi-equilibrium. **2** Hydrogen is an ideal gas.

Analysis (*a*) When the volume doubles, the spring force and the final pressure of H_2 becomes

$$F_s = kx_2 = k\frac{\Delta V}{A} = (15{,}000\text{lbf/ft})\frac{15\text{ft}^3}{3\text{ft}^2} = 75{,}000\text{lbf}$$

$$P_2 = P_1 + \frac{F_s}{A} = (14.7\text{psia}) + \frac{75{,}000\text{lbf}}{3\text{ft}^2}\left(\frac{1\text{ft}^2}{144\text{in}^2}\right) = \mathbf{188.3\ psia}$$



(*b*) The pressure of H_2 changes linearly with volume during this process, and thus the process curve on a *P-V* diagram will be a straight line. Then the boundary work during this process is simply the area under the process curve, which is a trapezoid. Thus,

$$W_{b,\text{out}} = \text{Area} = \frac{P_1 + P_2}{2}(V_2 - V_1)$$

$$= \frac{(188.3 + 14.7)\text{psia}}{2}(30 - 15)\text{ft}^3\left(\frac{1\text{Btu}}{5.4039\text{psia}\cdot\text{ft}^3}\right) = \mathbf{281.7\text{Btu}}$$

(*c*) If there were no spring, we would have a constant pressure process at P = 14.7 psia. The work done during this process would be

$$W_{b,\text{out,no spring}} = \int_1^2 P\,dV = P(V_2 - V_1)$$

$$= (14.7\text{psia})(30 - 15)\text{ft}^3\left(\frac{1\text{Btu}}{5.4039\text{psia}\cdot\text{ft}^3}\right) = \mathbf{40.8\ Btu}$$

Thus,

$$W_{\text{spring}} = W_b - W_{b,\text{no spring}} = 281.7 - 40.8 = \mathbf{240.9\ Btu}$$

Discussion The positive sign for boundary work indicates that work is done by the system (work output).

4-25 Water in a cylinder equipped with a spring is heated and evaporated. The vapor expands until it compresses the spring 20 cm. The final pressure and temperature, and the boundary work done are to be determined, and the process is to be shown on a *P-V* diagram. √

Assumptions The process is quasi-equilibrium.

Analysis (*a*) The final pressure is determined from

$$P_3 = P_2 + \frac{F_s}{A} = P_2 + \frac{kx}{A} = (150\text{kPa}) + \frac{(100\text{kN/m})(0.2\text{m})}{0.1\text{m}^2}\left(\frac{1\text{kPa}}{1\text{kN/m}^2}\right) = \mathbf{350kPa}$$

The specific and total volumes at the three states are

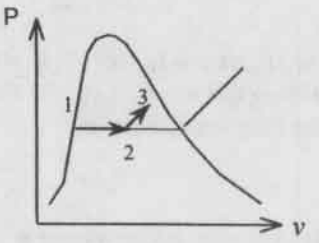
$$\left.\begin{array}{l} T_1 = 25\,^\circ\text{C} \\ P_1 = 150\text{kPa} \end{array}\right\} v_1 \cong v_{f@25^\circ\text{C}} = 0.001003\text{m}^3/\text{kg}$$

$$V_1 = mv_1 = (50\text{kg})(0.001003\text{m}^3/\text{kg}) = 0.05\text{m}^3$$

$$V_2 = 0.2\text{m}^3$$

$$V_3 = V_2 + x_{23}A_p = (0.2\text{m}^3) + (0.2\text{m})(0.1\text{m}^2) = 0.22\text{m}^3$$

$$v_3 = \frac{V_3}{m} = \frac{0.22\text{m}^3}{50\text{kg}} = 0.0044\ \text{m}^3/\text{kg}$$



At 350 kPa, $v_f = 0.0010$ m^3/kg and $v_g = 0.5243$ m^3/kg. Noting that $v_f < v_3 < v_g$, the final state is a saturated mixture and thus the final temperature is

$$T_3 = T_{\text{sat@350kPa}} = \mathbf{138.88\,^\circ\text{C}}$$

(*b*) The pressure remains constant during process 1-2 and changes linearly (a straight line) during process 2-3. Then the boundary work during this process is simply the total area under the process curve,

$$W_{b,\text{out}} = \text{Area} = P_1(V_2 - V_1) + \frac{P_2 + P_3}{2}(V_3 - V_2)$$

$$= \left((150\text{kPa})(0.2 - 0.05)\text{m}^3 + \frac{(150 + 350)\text{kPa}}{2}(0.22 - 0.2)\text{m}^3\right)\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$

$$= \mathbf{27.5kJ}$$

Discussion The positive sign indicates that work is done by the system (work output).

4-26 Refrigerant-134a in a cylinder equipped with a set of stops is heated and evaporated. The vapor expands until the piston hits the stops. The final temperature, and the boundary work done are to be determined, and the process is to be shown on a *P-V* diagram.

Assumptions The process is quasi-equilibrium.

Analysis (a) This a constant pressure process. Initially the system contains a saturated mixture, and thus the pressure is

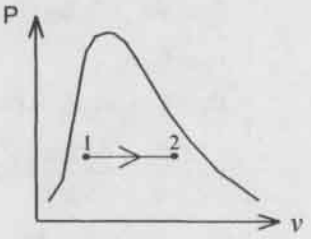
$$P_2 = P_1 = P_{sat@-8^\circ C} = 217.04 \text{ kPa}$$

The specific volume of the refrigerant at the final state is

$$v_2 = \frac{V_2}{m} = \frac{0.4 \text{ m}^3}{10 \text{ kg}} = 0.04 \text{ m}^3/\text{kg}$$

At 217.04 kPa (or -8 °C), $v_f = 0.0007569$ m^3/kg and $v_g = 0.0919$ m^3/kg. Noting that $v_f < v_2 < v_g$, the final state is a saturated mixture and thus the final temperature is

$$T_2 = T_{\text{sat@217.04kPa}} = \mathbf{-8^\circ C}$$



(*b*) The total initial volume is

$$V_1 = m_f v_f + m_g v_g = 8 \times 0.0007569 + 2 \times 0.0919 = 0.19 \text{ m}^3$$

Thus,

$$W_{b,\text{out}} = \int_1^2 P\,dV = P(V_2 - V_1)$$

$$= (217.04\text{kPa})(0.4 - 0.19)\text{m}^3 \left(\frac{1\text{kJ}}{1\text{kPa} \cdot \text{m}^3} \right)$$

$$= \mathbf{45.6kJ}$$

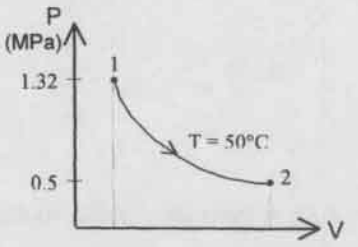
Discussion The positive sign indicates that work is done by the system (work output).

4-27 Saturated refrigerant-134a vapor in a cylinder is allowed to expand isothermally by gradually decreasing the pressure inside to 500 kPa. The boundary work done during this process is to be determined by using property data from the refrigerant tables, and by treating the refrigerant vapor as an ideal gas.

Assumptions The process is quasi-equilibrium.

Analysis From the refrigerant tables, the specific volume of the refrigerant at various pressures at 50°C are determined to be

P, MPa	v, m^3/kg
1.320	0.01503
1.200	0.01712
1.0	0.02171
0.9	0.02472
0.8	0.02846
0.7	0.03324
0.6	0.03958
0.5	0.04842



Plotting these on a P-V diagram and finding the area under the process curve, the boundary work during this isothermal process is determined to be

$$W_b = \mathbf{262.9\ kJ}$$

(*b*) Treating the refrigerant as an ideal gas, the boundary work for this isothermal process can be determined from

$$W_{b,\text{out}} = \int_1^2 P\,dV = P_1V_1 \ln\frac{V_2}{V_1} = mRT\ln\frac{P_1}{P_2}$$

$$= (10\text{kg})(0.08149\text{kJ/kg}\cdot\text{K})(323\text{K})\ln\frac{1320\text{kPa}}{500\text{kPa}}$$

$$= \mathbf{255.5kJ}$$

which is sufficiently close to the experimental value.

Discussion The positive sign indicates that work is done by the system (work output).

4-28 Several sets of pressure and volume data are taken as a gas expands. The boundary work done during this process is to be determined using the experimental data.

Assumptions The process is quasi-equilibrium.

Analysis Plotting the given data on a *P-V* diagram on a graph paper and evaluating the area under the process curve, the work done is determined to be **0.25 kJ**.

Other Forms of Work

4-29C The work done is the same, but the power is different.

4-30C The work done is the same, but the power is different.

4-31 A car is accelerated from rest to 100 km/h. The work needed to achieve this is to be determined.

Analysis The work needed to accelerate a body the change in kinetic energy of the body,

$$W_a = \frac{1}{2}m\left(\mathbf{V}_2^2 - \mathbf{V}_1^2\right) = \frac{1}{2}(800\text{kg})\left(\left(\frac{100{,}000\text{m}}{3600\text{s}}\right)^2 - 0\right)\left(\frac{1\text{kJ}}{1000\text{kg}\cdot\text{m}^2/\text{s}^2}\right) = \mathbf{308.6kJ}$$

4-32 A car is accelerated from 20 to 70 km/h on an uphill road. The work needed to achieve this is to be determined. √

Analysis The total work required is the sum of the changes in potential and kinetic energies,

$$W_a = \frac{1}{2}m\left(\mathbf{V}_2^2 - \mathbf{V}_1^2\right) = \frac{1}{2}(2000\text{kg})\left(\left(\frac{70{,}000\text{m}}{3600\text{s}}\right)^2 - \left(\frac{20{,}000\text{m}}{3600\text{s}}\right)^2\right)\left(\frac{1\text{kJ}}{1000\text{kg}\cdot\text{m}^2/\text{s}^2}\right) = 347.2\text{kJ}$$

and,

$$W_g = mg\left(z_2 - z_1\right) = (2000\text{kg})(9.807\text{m/s}^2)(40\text{m})\left(\frac{1\text{kJ}}{1000\text{kg}\cdot\text{m}^2/\text{s}^2}\right) = 784.6\text{kJ}$$

Thus,

$$W_{\text{total}} = W_a + W_g = 347.2 + 784.6 = \mathbf{1131.8kJ}$$



4-33E A engine of a car develops 450 hp at 3000 rpm. The torque transmitted through the shaft is to be determined. √

Analysis The torque is determined from

$$T = \frac{\dot{W}_{sh}}{2\pi \dot{n}} = \frac{450 \text{ hp}}{2\pi(3000/60)/\text{s}}\left(\frac{550 \text{ lbf} \cdot \text{ft/s}}{1 \text{ hp}}\right) = \mathbf{787.8 \text{ lbf} \cdot \text{ft}}$$

4-34 A linear spring is elongated by 20 cm from its rest position. The work done is to be determined.

Analysis The spring work can be determined from

$$W_{spring} = \frac{1}{2}k\left(x_2^2 - x_1^2\right) = \frac{1}{2}(70\text{kN/m})(0.2^2 - 0)\text{m}^2 = 2.8\text{kN} \cdot \text{m} = \mathbf{2.8kJ}$$

4-35 The engine of a car develops 75 kW of power. The acceleration time of this car from rest to 85 km/h on a level road is to be determined.

Analysis The work needed to accelerate a body is the change in its kinetic energy,

$$W_a = \frac{1}{2}m\left(\mathbf{V}_2^2 - \mathbf{V}_1^2\right) = \frac{1}{2}(1500\text{kg})\left(\left(\frac{85{,}000\text{m}}{3600\text{s}}\right)^2 - 0\right)\left(\frac{1\text{kJ}}{1000\text{kg} \cdot \text{m}^2/\text{s}^2}\right) = 418.1\text{kJ}$$

Thus the time required is

$$\Delta t = \frac{W_a}{\dot{W}_a} = \frac{418.1\text{ kJ}}{75\text{ kJ/s}} = \mathbf{5.57\text{ s}}$$



This answer is not realistic because part of the power will be used against the air drag, friction, and rolling resistance.

4-36 A ski lift is operating steadily at 10 km/h. The power required to operate and also to accelerate this ski lift from rest to the operating speed are to be determined. √

Assumptions **1** Air drag and friction are negligible. **2** The average mass of each loaded chair is 250 kg. **3** The mass of chairs is small relative to the mass of people, and thus the contribution of returning empty chairs to the motion is disregarded (this provides a safety factor).

Analysis The lift is 1000 m long and the chairs are spaced 20 m apart. Thus at any given time there are 1000/20 = 50 chairs being lifted. Considering that the mass of each chair is 250 kg, the load of the lift at any given time is

$$\text{Load} = (50 \text{ chairs})(250 \text{ kg/chair}) = 12{,}500 \text{ kg}$$

Neglecting the work done on the system by the returning empty chairs, the work needed to raise this mass by 200 m is

$$W_g = mg(z_2 - z_1) = (12{,}500 \text{ kg})(9.81 \text{ m/s}^2)(200 \text{ m})\left(\frac{1 \text{ kJ}}{1000 \text{ kg}\cdot\text{m}^2/\text{s}^2}\right) = 24{,}525 \text{ kJ}$$

At 10 km/h, it will take

$$\Delta t = \frac{\text{distance}}{\text{velocity}} = \frac{1 \text{ km}}{10 \text{ km/h}} = 0.1 \text{ h} = 360 \text{ s}$$

to do this work. Thus the power needed is

$$\dot{W}_g = \frac{W_g}{\Delta t} = \frac{24{,}525 \text{ kJ}}{360 \text{ s}} = \mathbf{68.1 \text{ kW}}$$

The velocity of the lift during steady operation, and the acceleration during start up are

$$\mathbf{V} = (10 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 2.778 \text{ m/s}$$

$$a = \frac{\Delta \mathbf{V}}{\Delta t} = \frac{2.778 \text{ m/s} - 0}{5 \text{ s}} = 0.556 \text{ m/s}^2$$

During acceleration, the power needed is

$$\dot{W}_a = \frac{1}{2} m\left(\mathbf{V}_2^2 - \mathbf{V}_1^2\right)/\Delta t = \frac{1}{2}(12{,}500 \text{ kg})\left((2.778 \text{ m/s})^2 - 0\right)\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right)/(5 \text{ s}) = 9.6 \text{ kW}$$

Assuming the power applied is constant, the acceleration will also be constant and the vertical distance traveled during acceleration will be

$$h = \frac{1}{2} at^2 \sin\alpha = \frac{1}{2} at^2 \frac{200\text{m}}{1000\text{m}} = \frac{1}{2}(0.556 \text{ m/s}^2)^2 (5 \text{ s})^2 (0.2) = 1.39 \text{ m}$$

and

$$\dot{W}_g = mg(z_2 - z_1)/\Delta t = (12{,}500 \text{ kg})(9.81 \text{ m/s}^2)(1.39 \text{ m})\left(\frac{1 \text{ kJ/kg}}{1000 \text{ kg}\cdot\text{m}^2/\text{s}^2}\right)/(5 \text{ s}) = 34.1 \text{ kW}$$

Thus,

$$\dot{W}_{\text{total}} = \dot{W}_a + \dot{W}_g = 9.6 + 34.1 = \mathbf{43.7 \text{ kW}}$$

4-37 A car is to climb a hill in 10 s. The power needed is to be determined for three different cases.

Assumptions Air drag, friction, and rolling resistance are negligible.

Analysis The total power required for each case is the sum of the rates of changes in potential and kinetic energies. That is,

$$\dot{W}_{\text{total}} = \dot{W}_a + \dot{W}_g$$

(*a*) $\dot{W}_a = 0$ since the velocity is constant. Also, the vertical rise is h = (100 m)(sin 30°) = 50 m. Thus,

$$\dot{W}_g = mg(z_2 - z_1)/\Delta t = (2000\text{kg})(9.8\text{m/s}^2)(50\text{m})\left(\frac{1\text{kJ}}{1000\text{kg}\cdot\text{m}^2/\text{s}^2}\right)/(10\text{s}) = 98.07\text{kW}$$

and

$$\dot{W}_{\text{total}} = \dot{W}_a + \dot{W}_g = 0 + 98.07 = \mathbf{98.07kW}$$

(*b*) The power needed to accelerate is

$$\dot{W}_a = \frac{1}{2}m\left(\mathbf{V}_2^2 - \mathbf{V}_1^2\right)/\Delta t = \frac{1}{2}(2000\text{kg})\left((30\text{m/s})^2 - 0\right)\left(\frac{1\text{kJ}}{1000\text{kg}\cdot\text{m}^2/\text{s}^2}\right)/(10\text{s}) = 90\text{kW}$$

and

$$\dot{W}_{\text{total}} = \dot{W}_a + \dot{W}_g = 90 + 98.07 = \mathbf{188.07kW}$$

(*c*) The power needed to accelerate is

$$\dot{W}_a = \frac{1}{2}m\left(\mathbf{V}_2^2 - \mathbf{V}_1^2\right)/\Delta t = \frac{1}{2}(2000\text{kg})\left((5\text{m/s})^2 - (35\text{m/s})^2\right)\left(\frac{1\text{kJ}}{1000\text{kg}\cdot\text{m}^2/\text{s}^2}\right)/(10\text{s}) = -120\text{kW}$$

and

$$\dot{W}_{\text{total}} = \dot{W}_a + \dot{W}_g = -120 + 98.07 = \mathbf{-21.93\ kW}$$



4-38 A damaged car is being towed by a truck. The extra power needed is to be determined for three different cases. √

Assumptions Air drag, friction, and rolling resistance are negligible.

Analysis The total power required for each case is the sum of the rates of changes in potential and kinetic energies. That is,

$$\dot{W}_{\text{total}} = \dot{W}_a + \dot{W}_g$$



(*a*) Zero.

(*b*) $\dot{W}_a = 0$. Thus,

$$\dot{W}_{\text{total}} = \dot{W}_g = mg(z_2 - z_1)/\Delta t = mg\frac{\Delta z}{\Delta t} = mg\mathbf{V}_z = mg\mathbf{V}\sin 30^\circ$$

$$= (1200\text{kg})(9.8\text{m/s}^2)\left(\frac{50{,}000\text{m}}{3600\text{s}}\right)\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right)(0.5) = \mathbf{81.7kW}$$

(*c*) $\dot{W}_g = 0$. Thus,

$$\dot{W}_{\text{total}} = \dot{W}_a = \frac{1}{2}m\left(\mathbf{V}_2^2 - \mathbf{V}_1^2\right)/\Delta t$$

$$= \frac{1}{2}(1200\text{kg})\left(\left(\frac{90{,}000\text{m}}{3600\text{s}}\right)^2 - 0\right)\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right)/(12\text{s}) = \mathbf{31.25kW}$$

Flow Work and Energy Transfer by Mass

4-39C Energy can be transferred to or from a control volume as heat, various forms of work, and by mass.

4-40C Flow energy or flow work is the energy needed to push a fluid into or out of a control volume. Fluids at rest do not possess any flow energy.

4-41C Flowing fluids possess flow energy in addition to the forms of energy a fluid at rest possesses. The total energy of a fluid at rest consists of internal, kinetic, and potential energies. The total energy of a flowing fluid consists of internal, kinetic, potential, and flow energies.

4-42E Steam is leaving a pressure cooker at a specified pressure. The velocity, flow rate, the total and flow energies, and the rate of energy transfer by mass are to be determined.

Assumptions **1** The flow is steady, and the initial start-up period is disregarded. **2** The kinetic and potential energies are negligible, and thus they are not considered. **3** Saturation conditions exist within the cooker at all times so that steam leaves the cooker as a saturated vapor at 30 psia.

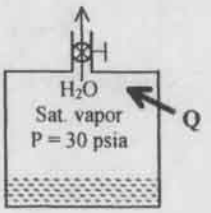
Properties The properties of saturated liquid water and water vapor at 30 psia are $v_f = 0.017004$ ft^3/lbm, $v_g = 13.748$ ft^3/lbm, $u_g = 1088.0$ Btu/lbm, and $h_g = 1164.3$ Btu/lbm (Table A-5E).

Analysis (*a*) Saturation conditions exist in a pressure cooker at all times after the steady operating conditions are established. Therefore, the liquid has the properties of saturated liquid and the exiting steam has the properties of saturated vapor at the operating pressure. The amount of liquid that has evaporated, the mass flow rate of the exiting steam, and the exit velocity are

$$m = \frac{\Delta V_{\text{liquid}}}{v_f} = \frac{0.4\text{ gal}}{0.017004\text{ ft}^3/\text{lbm}}\left(\frac{0.13368\text{ ft}^3}{1\text{ gal}}\right) = 3.145\text{ lbm}$$

$$\dot{m} = \frac{m}{\Delta t} = \frac{3.145\text{ lbm}}{45\text{ min}} = 0.0699\text{ lbm/min} = \mathbf{1.165\times10^{-3}\ lbm/s}$$

$$\mathbf{V} = \frac{\dot{m}}{\rho_g A_c} = \frac{\dot{m}v_g}{A_c} = \frac{(1.65\times10^{-3}\text{ lbm/s})(13.748\text{ ft}^3/\text{lbm})}{0.15\text{ in}^2}\left(\frac{144\text{ in}^2}{1\text{ ft}^2}\right) = \mathbf{21.8\ ft/s}$$



(*b*) Noting that $h = u + Pv$ and that the kinetic and potential energies are disregarded, the flow and total energies of the exiting steam are

$$e_{\text{flow}} = Pv = h - u = 1164.3 - 1088.0 = \mathbf{76.3\ Btu/lbm}$$

$$\theta = h + ke + pe \cong h = \mathbf{1164.3\ Btu/lbm}$$

Note that the kinetic energy in this case is ke = $\mathbf{V}^2/2$ = (21.8 ft/s)^2 = 475 ft^2/s^2 = 0.019 Btu/lbm, which is very small compared to enthalpy.

(*c*) The rate at which energy is leaving the cooker by mass is simply the product of the mass flow rate and the total energy of the exiting steam per unit mass,

$$\dot{E}_{\text{mass}} = \dot{m}\theta = (1.165\times10^{-3}\text{ lbm/s})(1164.3\text{ Btu/lbm}) = \mathbf{1.356\ Btu/s}$$

Discussion The numerical value of the energy leaving the cooker with steam alone does not mean much since this value depends on the reference point selected for enthalpy (it could even be negative). The significant quantity is the difference between the enthalpies of the exiting vapor and the liquid inside (which is h_{fg}) since it relates directly to the amount of energy supplied to the cooker.

4-43 Refrigerant-134a enters a compressor as a saturated vapor at a specified pressure, and leaves as superheated vapor at a specified rate. The rates of energy transfer by mass into and out of the compressor are to be determined.

Assumptions **1** The flow of the refrigerant through the compressor is steady. **2** The kinetic and potential energies are negligible, and thus they are not considered.

Properties The enthalpy of refrigerant-134a at the inlet and the exit are (Tables A-12 and A-13)

$$h_1 = h_{g@0.14\text{MPa}} = 236.04\text{kJ/kg}$$

$$\left.\begin{array}{l} P_2 = 0.14\text{MPa} \\ T_2 = 50°\text{C} \end{array}\right\} h_2 = 284.36\text{kJ/kg}$$

Analysis Noting that the total energy of a flowing fluid is equal to its enthalpy when the kinetic and potential energies are negligible, and that the rate of energy transfer by mass is equal to the product of the mass flow rate and the total energy of the fluid per unit mass, the rates of energy transfer by mass into and out of the compressor are

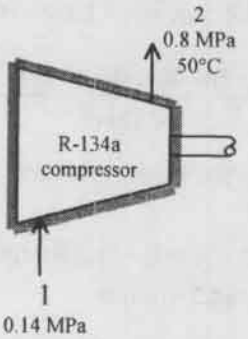
$$\dot{E}_{\text{mass, in}} = \dot{m}\theta_{in} = \dot{m}h_1 = (0.04\text{ kg/s})(236.04\text{ kJ/kg}) = 9.442\text{ kJ/s} = \mathbf{9.44\,kW}$$

$$\dot{E}_{\text{mass, out}} = \dot{m}\theta_{out} = \dot{m}h_2 = (0.04\text{ kg/s})(284.36\text{ kJ/kg}) = 11.37\text{ kJ/s} = \mathbf{11.37\,kW}$$

Discussion The numerical values of the energy entering or leaving a device by mass alone does not mean much since this value depends on the reference point selected for enthalpy (it could even be negative). The significant quantity here is the difference between the outgoing and incoming energy flow rates, which is

$$\Delta\dot{E}_{\text{mass}} = \dot{E}_{\text{mass, out}} - \dot{E}_{\text{mass, in}} = 11.37 - 9.44 = 1.93\,\text{kW}$$

This quantity represents the rate of energy transfer to the refrigerant in the compressor.



4-44 Warm air in a house is forced to leave by the infiltrating cold outside air at a specified rate. The net energy loss due to mass transfer is to be determined.

Assumptions **1** The flow of the air into and out of the house through the cracks is steady. **2** The kinetic and potential energies are negligible. **3** Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is $R = 0.287$ kPa·m^3/kg·K (Table A-1). The constant pressure specific heat of air at room temperature is $C_p = 1.005$ kJ/kg·°C (Table A-2).

Analysis The density of air at the indoor conditions and its mass flow rate are

$$\rho = \frac{P}{RT} = \frac{101.325\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(24+273)\text{K}} = 1.189\,\text{kg/m}^3$$

$$\dot{m} = \rho\dot{V} = (1.189\,\text{kg/m}^3)(150\,\text{m}^3/\text{h}) = 178.35\,\text{kg/h} = 0.0495\,\text{kg/s}$$

Noting that the total energy of a flowing fluid is equal to its enthalpy when the kinetic and potential energies are negligible, and that the rate of energy transfer by mass is equal to the product of the mass flow rate and the total energy of the fluid per unit mass, the rates of energy transfer by mass into and out of the house by air re

$$\dot{E}_{\text{mass, in}} = \dot{m}\theta_{in} = \dot{m}h_1$$

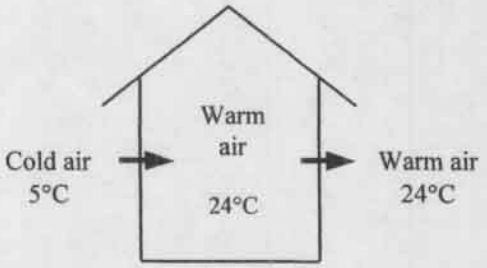
$$\dot{E}_{\text{mass, out}} = \dot{m}\theta_{out} = \dot{m}h_2$$

The net energy loss by air infiltration is equal to the difference between the outgoing and incoming energy flow rates, which is

$$\begin{aligned}\Delta\dot{E}_{\text{mass}} &= \dot{E}_{\text{mass, out}} - \dot{E}_{\text{mass, in}} = \dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)\\ &= (0.0495\,\text{kg/s})(1.005\,\text{kJ/kg}\cdot\text{°C})(24-5)\text{°C} = 0.945\,\text{kJ/s}\\ &= \mathbf{0.945\,kW}\end{aligned}$$

This quantity represents the rate of energy transfer to the refrigerant in the compressor.

Discussion The rate of energy loss by infiltration will be less in reality since some air will leave the house before it is fully heated to 24°C.



Review Problems

4-45 The weight of the cabin of an elevator is balanced by a counterweight. The power needed when the fully loaded cabin is rising , and when the empty cabin is descending at a constant speed are to be determined. √

Assumptions **1** The weight of the cables is negligible. **2** The guide rails and pulleys are frictionless. **3** Air drag is negligible.

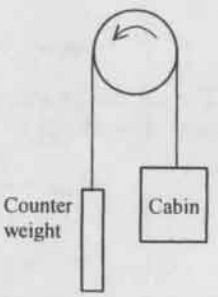
Analysis (*a*) When the cabin is fully loaded, half of the weight is balanced by the counterweight. The power required to raise the cabin at a constant speed of 2 m/s is

$$\dot{W} = \frac{mgz}{\Delta t} = mg\mathbf{V} = (400\text{kg})(9.8\text{m/s}^2)(2\text{m/s})\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{kW}}{1000\text{N}\cdot\text{m/s}}\right) = \mathbf{7.84kW}$$

If no counterweight is used, the mass would double to 800 kg and the power would be 2×7.84 = **15.68 kW**.

(*b*) When the empty cabin is descending (and the counterweight is ascending) there is mass imbalance of 400-150 = 250 kg. The power required to raise this mass at a constant speed of 2 m/s is

$$\dot{W} = \frac{mgz}{\Delta t} = mg\mathbf{V} = (250\text{ kg})(9.81\text{ m/s}^2)(2\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right) = \mathbf{4.9\ kW}$$



If a friction force of 1200 N develops between the cabin and the guide rails, we will need

$$\dot{W}_{\text{friction}} = \frac{F_{\text{friction}}\, z}{\Delta t} = F_{\text{friction}}\,\mathbf{V} = (1200\text{ N})(2\text{ m/s})\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right) = 2.4\text{ kW}$$

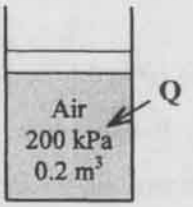
of additional power to combat friction which always acts in the opposite direction to motion. Therefore, the total power needed in this case is

$$\dot{W}_{\text{total}} = \dot{W} + \dot{W}_{\text{friction}} = 4.9 + 2.4 = \mathbf{7.3kW}$$

4-46 A cylinder equipped with an external spring is initially filled with air at a specified state. Heat is transferred to the air, and both the temperature and pressure rise. The total boundary work done by the air, and the amount of work done against the spring are to be determined, and the process is to be shown on a *P-v* diagram.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are negligible. **2** The thermal energy stored in the cylinder itself is negligible. **3** The compression or expansion process is quasi-equilibrium. **4** The spring is a linear spring.

Analysis (*a*) We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. Noting that the spring is not part of the system (it is external), the energy balance for this stationary closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} - W_{b,out} = \Delta U = m(u_2 - u_1) \quad \text{(since KE = PE = 0)}$$

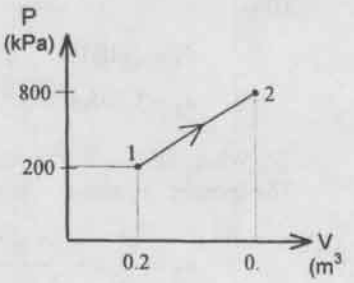
$$Q_{in} = m(u_2 - u_1) + W_{b,out}$$

The pressure of the gas changes linearly with volume during this process, and thus the process curve on a P-V diagram will be a straight line. Then the boundary work during this process is simply the area under the process curve, which is a trapezoidal. Thus,

$$W_{b,out} = Area = \frac{P_1 + P_2}{2}(V_2 - V_1)$$

$$= \frac{(200 + 800)\text{kPa}}{2}(0.5 - 0.2)\text{m}^3\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$

$$= \mathbf{150kJ}$$



(*b*) If there were no spring, we would have a constant pressure process at P = 200 kPa. The work done during this process is

$$W_{\text{b,out,nospring}} = \int_1^2 P\,dV = P(V_2 - V_1) = (200\text{kPa})(0.5 - 0.2)\text{m}^3/\text{kg}\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = 60\text{kJ}$$

Thus,

$$W_{\text{spring}} = W_b - W_{\text{b,nospring}} = 150 - 60 = \mathbf{90kJ}$$

4-47 A cylinder equipped with a set of stops for the piston is initially filled with saturated liquid-vapor mixture of water a specified pressure. Heat is transferred to the water until the volume increases by 20%. The initial and final temperature, the mass of the liquid when the piston starts moving, and the work done during the process are to be determined, and the process is to be shown on a *P-v* diagram.

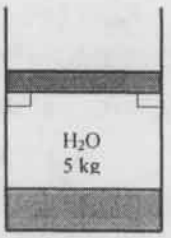
Assumptions **1** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **2** The thermal energy stored in the cylinder itself is negligible. **3** The compression or expansion process is quasi-equilibrium.

Analysis (*a*) Initially the system is a saturated mixture at 100 kPa pressure, and thus the initial temperature is

$$T_1 = T_{sat@100\,\text{kPa}} = \mathbf{99.63^\circ C}$$

The total initial volume is

$$V_1 = m_f v_f + m_g v_g = 2 \times 0.001044 + 3 \times 1.6729 = 5.02\ \text{m}^3$$



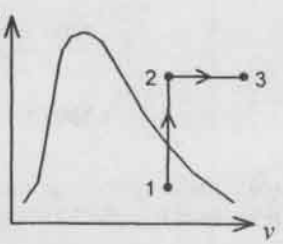
Then the total and specific volumes at the final state are

$$V_3 = 1.2V_1 = 1.2 \times 5.02 = 6.025\ \text{m}^3$$

$$v_3 = \frac{V_3}{m} = \frac{6.025\ \text{m}^3}{5\ \text{kg}} = 1.205\ \text{m}^3/\text{kg}$$

Thus,

$$\left.\begin{array}{l} P_3 = 200\text{kPa} \\ v_3 = 1.205\text{m}^3/\text{kg} \end{array}\right\} T_3 = \mathbf{252.6^\circ C}$$



(*b*) When the piston first starts moving, $P_2 = 200$ kPa and $V_2 = V_1 = 5.02\ \text{m}^3$. The specific volume at this state is

$$v_2 = \frac{V_2}{m} = \frac{5.02\ \text{m}^3}{5\ \text{kg}} = 1.004\ \text{m}^3/\text{kg}$$

which is greater than $v_g = 0.8857\ \text{m}^3/\text{kg}$ at 200 kPa. Thus **no liquid** is left in the cylinder when the piston starts moving.

(*c*) No work is done during process 1-2 since $V_1 = V_2$. The pressure remains constant during process 2-3 and the work done during this process is

$$W_b = \int_2^3 P\,dV = P_2(V_3 - V_2) = (200\text{kPa})(6.025 - 5.02)\text{m}^3\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = \mathbf{201kJ}$$

4-48E A spherical balloon is initially filled with air at a specified state. The pressure inside is proportional to the square of the diameter. Heat is transferred to the air until the volume doubles. The work done is to be determined.

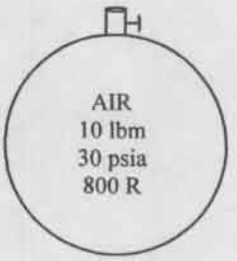
Assumptions **1** Air is an ideal gas. **2** The compression or expansion process is quasi-equilibrium.

Properties The gas constant of air is $R = 0.06855$ Btu/lbm.R (Table A-1E).

Analysis The dependence of pressure on volume can be expressed as

$$V = \frac{1}{6}\pi D^3 \longrightarrow D = \left(\frac{6V}{\pi}\right)^{1/3}$$

$$P \propto D^2 \longrightarrow P = kD^2 = k\left(\frac{6V}{\pi}\right)^{2/3}$$



or, $$k\left(\frac{6}{\pi}\right)^{2/3} = P_1 V_1^{-2/3} = P_2 V_2^{-2/3}$$

Also, $$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{2/3} = 2^{2/3} = 1.587$$

and $$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \longrightarrow T_2 = \frac{P_2 V_2}{P_1 V_1} T_1 = 1.587 \times 2 \times (800\text{R}) = 2539\text{R}$$

Thus,

$$W_b = \int_1^2 P\,dV = \int_1^2 k\left(\frac{6V}{\pi}\right)^{2/3} dV = \frac{3k}{5}\left(\frac{6}{\pi}\right)^{2/3}\left(V_2^{5/3} - V_1^{5/3}\right) = \frac{3}{5}\left(P_2 V_2 - P_1 V_1\right)$$

$$= \frac{3}{5} mR(T_2 - T_1) = \frac{3}{5}(10\text{lbm})(0.06855\text{Btu/lbm} \cdot \text{R})(2539 - 800)\text{R} = \mathbf{715.3Btu}$$

Chapter 5
THE FIRST LAW OF THERMODYNAMICS

Closed System Energy Balance: General Systems

5-1C No. This is the case for adiabatic systems only.

5-2C Warmer. Because energy is added to the room air in the form of electrical work.

5-3C Warmer. If we take the room which contains the refrigerator as our system, we will see that electrical work is supplied to this room to run the refrigerator, which is eventually dissipated to the room as waste heat.

5-4C Energy can be transferred to or from a control volume as heat, various forms of work, and by mass transport.

5-5 Water is heated in a pan on top of a range while being stirred. The energy of the water at the end of the process is to be determined.

Assumptions The pan is stationary and thus the changes in kinetic and potential energies are negligible.

Analysis We take the water in the pan as our system. This is a closed system since no mass enters or leaves. Applying the energy balance on this system gives

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{\text{in}} + W_{\text{pw,in}} - Q_{\text{out}} = \Delta U = U_2 - U_1$$

$$30\,\text{kJ} + 0.5\,\text{kJ} - 5\,\text{kJ} = U_2 - 10\,\text{kJ}$$

$$U_2 = \mathbf{35.5\,kJ}$$

Therefore, the final internal energy of the system is 35.5 kJ.

5-6E Water is heated in a cylinder on top of a range. The change in the energy of the water during this process is to be determined.

Assumptions The pan is stationary and thus the changes in kinetic and potential energies are negligible.

Analysis We take the water in the cylinder as the system. This is a closed system since no mass enters or leaves. Applying the energy balance on this system gives

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{\text{in}} - W_{\text{b,out}} - Q_{\text{out}} = \Delta U = U_2 - U_1$$

$$50\ \text{Btu} - 5\ \text{Btu} - 8\ \text{Btu} = \Delta U$$

$$\Delta U = U_2 - U_1 = \mathbf{37\ Btu}$$

Therefore, the energy content of the system increases by 37 Btu during this process.

5-7 A classroom is to be air-conditioned using window air-conditioning units. The cooling load is due to people, lights, and heat transfer through the walls and the windows. The number of 5-kW window air conditioning units required is to be determined.

Assumptions There are no heat dissipating equipment (such as computers, TVs, or ranges) in the room.

Analysis The total cooling load of the room is determined from

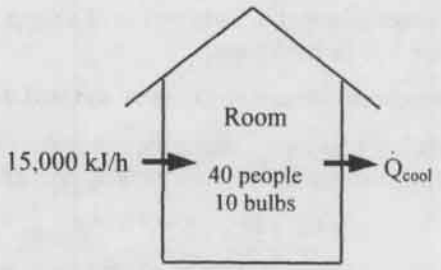
$$\dot{Q}_{\text{cooling}} = \dot{Q}_{\text{lights}} + \dot{Q}_{\text{people}} + \dot{Q}_{\text{heat gain}}$$

where

$$\dot{Q}_{\text{lights}} = 10 \times 100\ \text{W} = 1\ \text{kW}$$

$$\dot{Q}_{\text{people}} = 40 \times 360\ \text{kJ/h} = 4\ \text{kW}$$

$$\dot{Q}_{\text{heat gain}} = 15{,}000\ \text{kJ/h} = 4.17\ \text{kW}$$



Substituting,

$$\dot{Q}_{\text{cooling}} = 1 + 4 + 4.17 = 9.17\ \text{kW}$$

Thus the number of air-conditioning units required is

$$\frac{9.17\ \text{kW}}{5\ \text{kw/unit}} = 1.83 \longrightarrow \mathbf{2\ units}$$

5-8 An industrial facility is to replace its 40-W standard fluorescent lamps by their 35-W high efficiency counterparts. The amount of energy and money that will be saved a year as well as the simple payback period are to be determined.

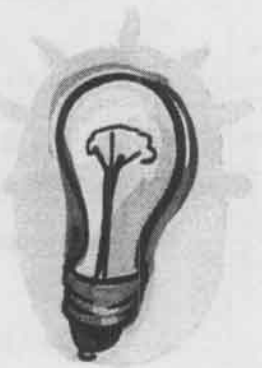
Analysis The reduction in the total electric power consumed by the lighting as a result of switching to the high efficiency fluorescent is

$$\begin{aligned}\text{Wattage reduction} &= (\text{Wattage reduction per lamp})(\text{Number of lamps})\\ &= (40 - 34\ \text{W/lamp})(700\ \text{lamps})\\ &= 4200\ \text{W}\end{aligned}$$

Then using the relations given earlier, the energy and cost savings associated with the replacement of the high efficiency fluorescent lamps are determined to be

$$\begin{aligned}\text{Energy Savings} &= (\text{Total wattage reduction})(\text{Ballast factor})(\text{Operating hours})\\ &= (4.2\ \text{kW})(1.1)(2800\ \text{h/year})\\ &= \mathbf{12{,}936\ kWh/year}\end{aligned}$$

$$\begin{aligned}\text{Cost Savings} &= (\text{Energy savings})(\text{Unit electricity cost})\\ &= (12{,}936\ \text{kWh/year})(\$0.08/\text{kWh})\\ &= \mathbf{\$1035}\end{aligned}$$



The implementation cost of this measure is simply the extra cost of the energy efficient fluorescent bulbs relative to standard ones, and is determined to be

$$\begin{aligned}\text{Implementation Cost} &= (\text{Cost difference of lamps})(\text{Number of lamps})\\ &= [(\$2.26-\$1.77)/\text{lamp}](700\ \text{lamps})\\ &= \$343\end{aligned}$$

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$343}{\$1035\,/\,\text{year}} = \mathbf{0.33\ year}\ (4.0\ \text{months})$$

Discussion Note that if all the lamps were burned out today and are replaced by high-efficiency lamps instead of the conventional ones, the savings from electricity cost would pay for the cost differential in about 4 months. The electricity saved will also help the environment by reducing the amount of CO_2, CO, NO_x, etc. associated with the generation of electricity in a power plant.

5-9 The lighting energy consumption of a storage room is to be reduced by installing motion sensors. The amount of energy and money that will be saved as well as the simple payback period are to be determined. √

Assumptions The electrical energy consumed by the ballasts is negligible.

Analysis The plant operates 12 hours a day, and thus currently the lights are on for the entire 12 hour period. The motion sensors installed will keep the lights on for 3 hours, and off for the remaining 9 hours every day. This corresponds to a total of 9×365 = 3285 off hours per year. Disregarding the ballast factor, the annual energy and cost savings become

Energy Savings = (Number of lamps)(Lamp wattage)(Reduction of annual operating hours)
= (24 lamps)(60 W/lamp)(3285 hours/year)
= **4730 kWh/year**

Cost Savings = (Energy Savings)(Unit cost of energy)
= (4730 kWh/year)($0.08/kWh)
= **$378/year**



The implementation cost of this measure is the sum of the purchase price of the sensor plus the labor,

Implementation Cost = Material + Labor = $32 + $40 = $72

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$72}{\$378/\text{year}} = \mathbf{0.19\ year}\ (2.3\ \text{months})$$

Therefore, the motion sensor will pay for itself in about 2 months.

5-10 The classrooms and faculty offices of a university campus are not occupied an average of 4 hours a day, but the lights are kept on. The amounts of electricity and money the campus will save per year if the lights are turned off during unoccupied periods are to be determined.

Analysis The total electric power consumed by the lights in the classrooms and faculty offices is

$$\dot{E}_{\text{lighting, classroom}} = (\text{Power consumed per lamp}) \times (\text{No. of lamps}) = (200 \times 12 \times 110\ \text{W}) = 264{,}000 = 264\ \text{kW}$$

$$\dot{E}_{\text{lighting, offices}} = (\text{Power consumed per lamp}) \times (\text{No. of lamps}) = (400 \times 6 \times 110\ \text{W}) = 264{,}000 = 264\ \text{kW}$$

$$\dot{E}_{\text{lighting, total}} = \dot{E}_{\text{lighting, classroom}} + \dot{E}_{\text{lighting, offices}} = 264 + 264 = 528\ \text{kW}$$

Noting that the campus is open 240 days a year, the total number of unoccupied work hours per year is

Unoccupied hours = (4 hours/day)(240 days/year) = 960 h/yr

Then the amount of electrical energy consumed per year during unoccupied work period and its cost are

Energy savings = ($\dot{E}_{\text{lighting, classroom}}$)(Unoccupied hours) = (528 kW)(960 h/yr) = **506,880 kWh**

Cost savings = (Energy savings)(Unit cost of energy) = (506,880 kWh)($0.075/kWh) = **$38,016/yr**

Discussion Note that simple conservation measures can result in significant energy and cost savings.

5-11 The radiator of a steam heating system is initially filled with superheated steam. The valves are closed, and steam is allowed to cool until the pressure drops to a specified value by transferring heat to the room. The amount of heat transfer is to be determined, and the process is to be shown on a *P-v* diagram.

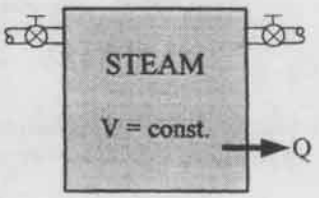
Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions.

Analysis We take the radiator as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{\text{out}} = \Delta U = m(u_2 - u_1) \quad (\text{since } W = \text{KE} = \text{PE} = 0)$$

$$Q_{\text{out}} = m(u_1 - u_2)$$



Using data from the steam tables (Tables A-4 through A-6), some properties are determined to be

$$\left.\begin{aligned} P_1 &= 300\text{kPa} \\ T_1 &= 250^\circ\text{C} \end{aligned}\right\} \begin{aligned} v_1 &= 0.7964\text{m}^3/\text{kg} \\ u_1 &= 2728.7\text{kJ/kg} \end{aligned}$$

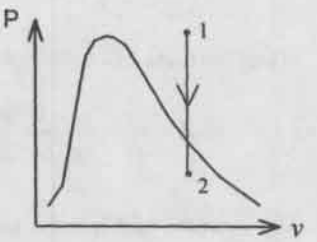
$$P_2 = 100\text{kPa} \rightarrow \begin{aligned} v_f &= 0.001043, \quad v_g = 1.6940\text{m}^3/\text{kg} \\ u_f &= 417.46, \quad u_{fg} = 2088.7\text{kJ/kg} \end{aligned}$$

Noting that $v_1 = v_2$ and $v_f < v_2 < v_g$, the mass and the final internal energy becomes

$$m = \frac{V_1}{v_1} = \frac{0.020\ \text{m}^3}{0.7964\ \text{m}^3/\text{kg}} = 0.0251\ \text{kg}$$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.7964 - 0.001043}{1.6940 - 0.001043} = 0.470$$

$$u_2 = u_f + x_2 u_{fg} = 417.46 + (0.470 \times 2088.7) = 1399.1\ \text{kJ/kg}$$



Substituting,

$$\begin{aligned} Q_{\text{out}} &= m(u_1 - u_2) \\ &= (0.0251\,\text{kg})(2728.7 - 1399.1)\,\text{kJ/kg} \\ &= \mathbf{33.4\,kJ} \end{aligned}$$

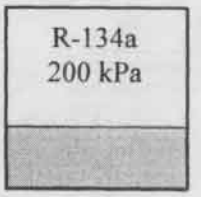
5-12 A rigid tank is initially filled with superheated R-134a. Heat is transferred to the tank until the pressure inside rises to a specified value. The mass of the refrigerant and the amount of heat transfer are to be determined, and the process is to be shown on a *P-v* diagram. √

Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions.

Analysis (*a*) We take the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{\text{in}} = \Delta U = m(u_2 - u_1) \quad (\text{since } W = \text{KE} = \text{PE} = 0)$$



Using data from the refrigerant tables (Tables A-11 through A-13), the properties of R-134a are determined to be

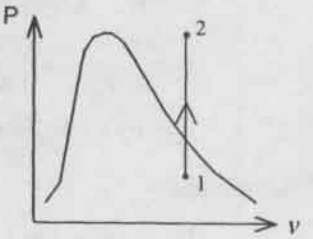
$$\left.\begin{array}{l} P_1 = 200\text{kPa} \\ x_1 = 0.4 \end{array}\right\} \begin{array}{l} v_f = 0.0007532, \quad v_g = 0.0993\text{m}^3/\text{kg} \\ u_f = 36.69, \quad\quad u_g = 221.43\text{kJ/kg} \end{array}$$

$$v_1 = v_f + x_1 v_{fg} = 0.0007532 + [0.4 \times (0.0993 - 0.0007532)] = 0.04017\text{m}^3/\text{kg}$$
$$u_1 = u_f + x_1 u_{fg} = 36.69 + [0.4 \times (221.43 - 36.69)] = 110.59\text{kJ/kg}$$

$$\left.\begin{array}{l} P_2 = 800\text{kPa} \\ (v_2 = v_1) \end{array}\right\} u_2 = 349.82\text{kJ/kg} \ (\text{Superheated vapor})$$

Then the mass of the refrigerant is determined to be

$$m = \frac{V_1}{v_1} = \frac{0.5\text{m}^3}{0.04017\text{m}^3/\text{kg}} = \mathbf{12.45kg}$$



(*b*) Then the heat transfer to the tank becomes

$$\begin{aligned} Q_{\text{in}} &= m(u_2 - u_1) \\ &= (12.45\text{ kg})(349.82 - 110.59)\text{ kJ/kg} \\ &= \mathbf{2978\text{ kJ}} \end{aligned}$$

5-13E A rigid tank is initially filled with saturated R-134a vapor. Heat is transferred from the refrigerant until the pressure inside drops to a specified value. The final temperature, the mass of the refrigerant that has condensed, and the amount of heat transfer are to be determined. Also, the process is to be shown on a *P-v* diagram.

Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions.

Analysis (*a*) We take the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{\text{out}} = \Delta U = m(u_2 - u_1) \quad (\text{since } W = \text{KE} = \text{PE} = 0)$$
$$Q_{\text{out}} = m(u_1 - u_2)$$

Using data from the refrigerant tables (Tables A-11 through A-13), the properties of R-134a are determined to be

$$\left.\begin{array}{l} P_1 = 120\text{psia} \\ \text{sat. vapor} \end{array}\right\} \begin{array}{l} v_1 = v_{g@120\text{psia}} = 0.3941\text{ft}^3/\text{lbm} \\ u_1 = u_{g@120\text{psia}} = 105.06\text{Btu/lbm} \end{array}$$

$$\left.\begin{array}{l} P_2 = 30\text{psia} \\ (v_2 = v_1) \end{array}\right\} \begin{array}{l} v_f = 0.01209, \quad v_g = 1.5408\text{ft}^3/\text{lbm} \\ u_f = 16.24, \quad u_g = 95.40\text{Btu/lbm} \end{array}$$

R-134a
120 psia
Sat. vapor

The final state is saturated mixture. Thus,

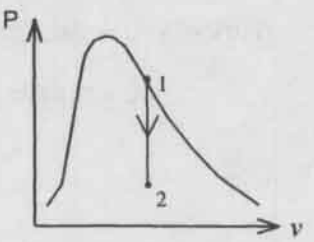
$$T_2 = T_{\text{sat @ 30 psia}} = \mathbf{15.38\ °F}$$

(*b*) The total mass and the amount of refrigerant that has condensed are

$$m = \frac{V_1}{v_1} = \frac{20\text{ ft}^3}{0.3941\text{ ft}^3/\text{lbm}} = 50.8\text{ lbm}$$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.3941 - 0.01209}{1.5408 - 0.01209} = 0.250$$

$$m_f = (1 - x_2)m = (1 - 0.250)(50.8\text{ lbm}) = \mathbf{38.1\ lbm}$$



Also,

$$u_2 = u_f + x_2 u_{fg} = 16.24 + [0.250 \times (95.40 - 16.24)] = 36.03\text{ Btu/lbm}$$

(*c*) Substituting,

$$Q_{\text{out}} = m(u_1 - u_2)$$
$$= (50.8\text{ lbm})(105.06 - 36.03)\text{ Btu/lbm}$$
$$= \mathbf{3507\ Btu}$$

5-14 An insulated rigid tank is initially filled with a saturated liquid-vapor mixture of water. An electric heater in the tank is turned on, and the entire liquid in the tank is vaporized. The length of time the heater was kept on is to be determined, and the process is to be shown on a *P-v* diagram.

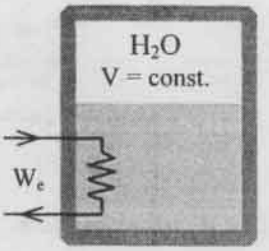
Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** The device is well-insulated and thus heat transfer is negligible. **3** The energy stored in the resistance wires, and the heat transferred to the tank itself is negligible.

Analysis We take the contents of the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{e,\text{in}} = \Delta U = m(u_2 - u_1) \quad (\text{since } Q = \text{KE} = \text{PE} = 0)$$

$$VI\Delta t = m(u_2 - u_1)$$

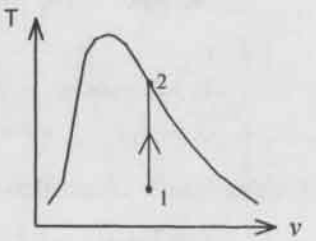


The properties of water are (Tables A-4 through A-6)

$$\left.\begin{array}{l} P_1 = 100\text{kPa} \\ x_1 = 0.25 \end{array}\right\} \begin{array}{l} v_f = 0.001043, \quad v_g = 1.6940\text{m}^3/\text{kg} \\ u_f = 417.36, \quad u_{fg} = 2088.7\text{kJ/kg} \end{array}$$

$$v_1 = v_f + x_1 v_{fg} = 0.001043 + [0.25 \times (1.6940 - 0.001043)] = 0.42428\text{m}^3/\text{kg}$$

$$u_1 = u_f + x_1 u_{fg} = 417.36 + (0.25 \times 2088.7) = 939.5\text{kJ/kg}$$



$$\left.\begin{array}{l} v_2 = v_1 = 0.42428\text{m}^3/\text{kg} \\ \text{sat.vapor} \end{array}\right\} u_2 = u_{g@0.42428\text{m}^3/\text{kg}} = 2556.7\text{kJ/kg}$$

Substituting,

$$(110\text{V})(8\text{A})\Delta t = (5\text{kg})(2556.7 - 939.5)\,\text{kJ/kg}\left(\frac{1000\text{VA}}{1\text{kJ/s}}\right)$$

$$\Delta t = 9189\text{s} \cong \mathbf{153.2min}$$

5-15 One part of an insulated tank contains compressed liquid while the other side is evacuated. The partition is then removed, and water is allowed to expand into the entire tank. The final temperature and the volume of the tank are to be determined.

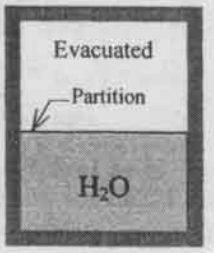
Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** The tank is insulated and thus heat transfer is negligible. **3** There are no work interactions.

Analysis We take the entire contents of the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$0 = \Delta U = m(u_2 - u_1) \quad (\text{since } W = Q = \text{KE} = \text{PE} = 0)$$

$$u_1 = u_2$$



The properties of water are (Tables A-4 through A-6)

$$\left.\begin{array}{l} P_1 = 600\text{kPa} \\ T_1 = 60^\circ\text{C} \end{array}\right\} \begin{array}{l} v_1 \cong v_{f@60^\circ\text{C}} = 0.001017\text{m}^3/\text{kg} \\ u_1 \cong u_{f@60^\circ\text{C}} = 251.1\,\text{kJ/kg} \end{array}$$

We now assume the final state in the tank is saturated liquid-vapor mixture and determine quality. This assumption will be verified if we get a quality between 0 and 1.

$$\left.\begin{array}{l} P_1 = 10\text{kPa} \\ (u_2 = u_1) \end{array}\right\} \begin{array}{l} v_f = 0.00101, \quad v_g = 14.67\text{m}^3/\text{kg} \\ u_f = 191.82, \quad u_{fg} = 2246.1\text{kJ/kg} \end{array}$$

$$x_2 = \frac{u_2 - u_f}{u_{fg}} = \frac{251.11 - 191.82}{2246.1} = 0.0264$$

Thus,

$$T_2 = = T_{\text{sat @ 10 kPa}} = \mathbf{45.81\ °C}$$

$$v_2 = v_f + x_2 v_{fg} = 0.0010 + [0.0264 \times (14.67 - 0.00101)] = 0.388\text{m}^3/\text{kg}$$

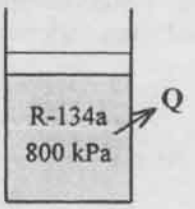
and,

$$V = mv_2 = (2.5\text{ kg})(0.388\text{ m}^3/\text{kg}) = \mathbf{0.97\ m^3}$$

5-16 A cylinder is initially filled with R-134a at a specified state. The refrigerant is cooled at constant pressure. The amount of heat loss is to be determined, and the process is to be shown on a *T*-*v* diagram.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions involved other than the boundary work. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

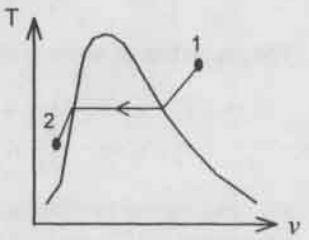


$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{out} - W_{b,out} = \Delta U = m(u_2 - u_1) \quad \text{(since KE = PE = 0)}$$

$$-Q_{out} = m(h_2 - h_1)$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process. The properties of R-134a are (Tables A-11 through A-13)



$$\left.\begin{matrix} P_1 = 800\text{kPa} \\ T_1 = 60^\circ\text{C} \end{matrix}\right\} h_1 = 294.98\text{kJ/kg}$$

$$\left.\begin{matrix} P_1 = 800\text{kPa} \\ T_1 = 20^\circ\text{C} \end{matrix}\right\} h_2 = h_{f@20^\circ\text{C}} = 77.26\text{kJ/kg}$$

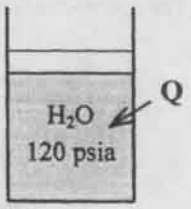
Substituting,

$$Q_{out} = -(5\text{ kg})(77.26 - 294.98)\text{ kJ/kg} = \mathbf{1089\ kJ}$$

5-17E A cylinder contains water initially at a specified state. The water is heated at constant pressure. The final temperature of the water is to be determined, and the process is to be shown on a *T-v* diagram.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** The thermal energy stored in the cylinder itself is negligible. **3** The compression or expansion process is quasi-equilibrium.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

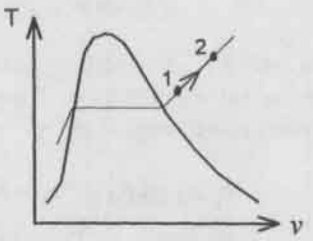


$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{\text{in}} - W_{b,\text{out}} = \Delta U = m(u_2 - u_1) \quad \text{(since KE = PE = 0)}$$

$$Q_{\text{in}} = m(h_2 - h_1)$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process. The properties of water are (Tables A-11 through A-13)



$$v_1 = \frac{V_1}{m} = \frac{2\text{ft}^3}{0.5\text{lbm}} = 4\text{ft}^3/\text{lbm}$$

$$\left.\begin{aligned} P_1 &= 120\text{psia} \\ v_1 &= 4\text{ft}^3/\text{lbm} \end{aligned}\right\} h_1 = 1216.9\text{Btu/lbm}$$

Substituting,

$$180\text{kJ} = (0.5\text{lbm})(h_2 - 1216.9)\text{Btu/lbm}$$

$$h_2 = 1616.9\text{Btu/lbm}$$

Then,

$$\left.\begin{aligned} P_2 &= 120\text{psia} \\ h_2 &= 1616.9\text{Btu/lbm} \end{aligned}\right\} T_2 = \mathbf{1161.4\,^\circ F}$$

5-18 A cylinder is initially filled with saturated liquid water at a specified pressure. The water is heated electrically as it is stirred by a paddle-wheel at constant pressure. The voltage of the current source is to be determined, and the process is to be shown on a *P-v* diagram.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** The cylinder is well-insulated and thus heat transfer is negligible. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

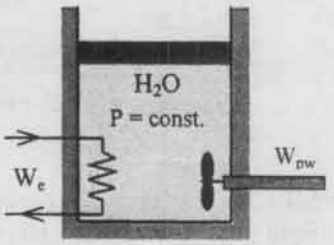
Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

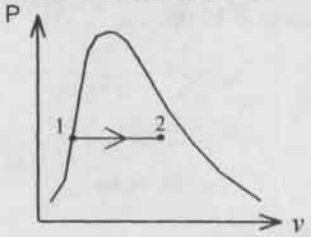
$$W_{e,in} + W_{pw,in} - W_{b,out} = \Delta U \quad (\text{since } Q = \text{KE} = \text{PE} = 0)$$

$$W_{e,in} + W_{pw,in} = m(h_2 - h_1)$$

$$(VI\Delta t) + W_{pw,in} = m(h_2 - h_1)$$



since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process. The properties of water are (Tables A-4 through A-6)



$$\left.\begin{array}{l} P_1 = 150\text{kPa} \\ \text{sat.liquid} \end{array}\right\} \begin{array}{l} h_1 = h_{f@150\text{kPa}} = 467.11\text{kJ/kg} \\ v_1 = v_{f@150\text{kPa}} = 0.0010528\text{m}^3/\text{kg} \end{array}$$

$$\left.\begin{array}{l} P_1 = 150\text{kPa} \\ x_2 = 0.5 \end{array}\right\} h_2 = h_f + x_2 h_{fg} = 467.11 + (0.5 \times 2226.5) = 1580.36\text{kJ/kg}$$

$$m = \frac{V_1}{v_1} = \frac{0.005\text{m}^3}{0.0010528\text{m}^3/\text{kg}} = 4.75\text{kg}$$

Substituting,

$$VI\Delta t + (300\text{kJ}) = (4.75\text{kg})(1580.36 - 467.11)\text{kJ/kg}$$

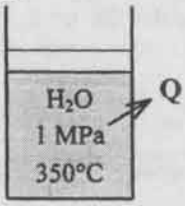
$$VI\Delta t = 4988\text{kJ}$$

$$V = \frac{4988\text{kJ}}{(8\text{A})(45 \times 60\text{s})}\left(\frac{1000\text{VA}}{1\text{kJ/s}}\right) = \mathbf{230.9V}$$

5-19 A cylinder is initially filled with steam at a specified state. The steam is cooled at constant pressure. The mass of the steam, the final temperature, and the amount of heat transfer are to be determined, and the process is to be shown on a *T-v* diagram.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions involved other than the boundary work. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

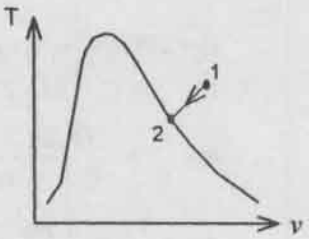


$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{out} - W_{b,out} = \Delta U = m(u_2 - u_1) \quad \text{(since KE = PE = 0)}$$

$$-Q_{out} = m(h_2 - h_1)$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process. The properties of water are (Tables A-4 through A-6)



$$\left.\begin{array}{l} P_1 = 1\text{MPa} \\ T_2 = 350\,^\circ\text{C} \end{array}\right\} \begin{array}{l} v_1 = 0.2825\text{m}^3/\text{kg} \\ h_1 = 3157.7\text{kJ/kg} \end{array}$$

$$m = \frac{V_1}{v_1} = \frac{1.5\text{m}^3}{0.2825\text{m}^3/\text{kg}} = \mathbf{5.31kg}$$

(*b*) The final temperature is determined from

$$\left.\begin{array}{l} P_2 = 1\text{MPa} \\ sat.vapor \end{array}\right\} \begin{array}{l} T_2 = T_{sat@1\text{MPa}} = \mathbf{179.91\,^\circ C} \\ h_2 = h_{g@1\text{MPa}} = 2778.1\text{kJ/kg} \end{array}$$

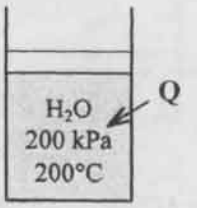
(*c*) Substituting, the energy balance gives

$$Q_{out} = -(5.31\text{ kg})(2778.1 - 3157.7)\text{ kJ/kg} = \mathbf{2016\text{ kJ}}$$

5-20 A cylinder equipped with an external spring is initially filled with steam at a specified state. Heat is transferred to the steam, and both the temperature and pressure rise. The final temperature, the boundary work done by the steam, and the amount of heat transfer are to be determined, and the process is to be shown on a *P-v* diagram.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** The thermal energy stored in the cylinder itself is negligible. **3** The compression or expansion process is quasi-equilibrium. **4** The spring is a linear spring.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. Noting that the spring is not part of the system (it is external), the energy balance for this stationary closed system can be expressed as

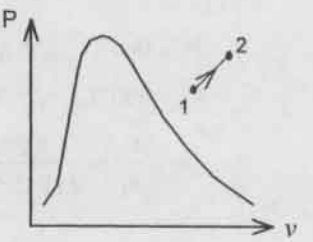


$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{in} - W_{b,\text{out}} = \Delta U = m(u_2 - u_1) \quad (\text{since KE} = \text{PE} = 0)$$

$$Q_{\text{in}} = m(u_2 - u_1) + W_{b,\text{out}}$$

The properties of steam are (Tables A-4 through A-6)



$$\left.\begin{aligned} P_1 &= 200\text{kPa} \\ T_1 &= 200^\circ\text{C} \end{aligned}\right\} \begin{aligned} v_1 &= 1.0803\text{m}^3/\text{kg} \\ u_1 &= 2654.4\text{kJ/kg} \end{aligned}$$

$$m = \frac{V_1}{v_1} = \frac{0.5\text{m}^3}{1.0803\text{m}^3/\text{kg}} = 0.463\text{kg}$$

$$v_2 = \frac{V_2}{m} = \frac{0.6\text{m}^3}{0.463\text{kg}} = 1.296\text{m}^3/\text{kg}$$

$$\left.\begin{aligned} P_2 &= 500\text{kPa} \\ v_2 &= 1.296\text{m}^3/\text{kg} \end{aligned}\right\} \begin{aligned} T_2 &= \mathbf{1131^\circ C} \\ u_2 &= 4321.9\text{kJ/kg} \end{aligned}$$

(*b*) The pressure of the gas changes linearly with volume, and thus the process curve on a P-V diagram will be a straight line. The boundary work during this process is simply the area under the process curve, which is a trapezoidal. Thus,

$$W_b = Area = \frac{P_1 + P_2}{2}(V_2 - V_1) = \frac{(200+500)\text{kPa}}{2}(0.6-0.5)\text{m}^3\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = \mathbf{35kJ}$$

(*c*) From the energy balance we have

$$Q_{\text{in}} = (0.463\text{ kg})(4321.9 - 2654.4)\text{kJ/kg} + 35\text{ kJ} = \mathbf{807\text{ kJ}}$$

5-21 A cylinder equipped with a set of stops for the piston to rest on is initially filled with saturated water vapor at a specified pressure. Heat is transferred to water until the volume doubles. The final temperature, the boundary work done by the steam, and the amount of heat transfer are to be determined, and the process is to be shown on a *P-v* diagram. √

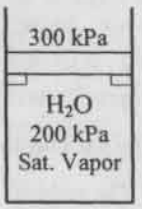
Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions involved other than the boundary work. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{\text{in}} - W_{b,\text{out}} = \Delta U = m(u_3 - u_1) \quad (\text{since KE} = \text{PE} = 0)$$

$$Q_{\text{in}} = m(u_3 - u_1) + W_{b,\text{out}}$$



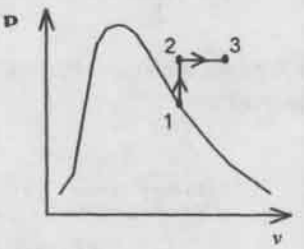
The properties of steam are (Tables A-4 through A-6)

$$\left.\begin{array}{l} P_1 = 200\text{kPa} \\ sat.vapor \end{array}\right\} \begin{array}{l} v_1 = v_{g@200\text{kPa}} = 0.8857\text{m}^3/\text{kg} \\ u_1 = u_{g@200\text{kPa}} = 2529.5\text{kJ/kg} \end{array}$$

$$m = \frac{V_1}{v_1} = \frac{0.5\text{m}^3}{0.8857\text{m}^3/\text{kg}} = 0.5645\text{kg}$$

$$v_3 = \frac{V_3}{m} = \frac{1\text{m}^3}{0.5645\text{kg}} = 1.7715\text{m}^3/\text{kg}$$

$$\left.\begin{array}{l} P_3 = 300\text{kPa} \\ v_3 = 1.7715\text{m}^3/\text{kg} \end{array}\right\} \begin{array}{l} T_3 = \mathbf{878.9\,°C} \\ u_3 = 3813.8\text{kJ/kg} \end{array}$$



(*b*) The work done during process 1-2 is zero (since V = const) and the work done during the constant pressure process 2-3 is

$$W_{b,\text{out}} = \int_2^3 P\,dV = P(V_3 - V_2) = (300\text{kPa})(1.0 - 0.5)\text{m}^3\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = \mathbf{150\ kJ}$$

(*c*) Heat transfer is determined from the energy balance,

$$Q_{\text{in}} = m(u_3 - u_1) + W_{b,\text{out}}$$
$$= (0.5645\,\text{kg})(3813.8 - 2529.5)\,\text{kJ/kg} + 150\,\text{kJ} = 875.0\,\text{kJ}$$

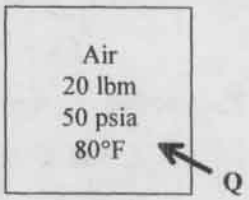
Closed System Energy Analysis: Ideal Gases

5-22C No, it isn't. This is because the first law relation $Q - W = \Delta U$ reduces to $W = 0$ in this case since the system is adiabatic ($Q = 0$) and $\Delta U = 0$ for the isothermal processes of ideal gases. Therefore, this adiabatic system cannot receive any net work at constant temperature.

5-23E The air in a rigid tank is heated until its pressure doubles. The volume of the tank and the amount of heat transfer are to be determined.

Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta pe \cong \Delta ke \cong 0$. **3** Constant specific heats at room temperature can be used for air. This assumption results in negligible error in heating and air-conditioning applications.

Properties The gas constant of air is $R = 0.3705$ psia.ft^3/lbm.R (Table A-1E).



Analysis (*a*) The volume of the tank can be determined from the ideal gas relation,

$$V = \frac{mRT_1}{P_1} = \frac{(20\ \text{lbm})(0.3705\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(540\ \text{R})}{50\ \text{psia}} = \mathbf{80.0\ ft^3}$$

(b) We take the air in the tank as our system. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{\text{in}} = \Delta U$$

$$Q_{\text{in}} = m(u_2 - u_1) \cong mC_v(T_2 - T_1)$$

The final temperature of air is

$$\frac{P_1V}{T_1} = \frac{P_2V}{T_2} \quad \longrightarrow \quad T_2 = \frac{P_2}{P_1}T_1 = 2 \times (540\ \text{R}) = 1080\ \text{R}$$

The internal energies are (Table A-17E)

$$u_1 = u_{@\,540\ \text{R}} = 92.04\ \text{Btu/lbm}$$

$$u_2 = u_{@\,1080\ \text{R}} = 186.93\ \text{Btu/lbm}$$

Substituting,

$$Q_{\text{in}} = (20\ \text{lbm})(186.93 - 92.04)\text{Btu/lbm} = \mathbf{1898\ Btu}$$

Alternative solutions The specific heat of air at the average temperature of $T_{\text{ave}} = (540+1080)/2 = 810$ R = 350°F is, from Table A-2Eb, $C_{v,\text{ave}} = 0.175$ Btu/lbm.R. Substituting,

$$Q_{\text{in}} = (20\ \text{lbm})(0.175\ \text{Btu/lbm.R})(1080 - 540)\ \text{R} = \mathbf{1890\ Btu}$$

Discussion Both approaches resulted in almost the same solution in this case.

5-24 The hydrogen gas in a rigid tank is cooled until its temperature drops to 300 K. The final pressure in the tank and the amount of heat transfer are to be determined. √

Assumptions **1** Hydrogen is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -240°C and 1.30 MPa. **2** The tank is stationary, and thus the kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$.

Properties The gas constant of hydrogen is R = 4.124 kPa.m^3/kg.K (Table A-1). The constant volume specific heat of hydrogen at the average temperature of 400 K is , $C_{v,ave}$ = 10.352 kJ/kg.K (Table A-2).

Analysis (*a*) The final pressure of hydrogen can be determined from the ideal gas relation,

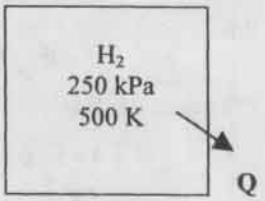
$$\frac{P_1V}{T_1} = \frac{P_2V}{T_2} \quad \longrightarrow \quad P_2 = \frac{T_2}{T_1}P_1 = \frac{300\ \text{K}}{500\ \text{K}}(250\ \text{kPa}) = \mathbf{150\ kPa}$$

(*b*) We take the hydrogen in the tank as the system. This is a *closed system* since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{\text{out}} = \Delta U$$

$$Q_{\text{out}} = -\Delta U = -m(u_2 - u_1) \cong mC_v(T_1 - T_2)$$



where

$$m = \frac{P_1V}{RT_1} = \frac{(250\ \text{kPa})(1.0\ \text{m}^3)}{(4.124\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(500\ \text{K})} = 0.1212\ \text{kg}$$

Substituting into the energy balance,

$$Q_{\text{out}} = (0.1212\ \text{kg})(10.352\ \text{kJ/kg·K})(500 - 300)\text{K} = \mathbf{251.0\ kJ}$$

5-25 A resistance heater is to raise the air temperature in the room from 7 to 25°C within 20 min. The required power rating of the resistance heater is to be determined. √

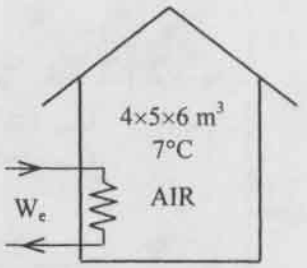
Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** Constant specific heats at room temperature can be used for air. This assumption results in negligible error in heating and air-conditioning applications. **4** Heat losses from the room are negligible. **5** The room is air-tight so that no air leaks in and out during the process.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1). Also, $C_v = 0.718$ kJ/kg.K for air at room temperature (Table A-2).

Analysis We take the air in the room to be the system. This is a closed system since no mass crosses the system boundary. The energy balance for this stationary constant-volume closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{e,\text{in}} = \Delta U \cong mC_{v,ave}(T_2 - T_1) \ (\text{since } Q = KE = PE = 0)$$



or,

$$\dot{W}_{e,in}\Delta t = mC_{v,ave}(T_2 - T_1)$$

The mass of air is

$$V = 4 \times 5 \times 6 = 120 \text{ m}^3$$

$$m = \frac{P_1 V}{RT_1} = \frac{(100 \text{ kPa})(120 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(280 \text{ K})} = 149.3 \text{ kg}$$

Substituting, the power rating of the heater becomes

$$\dot{W}_{e,in} = (149.3 \text{ kg})(0.718 \text{ kJ/kg}\cdot^\circ\text{C})(25-7)^\circ\text{C}/(15 \times 60 \text{ s}) = \mathbf{1.91 \text{ kW}}$$

Discussion In practice, the pressure in the room will remain constant during this process rather than the volume, and some air will leak out as the air expands. As a result, the air in the room will undergo a constant pressure expansion process. Therefore, it is more proper to be conservative and to use ΔH instead of using ΔU in heating and air-conditioning applications.

5-26 A room is heated by a radiator, and the warm air is distributed by a fan. Heat is lost from the room. The time it takes for the air temperature to rise to 20°C is to be determined. √

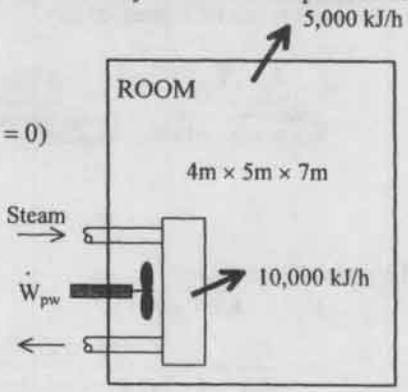
Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** Constant specific heats at room temperature can be used for air. This assumption results in negligible error in heating and air-conditioning applications. **4** The local atmospheric pressure is 100 kPa. **5** The room is air-tight so that no air leaks in and out during the process.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1). Also, $C_v = 0.718$ kJ/kg.K for air at room temperature (Table A-2).

Analysis We take the air in the room to be the system. This is a closed system since no mass crosses the system boundary. The energy balance for this stationary constant-volume closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{in} + W_{fan,in} - Q_{out} = \Delta U \cong mC_{v,ave}(T_2 - T_1) \text{ (since } KE = PE = 0)$$



or,

$$(\dot{Q}_{in} + \dot{W}_{fan,in} - \dot{Q}_{out})\Delta t = mC_{v,ave}(T_2 - T_1)$$

The mass of air is

$$V = 4 \times 5 \times 7 = 140 \text{ m}^3$$

$$m = \frac{P_1 V}{RT_1} = \frac{(100 \text{ kPa})(140 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(283 \text{ K})} = 172.4 \text{ kg}$$

Using the C_v value at room temperature,

$$[(10{,}000 - 5{,}000)/3600 \text{ kJ/s} + 0.1 \text{ kJ/s}]\Delta t = (172.4\text{kg})(0.718 \text{ kJ/kg}\cdot^\circ\text{C})(20 - 10)^\circ\text{C}$$

It yields

$$\Delta t = \mathbf{831\ s}$$

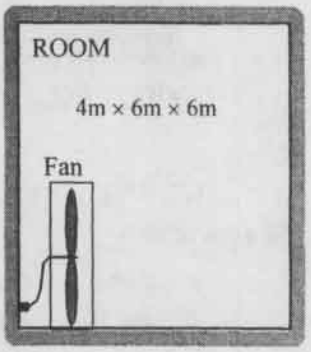
Discussion In practice, the pressure in the room will remain constant during this process rather than the volume, and some air will leak out as the air expands. As a result, the air in the room will undergo a constant pressure expansion process. Therefore, it is more proper to be conservative and to using ΔH instead of use ΔU in heating and air-conditioning applications.

5-27 A student living in a room turns her 150-W fan on in the morning. The temperature in the room when she comes back 10 h later is to be determined. √

Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** Constant specific heats at room temperature can be used for air. This assumption results in negligible error in heating and air-conditioning applications. **4** All the doors and windows are tightly closed, and heat transfer through the walls and the windows is disregarded.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1). Also, $C_v = 0.718$ kJ/kg.K for air at room temperature (Table A-2).

Analysis We take the room as the system. This is a *closed system* since the doors and the windows are said to be tightly closed, and thus no mass crosses the system boundary during the process. The energy balance for this system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{e,in} = \Delta U$$

$$W_{e,in} = m(u_2 - u_1) \cong mC_v(T_2 - T_1)$$

The mass of air is

$$V = 4 \times 6 \times 6 = 144 \text{ m}^3$$

$$m = \frac{P_1 V}{RT_1} = \frac{(100 \text{ kPa})(144 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(288 \text{ K})} = 174.2 \text{ kg}$$

The electrical work done by the fan is

$$W_e = \dot{W}_e \Delta t = (0.15 \text{ kJ/s})(10 \times 3600 \text{ s}) = 5400 \text{ kJ}$$

Substituting and using the C_v value at room temperature,

$$5400 \text{ kJ} = (174.2 \text{ kg})(0.718 \text{ kJ/kg·°C})(T_2 - 15)°\text{C}$$

$$T_2 = \mathbf{58.2°C}$$

5-28E A paddle wheel in an oxygen tank is rotated until the pressure inside rises to 20 psia while some heat is lost to the surroundings. The paddle wheel work done is to be determined.

Assumptions **1** Oxygen is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -181°F and 736 psia. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** The energy stored in the paddle wheel is negligible. **4** This is a rigid tank and thus its volume remains constant.

Properties The gas constant and molar mass of oxygen are $R = 0.3353$ psia.ft^3/lbm.R and $M = 32$ lbm/lbmol (Table A-1E). The specific heat of oxygen at the average temperature of $T_{ave} = (735+540)/2 = 638$ R is $C_{v,ave} = 0.160$ Btu/lbm.R (Table A-2E).

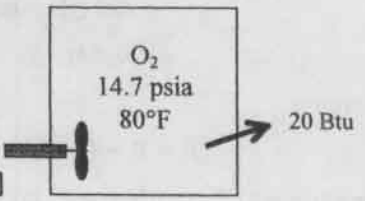
Analysis We take the oxygen in the tank as our system. This is a *closed system* since no mass enters or leaves. The energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{pw,in} - Q_{out} = \Delta U$$

$$W_{pw,in} = Q_{out} + m(u_2 - u_1) = Q_{out} + m[(\bar{u}_2 - \bar{u}_1)/M]$$

$$\cong Q_{out} + mC_v(T_2 - T_1)$$



The final temperature and the number of moles of oxygen are

$$\frac{P_1V}{T_1} = \frac{P_2V}{T_2} \longrightarrow T_2 = \frac{P_2}{P_1}T_1 = \frac{20 \text{ psia}}{14.7 \text{ psia}}(540 \text{ R}) = 735 \text{ R}$$

$$m = \frac{P_1V}{RT_1} = \frac{(14.7 \text{ psia})(10 \text{ ft}^3)}{(0.3353 \text{ psia} \cdot \text{ft}^3/\text{lbmol} \cdot \text{R})(540 \text{ R})} = 0.812 \text{ lbm}$$

Substituting,

$$W_{pw,in} = (20 \text{ Btu}) + (0.812 \text{ lbm})(0160 \text{ Btu/lbm.R})(735 - 540) \text{ R} = \mathbf{45.3 \text{ Btu}}$$

Discussion Note that a "cooling" fan actually causes the internal temperature of a confined space to rise. In fact, a 100-W fan supplies a room with as much energy as a 100-W resistance heater.

5-29 One part of an insulated rigid tank contains an ideal gas while the other side is evacuated. The final temperature and pressure in the tank are to be determined when the partition is removed. √

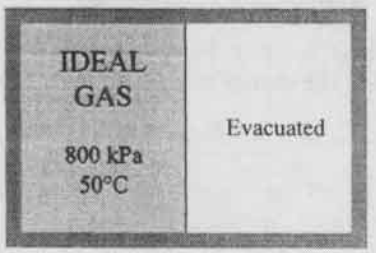
Assumptions **1** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **2** The tank is insulated and thus heat transfer is negligible.

Analysis We take the entire tank as the system. This is a *closed system* since no mass crosses the boundaries of the system. The energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$0 = \Delta U = m(u_2 - u_1)$$

$$u_2 = u_1$$



Therefore,

$$T_2 = T_1 = \mathbf{50°C}$$

Since $u = u(T)$ for an ideal gas. Then,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \longrightarrow P_2 = \frac{V_1}{V_2}P_1 = \frac{1}{2}(800 \text{ kP}a) = \mathbf{400kPa}$$

5-30 A cylinder equipped with a set of stops for the piston to rest on is initially filled with helium gas at a specified state. The amount of heat that must be transferred to raise the piston is to be determined. √

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** There are no work interactions involved. **4** The thermal energy stored in the cylinder itself is negligible.

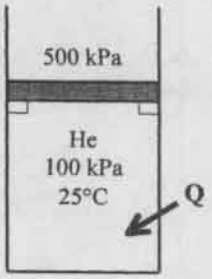
Properties The specific heat of helium at room temperature is $C_v = 3.1156$ kJ/kg.K (Table A-2).

Analysis We take the helium gas in the cylinder as the system. This is a closed system since no mass crosses the boundary of the system. The energy balance for this constant volume closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} = \Delta U = m(u_2 - u_1)$$

$$Q_{in} = m(u_2 - u_1) = mC_v(T_2 - T_1)$$



The final temperature of helium can be determined from the ideal gas relation to be

$$\frac{P_1V}{T_1} = \frac{P_2V}{T_2} \longrightarrow T_2 = \frac{P_2}{P_1}T_1 = \frac{500\text{ kPa}}{100\text{ kPa}}(298\text{ K}) = 1490\text{ K}$$

Substituting into the energy balance relation gives

$$Q_{in} = (0.5\text{ kg})(3.1156\text{ kJ/kg·K})(1490 - 298)\text{K} = \mathbf{1857\text{ kJ}}$$

5-31 An insulated cylinder is initially filled with air at a specified state. A paddle-wheel in the cylinder stirs the air at constant pressure. The final temperature of air is to be determined.

Assumptions **1** Air is an ideal gas with variable specific heats. **2** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **3** There are no work interactions involved other than the boundary work. **4** The cylinder is well-insulated and thus heat transfer is negligible. **5** The thermal energy stored in the cylinder itself and the paddle-wheel is negligible. **6** The compression or expansion process is quasi-equilibrium.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1). Also, $C_p = 1.005$ kJ/kg.K for air at room temperature (Table A-2). The enthalpy of air at the initial temperature is

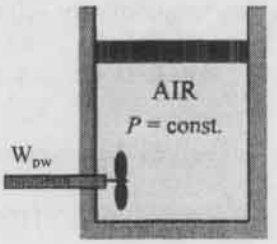
$$h_1 = h_{@298\ K} = 298.18\ \text{kJ/kg}$$

Analysis We take the air in the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{pw,in} - W_{b,out} = \Delta U$$

$$W_{pw,in} = m(h_2 - h_1)$$



since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process. The mass of air is

$$m = \frac{P_1 V}{RT_1} = \frac{(400\ \text{kPa})(0.1\ \text{m}^3)}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\ \text{K})} = 0.468\ \text{kg}$$

Substituting into the energy balance,

$$15\ \text{kJ} = (0.468\ \text{kg})(h_2 - 298.18\ \text{kJ/kg})$$
$$h_2 = 330.23\ \text{kJ/kg}$$

From Table A-17,

$$T_2 = \mathbf{329.9\ K}$$

Alternative solution Using specific heats at room temperature, $C_p = 1.005$ kJ/kg.°C, the final temperature is determined to be

$$W_{pw,in} = m(h_2 - h_1) \cong mC_p(T_2 - T_1)$$
$$15\ \text{kJ} = (0.468\ \text{kg})(1.005\ \text{kJ/kg.°C})(T_2 - 25)°\text{C}$$

which gives

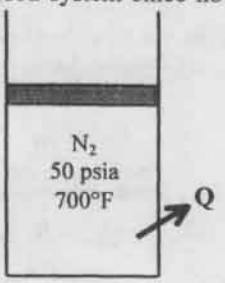
$$T_2 = \mathbf{56.9°C}$$

5-32E A cylinder is initially filled with nitrogen gas at a specified state. The gas is cooled by transferring heat from it. The amount of heat transfer is to be determined.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions involved other than the boundary work. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium. **5** Nitrogen is an ideal gas with variable specific heats.

Properties The specific heat of nitrogen at the average temperature of T_{ave} = (700+ 140)/2 = 420°F is C_{pave} = 0.252 Btu/lbm.°F (Table A-2Eb).

Analysis We take the nitrogen gas in the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} - W_{b,out} = \Delta U = m(u_2 - u_1)$$

$$-Q_{out} = m(h_2 - h_1) = mC_p(T_2 - T_1)$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process.
The mass of nitrogen is

$$m = \frac{P_1 V}{RT_1} = \frac{(50 \text{ psia})(25 \text{ ft}^3)}{(0.3830 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(1160 \text{ R})} = 2.814 \text{ lbm}$$

Substituting,

$$Q_{out} = (0.252 \text{ lbm})(0.252 \text{ Btu/lbm.°F})(700 - 140)°\text{F} = \mathbf{397\ Btu}$$

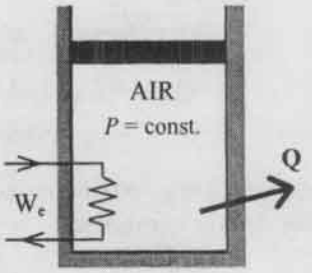
5-33 A cylinder is initially filled with air at a specified state. Air is heated electrically at constant pressure, and some heat is lost in the process. The amount of electrical energy supplied is to be determined. √

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** Air is an ideal gas with variable specific heats. **3** The thermal energy stored in the cylinder itself and the resistance wires is negligible. **4** The compression or expansion process is quasi-equilibrium.

Properties The initial and final enthalpies of air are (Table A-17)

$$h_1 = h_{@298\,\mathrm{K}} = 298.18\ \mathrm{kJ/kg}$$
$$h_2 = h_{@350\,\mathrm{K}} = 350.49\ \mathrm{kJ/kg}$$

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{e,in} - Q_{out} - W_{b,out} = \Delta U$$
$$W_{e,in} = m(h_2 - h_1) + Q_{out}$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process. Substituting,

$$W_{e,in} = (15\ \mathrm{kg})(350.49 - 298.18)\mathrm{kJ/kg} + (60\ \mathrm{kJ}) = 845\ \mathrm{kJ}$$

or,

$$W_{e,in} = (845\mathrm{kJ})\left(\frac{1\ \mathrm{kWh}}{3600\ \mathrm{kJ}}\right) = \mathbf{0.235\ kWh}$$

Alternative solution The specific heat of air at the average temperature of $T_{ave} = (25+ 77)/2$ 51°C = 324 K is, from Table A-2b, $C_{p,ave} = 1.0065$ kJ/kg.°C. Substituting,

$$W_{e,in} = mC_p(T_2 - T_1) + Q_{out}$$
$$= (15\ \mathrm{kg})(1.0065\ \mathrm{kJ/kg.°C})(77 - 25)°\mathrm{C} + 60\ \mathrm{kJ} = 845\ \mathrm{kJ}$$

or,

$$W_{e,in} = (845\ \mathrm{kJ})\left(\frac{1\ \mathrm{kWh}}{3600\ \mathrm{kJ}}\right) = \mathbf{0.235\ kWh}$$

Discussion Note that for small temperature differences, both approaches give the same result.

5-34 An insulated cylinder initially contains CO_2 at a specified state. The CO_2 is heated electrically for 10 min at constant pressure until the volume doubles. The electric current is to be determined.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** The CO_2 is an ideal gas with variable specific heats. **3** The thermal energy stored in the cylinder itself and the resistance wires is negligible. **4** The compression or expansion process is quasi-equilibrium.

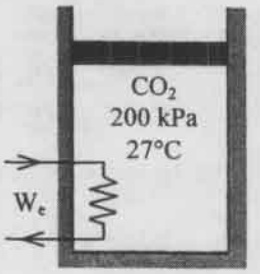
Properties The gas constant and molar mass of CO_2 are $R = 0.1889$ kPa.m^3/kg.K and $M = 44$ kg/kmol (Table A-1). The specific heat of CO_2 at the average temperature of $T_{ave} = (300 + 600)/2 = 450$ K is $C_{p,ave} = 0.978$ kJ/kg.°C (Table A-2b).

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{e,in} - W_{b,out} = \Delta U$$

$$W_{e,in} = m(h_2 - h_1) \cong mC_p(T_2 - T_1)$$



since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process. The final temperature of CO_2 is

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \longrightarrow T_2 = \frac{P_2}{P_1}\frac{V_2}{V_1}T_1 = 1 \times 2 \times (300\text{K}) = 600\text{K}$$

The mass of CO_2 is

$$m = \frac{P_1V_1}{RT_1} = \frac{(200\text{ kPa})(0.3\text{ m}^3)}{(0.1889\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{ K})} = 1.059\text{ kg}$$

Substituting,

$$W_{e,in} = (1.059\text{ kg})\ 0.978\text{ kJ/kg.K.})(600 - 300)\text{K} = 311\text{ kJ}$$

Then,

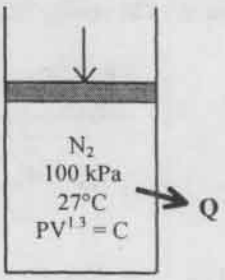
$$I = \frac{W_{e,in}}{V\Delta t} = \frac{311\text{kJ}}{(110\text{V})(10 \times 60\text{s})}\left(\frac{1000\text{VA}}{1\text{kJ/s}}\right) = \mathbf{4.71A}$$

5-35 A cylinder initially contains nitrogen gas at a specified state. The gas is compressed polytropically until the volume is reduced by one-half. The work done and the heat transfer are to be determined.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** The N_2 is an ideal gas with constant specific heats. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

Properties The gas constant of N_2 are $R = 0.2968$ kPa.m^3/kg.K (Table A-1). The C_v value of N_2 at the average temperature (369+300)/2 = 335 K is 0.744 kJ/kg.K (Table A-2b).

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass crosses the system boundary. The energy balance for this closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{b,in} - Q_{out} = \Delta U = m(u_2 - u_1)$$

$$W_{b,in} - Q_{out} = mC_v(T_2 - T_1)$$

The final pressure and temperature of nitrogen are

$$P_2V_2^{1.3} = P_1V_1^{1.3} \longrightarrow P_2 = \left(\frac{V_1}{V_2}\right)^{1.3} P_1 = 2^{1.3}(100\text{kPa}) = 246.2\text{kPa}$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \longrightarrow T_2 = \frac{P_2}{P_1}\frac{V_2}{V_1}T_1 = \frac{246.2\text{kPa}}{100\text{kPa}} \times 0.5 \times (300\text{K}) = 369.3\text{K}$$

Then the boundary work for this polytropic process can be determined from Eq. 5-12,

$$W_{b,in} = -\int_1^2 P\,dV = -\frac{P_2V_2 - P_1V_1}{1-n} = -\frac{mR(T_2 - T_1)}{1-n}$$

$$= -\frac{(0.8\text{kg})(0.2968\text{kJ/kg}\cdot\text{K})(369.3 - 300)\text{K}}{1 - 1.3} = \mathbf{54.8kJ}$$

Substituting into the energy balance gives

$$Q_{out} = W_{b,in} - mC_v(T_2 - T_1)$$

$$= 54.8\text{ kJ} - (0.8\text{ kg})(0.744\text{ kJ/kg.K})(369.3 - 360)\text{K}$$

$$= \mathbf{13.6\ kJ}$$

5-36 It is observed that the air temperature in a room heated by electric baseboard heaters remains constant even though the heater operates continuously when the heat losses from the room amount to 8,000 kJ/h. The power rating of the heater is to be determined.

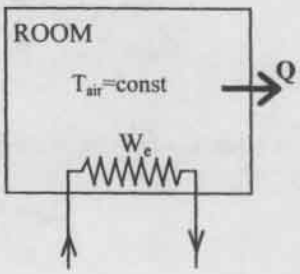
Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** The temperature of the room is said to remain constant during this process.

Analysis We take the room as the system. This is a closed system since no mass crosses the boundary of the system. The energy balance for this system reduces to

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{e,in} - Q_{out} = \Delta U = 0$$

$$W_{e,in} = Q_{out}$$



since $\Delta U = mC_v\Delta T = 0$ for isothermal processes of ideal gases. Thus,

$$\dot{W}_{e,in} = \dot{Q}_{out} = 8{,}000\text{kJ/h}\left(\frac{1\text{kW}}{3600\text{kJ/h}}\right) = \mathbf{2.22kW}$$

5-37E A cylinder initially contains air at a specified state. Heat is transferred to the air, and air expands isothermally. The boundary work done is to be determined.

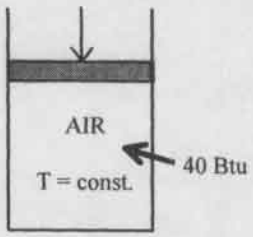
Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** The air is an ideal gas with constant specific heats. **3** The compression or expansion process is quasi-equilibrium.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass crosses the system boundary. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{in} - W_{b,out} = \Delta U = m(u_2 - u_1)$$

$$= mC_v(T_2 - T_1) = 0$$



since $u = u(T)$ for ideal gases, and thus $u_2 = u_1$ when $T_1 = T_2$. Therefore,

$$W_{b,out} = Q_{in} = \mathbf{40\ Btu}$$

5-38 A cylinder initially contains argon gas at a specified state. The gas is stirred while being heated and expanding isothermally. The amount of heat transfer is to be determined.

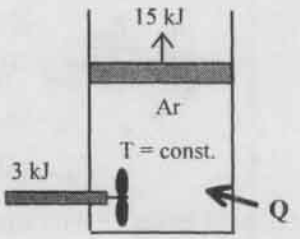
Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** The air is an ideal gas with constant specific heats. **3** The compression or expansion process is quasi-equilibrium.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass crosses the system boundary. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{in} + W_{pw,in} - W_{b,out} = \Delta U = m(u_2 - u_1)$$
$$= mC_v(T_2 - T_1) = 0$$

since $u = u(T)$ for ideal gases, and thus $u_2 = u_1$ when $T_1 = T_2$. Therefore,



$$Q_{in} = W_{b,out} - W_{pw,in} = 15 - 3 = \mathbf{12\ kJ}$$

5-39 A cylinder equipped with a set of stops for the piston is initially filled with air at a specified state. Heat is transferred to the air until the volume doubled. The work done by the air and the amount of heat transfer are to be determined, and the process is to be shown on a *P-v* diagram.

Assumptions **1** Air is an ideal gas with variable specific heats. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

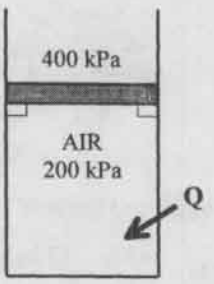
Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1).

Analysis We take the air in the cylinder as the system. This is a closed system since no mass crosses the boundary of the system. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} - W_{b,out} = \Delta U = m(u_3 - u_1)$$

$$Q_{in} = m(u_3 - u_1) + W_{b,out}$$

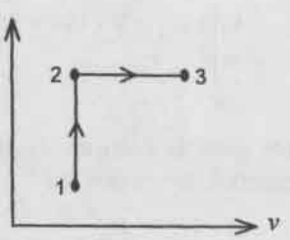


The initial and the final volumes and the final temperature of air are

$$V_1 = \frac{mRT_1}{P_1} = \frac{(3\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{ K})}{200\text{ kPa}} = 1.29\text{ m}^3$$

$$V_3 = 2V_1 = 2 \times 1.29 = 2.58\text{ m}^3$$

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \longrightarrow T_3 = \frac{P_3}{P_1}\frac{V_3}{V_1}T_1 = \frac{400\text{ kPa}}{200\text{ kPa}} \times 2 \times (300\text{ K}) = 1200\text{ K}$$



No work is done during process 1-2 since $V_1 = V_2$. The pressure remains constant during process 2-3 and the work done during this process is

$$W_{b,out} = \int_2^3 P\,dV = P_2(V_3 - V_2) = (400\text{kPa})(2.58 - 1.29)\text{m}^3 = \mathbf{516kJ}$$

The initial and final internal energies of air are (Table A-17)

$$u_1 = u_{@300\text{ K}} = 214.07\text{ kJ/kg}$$

$$u_2 = u_{@1200\text{ K}} = 933.33\text{ kJ/kg}$$

Then from the energy balance,

$$Q_{in} = (3\text{ kg})(933.33 - 214.07)\text{kJ/kg} + 516\text{ kJ} = \mathbf{2674\text{ kJ}}$$

Alternative solution The specific heat of air at the average temperature of $T_{ave} = (300 + 1200)/2 = 750$ K is, from Table A-2b, $C_{vave} = 0.800$ kJ/kg.K. Substituting,

$$Q_{in} = m(u_3 - u_1) + W_{b,out} \cong mC_v(T_3 - T_1) + W_{b,out}$$

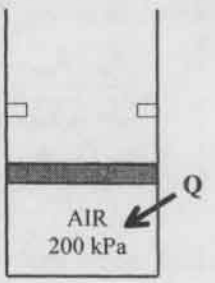
$$Q_{in} = (3\text{ kg})(0.800\text{ kJ/kg.K})(1200 - 300)\text{ K} + 516\text{ kJ} = \mathbf{2676\text{ kJ}}$$

5-40 A cylinder equipped with a set of stops on the top is initially filled with air at a specified state. Heat is transferred to the air until the piston hits the stops, and then the pressure doubles. The work done by the air and the amount of heat transfer are to be determined, and the process is to be shown on a *P-v* diagram.

Assumptions **1** Air is an ideal gas with variable specific heats. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** There are no work interactions involved. **3** The thermal energy stored in the cylinder itself is negligible.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1).

Analysis We take the air in the cylinder to be the system. This is a closed system since no mass crosses the boundary of the system. The energy balance for this closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} - W_{b,out} = \Delta U = m(u_3 - u_1)$$

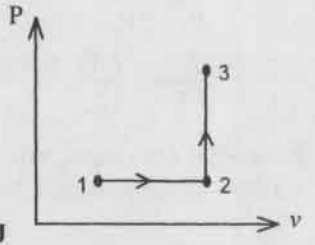
$$Q_{in} = m(u_3 - u_1) + W_{b,out}$$

The initial and the final volumes and the final temperature of air are determined from

$$V_1 = \frac{mRT_1}{P_1} = \frac{(3\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{ K})}{200\text{ kPa}} = 1.29\text{ m}^3$$

$$V_3 = 2V_1 = 2 \times 1.29 = 2.58\text{ m}^3$$

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \quad \longrightarrow \quad T_3 = \frac{P_3}{P_1}\frac{V_3}{V_1}T_1 = \frac{400\text{ kPa}}{200\text{ kPa}} \times 2 \times (300\text{ K}) = 1200\text{ K}$$



No work is done during process 2-3 since $V_2 = V_3$. The pressure remains constant during process 1-2 and the work done during this process is

$$W_b = \int_2^3 P\,dV = P_2(V_3 - V_2) = (200\text{kPa})(2.58 - 1.29)\text{m}^3\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = \mathbf{258kJ}$$

The initial and final internal energies of air are (Table A-17)

$$u_1 = u_{@300\text{ K}} = 214.07\text{ kJ/kg}$$

$$u_2 = u_{@1200\text{ K}} = 933.33\text{ kJ/kg}$$

Substituting,

$$Q_{\text{in}} = (3\text{ kg})(933.33 - 214.07)\text{kJ/kg} + 258\text{ kJ} = \mathbf{2416\text{ kJ}}$$

Alternative solution The specific heat of air at the average temperature of $T_{\text{ave}} = (300 + 1200)/2 = 750$ K is, from Table A-2b, $C_{\text{vave}} = 0.800$ kJ/kg.K. Substituting

$$Q_{in} = m(u_3 - u_1) + W_{b,out} \cong mC_v(T_3 - T_1) + W_{b,out}$$

$$= (3\text{ kg})(0.800\text{ kJ/kg.K})(1200 - 300)\text{ K} + 258\text{ kJ} = \mathbf{2418\text{ kJ}}$$

Closed System Energy Analysis: Solids and Liquids

5-41 A number of brass balls are to be quenched in a water bath at a specified rate. The rate at which heat needs to be removed from the water in order to keep its temperature constant is to be determined.

Assumptions **1** The thermal properties of the balls are constant. **2** The balls are at a uniform temperature before and after quenching. **3** The changes in kinetic and potential energies are negligible.

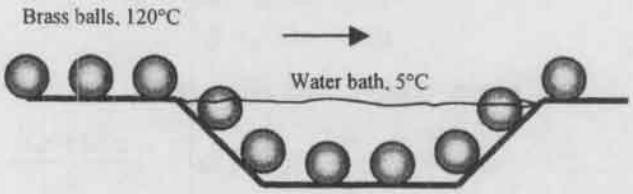
Properties The density and specific heat of the brass balls are given to be $\rho = 8522$ kg/m^3 and $C_p = 0.385$ kJ/kg.°C.

Analysis We take a single ball as the system. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U_{ball} = m(u_2 - u_1)$$

$$Q_{out} = mC(T_1 - T_2)$$



The total amount of heat transfer from a ball is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (8522 \text{ kg/m}^3)\frac{\pi(0.05 \text{ m})^3}{6} = 0.558 \text{ kg}$$

$$Q_{out} = mC(T_1 - T_2) = (0.558 \text{ kg})(0.385 \text{ kJ/kg.°C})(120 - 74)°\text{C} = 9.88 \text{ kJ/ball}$$

Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{\text{ball}} Q_{\text{ball}} = (100 \text{ balls/min}) \times (9.88 \text{ kJ/ball}) = \mathbf{988 \text{ kJ/min}}$$

Therefore, heat must be removed from the water at a rate of 988 kJ/min in order to keep its temperature constant at 50°C since energy input must be equal to energy output for a system whose energy level remains constant. That is, $E_{in} = E_{out}$ when $\Delta E_{\text{system}} = 0$.

5-42 A number of aluminum balls are to be quenched in a water bath at a specified rate. The rate at which heat needs to be removed from the water in order to keep its temperature constant is to be determined.

Assumptions **1** The thermal properties of the balls are constant. **2** The balls are at a uniform temperature before and after quenching. **3** The changes in kinetic and potential energies are negligible.

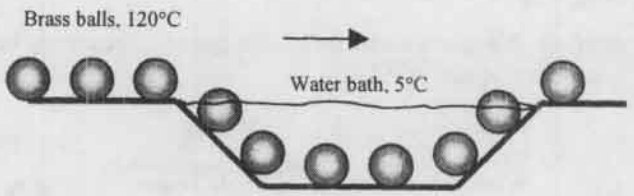
Properties The density and specific heat of aluminum at the average temperature of (120+74)/2 = 97°C = 370 K are $\rho = 2700$ kg/m^3 and $C_p = 0.959$ kJ/kg.°C (Table A-3).

Analysis We take a single ball as the system. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U_{ball} = m(u_2 - u_1)$$

$$Q_{out} = mC(T_1 - T_2)$$



The total amount of heat transfer from a ball is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (2700 \text{ kg/m}^3)\frac{\pi(0.05 \text{ m})^3}{6} = 0.1767 \text{ kg}$$

$$Q_{out} = mC(T_1 - T_2) = (0.1767 \text{ kg})(0.959 \text{ kJ/kg.°C})(120 - 74)°\text{C} = 7.80 \text{ kJ/ball}$$

Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{\text{ball}} Q_{\text{ball}} = (100 \text{ balls/min}) \times (7.80 \text{ kJ/ball}) = \mathbf{780 \text{ kJ/min}}$$

Therefore, heat must be removed from the water at a rate of 780 kJ/min in order to keep its temperature constant at 50°C since energy input must be equal to energy output for a system whose energy level remains constant. That is, $E_{in} = E_{out}$ when $\Delta E_{\text{system}} = 0$.

5-43E A person shakes a canned of drink in a iced water to cool it. The mass of the ice that will melt by the time the canned drink is cooled to a specified temperature is to be determined.

Assumptions **1** The thermal properties of the drink are constant, and are taken to be the same as those of water. **2** The effect of agitation on the amount of ice melting is negligible. **3** The thermal energy capacity of the aluminum can itself is negligible, and thus it does not need to be considered in the analysis.

Properties The density and specific heat of water at the average temperature of (75+45)/2 = 60°F are ρ = 62.3 lbm/ft^3, and C_p = 1.0 Btu/lbm.°F (Table A-3E). The heat of fusion of water is 143.5 Btu/lbm.

Analysis We take a canned drink as the system. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U_{\text{canned drink}} = m(u_2 - u_1)$$

$$Q_{out} = mC(T_1 - T_2)$$



Noting that 1 gal = 128 oz and 1 ft^3 = 7.48 gal = 957.5 oz, the total amount of heat transfer from a ball is

$$m = \rho V = (62.3\ \text{lbm/ft}^3)(12\ \text{oz/can})\left(\frac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)\left(\frac{1\ \text{gal}}{128\ \text{fluid oz}}\right) = 0.781\ \text{lbm/can}$$

$$Q_{out} = mC(T_1 - T_2) = (0.781\ \text{lbm/can})(1.0\ \text{Btu/lbm.°F})(75 - 45)\text{°F} = 23.4\ \text{Btu/can}$$

Noting that the heat of fusion of water is 14.5 Btu/lbm, the amount of ice that will melt to cool the drink is

$$m_{ice} = \frac{Q_{out}}{h_{if}} = \frac{23.4\ \text{Btu/can}}{143.5\ \text{Btu/lbm}} = \mathbf{0.163\ lbm}\quad \text{(per can of drink)}$$

since heat transfer to the ice must be equal to heat transfer from the can.

Discussion The actual amount of ice melted will be greater since agitation will also cause some ice to melt.

5-44 An iron whose base plate is made of an aluminum alloy is turned on. The minimum time for the plate to reach a specified temperature is to be determined.

Assumptions **1** It is given that 85 percent of the heat generated in the resistance wires is transferred to the plate. **2** The thermal properties of the plate are constant. **3** Heat loss from the plate during heating is disregarded since the minimum heating time is to be determined. **4** There are no changes in kinetic and potential energies. **5** The plate is at a uniform temperature at the end of the process.

Properties The density and specific heat of the aluminum alloy plate are given to be $\rho = 2770$ kg/m^3 and $C_p = 875$ kJ/kg.°C.

Analysis The mass of the iron's base plate is

$$m = \rho V = \rho LA = (2770 \text{ kg/m}^3)(0.005 \text{ m})(0.03 \text{ m}^2) = 0.4155 \text{ kg}$$

Noting that only 85 percent of the heat generated is transferred to the plate, the rate of heat transfer to the iron's base plate is

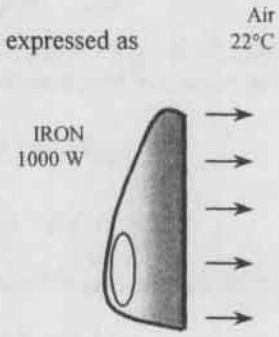
$$\dot{Q}_{\text{in}} = 0.85 \times 1000 \text{ W} = 850 \text{ W}$$

We take plate to be the system. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} = \Delta U_{\text{plate}} = m(u_2 - u_1)$$

$$\dot{Q}_{in}\Delta t = mC(T_2 - T_1)$$



Solving for Δt and substituting,

$$\Delta t = \frac{mC\Delta T_{\text{plate}}}{\dot{Q}_{\text{in}}} = \frac{(0.4155 \text{ kg})(875 \text{ J/kg.°C})(140 - 22)°\text{C}}{850 \text{ J/s}} = \mathbf{51.8 \text{ s}}$$

which is the time required for the plate temperature to reach 140 °C.

5-45 Stainless steel ball bearings leaving the oven at a specified uniform temperature at a specified rate are exposed to air and are cooled before they are dropped into the water for quenching. The rate of heat transfer from the ball bearing to the air is to be determined. √

Assumptions **1** The thermal properties of the bearing balls are constant. **2** The kinetic and potential energy changes of the balls are negligible. **3** The balls are at a uniform temperature at the end of the process

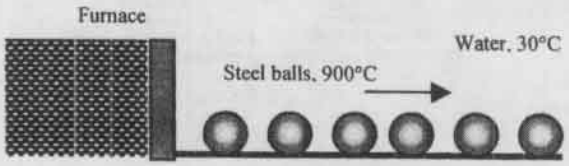
Properties The density and specific heat of the ball bearings are given to be $\rho = 8085$ kg/m^3 and $C_p = 0.480$ kJ/kg.°C.

Analysis We take a single bearing ball as the system. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U_{ball} = m(u_2 - u_1)$$

$$Q_{out} = mC(T_1 - T_2)$$



The total amount of heat transfer from a ball is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (8085 \text{ kg/m}^3)\frac{\pi(0.012 \text{ m})^3}{6} = 0.007315 \text{ kg}$$

$$Q_{out} = mC(T_1 - T_2) = (0.007315 \text{ kg})(0.480 \text{ kJ/kg.°C})(900 - 850)°\text{C} = 0.1756 \text{ kJ/ball}$$

Then the rate of heat transfer from the balls to the air becomes

$$\dot{Q}_{total} = \dot{n}_{\text{ball}} Q_{\text{out (per ball)}} = (1400 \text{ balls/min}) \times (0.1756 \text{ kJ/ball}) = \mathbf{245.8 \text{ kJ/min} = 4.10 \text{ kW}}$$

Therefore, heat is lost to the air at a rate of 4.10 kW.

5-46 Carbon steel balls are to be annealed at a rate of 2500/h by heating them first and then allowing them to cool slowly in ambient air at a specified rate. The total rate of heat transfer from the balls to the ambient air is to be determined.

Assumptions **1** The thermal properties of the balls are constant. **2** There are no changes in kinetic and potential energies. **3** The balls are at a uniform temperature at the end of the process

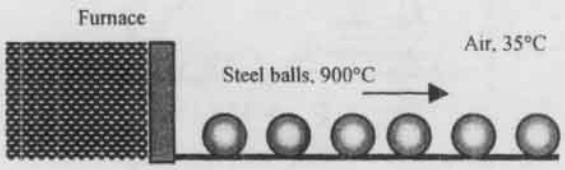
Properties The density and specific heat of the balls are given to be $\rho = 7833$ kg/m^3 and $C_p = 0.465$ kJ/kg.°C.

Analysis We take a single ball as the system. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U_{ball} = m(u_2 - u_1)$$

$$Q_{out} = mC(T_1 - T_2)$$



(*b*) The amount of heat transfer from a single ball is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (7833 \text{ kg/m}^3)\frac{\pi(0.008 \text{ m})^3}{6} = 0.00210 \text{ kg}$$

$$Q_{out} = mC_p(T_1 - T_2) = (0.0021 \text{ kg})(0.465 \text{ kJ/kg.°C})(900 - 100)°\text{C} = 781 \text{ J} = 0.781 \text{ kJ (per ball)}$$

Then the total rate of heat transfer from the balls to the ambient air becomes

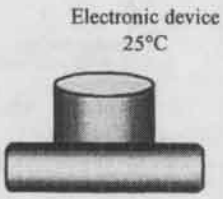
$$\dot{Q}_{out} = \dot{n}_{\text{ball}} Q_{out} = (2500 \text{ balls/h}) \times (0.781 \text{ kJ/ball}) = 1{,}953 \text{ kJ/h} = \mathbf{542\ W}$$

5-47 An electronic device is on for 5 minutes, and off for several hours. The temperature of the device at the end of the 5-min operating period is to be determined for the cases of operation with and without a heat sink.

Assumptions **1** The device and the heat sink are isothermal. **2** The thermal properties of the device and of the sink are constant. **3** Heat loss from the device during on time is disregarded since the highest possible temperature is to be determined.

Properties The specific heat of the device is given to be C_p = 850 J/kg.°C. The specific heat of aluminum at room temperature of 300 K is 902 J/kg.°C (Table A-3).

Analysis We take the device to be the system. Noting that electrical energy is supplied, the energy balance for this closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{e,in} = \Delta U_{\text{device}} = m(u_2 - u_1)$$

$$\dot{W}_{e,in}\Delta t = mC(T_2 - T_1)$$

Substituting, the temperature of the device at the end of the process is determined to be

$$(30\ \text{J/s})(5 \times 60\ \text{s}) = (0.020\ \text{kg})(850\ \text{J/kg.°C})(T_2 - 25)°\text{C} \quad \rightarrow \quad T_2 = \mathbf{554°C} \text{ (without the heat sink)}$$

Case 2 When a heat sink is attached, the energy balance can be expressed as

$$W_{e,in} = \Delta U_{\text{device}} + \Delta U_{\text{heat sink}}$$

$$\dot{W}_{e,in}\Delta t = mC(T_2 - T_1)_{\text{device}} + mC(T_2 - T_1)_{\text{heat sink}}$$

Substituting, the temperature of the device-heat sink combination is determined to be

$$(30\ \text{J/s})(5 \times 60\ \text{s}) = (0.020\ \text{kg})(850\ \text{J/kg.°C})(T_2 - 25)°\text{C} + (0.200\ \text{kg})(902\ \text{J/kg.°C})(T_2 - 25)°\text{C}$$

$$T_2 = \mathbf{70.6°C} \text{ (with heat sink)}$$

Discussion These are the maximum temperatures. In reality, the temperatures will be lower because of the heat losses to the surroundings.

5-48 An egg is dropped into boiling water. The amount of heat transfer to the egg by the time it is cooked is to be determined. √

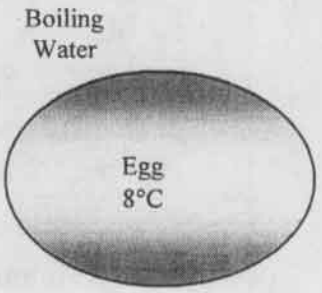
Assumptions **1** The egg is spherical in shape with a radius of $r_0 = 2.75$ cm. **2** The thermal properties of the egg are constant. **3** Energy absorption or release associated with any chemical and/or phase changes within the egg is negligible. **4** There are no changes in kinetic and potential energies.

Properties The density and specific heat of the egg are given to be $\rho = 1020$ kg/m^3 and $C_p = 3.32$ kJ/kg.°C.

Analysis We take the egg as the system. This is a closes system since no mass enters or leaves the egg. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} = \Delta U_{egg} = m(u_2 - u_1) = mC(T_2 - T_1)$$



Then the mass of the egg and the amount of heat transfer become

$$m = \rho V = \rho \frac{\pi D^3}{6} = (1020 \text{ kg/m}^3)\frac{\pi(0.055 \text{ m})^3}{6} = 0.0889 \text{ kg}$$

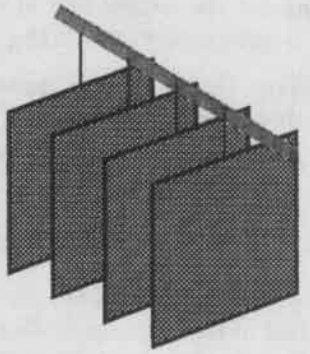
$$Q_{in} = mC_p(T_2 - T_1) = (0.0889 \text{ kg})(3.32 \text{ kJ/kg.°C})(70 - 8)°\text{C} = \mathbf{18.3 \text{ kJ}}$$

5-49E Large brass plates are heated in an oven at a rate of 300/min. The rate of heat transfer to the plates in the oven is to be determined.

Assumptions **1** The thermal properties of the plates are constant. **2** The changes in kinetic and potential energies are negligible.

Properties The density and specific heat of the brass are given to be $\rho = 532.5$ lbm/ft^3 and $C_p = 0.091$ Btu/lbm.°F.

Analysis We take the plate to be the system. The energy balance for this closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} = \Delta U_{\text{plate}} = m(u_2 - u_1) = mC(T_2 - T_1)$$

The mass of each plate and the amount of heat transfer to each plate is

$$m = \rho V = \rho LA = (532.5 \text{ lbm/ft}^3)[(1.2/12 \text{ ft})(2 \text{ ft})(2 \text{ ft})] = 213 \text{ lbm}$$

$$Q_{in} = mC(T_2 - T_1) = (213 \text{ lbm/plate})(0.091 \text{ Btu/lbm.°F})(1000 - 75)°\text{F} = 17{,}930 \text{ Btu/plate}$$

Then the total rate of heat transfer to the plates becomes

$$\dot{Q}_{total} = \dot{n}_{\text{plate}} Q_{\text{in, per plate}} = (300 \text{ plates/min}) \times (17{,}930 \text{ Btu/plate}) = \mathbf{5{,}379{,}000 \text{ Btu/min} = 89{,}650 \text{ Btu/s}}$$

5-50 Long cylindrical steel rods are heat-treated in an oven. The rate of heat transfer to the rods in the oven is to be determined.

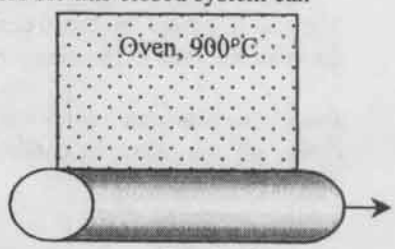
Assumptions **1** The thermal properties of the rods are constant. **2** The changes in kinetic and potential energies are negligible.

Properties The density and specific heat of the steel rods are given to be $\rho = 7833$ kg/m^3 and $C_p = 0.465$ kJ/kg.°C.

Analysis Noting that the rods enter the oven at a velocity of 3 m/min and exit at the same velocity, we can say that a 5-m long section of the rod is heated in the oven in 1 min. Then the mass of the rod heated in 1 minute is

$$m = \rho V = \rho LA = \rho L(\pi D^2/4) = (7833 \text{ kg/m}^3)(3 \text{ m})[\pi(0.1 \text{ m})^2/4] = 184.6 \text{ kg}$$

We take the 5-m section of the rod in the oven as the system. The energy balance for this closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} = \Delta U_{\text{rod}} = m(u_2 - u_1) = mC(T_2 - T_1)$$

Substituting,

$$Q_{in} = mC(T_2 - T_1) = (184.6 \text{ kg})(0.465 \text{ kJ/kg.°C})(700 - 30)°\text{C} = 57{,}512 \text{ kJ}$$

Noting that this much heat is transferred in 1 min, the rate of heat transfer to the rod becomes

$$\dot{Q}_{in} = Q_{\text{in}}/\Delta t = (57{,}512 \text{ kJ})/(1 \text{ min}) = 57{,}512 \text{ kJ/min} = \mathbf{958.5 \text{ kW}}$$

5-51 Fresh strawberries with a water content of 88% in nylon boxes are to be cooled to 4°C at a rate of 60 boxes/h. The rate of heat removal from the strawberries together with their boxes, and the percent error involved if the strawberry boxes were ignored in calculations are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of strawberries and boxes are constant.

Properties The specific heat of strawberries is given to be C_p = 3.89 kJ/kg.°C. The specific heat of nylon box is given to be C_p = 1.7 kJ/kg.°C. The heat of respiration of strawberries is given to be 0.21 W/kg.

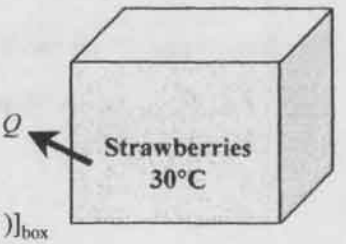
Analysis Noting that the strawberries are cooled at a rate of 60 boxes per hour, the total amounts of strawberries and the box material cooled per hour are

$$m_{\text{strawberry}} = (\text{Mass per box})(\text{Number of boxes per hour}) = (23\ \text{kg/box})(60\ \text{box}) = 1380\ \text{kg}$$

$$m_{\text{box}} = (\text{Mass per box})(\text{Number of boxes per hour}) = (0.8\ \text{kg/box})(60\ \text{box/h}) = 48\ \text{kg}$$

Analysis We take the strawberries together with their boxes as the system. A practical way of considering the heat of respiration is to disregard the chemical changes that occur in the food, and to treat the heat of respiration as heat input. Then the energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$



$$\begin{aligned}
Q_{\text{respiration,in}} - Q_{out} &= \Delta U_{\text{total}} \\
&= \Delta U_{\text{strawberries}} + \Delta U_{\text{box}} \\
&= [mC(T_2 - T_1)]_{\text{strawberries}} + [mC(T_2 - T_1)]_{\text{box}} \\
Q_{out} &= Q_{\text{respiration,in}} - [mC(T_2 - T_1)]_{\text{strawberries}} - [mC(T_2 - T_1)]_{\text{box}} \\
&= \dot{Q}_{\text{respiration,in}}\Delta t + [mC(T_1 - T_2)]_{\text{strawberries}} + [mC(T_1 - T_2)]_{\text{box}}
\end{aligned}$$

Dividing by Δt, it can be expressed in the rate form as

$$\dot{Q}_{out} = (m_{\text{strawberries}}\dot{q}_{\text{respiration}}) + [mC(T_1 - T_2)/\Delta t]_{\text{strawberries}} + [mC(T_1 - T_2)/\Delta t]_{\text{box}}$$

Substituting, the rate of heat transfer is determined to be

$$\begin{aligned}
\dot{Q}_{\text{out}} &= (1380\ \text{kg})(0.00021\ \text{kW/kg}) + [(1380\ \text{kg})(3.89\ \text{kJ/kg.°C})(30 - 4)°\text{C}/1\ \text{h}] \\
&\quad + [(48\ \text{kg})(1.7\ \text{kJ/kg.°C})(30 - 4)°\text{C}]/(1\ \text{h}) \\
&= 0.290\ \text{kJ/s}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) + 139{,}573\ \text{kJ/h} + 2122\ \text{kJ/h} \\
&= \mathbf{142{,}739\ kJ/h} = 39.6\ \text{kW}
\end{aligned}$$

The heat loss from the boxes constitute 2122/142,739 = 0.015 or just 1.5% of the total heat loss. Therefore, the error involved in ignoring the boxes in calculations is **negligible**.

Discussion Note that most of the cooling load is due to the removal of the latent heat during the phase change process. Also, the cooling load due to the box is negligible.

5-52 Lettuce is to be vacuum cooled in an insulated spherical vacuum chamber. The final pressure in the vacuum chamber and the amount of moisture removed from the lettuce are to be determined.

Assumptions **1** The thermal properties of lettuce are constant. **2** Heat transfer through the walls of the vacuum chamber is negligible. **3** All the air in the chamber is sucked out by the vacuum pump so that there is only vapor in the chamber at the end of the process.

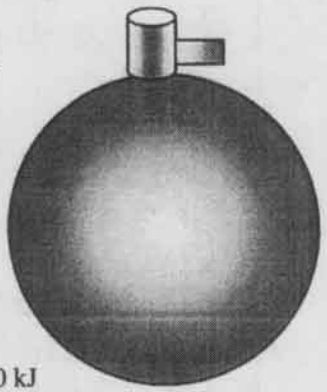
Properties The specific heat of the lettuce is 4.02 kJ/kg.°C. The saturation pressure of water at 2°C is 0.712 kPa. At the average temperature of (24 + 2)/2 = 13°C, the latent heat of vaporization of water is h_{fg} = 2470 kJ/kg (Table A-4).

Analysis (*a*) The final vapor pressure in the vacuum chamber will be the saturation pressure at the final temperature of 2°C, which is determined from the saturated water tables to be

$$P_{final} = P_{sat\ @\ 2°C} = \mathbf{0.712\ kPa}$$

Assuming all the air in the chamber is already sucked out by the vacuum pump, this vapor pressure will be equivalent to the final pressure in the chamber.

(*b*) We take the lettuce as the system. Disregarding the small amount of water that evaporates, the energy balance for this closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U = [mC(T_2 - T_1)]_{lettuce}$$

Then the amount of heat transfer to cool the lettuce from 24 to 2°C becomes

$$Q_{lettuce,out} = [mC(T_1 - T_2)]_{lettuce} = (5000\ \text{kg})(4.02\ \text{kJ/kg.°C})(24 - 2)°\text{C} = 442{,}200\ \text{kJ}$$

Each kg of water in lettuce absorbs 2470 kJ of heat as it evaporates. Disregarding any heat gain through the walls of the vacuum chamber, the amount of moisture removed is determined to be

$$Q_{lettuce} = m_{evap} h_{fg} \rightarrow m_{evap} = \frac{Q_{lettuce}}{h_{fg}} = \frac{442{,}200\ \text{kJ}}{2470\ \text{kJ/kg}} = \mathbf{179.0\ kg}$$

5-53 A box of shrimp is to be frozen in a freezer. The amount of heat that needs to be removed is to be determined. √

Assumptions **1** The thermal properties of fresh and frozen shrimp are constant. **2** The entire water content of the shrimps freezes during the process.

Properties For shrimp, the freezing temperature is -2.2°C, the latent heat of fusion is 277 kJ/kg, the specific heat is 3.62 kJ/kg·°C above freezing and 1.89 kJ/kg·°C below freezing (given). The specific heat of polyethylene box is given to be 2.33 kJ/kg·°C.

Analysis We take the shrimp together with their boxes as the system. The energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U_{\text{total}} = [\Delta U_{\text{fresh}} + \Delta U_{\text{during freezing}} + \Delta U_{\text{frozen}}]_{\text{shrimp}} + \Delta U_{\text{box}}$$

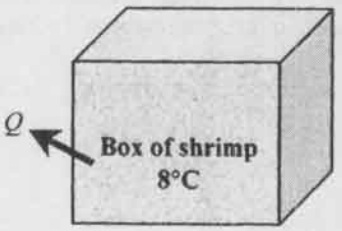
$$Q_{out} = -[\Delta U_{\text{fresh}} + \Delta U_{\text{during freezing}} + \Delta U_{\text{frozen}}]_{\text{shrimp}} - \Delta U_{\text{box}}$$

$$= [mC(T_1 - T_{fp})_{\text{fresh}} + mh_{latent} + mC(T_{fp} - T_2)_{\text{frozen}}]_{\text{shrimp}} + [mC(T_1 - T_2)]_{\text{box}}$$

Substituting, the total heat transfer is determined to be

$$\begin{aligned} Q_{\text{out}} &= (40\text{ kg})(3.62\text{ kJ/kg.°C})[8-(-2.2)]\text{°C} + (40\text{ kg})(277\text{ kJ/kg}) \\ &\quad + (40\text{ kg})(1.89\text{ kJ/kg.°C})[\text{-}2.2-(-18)]\text{°C} + (1.2\text{ kg})(2.3\text{ kJ/kg.°C})[8-(-18)\text{°C} \\ &= 1477 + 11{,}080 + 1195 + 72 \\ &= \mathbf{13{,}800\text{ kJ}} \end{aligned}$$

Discussion Note that most of the cooling load (80 percent of it) is due to the removal of the latent heat during the phase change process. Also, the cooling load due to the box is negligible (less than 1 percent).



5-54 A box of shrimp is to be frozen in a freezer. The amount of heat that needs to be removed is to be determined. √

Assumptions **1** The thermal properties of fresh and frozen shrimp are constant. **2** The entire water content of the shrimps freezes during the process. **3** The effect of the box is negligible.

Properties For shrimp, the freezing temperature is -2.2°C, the latent heat of fusion is 277 kJ/kg, the specific heat is 3.62 kJ/kg·°C above freezing and 1.89 kJ/kg·°C below freezing (given).

Analysis We take the shrimp within the boxes as the system. The energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U_{\text{total}} = [\Delta U_{\text{fresh}} + \Delta U_{\text{during freezing}} + \Delta U_{\text{frozen}}]_{\text{shrimp}}$$

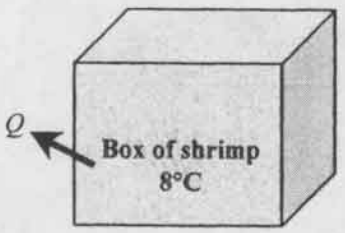
$$Q_{out} = -[\Delta U_{\text{fresh}} + \Delta U_{\text{during freezing}} + \Delta U_{\text{frozen}}]_{\text{shrimp}}$$

$$= [mC(T_1 - T_{fp})_{\text{fresh}} + mh_{latent} + mC(T_{fp} - T_2)_{\text{frozen}}]_{\text{shrimp}}$$

Substituting, the total heat transfer is determined to be

$$\begin{aligned} Q_{\text{out}} &= (40\text{ kg})(3.62\text{ kJ/kg.°C})[8-(-2.2)]\text{°C} + (40\text{ kg})(277\text{ kJ/kg}) \\ &\quad + (40\text{ kg})(1.89\text{ kJ/kg.°C})[-2.2-(-18)]\text{°C} \\ &= 1477 + 11{,}080 + 1195 + 72 \\ &= \mathbf{13{,}750\text{ kJ}} \end{aligned}$$

Discussion Note that most of the cooling load (80 percent of it) is due to the removal of the latent heat during the phase change process. Also, it can be shown that the cooling load due to the box is negligible (less than 1 percent).



Steady Flow Energy Balance: Nozzles and Diffusers

5-55C A steady-flow system involves no changes with time anywhere within the system or at the system boundaries

5-56C No.

5-57C It is mostly converted to internal energy as shown by a rise in the fluid temperature.

5-58C The kinetic energy of a fluid increases at the expense of the internal energy as evidenced by a decrease in the fluid temperature.

5-59C Heat transfer to the fluid as it flows through a nozzle is desirable since it will probably increase the kinetic energy of the fluid. Heat transfer from the fluid will decrease the exit velocity.

5-60 Air is accelerated in a nozzle from 30 m/s to 180 m/s. The mass flow rate, the exit temperature, and the exit area of the nozzle are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with constant specific heats. **3** Potential energy changes are negligible. **4** The device is adiabatic and thus heat transfer is negligible. **5** There are no work interactions.

Properties The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1). The specific heat of air at the anticipated average temperature of 450 K is C_p = 1.02 kJ/kg.°C (Table A-2).

Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Using the ideal gas relation, the specific volume and the mass flow rate of air are determined to be

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(473\text{K})}{300\text{kPa}} = 0.4525\text{m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{v_1}A_1\mathbf{V}_1 = \frac{1}{0.4525\text{m}^3/\text{kg}}(0.008\text{m}^2)(30\text{m/s}) = \mathbf{0.5304kg/s}$$

(*b*) We take nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta\text{pe} \cong 0)$$

$$0 = h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} \longrightarrow 0 = C_{p,ave}(T_2 - T_1) + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}$$

Substituting,

$$0 = (1.02\text{kJ/kg}\cdot\text{K})(T_2 - 200^\circ\text{C}) + \frac{(180\text{m/s})^2 - (30\text{m/s})^2}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right)$$

It yields $T_2 = \mathbf{184.6°C}$

(*c*) The specific volume of air at the nozzle exit is

$$v_2 = \frac{RT_2}{P_2} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(184.6 + 273\text{K})}{100\text{kPa}} = 1.313\text{m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{v_2}A_2\mathbf{V}_2 \longrightarrow 0.5304\text{kg/s} = \frac{1}{1.313\text{m}^3/\text{kg}}A_2(180\text{m/s})$$

$$A_2 = 0.00387\ \text{m}^2 = \mathbf{38.7\ cm^2}$$



5-61 Steam is accelerated in a nozzle from a velocity of 80 m/s. The mass flow rate, the exit velocity, and the exit area of the nozzle are to be determined.

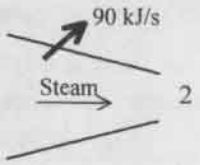
Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** There are no work interactions.

Properties From the steam tables (Table A-6)

$$\left.\begin{matrix} P_1 = 5\text{MPa} \\ T_1 = 500^\circ\text{C} \end{matrix}\right\} \begin{matrix} v_1 = 0.06857\text{m}^3/\text{kg} \\ h_1 = 3433\text{kJ/kg} \end{matrix}$$

and

$$\left.\begin{matrix} P_2 = 2\text{MPa} \\ T_2 = 400^\circ\text{C} \end{matrix}\right\} \begin{matrix} v_2 = 0.15120\text{m}^3/\text{kg} \\ h_2 = 3247.6\text{kJ/kg} \end{matrix}$$



Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

The mass flow rate of steam is

$$\dot{m} = \frac{1}{v_1}\mathbf{V}_1 A_1 = \frac{1}{0.06857\text{m}^3/\text{kg}}(80\text{m/s})(50\times10^{-4}\text{m}^2) = \mathbf{5.833kg/s}$$

(*b*) We take nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\begin{matrix}\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}\end{matrix}} = \underbrace{\Delta\dot{E}_{\text{system}}}^{\nearrow 0 \text{ (steady)}}_{\begin{matrix}\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}\end{matrix}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{Q}_{\text{out}} + \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{\text{W}} \cong \Delta\text{pe} \cong 0)$$

$$-\dot{Q}_{\text{out}} = \dot{m}\left(h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right)$$

Substituting, the exit velocity of the steam is determined to be

$$-90\text{kJ/s} = (5.833\text{kg/s})\left(3247.6 - 3433.8 + \frac{\mathbf{V}_2^2 - (80\text{m/s})^2}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right)\right)$$

It yields $\mathbf{V_2 = 585.2\ m/s}$

(*c*) The exit area of the nozzle is determined from

$$\dot{m} = \frac{1}{v_2}\mathbf{V}_2 A_2 \longrightarrow A_2 = \frac{\dot{m}v_2}{\mathbf{V}_2} = \frac{(5.833\text{kg/s})(0.1512\text{m}^3/\text{kg})}{585.2\text{m/s}} = \mathbf{15.1\times10^{-4}\,m^2}$$

5-62E Air is accelerated in a nozzle from 150 ft/s to 900 ft/s. The exit temperature of air and the exit area of the nozzle are to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with variable specific heats. **3** Potential energy changes are negligible. **4** There are no work interactions.

Properties The enthalpy of air at the inlet is $h_1 = 243.47$ Btu/lbm (Table A-17E).

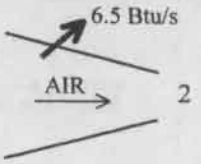
Analysis (a) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}{}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{Q}_{\text{out}} + \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{W} \cong \Delta pe \cong 0)$$

$$-\dot{Q}_{\text{out}} = \dot{m}\left(h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right)$$



or,

$$h_2 = q_{\text{out}} + h_1 - \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}$$

$$= -6.5\text{Btu/lbm} + 143.47\text{Btu/lbm} - \frac{(900\text{ft/s})^2 - (150\text{ft/s})^2}{2}\left(\frac{1\text{Btu/lbm}}{25{,}037\text{ft}^2/\text{s}^2}\right)$$

$$= 121.3 \text{ Btu/lbm}$$

Thus, from Table A-17E, $T_2 = \mathbf{508\ R}$

(*b*) The exit area is determined from the conservation of mass relation,

$$\frac{1}{v_2} A_2 \mathbf{V}_2 = \frac{1}{v_1} A_1 \mathbf{V}_1 \longrightarrow A_2 = \frac{v_2}{v_1}\frac{\mathbf{V}_1}{\mathbf{V}_2} A_1 = \left(\frac{RT_2/P_2}{RT_1/P_1}\right)\frac{\mathbf{V}_1}{\mathbf{V}_2} A_1$$

$$A_2 = \frac{(508/14.7)(150\text{ft/s})}{(600/50)(900\text{ft/s})}\left(0.1\text{ft}^2\right) = \mathbf{0.048ft^2}$$

5-63 Steam is accelerated in a nozzle from a velocity of 40 m/s to 300 m/s. The exit temperature and the ratio of the inlet-to-exit area of the nozzle are to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** There are no work interactions. **4** The device is adiabatic and thus heat transfer is negligible.

Properties From the steam tables (Table A-6),

$$\left.\begin{matrix} P_1 = 3\text{MPa} \\ T_1 = 400^\circ\text{C} \end{matrix}\right\} \begin{matrix} v_1 = 0.09936\text{m}^3/\text{kg} \\ h_1 = 3230.9\text{kJ/kg} \end{matrix}$$

Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}{}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$



$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta pe \cong 0)$$

$$0 = h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}$$

or,

$$h_2 = h_1 - \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} = 3230.9\text{kJ/kg} - \frac{(300\text{m/s})^2 - (20\text{m/s})^2}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right) = 3186.7\text{kJ/kg}$$

Thus, $\left.\begin{matrix} P_2 = 2.5\text{MPa} \\ h_2 = 3186.7\text{kJ/kg} \end{matrix}\right\} \begin{matrix} T_2 = \mathbf{376.7^\circ C} \\ v_2 = 0.1153\text{m}^3/\text{kg} \end{matrix}$

(*b*) The ratio of the inlet to exit area is determined from the conservation of mass relation,

$$\frac{1}{v_2} A_2 \mathbf{V}_2 = \frac{1}{v_1} A_1 \mathbf{V}_1 \longrightarrow \frac{A_1}{A_2} = \frac{v_2}{v_1}\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{(0.09936\text{m}^3/\text{kg})(300\text{m/s})}{(0.1153\text{m}^3/\text{kg})(40\text{m/s})} = \mathbf{6.46}$$

5-64 Air is accelerated in a nozzle from 120 m/s to 380 m/s. The exit temperature and pressure of air are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with variable specific heats. **3** Potential energy changes are negligible. **4** The device is adiabatic and thus heat transfer is negligible. **5** There are no work interactions.

Properties The enthalpy of air at the inlet temperature of 500 K is $h_1 = 503.02$ kJ/kg (Table A-17).

Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta pe \cong 0)$$

$$0 = h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}$$

or,

$$h_2 = h_1 - \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} = 503.02\text{kJ/kg} - \frac{(380\text{m/s})^2 - (120\text{m/s})^2}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right) = 438.02\text{kJ/kg}$$

Then from Table A-17 we read $T_2 = \mathbf{436.5\ K}$

(*b*) The exit pressure is determined from the conservation of mass relation,

$$\frac{1}{v_2} A_2 \mathbf{V}_2 = \frac{1}{v_1} A_1 \mathbf{V}_1 \quad \longrightarrow \quad \frac{1}{RT_2/P_2} A_2 \mathbf{V}_2 = \frac{1}{RT_1/P_1} A_1 \mathbf{V}_2$$

Thus,

$$P_2 = \frac{A_1 T_2 \mathbf{V}_1}{A_2 T_1 \mathbf{V}_2} P_1 = \frac{2}{1}\frac{(436.5\text{K})(120\text{m/s})}{(500\text{K})(380\text{m/s})}(600\text{kPa}) = \mathbf{330.8kPa}$$

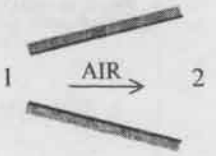
5-65 Air is decelerated in a diffuser from 230 m/s to 3 m/s. The exit temperature of air and the exit area of the diffuser are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with variable specific heats. **3** Potential energy changes are negligible. **4** The device is adiabatic and thus heat transfer is negligible. **5** There are no work interactions.

Properties The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1). The enthalpy of air at the inlet temperature of 400 K is $h_1 = 400.98$ kJ/kg (Table A-17).

Analysis (a) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take diffuser as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta pe \cong 0)$$

$$0 = h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2},$$

or,

$$h_2 = h_1 - \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} = 400.98\text{kJ/kg} - \frac{(30\text{m/s})^2 - (230\text{m/s})^2}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right) = 426.98\text{kJ/kg}$$

From Table A-17, $T_2 = \mathbf{425.6\ K}$

(*b*) The specific volume of air at the diffuser exit is

$$v_2 = \frac{RT_2}{P_2} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(425.6\text{K})}{(100\text{kPa})} = 1.221\text{m}^3/\text{kg}$$

From conservation of mass,

$$\dot{m} = \frac{1}{v_2} A_2 \mathbf{V}_2 \longrightarrow A_2 = \frac{\dot{m} v_2}{\mathbf{V}_2} = \frac{(6000/3600\text{kg/s})(1.221\text{m}^3/\text{kg})}{30\text{m/s}} = \mathbf{0.0678m^2}$$

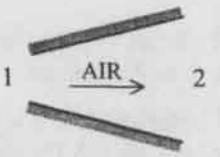
5-66E Air is decelerated in a diffuser from 600 ft/s to a low velocity. The exit temperature and the exit velocity of air are to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with variable specific heats. **3** Potential energy changes are negligible. **4** The device is adiabatic and thus heat transfer is negligible. **5** There are no work interactions.

Properties The enthalpy of air at the inlet temperature of 20°F is h_1 = 114.69 Btu/lbm (Table A-17E).

Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take diffuser as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{\nearrow 0 \text{ (steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta pe \cong 0)$$

$$0 = h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}$$

or,

$$h_2 = h_1 - \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} = 114.69\text{Btu/lbm} - \frac{0 - (600\text{ft/s})^2}{2}\left(\frac{1\text{Btu/lbm}}{25{,}037\text{ft}^2/\text{s}^2}\right) = 121.88\text{Btu/lbm}$$

From Table A-17E, T_2 = **510.0 R**

(*b*) The exit velocity of air is determined from the conservation of mass relation,

$$\frac{1}{v_2} A_2 \mathbf{V}_2 = \frac{1}{v_1} A_1 \mathbf{V}_1 \quad \longrightarrow \quad \frac{1}{RT_2/P_2} A_2 \mathbf{V}_2 = \frac{1}{RT_1/P_1} A_1 \mathbf{V}_2$$

Thus,

$$\mathbf{V}_2 = \frac{A_1 T_2 P_1}{A_2 T_1 P_2} \mathbf{V}_1 = \frac{1}{5}\frac{(510\text{R})(13\text{psia})}{(480\text{R})(14.5\text{psia})}(600\text{ft/s}) = \mathbf{114.3ft/s}$$

Turbines and Compressors

5-67C Yes.

5-68C The volume flow rate at the compressor inlet will be greater than that at the compressor exit.

5-69C Yes. Because energy (in the form of shaft work) is being added to the air.

5-70C No.

5-71 Steam expands in a turbine. The change in kinetic energy, the power output, and the turbine inlet area are to be determined.

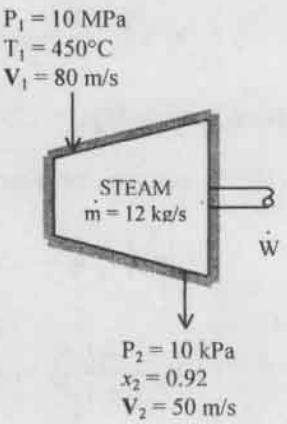
Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Properties From the steam tables (Tables A-4 through 6)

$$\left.\begin{matrix} P_1 = 10\text{MPa} \\ T_1 = 450^\circ\text{C} \end{matrix}\right\} \begin{matrix} v_1 = 0.02975\text{m}^3/\text{kg} \\ h_1 = 3240.9\text{kJ/kg} \end{matrix}$$

and

$$\left.\begin{matrix} P_2 = 100\text{kPa} \\ x_2 = 0.92 \end{matrix}\right\} h_2 = h_f + x_2 h_{fg} = 191.83 + 0.92 \times 2392.8 = 2393.2\text{kJ/kg}$$



Analysis (a) The change in kinetic energy is determined from

$$\Delta ke = \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} = \frac{(50\text{m/s})^2 - (80\text{m/s})^2}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right) = \mathbf{-1.95kJ/kg}$$

(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{W}_{\text{out}} + \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{out}} = -\dot{m}\left(h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right)$$

Then the power output of the turbine is determined by substitution to be

$$\dot{W}_{\text{out}} = -(12\text{kg/s})(2393.2 - 3240.9 - 1.95)\text{kJ/kg} = \mathbf{10.2MW}$$

(*c*) The inlet area of the turbine is determined from the mass flow rate relation,

$$\dot{m} = \frac{1}{v_1} A_1 \mathbf{V}_1 \longrightarrow A_1 = \frac{\dot{m} v_1}{\mathbf{V}_1} = \frac{(12\text{kg/s})(0.02975\text{m}^3/\text{kg})}{80\text{m/s}} = \mathbf{0.00446m^2}$$

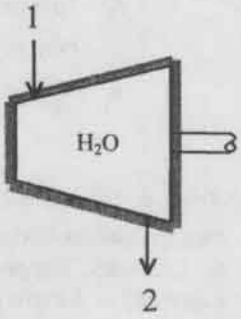
5-72 Steam expands in a turbine. The mass flow rate of steam for a power output of 5 MW is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Properties From the steam tables (Tables A-4 through 6)

$$\left.\begin{matrix} P_1 = 10\text{MPa} \\ T_1 = 400^\circ\text{C} \end{matrix}\right\} h_1 = 3096.5\text{kJ/kg}$$

$$\left.\begin{matrix} P_2 = 20\text{kPa} \\ x_2 = 0.90 \end{matrix}\right\} h_2 = h_f + x_2 h_{fg} = 251.40 + 0.90 \times 2358.3 = 2373.9\text{kJ/kg}$$



Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{out}} = -\dot{m}(h_2 - h_1)$$

Substituting, the required mass flow rate of the steam is determined to be

$$5000\text{kJ/s} = -\dot{m}(2373.9 - 3096.5)\text{kJ/kg}$$

$$\dot{m} = \mathbf{6.919\,kg/s}$$

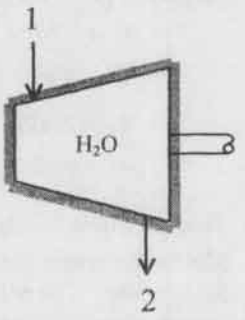
5-73E Steam expands in a turbine. The rate of heat loss from the steam for a power output of 4 MW is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible.

Properties From the steam tables (Tables A-4 through 6)

$$\left.\begin{matrix} P_1 = 1000\text{psia} \\ T_1 = 900^\circ\text{F} \end{matrix}\right\} h_1 = 1448.1\text{Btu/lbm}$$

$$\left.\begin{matrix} P_2 = 5\text{psia} \\ sat.vapor \end{matrix}\right\} h_2 = 1131.0\text{Btu/lbm}$$



Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{\text{out}} = -\dot{m}(h_2 - h_1) - \dot{W}_{\text{out}}$$

Substituting,

$$\dot{Q}_{\text{out}} = -(45000/3600\text{lbm/s})(1131.0 - 1448.1)\text{Btu/lbm} - 4000\,\text{kJ/s}\left(\frac{1\text{Btu}}{1.055\text{kJ}}\right)$$

$$= \mathbf{172.3Btu/s}$$

5-74 Steam expands in a turbine. The exit temperature of the steam for a power output of 2 MW is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Properties From the steam tables (Tables A-4 through 6)

$$\left.\begin{array}{l} P_1 = 10\text{MPa} \\ T_1 = 500^\circ\text{C} \end{array}\right\} h_1 = 3373.7\text{kJ/kg}$$

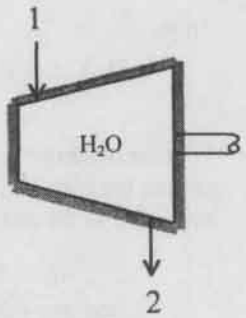
Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{out}} = \dot{m}(h_1 - h_2)$$



Substituting,

$$2000\text{kJ/s} = (3\text{kg/s})(3373.7 - h_2)\text{kJ/kg}$$

$$h_2 = 2707\text{kJ/kg}$$

Then the exit temperature becomes

$$\left.\begin{array}{l} P_2 = 20\text{kPa} \\ h_2 = 2707\text{kJ/kg} \end{array}\right\} T_2 = \mathbf{110.8^\circ C}$$

5-75 Argon gas expands in a turbine. The exit temperature of the argon for a power output of 250 kW is to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Argon is an ideal gas with constant specific heats.

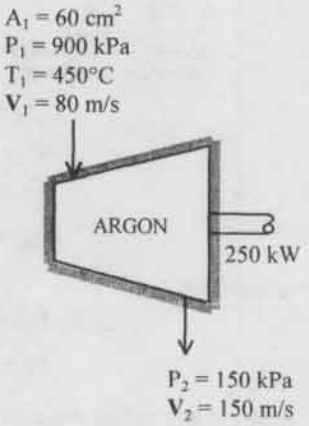
Properties The gas constant of Ar is $R = 0.2081$ kPa.m^3/kg.K. The constant pressure specific heat of Ar is $C_p = 0.5203$ kJ/kg·°C (Tables A-2a)

Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The inlet specific volume of argon and its mass flow rate are

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.2081\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(723\text{K})}{900\text{kPa}} = 0.167\text{m}^3/\text{kg}$$

Thus,

$$\dot{m} = \frac{1}{v_1}A_1V_1 = \frac{1}{0.167\text{m}^3/\text{kg}}(0.006\text{m}^2)(80\text{m/s}) = 2.874\text{kg/s}$$



We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{W}_{\text{out}} + \dot{m}(h_2 + \mathbf{V}_2^2/2) \quad (\text{since } \dot{Q} \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{out}} = -\dot{m}\left(h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right)$$

Substituting,

$$250\text{ kJ/s} = -(2.874\text{ kg/s})\left[(0.5203\text{ kJ/kg}\cdot^\circ\text{C})(T_2 - 400^\circ\text{C}) + \frac{(150\text{ m/s})^2 - (80\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)\right]$$

It yields $T_2 = \mathbf{217.3°C}$

5-76E Air expands in a turbine. The mass flow rate of air and the power output of the turbine are to be determined.

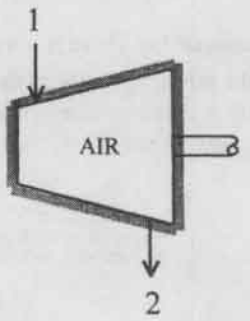
Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Air is an ideal gas with constant specific heats.

Properties The gas constant of air is $R = 0.3704$ psia.ft^3/lbm.R. The constant pressure specific heat of air at the average temperature of (900 + 300)/2 = 600°F is $C_p = 0.25$ Btu/lbm·°F (Tables A-2a)

Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The inlet specific volume of air and its mass flow rate are

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.3704\text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(1360\text{R})}{150\text{psia}} = 3.358\text{ft}^3/\text{lbm}$$

$$\dot{m} = \frac{1}{v_1}A_1V_1 = \frac{1}{3.358\text{ft}^3/\text{lbm}}(0.1\text{ft}^2)(350\text{ft/s}) = \mathbf{10.42lbm/s}$$



(*b*) We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{W}_{\text{out}} + \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{out}} = -\dot{m}\left(h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right) = -\dot{m}\left(C_p(T_2 - T_1) + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right)$$

Substituting,

$$\dot{W}_{\text{out}} = -(10.42\text{lbm/s})\left[(0.250\text{Btu/lbm}\cdot^\circ\text{F})(300 - 900)^\circ\text{F} + \frac{(700\text{ft/s})^2 - (350\text{ft/s})^2}{2}\left(\frac{1\text{Btu/lbm}}{25{,}037\text{ft}^2/\text{s}^2}\right)\right]$$

$$= 1486.5\text{Btu/s} = \mathbf{1568kW}$$

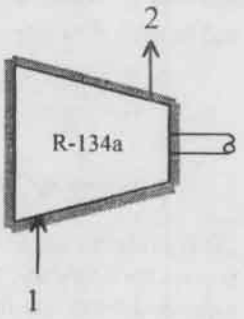
5-77 Refrigerant-134a is compressed steadily by a compressor. The power input to the compressor and the volume flow rate of the refrigerant at the compressor inlet are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Properties From the refrigerant tables (Tables A-11 through 13)

$$\left.\begin{array}{l} T_1 = -20^\circ\text{C} \\ \textit{sat.vapor} \end{array}\right\} \begin{array}{l} v_1 = 0.1464\text{m}^3/\text{kg} \\ h_1 = 235.31\text{kJ/kg} \end{array}$$

$$\left.\begin{array}{l} P_2 = 0.7\text{MPa} \\ T_2 = 70^\circ\text{C} \end{array}\right\} h_2 = 307.01\text{kJ/kg}$$



Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1)$$

Substituting,

$$\dot{W}_{in} = (1.2\text{kg/s})(307.01 - 235.31)\text{kJ/kg}$$
$$= \mathbf{86.04kJ/s}$$

(*b*) The volume flow rate of the refrigerant at the compressor inlet is

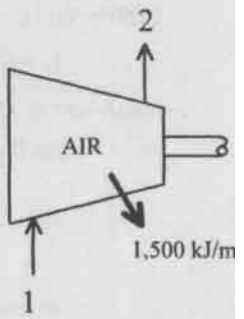
$$\dot{V}_1 = \dot{m}v_1 = (1.2\text{kg/s})(0.1464\text{m}^3/\text{kg}) = \mathbf{0.176m^3/s}$$

5-78 Air is compressed by a compressor. The mass flow rate of air through the compressor is to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The inlet and exit enthalpies of air are (Table A-17)

$$T_1 = 25°C = 298\ K \quad \rightarrow \quad h_1 = h_{@\,298\ K} = 298.2\ kJ/kg$$
$$T_2 = 347°C = 620\ K \quad \rightarrow \quad h_2 = h_{@\,620\ K} = 628.07\ kJ/kg$$



Analysis We take the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{Q}_{\text{out}} + \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \Delta pe \equiv 0)$$

$$\dot{W}_{\text{in}} - \dot{Q}_{\text{out}} = \dot{m}\left(h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right)$$

Substituting, the mass flow rate is determined to be

$$250\ \text{kJ/s} - (1500/60\ \text{kJ/s}) = \dot{m}\left[628.07 - 298.2 + \frac{(90\text{m/s})^2 - 0}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right)\right]$$

$$\dot{m} = \mathbf{0.674\ kg/s}$$

5-79E Air is compressed by a compressor. The mass flow rate of air through the compressor and the exit temperature of air are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

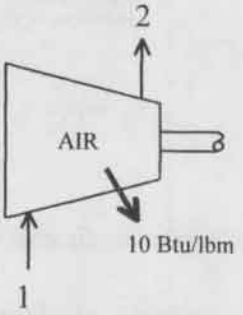
Properties The gas constant of air is R = 0.3704 psia.ft^3/lbm.R (Table A-1). The inlet enthalpy of air is (Table A-17E)

$$T_1 = 60°\text{F} = 520\ \text{R} \qquad \rightarrow \qquad h_1 = h_{@\ 520\ \text{R}} = 124.27\ \text{Btu/lbm}$$

Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The inlet specific volume of air and its mass flow rate are

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.3704\text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(520\text{R})}{14.7\text{psia}} = 13.1\text{ft}^3/\text{lbm}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{5000\ \text{ft}^3/\text{min}}{13.1\ \text{ft}^3/\text{lbm}} = 381.7\ \text{lbm/min} = \mathbf{6.36\ lbm/s}$$



(*b*) We take the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{\text{in}} - \dot{Q}_{\text{out}} = \dot{m}(h_2 - h_1)$$

Substituting,

$$(700\text{hp})\left(\frac{0.7068\text{Btu/s}}{1\text{hp}}\right) - (6.36\ \text{lbm/s}) \times (10\text{Btu/lbm}) = (6.36\text{lbm/s})(h_2 - 124.27\text{Btu/lbm})$$

$$h_2 = 192.06\text{Btu/lbm}$$

Then the exit temperature is determined from Table A-17E to be

$$T_2 = 781\ \text{R} = \mathbf{321°F}$$

5-80 Helium is compressed by a compressor. For a mass flow rate of 90 kg/min, the power input required is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** Helium is an ideal gas with constant specific heats.

Properties The constant pressure specific heat of helium is $C_p = 5.1926$ kJ/kg·K (Table A-2a).

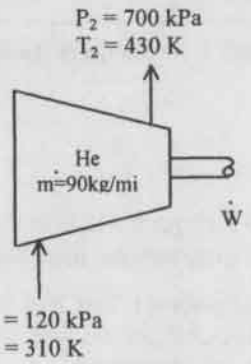
Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{\nearrow 0 \text{ (steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{\text{in}} - \dot{Q}_{\text{out}} = \dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)$$



Thus,

$$\dot{W}_{in} = \dot{Q}_{out} + \dot{m}C_p(T_2 - T_1)$$

$$= (90/60\text{kg/s})(20\text{ kJ/kg}) + (90/60\text{kg/s})(5.1926\text{kJ/kg}\cdot\text{K})(430 - 310)\text{K}$$

$$= \mathbf{965kW}$$

Throttling Valves

5-81C Because usually there is a large temperature drop associated with the throttling process.

5-82C Yes.

5-83C No. Because air is an ideal gas and h = h(T) for ideal gases. Thus if h remains constant, so does the temperature.

5-84C If it remains in the liquid phase, no. But if some of the liquid vaporizes during throttling, then yes.

5-85 Refrigerant-134a is throttled by a valve. The temperature drop of the refrigerant and specific volume after expansion are to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** Heat transfer to or from the fluid is negligible. **4** There are no work interactions involved.

Properties The inlet enthalpy of R-134a is, from the refrigerant tables (Tables A-11 through 13),

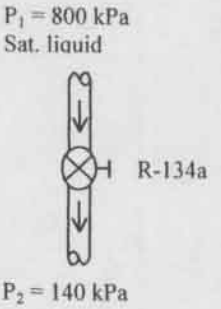
$$\left.\begin{array}{l} P_1 = 0.8MPa \\ sat.liquid \end{array}\right\} \begin{array}{l} T_1 = T_{sat} = 31.33^\circ C \\ h_1 = h_f = 93.42kJ/kg \end{array}$$

Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the throttling valve as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \quad \rightarrow \quad \dot{E}_{in} = \dot{E}_{out} \quad \rightarrow \quad \dot{m}h_1 = \dot{m}h_2 \quad \rightarrow \quad h_1 = h_2$$

since $\dot{Q} \cong \dot{W} = \Delta ke \cong \Delta pe \cong 0$. Then,

$$\left.\begin{array}{l} P_2 = 0.14\text{MPa} \\ (h_2 = h_1) \end{array}\right\} \begin{array}{l} h_f = 25.77\text{kJ/kg}, \quad T_{sat} = -18.8^\circ\text{C} \\ h_g = 236.04\text{kJ/kg} \end{array}$$



Obviously $h_f < h_2 < h_g$, thus the refrigerant exists as a saturated mixture at the exit state and thus $T_2 = T_{sat} = -18.8°C$. Then the temperature drop becomes

$$\Delta T = T_2 - T_1 = -18.8 - 3133 = \mathbf{-50.13^\circ C}$$

The quality at this state is determined from

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{93.42 - 25.77}{210.27 - 25.77} = 0.322$$

Thus,

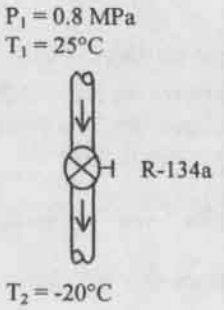
$$v_2 = v_f + x_2 v_{fg} = 0.0007381 + 0.322 \times 0.13876 = \mathbf{0.0454 m^3/kg}$$

5-86 Refrigerant-134a is throttled by a valve. The temperature drop of the refrigerant and internal energy after expansion are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** Heat transfer to or from the fluid is negligible. **4** There are no work interactions involved.

Properties The inlet enthalpy of R-134a is, from the refrigerant tables (Tables A-11 through 13),

$$\left.\begin{array}{l} P_1 = 0.8\text{MPa} \\ T_1 = 25^\circ\text{C} \end{array}\right\} h_1 \cong h_{f@25^\circ\text{C}} = 84.33\text{kJ/kg}$$



Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the throttling valve as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \quad \rightarrow \quad \dot{E}_{in} = \dot{E}_{out} \quad \rightarrow \quad \dot{m}h_1 = \dot{m}h_2 \quad \rightarrow \quad h_1 = h_2$$

since $\dot{Q} \cong \dot{W} = \Delta ke \cong \Delta pe \cong 0$. Then,

$$\left.\begin{array}{l} T_2 = -20^\circ\text{C} \\ (h_2 = h_1) \end{array}\right\} \begin{array}{l} h_f = 24.26\text{kJ/kg}, \quad u_f = 24.17\text{kJ/kg} \\ h_g = 235.31\text{kJ/kg} \quad u_g = 215.84\text{kJ/kg} \end{array}$$

Obviously $h_f < h_2 < h_g$, thus the refrigerant exists as a saturated mixture at the exit state, and thus

$$P_2 = P_{\text{sat @ -20°C}} = \mathbf{0.13299\ MPa}$$

Also, $$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{84.33 - 24.26}{211.05} = 0.285$$

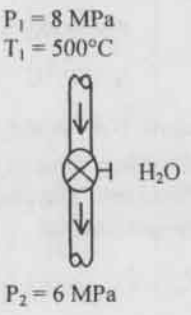
Thus, $$u_2 = u_f + x_2 u_{fg} = 24.17 + 0.285 \times (215.84 - 24.17) = \mathbf{78.8kJ/kg}$$

5-87 Steam is throttled by a well-insulated valve. The temperature drop of the steam after the expansion is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** Heat transfer to or from the fluid is negligible. **4** There are no work interactions involved.

Properties The inlet enthalpy of steam is (Tables A-6),

$$\left.\begin{matrix} P_1 = 8\text{MPa} \\ T_1 = 500^\circ\text{C} \end{matrix}\right\} h_1 = 3398.3\text{kJ/kg}$$



Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the throttling valve as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \quad \rightarrow \quad \dot{E}_{in} = \dot{E}_{out} \quad \rightarrow \quad \dot{m}h_1 = \dot{m}h_2 \quad \rightarrow \quad h_1 = h_2$$

since $\dot{Q} \cong \dot{W} = \Delta ke \cong \Delta pe \cong 0$. Then the exit temperature of steam becomes

$$\left.\begin{matrix} P_2 = 6\text{MPa} \\ (h_2 = h_1) \end{matrix}\right\} T_2 = \mathbf{490.1^\circ C}$$

5-88E High-pressure air is throttled to atmospheric pressure. The temperature of air after the expansion is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** Heat transfer to or from the fluid is negligible. **4** There are no work interactions involved. **5** Air is an ideal gas.

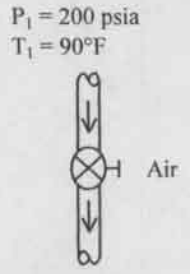
Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the throttling valve as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \quad \rightarrow \quad \dot{E}_{in} = \dot{E}_{out} \quad \rightarrow \quad \dot{m}h_1 = \dot{m}h_2 \quad \rightarrow \quad h_1 = h_2$$

since $\dot{Q} \cong \dot{W} = \Delta ke \cong \Delta pe \cong 0$. For an ideal gas, $h = h(T)$.

Therefore,

$$T_2 = T_1 = \mathbf{90°F}$$



Mixing Chambers and Heat Exchangers

5-89C Yes, if the mixing chamber is losing heat to the surrounding medium.

5-90C Under the conditions of no heat and work interactions between the mixing chamber and the surrounding medium.

5-91C Under the conditions of no heat and work interactions between the heat exchanger and the surrounding medium.

5-92 A hot water stream is mixed with a cold water stream. For a specified mixture temperature, the mass flow rate of cold water is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The mixing chamber is well-insulated so that heat loss to the surroundings is negligible. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant. **5** There are no work interactions.

Properties Noting that $T < T_{sat\ @\ 250\ kPa} = 127.44°C$, the water in all three streams exists as a compressed liquid, which can be approximated as a saturated liquid at the given temperature. Thus,

$$h_1 \cong h_{f@\,80°C} = 334.91\ \text{kJ/kg}$$
$$h_2 \cong h_{f@\,20°C} = 83.96\ \text{kJ/kg}$$
$$h_3 \cong h_{f@\,42°C} = 175.92\ \text{kJ/kg}$$

Analysis We take the mixing chamber as the system, which is a control volume. The mass and energy balances for this steady-flow system can be expressed in the rate form as

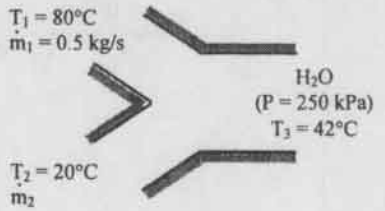
Mass balance: $\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}} = 0 \longrightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy balance:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} = \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$



Combining the two relations and solving for $\dot{m}_2$ gives

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

$$\dot{m}_2 = \frac{h_1 - h_3}{h_3 - h_2}\dot{m}_1$$

Substituting, the mass flow rate of cold water stream is determined to be

$$\dot{m}_2 = \frac{(334.91 - 175.92)\text{kJ/kg}}{(175.92 - 83.96)\text{kJ/kg}}(0.5\text{kg/s}) = \mathbf{0.864\,kg/s}$$

5-93 Liquid water is heated in a chamber by mixing it with superheated steam. For a specified mixing temperature, the mass flow rate of the steam is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** The device is adiabatic and thus heat transfer is negligible.

Properties Noting that $T < T_{sat\ @\ 300\ kPa} = 133.55°C$, the cold water stream and the mixture exist as a compressed liquid, which can be approximated as a saturated liquid at the given temperature. Thus,

$$h_1 \cong h_{f\ @\ 20°C} = 83.96\ kJ/kg$$
$$h_3 \cong h_{f\ @\ 60°C} = 251.13\ kJ/kg$$

and

$$\left.\begin{matrix} P_2 = 300 kPa \\ T_2 = 300°C \end{matrix}\right\} h_2 = 3069.3 kJ/kg$$

Analysis We take the mixing chamber as the system, which is a control volume since mass crosses the boundary. The mass and energy balances for this steady-flow system can be expressed in the rate form as

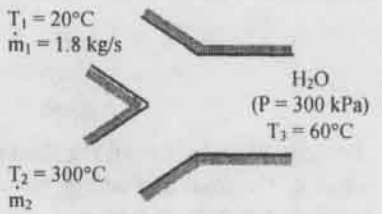
Mass balance: $$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{system}^{\nearrow 0\ (steady)} = 0 \quad \rightarrow \quad \dot{m}_{in} = \dot{m}_{out} \quad \rightarrow \quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Energy balance:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}^{\nearrow 0\ (steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta ke \cong \Delta pe \cong 0)$$



Combining the two, $$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

Solving for $\dot{m}_2$: $$\dot{m}_2 = \frac{h_1 - h_3}{h_3 - h_2} \dot{m}_1$$

Substituting,

$$\dot{m}_2 = \frac{(83.96 - 251.13) kJ/kg}{(251.13 - 3069.3) kJ/kg} (1.8 kg/s) = \mathbf{0.107 kg/s}$$

5-94 Feedwater is heated in a chamber by mixing it with superheated steam. If the mixture is saturated liquid, the ratio of the mass flow rates of the feedwater and the superheated vapor is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** The device is adiabatic and thus heat transfer is negligible.

Properties Noting that $T < T_{sat\ @\ 800\ kPa} = 170.43°C$, the cold water stream and the mixture exist as a compressed liquid, which can be approximated as a saturated liquid at the given temperature. Thus,

$$h_1 \cong h_{f@50°C} = 209.33\ \text{kJ/kg}$$
$$h_3 \cong h_{f@800\ kPa} = 721.11\ \text{kJ/kg}$$

and

$$\left.\begin{matrix} P_2 = 800\text{kPa} \\ T_2 = 200^\circ\text{C} \end{matrix}\right\} h_2 = 2839.3\text{kJ/kg}$$

Analysis We take the mixing chamber as the system, which is a control volume since mass crosses the boundary. The mass and energy balances for this steady-flow system can be expressed in the rate form as

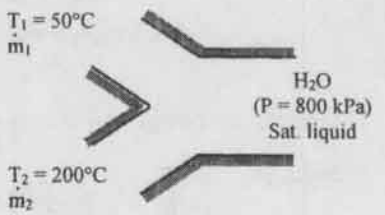
Mass balance: $$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{\text{system}}^{\nearrow 0\ \text{(steady)}} = 0 \quad \rightarrow \quad \dot{m}_{in} = \dot{m}_{out} \quad \rightarrow \quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Energy balance:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$



Combining the two, $$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

Dividing by $\dot{m}_2$ yields $$y h_1 + h_2 = (y + 1) h_3$$

Solving for y: $$y = \frac{h_3 - h_2}{h_1 - h_3}$$

where $y = \dot{m}_1 / \dot{m}_2$ is the desired mass flow rate ratio. Substituting,

$$y = \frac{(2839.3 - 721.11)\text{kJ/kg}}{(721.11 - 209.33)\text{kJ/kg}} = \mathbf{4.14}$$

5-95E Liquid water is heated in a chamber by mixing it with saturated water vapor. If both streams enter at the same rate, the temperature or quality (if saturated) of the exit stream is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** The device is adiabatic and thus heat transfer is negligible.

Properties From steam tables (Tables A-5 through A-6),

$$h_1 \cong h_{f\,@\,50°F} = 18.06 \text{ Btu/lbm}$$
$$h_2 = h_{g\,@\,50 \text{ psia}} = 1174.4 \text{ Btu/lbm}$$

Analysis We take the mixing chamber as the system, which is a control volume since mass crosses the boundary. The mass and energy balances for this steady-flow system can be expressed in the rate form as

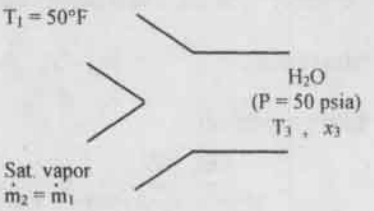
Mass balance: $\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \rightarrow \dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 2\dot{m} \rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$

Energy balance:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta \text{ke} \cong \Delta \text{pe} \cong 0)$$



Combining the two gives $\dot{m}_1 h_1 + \dot{m}_2 h_2 = 2\dot{m} h_3$ or $h_3 = (h_1 + h_2)/2$

Substituting,

$$h_3 = (18.06 + 1174.4)/2 = 596.23 \text{ Btu/lbm}$$

At 50 psia, $h_f = 250.24$ Btu/lbm and $h_g = 1174.4$ Btu/lbm. Thus the exit stream is a saturated mixture since $h_f < h_3 < h_g$. Therefore,

$$T_3 = T_{\text{sat @ 50 psia}} = \mathbf{281.03°F}$$

and

$$x_3 = \frac{h_3 - h_f}{h_{fg}} = \frac{596.23 - 250.24}{974.2} = \mathbf{0.374}$$

5-96 Two streams of refrigerant-134a are mixed in a chamber. If the cold stream enters at twice the rate of the hot stream, the temperature and quality (if saturated) of the exit stream are to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** The device is adiabatic and thus heat transfer is negligible.

Properties From R-134a tables (Tables A-11 through A-13),

$$h_1 \cong h_{f@\,12°C} = 66.18 \text{ kJ/kg}$$
$$h_2 = h_{g@\,1\text{ MPa},\ 60°C} = 291.36 \text{ kJ/kg}$$

Analysis We take the mixing chamber as the system, which is a control volume since mass crosses the boundary. The mass and energy balances for this steady-flow system can be expressed in the rate form as

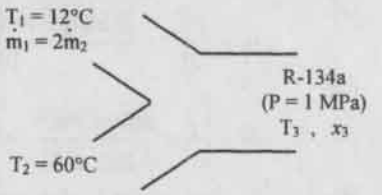
Mass balance: $\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \rightarrow \dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 3\dot{m}_2$ since $\dot{m}_1 = 2\dot{m}_2$

Energy balance:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$



Combining the two gives $2\dot{m}_2 h_1 + \dot{m}_2 h_2 = 3\dot{m}_2 h_3$ *or* $h_3 = (2h_1 + h_2)/3$

Substituting,

$$h_3 = (2\times 66.18 + 291.36)/3 = 141.24 \text{ kJ/kg}$$

At 1 MPa, $h_f = 105.29$ kJ/kg and $h_g = 267.97$ kJ/kg. Thus the exit stream is a saturated mixture since $h_f < h_3 < h_g$. Therefore,

$$T_3 = T_{\text{sat @ 1 MPa}} = \mathbf{39.39°C}$$

and

$$x_3 = \frac{h_3 - h_f}{h_{fg}} = \frac{141.24 - 105.29}{162.68} = \mathbf{0.221}$$

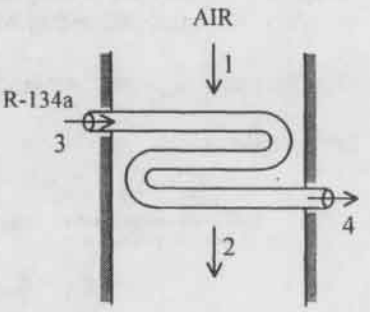
5-97 Refrigerant-134a is to be cooled by air in the condenser. For a specified volume flow rate of air, the mass flow rate of the refrigerant is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** Heat loss from the device to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **5** Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1). The constant pressure specific heat of air is C_p = 1.005 kJ/kg·°C (Table A-2). The enthalpies of the R-134a at the inlet and the exit states are (Tables A-11 through A-13)

$$\left.\begin{matrix} P_3 = 1\text{MPa} \\ T_3 = 80^\circ\text{C} \end{matrix}\right\} h_3 = 313.20\text{kJ/kg}$$

$$\left.\begin{matrix} P_4 = 1\text{MPa} \\ T_4 = 30^\circ\text{C} \end{matrix}\right\} h_4 \cong h_{f@30^\circ C} = 91.49\text{kJ/kg}$$



Analysis The inlet specific volume and the mass flow rate of air are

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})}{100\text{kPa}} = 0.861\text{m}^3/\text{kg}$$

and

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{800\text{m}^3/\text{min}}{0.861\text{m}^3/\text{kg}} = 929.2\text{kg/min}$$

We take the entire heat exchanger as the system, which is a control volume. The mass and energy balances for this steady-flow system can be expressed in the rate form as

Mass balance (*for each fluid stream*):

$$\dot{m}_{in} - \dot{m}_{out} = \Delta\dot{m}_{\text{system}}^{\nearrow 0\ \text{(steady)}} = 0 \rightarrow \dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_a \quad \text{and} \quad \dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

Energy balance (for the entire heat exchanger):

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4 \quad (\text{since } \dot{Q} = \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

Combining the two, $\dot{m}_a(h_2 - h_1) = \dot{m}_R(h_3 - h_4)$

Solving for $\dot{m}_R$: $\dot{m}_R = \dfrac{h_2 - h_1}{h_3 - h_4}\dot{m}_a \cong \dfrac{C_p(T_2 - T_1)}{h_3 - h_4}\dot{m}_a$

Substituting,

$$\dot{m}_R = \frac{(1.005\text{kJ/kg}\cdot^\circ\text{C})(60 - 27)^\circ\text{C}}{(313.20 - 91.49)\text{kJ/kg}}(929.2\text{kg/min}) = \mathbf{139.0\text{kg/min}}$$

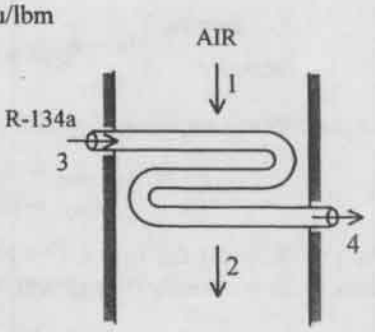
5-98E Refrigerant-134a is vaporized by air in the evaporator of an air-conditioner. For specified flow rates, the exit temperature of the air and the rate of heat transfer from the air are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** Heat loss from the device to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **5** Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is 0.3704 psia.ft^3/lbm.R (Table A-1E). The constant pressure specific heat of air is C_p = 0.240 Btu/lbm·°F (Table A-2E). The enthalpies of the R-134a at the inlet and the exit states are (Tables A-11E through A-13E)

$$\left.\begin{matrix} P_3 = 20\text{psia} \\ x_3 = 0.3 \end{matrix}\right\} h_3 = h_f + x_3 h_{fg} = 6.77 + 0.3 \times 69.63 = 27.66\text{Btu/lbm}$$

$$\left.\begin{matrix} P_4 = 20\text{psia} \\ sat.vapor \end{matrix}\right\} h_4 = h_{g@20\text{psia}} = 76.40\text{Btu/lbm}$$



Analysis The inlet specific volume and the mass flow rate of air are

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.3704\text{psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(550\text{R})}{14.7\text{psia}} = 13.86\text{ft}^3/\text{lbm}$$

and

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{200\text{ft}^3/\text{min}}{13.86\text{ft}^3/\text{lbm}} = 14.43\text{lbm/min}$$

We take the entire heat exchanger as the system, which is a control volume. The mass and energy balances for this steady-flow system can be expressed in the rate form as

Mass balance (*for each fluid stream*):

$$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \rightarrow \dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_a \text{ and } \dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

Energy balance (for the entire heat exchanger):

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4 \quad (\text{since } \dot{Q} = \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

Combining the two,

$$\dot{m}_R(h_3 - h_4) = \dot{m}_a(h_2 - h_1) = \dot{m}_a C_p(T_2 - T_1)$$

Solving for T_2:

$$T_2 = T_1 + \frac{\dot{m}_R(h_3 - h_4)}{\dot{m}_a C_p}$$

Substituting,

$$T_2 = 90^\circ\text{F} + \frac{(4\text{lbm/min})(27.66 - 76.40)\text{Btu/lbm}}{(14.43\text{Btu/min})(0.24\text{Btu/lbm}\cdot^\circ\text{F})} = \mathbf{33.7^\circ F}$$

(*b*) The rate of heat transfer from the air to the refrigerant is determined from the steady-flow energy balance applied to the air only. It yields

$$-\dot{Q}_{air,out} = \dot{m}_a(h_2 - h_1) = \dot{m}_a C_p(T_2 - T_1)$$

$$\dot{Q}_{air,out} = -(14.43\text{ lbm/min})(0.24\text{ Btu/lbm}\cdot^\circ\text{F})(33.7 - 90)^\circ\text{F}$$

$$= \mathbf{195\ Btu/min}$$

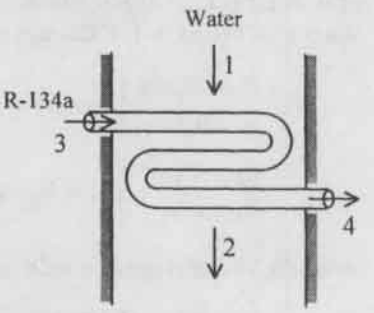
5-99 Refrigerant-134a is condensed in a water-cooled condenser. The mass flow rate of the cooling water required is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** Heat loss from the device to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid.

Properties The enthalpies of R-134a at the inlet and the exit states are (Tables A-5 and A-6)

$$\left.\begin{matrix} P_3 = 800\text{kPa} \\ T_3 = 70^\circ\text{C} \end{matrix}\right\} h_3 = 305.50\text{kJ/kg}$$

$$\left.\begin{matrix} P_4 = 800\text{kPa} \\ sat.liquid \end{matrix}\right\} h_4 = h_{f@800\text{kPa}} = 93.42\text{kJ/kg}$$



Water exists as compressed liquid at both states, and thus

$$h_1 \cong h_{f@15^\circ\text{C}} = 62.99\ \text{kJ/kg}$$
$$h_2 \cong h_{f@30^\circ\text{C}} = 125.79\ \text{kJ/kg}$$

Analysis We take the heat exchanger as the system, which is a control volume. The mass and energy balances for this steady-flow system can be expressed in the rate form as

Mass balance (for each fluid stream):

$$\dot{m}_{in} - \dot{m}_{out} = \Delta\dot{m}_{\text{system}}^{\nearrow 0\ \text{(steady)}} = 0 \rightarrow \dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_w \quad \text{and} \quad \dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

Energy balance (for the heat exchanger):

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4 \quad (\text{since } \dot{Q} = \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

Combining the two, $$\dot{m}_w (h_2 - h_1) = \dot{m}_R (h_3 - h_4)$$

Solving for $\dot{m}_w$: $$\dot{m}_w = \frac{h_3 - h_4}{h_2 - h_1}\dot{m}_R$$

Substituting,

$$\dot{m}_w = \frac{(305.50 - 93.42)\,\text{kJ/kg}}{(125.79 - 62.99)\,\text{kJ/kg}}(8\ \text{kg/min}) = \mathbf{27.0\ kg/min}$$

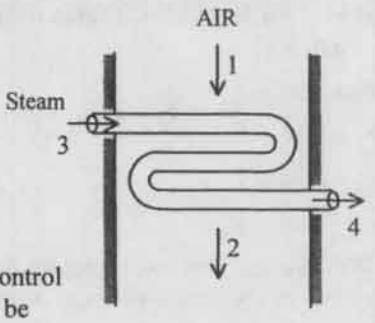
5-100E Air is heated in a steam heating system. For specified flow rates, the volume flow rate of air at the inlet is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** Heat loss from the device to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **5** Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is 0.3704 psia.ft^3/lbm.R (Table A-1E). The constant pressure specific heat of air is C_p = 0.240 Btu/lbm·°F (Table A-2E). The enthalpies of the R-134a at the inlet and the exit states are (Tables A-11E through A-13E)

$$\left.\begin{matrix} P_3 = 30\text{psia} \\ T_3 = 400^\circ\text{F} \end{matrix}\right\} h_3 = 1237.8\text{Btu/lbm}$$

$$\left.\begin{matrix} P_4 = 25\text{psia} \\ T_4 = 212^\circ F \end{matrix}\right\} h_4 \cong h_{f@212^\circ\text{F}} = 180.16\text{Btu/lbm}$$



Analysis We take the entire heat exchanger as the system, which is a control volume. The mass and energy balances for this steady-flow system can be expressed in the rate form as

Mass balance (*for each fluid stream*):

$$\dot{m}_{in} - \dot{m}_{out} = \Delta\dot{m}_{\text{system}}^{\nearrow 0\ \text{(steady)}} = 0 \rightarrow \dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_a \quad \text{and} \quad \dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

Energy balance (for the entire heat exchanger):

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4 \quad (\text{since } \dot{Q} = \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

Combining the two, $$\dot{m}_a(h_2 - h_1) = \dot{m}_s(h_3 - h_4)$$

Solving for $\dot{m}_a$: $$\dot{m}_a = \frac{h_3 - h_4}{h_2 - h_1}\dot{m}_s \cong \frac{h_3 - h_4}{C_p(T_2 - T_1)}\dot{m}_s$$

Substituting,

$$\dot{m}_a = \frac{(1237.8 - 180.16)\ \text{Btu/lbm}}{(0.240\ \text{Btu/lbm}\cdot^\circ\text{F})(130 - 80)^\circ\text{F}}(15\ \text{lbm/min}) = 1322\ \text{lbm/min} = 22.03\ \text{lbm/s}$$

Also, $$v_1 = \frac{RT_1}{P_1} = \frac{(0.3704\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(540\ \text{R})}{14.7\ \text{psia}} = 13.61\ \text{ft}^3/\text{lbm}$$

Then the volume flow rate of air at the inlet becomes

$$\dot{V}_1 = \dot{m}_a v_1 = (22.03\ \text{lbm/s})(13.61\ \text{ft}^3/\text{lbm}) = \mathbf{299.8\ ft^3/s}$$

5-101 Steam is condensed by cooling water in the condenser of a power plant. If the temperature rise of the cooling water is not to exceed 10°C, the minimum mass flow rate of the cooling water required is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** Heat loss from the device to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **5** Liquid water is an incompressible substance with constant specific heats at room temperature.

Properties The cooling water exists as compressed liquid at both states, and its specific heat at room temperature is $C = 4.18$ kJ/kg·°C (Table A-3). The enthalpies of the steam at the inlet and the exit states are (Tables A-5 and A-6)

$$\left.\begin{array}{l} P_3 = 20\text{kPa} \\ x_3 = 0.95 \end{array}\right\} h_3 = h_f + x_3 h_{fg} = 251.40 + 0.95 \times 2358.3 = 2491.8\text{kJ/kg}$$

$$\left.\begin{array}{l} P_4 = 20\text{kPa} \\ sat.liquid \end{array}\right\} h_4 \cong h_{f@20kPa} = 251.40\text{kJ/kg}$$

Analysis We take the heat exchanger as the system, which is a control volume. The mass and energy balances for this steady-flow system can be expressed in the rate form as

Mass balance (for each fluid stream):

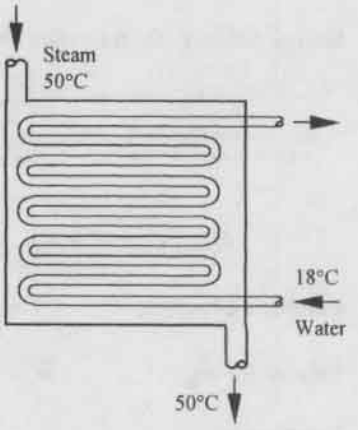
$$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \rightarrow \dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_w \quad \text{and} \quad \dot{m}_3 = \dot{m}_4 = \dot{m}_s$$

Energy balance (for the heat exchanger):

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4 \quad (\text{since } \dot{Q} = \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$



Combining the two, $\dot{m}_w (h_2 - h_1) = \dot{m}_s (h_3 - h_4)$

Solving for $\dot{m}_w$: $\dot{m}_w = \dfrac{h_3 - h_4}{h_2 - h_1} \dot{m}_s \cong \dfrac{h_3 - h_4}{C_p (T_2 - T_1)} \dot{m}_s$

Substituting,

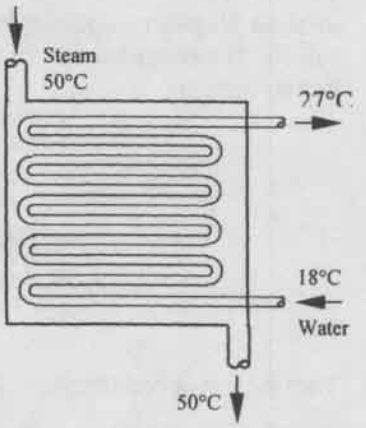
$$\dot{m}_w = \frac{(2491.8 - 251.4)\text{ kJ/kg}}{(4.18\text{ kJ/kg·°C})(10\text{°C})}(20{,}000/3600\text{ kg/s}) = \mathbf{298\ kg/s}$$

5-102 Steam is condensed by cooling water in the condenser of a power plant. The rate of condensation of steam is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The heat of vaporization of water at 50°C is given to be h_{fg} = 2305 kJ/kg and specific heat of cold water is given to be C_p = 4.20 kJ/kg.°C.

Analysis We take the cold water tubes as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{\nearrow 0 \text{ (steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$

Then the heat transfer rate to the cooling water in the condenser becomes

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{cooling water}}$$
$$= (101 \text{ kg/s})(4.20 \text{ kJ/kg.°C})(27°C - 18°C)$$
$$= 3818 \text{ kJ/s}$$

The rate of condensation of steam is determined to be

$$\dot{Q} = (\dot{m}h_{fg})_{\text{steam}} \longrightarrow \dot{m}_{\text{steam}} = \frac{\dot{Q}}{h_{fg}} = \frac{3818 \text{ kJ/s}}{2305 \text{ kJ/kg}} = \mathbf{1.656 \text{ kg/s}}$$

5-103 Water is heated in a heat exchanger by geothermal water. The rate of heat transfer to the water and the exit temperature of the geothermal water is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of water and geothermal fluid are given to be 4.18 and 4.31 kJ/kg.°C, respectively.

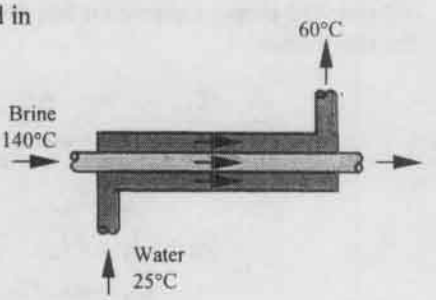
Analysis We take the cold water tubes as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



Then the rate of heat transfer to the cold water in the heat exchanger becomes

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg.°C})(60°\text{C} - 25°\text{C}) = \mathbf{29.26 \text{ kW}}$$

Noting that heat transfer to the cold water is equal to the heat loss from the geothermal water, the outlet temperature of the geothermal water is determined from

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{geot.water}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 140°\text{C} - \frac{29.26 \text{ kW}}{(0.3 \text{ kg/s})(4.31 \text{ kJ/kg.°C})} = \mathbf{117.4°\text{C}}$$

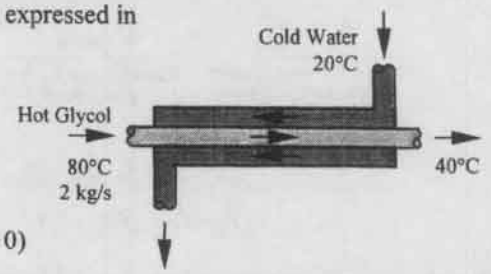
5-104 Ethylene glycol is cooled by water in a heat exchanger. The rate of heat transfer in the heat exchanger and the mass flow rate of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of water and ethylene glycol are given to be 4.18 and 2.56 kJ/kg.°C, respectively.

Analysis (a) We take the ethylene glycol tubes as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{\nearrow 0 \text{ (steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{m}C_p(T_1 - T_2)$$

Then the rate of heat transfer becomes

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{glycol}} = (2 \text{ kg/s})(2.56 \text{ kJ/kg.°C})(80°\text{C} - 40°\text{C}) = \mathbf{204.8 \text{ kW}}$$

(b) The rate of heat transfer from water must be equal to the rate of heat transfer to the glycol. Then,

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} \longrightarrow \dot{m}_{\text{water}} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})}$$

$$= \frac{204.8 \text{ kJ/s}}{(4.18 \text{ kJ/kg.°C})(55°\text{C} - 20°\text{C})} = \mathbf{1.4 \text{ kg/s}}$$

5-105 Oil is to be cooled by water in a thin-walled heat exchanger. The rate of heat transfer in the heat exchanger and the exit temperature of water is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of water and oil are given to be 4.18 and 2.20 kJ/kg.°C, respectively.

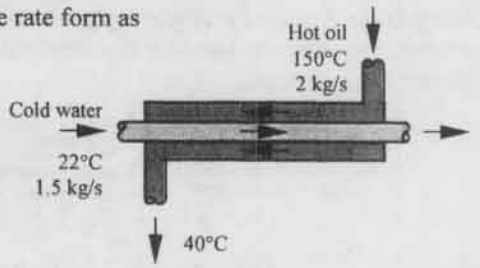
Analysis We take the oil tubes as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{m}C_p(T_1 - T_2)$$



Then the rate of heat transfer from the oil becomes

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{oil}} = (2\ \text{kg/s})(2.2\ \text{kJ/kg.°C})(150°\text{C} - 40°\text{C}) = \mathbf{484\ kW}$$

Noting that the heat lost by the oil is gained by the water, the outlet temperature of the water is determined from

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} \longrightarrow T_{\text{out}} = T_{\text{in}} + \frac{\dot{Q}}{\dot{m}_{\text{water}}C_p} = 22°\text{C} + \frac{484\ \text{kJ/s}}{(1.5\ \text{kg/s})(4.18\ \text{kJ/kg.°C})} = \mathbf{99.2°C}$$

5-106 Cold water is heated by hot water in a heat exchanger. The rate of heat transfer and the heat transfer surface area of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of cold and hot water are given to be 4.18 and 4.19 kJ/kg.°C, respectively.

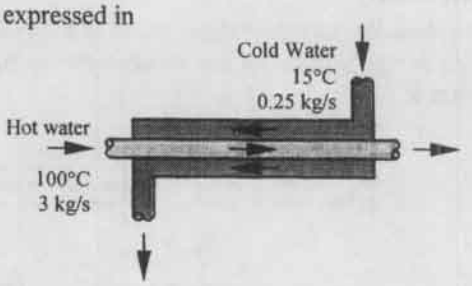
Analysis We take the cold water tubes as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



Then the rate of heat transfer to the cold water in this heat exchanger becomes

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{cold water}} = (0.25 \text{ kg/s})(4.18 \text{ kJ/kg.°C})(45°\text{C} - 15°\text{C}) = \mathbf{31.35 \text{ kW}}$$

Noting that heat gain by the cold water is equal to the heat loss by the hot water, the outlet temperature of the hot water is determined to be

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{hot water}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 100°\text{C} - \frac{31.35 \text{ kW}}{(3 \text{ kg/s})(4.19 \text{ kJ/kg.°C})} = \mathbf{97.5°\text{C}}$$

5-107 Air is preheated by hot exhaust gases in a cross-flow heat exchanger. The rate of heat transfer and the outlet temperature of the air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of air and combustion gases are given to be 1.005 and 1.10 kJ/kg.°C, respectively.

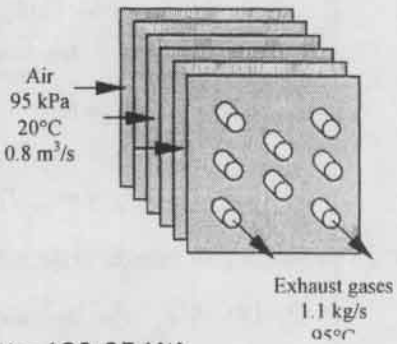
Analysis We take the exhaust pipes as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{m}C_p(T_1 - T_2)$$



Then the rate of heat transfer from the exhaust gases becomes

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{gas.}} = (1.1 \text{ kg/s})(1.1 \text{ kJ/kg.°C})(180°\text{C} - 95°\text{C}) = \mathbf{102.85 \text{ kW}}$$

The mass flow rate of air is

$$\dot{m} = \frac{P\dot{V}}{RT} = \frac{(95 \text{ kPa})(0.8 \text{ m}^3/\text{s})}{(0.287 \text{ kPa.m}^3/\text{kg.K}) \times 293 \text{ K}} = 0.904 \text{ kg/s}$$

Noting that heat loss by the exhaust gases is equal to the heat gain by the air, the outlet temperature of the air becomes

$$\dot{Q} = \dot{m}C_p(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 20°\text{C} + \frac{102.85 \text{ kW}}{(0.904 \text{ kg/s})(1.005 \text{ kJ/kg.°C})} = \mathbf{133.2°\text{C}}$$

5-108 Water is heated by hot oil in a heat exchanger. The rate of heat transfer in the heat exchanger and the outlet temperature of oil are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of water and oil are given to be 4.18 and 2.3 kJ/kg.°C, respectively.

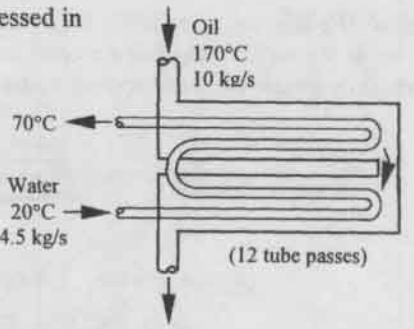
Analysis We take the cold water tubes as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



Then the rate of heat transfer to the cold water in this heat exchanger becomes

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} = (4.5 \text{ kg/s})(4.18 \text{ kJ/kg.°C})(70°\text{C} - 20°\text{C}) = \mathbf{940.5\ kW}$$

Noting that heat gain by the water is equal to the heat loss by the oil, the outlet temperature of the hot water is determined from

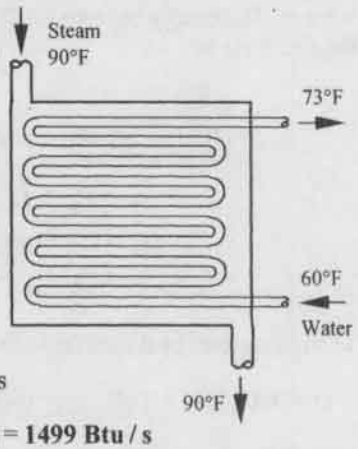
$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{oil}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p} = 170°\text{C} - \frac{940.5\text{ kW}}{(10\text{ kg/s})(2.3\text{ kJ/kg.°C})} = \mathbf{129.1°C}$$

5-109E Steam is condensed by cooling water in a condenser. The rate of heat transfer in the heat exchanger and the rate of condensation of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heat of water is given to be 1.0 Btu/lbm.°F. The heat of condensation of steam at 90°F is given to be 1043 Btu/lbm.

Analysis We take the tube-side of the heat exchanger where cold water is flowing as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$

Then the rate of heat transfer to the cold water in this heat exchanger becomes

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} = (115.3 \text{ lbm/s})(1.0 \text{ Btu/lbm.°F})(73°\text{F} - 60°\text{F}) = \mathbf{1499 \text{ Btu/s}}$$

Noting that heat gain by the water is equal to the heat loss by the condensing steam, the rate of condensation of the steam in the heat exchanger is determined from

$$\dot{Q} = (\dot{m}h_{fg})_{steam} \longrightarrow \dot{m}_{steam} = \frac{\dot{Q}}{h_{fg}} = \frac{1499 \text{ Btu/s}}{1043 \text{ Btu/lbm}} = \mathbf{1.44 \text{ lbm/s}}$$

Pipe and duct Flow

5-110 A desktop computer is to be cooled safely by a fan in hot environments and high elevations. The air flow rate of the fan and the diameter of the casing are to be determined.

Assumptions **1** Steady operation under worst conditions is considered. **2** Air is an ideal gas with constant specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The specific heat of air at the average temperature of T_{ave} = (45+60)/2 = 52.5°C = 325.5 K is C_p = 1.006 kJ/kg.°C (Table A-3)

Analysis The fan selected must be able to meet the cooling requirements of the computer at worst conditions. Therefore, we assume air to enter the computer at 66.63 kPa and 45°C, and leave at 60°C.

We take the air space in the computer as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



Then the required mass flow rate of air to absorb heat at a rate of 60 W is determined. to be

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in}) \rightarrow \dot{m} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})} = \frac{60 \text{ W}}{(1006 \text{ J/kg.°C})(60-45)°\text{C}}$$

$$= 0.00397 \text{ kg/s} = 0.239 \text{ kg/min}$$

The density of air entering the fan at the exit and its volume flow rate are

$$\rho = \frac{P}{RT} = \frac{66.63 \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg.K})(60+273)\text{K}} = 0.6972 \text{ kg/m}^3$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.239 \text{ kg/min}}{0.6972 \text{ kg/m}^3} = \mathbf{0.343 \text{ m}^3/\text{min}}$$

For an average exit velocity of 110 m/min, the diameter of the casing of the fan is determined from

$$\dot{V} = A_c V = \frac{\pi D^2}{4} V \rightarrow D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{(4)(0.343 \text{ m}^3/\text{min})}{\pi(110 \text{ m/min})}} = 0.063 \text{ m} = \mathbf{6.3 \text{ cm}}$$

111 A desktop computer is to be cooled safely by a fan in hot environments and high elevations. The air ow rate of the fan and the diameter of the casing are to be determined.

sumptions **1** Steady operation under worst conditions is considered. **2** Air is an ideal gas with constant ecific heats. **3** Kinetic and potential energy changes are negligible.

operties The specific heat of air at the average temperature of T_{ave} = (45+60)/2 = 52.5°C is C_p = 1.006 /kg.°C (Table A-3).

alysis The fan selected must be able to meet the cooling requirements of the computer at worst nditions. Therefore, we assume air to enter the computer at 66.63 kPa and 45°C, and leave at 60°C.

We take the air space in the computer as the system, which is a control volume. The energy lance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



en the required mass flow rate of air to absorb heat at a rate of 100 W is termined to be

$$= \dot{m}C_p(T_{out} - T_{in}) \rightarrow \dot{m} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})} = \frac{100 \text{ W}}{(1006 \text{ J/kg.°C})(60-45)°\text{C}}$$

$$= 0.006627 \text{ kg/s} = 0.398 \text{ kg/min}$$

e density of air entering the fan at the exit and its volume flow rate are

$$\rho = \frac{P}{RT} = \frac{66.63 \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg.K})(60+273)\text{K}} = 0.6972 \text{ kg/m}^3$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.398 \text{ kg/min}}{0.6972 \text{ kg/m}^3} = \mathbf{0.57 \text{ m}^3/\text{min}}$$

r an average exit velocity of 110 m/min, the diameter of the casing of the fan is determined from

$$\dot{V} = A_c V = \frac{\pi D^2}{4} V \longrightarrow D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{(4)(0.57 \text{ m}^3/\text{min})}{\pi(110 \text{ m/min})}} = 0.081 \text{ m} = \mathbf{8.1 \text{ cm}}$$

5-112E Electronic devices mounted on a cold plate are cooled by water. The amount of heat generated by the electronic devices is to be determined.

Assumptions **1** Steady operating conditions exist. **2** About 15 percent of the heat generated is dissipated from the components to the surroundings by convection and radiation. **3** Kinetic and potential energy changes are negligible.

Properties The properties of water at room temperature are ρ = 62.1 lbm/ft^3 and C_p = 1.00 Btu/lbm.°F (Table A-3).

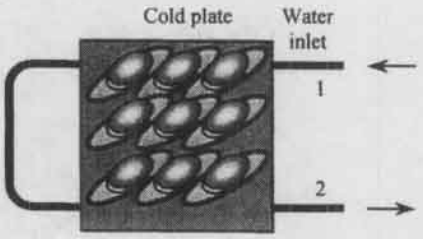
Analysis We take the tubes of the cold plate to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



Then mass flow rate of water and the rate of heat removal by the water are determined to be

$$\dot{m} = \rho A\mathbf{V} = \rho\frac{\pi D^2}{4}\mathbf{V} = (62.1 \text{ lbm/ft}^3)\frac{\pi(0.25/12 \text{ ft})^2}{4}(60 \text{ ft/min}) = 1.272 \text{ lbm/min} = 76.2 \text{ lbm/h}$$

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in}) = (76.2 \text{ lbm/h})(1.00 \text{ Btu/lbm.°F})(105 - 95)°\text{F} = 762 \text{ Btu/h}$$

which is 85 percent of the heat generated by the electronic devices. Then the total amount of heat generated by the electronic devices becomes

$$\dot{Q} = \frac{762 \text{ Btu/h}}{0.85} = \mathbf{896 \text{ Btu/h} = 263 \text{ W}}$$

5-113 A sealed electronic box is to be cooled by tap water flowing through channels on two of its sides. The mass flow rate of water and the amount of water used per year are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Entire heat generated is dissipated by water. **3** Water is an incompressible substance with constant specific heats at room temperature. **4** Kinetic and potential energy changes are negligible.

Properties The specific heat of water at room temperature is $C_p = 4.18$ kJ/kg.°C (Table A-3).

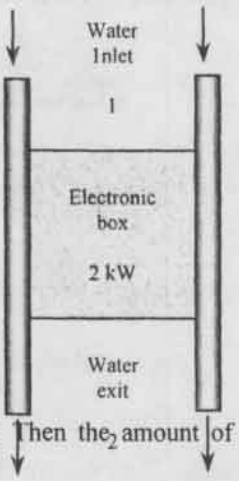
Analysis We take the water channels on the sides to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



Then the mass flow rate of tap water flowing through the electronic box becomes

$$\dot{Q} = \dot{m}C_p\Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p\Delta T} = \frac{2 \text{ kJ/s}}{(4.18 \text{ kJ/kg.°C})(4°\text{C})} = \mathbf{0.1196 \text{ kg/s}}$$

Therefore, 0.11962 kg of water is needed per second to cool this electronic box. Then the amount of cooling water used per year becomes

$$m = \dot{m}\Delta t = (0.1196 \text{ kg/s})(365 \text{ days/yr} \times 24 \text{ h/day} \times 3600 \text{ s/h})$$
$$= 3{,}772{,}000 \text{ kg/yr} = \mathbf{3{,}772 \text{ tons/yr}}$$

5-114 A sealed electronic box is to be cooled by tap water flowing through channels on two of its sides. The mass flow rate of water and the amount of water used per year are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Entire heat generated is dissipated by water. **3** Water is an incompressible substance with constant specific heats at room temperature. **4** Kinetic and potential energy changes are negligible

Properties The specific heat of water at room temperature is $C_p = 4.18$ kJ/kg.°C (Table A-3).

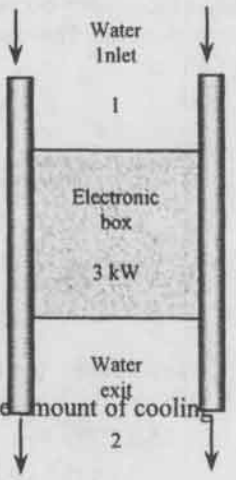
Analysis We take the water channels on the sides to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{\nearrow 0 \text{ (steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



Then the mass flow rate of tap water flowing through the electronic box becomes

$$\dot{Q} = \dot{m}C_p\Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p\Delta T} = \frac{3 \text{ kJ/s}}{(4.18 \text{ kJ/kg.°C})(4°\text{C})} = \mathbf{0.1794 \text{ kg/s}}$$

Therefore, 0.1794 kg of water is needed per second to cool this electronic box. Then the amount of cooling water used per year becomes

$$m = \dot{m}\Delta t = (0.1794 \text{ kg/s})(365 \text{ days/yr} \times 24 \text{ h/day} \times 3600 \text{ s/h})$$
$$= 5{,}658{,}400 \text{ kg/yr} = \mathbf{5{,}658 \text{ tons/yr}}$$

5-115 A long roll of large 1-Mn manganese steel plate is to be quenched in an oil bath at a specified rate. The rate at which heat needs to be removed from the oil to keep its temperature constant is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of the roll are constant. **3** Kinetic and potential energy changes are negligible

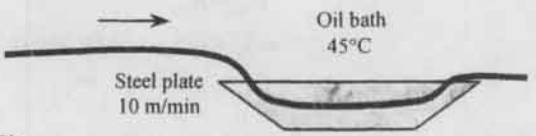
Properties The properties of the steel plate are given to be $\rho = 7854$ kg/m^3 and $C_p = 0.434$ kJ/kg.°C.

Analysis The mass flow rate of the sheet metal through the oil bath is

$$\dot{m} = \rho \dot{V} = \rho w t V = (785 \text{ kg/m}^3)(2 \text{ m})(0.005 \text{ m})(10 \text{ m/min}) = 78.5 \text{ kg/min}$$

We take the volume occupied by the sheet metal in the oil bath to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed I n the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{m}C_p(T_1 - T_2)$$

Then the rate of heat transfer from the sheet metal to the oil bath becomes

$$\dot{Q}_{out} = \dot{m}C_p[T_{in} - T_{out}]_{\text{metal}}$$
$$= (78.5 \text{ kg/min})(0.434 \text{ kJ/kg.°C})(820 - 51.1)°\text{C}$$
$$= 26{,}200 \text{ kJ/min} = \mathbf{437 \text{ kW}}$$

This is the rate of heat transfer from the metal sheet to the oil, which is equal to the rate of heat removal from the oil since the oil temperature is maintained constant at 45 °C.

5-116 The components of an electronic device located in a horizontal duct of rectangular cross section are cooled by forced air. The heat transfer from the outer surfaces of the duct is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible

Properties The gas constant of air is $R = 0.287$ kJ/kg.°C (Table A-1). The specific heat of air at room temperature is $C_p = 1.005$ kJ/kg.°C (Table A-2).

Analysis The density of air entering the duct and the mass flow rate are

$$\rho = \frac{P}{RT} = \frac{101.325 \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg.K})(30+273)\text{K}} = 1.165 \text{ kg/m}^3$$

$$\dot{m} = \rho \dot{V} = (1.165 \text{ kg/m}^3)(0.6 \text{ m}^3/\text{min}) = 0.700 \text{ kg/min}$$

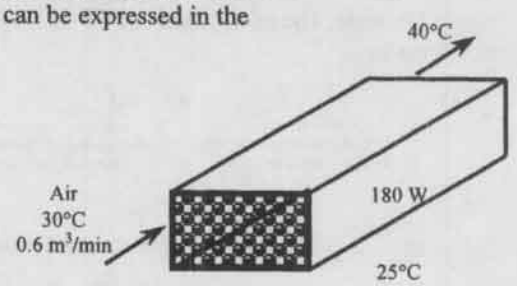
We take the channel, excluding the electronic components, to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



Then the rate of heat transfer to the air passing through the duct becomes

$$\dot{Q}_{air} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{air}} = (0.700/60 \text{ kg/s})(1.005 \text{ kJ/kg.°C})(40-30)\text{°C} = 0.117 \text{ kW} = 117 \text{ W}$$

The rest of the 180 W heat generated must be dissipated through the outer surfaces of the duct by natural convection and radiation,

$$\dot{Q}_{\text{external}} = \dot{Q}_{\text{total}} - \dot{Q}_{\text{internal}} = 180 - 117 = \mathbf{63 \text{ W}}$$

5-117 The components of an electronic device located in a horizontal duct of circular cross section is cooled by forced air. The heat transfer from the outer surfaces of the duct is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible

Properties The gas constant of air is R = 0.287 kJ/kg.°C (Table A-1). The specific heat of air at room temperature is C_p = 1.005 kJ/kg.°C (Table A-2).

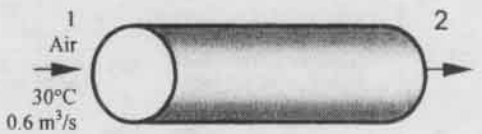
Analysis The density of air entering the duct and the mass flow rate are

$$\rho = \frac{P}{RT} = \frac{101.325 \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg.K})(30+273)\text{K}} = 1.165 \text{ kg/m}^3$$

$$\dot{m} = \rho \dot{V} = (1.165 \text{ kg/m}^3)(0.6 \text{ m}^3/\text{min}) = 0.700 \text{ kg/min}$$

We take the channel, excluding the electronic components, to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}{}^{\nearrow 0 \text{ (steady)}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$

Then the rate of heat transfer to the air passing through the duct becomes

$$\dot{Q}_{air} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{air}} = (0.700/60 \text{ kg/s})(1.005 \text{ kJ/kg.°C})(40-30)\text{°C} = 0.117 \text{ kW} = 117 \text{ W}$$

The rest of the 180 W heat generated must be dissipated through the outer surfaces of the duct by natural convection and radiation,

$$\dot{Q}_{\text{external}} = \dot{Q}_{\text{total}} - \dot{Q}_{\text{internal}} = 180 - 117 = \mathbf{63 \text{ W}}$$

5-118E Water is heated in a parabolic solar collector. The required length of parabolic collector and the surface temperature of the collector tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat loss from the tube is negligible so that the entire solar energy incident on the tube is transferred to the water. **3** Kinetic and potential energy changes are negligible

Properties The specific heat of water at room temperature is $C_p = 1.00$ Btu/lbm.°F (Table A-2E).

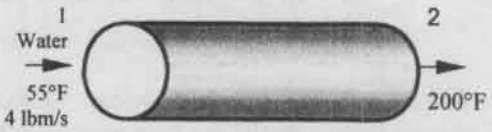
Analysis We take the thin aluminum tube to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{\nearrow 0 \text{ (steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}_{\text{water}} C_p (T_2 - T_1)$$



Then the total rate of heat transfer to the water flowing through the tube becomes

$$\dot{Q}_{\text{total}} = \dot{m}C_p(T_e - T_i) = (4 \text{ lbm/s})(1.00 \text{ Btu/lbm.°F})(200 - 55)°\text{F} = 580 \text{ Btu/s} = 2{,}088{,}000 \text{ Btu/h}$$

The length of the tube required is

$$L = \frac{\dot{Q}_{\text{total}}}{\dot{Q}} = \frac{2{,}088{,}000 \text{ Btu/h}}{350 \text{ Btu/h.ft}} = \mathbf{5966 \text{ ft}}$$

-119 Air enters a hollow-core printed circuit board. The exit temperature of the air is to be determined. √

ssumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant specific heats at room mperature. **3** The local atmospheric pressure is 1 atm. **4** Kinetic and potential energy changes are egligible.

roperties The gas constant of air is R = 0.287 kJ/kg.°C (Table A-1). The specific heat of air at room mperature is C_p = 1.005 kJ/kg.°C (Table A-2).

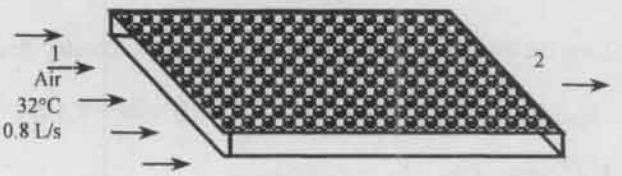
nalysis The density of air entering the duct and the mass flow rate are

$$\rho = \frac{P}{RT} = \frac{101.325 \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg.K})(32+273)\text{K}} = 1.16 \text{ kg/m}^3$$

$$\dot{m} = \rho \dot{V} = (1.16 \text{ kg/m}^3)(0.0008 \text{ m}^3/\text{s}) = 0.000928 \text{ kg/s}$$

/e take the hollow core to be the system, which is a control volume. The energy balance r this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$

hen the exit temperature of air leaving the hollow core becomes

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1) \rightarrow T_2 = T_1 + \frac{\dot{Q}_{in}}{\dot{m}C_p} = 32\,^\circ\text{C} + \frac{20 \text{ J/s}}{(0.000928 \text{ kg/s})(1005 \text{ J/kg.°C})} = \mathbf{53.4°C}$$

5-120 A computer is cooled by a fan blowing air through the case of the computer. The required flow rate of the air and the fraction of the temperature rise of air that is due to heat generated by the fan are to be determined. √

Assumptions **1** Steady flow conditions exist. **2** Air is an ideal gas with constant specific heats. **3** The pressure of air is 1 atm. **4** Kinetic and potential energy changes are negligible

Properties The specific heat of air at room temperature is C_p = 1.005 kJ/kg.°C (Table A-2).

Analysis (*a*) We take the air space in the computer as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} + \dot{W}_{in} = \dot{m}C_p(T_2 - T_1)$$

Noting that the fan power is 25 W and the 8 PCBs transfer a total of 80 W of heat to air, the mass flow rate of air is determined to be

$$\dot{Q}_{in} + \dot{W}_{in} = \dot{m}C_p(T_e - T_i) \rightarrow \dot{m} = \frac{\dot{Q}_{in} + \dot{W}_{in}}{C_p(T_e - T_i)} = \frac{25 + (8 \times 10)\ \text{W}}{(1005\ \text{J/kg.°C})(10°\text{C})} = \mathbf{0.0104\ kg/s}$$

(*b*) The fraction of temperature rise of air that is due to the heat generated by the fan and its motor can be determined from

$$\dot{Q} = \dot{m}C_p\Delta T \rightarrow \Delta T = \frac{\dot{Q}}{\dot{m}C_p} = \frac{25\ \text{W}}{(0.0104\ \text{kg/s})(1005\ \text{J/kg.°C})} = 2.4°\text{C}$$

$$f = \frac{2.4°\text{C}}{10°\text{C}} = 0.24 = \mathbf{24\%}$$

5-121 Hot water enters a pipe whose outer surface is exposed to cold air in a basement. The rate of heat loss from the water is to be determined.

Assumptions **1** Steady flow conditions exist. **2** Water is an incompressible substance with constant specific heats. **3** The changes in kinetic and potential energies are negligible.

Properties The properties of water at the average temperature of (90+88)/2 = 89°C are $\rho = 965\ \text{kg/m}^3$ and $C_p = 4.21$ kJ/kg.°C (Table A-3).

Analysis The mass flow rate of water is

$$\dot{m} = \rho A_c V = (965\ \text{kg/m}^3)\frac{\pi(0.04\ \text{m})^2}{4}(0.8\ \text{m/s}) = 0.970\ \text{kg/s}$$

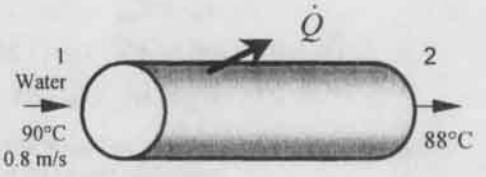
We take the section of the pipe in the basement to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{out} = \dot{m}C_p(T_1 - T_2)$$



Then the rate of heat transfer from the hot water to the surrounding air becomes

$$\dot{Q}_{out} = \dot{m}C_p[T_{in} - T_{out}]_{\text{water}}$$
$$= (0.970\ \text{kg/s})(4.21\ \text{kJ/kg.°C})(90 - 88)°\text{C}$$
$$= \mathbf{8.17\ kW}$$

5-122 A room is to be heated by an electric resistance heater placed in a duct in the room. The power rating of the electric heater and the temperature rise of air as it passes through the heater are to be determined.

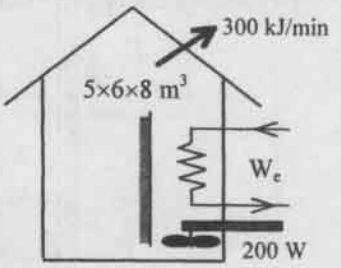
Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible. **4** The heating duct is adiabatic, and thus heat transfer through it is negligible. **5** No air leaks in and out of the room.

Properties The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1). The specific heats of air at room temperature are $C_p = 1.005$ and $C_v = 0.718$ kJ/kg·K (Table A-2).

Analysis The total mass of air in the room is

$$V = 5 \times 6 \times 8\text{m}^3 = 240\text{m}^3$$

$$m = \frac{P_1 V}{RT_1} = \frac{(98\text{kPa})(240\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(288\text{K})} = 285.7\text{kg}$$



We first take the *entire room* as our system, which is a closed system since no mass leaks in or out. The power rating of the electric heater is determined by applying the conservation of energy relation to this constant volume closed system:

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{e,in} + W_{fan,in} - Q_{out} = \Delta U \quad (\text{since } \Delta\text{KE} = \Delta\text{PE} = 0)$$

$$\Delta t\left(\dot{W}_{e,in} + \dot{W}_{fan,in} - \dot{Q}_{out}\right) = mC_{v,ave}(T_2 - T_1)$$

Solving for the electrical work input gives

$$\begin{aligned}\dot{W}_{e,in} &= \dot{Q}_{out} - \dot{W}_{fan,in} + mC_v(T_2 - T_1)/\Delta t \\ &= (200/60\text{kJ/s}) - (0.2\text{kJ/s}) + (284.7\text{kg})(0.718\text{kJ/kg}\cdot^\circ\text{C})(25-15)^\circ\text{C}/(15\times 60\text{s}) \\ &= \mathbf{5.40kW}\end{aligned}$$

We now take the *heating duct* as the system, which is a control volume since mass crosses the boundary. There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this adiabatic steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ (\text{steady})}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{W}_{fan,in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{e,in} + \dot{W}_{fan,in} = \dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)$$

Thus,

$$\Delta T = T_2 - T_1 = \frac{\dot{W}_{e,in} + \dot{W}_{fan,in}}{\dot{m}C_p} = \frac{(5.40 + 0.2)\text{kJ/s}}{(50/60\text{kg/s})(1.005\text{kJ/kg}\cdot\text{K})} = \mathbf{6.7^\circ C}$$

5-123 A house is heated by an electric resistance heater placed in a duct. The power rating of the electric heater is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible.

Properties The constant pressure specific heat of air at room temperature is C_p = 1.005 kJ/kg·K (Table A-2)

Analysis We take the *heating duct* as the system, which is a control volume since mass crosses the boundary. There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this steady-flow system can be expressed in the rate form as

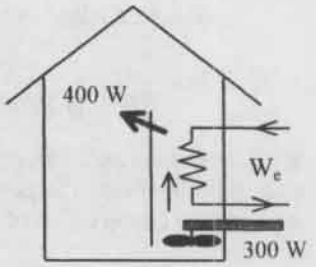
$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{W}_{fan,in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{e,in} + \dot{W}_{fan,in} = \dot{Q}_{out} + \dot{m}(h_2 - h_1)$$

$$= \dot{Q}_{out} + \dot{m}C_p(T_2 - T_1)$$



Substituting, the power rating of the heating element is determined to be

$$\dot{W}_{e,in} = \dot{Q}_{out} + \dot{m}C_p\Delta T - \dot{W}_{fan,in}$$

$$= (0.4\text{kJ/s}) + (0.6\text{kg/s})(1.005\text{kJ/kg}\cdot^\circ\text{C})(5^\circ\text{C}) - 0.3\text{kW}$$

$$= \mathbf{3.12kW}$$

5-124 A hair dryer consumes 1200 W of electric power when running. The inlet volume flow rate and the exit velocity of air are to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible. **4** The power consumed by the fan and the heat losses are negligible.

Properties The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1). The constant pressure specific heat of air at room temperature is C_p = 1.005 kJ/kg·K (Table A-2)

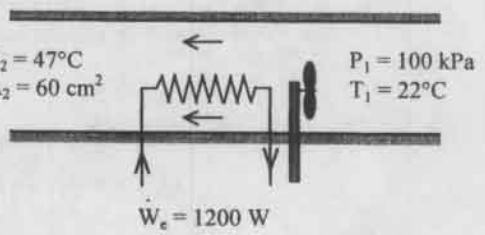
Analysis We take the *hair dryer* as the system, which is a control volume since mass crosses the boundary. There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q}_{out} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{e,in} = \dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)$$



Substituting, the mass and volume flow rates of air are determined to be

$$\dot{m} = \frac{\dot{W}_{e,in}}{C_p(T_2 - T_1)} = \frac{1.2\ \text{kJ/s}}{(1.005\text{kJ/kg}\cdot^\circ\text{C})(47-22)^\circ\text{C}} = 0.04776\text{kg/s}$$

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(295\ \text{K})}{(100\ \text{kPa})} = 0.8467\ \text{m}^3/\text{kg}$$

$$\dot{V}_1 = \dot{m}v_1 = (0.04776\ \text{kg/s})(0.8467\ \text{m}^3/\text{kg}) = \mathbf{0.0404\ m^3/s}$$

(*b*) The exit velocity of air is determined from the mass balance $\dot{m}_1 = \dot{m}_2 = \dot{m}$ to be

$$v_2 = \frac{RT_2}{P_2} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(320\text{K})}{(100\text{kPa})} = 0.9184\text{m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{v_2}A_2\mathbf{V}_2 \longrightarrow \mathbf{V}_2 = \frac{\dot{m}v_2}{A_2} = \frac{(0.04776\text{kg/s})(0.9187\text{m}^3/\text{kg})}{60\times10^{-4}\text{m}^2} = \mathbf{7.31m/s}$$

5-125 The ducts of a heating system pass through an unheated area. The rate of heat loss from the air in the ducts is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible. **4** There are no work interactions involved.

Properties The constant pressure specific heat of air at room temperature is C_p = 1.005 kJ/kg·K (Table A-2)

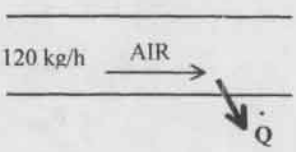
Analysis We take the *heating duct* as the system, which is a control volume since mass crosses the boundary. There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \dot{W} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{out} = \dot{m}(h_1 - h_2) = \dot{m}C_p(T_1 - T_2)$$



Substituting,

$$\dot{Q}_{out} = (120 \text{ kg/min})(1.005 \text{ kJ/kg·°C})(4\text{°C}) = \mathbf{482 \text{ kJ/min}}$$

5-126E (a) The ducts of an air-conditioning system pass through an unconditioned area. The inlet velocity and the exit temperature of air are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible. **4** There are no work interactions involved.

Properties The gas constant of air is 0.3704 psia.ft^3/lbm.R (Table A-1E). The constant pressure specific heat of air at room temperature is C_p = 0.240 Btu/lbm.R (Table A-2E)

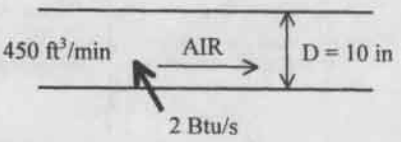
Analysis (*a*) The inlet velocity of air through the duct is

$$\mathbf{V}_1 = \frac{\dot{V}_1}{A_1} = \frac{\dot{V}_1}{\pi r^2} = \frac{450\text{ft}^3/\text{min}}{\text{p}(5/12\text{ft})^2} = \mathbf{825ft/min}$$

Then the mass flow rate of air becomes

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.3704\text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(510\text{R})}{(15\text{psia})} = 12.6\text{ft}^3/\text{lbm}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{450\text{ft}^3/\text{min}}{12.6\text{ft}^3/\text{lbm}} = 35.7\text{lbm/min} = 0.595\text{lbm/s}$$



(*b*) We take the *air-conditioning duct* as the system, which is a control volume since mass crosses the boundary. There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{W} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{in} = \dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)$$

Then the exit temperature of air becomes

$$T_2 = T_1 + \frac{\dot{Q}_{in}}{\dot{m}C_p} = 50^\circ\text{F} + \frac{2\text{Btu/s}}{(0.595\text{lbm/s})(0.24\text{Btu/lbm}\cdot^\circ\text{F})} = \mathbf{64.0^\circ F}$$

5-127 Water is heated by a 7-kW resistance heater as it flows through an insulated tube. The mass flow rate of water is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Water is an incompressible substance with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible. **4** The tube is adiabatic and thus heat losses are negligible.

Properties The specific heat of water at room temperature is $C = 4.18$ kJ/kg·°C (Table A-3).

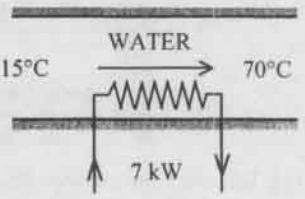
Analysis We take the *water pipe* as the system, which is a control volume since mass crosses the boundary. There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q}_{out} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{e,in} = \dot{m}(h_2 - h_1) = \dot{m}[C(T_2 - T_1) + v\Delta P^{\nearrow 0}] = \dot{m}C(T_2 - T_1)$$



Substituting, the mass flow rates of water is determined to be

$$\dot{m} = \frac{\dot{W}_{e,in}}{C(T_2 - T_1)} = \frac{7\text{kJ/s}}{(4.184\text{kJ/kg}\cdot^\circ\text{C})(70 - 15)^\circ\text{C}} = \mathbf{0.0304 kg/s}$$

5-128 Steam pipes pass through an unheated area, and the temperature of steam drops as a result of heat losses. The mass flow rate of steam and the rate of heat loss from are to be determined. √

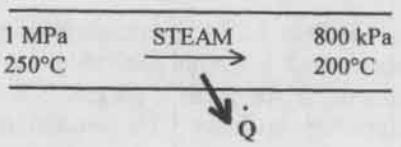
Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **4** There are no work interactions involved.

Properties From the steam tables (Table A-6),

$$\left.\begin{matrix} P_1 = 1\text{MPa} \\ T_1 = 250^\circ\text{C} \end{matrix}\right\} \begin{matrix} v_1 = 0.2327\text{m}^3/\text{kg} \\ h_1 = 2942.6\text{kJ/kg} \end{matrix}$$

and

$$\left.\begin{matrix} P_2 = 800\text{kPa} \\ T_2 = 200^\circ\text{C} \end{matrix}\right\} h_2 = 2839.3\text{kJ/kg}$$



Analysis (*a*) The mass flow rate of steam is determined directly from

$$\dot{m} = \frac{1}{v_1} A_1 \mathbf{V}_1 = \frac{1}{0.2327\text{m}^3/\text{kg}} \left[\pi(0.06\text{m})^2\right](2\text{m/s}) = \mathbf{0.0972\ kg/s}$$

(*b*) We take the *steam pipe* as the system, which is a control volume since mass crosses the boundary. There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\begin{matrix}\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}\end{matrix}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\begin{matrix}\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}\end{matrix}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \dot{W} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{out} = \dot{m}(h_1 - h_2)$$

Substituting, the rate of heat loss is determined to be

$$\dot{Q}_{loss} = (0.0972\text{ kg/s})(2942.6 - 2839.3)\text{ kJ/kg} = \mathbf{10.04\ kJ/s}$$

Energy Balance for Charging and Discharging Processes

5-129 An evacuated bottle is surrounded by atmospheric air. A valve is opened, and air is allowed to fill the bottle. The amount of heat transfer through the wall of the bottle when thermal and mechanical equilibrium is established is to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** Air is an ideal gas. **3** Kinetic and potential energies are negligible. **4** There are no work interactions involved. **5** The direction of heat transfer is to the air in the bottle (will be verified).

Properties The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1).

Analysis We take the bottle as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2 \quad (\text{since } m_{out} = m_{initial} = 0)$

Energy balance:

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} + m_i h_i = m_2 u_2 \quad (\text{since } W \cong E_{out} = E_{initial} = ke \cong pe \cong 0)$$

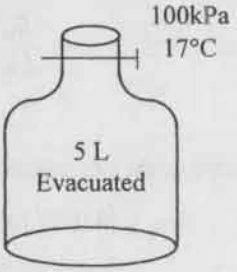
Combining the two balances:

$$Q_{in} = m_2 (u_2 - h_i)$$

where

$$m_2 = \frac{P_2 V}{R T_2} = \frac{(100\text{kPa})(0.005\text{m}^3)}{(0.287\text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(290\text{K})} = 0.0060\text{kg}$$

$$T_i = T_2 = 290\text{K} \xrightarrow{\text{Table A-17}} \begin{matrix} h_i = 290.16\text{kJ/kg} \\ u_2 = 206.91\text{kJ/kg} \end{matrix}$$



Substituting,

$$Q_{\text{in}} = (0.0060\text{ kg})(206.91 - 290.16)\text{ kJ/kg} = -0.5\text{ kJ} \rightarrow Q_{\text{out}} = \mathbf{0.5\ kJ}$$

Discussion The negative sign for heat transfer indicates that the assumed direction is wrong. Therefore, we reverse the direction.

5-130 An insulated rigid tank is evacuated. A valve is opened, and air is allowed to fill the tank until mechanical equilibrium is established. The final temperature in the tank is to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** Air is an ideal gas with constant specific heats. **3** Kinetic and potential energies are negligible. **4** There are no work interactions involved. **5** The device is adiabatic and thus heat transfer is negligible.

Properties The specific heat ratio air at room temperature is $k = 1.4$ (Table A-2).

Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2$ (since $m_{out} = m_{initial} = 0$)

Energy balance:

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$m_i h_i = m_2 u_2 \text{ (since } Q \cong W \cong E_{out} = E_{initial} = ke \cong pe \cong 0)$$



Combining the two balances:

$$u_2 = h_i \rightarrow C_v T_2 = C_p T_i \rightarrow T_2 = (C_p / C_v) T_i = kT_i$$

Substituting,

$$T_2 = 1.4 \times 290 \text{ K} = 406 \text{ K} = \mathbf{133^\circ C}$$

5-131 A rigid tank initially contains air at atmospheric conditions. The tank is connected to a supply line, and air is allowed to enter the tank until mechanical equilibrium is established. The mass of air that entered and the amount of heat transfer are to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energies are negligible. **4** There are no work interactions involved. **5** The direction of heat transfer is to the tank (will be verified).

Properties The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1). The properties of air are (Table A-17)

$$\begin{aligned} T_i = 295\ K &\longrightarrow h_i = 295.17\ kJ/kg \\ T_1 = 295\ K &\longrightarrow u_1 = 210.49\ kJ/kg \\ T_2 = 350\ K &\longrightarrow u_2 = 250.02\ kJ/kg \end{aligned}$$

Analysis (*a*) We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2 - m_1$

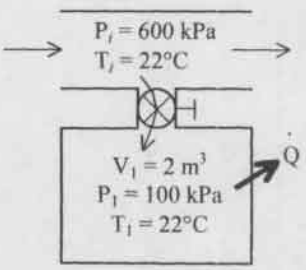
Energy balance: $\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$

$$Q_{in} + m_i h_i = m_2 u_2 - m_1 u_1 \ \text{(since } W \equiv ke \equiv pe \equiv 0)$$

The initial and the final masses in the tank are

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(100\text{kPa})(2\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(295\text{K})} = 2.362\text{kg}$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(600\text{kPa})(2\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(350\text{K})} = 11.946\text{kg}$$



Then from the mass balance,

$$m_i = m_2 - m_1 = 11.946 - 2.362 = \mathbf{9.584\ kg}$$

(*b*) The heat transfer during this process is determined from

$$\begin{aligned} Q_{in} &= -m_i h_i + m_2 u_2 - m_1 u_1 \\ &= -(9.584\text{kg})(295.17\text{kJ/kg}) + (11.946\text{kg})(250.02\text{kJ/kg}) - (2.362\text{kg})(210.49\text{kJ/kg}) \\ &= -339.3\text{kJ} \ \rightarrow \ Q_{out} = \mathbf{339.3\ kJ} \end{aligned}$$

Discussion The negative sign for heat transfer indicates that the assumed direction is wrong. Therefore, we reversed the direction.

5-132 A rigid tank initially contains saturated R-134a vapor. The tank is connected to a supply line, and R-134a is allowed to enter the tank. The final temperature in the tank, the mass of R-134a that entered, and the heat transfer are to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** Kinetic and potential energies are negligible. **3** There are no work interactions involved. **4** The direction of heat transfer is to the tank (will be verified).

Properties The properties of refrigerant are (Tables A-11 through A-13)

$$\left.\begin{array}{l} T_1 = 8^\circ\text{C} \\ x_1 = 0.6 \end{array}\right\} \begin{array}{l} v_1 = v_f + x_1 v_{fg} = 0.0007884 + 0.6 \times (0.0525 - 0.0007884) = 0.03182\text{m}^3/\text{kg} \\ u_1 = u_f + x_1 u_{fg} = 60.43 + 0.6 \times (231.46 - 60.43) = 163.05\text{kJ/kg} \end{array}$$

$$\left.\begin{array}{l} P_2 = 800\text{kPa} \\ sat.vapor \end{array}\right\} \begin{array}{l} v_2 = v_{g@800\text{kPa}} = 0.0255\text{m}^3/\text{kg} \\ u_2 = u_{g@800\text{kPa}} = 243.78\text{kJ/kg} \end{array}$$

$$\left.\begin{array}{l} P_i = 1.0\text{MPa} \\ T_i = 120^\circ C \end{array}\right\} h_i = 356.52\text{kJ/kg}$$

Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2 - m_1$

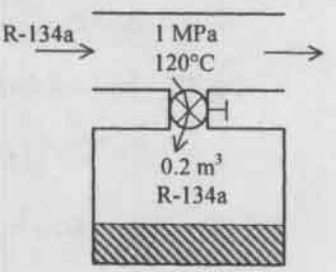
Energy balance:

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} + m_i h_i = m_2 u_2 - m_1 u_1 \quad (\text{since } W \cong ke \cong pe \cong 0)$$

(*a*) The tank contains saturated vapor at the final state at 800 kPa, and thus the final temperature is the saturation temperature at this pressure,

$$T_2 = T_{sat\,@\,800\,kPa} = \mathbf{31.33^\circ C}$$



(*b*) The initial and the final masses in the tank are

$$m_1 = \frac{V}{v_1} = \frac{0.2\ m^3}{0.03182\ m^3/kg} = 6.29\ kg$$

$$m_2 = \frac{V}{v_2} = \frac{0.2\ m^3}{0.0255\ m^3/kg} = 7.84\ kg$$

Then from the mass balance

$$m_i = m_2 - m_1 = 7.84 - 6.29 = \mathbf{1.55\ kg}$$

(*c*) The heat transfer during this process is determined from the energy balance to be

$$\begin{aligned} Q_{in} &= -m_i h_i + m_2 u_2 - m_1 u_1 \\ &= -(1.55\text{kg})(356.52\text{kJ/kg}) + (7.84\text{kg})(243.78\text{kJ/kg}) - (6.29\text{kg})(163.05\text{kJ/kg}) \\ &= \mathbf{333\text{kJ}} \end{aligned}$$

5-133E A rigid tank initially contains saturated water vapor. The tank is connected to a supply line, and water vapor is allowed to enter the tank until one-half of the tank is filled with liquid water. The final pressure in the tank, the mass of steam that entered, and the heat transfer are to be determined.

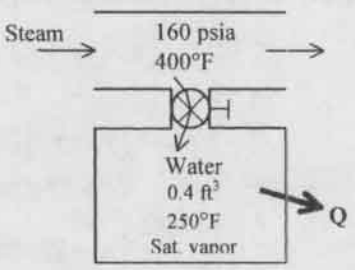
Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** Kinetic and potential energies are negligible. **3** There are no work interactions involved. **4** The direction of heat transfer is to the tank (will be verified).

Properties The properties of water are (Tables A-4 through A-6)

$$\left.\begin{array}{l} T_1 = 250^\circ\text{F} \\ \textit{sat.vapor} \end{array}\right\} \begin{array}{l} v_1 = v_{g@250^\circ\text{F}} = 13.826\,\text{ft}^3/\text{lbm} \\ u_1 = u_{g@250^\circ\text{F}} = 1087.9\,\text{Btu/lbm} \end{array}$$

$$\left.\begin{array}{l} T_2 = 250^\circ F \\ \textit{sat.mixture} \end{array}\right\} \begin{array}{l} v_f = 0.017001, \quad v_g = 13.826\,\text{ft}^3/\text{lbm} \\ u_f = 218.49, \quad u_g = 1087.9\,\text{Btu/lbm} \end{array}$$

$$\left.\begin{array}{l} P_i = 160\,\text{psia} \\ T_i = 400^\circ\text{F} \end{array}\right\} h_i = 1217.8\,\text{Btu/lbm}$$



Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2 - m_1$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$

$$Q_{in} + m_i h_i = m_2 u_2 - m_1 u_1 \quad (\text{since } W \cong ke \cong pe \cong 0)$$

(*a*) The tank contains saturated mixture at the final state at 250°F, and thus the exit pressure is the saturation pressure at this temperature,

$$P_2 = P_{sat\,@\,250^\circ F} = \mathbf{29.28\ psia}$$

(*b*) The initial and the final masses in the tank are

$$m_1 = \frac{V}{v_1} = \frac{4\ \text{ft}^3}{13.826\ \text{ft}^3/\text{lbm}} = 0.289\ \text{lbm}$$

$$m_2 = m_f + m_g = \frac{V_f}{v_f} + \frac{V_g}{v_g} = \frac{2\ \text{ft}^3}{0.0170\ \text{ft}^3/\text{lbm}} + \frac{2\ \text{ft}^3}{13.826\ \text{ft}^3/\text{lbm}} = 117.64 + 0.14 = 117.78\ \text{lbm}$$

Then from the mass balance

$$m_i = m_2 - m_1 = 117.78 - 0.289 = \mathbf{117.49\ lbm}$$

(*c*) The heat transfer during this process is determined from the energy balance to be

$$\begin{aligned} Q_{in} &= -m_i h_i + m_2 u_2 - m_1 u_1 \\ &= -(117.49\text{lbm})(1217.8\text{Btu/lbm}) + 25{,}855\text{Btu} - (0.289\text{lbm})(1087.9\text{Btu/lbm}) \\ &= -117{,}539\text{Btu} \rightarrow Q_{out} = \mathbf{117{,}539\,Btu} \end{aligned}$$

since $U_2 = m_2 u_2 = m_f u_f + m_g u_g = 117.64 \times 218.49 + 0.14 \times 1087.9 = 25{,}855$ Btu

Discussion A negative result for heat transfer indicates that the assumed direction is wrong, and should be reversed.

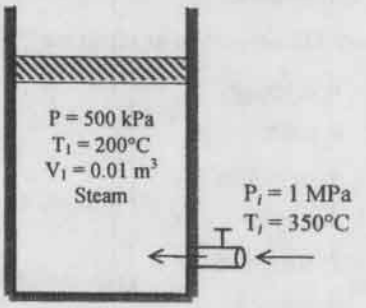
5-134 A cylinder initially contains superheated steam. The cylinder is connected to a supply line, and is superheated steam is allowed to enter the cylinder until the volume doubles at constant pressure. The final temperature in the cylinder and the mass of the steam that entered are to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** The expansion process is quasi-equilibrium. **3** Kinetic and potential energies are negligible. **3** There are no work interactions involved other than boundary work. **4** The device is insulated and thus heat transfer is negligible.

Properties The properties of steam are (Tables A-4 through A-6)

$$\left.\begin{aligned} P_1 &= 500\text{kPa} \\ T_1 &= 200^\circ\text{C} \end{aligned}\right\} \begin{aligned} v_1 &= 0.4249\text{m}^3/\text{kg} \\ u_1 &= 2642.9\text{kJ/kg} \end{aligned}$$

$$\left.\begin{aligned} P_i &= 1\text{MPa} \\ T_i &= 350^\circ\text{C} \end{aligned}\right\} h_i = 3157.7\text{kJ/kg}$$



Analysis (*a*) We take the cylinder as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2 - m_1$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$

$$m_i h_i = W_{b,out} + m_2 u_2 - m_1 u_1 \quad (\text{since } Q \cong ke \cong pe \cong 0)$$

Combining the two relations gives $0 = W_{b,out} - (m_2 - m_1)h_i + m_2 u_2 - m_1 u_1$

The boundary work done during this process is

$$W_{b,out} = \int_1^2 P\,dV = P(V_2 - V_1) = (500\text{kPa})(0.02 - 0.01)\text{m}^3\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = 5\text{kJ}$$

The initial and the final masses in the cylinder are

$$m_1 = \frac{V_1}{v_1} = \frac{0.01\text{ m}^3}{0.4249\text{ m}^3/\text{kg}} = 0.0235\text{ kg}$$

$$m_2 = \frac{V_2}{v_2} = \frac{0.02\text{ m}^3}{v_2}$$

Substituting,

$$= 5 - \left(\frac{0.02}{v_2} - 0.0235\right)(3157.7) + \frac{0.02}{v_2}u_2 - (0.0235)(2642.9)$$

Then by trial and error, $T_2 = \mathbf{262.6^\circ C}$ and $v_2 = 0.4865\text{ m}^3/\text{kg}$

(*b*) The final mass in the cylinder is

$$m_2 = \frac{V_2}{v_2} = \frac{0.02\text{ m}^3}{0.4865\text{ m}^3/\text{kg}} = 0.0411\text{ kg}$$

Then,

$$m_i = m_2 - m_1 = 0.0411 - 0.0235 = \mathbf{0.0176\text{ kg}}$$

5-135 A cylinder initially contains saturated liquid-vapor mixture of water. The cylinder is connected to a supply line, and the steam is allowed to enter the cylinder until all the liquid is vaporized. The final temperature in the cylinder and the mass of the steam that entered are to be determined. √

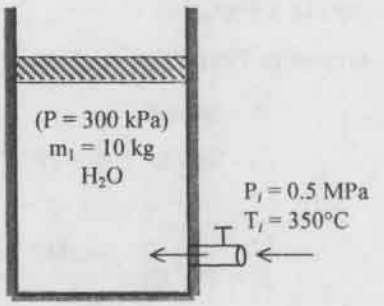
Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** The expansion process is quasi-equilibrium. **3** Kinetic and potential energies are negligible. **3** There are no work interactions involved other than boundary work. **4** The device is insulated and thus heat transfer is negligible.

Properties The properties of steam are (Tables A-4 through A-6)

$$\left.\begin{matrix} P_1 = 300\text{kPa} \\ x_1 = 0.8 \end{matrix}\right\} h_1 = h_f + x_1 h_{fg} = 561.47 + 0.8 \times 2163.8 = 2292.51\text{kJ/kg}$$

$$\left.\begin{matrix} P_2 = 300\text{kPa} \\ sat.vapor \end{matrix}\right\} h_2 = h_{g@300\text{kPa}} = 2725.3\text{kJ/kg}$$

$$\left.\begin{matrix} P_i = 0.5\text{MPa} \\ T_i = 350^\circ\text{C} \end{matrix}\right\} h_i = 3167.7\text{kJ/kg}$$



Analysis (*a*) The cylinder contains saturated vapor at the final state at a pressure of 300 kPa, thus the final temperature in the cylinder must be

$$T_2 = T_{\text{sat @ 300 kPa}} = \mathbf{133.6°C}$$

(*b*) We take the cylinder as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2 - m_1$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$

$$m_i h_i = W_{b,out} + m_2 u_2 - m_1 u_1 \quad (\text{since } Q \cong ke \cong pe \cong 0)$$

Combining the two relations gives $0 = W_{b,out} - (m_2 - m_1)h_i + m_2 u_2 - m_1 u_1$

or, $0 = -(m_2 - m_1)h_i + m_2 h_2 - m_1 h_1$

since the boundary work and ΔU combine into ΔH for constant pressure expansion and compression processes. Solving for m_2 and substituting,

$$m_2 = \frac{h_i - h_1}{h_i - h_2} m_1 = \frac{(3167.7 - 2292.51)\text{kJ/kg}}{(3167.7 - 2725.3)\text{kJ/kg}}(10\text{kg}) = 19.78\text{kg}$$

Thus,

$$m_i = m_2 - m_1 = 19.78 - 10 = \mathbf{9.78\ kg}$$

5-136 A rigid tank initially contains saturated R-134a vapor. The tank is connected to a supply line, and R-134a is allowed to enter the tank. The mass of the R-134a that entered and the heat transfer are to be determined.

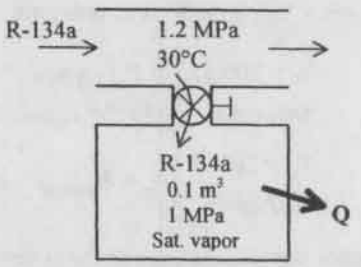
Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** Kinetic and potential energies are negligible. **3** There are no work interactions involved. **4** The direction of heat transfer is to the tank (will be verified).

Properties The properties of refrigerant are (Tables A-11 through A-13)

$$\left.\begin{matrix} P_1 = 1\text{MPa} \\ sat.vapor \end{matrix}\right\} \begin{matrix} v_1 = v_{g@1\text{MPa}} = 0.0202\text{m}^3/\text{kg} \\ u_1 = u_{g@1\text{MPa}} = 247.77\text{kJ/kg} \end{matrix}$$

$$\left.\begin{matrix} P_2 = 1.2\text{MPa} \\ sat\,liquid \end{matrix}\right\} \begin{matrix} v_2 = v_{f@1.2\text{MPa}} = 0.0008928\text{m}^3/\text{kg} \\ u_2 = u_{f@1.2\text{MPa}} = 114.69\text{kJ/kg} \end{matrix}$$

$$\left.\begin{matrix} P_i = 1.2\text{MPa} \\ T_i = 30^\circ\text{C} \end{matrix}\right\} h_i = u_{f@30^\circ\text{C}} = 91.49\text{kJ/kg}$$



Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2 - m_1$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\begin{matrix}\text{Net energy transfer}\\ \text{by heat, work, and mass}\end{matrix}} = \underbrace{\Delta E_{\text{system}}}_{\begin{matrix}\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}\end{matrix}}$

$$Q_{in} + m_i h_i = m_2 u_2 - m_1 u_1 \quad (\text{since } W \cong ke \cong pe \cong 0)$$

(*a*) The initial and the final masses in the tank are

$$m_1 = \frac{V_1}{v_1} = \frac{0.1\,\text{m}^3}{0.0202\,\text{m}^3/\text{kg}} = 4.95\,\text{kg}$$

$$m_2 = \frac{V_2}{v_2} = \frac{0.1\,\text{m}^3}{0.0008928\,\text{m}^3/\text{kg}} = 112.01\,\text{kg}$$

Then from the mass balance

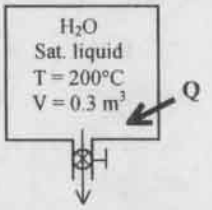
$$m_i = m_2 - m_1 = 112.01 - 4.95 = \mathbf{107.06\ kg}$$

(*c*) The heat transfer during this process is determined from the energy balance to be

$$\begin{aligned} Q_{in} &= -m_i h_i + m_2 u_2 - m_1 u_1 \\ &= -(107.06\text{kg})(91.49\text{kJ/kg}) + (112.01\text{kg})(114.69\text{kJ/kg}) - (4.95\text{kg})(247.77\text{kJ/kg}) \\ &= \mathbf{1825kJ} \end{aligned}$$

5-137 A rigid tank initially contains saturated liquid water. A valve at the bottom of the tank is opened, and half of the mass in liquid form is withdrawn from the tank. The temperature in the tank is maintained constant. The amount of heat transfer is to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid leaving the device remains constant. **2** Kinetic and potential energies are negligible. **3** There are no work interactions involved. **4** The direction of heat transfer is to the tank (will be verified).



Properties The properties of water are (Tables A-4 through A-6)

$$\left.\begin{array}{l} T_1 = 200^\circ\text{C} \\ sat.liquid \end{array}\right\} \begin{array}{l} v_1 = v_{f@200^\circ\text{C}} = 0.001157\text{m}^3/\text{kg} \\ u_1 = u_{f@200^\circ\text{C}} = 850.65\text{kJ/kg} \end{array}$$

$$\left.\begin{array}{l} T_e = 200^\circ\text{C} \\ sat.liquid \end{array}\right\} h_e = h_{f@200^\circ\text{C}} = 852.45\text{kJ/kg}$$

Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_e = m_1 - m_2$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$

$$Q_{in} = m_e h_e + m_2 u_2 - m_1 u_1 \quad (\text{since } W \cong ke \cong pe \cong 0)$$

The initial and the final masses in the tank are

$$m_1 = \frac{V_1}{v_1} = \frac{0.3\text{m}^3}{0.001157\text{m}^3/\text{kg}} = 259.3\text{kg}$$

$$m_2 = \tfrac{1}{2} m_1 = \tfrac{1}{2}(259.3\text{kg}) = 129.65\text{kg}$$

Then from the mass balance,

$$m_e = m_1 - m_2 = 259.3 - 129.65 = 129.65\text{ kg}$$

Now we determine the final internal energy,

$$v_2 = \frac{V}{m_2} = \frac{0.3\text{m}^3}{129.65\text{kg}} = 0.002314\text{m}^3/\text{kg}$$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.002314 - 0.001157}{0.12736 - 0.001157} = 0.00917$$

$$\left.\begin{array}{l} T_2 = 200^\circ\text{C} \\ x_2 = 0.00917 \end{array}\right\} u_2 = u_f + x_2 u_{fg} = 850.65 + (0.00917)(1744.7) = 866.65\text{kJ/kg}$$

Then the heat transfer during this process is determined from the energy balance by substitution to be

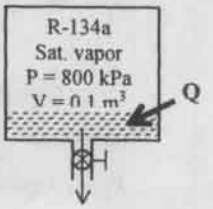
$$Q = (129.65\text{kg})(852.45\text{kJ/kg}) + (129.65\text{kg})(866.65\text{kJ/kg}) - (259.3\text{kg})(850.65\text{kJ/kg})$$
$$= \mathbf{2308kJ}$$

5-138 A rigid tank initially contains saturated liquid-vapor mixture of refrigerant-134a. A valve at the bottom of the tank is opened, and liquid is withdrawn from the tank at constant pressure until no liquid remains inside. The amount of heat transfer is to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid leaving the device remains constant. **2** Kinetic and potential energies are negligible. **3** There are no work interactions involved. **4** The direction of heat transfer is to the tank (will be verified).

Properties The properties of R-134a are (Tables A-11 through A-13)

$$P_1 = 800\text{kPa} \rightarrow v_f = 0.0008454\text{m}^3/\text{kg},\ v_g = 0.0255\text{m}^3/\text{kg}$$
$$u_f = 92.75\text{kJ/kg},\ u_g = 243.78\text{kJ/kg}$$
$$\left.\begin{matrix} P_2 = 800\text{kPa} \\ sat.vapor \end{matrix}\right\} \begin{matrix} v_2 = v_{g@800\text{kPa}} = 0.0255\text{m}^3/\text{kg} \\ u_2 = u_{g@800\text{kPa}} = 243.78\text{kJ/kg} \end{matrix}$$
$$\left.\begin{matrix} P_e = 800\text{kPa} \\ sat.liquid \end{matrix}\right\} h_e = h_{f@800\text{kPa}} = 93.42\text{kJ/kg}$$



Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_e = m_1 - m_2$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$

$$Q_{in} = m_e h_e + m_2 u_2 - m_1 u_1 \ \ (\text{since } W \cong ke \cong pe \cong 0)$$

The initial mass, initial internal energy, and final mass in the tank are

$$m_1 = m_f + m_g = \frac{V_f}{v_f} + \frac{V_g}{v_g} = \frac{0.1 \times 0.4\text{m}^3}{0.0008454\text{m}^3/\text{kg}} + \frac{0.1 \times 0.6\text{m}^3}{0.0255\text{m}^3/\text{kg}} = 47.32 + 2.35 = 49.67\text{kg}$$
$$U_1 = m_1 u_1 = m_f u_f + m_g u_g = (47.32)(92.75) + (2.35)(243.78) = 4962\text{kJ}$$
$$m_2 = \frac{V}{v_2} = \frac{0.1\text{m}^3}{0.0255\text{m}^3/\text{kg}} = 3.92\text{kg}$$

Then from the mass and energy balances,

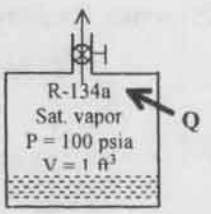
$$m_e = m_1 - m_2 = 49.67 - 3.92 = 45.75\ \text{kg}$$

$$Q_{in} = (45.75\text{kg})(93.42\text{kJ/kg}) + (3.92\text{kg})(243.78\text{kJ/kg}) - 4962\text{kJ} = \mathbf{267.6kJ}$$

5-139E A rigid tank initially contains saturated liquid-vapor mixture of R-134a. A valve at the top of the tank is opened, and vapor is allowed to escape at constant pressure until all the liquid in the tank disappears. The amount of heat transfer is to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid leaving the device remains constant. **2** Kinetic and potential energies are negligible. **3** There are no work interactions involved. **4** The direction of heat transfer is to the tank (will be verified).

Properties The properties of R-134a are (Tables A-11 through A-13)



$$P_1 = 100\text{psia} \rightarrow v_f = 0.01332\text{ft}^3/\text{lbm},\ v_g = 0.4747\text{ft}^3/\text{lbm}$$
$$u_f = 36.75\text{Btu/lbm},\ u_g = 103.68\text{Btu/lbm}$$

$$\left.\begin{array}{l} P_2 = 100\text{psia} \\ sat.vapor \end{array}\right\} \begin{array}{l} v_2 = v_{g@100\text{psia}} = 0.4747\text{ft}^3/\text{lbm} \\ u_2 = u_{g@100\text{psia}} = 103.68\text{Btu/lbm} \end{array}$$

$$\left.\begin{array}{l} P_e = 100\text{psia} \\ sat.vapor \end{array}\right\} h_e = h_{g@100\text{psia}} = 112.46\text{Btu/lbm}$$

Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_e = m_1 - m_2$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$

$$Q_{in} - m_e h_e = m_2 u_2 - m_1 u_1 \ \ (\text{since } W \cong ke \cong pe \cong 0)$$

The initial mass, initial internal energy, and final mass in the tank are

$$m_1 = m_f + m_g = \frac{V_f}{v_f} + \frac{V_g}{v_g} = \frac{4 \times 0.2\text{ft}^3}{0.01332\text{ft}^3/\text{lbm}} + \frac{4 \times 0.8\text{ft}^3}{0.4747\text{ft}^3/\text{lbm}} = 60.06 + 6.74 = 66.8\text{lbm}$$

$$U_1 = m_1 u_1 = m_f u_f + m_g u_g = (60.06)(36.75) + (6.74)(103.68) = 2906\text{Btu}$$

$$m_2 = \frac{V}{v_2} = \frac{4\text{ft}^3}{0.4747\text{ft}^3/\text{lbm}} = 8.426\text{lbm}$$

Then from the mass and energy balances,

$$m_e = m_1 - m_2 = 66.8 - 8.426 = 58.374\ \text{lbm}$$

$$\begin{aligned} Q_{in} &= m_e h_e + m_2 u_2 - m_1 u_1 \\ &= (58.374\text{lbm})(112.46\text{Btu/lbm}) + (8.426\text{lbm})(103.68\text{Btu/lbm}) - 2906\text{Btu} \\ &= \mathbf{4532Btu} \end{aligned}$$

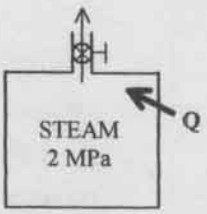
5-140 A rigid tank initially contains superheated steam. A valve at the top of the tank is opened, and vapor is allowed to escape at constant pressure until the temperature rises to 500°C. The amount of heat transfer is to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process by using constant average properties for the steam leaving the tank. **2** Kinetic and potential energies are negligible. **3** There are no work interactions involved. **4** The direction of heat transfer is to the tank (will be verified).

Properties The properties of water are (Tables A-4 through A-6)

$$\left.\begin{aligned} P_1 &= 2\text{MPa} \\ T_1 &= 300^\circ\text{C} \end{aligned}\right\} \begin{aligned} v_1 &= 0.12547\text{m}^3/\text{kg} \\ u_1 &= 2772.6\text{kJ/kg}, \quad h_1 = 3023.5\text{kJ/kg} \end{aligned}$$

$$\left.\begin{aligned} P_2 &= 2\text{MPa} \\ T_2 &= 500^\circ\text{C} \end{aligned}\right\} \begin{aligned} v_2 &= 0.17568\text{m}^3/\text{kg} \\ u_2 &= 3116.2\text{kJ/kg}, \quad h_2 = 3467.6\text{kJ/kg} \end{aligned}$$



Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \quad \rightarrow \quad m_e = m_1 - m_2$

Energy balance:

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} - m_e h_e = m_2 u_2 - m_1 u_1 \quad (\text{since } W \cong ke \cong pe \cong 0)$$

The state and thus the enthalpy of the steam leaving the tank is changing during this process. But for simplicity, we assume constant properties for the exiting steam at the average values. Thus,

$$h_e \cong \frac{h_1 + h_2}{2} = \frac{3023.5 + 3467.6\ \text{kJ/kg}}{2} = 3245.55\ \text{kJ/kg}$$

The initial and the final masses in the tank are

$$m_1 = \frac{V_1}{v_1} = \frac{0.2\ \text{m}^3}{0.12547\ \text{m}^3/\text{kg}} = 1.594\ \text{kg}$$

$$m_2 = \frac{V_2}{v_2} = \frac{0.2\ \text{m}^3}{0.17568\ \text{m}^3/\text{kg}} = 1.138\ \text{kg}$$

Then from the mass and energy balance relations,

$$m_e = m_1 - m_2 = 1.594 - 1.138 = 0.456\ \text{kg}$$

$$\begin{aligned} Q_{in} &= m_e h_e + m_2 u_2 - m_1 u_1 \\ &= (0.456\text{kg})(3245.55\text{kJ/kg}) + (1.138\text{kg})(3116.2\text{kJ/kg}) - (1.594\text{kg})(2772.6\text{kJ/kg}) \\ &= \mathbf{606.7\text{kJ}} \end{aligned}$$

5-141 A pressure cooker is initially half-filled with liquid water. If the pressure cooker is not to run out of liquid water for 1 h, the highest rate of heat transfer allowed is to be determined.

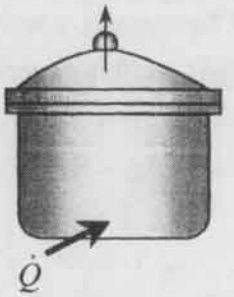
Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid leaving the device remains constant. **2** Kinetic and potential energies are negligible. **3** There are no work interactions involved.

Properties The properties of water are (Tables A-4 through A-6)

$$P_1 = 175kPa \rightarrow v_f = 0.001057\text{m}^3/\text{kg},\ v_g = 1.0036\text{m}^3/\text{kg}$$
$$u_f = 486.8\text{kJ/kg},\ u_g = 2524.9\,\text{kJ/kg}$$

$$\left.\begin{matrix} P_2 = 175\text{kPa} \\ sat.vapor \end{matrix}\right\} \begin{matrix} v_2 = v_{g@175\text{kPa}} = 1.0036\text{m}^3/\text{kg} \\ u_2 = u_{g@175\text{kPa}} = 2524.9\text{kJ/kg} \end{matrix}$$

$$\left.\begin{matrix} P_e = 175\text{kPa} \\ sat.vapor \end{matrix}\right\} h_e = h_{g@175\text{kPa}} = 2700.6\text{kJ/kg}$$



Analysis We take the cooker as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $$m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_e = m_1 - m_2$$

Energy balance: $$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} - m_e h_e = m_2 u_2 - m_1 u_1 \ \ (\text{since } W \cong ke \cong pe \cong 0)$$

The initial mass, initial internal energy, and final mass in the tank are

$$m_1 = m_f + m_g = \frac{V_f}{v_f} + \frac{V_g}{v_g} = \frac{0.002\text{m}^3}{0.001057\text{m}^3/\text{kg}} + \frac{0.002\text{m}^3}{1.0036\text{m}^3/\text{kg}} = 1.892 + 0.002 = 1.894\text{kg}$$

$$U_1 = m_1 u_1 = m_f u_f + m_g u_g = (1.892)(486.8) + (0.002)(2524.9) = 926.1\text{kJ}$$

$$m_2 = \frac{V}{v_2} = \frac{0.004\text{m}^3}{1.0036\text{m}^3/\text{kg}} = 0.004\text{kg}$$

Then from the mass and energy balances,

$$m_e = m_1 - m_2 = 1.894 - 0.004 = 1.890\ \text{kg}$$

$$Q_{in} = m_e h_e + m_2 u_2 - m_1 u_1$$
$$= (1.890\text{kg})(2700.6\text{kJ/kg}) + (0.004\text{kg})(2524.9\text{kJ/kg}) - 926.1\text{kJ} = 4188\text{kJ}$$

Thus,

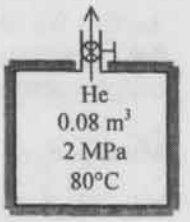
$$\dot{Q} = \frac{Q}{\Delta t} = \frac{4188\ \text{kJ}}{3600\ \text{s}} = \mathbf{1.163\ kW}$$

5-142 An insulated rigid tank initially contains helium gas at high pressure. A valve is opened, and half of the mass of helium is allowed to escape. The final temperature and pressure in the tank are to be determined. √

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process by using constant average properties for the helium leaving the tank. **2** Kinetic and potential energies are negligible. **3** There are no work interactions involved. **4** The tank is insulated and thus heat transfer is negligible. **5** Helium is an ideal gas with constant specific heats.

Properties The specific heat ratio of helium is k =1.667 (Table A-2).

Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as



Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_e = m_1 - m_2$

$m_2 = \frac{1}{2} m_1$ (given) $\longrightarrow$ $m_e = m_2 = \frac{1}{2} m_1$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$

$$-m_e h_e = m_2 u_2 - m_1 u_1 \quad (\text{since } W \cong Q \cong ke \cong pe \cong 0)$$

Note that the state and thus the enthalpy of helium leaving the tank is changing during this process. But for simplicity, we assume constant properties for the exiting steam at the average values.

Combining the mass and energy balances: $0 = \frac{1}{2} m_1 h_e + \frac{1}{2} m_1 u_2 - m_1 u_1$

Dividing by $2m_1$: $0 = h_e + u_2 - 2u_1$ *or* $0 = C_p \frac{T_1 + T_2}{2} + C_v T_2 - 2C_v T_1$

Dividing by C_v: $0 = k(T_1 + T_2) + 2T_2 - 4T_1$ since $k = C_p / C_v$

Solving for T_2: $T_2 = \frac{(4-k)}{(2+k)} T_1 = \frac{(4-1.667)}{(2+1.667)}(353\text{K}) = \mathbf{225K}$

The final pressure in the tank is

$$\frac{P_1 V}{P_2 V} = \frac{m_1 R T_1}{m_2 R T_2} \longrightarrow P_2 = \frac{m_2 T_2}{m_1 T_2} P_1 = \frac{1}{2} \frac{225}{353}(2000\text{kPa}) = \mathbf{637kPa}$$

5-143E An insulated rigid tank equipped with an electric heater initially contains pressurized air. A valve is opened, and air is allowed to escape at constant temperature until the pressure inside drops to 30 psia. The amount of electrical work transferred is to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the exit temperature (and enthalpy) of air remains constant. **2** Kinetic and potential energies are negligible. **3** The tank is insulated and thus heat transfer is negligible. **4** Air is an ideal gas with variable specific heats.

Properties The gas constant of air is R =0.3704 psia.ft^3/lbm.R (Table A-1E). The properties of air are (Table A-17E)

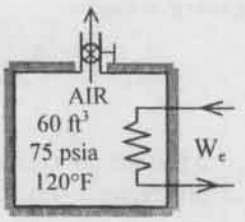
$$T_i = 580\ \text{R} \longrightarrow h_i = 138.66\ \text{Btu/lbm}$$
$$T_1 = 580\ \text{R} \longrightarrow u_1 = 98.90\ \text{Btu/lbm}$$
$$T_2 = 580\ \text{R} \longrightarrow u_2 = 98.90\ \text{Btu/lbm}$$

Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_e = m_1 - m_2$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$

$$W_{e,in} - m_e h_e = m_2 u_2 - m_1 u_1 \ (\text{since } Q \cong ke \cong pe \cong 0)$$



The initial and the final masses of air in the tank are

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(75\text{psia})(60\text{ft}^3)}{(0.3704\text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(580\text{R})} = 20.95\text{lbm}$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(30\text{psia})(60\text{ft}^3)}{(0.3704\text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(580\text{R})} = 8.38\text{lbm}$$

Then from the mass and energy balances,

$$m_e = m_1 - m_2 = 20.95 - 8.38 = 12.57\ \text{lbm}$$

$$\begin{aligned} W_{e,in} &= m_e h_e + m_2 u_2 - m_1 u_1 \\ &= (12.57\text{lbm})(138.66\text{Btu/lbm}) + (8.38\text{lbm})(98.90\text{Btu/lbm}) - (20.95\text{lbm})(98.90\text{Btu/lbm}) \\ &= \mathbf{500Btu} \end{aligned}$$

5-144 A vertical cylinder initially contains air at room temperature. Now a valve is opened, and air is allowed to escape at constant pressure and temperature until the volume of the cylinder goes down by half. The amount air that left the cylinder and the amount of heat transfer are to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the exit temperature (and enthalpy) of air remains constant. **2** Kinetic and potential energies are negligible. **3** There are no work interactions. **4** Air is an ideal gas with constant specific heats. **5** The direction of heat transfer is to the cylinder (will be verified).

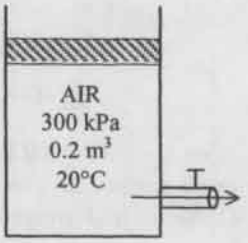
Properties The gas constant of air is R =0.287 kPa.m^3/kg.K.

Analysis (*a*) We take the cylinder as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_e = m_1 - m_2$

Energy balance: $\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$

$$Q_{in} + W_{b,in} - m_e h_e = m_2 u_2 - m_1 u_1 \quad (\text{since } ke \cong pe \cong 0)$$



The initial and the final masses of air in the cylinder are

$$m_1 = \frac{P_1 V_1}{RT_1} = \frac{(300\text{kPa})(0.2\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{K})} = 0.714\text{kg}$$

$$m_2 = \frac{P_2 V_2}{RT_2} = \frac{(300\text{kPa})(0.1\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{K})} = 0.357\text{kg} = \tfrac{1}{2} m_1$$

Then from the mass balance,

$$m_e = m_1 - m_2 = 0.714 - 0.357 = \mathbf{0.357\ kg}$$

(*b*) This is a constant pressure process, and thus the W_b and the ΔU terms can be combined into Δh to yield

$$Q = m_e h_e + m_2 h_2 - m_1 h_1$$

Noting that the temperature of the air remains constant during this process, we have $h_i = h_1 = h_2 = h$. Also, $m_e = m_2 = \frac{1}{2} m_1$. Thus,

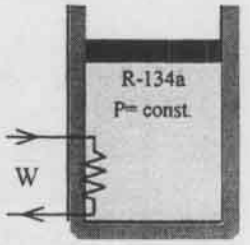
$$Q = \left(\tfrac{1}{2} m_1 + \tfrac{1}{2} m_1 - m_1\right) h = \mathbf{0}$$

Review Problems

5-145 A cylinder is initially filled with saturated R-134a vapor at a specified pressure. The refrigerant is heated both electrically and by heat transfer at constant pressure for 6 min. The electric current is to be determined, and the process is to be shown on a *T-v* diagram.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are negligible. **2** The thermal energy stored in the cylinder itself and the wires is negligible. **3** The compression or expansion process is quasi-equilibrium.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as



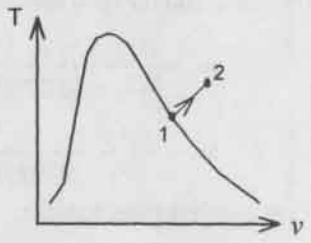
$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} + W_{e,in} - W_{b,out} = \Delta U \quad (\text{since } Q = \text{KE} = \text{PE} = 0)$$

$$Q_{in} + W_{e,in} = m(h_2 - h_1)$$

$$Q_{in} + (VI\Delta t) = m(h_2 - h_1)$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process. The properties of R-134a are (Tables A-11 through A-13)



$$\left.\begin{array}{l} P_1 = 200\text{kPa} \\ \text{sat.vapor} \end{array}\right\} h_1 = h_{g@200\text{kPa}} = 241.30\text{kJ/kg}$$

$$\left.\begin{array}{l} P_2 = 200\text{kPa} \\ T_1 = 70^\circ\text{C} \end{array}\right\} h_2 = 314.02\text{kJ/kg}$$

Substituting,

$$250{,}000\text{VA} + (110\text{V})(I)(6 \times 60\text{s}) = (12\text{kg})(314.02 - 241.30)\text{kJ/kg}\left(\frac{1000\text{VA}}{1\text{kJ/s}}\right)$$

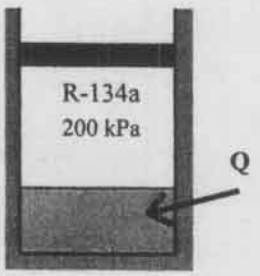
$$I = \mathbf{15.72A}$$

5-146 A cylinder is initially filled with saturated liquid-vapor mixture of R-134a at a specified pressure. Heat is transferred to the cylinder until the refrigerant vaporizes completely at constant pressure. The initial volume, the work done, and the total heat transfer are to be determined, and the process is to be shown on a *P-v* diagram.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are negligible. **2** The thermal energy stored in the cylinder itself is negligible. **3** The compression or expansion process is quasi-equilibrium.

Analysis (*a*) Using property data from R-134a tables (Tables A-11 through A-13), the initial volume of the refrigerant is determined to be

$$\left.\begin{array}{l} P_1 = 200\text{kPa} \\ x_1 = 0.25 \end{array}\right\} \begin{array}{l} v_f = 0.0007532, \quad v_g = 0.0993\text{m}^3/\text{kg} \\ u_f = 36.69, \quad\quad u_g = 221.43\text{kJ/kg} \end{array}$$

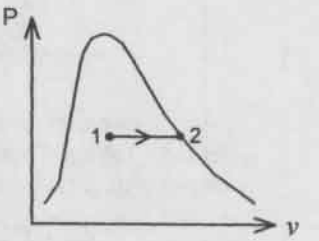


$$v_1 = v_f + x_1 v_{fg} = 0.0007532 + 0.25 \times (0.0993 - 0.0007532) = 0.02539\text{m}^3/\text{kg}$$
$$u_1 = u_f + x_1 u_{fg} = 36.69 + 0.25 \times (221.43 - 36.69) = 82.88\text{kJ/kg}$$

$$V_1 = mv_1 = (0.2\text{kg})(0.02539\text{m}^3/\text{kg}) = \mathbf{0.005078m^3}$$

(*b*) The work done during this constant pressure process is

$$\left.\begin{array}{l} P_2 = 200\text{kPa} \\ sat.vapor \end{array}\right\} \begin{array}{l} v_2 = v_{g@200\text{kPa}} = 0.0993\text{m}^3/\text{kg} \\ u_2 = u_{g@200\text{kPa}} = 221.43\text{kJ/kg} \end{array}$$



$$W_{b,out} = \int_1^2 P\,dV = P(V_2 - V_1) = mP(v_2 - v_1)$$
$$= (0.2\text{kg})(200\text{kPa})(0.0993 - 0.02539)\text{m}^3/\text{kg}\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = \mathbf{2.96kJ}$$

(*c*) We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} - W_{b,out} = \Delta U$$
$$Q_{in} = m(u_2 - u_1) + W_{b,out}$$

Substituting,

$$Q_{\text{in}} = (0.2\text{ kg})(221.43 - 82.88)\text{kJ/kg} + 2.96 = \mathbf{30.67\text{ kJ}}$$

5-147 A cylinder is initially filled with helium gas at a specified state. Helium is compressed polytropically to a specified temperature and pressure. The heat transfer during the process is to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** The cylinder is stationary and thus the kinetic and potential energy changes are negligible. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

Properties The gas constant of helium is $R = 2.0769$ kPa.m^3/kg.K (Table A-1). Also, $C_v = 3.1156$ kJ/kg.K (Table A-2).

Analysis The mass of helium and the exponent n are determined to be

$$m = \frac{P_1V_1}{RT_1} = \frac{(150\text{kPa})(0.5\text{m}^3)}{(2.0769\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{K})} = 0.123\text{kg}$$

$$\frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2} \longrightarrow V_2 = \frac{T_2P_1}{T_1P_2}V_1 = \frac{413\text{K}}{293\text{K}} \times \frac{150\text{kPa}}{400\text{kPa}} \times 0.5\text{m}^3 = 0.264\text{m}^3$$

$$P_2V_2^n = P_1V_1^n \longrightarrow \left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^n \longrightarrow \frac{400}{150} = \left(\frac{0.5}{0.264}\right)^n \longrightarrow n = 1.536$$

Then the boundary work for this polytropic process can be determined from

$$W_{b,in} = -\int_1^2 P\,dV = -\frac{P_2V_2 - P_1V_1}{1-n} = -\frac{mR(T_2 - T_1)}{1-n}$$

$$= -\frac{(0.123\text{kg})(2.0769\text{kJ/kg}\cdot\text{K})(413 - 293)\text{K}}{1 - 1.536} = \mathbf{57.2kJ}$$

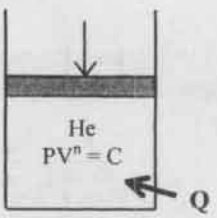
We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. Taking the direction of heat transfer to be to the cylinder, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{in} + W_{b,in} = \Delta U = m(u_2 - u_1)$$

$$Q_{in} = m(u_2 - u_1) - W_{b,in}$$

$$= mC_v(T_2 - T_1) - W_{b,in}$$



Substituting,

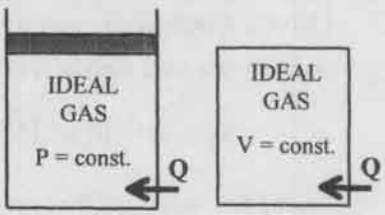
$$Q_{in} = (0.123\text{ kg})(3.1156\text{ kJ/kg·K})(413 - 293)\text{K} - (57.2\text{ kJ}) = \mathbf{-11.2\ kJ}$$

The negative sign indicates that heat is lost from the system.

5-148 A cylinder and a rigid tank initially contain the same amount of an ideal gas at the same state. The temperature of both systems is to be raised by the same amount. The amount of extra heat that must be transferred to the cylinder is to be determined.

Analysis In the absence of any work interactions, other than the boundary work, the ΔH and ΔU represent the heat transfer for ideal gases for constant pressure and constant volume processes, respectively. Thus the extra heat that must be supplied to the air maintained at constant pressure is

$$Q_{\text{in, extra}} = \Delta H - \Delta U = mC_p\Delta T - mC_v\Delta T = m(C_p - C_v)\Delta T = mR\Delta T$$



where

$$R = \frac{R_u}{M} = \frac{8.314\ \text{kJ/kmol}\cdot\text{K}}{25\ \text{kg/kmol}} = 0.3326\ \text{kJ/kg}\cdot\text{K}$$

Substituting,

$$Q_{\text{in, extra}} = (12\ \text{kg})(0.3326\ \text{kJ/kg}\cdot\text{K})(15\ \text{K}) = \mathbf{59.9\ kJ}$$

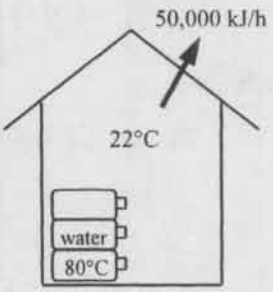
5-149 The heating of a passive solar house at night is to be assisted by solar heated water. The length of time that the electric heating system would run that night with or without solar heating are to be determined.

Assumptions **1** Water is an incompressible substance with constant specific heats. **2** The energy stored in the glass containers themselves is negligible relative to the energy stored in water. **3** The house is maintained at 22°C at all times.

Properties The density and specific heat of water at room temperature are ρ = 1 kg/L and C = 4.18 kJ/kg·°C (Table A-3).

Analysis The total mass of water is

$$m_w = \rho V = (1\text{kg/L})(50 \times 20\text{L}) = 1000\text{kg}$$



Taking the contents of the house, including the water as our system, the energy balance relation can be written as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$\begin{aligned} W_{e,in} - Q_{out} &= \Delta U = (\Delta U)_{\text{water}} + (\Delta U)_{\text{air}} \\ &= (\Delta U)_{\text{water}} \\ &= mC(T_2 - T_1)_{\text{water}} \end{aligned}$$

or,

$$\dot{W}_{e,in}\Delta t - Q_{out} = [mC(T_2 - T_1)]_{\text{water}}$$

Substituting,

$$(15\text{ kJ/s})\Delta t - (50{,}000\text{ kJ/h})(10\text{ h}) = (1000\text{ kg})(4.18\text{ kJ/kg·°C})(22 - 80)°\text{C}$$

It gives

$$\Delta t = 17{,}170\text{ s} = \mathbf{4.77\ h}$$

(b) If the house incorporated no solar heating, the energy balance relation above would simplify further to

$$\dot{W}_{e,in}\Delta t - Q_{out} = 0$$

Substituting,

$$(15\text{ kJ/s})\Delta t - (50{,}000\text{ kJ/h})(10\text{ h}) = 0$$

It gives

$$\Delta t = 33{,}333\text{ s} = \mathbf{9.26\ h}$$

5-150 An electric resistance heater is immersed in water. The time it will take for the electric heater to raise the water temperature to a specified temperature is to be determined.

Assumptions **1** Water is an incompressible substance with constant specific heats. **2** The energy stored in the container itself and the heater is negligible. **3** Heat loss from the container is negligible.

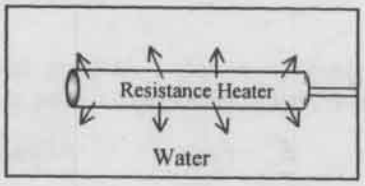
Properties The density and specific heat of water at room temperature are ρ = 1 kg/L and C = 4.18 kJ/kg·°C (Table A-3).

Analysis Taking the water in the container as the system, the energy balance can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{e,in} = (\Delta U)_{\text{water}}$$

$$\dot{W}_{e,in}\Delta t = mC(T_2 - T_1)_{\text{water}}$$



Substituting,

$$(800 \text{ J/s})\Delta t = (40 \text{ kg})(4184 \text{ J/kg·°C})(80 - 20)°\text{C}$$

Solving for Δt gives

$$\Delta t = \mathbf{12{,}552 \text{ s} = 209.2 \text{ min} = 3.487 \text{ h}}$$

5-151 One ton of liquid water at 80°C is brought into a room. The final equilibrium temperature in the room is to be determined.

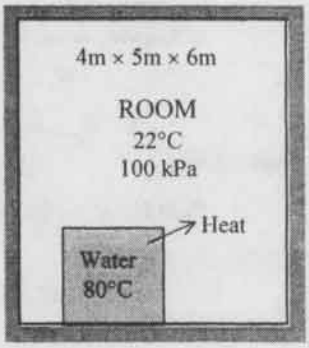
Assumptions **1** The room is well insulated and well sealed. **2** The thermal properties of water and air are constant.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1). The specific heat of water at room temperature is $C = 4.18$ kJ/kg·°C (Table A-3).

Analysis The volume and the mass of the air in the room are

$$V = 4\text{x}5\text{x}6 = 120 \text{ m}^3$$

$$m_{air} = \frac{P_1 V_1}{RT_1} = \frac{(100\text{kPa})(120\text{m}^3)}{(0.2870\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(295\text{K})} = 141.7\text{kg}$$



Taking the contents of the room, including the water, as our system, the energy balance can be written as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}} \rightarrow 0 = \Delta U = (\Delta U)_{\text{water}} + (\Delta U)_{\text{air}}$$

or $$[mC(T_2 - T_1)]_{\text{water}} + [mC_v(T_2 - T_1)]_{\text{air}} = 0$$

Substituting,

$$(1000\text{kg})(4.184\text{kJ/kg}\cdot^\circ\text{C})(T_f - 80)^\circ\text{C} + (147.7\text{kg})(0.718\text{kJ/kg}\cdot^\circ\text{C})(T_f - 22)^\circ\text{C} = 0$$

It gives $$T_f = \mathbf{78.6°C}$$

where T_f is the final equilibrium temperature in the room.

5-152 A room is to be heated by 1 ton of hot water contained in a tank placed in the room. The minimum initial temperature of the water is to be determined if it to meet the heating requirements of this room for a 24-h period.

Assumptions **1** Water is an incompressible substance with constant specific heats. **2** Air is an ideal gas with constant specific heats. **3** The energy stored in the container itself is negligible relative to the energy stored in water. **4** The room is maintained at 22°C at all times. **5** The hot water is to meet the heating requirements of this room for a 24-h period.

Properties The specific heat of water at room temperature is $C = 4.18$ kJ/kg·°C (Table A-3).

Analysis Heat loss from the room during a 24-h period is

$$Q_{loss} = (10{,}000 \text{ kJ/h})(24 \text{ h}) = 240{,}000 \text{ kJ}$$

Taking the contents of the room, including the water, as our system, the energy balance can be written as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}} \rightarrow -Q_{\text{out}} = \Delta U = (\Delta U)_{\text{water}} + \cancel{(\Delta U)_{\text{air}}}^{\,0}$$

or

$$-Q_{\text{out}} = [mC(T_2 - T_1)]_{\text{water}}$$

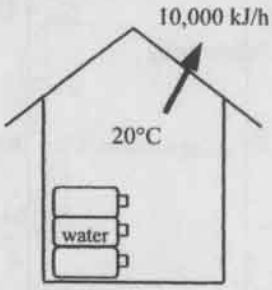
Substituting,

$$-240{,}000 \text{ kJ} = (1000 \text{ kg})(4.184 \text{ kJ/kg·°C})(20 - T_1)$$

It gives

$$T_1 = \mathbf{77.4°C}$$

where T_1 is the temperature of the water when it is first brought into the room.



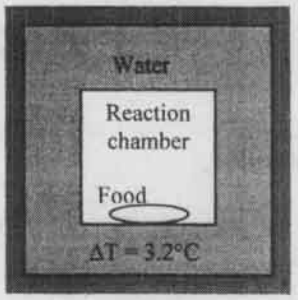
5-153 A sample of a food is burned in a bomb calorimeter, and the water temperature rises by 3.2°C when equilibrium is established. The energy content of the food is to be determined. √

Assumptions **1** Water is an incompressible substance with constant specific heats. **2** Air is an ideal gas with constant specific heats. **3** The energy stored in the reaction chamber is negligible relative to the energy stored in water. **4** The energy supplied by the mixer is negligible.

Properties The specific heat of water at room temperature is $C = 4.18$ kJ/kg·°C (Table A-3). The constant volume specific heat of air at room temperature is $C_v = 0.718$ kJ/kg·°C (Table A-2).

Analysis The chemical energy released during the combustion of the sample is transferred to the water as heat. Therefore, disregarding the change in the sensible energy of the reaction chamber, the energy content of the food is simply the heat transferred to the water. Taking the water as our system, the energy balance can be written as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}} \rightarrow Q_{in} = \Delta U$$



or $$Q_{in} = (\Delta U)_{\text{water}} = [mC(T_2 - T_1)]_{\text{water}}$$

Substituting ,

$$Q_{\text{in}} = (3 \text{ kg})(4.184 \text{ kJ/kg·°C})(3.2°\text{C}) = 40.166 \text{ kJ}$$

for a 2-g sample. Then the energy content of the food per unit mass is

$$\frac{40.166 \text{ kJ}}{2 \text{ g}}\left(\frac{1000 \text{ g}}{1 \text{ kg}}\right) = \mathbf{20{,}083 \text{ kJ/kg}}$$

To make a rough estimate of the error involved in neglecting the thermal energy stored in the reaction chamber, we treat the entire mass within the chamber as air and determine the change in sensible internal energy:

$$(\Delta U)_{\text{chamber}} = [mC_v(T_2 - T_1)]_{\text{chamber}} = (0.102 \text{ kg})(0.718 \text{ kJ/kg·°C})(3.2°\text{C}) = 0.23 \text{ kJ}$$

which is less than 1% of the internal energy change of water. Therefore, it is reasonable to disregard the change in the sensible energy content of the reaction chamber in the analysis.

5-154 A man drinks one liter of cold water at 3°C in an effort to cool down. The drop in the average body temperature of the person under the influence of this cold water is to be determined.

Assumptions **1** Thermal properties of the body and water are constant. **2** The effect of metabolic heat generation and the heat loss from the body during that time period are negligible.

Properties The density of water is very nearly 1 kg/L, and the specific heat of water at room temperature is $C = 4.18$ kJ/kg·°C (Table A-3). The average specific heat of human body is given 3.6 kJ/kg.°C.

Analysis. The mass of the water is

$$m_w = \rho V = (1\ \text{kg/L})(1\ \text{L}) = 1\ \text{kg}$$

We take the man and the water as our system, and disregard any heat and mass transfer and chemical reactions. Of course these assumptions may be acceptable only for very short time periods, such as the time it takes to drink the water. Then the energy balance can be written as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$



$$0 = \Delta U$$

$$0 = \Delta U_{\text{body}} + \Delta U_{\text{water}}$$

$$[mC_v(T_2 - T_1)]_{\text{body}} + [mC_v(T_2 - T_1)]_{\text{water}} = 0$$

Substituting,

$$(68\text{kg})(3.6\text{kJ/kg}\cdot^\circ\text{C})(T_f - 38)^\circ\text{C} + (1\text{kg})(4.184\text{kJ/kg}\cdot^\circ\text{C})(T_f - 3)^\circ\text{C} = 0$$

It gives

$$T_f = \mathbf{37.4°C}$$

Therefore, the average body temperature of this person should drop about half a degree Celsius.

5-155 A 0.2-L glass of water at 20°C is to be cooled with ice to 5°C. The amount of ice or cold water that needs to be added to the water is to be determined.

Assumptions **1** Thermal properties of the ice and water are constant. **2** Heat transfer to the glass is negligible. **3** There is no stirring by hand or a mechanical device (it will add energy).

Properties The density of water is 1 kg/L, and the specific heat of water at room temperature is $C = 4.18$ kJ/kg·°C (Table A-3). The specific heat of ice at about 0°C is $C = 2.11$ kJ/kg·°C (Table A-3). The melting temperature and the heat of fusion of ice at 1 atm are 0°C and 333.7 kJ/kg,.

Analysis (*a*) The mass of the water is

$$m_w = \rho V = (1\,\text{kg/L})(0.2\,\text{L}) = 0.2\text{kg}$$

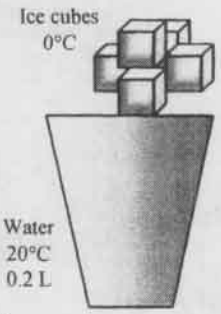
We take the ice and the water as our system, and disregard any heat and mass transfer. This is a reasonable assumption since the time period of the process is very short. Then the energy balance can be written as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$0 = \Delta U$$

$$0 = \Delta U_{\text{ice}} + \Delta U_{\text{water}}$$

$$[mC(0^\circ\text{C} - T_1)_{\text{solid}} + mh_{if} + mC(T_2 - 0^\circ\text{C})_{\text{liquid}}]_{\text{ice}} + [mC(T_2 - T_1)]_{\text{water}} = 0$$



Noting that $T_{1,\text{ice}} = 0$°C and $T_2 = 5$°C and substituting gives

$$m[0 + 333.7\,\text{kJ/kg} + (4.18\,\text{kJ/kg·°C})(5\text{-}0)°\text{C}] + (0.2\,\text{kg})(4.18\,\text{kJ/kg·°C})(5\text{-}20)°\text{C} = 0$$

$$m = 0.0354\,\text{kg} = \mathbf{35.4\ g}$$

(*b*) When $T_{1,\text{ice}} = -8$°C instead of 0°C, substituting gives

$$m[(2.11\,\text{kJ/kg·°C})[0\text{-}(-8)]°\text{C} + 333.7\,\text{kJ/kg} + (4.18\,\text{kJ/kg·°C})(5\text{-}0)°\text{C}] + (0.2\,\text{kg})(4.18\,\text{kJ/kg·°C})(5\text{-}20)°\text{C} = 0$$

$$m = 0.0338\,\text{kg} = \mathbf{33.8\ g}$$

Cooling with cold water can be handled the same way. All we need to do is replace the terms for ice by a term for cold water at 0°C:

$$(\Delta U)_{\text{cold water}} + (\Delta U)_{\text{water}} = 0$$

$$[mC(T_2 - T_1)]_{\text{cold water}} + [mC(T_2 - T_1)]_{\text{water}} = 0$$

Substituting,

$$[m_{\text{cold water}}\,(4.184\,\text{kJ/kg·°C})(5\text{ - }0)°\text{C}] + (0.2\,\text{kg})(4.184\,\text{kJ/kg·°C})(5\text{-}20)°\text{C} = 0$$

It gives $m = 0.6$ kg = **600 g**

Discussion Note that this is 17 times the amount of ice needed, and it explains why we use ice instead of water to cool drinks. Also, the temperature of ice does not seem to make a significant difference.

5-156 A 1- ton (1000 kg) of water is to be cooled in a tank by pouring ice into it. The final equilibrium temperature in the tank is to be determined.

Assumptions **1** Thermal properties of the ice and water are constant. **2** Heat transfer to the water tank is negligible. **3** There is no stirring by hand or a mechanical device (it will add energy).

Properties The specific heat of water at room temperature is $C = 4.18$ kJ/kg·°C, and the specific heat of ice at about 0°C is $C = 2.11$ kJ/kg·°C (Table A-3). The melting temperature and the heat of fusion of ice at 1 atm are 0°C and 333.7 kJ/kg..

Analysis We take the ice and the water as our system, and disregard any heat transfer between the system and the surroundings. Then the energy balance for this process can be written as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

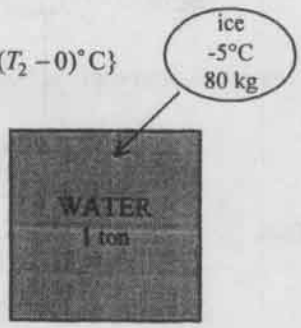
$$0 = \Delta U$$

$$0 = \Delta U_{\text{ice}} + \Delta U_{\text{water}}$$

$$[mC(0^\circ\text{C} - T_1)_{\text{solid}} + mh_{if} + mC(T_2 - 0^\circ\text{C})_{\text{liquid}}]_{\text{ice}} + [mC(T_2 - T_1)]_{\text{water}} = 0$$

Substituting,

$$(80\text{ kg})\{(2.11\text{ kJ/kg}\cdot^\circ\text{C})[0-(-5)]^\circ\text{C} + 333.7\text{ kJ/kg} + (4.18\text{ kJ/kg}\cdot^\circ\text{C})(T_2 - 0)^\circ\text{C}\}$$
$$+(1000\text{ kg})(4.18\text{ kJ/kg}\cdot^\circ\text{C})(T_2 - 20)^\circ\text{C} = 0$$



It gives $T_2 = \mathbf{12.4°C}$

which is the final equilibrium temperature in the tank.

5-157 An insulated cylinder initially contains a saturated liquid-vapor mixture of water at a specified temperature. The entire vapor in the cylinder is to be condensed isothermally by adding ice inside the cylinder. The amount of ice that needs to be added is to be determined.

Assumptions **1** Thermal properties of the ice are constant. **2** The cylinder is well-insulated and thus heat transfer is negligible. **3** There is no stirring by hand or a mechanical device (it will add energy).

Properties The specific heat of ice at about 0°C is $C = 2.11$ kJ/kg·°C (Table A-3). The melting temperature and the heat of fusion of ice at 1 atm are given to be 0°C and 333.7 kJ/kg.

Analysis (*a*) We take the contents of the cylinder (ice and saturated water) as our system, which is a closed system. Noting that the temperature and thus the pressure remains constant during this phase change process and thus $W_b + \Delta U = \Delta H$, the energy balance for this system can be written as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

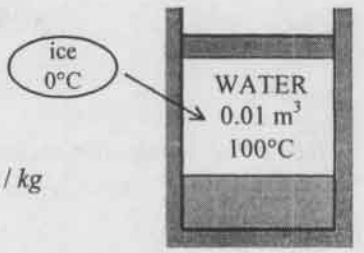
$$W_{b,in} = \Delta U \rightarrow \Delta H = 0$$

$$\Delta H_{\text{ice}} + \Delta H_{\text{water}} = 0$$

$$[mC(0^\circ\text{C} - T_1)_{\text{solid}} + mh_{if} + mC(T_2 - 0^\circ\text{C})_{\text{liquid}}]_{\text{ice}} + [m(h_2 - h_1)]_{\text{water}} = 0$$

The properties of water at 100°C are (Table A-4)

$$v_f = 0.001044, \quad v_g = 1.6729 \text{ m}^3/\text{kg}$$
$$h_f = 419.04, \quad h_{fg} = 2257.0 \text{ kJ.kg}$$



Then,

$$v_1 = v_f + x_1 v_{fg} = 0.001044 + 0.2 \times (1.6729 - 0.001044) = 0.3354 m^3/kg$$
$$h_1 = h_f + x_1 h_{fg} = 419.04 + 0.2 \times 2257.0 = 870.4 kJ/kg$$
$$h_2 = h_{f@100°C} = 419.04 \text{ kJ/kg}$$

$$m_{steam} = \frac{V_1}{v_1} = \frac{0.01\ m^3}{0.3354\ m^3/kg} = 0.0298\ kg$$

Noting that $T_{1,\text{ice}} = 0°\text{C}$ and $T_2 = 100°\text{C}$ and substituting gives

$$m[0 + 333.7 \text{ kJ/kg} + (4.18 \text{ kJ/kg·°C})(100\text{-}0)°\text{C}] + (0.0298 \text{ kg})(419.04 - 870.4) \text{ kJ/kg} = 0$$

$$m = 0.0179 \text{ kg} = \mathbf{17.9\ g\ ice}$$

5-158 The cylinder of a steam engine initially contains saturated vapor of water at 100 kPa. The cylinder is cooled by pouring cold water outside of it, and some of the steam inside condenses. If the piston is stuck at its initial position, the friction force acting on the piston and the amount of heat transfer are to be determined.

Assumptions The device is air-tight so that no air leaks into the cylinder as the pressure drops.

Analysis We take the contents of the cylinder (the saturated liquid-vapor mixture) as the system, which is a closed system. Noting that the volume remains constant during this phase change process, the energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U = m(u_2 - u_1)$$

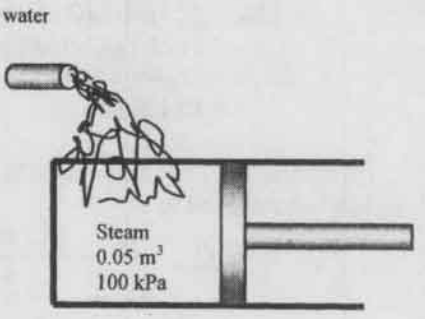
The saturation properties of water at 100 kPa and at 30°C are (Tables A-4 and A-5)

$$P_1 = 100 \text{ kPa} \longrightarrow \quad v_f = 0.001043 \text{ m}^3/\text{kg}, \; v_g = 1.6940 \text{ m}^3/\text{kg}$$
$$u_f = 417.36 \text{ kJ/kg}, \quad u_g = 2506.1 \text{ kJ/kg}$$

$$T_2 = 30°\text{C} \longrightarrow \quad v_f = 0.001004 \text{ m}^3/\text{kg}, \; v_g = 32.89 \text{ m}^3/\text{kg}$$
$$u_f = 125.78 \text{ kJ/kg}, \quad u_g = 2290.8 \text{ kJ/kg}$$
$$P_{sat} = 4.246 \; kPa$$

Then,

$$P_2 = P_{sat@30°\text{C}} = 4.246 \text{ kPa}$$
$$v_1 = v_{g@100 \text{ kPa}} = 1.694 \text{ m}^3/\text{kg}$$
$$u_1 = u_{g@100 \text{ kPa}} = 2506.1 \text{ kJ/kg}$$



and

$$m = \frac{V_1}{v_1} = \frac{0.05 \text{ m}^3}{1.6940 \text{ m}^3/\text{kg}} = 0.0295 \text{ kg}$$

$$v_2 = v_1 \longrightarrow \quad x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{1.694 - 0.001}{32.89 - 0.001} = 0.05147$$

$$u_2 = u_f + x_2 u_{fg} = 417.36 + 0.05147 \times 2290.8 = 243.7 \text{ kJ/kg}$$

The friction force that develops at the piston-cylinder interface balances the force acting on the piston, and is equal to

$$F = A(P_1 - P_2) = (0.1\text{m}^3)(100 - 4.246)\text{kPa}\left(\frac{1000\text{N/m}^2}{1\text{kPa}}\right) = \mathbf{9570N}$$

The heat transfer is determined from the energy balance to be

$$Q_{out} = m(u_1 - u_2)$$
$$= (0.0295\text{kg})(2506.1 - 24369)\text{kJ/kg}$$
$$= \mathbf{66.7kJ}$$

5-159 Cod fish in a polypropylene box is to be frozen in 4 h. The total amount of heat removed from the fish and the box and the average rate of heat removal are to be determined.

Assumptions **1** The specific heats of the box and the fish are constant. **2** The box and the fish are at uniform temperatures at the beginning and at the end of the process. **3** Evaporation from the fish is negligible.

Properties The specific heat of the cod fish is given to be 3.62 kJ/kg.°C above the freezing point of –2.2°C, and 1.89 kJ/kg.°C below the freezing point. The specific heat of the box is given to be 1.9 kJ/kg.°C. The latent heat of fish is given to be 277 kJ/kg.

Analysis (a) We take the fish and its box as the system. The energy balance for this fixed mass can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$-Q_{out} = \Delta U = \Delta U_{\text{fish}} + \Delta U_{\text{box}}$$

$$Q_{out} = -[\Delta U_{\text{below freezing point}} + \Delta U_{\text{during freezing}} + \Delta U_{\text{below freezing point}}]_{\text{fish}} - \Delta U_{\text{box}}$$

or,

$$Q_{out} = m_{\text{fish}}[C_{\text{fresh}}(T_1 - T_{fp}) + h_{\text{latent}} + C_{\text{frozen}}(T_{fp} - T_2)]_{\text{fish}} + [mC(T_1 - T_2)]_{\text{box}}$$

Noting that T_{fp} = -2.2°C and substituting, the amount of heat removed as the cod fish and its box are cooled from 16 to -20°C becomes

$$\begin{aligned} Q_{\text{out}} &= (32\text{ kg})\{(3.62\text{ kJ/kg·°C})[16 - (-2.2)]\text{°C} + 277\text{ kJ/kg} + (1.89\text{ kJ/kg·°C})(-2.2 - (-20)\text{°C}] \\ &\quad + (1.6\text{ kg})(1.9\text{ kJ/kg·°C})[16 - (-20)]\text{°C} = 0 \\ &= 12{,}049 + 109 \\ &= \mathbf{12{,}158\text{ kJ}} \end{aligned}$$

(b) Noting that 12,158 kJ of heat is removed in 4 h, the average rate of heat removal from the fish and its box is determined to be

$$\dot{Q}_{ave} = \frac{Q_{total}}{\Delta t} = \frac{12{,}158\text{ kJ}}{4\text{ h}} = \mathbf{3040\text{ kJ/h} = 0.844\text{ kW}}$$

5-160 Water is boiled at sea level (1 atm pressure) in a coffee maker, and half of the water evaporates in 25 min. The power rating of the electric heating element and the time it takes to heat the cold water to the boiling temperature are to be determined. √

Assumptions **1** The electric power consumption by the heater is constant. **2** Heat losses from the coffee maker are negligible.

Properties The enthalpy of vaporization of water at the saturation temperature of 100°C is h_{fg} = 2257 kJ/kg (Table A-4). At an average temperature of (100+18)/2 = 59°C, the specific heat of water is C = 4.18 kJ/kg.°C, and the density is about 1 kg/L (Table A-3).

Analysis The density of water at room temperature is very nearly 1 kg/L, and thus the mass of 1 L water at 18°C is nearly 1 kg. Noting that the enthalpy of vaporization represents the amount of energy needed to vaporize a liquid at a specified temperature, the amount of electrical energy needed to vaporize 0.5 kg of water in 25 min is

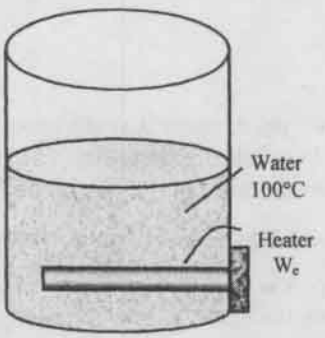
$$W_e = \dot{W}_e \Delta t = m h_{fg} \rightarrow \dot{W}_e = \frac{m h_{fg}}{\Delta t} = \frac{(0.5\,\text{kg})(2257\,\text{kJ/kg})}{(25\times 60\,\text{s})} = \mathbf{0.752\,kW}$$

Therefore, the electric heater consumes (and transfers to water) 0.752 kW of electric power.

Noting that the specific heat of water at the average temperature of (18+100)/2 = 59°C is C = 4.18 kJ/kg·°C, the time it takes for the entire water to be heated from 18°C to 100°C is determined to be

$$W_e = \dot{W}_e \Delta t = mC\Delta T \rightarrow \Delta t = \frac{mC\Delta T}{\dot{W}_e} = \frac{(1\,\text{kg})(4.18\,\text{kJ/kg}\cdot^\circ\text{C})(100-18)^\circ\text{C}}{0.752\,\text{kJ/s}} = 456\,\text{s} = \mathbf{7.60\,min}$$

Discussion We can also solve this problem using v_f data (instead of density), and h_f data instead of specific heat. At 100 °C, we have v_f = 0.001044 m³/kg and h_f = 419.04 kJ/kg. At 18°C, we have h_f = 75.57 kJ/kg (Table A-4). The two results will be practically the same.



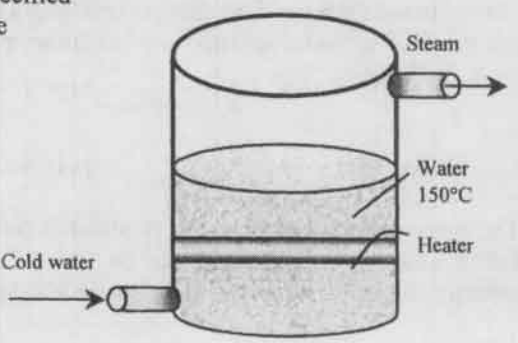
5-161 Water is boiled at a specified temperature by hot gases flowing through a stainless steel pipe submerged in water. The rate of evaporation of is are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the outer surfaces of the boiler are negligible.

Properties The enthalpy of vaporization of water at 150°C is h_{fg} = 2114.3 kJ/kg (Table A-4).

Analysis The rate of heat transfer to water is given to be 74 kJ/s. Noting that the enthalpy of vaporization represents the amount of energy needed to vaporize a unit mass of a liquid at a specified temperature, the rate of evaporation of water is determined to be

$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{74\,\text{kJ/s}}{2114.3\,\text{kJ/kg}} = \mathbf{0.0350\,kg/s}$$



5-162 Cold water enters a steam generator at 20°C, and leaves as saturated vapor at T_{sat} = 100°C. The fraction of heat used to preheat the liquid water from 20°C to saturation temperature of 100°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the steam generator are negligible. **3** The specific heat of water is constant at the average temperature.

Properties The heat of vaporization of water at 100°C is h_{fg} = 2257 kJ/kg (Table A-4), and the specific heat of liquid water at the average temperature of (20+100)/2 = 60°C is C = 4.18 kJ/kg.°C (Table A-3).

Analysis The heat of vaporization of water represents the amount of heat needed to vaporize a unit mass of liquid at a specified temperature. Using the average specific heat, the amount of heat transfer needed to preheat a unit mass of water from 20°C to 100°C is

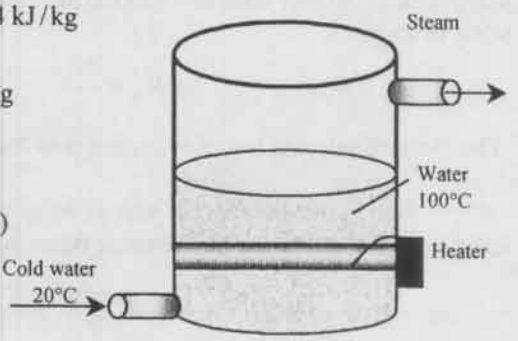
$$q_{\text{preheating}} = C\Delta T = (4.18\ \text{kJ/kg}\cdot^\circ\text{C})(100-20)^\circ\text{C} = 334.4\ \text{kJ/kg}$$

and

$$q_{\text{total}} = q_{\text{boiling}} + q_{\text{preheating}} = 2257 + 334.4 = 2591.4\ \text{kJ/kg}$$

Therefore, the fraction of heat used to preheat the water is

$$\text{Fraction to preheat} = \frac{q_{\text{preheating}}}{q_{\text{total}}} = \frac{334.4}{2591.4} = \mathbf{0.129}\ \text{(or } \mathbf{12.9\%}\text{)}$$



5-163 Cold water enters a steam generator at 20°C and is boiled, and leaves as saturated vapor at boiler pressure. The boiler pressure at which the amount of heat needed to preheat the water to saturation temperature which is equal to the heat of vaporization is to be determined.

Assumptions Heat losses from the steam generator are negligible.

Properties The enthalpy of liquid water at 20°C is 83.96 kJ/kg. Other properties needed to solve this problem are the heat of vaporization h_{fg} and the enthalpy of saturated liquid at the specified temperatures, and they can be obtained from Table A-4.

Analysis The heat of vaporization of water represents the amount of heat needed to vaporize a unit mass of liquid at a specified temperature, and Δh represents the amount of heat needed to preheat a unit mass of water from 20°C to the saturation temperature. Therefore,

$$q_{\text{preheating}} = q_{\text{boiling}}$$

$$(h_{f@T_{sat}} - h_{f@20^\circ\text{C}}) = h_{fg@T_{sat}}$$

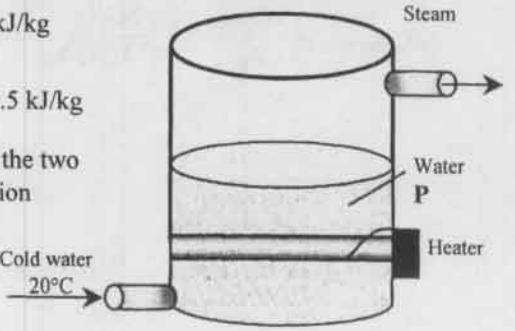
$$h_{f@T_{sat}} - 83.96\ \text{kJ/kg} = h_{fg@T_{sat}} \quad \rightarrow \quad h_{f@T_{sat}} - h_{fg@T_{sat}} = 83.96\ \text{kJ/kg}$$

The solution of this problem requires choosing a boiling temperature, reading h_f and h_{fg} at that temperature, and substituting the values into the relation above to see if it is satisfied. By trial and error, (Table A-4)

At 310°C: $h_{f@T_{sat}} - h_{fg@T_{sat}} = 1401.3 - 1326 = 75.3$ kJ/kg

At 315°C: $h_{f@T_{sat}} - h_{fg@T_{sat}} = 1431.0 - 1283.5 = 147.5$ kJ/kg

The temperature that satisfies this condition is determined from the two values above by interpolation to be 310.6°C. The saturation pressure corresponding to this temperature is **9.94 MPa.**



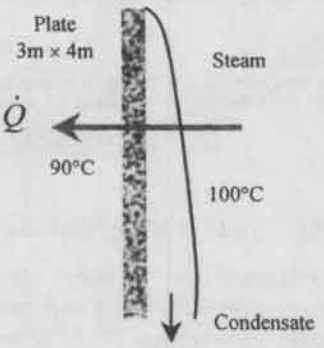
5-164 Saturated steam at 1 atm pressure and thus at a saturation temperature of T_{sat} = 100°C condenses on a vertical plate maintained at 90°C by circulating cooling water through the other side. The rate of condensation of steam and the average heat flux on the plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The steam condenses and the condensate drips off at 100°C. (In reality, the condensate temperature will be between 90 and 100, and the cooling of the condensate a few °C should be considered if better accuracy is desired).

Properties The enthalpy of vaporization of water at 100°C is h_{fg} = 2257 kJ/kg (Table A-4).

Analysis The rate of heat transfer during this condensation process is given to be 180 kJ/s. Noting that the heat of vaporization of water represents the amount of heat released as a unit mass of vapor at a specified temperature condenses, the rate of condensation of steam is determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h_{fg}} = \frac{180\ \text{kJ/s}}{2257\ \text{kJ/kg}} = \mathbf{0.0798\ kg/s}$$



The average heat flux on the plate is

$$\dot{q}_{\text{ave}} = \frac{\dot{Q}}{A} = \frac{180\ \text{kW}}{3 \times 4\ \text{m}^2} = \mathbf{15\ kW/m^2}$$

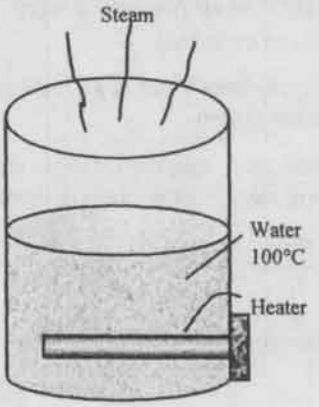
5-165 Water is boiled at T_{sat} = 100°C by an electric heater. The rate of evaporation of water and the heat flux at the surface of the wire are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the outer surfaces of the water tank are negligible.

Properties The enthalpy of vaporization of water at 100°C is h_{fg} = 2257 kJ/kg (Table A-4).

Analysis (*a*) Noting that the enthalpy of vaporization represents the amount of energy needed to vaporize a unit mass of a liquid at a specified temperature, the rate of evaporation of water is determined to be

$$\dot{m}_{\text{evaporation}} = \frac{\dot{W}_{\text{e,boiling}}}{h_{fg}} = \frac{5\ \text{kJ/s}}{2257\ \text{kJ/kg}} = \mathbf{0.00222\ kg/s = 7.98\ kg/h}$$



The heat transfer area of the heater wire is

$$A = \pi DL = \pi(0.002\ \text{m})(0.50\ \text{m}) = 0.003142\ \text{m}^2$$

Then the heat flux at the surface of the heater wire becomes

$$\dot{q}_s = \frac{\dot{Q}}{A} = \frac{\dot{W}_{\text{e,boiling}}}{A} = \frac{5\ \text{kJ/s}}{0.003142\ \text{m}^2} = \mathbf{1591\ kW/m^2}$$

5-166 An insulated cylinder initially contains saturated liquid-vapor mixture of water at a specified temperature. A cold copper block is dropped into the cylinder. The final equilibrium temperature inside the cylinder and the final mass of water vapor are to be determined.

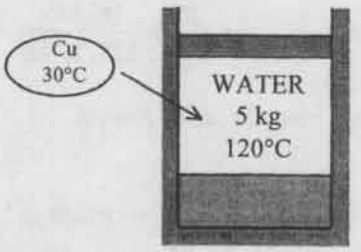
Assumptions **1** The copper block has a constant specific heat at the average temperature. **2** The cylinder is well-insulated and thus heat transfer is negligible. **3** Only part of the vapor in the cylinder condenses as a result of copper block being dropped into the cylinder, so that the temperature inside the cylinder remains constant (will be verified).

Properties The specific heat of copper at the average temperature of (20+120)/2 = 75°C is C = 0.391 kJ/kg·°C (Table A-3). The enthalpy of vaporization of water at 120°C is h_{fg} = 2202.6 kJ/kg (Table A-4).

Analysis We take the copper block as the system, which is closed system. Assuming the final temperature to be 120°C, the energy balance for copper can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} = \Delta U = [mC(T_2 - T_1)]_{\text{copper}}$$



Substituting,

$$Q_{\text{in}} = (5\ \text{kg})(0.391\ \text{kJ/kg.°C})(120 - 30)°\text{C} = 176.0\ \text{kJ}$$

This energy will come from the water vapor inside. Noting that the enthalpy of vaporization represents the amount of energy released when a unit mass of vapor condenses at a specified temperature, the amount of vapor that condenses to release 176 kJ of heat is

$$m_{\text{condensed}} = \frac{Q}{h_{fg}} = \frac{176\ \text{kJ}}{2202.6\ \text{kJ/kg}} = 0.080\ \text{kg}$$

Initially, the cylinder contained 1 kg of vapor. Then the amount of water vapor in the cylinder at the end of the process becomes

$$m_{\text{final}} = m_{\text{initial}} - m_{\text{condensed}} = 1.0 - 0.080 = \mathbf{0.920\ kg}$$

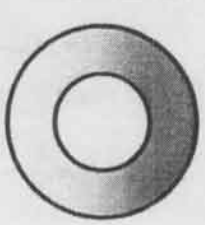
This verifies our assumption that the cylinder still contains some vapor at the end of the process. Then the final temperature in the cylinder must be the same as the initial temperature since saturation conditions still exist inside,

$$T_2 = T_1 = \mathbf{120°C}$$

5-167 The air pressure in a tire is measured before and after a trip. The temperature rise of air inside the tire is to be estimated.

Assumptions **1** Air is an ideal gas. **2** The volume of the tire remains constant. **3** No air leaks out of the tire during the trip.

Analysis Using the ideal gas relation between the two states, the final temperature in the tire is determined to be



$$\frac{P_1 V}{T_1} = \frac{P_2 V}{T_2} \quad \rightarrow \quad T_2 = \frac{P_2}{P_1} T_1 = \frac{(220 + 90)\ \text{kPa}}{(200 + 90)\ \text{kPa}} (25 + 273\ \text{K}) = 318.6\ \text{K} = 45.6°\text{C}$$

Therefore, the temperature rise of air in the tire during the trip is

$$\Delta T = T_2 - T_1 = 45.6 - 25 = \mathbf{20.6°C}$$

5-168 Two identical buildings in Los Angeles and Denver have the same infiltration rate. The ratio of the heat losses by infiltration at the two cities under identical conditions is to be determined.

Assumptions **1** Both buildings are identical and both are subjected to the same conditions except the atmospheric conditions. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Steady flow conditions exist.

Analysis We can view infiltration as a steady stream of air that is heated as it flows in an imaginary duct passing through the building. The energy balance for this imaginary steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

Los Angeles: 101 kPa
Denver: 83 kPa



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta \text{ke} \cong \Delta \text{pe} \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1) = \rho \dot{V} C_p(T_2 - T_1)$$

Then the sensible infiltration heat loss (heat gain for the infiltrating air) can be expressed

$$\dot{Q}_{\text{infiltration}} = \dot{m}_{\text{air}} C_p (T_i - T_o) = \rho_{o,\text{air}} (ACH)(V_{\text{building}}) C_p (T_i - T_o)$$

where *ACH* is the infiltration volume rate in *air changes per hour*. Therefore, the infiltration heat loss is proportional to the density of air, and thus the ratio of infiltration heat losses at the two cities is simply the densities of outdoor air at those cities,

$$\text{Infiltration heat loss ratio} = \frac{\dot{Q}_{\text{infiltration, Los Angeles}}}{\dot{Q}_{\text{infiltration, Denver}}} = \frac{\rho_{o,\text{air, Los Angeles}}}{\rho_{o,\text{air, Denver}}}$$

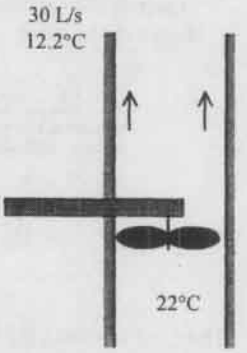
$$= \frac{(P_0 / RT_0)_{\text{Los Angeles}}}{(P_0 / RT_0)_{\text{Denver}}} = \frac{P_{o,\text{Los Angeles}}}{P_{0,\text{Denver}}}$$

$$= \frac{101 \text{ kPa}}{83 \text{ kPa}} = \mathbf{1.22}$$

Therefore, the infiltration heat loss in Los Angeles will be 22% higher than that in Denver under identical conditions.

5-169 The ventilating fan of the bathroom of an electrically heated building in San Francisco runs continuously. The amount and cost of the heat "vented out" per month in winter are to be determined.

Assumptions **1** We take the atmospheric pressure to be 1 atm = 101.3 kPa since San Francisco is at sea level. **2** The building is maintained at 22°C at all times. **3** The infiltrating air is heated to 22°C before it exfiltrates. **4** Air is an ideal gas with constant specific heats at room temperature. **5** Steady flow conditions exist.



Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg·K (Table A-1). The specific heat of air at room temperature is $C_p = 1.005$ kJ/kg·°C (Table A-2).

Analysis The density of air at the indoor conditions of 1 atm and 22°C is

$$\rho_o = \frac{P_o}{RT_o} = \frac{(101.3\,\text{kPa})}{(0.287\,\text{kPa.m}^3/\text{kg.K})(22+273\,\text{K})} = 1.20\,\text{kg/m}^3$$

Then the mass flow rate of air vented out becomes

$$\dot{m}_{\text{air}} = \rho \dot{V}_{\text{air}} = (1.20\,\text{kg/m}^3)(0.030\,\text{m}^3/\text{s}) = 0.036\,\text{kg/s}$$

We can view infiltration as a steady stream of air that is heated as it flows in an imaginary duct passing through the house. The energy balance for this imaginary steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$

Noting that the indoor air vented out at 22°C is replaced by infiltrating outdoor air at 12.2°C, the sensible infiltration heat loss (heat gain for the infiltrating air) due to venting by fans can be expressed

$$\dot{Q}_{\text{loss by fan}} = \dot{m}_{\text{air}} C_p (T_{\text{indoors}} - T_{\text{outdoors}})$$
$$= (0.036\,\text{kg/s})(1.005\,\text{kJ/kg.°C})(22-12.2)°\text{C} = 0.355\,\text{kJ/s} = 0.355\,\text{kW}$$

Then the amount and cost of the heat "vented out" per month (1 month = 30×24 = 720 h) becomes

Energy loss = $\dot{Q}_{\text{loss by fan}}\,\Delta t$ = (0.355 kW)(720 h/month) = **256 kWh/month**

Money loss = (Energy loss)(Unit cost of energy) = (256 kWh/month)($0.09/kWh) = **$23.0/month**

Discussion Note that the energy and money loss associated with ventilating fans can be very significant. Therefore, ventilating fans should be used with care.

5-170 Chilled air is to cool a room by removing the heat generated in a large insulated classroom by lights and students. The required flow rate of air that needs to be supplied to the room is to be determined.

Assumptions **1** The moisture produced by the bodies leave the room as vapor without any condensing, and thus the classroom has no latent heat load. **2** Heat gain through the walls and the roof is negligible. **4** Air is an ideal gas with constant specific heats at room temperature. **5** Steady operating conditions exist.

Properties The specific heat of air at room temperature is 1.005 kJ/kg·°C (Table A-2). The average rate of sensible heat generation by a person is given to be 60 W.

Analysis The rate of sensible heat generation by the people in the room and the total rate of sensible internal heat generation are

$$\dot{Q}_{\text{gen, sensible}} = \dot{q}_{\text{gen, sensible}}(\text{No. of people}) = (60\ \text{W/person})(150\ \text{persons}) = 9000\ \text{W}$$

$$\dot{Q}_{\text{total, sensible}} = \dot{Q}_{\text{gen, sensible}} + \dot{Q}_{\text{lighting}} = 9000 + 4000 = 13{,}000\ \text{W}$$

Both of these effects can be viewed as heat gain for the chilled air stream, which can be viewed as a steady stream of cool air that is heated as it flows in an imaginary duct passing through the room. The energy balance for this imaginary steady-flow system can be expressed in the rate form as



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}}^{\nearrow 0\ \text{(steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{in} = \dot{Q}_{\text{total, sensible}} = \dot{m}C_p(T_2 - T_1)$$

Then the required mass flow rate of chilled air becomes

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{total, sensible}}}{C_p \Delta T} = \frac{13\ \text{kJ/s}}{(1.005\ \text{kJ/kg·°C})(25-15)°\text{C}} = \mathbf{1.29\ kg/s}$$

Discussion The latent heat will be removed by the air-conditioning system as the moisture condenses outside the cooling coils.

5-171 A banana cooling room is being analyzed. The minimum flow rate of air needed to cool bananas at a rate of 0.4°C/h while keeping the temperature rise of cool air under 2°C as it flows through the room is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Thermal properties of air, bananas, and boxes are constant. **3** Air is an ideal gas with constant specific heats at room temperature. **4** The kinetic and potential energy changes are negligible.

Properties The specific heats of banana and the fiberboard are given to be 3.55 kJ/kg.°C and 1.7 kJ/kg.°C, respectively. The peak heat of respiration of bananas is given to be 0.3 W/kg. The density and specific heat of air are given to be 1.2 kg/m^3 and 1.0 kJ/kg.°C.

Analysis Noting that the banana room holds 432 boxes, the total mass of bananas and the boxes are determined to be

$$m_{\text{banana}} = (\text{Mass per box})(\text{Number of boxes}) = (19\ \text{kg / box})(864\ \text{box}) = 16{,}416\ \text{kg}$$

$$m_{\text{box}} = (\text{Mass per box})(\text{Number of boxes}) = (2.3\ \text{kg / box})(864\ \text{box}) = 1987\ \text{kg}$$

The total refrigeration load in this case is due to the heat of respiration, the cooling of the bananas and the boxes, and the heat gain through the walls, and is determined from

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{respiration}} + \dot{Q}_{\text{banana}} + \dot{Q}_{\text{box}} + \dot{Q}_{\text{wall}}$$

where

$$\dot{Q}_{\text{respiration}} = m_{\text{banana}}\dot{q}_{\text{respiration}} = (16{,}419\ \text{kg})(0.3\ \text{W / kg}) = 4925\ \text{W}$$

$$\dot{Q}_{\text{banana}} = (mC_p\Delta T / \Delta t)_{\text{banana}} = (16{,}416\ \text{kg})(3.55\ \text{kJ / kg.°C})(0.4\ \text{°C / h}) = 23{,}315\ \text{kJ / h} = 6476\ \text{W}$$

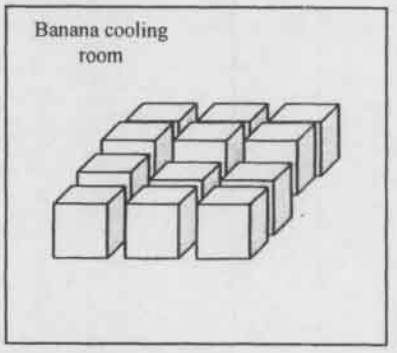
$$\dot{Q}_{\text{box}} = (mC_p\Delta T / \Delta t)_{\text{box}} = (1987\ \text{kg})(1.7\ \text{kJ / kg.°C})(0.4\ \text{°C / h}) = 1351\ \text{kJ / h} = 375\ \text{W}$$

$$\dot{Q}_{\text{wall}} = 1800\ \text{kJ / kg}) = 0.5\ \text{kW} = 500\ \text{W}$$

and the quantity $\Delta T / \Delta t$ is the rate of change of temperature of the products, and is given to be 0.4°C/W. Then the total rate of cooling becomes

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{respiration}} + \dot{Q}_{\text{banana}} + \dot{Q}_{\text{box}} + \dot{Q}_{\text{wall}} = 4925 + 6476 + 375 + 500 = 12{,}276\ \text{W}$$

All of these effects can be viewed as heat gain for the chilled air stream, which can be viewed as a steady stream of cool air that is heated as it flows in an imaginary duct passing through the room. The energy balance for this imaginary steady-flow system can be expressed in the rate form as



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}}{}^{\nearrow 0\ \text{(steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{in} = \dot{Q}_{\text{total}} = \dot{m}C_p(T_2 - T_1)$$

The temperature rise of air is limited to 2.0 K as it flows through the load. Noting that air picks up heat at a rate of 12,276 W, the minimum mass and volume flow rates of air are determined to be

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{air}}}{(C_p\Delta T)_{\text{air}}} = \frac{12{,}276\ \text{W}}{(1000\ \text{J / kg.°C})(2.0\text{°C})} = 6.14\ \text{kg / s}$$

$$\dot{V}_{\text{air}} = \frac{\dot{m}_{\text{air}}}{\rho_{\text{air}}} = \frac{6.14\ \text{kg / s}}{1.2\ \text{kg / m}^3} = \mathbf{5.12\ m^3 / s}$$

Therefore, the fan selected for the banana room must be large enough to circulate air at a rate of 5.12 m^3/s.

5-172 Chickens are to be cooled by chilled water in an immersion chiller. The rate of heat removal from the chicken and the mass flow rate of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of chickens and water are constant.

Properties The specific heat of chicken are given to be 3.54 kJ/kg.°C. The specific heat of water is 4.18 kJ/kg.°C (Table A-3).

Analysis (*a*) Chickens are dropped into the chiller at a rate of 500 per hour. Therefore, chickens can be considered to flow steadily through the chiller at a mass flow rate of

$$\dot{m}_{\text{chicken}} = (500 \text{ chicken / h})(2.2 \text{ kg / chicken}) = 1100 \text{ kg / h} = 0.3056 \text{ kg / s}$$

Taking the chicken flow stream in the chiller as the system, the energy balance for steadily flowing chickens can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta \text{ke} \cong \Delta \text{pe} \cong 0)$$

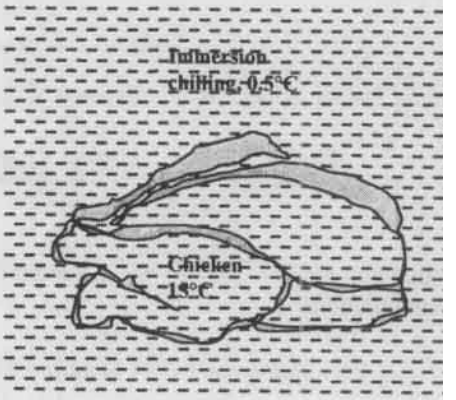
$$\dot{Q}_{out} = \dot{Q}_{\text{chicken}} = \dot{m}_{\text{chicken}} C_p (T_1 - T_2)$$

Then the rate of heat removal from the chickens as they are cooled from 15°C to 3°C becomes

$$\dot{Q}_{\text{chicken}} = (\dot{m} C_p \Delta T)_{\text{chicken}} = (0.3056 \text{ kg/s})(3.54 \text{ kJ/kg.°C})(15 - 3)°\text{C} = \mathbf{13.0 \text{ kW}}$$

The chiller gains heat from the surroundings at a rate of 200 kJ/h = 0.0556 kJ/s. Then the total rate of heat gain by the water is

$$\dot{Q}_{\text{water}} = \dot{Q}_{\text{chicken}} + \dot{Q}_{\text{heat gain}} = 13.0 + 0.056 = 13.056 \text{ kW}$$



Noting that the temperature rise of water is not to exceed 2°C as it flows through the chiller, the mass flow rate of water must be at least

$$\dot{m}_{\text{water}} = \frac{\dot{Q}_{\text{water}}}{(C_p \Delta T)_{\text{water}}} = \frac{13.056 \text{ kW}}{(4.18 \text{ kJ/kg.°C})(2°\text{C})} = \mathbf{1.56 \text{ kg/s}}$$

If the mass flow rate of water is less than this value, then the temperature rise of water will have to be more than 2°C.

173 A regenerator is considered to save heat during the cooling of milk in a dairy plant. The amounts of el and money such a generator will save per year are to be determined.

sumptions 1 Steady operating conditions exist. **2** The properties of the milk are constant.

operties The average density and specific heat of milk can be taken to be $\rho_{milk} \cong \rho_{water} = 1$ kg/L d $C_{p,\,milk} = 3.79$ kJ/kg.°C (Table A-3).

alysis The mass flow rate of the milk is

$$\dot{m}_{milk} = \rho \dot{V}_{milk} = (1\ \text{kg/L})(12\ \text{L/s}) = 12\ \text{kg/s} = 43{,}200\ \text{kg/h}$$

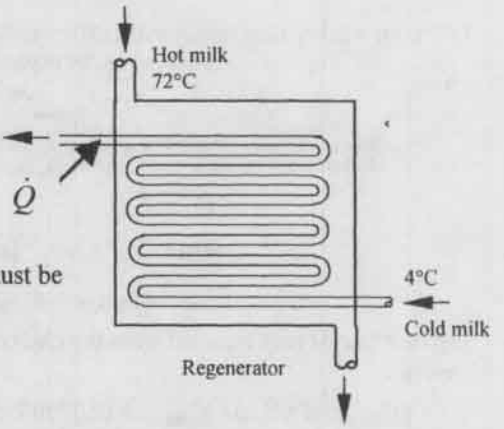
aking the pasteurizing section as the system, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}^{\nearrow 0\ \text{(steady)}}{}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}_{milk} C_p (T_2 - T_1)$$



erefore, to heat the milk from 4 to 72°C as being done currently, heat must be nsferred to the milk at a rate of

$$\dot{Q}_{current} = [\dot{m} C_p (T_{pasturization} - T_{refrigeration})]_{milk}$$
$$= (12\,\text{kg/s})(3.79\,\text{kJ/kg.°C})(72 - 4)°\text{C} = 3093\,\text{kJ/s}$$

e proposed regenerator has an effectiveness of $\varepsilon = 0.82$, and thus it will save 82 percent of this energy. erefore,

$$\dot{Q}_{saved} = \varepsilon \dot{Q}_{current} = (0.82)(3093\ \text{kJ/s}) = 2536\ \text{kJ/s}$$

ting that the boiler has an efficiency of $\eta_{boiler} = 0.82$, the energy savings above correspond to fuel savings

$$\text{Fuel Saved} = \frac{\dot{Q}_{saved}}{\eta_{boiler}} = \frac{(2536\ \text{kJ/s})}{(0.82)} \frac{(1\text{therm})}{(105{,}500\ \text{kJ})} = 0.02931\,\text{therm/s}$$

ting that 1 year = 365×24=8760 h and unit cost of natural gas is $0.52/therm, the annual fuel and money vings will be

Fuel Saved = (0.02931 therms/s)(8760×3600 s) = **924,450 therms/yr**

Money saved = (Fuel saved)(Unit cost of fuel)
= (924,450 therm/yr)($0.52/therm) = **$480,700/yr**

5-174E A refrigeration system is to cool eggs by chilled air at a rate of 10,000 eggs per hour. The rate of heat removal from the eggs, the required volume flow rate of air, and the size of the compressor of the refrigeration system are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The eggs are at uniform temperatures before and after cooling. **3** The cooling section is well-insulated. **4** The properties of eggs are constant. **5** The local atmospheric pressure is 1 atm.

Properties The properties of the eggs are given to $\rho = 67.4$ lbm/ft^3 and $C_p = 0.80$ Btu/lbm.°F. The specific heat of air at room temperature $C_p = 0.24$ Btu/lbm. °F (Table A-2E). The gas constant of air is $R = 0.3704$ psia.ft^3/lbm.R (Table A-1E).

Analysis (*a*) Noting that eggs are cooled at a rate of 10,000 eggs per hour, eggs can be considered to flow steadily through the cooling section at a mass flow rate of

$$\dot{m}_{egg} = (10{,}000 \text{ eggs/h})(0.14 \text{ lbm/egg}) = 1400 \text{ lbm/h} = 0.3889 \text{ lbm/s}$$

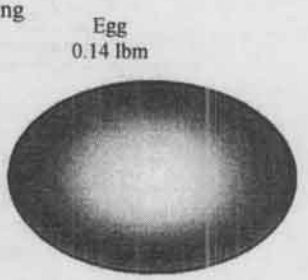
Taking the egg flow stream in the cooler as the system, the energy balance for steadily flowing eggs can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}^{\nearrow 0 \text{ (steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{Q}_{egg} = \dot{m}_{egg} C_p (T_1 - T_2)$$



Then the rate of heat removal from the eggs as they are cooled from 90°F to 50°F at this rate becomes

$$\dot{Q}_{egg} = (\dot{m}C_p \Delta T)_{egg} = (1400 \text{ lbm/h})(0.80 \text{ Btu/lbm.°F})(90 - 50)°\text{F} = \mathbf{44{,}800 \text{ Btu/h}}$$

(*b*) All the heat released by the eggs is absorbed by the refrigerated air since heat transfer through he walls of cooler is negligible, and the temperature rise of air is not to exceed 10°F. The minimum mass flow and volume flow rates of air are determined to be

$$\dot{m}_{air} = \frac{\dot{Q}_{air}}{(C_p \Delta T)_{air}} = \frac{44{,}800 \text{ Btu/h}}{(0.24 \text{ Btu/lbm.°F})(10°\text{F})} = 18{,}667 \text{ lbm/h}$$

$$\rho_{air} = \frac{P}{RT} = \frac{14.7 \text{ psia}}{(0.3704 \text{ psia.ft}^3/\text{lbm.R})(34 + 460)\text{R}} = 0.0803 \text{ lbm/ft}^3$$

$$\dot{V}_{air} = \frac{\dot{m}_{air}}{\rho_{air}} = \frac{18{,}667 \text{ lbm/h}}{0.0803 \text{ lbm/ft}^3} = \mathbf{232{,}500 \text{ ft}^3\text{/h}}$$

5-175 Dough is made with refrigerated water in order to absorb the heat of hydration and thus to control the temperature rise during kneading. The temperature to which the city water must be cooled before mixing with flour is to be determined to avoid temperature rise during kneading.

Assumptions **1** Steady operating conditions exist. **2** The dough is at uniform temperatures before and after cooling. **3** The kneading section is well-insulated. **4** The properties of water and dough are constant.

Properties The specific heats of the flour and the water are given to be 1.76 and 4.18 kJ/kg.°C, respectively. The heat of hydration of dough is given to be 15 kJ/kg.

Analysis It is stated that 2 kg of flour is mixed with 1 kg of water, and thus 3 kg of dough is obtained from each kg of water. Also, 15 kJ of heat is released for each kg of dough kneaded, and thus 3×15 = 45 kJ of heat is released from the dough made using 1 kg of water.

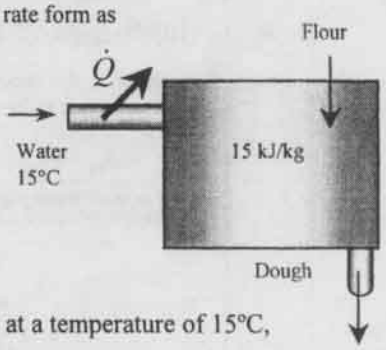
Taking the cooling section of water as the system, which is a steady-flow control volume, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{Q}_{\text{water}} = \dot{m}_{\text{water}} C_p (T_1 - T_2)$$



In order for water to absorb all the heat of hydration and end up at a temperature of 15°C, its temperature before entering the mixing section must be reduced to

$$Q_{in} = Q_{\text{dough}} = mC_p(T_2 - T_1) \rightarrow T_1 = T_2 - \frac{Q}{mC_p} = 15°\text{C} - \frac{45 \text{ kJ}}{(1 \text{ kg})(4.18 \text{ kJ/kg.°C})} = \mathbf{4.2°C}$$

That is, the water must be precooled to 4.2°C before mixing with the flour in order to absorb the entire heat of hydration.

5-176 Glass bottles are washed in hot water in an uncovered rectangular glass washing bath. The rates of heat and water mass that need to be supplied to the water are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The entire water body is maintained at a uniform temperature of 55°C. **3** Heat losses from the outer surfaces of the bath are negligible. **4** Water is an incompressible substance with constant properties.

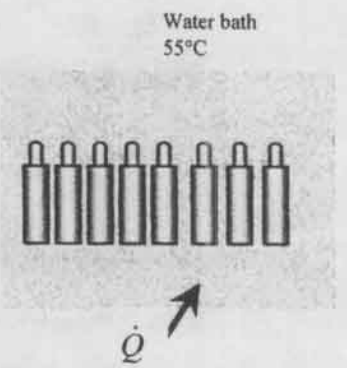
Properties The specific heat of water at room temperature is C_p = 4.18 kJ/kg.°C. Also, the specific heat of glass is 0.80 kJ/kg.°C (Table A-3).

Analysis (*a*) The mass flow rate of glass bottles through the water bath in steady operation is

$$\dot{m}_{\text{bottle}} = m_{\text{bottle}} \times \text{Bottle flow rate} = (0.150\ \text{kg/bottle})(800\ \text{bottles/min}) = 120\ \text{kg/min} = 2\ \text{kg/s}$$

Taking the bottle flow section as the system, which is a steady-flow control volume, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta \text{ke} \cong \Delta \text{pe} \cong 0)$$

$$\dot{Q}_{in} = \dot{Q}_{\text{bottle}} = \dot{m}_{\text{water}} C_p (T_2 - T_1)$$

Then the rate of heat removal by the bottles as they are heated from 20 to 55°C is

$$\dot{Q}_{\text{bottle}} = \dot{m}_{\text{bottle}} C_p \Delta T = (2\text{kg/s})(0.8\text{kJ/kg.°C})(55-20)\text{°C} = 56{,}000\ \text{W}$$

The amount of water removed by the bottles is

$$\dot{m}_{\text{water,out}} = (\text{Flow rate of bottles})(\text{Water removed per bottle})$$

$$= (800\ \text{bottles/min})(0.2\ \text{g/bottle}) = 160\ \text{g/min} = \mathbf{2.67 \times 10^{-3}\ kg/s}$$

Noting that the water removed by the bottles is made up by fresh water entering at 15°C, the rate of heat removal by the water that sticks to the bottles is

$$\dot{Q}_{\text{water removed}} = \dot{m}_{\text{water removed}} C_p \Delta T = (2.67 \times 10^{-3}\ \text{kg/s})(4180\ \text{J/kg.°C})(55-15)\text{°C} = 446\ \text{W}$$

Therefore, the total amount of heat removed by the wet bottles is

$$\dot{Q}_{\text{total, removed}} = \dot{Q}_{\text{glass removed}} + \dot{Q}_{\text{water removed}} = 56{,}000 + 446 = \mathbf{56{,}446\ W}$$

Discussion In practice, the rates of heat and water removal will be much larger since the heat losses from the tank and the moisture loss from the open surface are not considered.

5-177 Glass bottles are washed in hot water in an uncovered rectangular glass washing bath. The rates of heat and water mass that need to be supplied to the water are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The entire water body is maintained at a uniform temperature of 50°C. **3** Heat losses from the outer surfaces of the bath are negligible. **4** Water is an incompressible substance with constant properties.

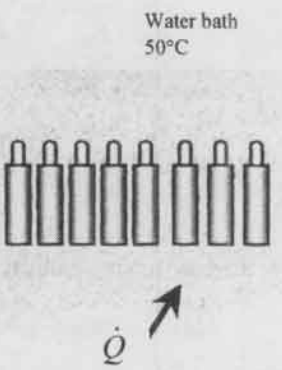
Properties The specific heat of water at room temperature is C_p = 4.18 kJ/kg.°C. Also, the specific heat of glass is 0.80 kJ/kg.°C (Table A-3).

Analysis (*a*) The mass flow rate of glass bottles through the water bath in steady operation is

$$\dot{m}_{\text{bottle}} = m_{\text{bottle}} \times \text{Bottle flow rate} = (0.150\ \text{kg/bottle})(800\ \text{bottles/min}) = 120\ \text{kg/min} = 2\ \text{kg/s}$$

Taking the bottle flow section as the system, which is a steady-flow control volume, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{Q}_{\text{bottle}} = \dot{m}_{\text{water}} C_p (T_2 - T_1)$$

Then the rate of heat removal by the bottles as they are heated from 20 to 50°C is

$$\dot{Q}_{\text{bottle}} = \dot{m}_{\text{bottle}} C_p \Delta T = (2\ \text{kg/s})(0.8\ \text{kJ/kg.°C})(50-20)°\text{C} = 48{,}000\ \text{W}$$

The amount of water removed by the bottles is

$$\dot{m}_{\text{water,out}} = (\text{Flow rate of bottles})(\text{Water removed per bottle})$$

$$= (800\ \text{bottles/min})(0.2\ \text{g/bottle}) = 160\ \text{g/min} = \mathbf{2.67 \times 10^{-3}\ kg/s}$$

Noting that the water removed by the bottles is made up by fresh water entering at 15°C, the rate of heat removal by the water that sticks to the bottles is

$$\dot{Q}_{\text{water removed}} = \dot{m}_{\text{water removed}} C_p \Delta T = (82.67 \times 10^{-3}\ \text{kg/s})(4180\ \text{J/kg·°C})(50-15)°\text{C} = 391\ \text{W}$$

Therefore, the total amount of heat removed by the wet bottles is

$$\dot{Q}_{\text{total, removed}} = \dot{Q}_{\text{glass removed}} + \dot{Q}_{\text{water removed}} = 48{,}000 + 391 = \mathbf{48{,}391\ W}$$

Discussion In practice, the rates of heat and water removal will be much larger since the heat losses from the tank and the moisture loss from the open surface are not considered.

5-178 Long aluminum wires are extruded at a velocity of 10 m/min, and are exposed to atmospheric air. The rate of heat transfer from the wire is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of the wire are constant.

Properties The properties of aluminum are given to be $\rho = 2702$ kg/m^3 and $C_p = 0.896$ kJ/kg.°C.

Analysis The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho \dot{V} = \rho(\pi r_0^2 / 4)V = (2702 \text{ kg/m}^3)\pi(0.0015 \text{ m})^2(10 \text{ m/min}) = 0.191 \text{ kg/min}$$

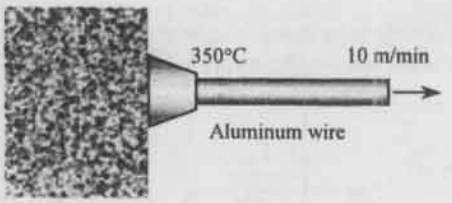
Taking the volume occupied by the extruded wire as the system, which is a steady-flow control volume, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}{}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{Q}_{\text{wire}} = \dot{m}_{\text{wire}} C_p (T_1 - T_2)$$



Then the rate of heat transfer from the wire to the air becomes

$$\dot{Q} = \dot{m}C_p[T(t) - T_\infty] = (0.191 \text{ kg/min})(0.896 \text{ kJ/kg.°C})(350 - 50)°\text{C} = 51.3 \text{ kJ/min} = \mathbf{0.856 \text{ kW}}$$

5-179 Long copper wires are extruded at a velocity of 10 m/min, and are exposed to atmospheric air. The rate of heat transfer from the wire is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of the wire are constant.

Properties The properties of copper are given to be $\rho = 8950$ kg/m^3 and $C_p = 0.383$ kJ/kg.°C.

Analysis The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho \dot{V} = \rho(\pi r_0^2 / 4)V = (8950 \text{ kg/m}^3)\pi(0.0015 \text{ m})^2(10 \text{ m/min}) = 0.633 \text{ kg/min}$$

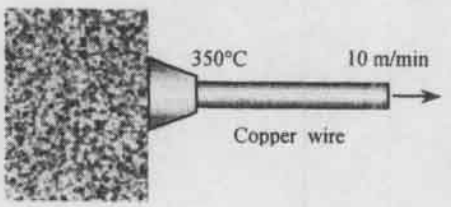
Taking the volume occupied by the extruded wire as the system, which is a steady-flow control volume, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}{}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{Q}_{\text{wire}} = \dot{m}_{\text{wire}} C_p (T_1 - T_2)$$



Then the rate of heat transfer from the wire to the air becomes

$$\dot{Q} = \dot{m}C_p[T(t) - T_\infty] = (0.633 \text{ kg/min})(0.383 \text{ kJ/kg.°C})(350 - 50)°\text{C} = 72.7 \text{ kJ/min} = \mathbf{1.21 \text{ kW}}$$

5-180 Steam at a saturation temperature of T_{sat} = 40°C condenses on the outside of a thin horizontal tube. Heat is transferred to the cooling water that enters the tube at 25°C and exits at 35°C. The rate of condensation of steam is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Water is an incompressible substance with constant properties at room temperature. **3** The changes in kinetic and potential energies are negligible.

Properties The properties of water at room temperature are ρ = 997 kg/m^3 and C_p = 4.18 kJ/kg.°C. The enthalpy of vaporization of water at a saturation temperature of 40°C is h_{fg} = 2406.7 kJ/kg (Table A-4).

Analysis The mass flow rate of water through the tube is

$$\dot{m}_{water} = \rho V A_c = (997\ \text{kg/m}^3)(2\ \text{m/s})[\pi(0.03\ \text{m})^2/4] = 1.409\ \text{kg/s}$$

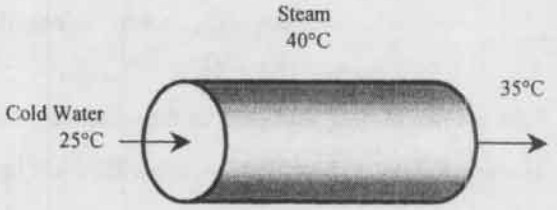
Taking the volume occupied by the cold water in the tube as the system, which is a steady-flow control volume, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}^{\nearrow 0\ \text{(steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{Q}_{water} = \dot{m}_{water} C_p (T_2 - T_1)$$



Then the rate of heat transfer to the water and the rate of evaporation become

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in}) = (1.409\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot°\text{C})(35-25)°\text{C} = 58.9\ \text{kW}$$

$$\dot{Q} = \dot{m}_{evap} h_{fg} \rightarrow \dot{m}_{evap} = \frac{\dot{Q}}{h_{fg}} = \frac{58.9\ \text{kJ/s}}{2406.7\ \text{kJ/kg}} = \mathbf{0.0245\ kg/s}$$

5-181E Saturated steam at a saturation pressure of 0.95 psia and thus at a saturation temperature of T_{sat} = 100°F (Table A-4E) condenses on the outer surfaces of 144 horizontal tubes that are maintained at 80°F by circulating cooling water and arranged in a 12 × 12 square array. The rate of heat transfer to the cooling water and the rate of condensation of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tubes are isothermal. **3** Water is an incompressible substance with constant properties at room temperature. **4** The changes in kinetic and potential energies are negligible.

Properties The properties of water at room temperature are ρ = 62.1 lbm/ft^3 and C_p = 1.00 Btu/lbm.°F. The enthalpy of vaporization of water at a saturation temperature of 100°F is h_{fg} = 1037 Btu/lbm (Table A-4E).

Analysis The rate of heat transfer from the steam to the cooling water is equal to the heat of vaporization released as the vapor condenses at the specified temperature,

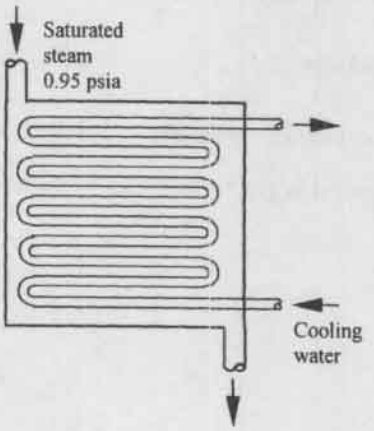
$$\dot{Q} = \dot{m}h_{fg} = (6800 \text{ lbm/h})(1037 \text{ Btu/lbm}) = \mathbf{7{,}051{,}600 \text{ Btu/h} = 1959 \text{ Btu/s}}$$

All of this energy is transferred to the cold water. Therefore, the mass flow rate of cold water must be

$$\dot{Q} = \dot{m}_{water} C_p \Delta T \quad \rightarrow \quad \dot{m}_{water} = \frac{\dot{Q}}{C_p \Delta T} = \frac{1959 \text{ Btu/s}}{(1.00 \text{ Btu/lbm.°F})(8°\text{F})} = 244.8 \text{ lbm/s}$$

Then the average velocity of the cooling water through the 144 tubes becomes

$$\dot{m} = \rho A \mathbf{V} \quad \rightarrow \quad \mathbf{V} = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(n\pi D^2/4)} = \frac{244.8 \text{ lbm/s}}{(62.1 \text{ lbm/ft}^3)[144\pi(2/12 \text{ ft})^2/4]} = \mathbf{1.26 \text{ ft/s}}$$



5-182 Saturated refrigerant-134a vapor at a saturation temperature of T_{sat} = 30°C condenses inside a horizontal tube. The rate of heat transfer from the refrigerant for the condensate exit temperatures of 30°C and 20°C are to be determined.

Assumptions **1** Steady flow conditions exist. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions involved.

Properties The properties of saturated refrigerant-134a at 30°C are h_f = 91.49 kJ/kg, are h_g = 263.50 kJ/kg, and are h_{fg} = 172.00 kJ/kg. The enthalpy of saturated liquid refrigerant at 20°C is h_f = 77.26 kJ/kg, (Table A-11).

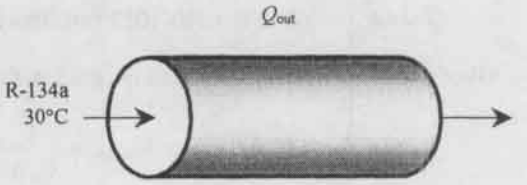
Analysis We take the *tube and the refrigerant in it* as the system. This is a *control volume* since mass crosses the system boundary during the process. We note that there is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Noting that heat is lost from the system, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{m}(h_1 - h_2)$$



where at the inlet state $h_1 = h_g$ = 263.50 kJ/kg. Then the rates of heat transfer during this condensation process for both cases become

Case 1: T_2 = 30°C: $h_2 = h_{f@30°C}$ = 91.49 kJ/kg.

$$\dot{Q}_{out} = (0.1 \text{ kg/min})(263.5 - 91.49) \text{ kJ/kg} = \mathbf{17.2\ kW}$$

Case 2: T_2 = 20°C: $h_2 \cong h_{f@20°C}$ = 77.26 kJ/kg.

$$\dot{Q}_{out} = (0.1 \text{ kg/min})(263.5 - 77.26) \text{ kJ/kg} = \mathbf{18.6\ kW}$$

Discussion Note that the rate of heat removal is greater in the second case since the liquid is subcooled in that case.

5-183E A winterizing project is to reduce the infiltration rate of a house from 2.2 ACH to 1.1 ACH. The resulting cost savings are to be determined.

Assumptions **1** The house is maintained at 72°F at all times. **2** The latent heat load during the heating season is negligible. **3** The infiltrating air is heated to 72°F before it exfiltrates. **4** Air is an ideal gas with constant specific heats at room temperature. **5** The changes in kinetic and potential energies are negligible. **6** Steady flow conditions exist.

Properties The gas constant of air is 0.3704 psia.ft^3/lbm·R (Table A-1E). The specific heat of air at room temperature is 0.24 Btu/lbm·°F (Table A-2E).

Analysis The density of air at the outdoor conditions is

$$\rho_o = \frac{P_o}{RT_o} = \frac{13.5\ \text{psia}}{(0.3704\ \text{psia.ft}^3/\text{lbm.R})(496.5\ \text{R})} = 0.0734\ \text{lbm/ft}^3$$

The volume of the house is

$$V_{\text{building}} = (\text{Floor area})(\text{Height}) = (3000\ \text{ft}^2)(9\ \text{ft}) = 27{,}000\ \text{ft}^3$$

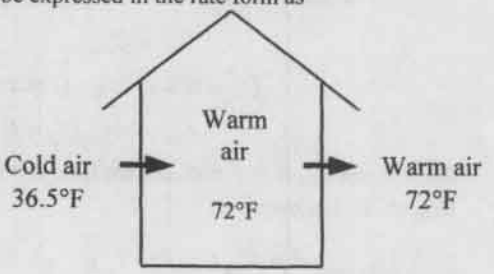
We can view infiltration as a steady stream of air that is heated as it flows in an imaginary duct passing through the house. The energy balance for this imaginary steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1) = \rho \dot{V} C_p(T_2 - T_1)$$



The reduction in the infiltration rate is 2.2 – 1.1 = 1.1 ACH.
The reduction in the sensible infiltration heat load corresponding to it is

$$\dot{Q}_{\text{infiltration, saved}} = \rho_o C_p (ACH_{\text{saved}})(V_{\text{building}})(T_i - T_o)$$
$$= (0.0734\ \text{lbm/ft}^3)(0.24\ \text{Btu/lbm.°F})(1.1/\text{h})(27{,}000\ \text{ft}^3)(72 - 36.5)°\text{F}$$
$$= 18{,}573\ \text{Btu/h} = 0.18573\ \text{therm/h}$$

since 1 therm = 100,000 Btu. The number of hours during a six month period is 6×30×24 = 4320 h. Noting that the furnace efficiency is 0.65 and the unit cost of natural gas is $0.62/therm, the energy and money saved during the 6-month period are

$$\text{Energy savings} = (\dot{Q}_{\text{infiltration, saved}})(\text{No. of hours per year})/\text{Efficiency}$$
$$= (0.18573\ \text{therm/h})(4320\ \text{h/year})/0.65$$
$$= 1234\ \text{therms/year}$$

$$\text{Cost savings} = (\text{Energy savings})(\text{Unit cost of energy})$$
$$= (1234\ \text{therms/year})(\$0.62/\text{therm})$$
$$= \mathbf{\$765/year}$$

Therefore, reducing the infiltration rate by one-half will reduce the heating costs of this homeowner by $765 per year.

5-184 Outdoor air at -10°C and 90 kPa enters the building at a rate of 35 L/s while the indoors is maintained at 22°C. The rate of sensible heat loss from the building due to infiltration is to be determined.

Assumptions **1** The house is maintained at 22°C at all times. **2** The latent heat load is negligible. **3** The infiltrating air is heated to 22°C before it exfiltrates. **4** Air is an ideal gas with constant specific heats at room temperature. **5** The changes in kinetic and potential energies are negligible. **6** Steady flow conditions exist.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg·K (Table A-1). The specific heat of air at room temperature is $C_p = 1.0$ kJ/kg·°C (Table A-2).

Analysis The density of air at the outdoor conditions is

$$\rho_o = \frac{P_o}{RT_o} = \frac{90\ \text{kPa}}{(0.287\ \text{kPa.m}^3/\text{kg.K})(-10+273\ \text{K})} = 1.19\ \text{kg/m}^3$$

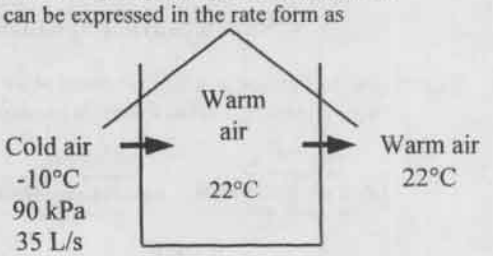
We can view infiltration as a steady stream of air that is heated as it flows in an imaginary duct passing through the building. The energy balance for this imaginary steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{in} = \dot{m}C_p(T_2 - T_1)$$



Then the sensible infiltration heat load corresponding to an infiltration rate of 35 L/s becomes

$$\dot{Q}_{\text{infiltration}} = \rho_o \dot{V}_{air} C_p (T_i - T_o)$$
$$= (1.19\ \text{kg/m}^3)(0.035\ \text{m}^3/\text{s})(1.005\ \text{kJ/kg.°C})[22-(-10)]\text{°C}$$
$$= \mathbf{1.335\ kW}$$

Therefore, sensible heat will be lost at a rate of 1.335 kJ/s due to infiltration.

5-185 The maximum flow rate of a standard shower head can be reduced from 13.3 to 10.5 L/min by switching to low-flow shower heads. The ratio of the hot-to-cold water flow rates and the amount of electricity saved by a family of four per year by replacing the standard shower heads by the low-flow ones are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$. **2** The kinetic and potential energies are negligible, $ke \cong pe \cong 0$. **3** Heat losses from the system are negligible and thus $\dot{Q} \cong 0$. **4** There are no work interactions involved. **5** .Showers operate at maximum flow conditions during the entire shower. **6** Each member of the household takes a 5-min shower every day. **7** Water is an incompressible substance with constant properties. **8** The efficiency of the electric water heater is 100%.

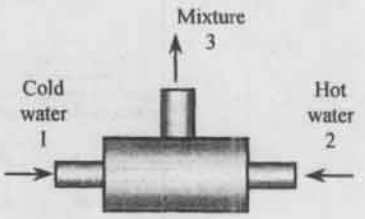
Properties The density and specific heat of water at room temperature are ρ = 1 kg/L and C = 4.18 kJ/kg.°C (Table A-3).

Analysis (*a*) We take the *mixing chamber* as the system. This is a *control volume* since mass crosses the system boundary during the process. We note that there are two inlets and one exit. The mass and energy balances for this steady-flow system can be expressed in the rate form as follows:

Mass balance:

$$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{system} \overset{\nearrow 0 \text{ (steady)}}{} = 0$$

$$\dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$



Energy balance:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system} \overset{\nearrow 0 \text{ (steady)}}{}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong 0,\ \dot{W} = 0,\ ke \cong pe \cong 0)$$

Combining the mass and energy balances and rearranging,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

$$\dot{m}_2 (h_2 - h_3) = \dot{m}_1 (h_3 - h_1)$$

Then the ratio of the mass flow rates of the hot water to cold water becomes

$$\frac{\dot{m}_2}{\dot{m}_1} = \frac{h_3 - h_1}{h_2 - h_3} = \frac{C(T_3 - T_1)}{C(T_2 - T_3)} = \frac{T_3 - T_1}{T_2 - T_3} = \frac{(42 - 15)^\circ C}{(55 - 42)^\circ C} = \mathbf{2.08}$$

(*b*) The low-flow heads will save water at a rate of

$$\dot{V}_{saved} = [(13.3 - 10.5)\ L/min](5\ min/person.day)(4\ persons)(365\ days/yr) = 20{,}440\ L/year$$

$$\dot{m}_{saved} = \rho \dot{V}_{saved} = (1\ kg/L)(20{,}440\ L/year) = 20{,}440\ kg/year$$

Then the energy saved per year becomes

$$\text{Energy saved} = \dot{m}_{saved} C \Delta T = (20{,}440\ kg/year)(4.18\ kJ/kg.^\circ C)(42 - 15)^\circ C$$

$$= 2{,}307{,}000\ kJ/year$$

$$= \mathbf{641\ kWh} \quad (\text{since } 1\ kWh = 3600\ kJ)$$

Therefore, switching to low-flow shower heads will save about 641 kWh of electricity per year.

5-186 A fan is powered by a 0.5 hp motor, and delivers air at a rate of 130 m^3/min. The highest possible air velocity at the fan exit is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$. **2** The inlet velocity and the change in potential energy are negligible, $\mathbf{V}_1 \cong 0$ and $\Delta pe \cong 0$. **3** There are no heat and work interactions other than the electrical power consumed by the fan motor. **4** The efficiencies of the motor and the fan are 100% since best possible operation is assumed. **5** Air is an ideal gas with constant specific heats at room temperature.

Properties The density of air is given to be ρ = 1.18 kg/m^3. The constant pressure specific heat of air at room temperature is C_p = 1.005 kJ/kg.°C (Table A-2).

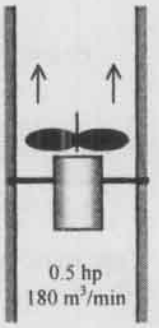
Analysis We take the *fan-motor assembly* as the system. This is a *control volume* since mass crosses the system boundary during the process. We note that there is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

The velocity of air leaving the fan will be highest when all of the entire electrical energy drawn by the motor is converted to kinetic energy, and the friction between the air layers is zero. In this best possible case, no energy will be converted to thermal energy, and thus the temperature change of air will be zero, $T_2 = T_1$. Then the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{m}(h_2 + \mathbf{V}_2^2/2) \quad (\text{since } \mathbf{V}_1 \cong 0 \text{ and } \Delta pe \cong 0)$$



Noting that the temperature and thus enthalpy remains constant, the relation above simplifies further to

$$\dot{W}_{e,in} = \dot{m}\mathbf{V}_2^2/2$$

where

$$\dot{m} = \rho \dot{V} = (1.18 \text{ kg/m}^3)(130 \text{ m}^3/\text{min}) = 153.4 \text{ kg/min} = 2.56 \text{ kg/s}$$

Solving for $\mathbf{V}_2$ and substituting gives

$$\mathbf{V}_2 = \sqrt{\frac{2\dot{W}_{e,in}}{\dot{m}}} = \sqrt{\frac{2(0.5 \text{ hp})}{2.56 \text{ kg/s}}\left(\frac{745.7 \text{ W}}{1 \text{ hp}}\right)\left(\frac{1 \text{ m}^2/\text{s}^2}{1 \text{ W}}\right)} = \mathbf{17.1 \text{ m/s}}$$

Discussion In reality, the velocity will be less because of the inefficiencies of the motor and the fan.

5-187 The average air velocity in the circular duct of an air-conditioning system is not to exceed 10 m/s. If the fan converts 70 percent of the electrical energy into kinetic energy, the size of the fan motor needed and the diameter of the main duct are to be determined.

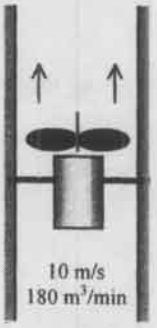
Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$. **2** The inlet velocity is negligible, $\mathbf{V}_1 \cong 0$.. **3** There are no heat and work interactions other than the electrical power consumed by the fan motor. **4** Air is an ideal gas with constant specific heats at room temperature.

Properties The density of air is given to be $\rho = 1.20$ kg/m^3. The constant pressure specific heat of air at room temperature is $C_p = 1.005$ kJ/kg.°C (Table A-2).

Analysis We take the *fan-motor assembly* as the system. This is a *control volume* since mass crosses the system boundary during the process. We note that there is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The change in the kinetic energy of air as it is accelerated from zero to 10 m/s at a rate of 130 m^3/s is

$$\dot{m} = \rho \dot{V} = (1.20\,\text{kg/m}^3)(180\,\text{m}^3/\text{min}) = 216\,\text{kg/min} = 3.6\,\text{kg/s}$$

$$\Delta \dot{KE} = \dot{m}\frac{V_2^2 - V_1^2}{2} = (3.6\,\text{kg/s})\frac{(10\,\text{m/s})^2 - 0}{2}\left(\frac{1\,\text{kJ/kg}}{1000\,\text{m}^2/\text{s}^2}\right) = 0.18\,\text{kW}$$



It is stated that this represents 70% of the electrical energy consumed by the motor. Then the total electrical power consumed by the motor is determined to be

$$0.7\dot{W}_{\text{motor}} = \Delta \dot{KE} \quad \rightarrow \quad \dot{W}_{\text{motor}} = \frac{\Delta \dot{KE}}{0.7} = \frac{0.18\,\text{kW}}{0.7} = \mathbf{0.257\,kW}$$

The diameter of the main duct is

$$\dot{V} = \mathbf{V}A = \mathbf{V}(\pi D^2/4) \rightarrow D = \sqrt{\frac{4\dot{V}}{\pi \mathbf{V}}} = \sqrt{\frac{4(180\,\text{m}^3/\text{min})}{\pi(10\,\text{m/s})}\left(\frac{1\,\text{min}}{60\,\text{s}}\right)} = \mathbf{0.618\,m}$$

Therefore, the motor should have a rated power of at least 0.257 kW, and the diameter of the duct should be at least 61.8 cm

5-188 An evacuated bottle is surrounded by atmospheric air. A valve is opened, and air is allowed to fill the bottle. The amount of heat transfer through the wall of the bottle when thermal and mechanical equilibrium is established is to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** Air is an ideal gas. **3** Kinetic and potential energies are negligible. **4** There are no work interactions involved. **5** The direction of heat transfer is to the air in the bottle (will be verified).

Analysis We take the bottle as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{\text{system}} \rightarrow m_i = m_2$ (since $m_{out} = m_{initial} = 0$)

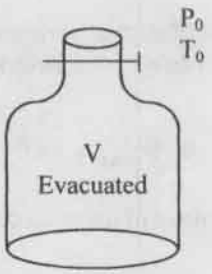
Energy balance:

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} + m_i h_i = m_2 u_2 \text{ (since } W \cong E_{out} = E_{initial} = ke \cong pe \cong 0)$$

Combining the two balances:

$$Q_{in} = m_2(u_2 - h_i) = m_2(C_v T_2 - C_p T_i)$$



But $T_i = T_2 = T_0$ and $C_p - C_v = R$. Substituting,

$$Q_{in} = m_2(C_v - C_p)T_0 = -m_2 R T_0 = -\frac{P_0 V}{R T_0} R T_0 = -P_0 V$$

Therefore,

$$Q_{\text{out}} = \boldsymbol{P_0 V} \quad \text{(Heat is lost from the tank)}$$

5-189 An adiabatic air compressor is powered by a direct-coupled steam turbine, which is also driving a generator. The net power delivered to the generator is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The devices are adiabatic and thus heat transfer is negligible. **4** Air is an ideal gas with variable specific heats.

Properties From the steam tables (Tables A-4 through 6)

$$\left.\begin{array}{l} P_3 = 12.5\text{MPa} \\ T_3 = 500^\circ\text{C} \end{array}\right\} h_3 = 3341.8\text{kJ/kg}$$

and

$$\left.\begin{array}{l} P_4 = 10\text{kPa} \\ x_4 = 0.92 \end{array}\right\} h_4 = h_f + x_4 h_{fg} = 191.83 + (0.92)(2392.8) = 2393.2\text{kJ/kg}$$

From the air table (Table A-17),

$$T_1 = 295\text{ K} \longrightarrow h_1 = 295.17\text{ kJ/kg}$$

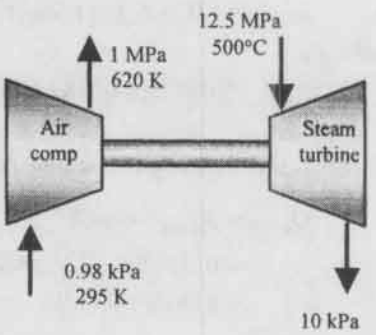
$$T_2 = 620\text{ K} \longrightarrow h_2 = 628.07\text{ kJ/kg}$$

Analysis There is only one inlet and one exit for either device, and thus $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$. We take either the turbine or the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for either steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

For the turbine and the compressor it becomes



Compressor: $\dot{W}_{\text{comp, in}} + \dot{m}_{\text{air}} h_1 = \dot{m}_{\text{air}} h_2 \rightarrow \dot{W}_{\text{comp, in}} = \dot{m}_{\text{air}} (h_2 - h_1)$

Turbine: $\dot{m}_{\text{steam}} h_3 = \dot{W}_{\text{turb, out}} + \dot{m}_{\text{steam}} h_4 \rightarrow \dot{W}_{\text{turb, out}} = \dot{m}_{\text{steam}} (h_3 - h_4)$

Substituting,

$$\dot{W}_{\text{comp,in}} = (10\text{kg/s})(628.07 - 295.17)\text{kJ/kg} = 3329\text{ kW}$$

$$\dot{W}_{\text{turb,out}} = (25\text{kg/s})(3341.8 - 2393.2)\text{kJ/kg} = 23{,}715\text{ kW}$$

Therefore,

$$\dot{W}_{\text{net,out}} = \dot{W}_{\text{turb,out}} - \dot{W}_{\text{comp,in}} = 23{,}715 - 3329 = \mathbf{20{,}386\text{ kW}}$$

5-190 Water is heated from 16°C to 43°C by an electric resistance heater placed in the water pipe as it flows through a showerhead steadily at a rate of 10 L/min. The electric power input to the heater, and the money that will be saved during a 10-min shower by installing a heat exchanger with an effectiveness of 0.50 are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point within the system and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$,. **2** Water is an incompressible substance with constant specific heats. **3** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **4** Heat losses from the pipe are negligible.

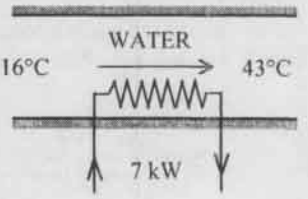
Properties The density and specific heat of water at room temperature are ρ = 1 kg/L and C = 4.18 kJ/kg·°C (Table A-3).

Analysis We take the pipe as the system. This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Then the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0 \quad \rightarrow \quad \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{e,in} = \dot{m}(h_2 - h_1) = \dot{m}[C(T_2 - T_1) + v(P_2 - P_1)^{\nearrow 0}] = \dot{m}C(T_2 - T_1)$$



where

$$\dot{m} = \rho \dot{V} = (1\ \text{kg/L})(10\ \text{L/min}) = 10\ \text{kg/min}$$

Substituting,

$$\dot{W}_{e,in} = (10/60\text{kg/s})(4.18\ \text{kJ/kg.}^\circ\text{C})(43 - 16)^\circ\text{C} = \mathbf{18.8\ kW}$$

The energy recovered by the heat exchanger is

$$\dot{Q}_{\text{saved}} = \varepsilon \dot{Q}_{\max} = \varepsilon \dot{m} C(T_{\max} - T_{\min})$$
$$= 0.5(10/60\text{kg/s})(4.18\ \text{kJ/kg.}^\circ\text{C})(39 - 16)^\circ\text{C}$$
$$= 8.0\text{kJ/s} = 8.0\text{kW}$$

Therefore, 8.0 kW less energy is needed in this case, and the required electric power in this case reduces to

$$\dot{W}_{\text{in,new}} = \dot{W}_{\text{in,old}} - \dot{Q}_{\text{saved}} = 18.8 - 8.0 = \mathbf{10.8\ kW}$$

The money saved during a 10-min shower as a result of installing this heat exchanger is

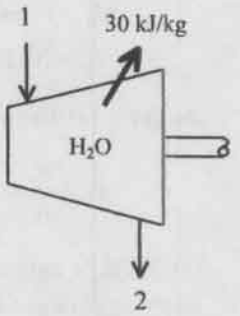
$$(8.0\ \text{kW})(10/60\ \text{h})(8.5\ \text{cents/kWh}) = \mathbf{11.3\ cents}$$

5-191 Steam expands in a turbine steadily. The mass flow rate of the steam, the exit velocity, and the power output are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible.

Properties From the steam tables (Tables A-4 through 6)

$$\left.\begin{array}{l} P_1 = 10\text{MPa} \\ T_1 = 550^\circ\text{C} \end{array}\right\} \begin{array}{l} v_1 = 0.03564\text{m}^3/\text{kg} \\ h_1 = 3500.9\text{kJ/kg} \end{array}$$



and

$$\left.\begin{array}{l} P_2 = 25kPa \\ x_2 = 0.95 \end{array}\right\} \begin{array}{l} v_2 = v_f + x_2 v_{fg} = 0.00102 + (0.95)(6.203) = 5.894\text{m}^3/\text{kg} \\ h_2 = h_f + x_2 h_{fg} = 271.93 + (0.95)(2346.3) = 2393.2\text{kJ/kg} \end{array}$$

Analysis (*a*) The mass flow rate of the steam is

$$\dot{m} = \frac{1}{v_1}\mathbf{V}_1 A_1 = \frac{1}{0.03564\text{m}^3/\text{kg}}(60\text{m/s})(0.015\text{m}^2) = \mathbf{25.3kg/s}$$

(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Then the exit velocity is determined from

$$\dot{m} = \frac{1}{v_2}\mathbf{V}_2 A_2 \quad \longrightarrow \quad \mathbf{V}_2 = \frac{\dot{m}v_2}{A_2} = \frac{(25.3\text{ kg/s})(5.894\text{ m}^3/\text{kg})}{0.14\text{ m}^2} = \mathbf{1065\text{ m/s}}$$

(*c*) We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}}^{\nearrow 0\ \text{(steady)}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{W}_{\text{out}} + \dot{Q}_{\text{out}} + \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \Delta\text{pe} \cong 0)$$

$$\dot{W}_{\text{out}} = -\dot{Q}_{\text{out}} - \dot{m}\left(h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right)$$

Then the power output of the turbine is determined by substituting to be

$$\dot{W}_{\text{out}} = -(25.3\times 30)\text{kJ/s} - (25.3\text{kg/s})\left(2500.9 - 3500.9 + \frac{(1065\text{m/s})^2 - (60\text{m/s})^2}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right)\right)$$

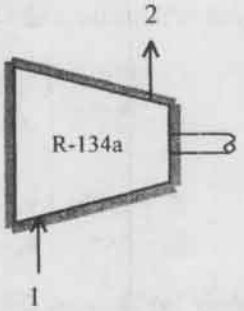
$$= \mathbf{10{,}240kW}$$

5-192E Refrigerant-134a is compressed steadily by a compressor. The mass flow rate of the refrigerant and the exit temperature are to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Properties From the refrigerant tables (Tables A-11E through A-13E)

$$\left.\begin{aligned} P_1 &= 15 \text{ psia} \\ T_1 &= 20^\circ\text{F} \end{aligned}\right\} \begin{aligned} v_1 &= 3.2468 \text{ ft}^3/\text{lbm} \\ h_1 &= 106.34 \text{ Btu/lbm} \end{aligned}$$



Analysis (*a*) The mass flow rate of refrigerant is

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{10 \text{ ft}^3/\text{s}}{3.2468 \text{ ft}^3/\text{lbm}} = \mathbf{3.08 \text{ lbm/s}}$$

(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1)$$

Substituting,

$$(60 \text{ hp})\left(\frac{0.7068 \text{Btu/s}}{1 \text{hp}}\right) = (3.08 \text{lbm/s})(h_2 - 106.34)\text{Btu/lbm}$$

$$h_2 = 120.11 \text{Btu/lbm}$$

Then the exit temperature becomes

$$\left.\begin{aligned} P_2 &= 120 \text{psia} \\ h_2 &= 120.11 \text{Btu/lbm} \end{aligned}\right\} T_2 = \mathbf{114.6^\circ F}$$

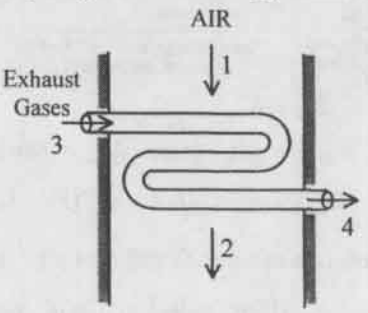
5-193 Air is preheated by the exhaust gases of a gas turbine in a regenerator. For a specified heat transfer rate, the exit temperature of air and the mass flow rate of exhaust gases are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** There are no work interactions. **4** Heat loss from the regenerator to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **5** Exhaust gases can be treated as air. **6** Air is an ideal gas with variable specific heats.

Properties The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1). The enthalpies of air are

$$T_1 = 550 \text{ K} \rightarrow h_1 = 554.71 \text{ kJ/kg}$$
$$T_3 = 800 \text{ K} \rightarrow h_3 = 607.02 \text{ kJ/kg}$$
$$T_4 = 600 \text{ K} \rightarrow h_4 = 821.95 \text{ kJ/kg}$$

Analysis (*a*) We take the *air side* of the heat exchanger as the system, which is a control volume since mass crosses the boundary. There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this steady-flow system can be expressed in the rate form as



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{Q}_{in} + \dot{m}_{\text{air}} h_1 = \dot{m}_{\text{air}} h_2 \quad (\text{since } \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$
$$\dot{Q}_{in} = \dot{m}_{\text{air}} (h_2 - h_1)$$

Substituting,

$$3200\text{kJ/s} = (800/60\text{kg/s})(h_2 - 554.71\text{kJ/kg}) \rightarrow h_2 = 794.74 \text{ kJ/kg}$$

Then from Table A-17 we read $T_2 =$ **775.2 K**

(*b*) Treating the exhaust gases as an ideal gas, the mass flow rate of the exhaust gases is determined from the steady-flow energy relation applied only to the exhaust gases,

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m}_{\text{exhaust}} h_3 = \dot{Q}_{out} + \dot{m}_{\text{exhaust}} h_4 \quad (\text{since } \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$
$$\dot{Q}_{out} = \dot{m}_{\text{exhaust}} (h_3 - h_4)$$

$$3200\text{kJ/s} = \dot{m}_{\text{exhaust}} (821.95 - 607.02)\text{kJ/kg}$$

It yields

$$\dot{m}_{\text{exhaust}} = \mathbf{14.9 \text{ kg/s}}$$

5-194 Water is to be heated steadily from 15°C to 50°C by an electrical resistor inside an insulated pipe. The power rating of the resistance heater and the average velocity of the water are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point within the system and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$. **2** Water is an incompressible substance with constant specific heats. **3** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **4** The pipe is insulated and thus the heat losses are negligible.

Properties The density and specific heat of water at room temperature are ρ = 1000 kg/m^3 and C = 4.18 kJ/kg·°C (Table A-3).

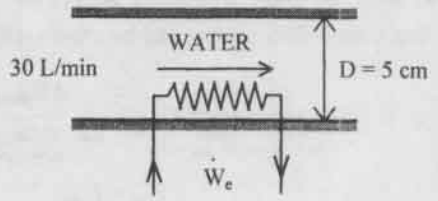
Analysis (*a*) We take the pipe as the system. This is a *control volume* since mass crosses the system boundary during the process. Also, there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q}_{out} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{e,in} = \dot{m}(h_2 - h_1) = \dot{m}[C(T_2 - T_1) + v\Delta P^{\nearrow 0}] = \dot{m}C(T_2 - T_1)$$



The mass flow rate of water through the pipe is

$$\dot{m} = \rho \dot{V}_1 = (1000 \text{ kg/m}^3)(0.030 \text{ m}^3/\text{min}) = 30 \text{ kg/min}$$

Therefore,

$$\dot{W}_{e,in} = \dot{m}C(T_2 - T_1) = (30/60 \text{ kg/s})(4.184 \text{ kJ/kg·°C})(50 - 15)°\text{C} = \mathbf{73.2\ kW}$$

(*b*) The average velocity of water through the pipe is determined from

$$\mathbf{V}_1 = \frac{\dot{V}_1}{A_1} = \frac{\dot{V}}{\pi r^2} = \frac{0.030 \text{ m}^3/\text{min}}{\pi(0.025 \text{ m})^2} = \mathbf{15.3\ m/min}$$

Chapter 6
THE SECOND LAW OF THERMODYNAMICS

Introduction to the Second Law of Thermodynamics and Thermal Energy Reservoirs

6-1C Water is not a fuel; thus the claim is false.

6-2C Transferring 5 kWh of heat to an electric resistance wire and producing 5 kWh of electricity.

6-3C An electric resistance heater which consumes 5 kWh of electricity and supplies 6 kWh of heat to a room.

6-4C Transferring 5 kWh of heat to an electric resistance wire and producing 6 kWh of electricity.

6-5C No. Heat cannot flow from a low-temperature medium to a higher temperature medium.

6-6C The surrounding air in the room which houses the computer.

6-7C A thermal-energy reservoir is a body which can supply or absorb finite quantities of heat isothermally. Some examples are the oceans, the lakes, and the atmosphere.

6-8C Yes. Because the temperature of the oven remains constant no matter how much heat is transferred to the potatoes.

6-9C The surrounding air in the room which houses the TV set.

Heat Engines and Thermal Efficiency

6-10C No. Such an engine violates the Kelvin-Planck statement of the second law of thermodynamics.

6-11C Heat engines are cyclic devices which receive heat from a source, convert some of it to work, and reject the rest to a sink.

6-12C They are $W_{net,\,out} = W_{out} - W_{in}$ and $W_{net,\,out} = Q_{in} - Q_{out}$.

6-13C The efficiency for this baking process can be defined as the ratio of the heat absorbed by the potato (desired output) to the total energy supplied to the oven (required input).

6-14C Method (b). With the heating element in the water, heat losses to the surrounding air are minimized, and thus the desired heating can be achieved with less electrical energy input.

6-15C Using the fluorescent tube. It is well known that a fluorescent tube converts a greater fraction of the electrical energy to light.

6-16C No. Because 100% of the work can be converted to heat.

6-17C It is expressed as "No heat engine can exchange heat with a single reservoir, and produce an equivalent amount of work".

6-18C (a) No, (b) Yes. According to the second law, no heat engine can have and efficiency of 100%.

6-19C No. Such an engine violates the Kelvin-Planck statement of the second law of thermodynamics.

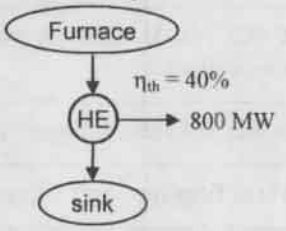
6-20C No. The Kelvin-Plank limitation applies only to heat engines; engines that receive heat and convert some of it to work.

6-21 The power output and thermal efficiency of a power plant are given. The rate of heat rejection is to be determined, and the result is to be compared to the actual case in practice.

Assumptions **1** The plant operates steadily. **2** Heat losses from the working fluid at the pipes and other components are negligible.

Analysis The rate of heat supply to the power plant is determined from the thermal efficiency relation,

$$\dot{Q}_H = \frac{\dot{W}_{net,out}}{\eta_{th}} = \frac{800\text{MW}}{0.4} = 2000\text{MW}$$



The rate of heat transfer to the river water is determined from the first law relation for a heat engine,

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{net,out} = 2000 - 800 = \mathbf{1200MW}$$

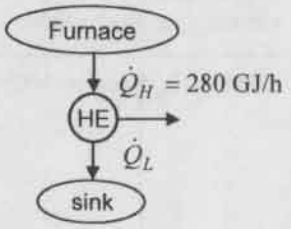
In reality the amount of heat rejected to the river will be **lower** since part of the heat will be lost to the surrounding air from the working fluid as it passes through the pipes and other components.

6-22 The rates of heat supply and heat rejection of a power plant are given. The power output and the thermal efficiency of this power plant are to be determined.

Assumptions **1** The plant operates steadily. **2** Heat losses from the working fluid at the pipes and other components are taken into consideration.

Analysis (*a*) The total heat rejected by this power plant is

$$\dot{Q}_L = 145 + 8 = 153\text{GJ/h}$$



Then the net power output of the plant becomes

$$\dot{W}_{net,out} = \dot{Q}_H - \dot{Q}_L = 280 - 153 = 127\text{GJ/h} = \mathbf{35.3MW}$$

(*b*) The thermal efficiency of the plant is determined from its definition,

$$\eta_{th} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{127\text{GJ/h}}{280\text{GJ/h}} = 0.454 = \mathbf{45.4\%}$$

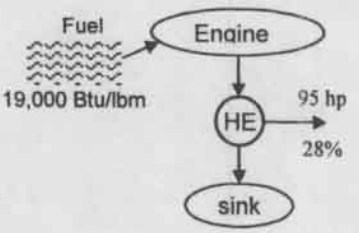
6-23E The power output and thermal efficiency of a car engine are given. The rate of fuel consumption is to be determined.

Assumptions The car operates steadily.

Properties The heating value of the fuel is given to be 19,000 Btu/lbm.

Analysis This car engine is converting 20% of the chemical energy released during the combustion process into work. The amount of energy input required to produce a power output of 90 hp is determined from the definition of thermal efficiency to be

$$\dot{Q}_H = \frac{\dot{W}_{net,out}}{\eta_{th}} = \frac{95\text{hp}}{0.28}\left(\frac{2545\text{Btu/h}}{1\text{hp}}\right) = 863{,}482\text{Btu/h}$$



To supply energy at this rate, the engine must burn fuel at a rate of

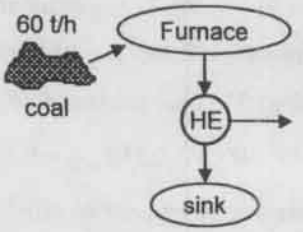
$$\dot{m} = \frac{863{,}482\text{Btu/h}}{19{,}000\text{Btu/lbm}} = \mathbf{45.4lbm/h}$$

since 19,000 Btu of thermal energy is released for each lbm of fuel burned.

6-24 The power output and fuel consumption rate of a power plant are given. The thermal efficiency is to be determined.

Assumptions The plant operates steadily.

Properties The heating value of coal is given to be 30,000 kJ/kg.



Analysis The rate of heat supply to this power plant is

$$\dot{Q}_H = \dot{m}_{coal} u_{coal} = (60{,}000\text{kg/h})(30{,}000\text{kJ/kg}) = 1.8\times10^9\,\text{kJ/h} = 500\text{MW}$$

Then the thermal efficiency of the plant becomes

$$\eta_{th} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{150\text{MW}}{500\text{MW}} = 0.300 = \mathbf{30.0\%}$$

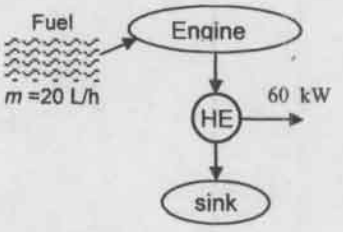
6-25 The power output and fuel consumption rate of a car engine are given. The thermal efficiency of the engine is to be determined.

Assumptions The car operates steadily.

Properties The heating value of the fuel is given to be 44,000 kJ/kg.

Analysis The mass consumption rate of the fuel is

$$\dot{m}_{fuel} = (\rho\dot{V})_{fuel} = (0.8\text{kg/L})(20\text{L/h}) = 16\text{kg/h}$$



The rate of heat supply to the car is

$$\dot{Q}_H = \dot{m}_{coal} u_{coal} = (16\text{kg/h})(44{,}000\text{kJ/kg}) = 704{,}000\text{kJ/h} = 195.6\text{kW}$$

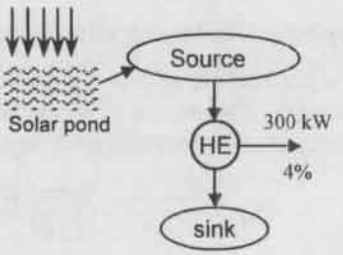
Then the thermal efficiency of the car becomes

$$\eta_{th} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{60\text{kW}}{195.6\text{kW}} = 0.307 = \mathbf{30.7\%}$$

6-26E The power output and thermal efficiency of a solar pond power plant are given. The rate of solar energy collection is to be determined.

Assumptions The plant operates steadily.

Analysis The rate of solar energy collection or the rate of heat supply to the power plant is determined from the thermal efficiency relation to be



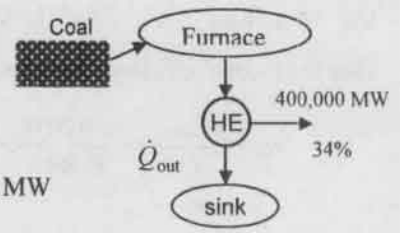
$$\dot{Q}_H = \frac{\dot{W}_{net,out}}{\eta_{th}} = \frac{300\text{kW}}{0.04}\left(\frac{1\text{Btu}}{1.055\text{kJ}}\right)\left(\frac{3600\text{s}}{1\text{h}}\right) = \mathbf{2.56\times10^7\ Btu/h}$$

6-27 The United States produces about 55 percent of its electricity from coal at a conversion efficiency of about 34 percent. The amount of heat rejected by the coal-fired power plants per year is to be determined.

Assumptions Power is generated continuously at an average rate of 400 MW.

Analysis The total amount of power generated by coal power plants is

$$\dot{W}_{\text{coal}} = 0.55\dot{W}_{\text{total}} = 0.55\times(400\text{ MW}) = 220\text{ MW}$$



Noting that the conversion efficiency is 34%, the rate of heat rejection is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{coal}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{coal}}}{\dot{Q}_{\text{out}} + \dot{W}_{\text{coal}}} \rightarrow \dot{Q}_{\text{out}} = \frac{\dot{W}_{\text{coal}}}{\eta_{\text{th}}} - \dot{W}_{\text{coal}} = \frac{220}{0.34} - 220 = 427\text{ MW}$$

Then the amount of heat rejected by the coal plants per year becomes

$$Q_{\text{out}} = \dot{Q}_{\text{out}}(1\text{ year}) = (427{,}000\text{ kJ/s})(365\times24\times3600\text{ s}) = \mathbf{1.35\times10^{13}\ kJ/year}$$

6-28 The projected power needs of the United States is to be met by building inexpensive but inefficient coal plants or by building expensive but efficient IGCC plants. The price of coal that will enable the IGCC plants to recover their cost difference from fuel savings in 5 years is to be determined.

Assumptions **1** Power is generated continuously by either plant at full capacity. **2** The time value of money (interest, inflation, etc.) is not considered.

Properties The heating value of the coal is given to be 28×10^6 kJ/ton.

Analysis For a power generation capacity of 150,000 MW, the construction costs of coal and IGCC plants and their difference are

$$\text{Construction cost}_{\text{coal}} = (150{,}000{,}000\,\text{kW})(\$1300/\text{kW}) = \$195\times10^9$$

$$\text{Construction cost}_{\text{IGCC}} = (150{,}000{,}000\,\text{kW})(\$1500/\text{kW}) = \$225\times10^9$$

$$\text{Construction cost difference} = \$225\times10^9 - \$195\times10^9 = \$30\times10^9$$

The amount of electricity produced by either plant in 5 years is

$$W_e = \dot{W}\Delta t = (150{,}000{,}000\,\text{kW})(5\times365\times24\,\text{h}) = 6.570\times10^{12}\ \text{kWh}$$

The amount of fuel needed to generate a specified amount of power can be determined from

$$\eta = \frac{W_e}{Q_{\text{in}}} \rightarrow Q_{\text{in}} = \frac{W_e}{\eta} \quad \text{or} \quad m_{\text{fuel}} = \frac{Q_{\text{in}}}{\text{Heating value}} = \frac{W_e}{\eta(\text{Heating value})}$$

Then the amount of coal needed to generate this much electricity by each plant and their difference are

$$m_{\text{coal, coal plant}} = \frac{W_e}{\eta(\text{Heating value})} = \frac{6.570\times10^{12}\ \text{kWh}}{(0.34)(28\times10^6\ \text{kJ/ton})}\left(\frac{3600\,\text{kJ}}{1\,\text{kWh}}\right) = 2.484\times10^9\ \text{tons}$$

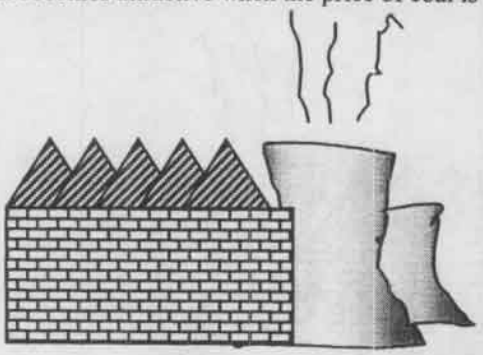
$$m_{\text{coal, IGCC plant}} = \frac{W_e}{\eta(\text{Heating value})} = \frac{6.570\times10^{12}\ \text{kWh}}{(0.45)(28\times10^6\ \text{kJ/ton})}\left(\frac{3600\,\text{kJ}}{1\,\text{kWh}}\right) = 1.877\times10^9\ \text{tons}$$

$$\Delta m_{\text{coal}} = m_{\text{coal, coal plant}} - m_{\text{coal, IGCC plant}} = 2.484\times10^9 - 1.877\times10^9 = 0.607\times10^9\ \text{tons}$$

For Δm_{coal} to pay for the construction cost difference of \$30 billion, the price of coal should be

$$\text{Unit cost of coal} = \frac{\text{Construction cost difference}}{\Delta m_{\text{coal}}} = \frac{\$30\times10^9}{0.607\times10^9\ \text{tons}} = \mathbf{\$49.4/ton}$$

Therefore, the IGCC plant becomes attractive when the price of coal is above \$49.4 per ton.



6-29 The projected power needs of the United States is to be met by building inexpensive but inefficient coal plants or by building expensive but efficient IGCC plants. The price of coal that will enable the IGCC plants to recover their cost difference from fuel savings in 3 years is to be determined.

Assumptions **1** Power is generated continuously by either plant at full capacity. **2** The time value of money (interest, inflation, etc.) is not considered.

Properties The heating value of the coal is given to be 28×10^6 kJ/ton.

Analysis For a power generation capacity of 150,000 MW, the construction costs of coal and IGCC plants and their difference are

$$\text{Construction cost}_{\text{coal}} = (150{,}000{,}000\,\text{kW})(\$1300/\text{kW}) = \$195\times10^9$$

$$\text{Construction cost}_{\text{IGCC}} = (150{,}000{,}000\,\text{kW})(\$1500/\text{kW}) = \$225\times10^9$$

$$\text{Construction cost difference} = \$225\times10^9 - \$195\times10^9 = \$30\times10^9$$

The amount of electricity produced by either plant in 3 years is

$$W_e = \dot{W}\Delta t = (150{,}000{,}000\,\text{kW})(\times365\times24\,\text{h}) = 3.942\times10^{12}\ \text{kWh}$$

The amount of fuel needed to generate a specified amount of power can be determined from

$$\eta = \frac{W_e}{Q_{\text{in}}} \rightarrow Q_{\text{in}} = \frac{W_e}{\eta} \quad \text{or} \quad m_{\text{fuel}} = \frac{Q_{\text{in}}}{\text{Heating value}} = \frac{W_e}{\eta(\text{Heating value})}$$

Then the amount of coal needed to generate this much electricity by each plant and their difference are

$$m_{\text{coal, coal plant}} = \frac{W_e}{\eta(\text{Heating value})} = \frac{3.942\times10^{12}\ \text{kWh}}{(0.34)(28\times10^6\ \text{kJ/ton})}\left(\frac{3600\,\text{kJ}}{1\,\text{kWh}}\right) = 1.491\times10^9\ \text{tons}$$

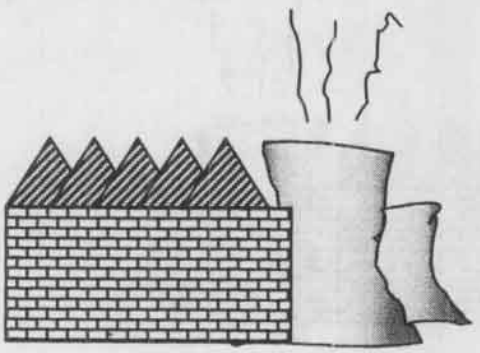
$$m_{\text{coal, IGCC plant}} = \frac{W_e}{\eta(\text{Heating value})} = \frac{3.942\times10^{12}\ \text{kWh}}{(0.45)(28\times10^6\ \text{kJ/ton})}\left(\frac{3600\,\text{kJ}}{1\,\text{kWh}}\right) = 1.126\times10^9\ \text{tons}$$

$$\Delta m_{\text{coal}} = m_{\text{coal, coal plant}} - m_{\text{coal, IGCC plant}} = 1.491\times10^9 - 1.126\times10^9 = 0.365\times10^9\ \text{tons}$$

For Δm_{coal} to pay for the construction cost difference of \$30 billion, the price of coal should be

$$\text{Unit cost of coal} = \frac{\text{Construction cost difference}}{\Delta m_{\text{coal}}} = \frac{\$30\times10^9}{0.365\times10^9\ \text{tons}} = \mathbf{\$82.2/ton}$$

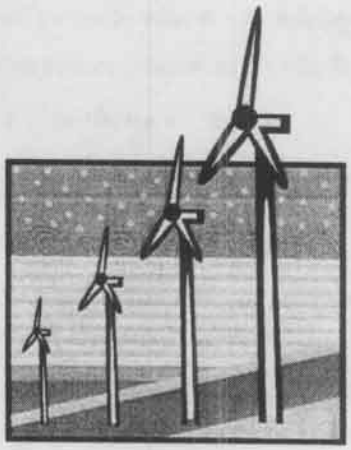
Therefore, the IGCC plant becomes attractive when the price of coal is above \$82.2 per ton.



6-30 A wind turbine is rotating at 20 rpm under steady winds of 30 km/h. The power produced, the tip speed of the blade, and the revenue generated by the wind turbine per year are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The wind turbine operates continuously during the entire year at the specified conditions.

Properties The density of air is given to be $\rho = 1.20\ \text{kg/m}^3$.



Analysis (*a*) The blade span area and the mass flow rate of air through the turbine are

$$A = \pi D^2/4 = \pi(80\ \text{m})^2/4 = 5027\ \text{m}^2$$

$$\mathbf{V} = (30\ \text{km/h})\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 8.333\ \text{m/s}$$

$$\dot{m} = \rho A\mathbf{V} = (1.2\ \text{kg/m}^3)(5027\ \text{m}^2)(8.333\ \text{m/s}) = 50{,}270\ \text{kg/s}$$

Noting that the kinetic energy of a unit mass is $\mathbf{V}^2/2$ and the wind turbine captures 35% of this energy, the power generated by this wind turbine becomes

$$\dot{W} = \eta\left(\frac{1}{2}\dot{m}\mathbf{V}^2\right) = (0.35)\frac{1}{2}(50{,}270\ \text{kg/s})(8.333\ \text{m/s})^2\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{610.9\ kW}$$

(*b*) Noting that the tip of blade travels a distance of πD per revolution, the tip velocity of the turbine blade for an rpm of $\dot{n}$ becomes

$$\mathbf{V}_{\text{tip}} = \pi D\dot{n} = \pi(80\ \text{m})(20/\text{min}) = 5027\ \text{m/min} = 83.8\ \text{m/s} = \mathbf{302\ km/h}$$

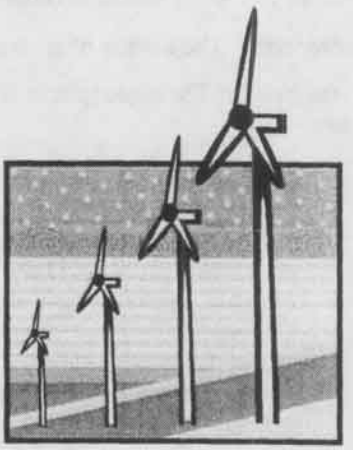
(*c*) The amount of electricity produced and the revenue generated per year are

$$\text{Electricity produced} = \dot{W}\Delta t = (610.9\ \text{kW})(365 \times 24\ \text{h/year})$$
$$= 5.351 \times 10^6\ \text{kWh/year}$$

$$\text{Revenue generated} = (\text{Electricity produced})(\text{Unit price}) = (5.351 \times 10^6\ \text{kWh/year})(\$0.06/\text{kWh})$$
$$= \mathbf{\$321{,}100/year}$$

1 A wind turbine is rotating at 20 rpm under steady winds of 25 km/h. The power produced, the tip ed of the blade, and the revenue generated by the wind turbine per year are to be determined.

sumptions **1** Steady operating conditions exist. **2** The wind turbine operates continuously during the ire year at the specified conditions.



perties The density of air is given to be $\rho = 1.20$ kg/m^3.

alysis (*a*) The blade span area and the mass flow rate of air through the turbine are

$$A = \pi D^2/4 = \pi(80\text{ m})^2/4 = 5027\text{ m}^2$$

$$\mathbf{V} = (25\text{ km/h})\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 6.944\text{ m/s}$$

$$\dot{m} = \rho A\mathbf{V} = (1.2\text{ kg/m}^3)(5027\text{ m}^2)(6.944\text{ m/s}) = 41{,}890\text{ kg/s}$$

ting that the kinetic energy of a unit mass is $\mathbf{V}^2/2$ and the wind turbine captures % of this energy, the power generated by this wind turbine becomes

$$\dot{W} = \eta\left(\frac{1}{2}\dot{m}\mathbf{V}^2\right) = (0.35)\frac{1}{2}(41{,}890\text{ kg/s})(6.944\text{ m/s})^2\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{353.5\text{ kW}}$$

Noting that the tip of blade travels a distance of πD per revolution, the tip velocity of the turbine blade an rpm of $\dot{n}$ becomes

$$\mathbf{V}_{\text{tip}} = \pi D\dot{n} = \pi(80\text{ m})(20/\text{min}) = 5027\text{ m/min} = 83.8\text{ m/s} = \mathbf{302\text{ km/h}}$$

The amount of electricity produced and the revenue generated per year are

$$\text{Electricity produced} = \dot{W}\Delta t = (353.5\text{ kW})(365\times 24\text{ h/year})$$
$$= 3.097\times 10^6\text{ kWh/year}$$

$$\text{Revenue generated} = (\text{Electricity produced})(\text{Unit price}) = (3.097\times 10^6\text{ kWh/year})(\$0.06/\text{kWh})$$
$$= \mathbf{\$185{,}780/year}$$

6-32E An OTEC power plant operates between the temperature limits of 86°F and 41°F. The cooling water experiences a temperature rise of 6°F in the condenser. The amount of power that can be generated by this OTEC plans is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Water is an incompressible substance with constant properties at room temperature

Properties The density and specific heat of water at room temperature $\rho = 62.1$ lbm/ft^3 and $C = 1.00$ Btu/lbm.°F (Table A-3E).

Analysis The mass flow rate of the cooling water is

$$\dot{m}_{\text{water}} = \rho \dot{V}_{\text{water}} = (62.1\,\text{lbm/ft}^3)(13{,}300\,\text{gal/min})\left(\frac{1\,\text{ft}^3}{7.48\,\text{gal}}\right) = 110{,}950\,\text{lbm/min} = 1849\,\text{lbm/s}$$

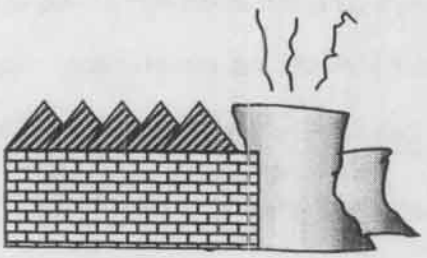
The rate of heat rejection to the cooling water is

$$\dot{Q}_{\text{out}} = \dot{m}_{\text{water}} C(T_{\text{out}} - T_{\text{in}}) = (1849\ \text{lbm/s})(1.0\ \text{Btu/lbm.°F})(6°\text{F}) = 11{,}094\ \text{Btu/s}$$

Noting that the thermal efficiency of this plant is 3%, the power generation is determined to be

$$\eta = \frac{\dot{W}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}}{\dot{W} + \dot{Q}_{\text{out}}} \quad \rightarrow \quad 0.03 = \frac{\dot{W}}{\dot{W} + (11{,}094\ \text{Btu/s})} \quad \rightarrow \quad \dot{W} = 343\ \text{Btu/s} = \mathbf{362\ kW}$$

since 1 kW = 0.9478 Btu/s. Therefore, this OTEC power plant generates 342 kW of power.



Energy Conversion Efficiencies

6-33 A hooded electric open burner and a gas burner are considered. The amount of the electrical energy used directly for cooking and the cost of energy per "utilized" kWh are to be determined. √

Analysis The efficiency of the electric heater is given to be 78 percent. Therefore, a burner that consumes 3-kW of electrical energy will supply

$\eta_{gas} = 38\%$

$\eta_{electric} = 73\%$



$$\dot{Q}_{utilized} = (\text{Energy input}) \times (\text{Efficiency}) = (3\ \text{kW})(0.73) = \mathbf{2.19\ kW}$$

of useful energy. The unit cost of utilized energy is inversely proportional to the efficiency, and is determined from

$$\text{Cost of utilized energy} = \frac{\text{Cost of energy input}}{\text{Efficiency}} = \frac{\$0.07/\text{kWh}}{0.73} = \mathbf{\$0.096/kWh}$$

Noting that the efficiency of a gas burner is 38 percent, the energy input to a gas burner that supplies utilized energy at the same rate (2.19 kW) is

$$\dot{Q}_{input,\,gas} = \frac{\dot{Q}_{utilized}}{\text{Efficiency}} = \frac{2.19\,\text{kW}}{0.38} = \mathbf{5.76\,kW}\ (= 19{,}660\ \text{Btu/h})$$

since 1 kW = 3412 Btu/h. Therefore, a gas burner should have a rating of at least 19,660 Btu/h to perform as well as the electric unit.

Noting that 1 therm = 29.3 kWh, the unit cost of utilized energy in the case of gas burner is determined the same way to be

$$\text{Cost of utilized energy} = \frac{\text{Cost of energy input}}{\text{Efficiency}} = \frac{\$0.60/(29.3\,\text{kWh})}{0.38} = \mathbf{\$0.054/kWh}$$

which is about one-quarter of the unit cost of utilized electricity.

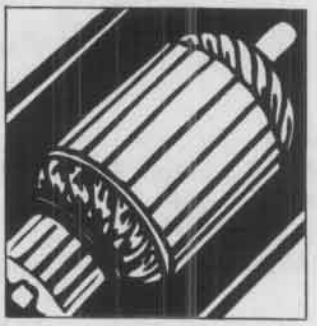
6-34 A worn out standard motor is replaced by a high efficiency one. The reduction in the internal heat gain due to the higher efficiency under full load conditions is to be determined.

Assumptions **1** The motor and the equipment driven by the motor are in the same room. **2** The motor operates at full load so that $f_{load} = 1$.

Analysis The heat generated by a motor is due to its inefficiency, and the difference between the heat generated by two motors that deliver the same shaft power is simply the difference between the electric power drawn by the motors,

$$\dot{W}_{\text{in, electric, standard}} = \dot{W}_{\text{shaft}} / \eta_{\text{motor}} = (75 \times 746 \text{ W})/0.91 = 61{,}484 \text{ W}$$

$$\dot{W}_{\text{in, electric, efficient}} = \dot{W}_{\text{shaft}} / \eta_{\text{motor}} = (75 \times 746 \text{ W})/0.954 = 58{,}648 \text{ W}$$



Then the reduction in heat generation becomes

$$\dot{Q}_{\text{reduction}} = \dot{W}_{\text{in, electric, standard}} - \dot{W}_{\text{in, electric, efficient}} = 61{,}484 - 58{,}648 = \mathbf{2836 \text{ W}}$$

6-35 An electric car is powered by an electric motor mounted in the engine compartment. The rate of heat supply by the motor to the engine compartment at full load conditions is to be determined.

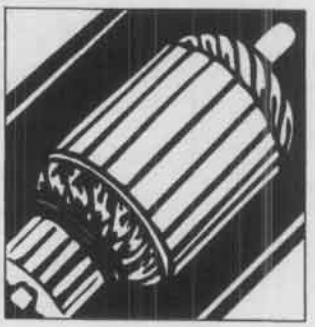
Assumptions The motor operates at full load so that the load factor is 1.

Analysis The heat generated by a motor is due to its inefficiency, and is equal to the difference between the electrical energy it consumes and the shaft power it delivers,

$$\dot{W}_{\text{in, electric}} = \dot{W}_{\text{shaft}} / \eta_{\text{motor}} = (60 \text{ hp})/0.88 = 68.18 \text{ hp}$$

$$\dot{Q}_{\text{generation}} = \dot{W}_{\text{in, electric}} - \dot{W}_{\text{shaft out}} = 68.18 - 60 = 8.18 \text{ hp} = \mathbf{6.10 \text{ kW}}$$

since 1 hp = 0.746 kW.



Discussion The motor will supply as much heat to the compartment as a 6.1 kW resistance heater.

6-36 A worn out standard motor is to be replaced by a high efficiency one. The amount of electrical energy and money savings as a result of installing the high efficiency motor instead of the standard one as well as the simple payback period are to be determined.

Assumptions The load factor of the motor remains constant at 0.75.

Analysis The electric power drawn by each motor and their difference can be expressed as

$$\dot{W}_{\text{electric in, standard}} = \dot{W}_{\text{shaft}} / \eta_{\text{standard}} = (\text{Power rating})(\text{Load factor}) / \eta_{\text{standard}}$$

$$\dot{W}_{\text{electric in, efficient}} = \dot{W}_{\text{shaft}} / \eta_{\text{efficient}} = (\text{Power rating})(\text{Load factor}) / \eta_{\text{efficient}}$$

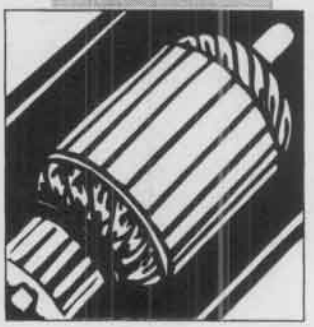
$$\text{Power savings} = \dot{W}_{\text{electric in, standard}} - \dot{W}_{\text{electric in, efficient}}$$

$$= (\text{Power rating})(\text{Load factor})[1/\eta_{\text{standard}} - 1/\eta_{\text{efficient}}]$$

where η_{standard} is the efficiency of the standard motor, and $\eta_{\text{efficient}}$ is the efficiency of the comparable high efficiency motor. Then the annual energy and cost savings associated with the installation of the high efficiency motor are determined to be

$\eta_{\text{old}} = 91.0\%$

$\eta_{\text{new}} = 95.4\%$



Energy Savings = (Power savings)(Operating Hours)

= (Power Rating)(Operating Hours)(Load Factor)($1/\eta_{\text{standard}}$- $1/\eta_{\text{efficient}}$)

= (75 hp)(0.746 kW/hp)(4,368 hours/year)(0.75)(1/0.91 - 1/0.954)

= **9,290 kWh/year**

Cost Savings = (Energy savings)(Unit cost of energy)

= (9,290 kWh/year)($0.08/kWh)

= **$743/year**

The implementation cost of this measure consists of the excess cost the high efficiency motor over the standard one. That is,

Implementation Cost = Cost differential = $5,520 - $5,449 = $71

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$71}{\$743/\text{year}} = \mathbf{0.096\ year}\ (\text{or } 1.1 \text{ months})$$

Therefore, the high-efficiency motor will pay for its cost differential in about one month.

6-37E The combustion efficiency of a furnace is raised from 0.7 to 0.8 by tuning it up. The annual energy and cost savings as a result of tuning up the boiler are to be determined. √

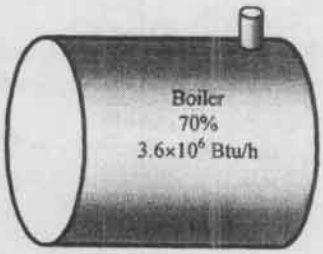
Assumptions The boiler operates at full load while operating.

Analysis The heat output of boiler is related to the fuel energy input to the boiler by

$$\text{Boiler output} = (\text{Boiler input})(\text{Combustion efficiency}) \quad \text{or} \quad \dot{Q}_{\text{out}} = \dot{Q}_{\text{in}}\eta_{\text{furnace}}$$

The current rate of heat input to the boiler is given to be $\dot{Q}_{\text{in, current}} = 3.6\times10^6$ Btu/h.
Then the rate of useful heat output of the boiler becomes

$$\dot{Q}_{\text{out}} = (\dot{Q}_{\text{in}}\eta_{\text{furnace}})_{\text{current}} = (3.6\times10^6 \text{ Btu/h})(0.7) = 2.52\times10^6 \text{ Btu/h}$$



The boiler must supply useful heat at the same rate after the tune up. Therefore, the rate of heat input to the boiler after the tune up and the rate of energy savings become

$$\dot{Q}_{\text{in, new}} = \dot{Q}_{\text{out}} / \eta_{\text{furnace, new}} = (2.52\times10^6 \text{ Btu/h})/0.8 = 3.15\times10^6 \text{ Btu/h}$$

$$\dot{Q}_{\text{in, saved}} = \dot{Q}_{\text{in, current}} - \dot{Q}_{\text{in, new}} = 3.6\times10^6 - 3.15\times10^6 = 0.45\times10^6 \text{ Btu/h}$$

Then the annual energy and cost savings associated with tuning up the boiler become

$$\text{Energy Savings} = \dot{Q}_{\text{in, saved}}(\text{Operation hours})$$
$$= (0.45\times10^6 \text{ Btu/h})(1500 \text{ h/year}) = \mathbf{675\times10^6 \text{ Btu/yr}}$$

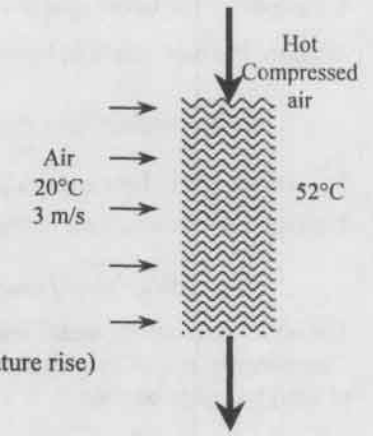
$$\text{Cost Savings} = (\text{Energy Savings})(\text{Unit cost of energy})$$
$$= (675\times10^6 \text{ Btu/yr})(\$4.35 \text{ per } 10^6 \text{ Btu}) = \mathbf{\$2936/year}$$

Discussion Notice that tuning up the boiler will save $2936 a year, which is a significant amount. The implementation cost of this measure is negligible if the adjustment can be made by in-house personnel. Otherwise it is worthwhile to have an authorized representative of the boiler manufacturer to service the boiler twice a year.

6-38 The gas space heating of a facility is to be supplemented by air heated in a liquid-to-air heat exchanger of a compressor. The amount of money that will be saved by diverting the compressor waste heat into the facility during the heating season is to be determined.

Assumptions The atmospheric pressure at that location is 1 atm.

Analysis The mass flow rate of air through the liquid-to-air heat exchanger is



$$\begin{aligned}\text{Mass flow rate of air} &= (\text{Density of air})(\text{Average velocity})(\text{Flow area})\\ &= (1.21\ \text{kg/m}^3)(3\ \text{m/s})(1.0\ \text{m}^2)\\ &= 3.63\ \text{kg/s} = 13{,}068\ \text{kg/h}\end{aligned}$$

Noting that the exit temperature of air is 52°C, the rate at which heat can be recovered (or the rate at which heat is transferred to air) is

$$\begin{aligned}\text{Rate of Heat Recovery} &= (\text{Mass flow rate of air})(\text{Specific heat of air})(\text{Temperature rise})\\ &= (13{,}068\ \text{kg/h})(1.0\ \text{kJ/kg.°C})(52-20)°\text{C}\\ &= 418{,}176\ \text{kJ/h}\end{aligned}$$

The number of operating hours of this compressor during the heating season is

$$\text{Operating hours} = (20\ \text{hours/day})(5\ \text{days/week})(26\ \text{weeks/year}) = 2600\ \text{hours/year}$$

Then the annual energy and cost savings become

$$\begin{aligned}\text{Energy Savings} &= (\text{Rate of Heat Recovery})(\text{Annual Operating Hours})/\text{Efficiency}\\ &= (418{,}176\ \text{kJ/h})(2600\ \text{h/year})/0.8\\ &= 1{,}359{,}100{,}000\ \text{kJ/year}\\ &= 12{,}882\ \text{therms/year}\end{aligned}$$

$$\begin{aligned}\text{Cost Savings} &= (\text{Energy savings})(\text{Unit cost of energy saved})\\ &= (12{,}882\ \text{therms/year})(\$0.50/\text{therm})\\ &= \$6441/\text{year}\end{aligned}$$

Discussion Notice that utilizing the waste heat from the compressor will save $6441 per year from the heating costs. The implementation of this measure requires the installation of an ordinary sheet metal duct from the outlet of the heat exchanger into the building. The installation cost associated with this measure is relatively low. Several manufacturing facilities already have this conservation system in place. A damper is used to direct the air into the building in winter and to the ambient in summer. Combined compressor/heat-recovery systems are available in the market for both air-cooled (greater than 50 hp) and water cooled (greater than 125 hp) systems.

6-39 Several people are working out in an exercise room. The rate of heat gain from people and the equipment is to be determined.

Assumptions The average rate of heat dissipated by people in an exercise room is 525 W.

Analysis The 8 weight lifting machines do not have any motors, and thus they do not contribute to the internal heat gain directly. The usage factors of the motors of the treadmills are taken to be unity since they are used constantly during peak periods. Noting that 1 hp = 746 W, the total heat generated by the motors is

$$\dot{Q}_{\text{motors}} = (\text{No. of motors}) \times \dot{W}_{\text{motor}} \times f_{\text{load}} \times f_{\text{usage}} / \eta_{\text{motor}}$$
$$= 4 \times (2.5 \times 746 \text{ W}) \times 0.70 \times 1.0 / 0.77 = 6782 \text{ W}$$



The heat gain from 14 people is

$$\dot{Q}_{\text{people}} = (\text{No. of people}) \times \dot{Q}_{\text{person}} = 14 \times (525 \text{ W}) = 7350 \text{ W}$$

Then the total rate of heat gain of the exercise room during peak period becomes

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{motors}} + \dot{Q}_{\text{people}} = 6782 + 7350 = \mathbf{14{,}132 \text{ W}}$$

6-40 A classroom has 40 students, one instructor, and 18 fluorescent light bulbs. The rate of internal heat generation in this classroom is to be determined.

Assumptions **1** There is a mix of men, women, and children in the classroom. **2** The amount of light (and thus energy) leaving the room through the windows is negligible.

Properties The average rate of heat generation from people seated in a room/office is 115 W (Table 12-8).

Analysis The amount of heat dissipated by the lamps is equal to the amount of electrical energy consumed by the lamps, including the 10% additional electricity consumed by the ballasts. Therefore,

$$\dot{Q}_{\text{lighting}} = (\text{Energy consumed per lamp}) \times (\text{No. of lamps})$$
$$= (40\ \text{W})(1.1)(18) = 792\ \text{W}$$
$$\dot{Q}_{\text{people}} = (\text{No. of people}) \times \dot{Q}_{\text{person}} = 41 \times (100\ \text{W}) = 4100\ \text{W}$$



Then the total rate of heat gain (or the internal heat load) of the classroom from the lights and people become

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{lighting}} + \dot{Q}_{\text{people}} = 792 + 4100 = \mathbf{4892\ W}$$

6-41 A room is cooled by circulating chilled water through a heat exchanger, and the air is circulated through the heat exchanger by a fan. The contribution of the fan-motor assembly to the cooling load of the room is to be determined.

Assumptions The fan motor operates at full load so that $f_{\text{load}} = 1$.

Analysis The entire electrical energy consumed by the motor, including the shaft power delivered to the fan, is eventually dissipated as heat. Therefore, the contribution of the fan-motor assembly to the cooling load of the room is equal to the electrical energy it consumes,



$$\dot{Q}_{\text{internal generation}} = \dot{W}_{\text{in, electric}} = \dot{W}_{\text{shaft}} / \eta_{\text{motor}}$$
$$= (0.25\ \text{hp})/0.54 = 0.463\ \text{hp} = \mathbf{345\ W}$$

since 1 hp = 746 W.

Refrigerators and Heat Pumps

6-42C The difference between the two devices is one of purpose. The purpose of a refrigerator is to remove heat from a cold medium whereas the purpose of a heat pump is to supply heat to a warm medium.

6-43C The difference between the two devices is one of purpose. The purpose of a refrigerator is to remove heat from a refrigerated space whereas the purpose of an air-conditioner is remove heat from a living space.

6-44C No. Because the refrigerator consumes work to accomplish this task.

6-45C No. Because the heat pump consumes work to accomplish this task.

6-46C The coefficient of performance of a refrigerator represents the amount of heat removed from the refrigerated space for each unit of work supplied. It can be greater than unity.

6-47C The coefficient of performance of a heat pump represents the amount of heat supplied to the heated space for each unit of work supplied. It can be greater than unity.

6-48C No. The heat pump captures energy from a cold medium and carries it to a warm medium. It does not create it.

6-49C No. The refrigerator captures energy from a cold medium and carries it to a warm medium. It does not create it.

6-50C No device can transfer heat from a cold medium to a warm medium without requiring a heat or work input from the surroundings.

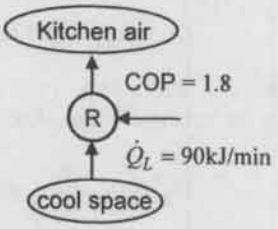
6-51C The violation of one statement leads to the violation of the other one, as shown in Sec. 6-4, and thus we conclude that the two statements are equivalent.

6-52 The COP and the refrigeration rate of a refrigerator are given. The power consumption and the rate of heat rejection are to be determined.

Assumptions The refrigerator operates steadily.

Analysis (*a*) Using the definition of the coefficient of performance, the power input to the refrigerator is determined to be

$$\dot{W}_{net,in} = \frac{\dot{Q}_L}{COP_R} = \frac{90\ \text{kJ/min}}{1.8} = 50\ \text{kJ/min} = \mathbf{0.83\ kW}$$



(*b*) The heat transfer rate to the kitchen air is determined from the energy balance,

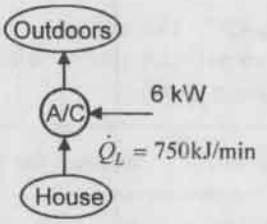
$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{net,in} = 90 + 50 = \mathbf{140\ kJ/min}$$

6-53 The power consumption and the cooling rate of an air conditioner are given. The COP and the rate of heat rejection are to be determined.

Assumptions The air conditioner operates steadily.

Analysis (*a*) The coefficient of performance of the air-conditioner (or refrigerator) is determined from its definition,

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{750\text{kJ/min}}{6\text{kW}}\left(\frac{1\text{kW}}{60\text{kJ/min}}\right) = \mathbf{2.08}$$



(*b*) The rate of heat discharge to the outside air is determined from the energy balance,

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{net,in} = (750\text{kJ/min}) + (6)(60\text{kJ/min}) = \mathbf{1110kJ/min}$$

6-54 The COP and the refrigeration rate of a refrigerator are given. The power consumption of the refrigerator is to be determined.

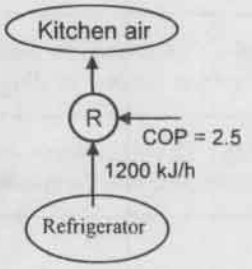
Assumptions The refrigerator operates steadily.

Analysis Since the refrigerator runs one-fourth of the time and removes heat from the food compartment at an average rate of 1200 kJ/h, the refrigerator removes heat at a rate of

$$\dot{Q}_L = (4)(1200\text{kJ/h}) = 4800\text{kJ/h}$$

when running. Thus the power the refrigerator draws when it is running is

$$\dot{W}_{net,in} = \frac{\dot{Q}_L}{COP_R} = \frac{4800\text{ kJ/h}}{2.5} = 1920\text{ kJ/h} = \mathbf{0.533\ kW}$$



6-55E The COP and the refrigeration rate of an ice machine are given. The power consumption is to be determined.

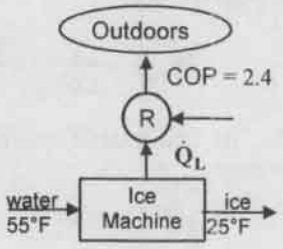
Assumptions The ice machine operates steadily.

Analysis The cooling load of this ice machine is

$$\dot{Q}_L = \dot{m}q_L = (20\text{lbm/h})(169\text{Btu/lbm}) = 3380\text{Btu/h}$$

Using the definition of the coefficient of performance, the power input to the ice machine system is determined to be

$$\dot{W}_{net,in} = \frac{\dot{Q}_L}{COP_R} = \frac{3380\text{Btu/h}}{2.4}\left(\frac{1\text{hp}}{2545\text{Btu/h}}\right) = \mathbf{0.553hp}$$



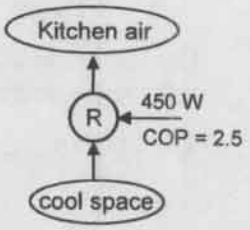
6-56 The COP and the power consumption of a refrigerator are given. The time it will take to cool 5 watermelons is to be determined.

Assumptions **1** The refrigerator operates steadily. **2** The heat gain of the refrigerator through its walls, door, etc. is negligible. **3** The watermelons are the only items in the refrigerator to be cooled.

Properties The specific heat of watermelons is given to be $C = 4.2$ kJ/kg.°C.

Analysis The total amount of heat that needs to be removed from the watermelons is

$$Q_L = (mC\Delta T)_{watermelons} = 5 \times (10\text{kg})(4.2\text{kJ/kg}\cdot^\circ\text{C})(20-8)^\circ\text{C} = 2520\text{kJ}$$



The rate at which this refrigerator removes heat is

$$\dot{Q}_L = (COP_R)(\dot{W}_{net,in}) = (2.5)(0.45\text{kW}) = 1.125\text{kW}$$

That is, this refrigerator can remove 1.125 kJ of heat per second. Thus the time required to remove 2520 kJ of heat is

$$\Delta t = \frac{Q_L}{\dot{Q}_L} = \frac{2520\text{ kJ}}{1.125\text{ kJ/s}} = 2240\text{ s} = \mathbf{37.3\ min}$$

This answer is optimistic since the refrigerated space will gain some heat during this process from the surrounding air, which will increase the work load. Thus, in reality, it will take longer to cool the watermelons.

6-57 An air conditioner with a known COP cools a house to desired temperature in 15 min. The power consumption of the air conditioner is to be determined.

Assumptions **1** The air conditioner operates steadily. **2** The house is well-sealed so that no air leaks in or out during cooling. **3** Air is an ideal gas with constant specific heats at room temperature.

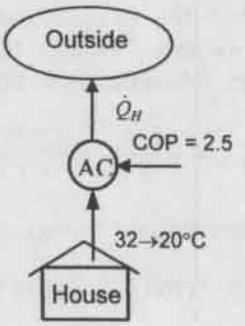
Properties The constant volume specific heat of air is given to be $C_v = 0.72$ kJ/kg.°C.

Analysis Since the house is well-sealed (constant volume), the total amount of heat that needs to be removed from the house is

$$Q_L = (mC_v\Delta T)_{House} = (800\text{kg})(0.72\text{kJ/kg}\cdot^\circ\text{C})(32-20)^\circ\text{C} = 6912\text{kJ}$$

This heat is removed in 15 minutes. Thus the average rate of heat removal from the house is

$$\dot{Q}_L = \frac{Q_L}{\Delta t} = \frac{6912\text{ kJ}}{15 \times 60\text{ s}} = 7.68\text{ kW}$$



Using the definition of the coefficient of performance, the power input to the air-conditioner is determined to be

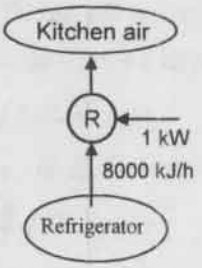
$$\dot{W}_{net,in} = \frac{\dot{Q}_L}{COP_R} = \frac{7.68\text{ kW}}{2.5} = \mathbf{3.07\ kW}$$

6-58 The heat removal rate of a refrigerator per kW of power it consumes is given. The COP and the rate of heat rejection are to be determined.

Assumptions The refrigerator operates steadily.

Analysis The coefficient of performance of the refrigerator is determined from its definition,

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{8000\,\text{kJ/h}}{1\,\text{kW}}\left(\frac{1\,\text{kW}}{3600\,\text{kJ/h}}\right) = \mathbf{2.22}$$



The rate of heat rejection to the surrounding air, per kW of power consumed, is determined from the energy balance,

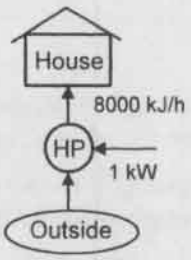
$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{net,in} = (8{,}000\,\text{kJ/h}) + (1)(3600\,\text{kJ/h}) = \mathbf{11{,}600\,kJ/h}$$

6-59 The rate of heat supply of a heat pump per kW of power it consumes is given. The COP and the rate of heat absorption from the cold environment are to be determined.

Assumptions The heat pump operates steadily.

Analysis The coefficient of performance of the refrigerator is determined from its definition,

$$COP_{HP} = \frac{\dot{Q}_{HP}}{\dot{W}_{net,in}} = \frac{8000\,\text{kJ/h}}{1\,\text{kW}}\left(\frac{1\,\text{kW}}{3600\,\text{kJ/h}}\right) = \mathbf{2.22}$$



The rate of heat absorption from the surrounding air, per kW of power consumed, is determined from the energy balance,

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{net,in} = (8{,}000\,\text{kJ/h}) - (1)(3600\,\text{kJ/h}) = 4400\,\mathbf{kJ/h}$$

6-60 A house is heated by resistance heaters, and the amount of electricity consumed during a winter month is given. The amount of money that would be saved if this house were heated by a heat pump with a knowm COP is to be determined.

Assumptions The heat pump operates steadily.

Analysis The amount of heat the resistance heaters supply to the house is equal to he amount of electricity they consume. Therefore, to achieve the same heating effect, the house must be supplied with 1200 kWh of energy. A heat pump that supplied this much heat will consume electrical power in the amount of

$$\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{1200\,\text{kWh}}{2.4} = 500\ \text{kWh}$$

which represent a savings of 1200 – 500 = 700 kWh. Thus the homeowner would have saved

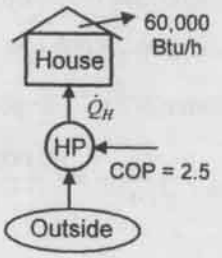
$$(700\ \text{kWh})(0.085\ \$/\text{kWh}) = \mathbf{\$59.50}$$

6-61E The rate of heat supply and the COP of a heat pump are given. The power consumption and the rate of heat absorption from the outside air are to be determined.

Assumptions The heat pump operates steadily.

Analysis (a) The power consumed by this heat pump can be determined from the definition of the coefficient of performance of a heat pump to be

$$\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{60{,}000 \text{ Btu/h}}{2.5} = 24{,}000 \text{ Btu/h} = \mathbf{9.43\ hp}$$



(*b*) The rate of heat transfer from the outdoor air is determined from the conservation of energy principle,

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{net,in} = (60{,}000 - 24{,}000 \text{Btu/h}) = \mathbf{36{,}000\ Btu/h}$$

6-62 The rate of heat loss from a house and the COP of the heat pump are given. The power consumption of the heat pump when it is running is to be determined.

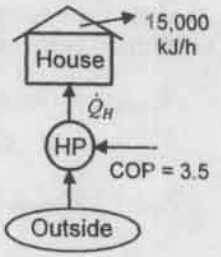
Assumptions The heat pump operates one-third of the time.

Analysis Since the heat pump runs one-third of the time and must supply heat to the house at an average rate of 15,000 kJ/h, the heat pump supplies heat at a rate of

$$\dot{Q}_H = (3)(15{,}000 \text{kJ/h}) = 45{,}000 \text{kJ/h}$$

when running. Thus the power the heat pump draws when it is running is

$$\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{45{,}000 \text{kJ/h}}{3.5}\left(\frac{1 \text{kW}}{3600 \text{kJ/h}}\right) = \mathbf{3.57 kW}$$

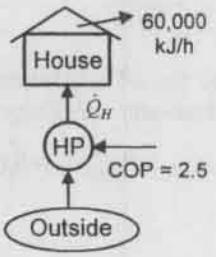


6-63 The rate of heat loss, the rate of internal heat gain, and the COP of a heat pump are given. The power input to the heat pump is to be determined.

Assumptions The heat pump operates steadily.

Analysis The heating load of this heat pump system is the difference between the heat lost to the outdoors and the heat generated in the house from the people, lights, and appliances,

$$\dot{Q}_H = 60{,}000 - 4{,}000 = 56{,}000 \text{ kJ/h}$$



Using the definition of COP, the power input to the heat pump is determined to be

$$\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{56{,}000\text{kJ/h}}{2.5}\left(\frac{1\text{kW}}{3600\text{kJ/h}}\right) = \mathbf{6.22kW}$$

6-64 An office that is being cooled adequately by a 12,000 Btu/h window air-conditioner is converted to a computer room. The number of additional air-conditioners that need to be installed is to be determined.

Assumptions **1** The computer are operated by 4 adult men. **2** The computers consume 40 percent of their rated power at any given time.

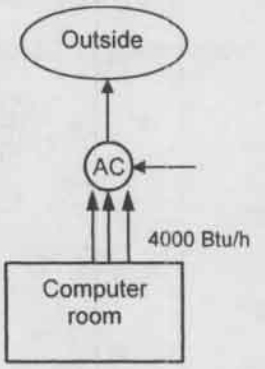
Properties The average rate of heat generation from a person seated in a room/office is 100 W (given).

Analysis The amount of heat dissipated by the computers is equal to the amount of electrical energy they consume. Therefore,

$$\dot{Q}_{\text{computers}} = (\text{Rated power}) \times (\text{Usage factor}) = (3.5 \text{ kW})(0.4) = 1.4 \text{ kW}$$

$$\dot{Q}_{\text{people}} = (\text{No. of people}) \times \dot{Q}_{\text{person}} = 4 \times (100 \text{ W}) = 400 \text{ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{computers}} + \dot{Q}_{\text{people}} = 1400 + 400 = 1800 \text{ W} = 6142 \text{ Btu/h}$$



since 1 W = 3.412 Btu/h. Then noting that each available air conditioner provides 4,000 Btu/h cooling, the number of air-conditioners needed becomes

$$\text{No. of air conditioners} = \frac{\text{Cooling load}}{\text{Cooling capacity of A/C}} = \frac{6142 \text{ Btu/h}}{4000 \text{ Btu/h}}$$

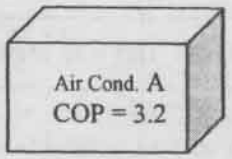
$$= 1.5 \approx \mathbf{2 \text{ Air conditioners}}$$

6-65 A decision is to be made between a cheaper but inefficient air-conditioner and an expensive but efficient air-conditioner for a building. The better buy is to be determined.

Assumptions The two air conditioners are comparable in all aspects other than the initial cost and the efficiency.

Analysis The unit that will cost less during its lifetime is a better buy. The total cost of a system during its lifetime (the initial, operation, maintenance, etc.) can be determined by performing a life cycle cost analysis. A simpler alternative is to determine the simple payback period. The energy and cost savings of the more efficient air conditioner in this case is

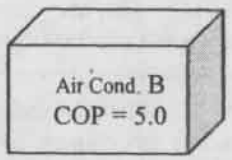
$$\begin{aligned}\text{Energy savings} &= (\text{Annual energy usage of A}) - (\text{Annual energy usage of B})\\ &= (\text{Annual cooling load})(1/\text{COP}_A - 1/\text{COP}_B)\\ &= (120{,}000\ \text{kWh/year})(1/3.2 - 1/5.0)\\ &= 13{,}500\ \text{kWh/year}\end{aligned}$$



$$\begin{aligned}\text{Cost savings} &= (\text{Energy savings})(\text{Unit cost of energy})\\ &= (13{,}500\ \text{kWh/year})(\$0.10/\text{kWh}) = \mathbf{\$1350/year}\end{aligned}$$

The installation cost difference between the two air-conditioners is

$$\text{Cost difference} = \text{Cost of B} - \text{cost of A} = 7000 - 5500 = \$1500$$



Therefore, the more efficient air-conditioner B will pay for the $1500 cost differential in this case in about 1 year.

Discussion A cost conscious consumer will have no difficulty in deciding that the more expensive but more efficient air-conditioner B is clearly the better buy in this case since air conditioners last at least 15 years. But the decision would not be so easy if the unit cost of electricity at that location was much less than $0.10/kWh, or if the annual air-conditioning load of the house was much less than 120,000 kWh.

Perpetual-Motion Machines

6-66C This device creates energy, and thus it is a PMM1.

6-67C This device creates energy, and thus it is a PMM1.

Reversible and Irreversible Processes

6-68C No. Because it involves heat transfer through a finite temperature difference.

6-69C This is an irreversible process since it involves heat transfer through a finite temperature difference.

6-70C Because reversible processes can be approached in reality, and they form the limiting cases. Work producing devices which operate on reversible processes deliver the most work, and work consuming devices which operate on reversible processes consume the least work.

6-71C The irreversible case. Because work consuming devices consume the minimum work when reversible processes are used instead of irreversible processes.

6-72C When the compression process is non-quasiequilibrium, the molecules before the piston face cannot escape fast enough, forming a high pressure region in front of the piston. It takes more work to move the piston against this high pressure region.

6-73C When an expansion process is non-quasiequilibrium, the molecules before the piston face cannot follow the piston fast enough, forming a low pressure region behind the piston. The lower pressure which pushes the piston produces less work.

6-74C The irreversibilities that occur within the system boundaries are **internal** irreversibilities; those which occur outside the system boundaries are **external** irreversibilities.

6-75C A reversible expansion or compression process cannot involve unrestrained expansion or sudden compression, and thus it is quasi-equilibrium. A quasi-equilibrium expansion or compression process, on the other hand, may involve external irreversibilities (such as heat transfer through a finite temperature difference), and thus is not necessarily reversible.

The Carnot Cycle and Carnot's Principle

6-76C The four processes that make up the Carnot cycle are isothermal expansion, reversible adiabatic expansion, isothermal compression, and reversible adiabatic compression.

6-77C They are (1) the thermal efficiency of an irreversible heat engine is lower than the efficiency of a reversible heat engine operating between the same two reservoirs, and (2) the thermal efficiency of all the reversible heat engines operating between the same two reservoirs are equal.

6-78C False. The second Carnot principle states that no heat engine cycle can have a higher thermal efficiency than the Carnot cycle operating between the same temperature limits.

6-79C Yes. The second Carnot principle states that all reversible heat engine cycles operating between the same temperature limits have the thermal efficiency.

6-80C (a) No, (b) No. They would violate the Carnot principle.

Carnot Heat Engines

6-81C No.

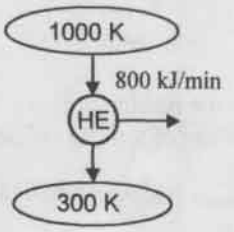
6-82C The one that has a source temperature of 600°C. This is true because the higher the temperature at which heat is supplied to the working fluid of a heat engine, the higher the thermal efficiency.

6-83 The source and sink temperatures of a Carnot heat engine and the rate of heat supply are given. The thermal efficiency and the power output are to be determined.

Assumptions The Carnot heat engine operates steadily.

Analysis (a) The thermal efficiency of a Carnot heat engine depends on the source and the sink temperatures only, and is determined from

$$\eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\ \text{K}}{1000\ \text{K}} = 0.70 \quad \text{or} \quad \mathbf{70\%}$$



(b) The power output of this heat engine is determined from the definition of thermal efficiency,

$$\dot{W}_{net,out} = \eta_{th}\dot{Q}_H = (0.70)(800\text{kJ/min}) = 560\text{kJ/min} = \mathbf{9.33kW}$$

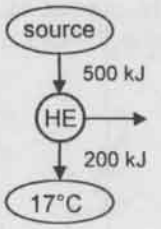
6-84 The sink temperature of a Carnot heat engine and the rates of heat supply and heat rejection are given. The source temperature and the thermal efficiency of the engine are to be determined.

Assumptions The Carnot heat engine operates steadily.

Analysis (a) For reversible cyclic devices we have $\left(\frac{Q_H}{Q_L}\right)_{rev} = \left(\frac{T_H}{T_L}\right)$

Thus the temperature of the source T_H must be

$$T_H = \left(\frac{Q_H}{Q_L}\right)_{rev} T_L = \left(\frac{500\text{kJ}}{200\text{kJ}}\right)(290\text{K}) = \mathbf{725K}$$



(b) The thermal efficiency of a Carnot heat engine depends on the source and the sink temperatures only, and is determined from

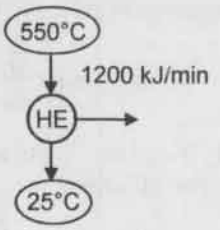
$$\eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{290\text{ K}}{725\text{ K}} = 0.60 \quad \text{or} \quad \mathbf{60\%}$$

6-85 The source and sink temperatures of a heat engine and the rate of heat supply are given. The maximum possible power output of this engine is to be determined.

Assumptions The heat engine operates steadily.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{th,\max} = \eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{298\text{ K}}{823\text{ K}} = 0.638 \quad \text{or} \quad \mathbf{63.8\%}$$



Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

$$\dot{W}_{net,out} = \eta_{th}\dot{Q}_H = (0.638)(1200\text{kJ/min}) = 765.6kJ/\text{min} = \mathbf{12.8kW}$$

6-86E The sink temperature of a Carnot heat engine, the rate of heat rejection, and the thermal efficiency are given. The power output of the engine and the source temperature are to be determined.

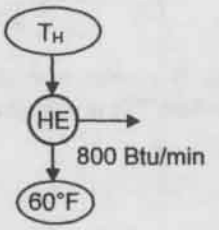
Assumptions The Carnot heat engine operates steadily.

Analysis (a) The rate of heat input to this heat engine is determined from the definition of thermal efficiency,

$$\eta_{th} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} \longrightarrow 0.55 = 1 - \frac{800\text{Btu/min}}{\dot{Q}_H} \longrightarrow \dot{Q}_H = 1777.8\text{Btu/min}$$

Then the power output of this heat engine can be determined from

$$\dot{W}_{net,out} = \eta_{th}\dot{Q}_H = (0.55)(1777.8\text{Btu/min}) = 977.8\text{Btu/min} = \mathbf{23.1hp}$$



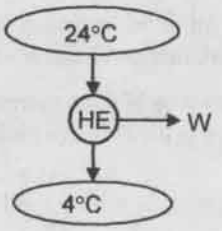
(*b*) For reversible cyclic devices we have $\left(\frac{\dot{Q}_H}{\dot{Q}_L}\right)_{rev} = \left(\frac{T_H}{T_L}\right)$

Thus the temperature of the source T_H must be

$$T_H = \left(\frac{\dot{Q}_H}{\dot{Q}_L}\right)_{rev} T_L = \left(\frac{1777.8\text{Btu/min}}{800\text{Btu/min}}\right)(520\text{R}) = \mathbf{1155.6R}$$

6-87 The source and sink temperatures of a OTEC (Ocean Thermal Energy Conversion) power plant are given. The maximum thermal efficiency is to be determined.

Assumptions The power plant operates steadily.

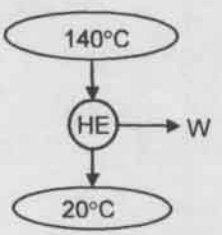


Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{th,\max} = \eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{277\text{K}}{297\text{K}} = 0.067 \text{ or } \mathbf{6.7\%}$$

6-88 The source and sink temperatures of a geothermal power plant are given. The maximum thermal efficiency is to be determined. √

Assumptions The power plant operates steadily.



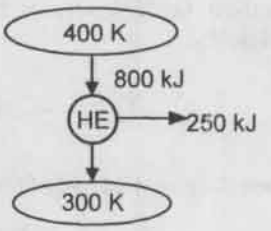
Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{th,\max} = \eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{20 + 273\text{ K}}{140 + 273\text{ K}} = 0.291 \text{ or } \mathbf{29.1\%}$$

6-89 An inventor claims to have developed a heat engine. The inventor reports temperature, heat transfer, and work output measurements. The claim is to be evaluated.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{th,\max} = \eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\ \text{K}}{400\ \text{K}} = 0.25 \quad or \quad \mathbf{25\%}$$



The actual thermal efficiency of the heat engine in question is

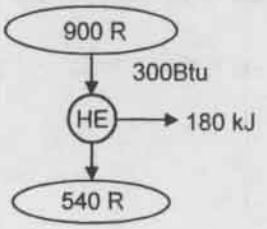
$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{250\ \text{kJ}}{800\ \text{kJ}} = 0.313 \quad \text{or} \quad \mathbf{31.3\%}$$

which is greater than the maximum possible thermal efficiency. Therefore, this heat engine is a PMM2 and the claim is **false**.

6-90E An inventor claims to have developed a heat engine. The inventor reports temperature, heat transfer, and work output measurements. The claim is to be evaluated.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{th,\max} = \eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{540\ \text{R}}{900\ \text{R}} = 0.40 \quad or \quad \mathbf{40\%}$$



The actual thermal efficiency of the heat engine in question is

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{160\ \text{Btu}}{300\ \text{Btu}} = 0.533 \quad \text{or} \quad \mathbf{53.3\%}$$

which is greater than the maximum possible thermal efficiency. Therefore, this heat engine is a PMM2 and the claim is **false**.

Carnot Refrigerators and Heat Pumps

6-91C By increasing T_L or by decreasing T_H.

6-92C It is the COP that a Carnot refrigerator would have, $COP_R = \dfrac{1}{T_H / T_L - 1}$.

6-93C No. At best (when everything is reversible), the increase in the work produced will be equal to the work consumed by the refrigerator. In reality, the work consumed by the refrigerator will always be greater than the additional work produced, resulting in a decrease in the thermal efficiency of the power plant.

6-94C No. At best (when everything is reversible), the increase in the work produced will be equal to the work consumed by the refrigerator. In reality, the work consumed by the refrigerator will always be greater than the additional work produced, resulting in a decrease in the thermal efficiency of the power plant.

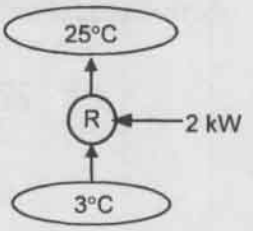
6-95C Bad idea. At best (when everything is reversible), the increase in the work produced will be equal to the work consumed by the heat pump. In reality, the work consumed by the heat pump will always be greater than the additional work produced, resulting in a decrease in the thermal efficiency of the power plant.

6-96 The refrigerated space and the environment temperatures of a Carnot refrigerator and the power consumption are given. The rate of heat removal from the refrigerated space is to be determined. √

Assumptions The Carnot refrigerator operates steadily.

Analysis The coefficient of performance of a Carnot refrigerator depends on the temperature limits in the cycle only, and is determined from

$$COP_{R,C} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(25 + 273\text{K})/(3 + 273\text{K}) - 1} = 12.5$$



The rate of heat removal from the refrigerated space is determined from the definition of the coefficient of performance of a refrigerator,

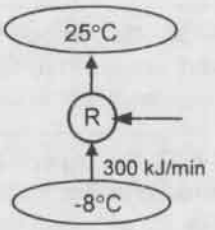
$$\dot{Q}_L = COP_R \times \dot{W}_{net,in} = (12.5)(2\text{kW}) = 25.0\ \text{kW} = \mathbf{1500kJ/min}$$

6-97 The refrigerated space and the environment temperatures for a refrigerator and the rate of heat removal from the refrigerated space are given. The minimum power input required is to be determined.

Assumptions The refrigerator operates steadily.

Analysis The power input to a refrigerator will be a minimum when the refrigerator operates in a reversible manner. The coefficient of performance of a reversible refrigerator depends on the temperature limits in the cycle only, and is determined from

$$COP_{R,rev} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(25 + 273\text{K})/(-8 + 273\text{K}) - 1} = 8.03$$



The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,

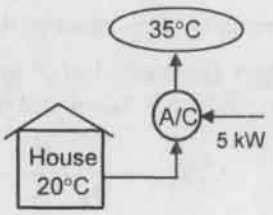
$$\dot{W}_{net,in,\min} = \frac{\dot{Q}_L}{COP_{R,\max}} = \frac{300\text{ kJ/min}}{8.03} = 37.36\text{ kJ/min} = \mathbf{0.623\ kW}$$

6-98 The cooled space and the outdoors temperatures for a Carnot air-conditioner and the rate of heat removal from the air-conditioned room are given. The power input required is to be determined.

Assumptions The air-conditioner operates steadily.

Analysis The COP of a Carnot air conditioner (or Carnot refrigerator) depends on the temperature limits in the cycle only, and is determined from

$$COP_{R,C} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(35 + 273\text{K})/(20 + 273\text{K}) - 1} = 19.5$$



The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,

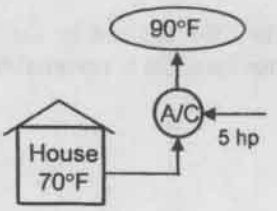
$$\dot{W}_{net,in} = \frac{\dot{Q}_L}{COP_{R,\max}} = \frac{750\text{ kJ/min}}{19.5} = 38.5\text{ kJ/min} = \mathbf{0.64\ kW}$$

6-99E The cooled space and the outdoors temperatures for an air-conditioner and the power consumption are given. The maximum rate of heat removal from the air-conditioned space is to be determined.

Assumptions The air-conditioner operates steadily.

Analysis The rate of heat removal from a house will be a maximum when the air-conditioning system operates in a reversible manner. The coefficient of performance of a reversible air-conditioner (or refrigerator) depends on the temperature limits in the cycle only, and is determined from

$$COP_{R,rev} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(90 + 460\text{R})/(70 + 460\text{R}) - 1} = 26.5$$



The rate of heat removal from the house is determined from the definition of the coefficient of performance of a refrigerator,

$$\dot{Q}_L = COP_R \times \dot{W}_{net,in} = (26.5)(5\text{hp})\left(\frac{42.41\text{Btu/min}}{1\text{hp}}\right) = \mathbf{5619Btu/min}$$

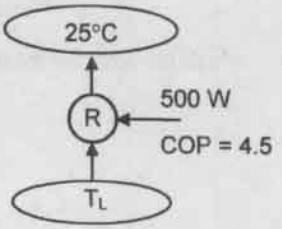
6-100 The refrigerated space temperature, the COP, and the power input of a Carnot refrigerator are given. The rate of heat removal from the refrigerated space and its temperature are to be determined.

Assumptions The refrigerator operates steadily.

Analysis (*a*) The rate of heat removal from the refrigerated space is determined from the definition of the COP of a refrigerator,

$$\dot{Q}_L = COP_R \times \dot{W}_{net,in} = (4.5)(0.5\text{kW}) = 2.25\text{kW} = \mathbf{135kJ/min}$$

(*b*) The temperature of the refrigerated space T_L is determined from the coefficient of performance relation for a Carnot refrigerator,



$$COP_{R,rev} = \frac{1}{(T_H / T_L) - 1} \longrightarrow 4.5 = \frac{1}{(25 + 273\text{K})/T_L - 1}$$

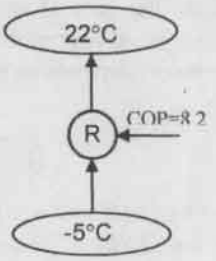
It yields

$$T_L = 243.8\text{ K} = \mathbf{-29.2°C}$$

6-101 An inventor claims to have developed a refrigerator. The inventor reports temperature and COP measurements. The claim is to be evaluated.

Analysis The highest coefficient of performance a refrigerator can have when removing heat from a cool medium at -5°C to a warmer medium at 22°C is

$$COP_{R,\max} = COP_{R,rev} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(22 + 273\text{K})/(-5 + 273\text{K}) - 1} = 9.9$$

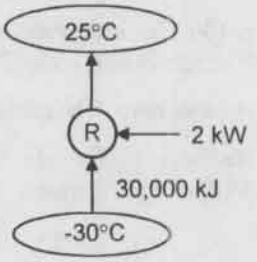


The COP claimed by the inventor is below this maximum value, thus the claim is **reasonable**.

6-102 An experimentalist claims to have developed a refrigerator. The experimentalist reports temperature, heat transfer, and work input measurements. The claim is to be evaluated.

Analysis The highest coefficient of performance a refrigerator can have when removing heat from a cool medium at -30°C to a warmer medium at 25°C is

$$COP_{R,\max} = COP_{R,rev} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(25 + 273\text{K})/(-30 + 273\text{K}) - 1} = 4.42$$



The work consumed by the actual refrigerator during this experiment is

$$W_{net,in} = \dot{W}_{net,in}\Delta t = (2\text{kJ/s})(20 \times 60\text{s}) = 2400\text{kJ}$$

Then the coefficient of performance of this refrigerator becomes

$$COP_R = \frac{Q_L}{W_{net,in}} = \frac{30{,}000\text{ kJ}}{2400\text{ kJ}} = 12.5$$

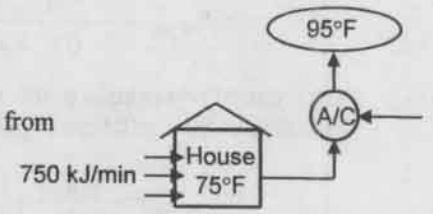
which is above the maximum value. Therefore, these measurements are **not reasonable**.

6-103E An air-conditioning system maintains a house at a specified temperature. The rate of heat gain of the house and the rate of internal heat generation are given. The maximum power input required is to be determined.

Assumptions The air-conditioner operates steadily.

Analysis The power input to an air-conditioning system will be a minimum when the air-conditioner operates in a reversible manner. The coefficient of performance of a reversible air-conditioner (or refrigerator) depends on the temperature limits in the cycle only, and is determined from

$$COP_{R,rev} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(95 + 460\text{R})/(75 + 460\text{R}) - 1} = 26.75$$



The cooling load of this air-conditioning system is the sum of the heat gain from the outside and the heat generated within the house,

$$\dot{Q}_L = 750 + 150 = 900 \text{ Btu/min}$$

The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,

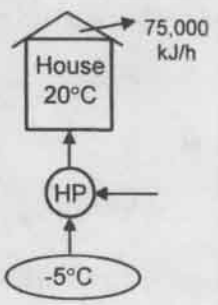
$$\dot{W}_{net,in,\min} = \frac{\dot{Q}_L}{COP_{R,\max}} = \frac{900 \text{ Btu/min}}{26.75} = 33.6 \text{ Btu/min} = \mathbf{0.79\ hp}$$

6-104 A heat pump maintains a house at a specified temperature. The rate of heat loss of the house is given. The minimum power input required is to be determined.

Assumptions The heat pump operates steadily.

Analysis The power input to a heat pump will be a minimum when the heat pump operates in a reversible manner. The COP of a reversible heat pump depends on the temperature limits in the cycle only, and is determined from

$$COP_{HP,rev} = \frac{1}{1 - (T_L / T_H)} = \frac{1}{1 - (-5 + 273\text{K})/(20 + 273\text{K})} = 11.7$$



The required power input to this reversible heat pump is determined from the definition of the coefficient of performance to be

$$\dot{W}_{net,in,\min} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{75{,}000\text{kJ/h}}{11.7}\left(\frac{1\text{h}}{3600\text{s}}\right) = \mathbf{1.78kW}$$

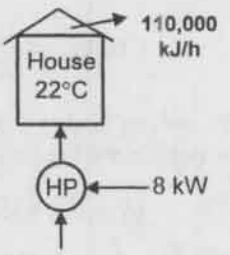
which is the *minimum* power input required.

6-105 A heat pump maintains a house at a specified temperature. The rate of heat loss of the house and the power consumption of the heat pump are given. It is to be determined if this heat pump can do the job.

Assumptions The heat pump operates steadily.

Analysis The power input to a heat pump will be a minimum when the heat pump operates in a reversible manner. The coefficient of performance of a reversible heat pump depends on the temperature limits in the cycle only, and is determined from

$$COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(2+273\text{K})/(22+273\text{K})} = 14.75$$



The required power input to this reversible heat pump is determined from the definition of the coefficient of performance to be

$$\dot{W}_{net,in,\min} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{110{,}000\text{kJ/h}}{14.75}\left(\frac{1\text{h}}{3600\text{s}}\right) = \mathbf{2.07kW}$$

This heat pump is **powerful enough** since 8 kW > 2.07 kW.

6-106 A heat pump that consumes 6-kW of power when operating maintains a house at a specified temperature. The house is losing heat in proportion to the temperature difference between the indoors and the outdoors. The lowest outdoor temperature for which this heat pump can do the job is to be determined.

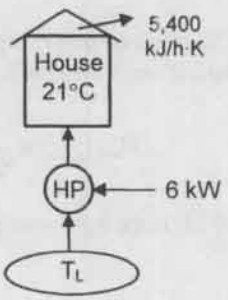
Assumptions The heat pump operates steadily.

Analysis Denoting the outdoor temperature by T_L, the heating load of this house can be expressed as

$$\dot{Q}_H = (5400\text{kJ/h}\cdot\text{K})(294 - T_L) = (1.5\text{kW/K})(294 - T_L)$$

The coefficient of performance of a Carnot heat pump depends on the temperature limits in the cycle only, and can be expressed as

$$COP_{HP} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-T_L/294\text{K}}$$



or, as

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_{net,in}} = \frac{(1.5\text{kW/K})(294 - T_L)}{6\text{kW}}$$

Equating the two relations above and solving for T_L, we obtain

$$T_L = 259.7\text{ K} = \mathbf{-13.3°C}$$

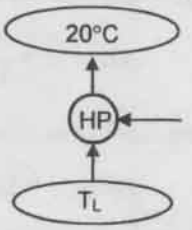
6-107 A heat pump maintains a house at a specified temperature in winter. The maximum COPs of the heat pump for different outdoor temperatures are to be determined.

Analysis The coefficient of performance of a heat pump will be a maximum when the heat pump operates in a reversible manner. The coefficient of performance of a reversible heat pump depends on the temperature limits in the cycle only, and is determined for all three cases above to be

$$COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(10+273\text{K})/(20+273\text{K})} = \mathbf{29.3}$$

$$COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(-5+273\text{K})/(20+273\text{K})} = \mathbf{11.7}$$

$$COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(-30+273\text{K})/(20+273\text{K})} = \mathbf{5.86}$$



6-108E A heat pump maintains a house at a specified temperature. The rate of heat loss of the house is given. The minimum power inputs required for different source temperatures are to be determined.

Assumptions The heat pump operates steadily.

Analysis (*a*) The power input to a heat pump will be a minimum when the heat pump operates in a reversible manner. If the outdoor air at 25°F is used as the heat source, the COP of the heat pump and the required power input are determined to be

$$COP_{HP,\max} = COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(25+460\text{R})/(78+460\text{R})} = 10.15$$

and

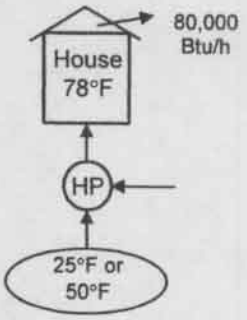
$$\dot{W}_{net,in,\min} = \frac{\dot{Q}_H}{COP_{HP,\max}} = \frac{80{,}000\text{Btu/h}}{10.15}\left(\frac{1\text{hp}}{2545\text{Btu/h}}\right) = \mathbf{3.1\ hp}$$

(*b*) If the well-water at 50°F is used as the heat source, the COP of the heat pump and the required power input are determined to be

$$COP_{HP,\max} = COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(50+460\text{R})/(78+460\text{R})} = 19.2$$

and

$$\dot{W}_{net,in,\min} = \frac{\dot{Q}_H}{COP_{HP,\max}} = \frac{80{,}000\text{Btu/h}}{19.2}\left(\frac{1\text{hp}}{2545\text{Btu/h}}\right) = \mathbf{1.64\ hp}$$



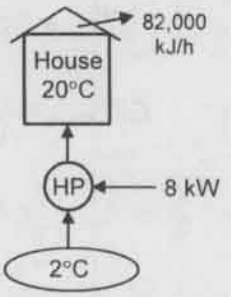
6-109 A Carnot heat pump consumes 8-kW of power when operating, and maintains a house at a specified temperature. The average rate of heat loss of the house in a particular day is given. The actual running time of the heat pump that day, the heating cost, and the cost if resistance heating is used instead are to be determined.

Analysis (*a*) The coefficient of performance of this Carnot heat pump depends on the temperature limits in the cycle only, and is determined from

$$COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(2+273\text{K})/(20+273\text{K})} = 16.3$$

The amount of heat the house lost that day is

$$Q_H = \dot{Q}_H(1\text{day}) = (82{,}000\text{kJ/h})(24\text{h}) = 1{,}968{,}000\text{kJ}$$



Then the required work input to this Carnot heat pump is determined from the definition of the coefficient of performance to be

$$W_{net,in} = \frac{Q_H}{COP_{HP}} = \frac{1{,}968{,}000\text{ kJ}}{16.3} = 120{,}736\text{ kJ}$$

Thus the length of time the heat pump ran that day is

$$\Delta t = \frac{W_{net,in}}{\dot{W}_{net,in}} = \frac{120{,}736\text{ kJ}}{8\text{ kJ/s}} = 15{,}092\text{ s} = \mathbf{4.19\ h}$$

(*b*) The total heating cost that day is

$$\text{Cost} = W \times \text{price} = (\dot{W}_{net,in} \times \Delta t)(\text{price}) = (8\text{kW})(4.19\text{h})(0.085\$/\text{kWh}) = \mathbf{\$2.85}$$

(*c*) If resistance heating were used, the entire heating load for that day would have to be met by electrical energy. Therefore, the heating system would consume 1,968,000 kJ of electricity which would cost

$$\text{New Cost} = Q_H \times \text{price} = (1{,}968{,}000\text{kJ})\left(\frac{1\text{kWh}}{3600\text{kJ}}\right)(0.085\$/\text{kWh}) = \mathbf{\$46.47}$$

6-110 A Carnot heat engine is used to drive a Carnot refrigerator. The maximum rate of heat removal from the refrigerated space and the total rate of heat rejection to the ambient air are to be determined.

Assumptions The heat engine and the refrigerator operate steadily.

Analysis (a) The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{th,\max} = \eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\ \text{K}}{1173\ \text{K}} = 0.744$$

Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

$$\dot{W}_{net,out} = \eta_{th}\dot{Q}_H = (0.744)(800\text{kJ/min}) = 595.2\text{kJ/min}$$

which is also the power input to the refrigerator, $\dot{W}_{net,in}$.

The rate of heat removal from the refrigerated space will be a maximum if a Carnot refrigerator is used. The COP of the Carnot refrigerator is

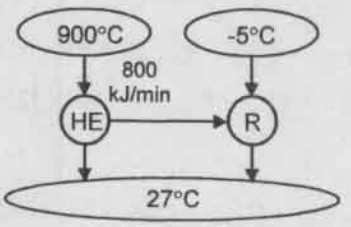
$$COP_{R,rev} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(27+273\text{K})/(-5+273\text{K}) - 1} = 8.37$$

Then the rate of heat removal from the refrigerated space becomes

$$\dot{Q}_{L,R} = (COP_{R,rev})(\dot{W}_{net,in}) = (8.37)(595.2\text{kJ/min}) = \mathbf{4982\text{kJ/min}}$$

(b) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,HE}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,R}$),

$$\dot{Q}_{L,HE} = \dot{Q}_{H,R} - \dot{W}_{net,out} = 800 - 595.2 = 204.8\ \text{kJ/min}$$
$$\dot{Q}_{H,R} = \dot{Q}_{L,R} - \dot{W}_{net,in} = 4982 - 595.2 = 5577.2\ \text{kJ/min}$$



and

$$\dot{Q}_{Ambient} = \dot{Q}_{L,HE} + \dot{Q}_{H,R} = 204.8 + 5577.2 = \mathbf{5782\ kJ/min}$$

6-111E A Carnot heat engine is used to drive a Carnot refrigerator. The maximum rate of heat removal from the refrigerated space and the total rate of heat rejection to the ambient air are to be determined.

Assumptions The heat engine and the refrigerator operate steadily. √

Analysis (*a*) The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{th,\max} = \eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{540\text{R}}{2160\text{R}} = 0.75$$

Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

$$\dot{W}_{net,out} = \eta_{th}\dot{Q}_H = (0.75)(700\ \text{Btu/min}) = 525\ \text{Btu/min}$$

which is also the power input to the refrigerator, $\dot{W}_{net,in}$.

The rate of heat removal from the refrigerated space will be a maximum if a Carnot refrigerator is used. The COP of the Carnot refrigerator is

$$COP_{R,rev} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(80 + 460\text{R})/(20 + 460\text{R}) - 1} = 8.0$$

Then the rate of heat removal from the refrigerated space becomes

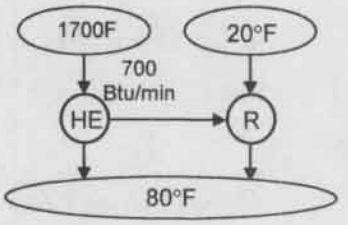
$$\dot{Q}_{L,R} = (COP_{R,rev})(\dot{W}_{net,in}) = (8.0)(525\ \text{Btu/min}) = \mathbf{4200\ Btu/min}$$

(*b*) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,HE}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,R}$),

$$\dot{Q}_{L,HE} = \dot{Q}_{H,R} - \dot{W}_{net,out} = 700 - 525 = 175\ \text{Btu/min}$$
$$\dot{Q}_{H,R} = \dot{Q}_{L,R} - \dot{W}_{net,in} = 4200 - 525 = 3675\ \text{Btu/min}$$

and

$$\dot{Q}_{Ambient} = \dot{Q}_{L,HE} + \dot{Q}_{H,R} = 175 + 3850 = \mathbf{3850\ Btu/min}$$



Household Refrigerators

6-112C It is a bad idea to overdesign the refrigeration system of a supermarket so that the entire air-conditioning needs of the store can be met by refrigerated air without installing any air-conditioning system. This is because the refrigerators cool the air to a much lower temperature than needed for air conditioning, and thus their efficiency is much lower, and their operating cost is much higher.

6-113C It is a bad idea to meet the entire refrigerator/freezer requirements of a store by using a large freezer that supplies sufficient cold air at -20°C instead of installing separate refrigerators and freezers . This is because the freezers cool the air to a much lower temperature than needed for refrigeration, and thus their efficiency is much lower, and their operating cost is much higher.

6-114C The energy consumption of a household refrigerator can be reduced by practicing good conservation measures such as (1) opening the refrigerator door the fewest times possible and for the shortest duration possible, (2) cooling the hot foods to room temperature first before putting them into the refrigerator, (3) cleaning the condenser coils behind the refrigerator, (4) checking the door gasket for air leaks, (5) avoiding unnecessarily low temperature settings, (6) avoiding excessive ice build-up on the interior surfaces of the evaporator, (7) using the power-saver switch that controls the heating coils that prevent condensation on the outside surfaces in humid environments, and (8) not blocking the air flow passages to and from the condenser coils of the refrigerator.

6-115C It is important to clean the condenser coils of a household refrigerator a few times a year since the dust that collects on them serves as insulation and slows down heat transfer. Also, it is important not to block air flow through the condenser coils since heat is rejected through them by natural convection, and blocking the air flow will interfere with this heat rejection process. A refrigerator cannot work unless it can reject the waste heat.

6-116C Today's refrigerators are much more efficient than those built in the past as a result of using smaller and higher efficiency motors and compressors, better insulation materials, larger coil surface areas, and better door seals.

6-117 A refrigerator consumes 300 W when running, and $83 worth of electricity per year under normal use. The fraction of the time the refrigerator will run in a year is to be determined.

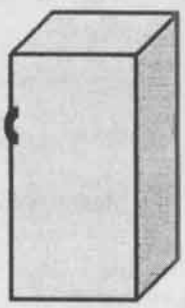
Assumptions The electricity consumed by the light bulb is negligible.

Analysis The total amount of electricity the refrigerator uses a year is

$$\text{Total electric energy used} = W_{e,\text{total}} = \frac{\text{Total cost of energy}}{\text{Unit cost of energy}} = \frac{\$83/\text{year}}{\$0.08/\text{kWh}} = 1037.5\ \text{kWh/year}$$

The number of hours the refrigerator is on per year is

$$\text{Total operating hours} = \Delta t = \frac{W_{e,\text{total}}}{\dot{W}_e} = \frac{1037.5\ \text{kWh}}{0.3\ \text{kW}} = 3458\ \text{h/year}$$



Noting that there are 365×24=8760 hours in a year, the fraction of the time the refrigerator is on during a year is determined to be

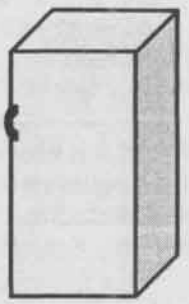
$$\text{Time fraction on} = \frac{\text{Total operating hours}}{\text{Total hours per year}} = \frac{3458/\text{year}}{8760\ \text{h/year}} = \mathbf{0.395}$$

Therefore, the refrigerator remained on 39.5% of the time.

6-118 The light bulb of a refrigerator is to be replaced by a $25 energy efficient bulb that consumes less than half the electricity. It is to be determined if the energy savings of the efficient light bulb justify its cost.

Assumptions The new light bulb remains on the same number of hours a year.

Analysis The lighting energy saved a year by the energy efficient bulb is



$$\begin{aligned}\text{Lighting energy saved} &= (\text{Lighting power saved})(\text{Operating hours})\\ &= [(40-18)\text{W}](60\,\text{h/year})\\ &= 1320\ \text{Wh} = 1.32\,\text{kWh}\end{aligned}$$

This means 1.32 kWh less heat is supplied to the refrigerated space by the light bulb, which must be removed from the refrigerated space. This corresponds to a refrigeration savings of

$$\text{Refrigeration energy saved} = \frac{\text{Lighting energy saved}}{\text{COP}} = \frac{1.32\,\text{kWh}}{1.3} = 1.02\ \text{kWh}$$

Then the total electrical energy and money saved by the energy efficient light bulb become

$$\text{Total energy saved} = (\text{Lighting} + \text{Refrigeration})\ \text{energy saved} = 1.32 + 1.02 = 2.34\ \text{kWh/year}$$

$$\begin{aligned}\text{Money saved} &= (\text{Total energy saved})(\text{Unit cost of energy}) = (2.34\,\text{kWh/year})(\$0.08/\text{kWh})\\ &= \mathbf{\$0.19/year}\end{aligned}$$

That is, the light bulb will save only 19 cents a year in energy costs, and it will take $25/$0.19 = 134 years for it to pay for itself from the energy it saves. Therefore, it is **not justified** in this case.

6-119 A person cooks twice a week and places the food into the refrigerator before cooling it first. The amount of money this person will save a year by cooling the hot foods to room temperature before refrigerating them is to be determined. √

Assumptions **1** The heat stored in the pan itself is negligible. **2** The specific heat of the food is constant.

Properties The specific heat of food is $C = 3.90$ kJ/kg.°C (given).

Analysis The amount of hot food refrigerated per year is

$$m_{\text{food}} = (5\,\text{kg/pan})(2\,\text{pans/week})(52\,\text{weeks/year}) = 520\,\text{kg/year}$$

The amount of energy removed from food as it is unnecessarily cooled to room temperature in the refrigerator is

$$\text{Energy removed} = Q_{\text{out}} = m_{\text{food}} C\Delta T = (520\,\text{kg/year})(3.90\,\text{kJ/kg.°C})(95-20)°\text{C} = 152{,}100\,\text{kJ/year}$$

$$\text{Energy saved} = E_{\text{saved}} = \frac{\text{Energy removed}}{\text{COP}} = \frac{152{,}100\,\text{kJ/year}}{1.2}\left(\frac{1\,\text{kWh}}{3600\,\text{kJ}}\right) = 35.2\,\text{kWh/year}$$

$$\text{Money saved} = (\text{Energy saved})(\text{Unit cost of energy}) = (35.2\,\text{kWh/year})(\$0.10/\text{kWh}) = \mathbf{\$3.52/year}$$

Therefore, cooling the food to room temperature before putting it into the refrigerator will save about three and a half dollars a year.

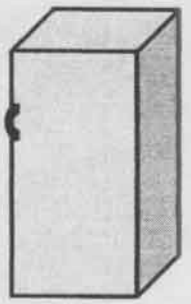
6-120 The door of a refrigerator is opened 8 times a day, and half of the cool air inside is replaced by the warmer room air. The cost of the energy wasted per year as a result of opening the refrigerator door is to be determined for the cases of moist and dry air in the room. √

Assumptions **1** The room is maintained at 20°C and 95 kPa at all times. **2** Air is an ideal gas with constant specific heats at room temperature. **3** The moisture is condensed at an average temperature of 4°C. **4** Half of the air volume in the refrigerator is replaced by the warmer kitchen air each time the door is opened.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg·K (Table A-1). The specific heat of air at room temperature is $C_p = 1.005$ kJ/kg·°C (Table A-2a). The heat of vaporization of water at 4°C is $h_{fg} = 2492$ kJ/kg.

Analysis The volume of the refrigerated air replaced each time the refrigerator is opened is 0.3 m^3 (half of the 0.6 m^3 air volume in the refrigerator). Then the total volume of refrigerated air replaced by room air per year is

$$\dot{V}_{\text{air, replaced}} = (0.3\ \text{m}^3)(8/\text{day})(365\ \text{days/year}) = 876\ \text{m}^3/\text{year}$$



The density of air at the refrigerated space conditions of 95 kPa and 4°C and the mass of air replaced per year are

$$\rho_o = \frac{P_o}{RT_o} = \frac{95\ \text{kPa}}{(0.287\ \text{kPa.m}^3/\text{kg.K})(4+273\ \text{K})} = 1.195\ \text{kg/m}^3$$

$$m_{\text{air}} = \rho V_{\text{air}} = (1.195\ \text{kg/m}^3)(876\ \text{m}^3/\text{year}) = 1047\ \text{kg/year}$$

The amount of moisture condensed and removed by the refrigerator is

$$m_{\text{moisture}} = m_{\text{air}}\,(\text{moisture removed per kg air}) = (1047\ \text{kg air/year})(0.006\ \text{kg/kg air})$$
$$= 6.28\ \text{kg/year}$$

The sensible, latent, and total heat gains of the refrigerated space become

$$Q_{\text{gain,sensible}} = m_{\text{air}} C_p (T_{\text{room}} - T_{\text{refrig}})$$
$$= (1047\ \text{kg/year})(1.005\ \text{kJ/kg.°C})(20-4)°\text{C} = 16{,}836\ \text{kJ/year}$$
$$Q_{\text{gain,latent}} = m_{\text{moisture}} h_{fg}$$
$$= (6.28\ \text{kg/year})(2492\ \text{kJ/kg}) = 15{,}650\ \text{kJ/year}$$
$$Q_{\text{gain,total}} = Q_{\text{gain,sensible}} + Q_{\text{gain,latent}} = 16{,}836 + 15{,}650 = 32{,}486\ \text{kJ/year}$$

For a COP of 1.4, the amount of electrical energy the refrigerator will consume to remove this heat from the refrigerated space and its cost are

$$\text{Electrical energy used (total)} = \frac{Q_{\text{gain,total}}}{\text{COP}} = \frac{32{,}486\ \text{kJ/year}}{1.4}\left(\frac{1\ \text{kWh}}{3600\ \text{kJ}}\right) = 6.45\ \text{kWh/year}$$

$$\text{Cost of energy used (total)} = (\text{Energy used})(\text{Unit cost of energy})$$
$$= (6.45\ \text{kWh/year})(\$0.075/\text{kWh}) = \mathbf{\$0.48/year}$$

If the room air is very dry and thus latent heat gain is negligible, then the amount of electrical energy the refrigerator will consume to remove the sensible heat from the refrigerated space and its cost become

$$\text{Electrical energy used (sensible)} = \frac{Q_{\text{gain,sensible}}}{\text{COP}} = \frac{16{,}836\ \text{kJ/year}}{1.4}\left(\frac{1\ \text{kWh}}{3600\ \text{kJ}}\right) = 3.34\ \text{kWh/year}$$

$$\text{Cost of energy used (sensible)} = (\text{Energy used})(\text{Unit cost of energy})$$
$$= (3.34\ \text{kWh/year})(\$0.075/\text{kWh}) = \mathbf{\$0.25/year}$$

Review Problems

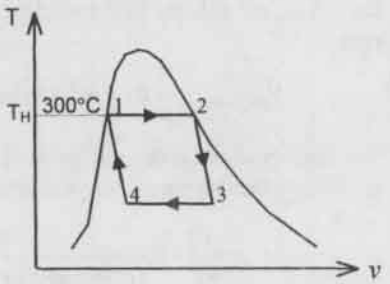
6-121 A Carnot heat engine cycle is executed in a steady-flow system with steam. The thermal efficiency and the mass flow rate of steam are given. The net power output of the engine is to be determined.

Assumptions All components operate steadily.

Properties The enthalpy of vaporization h_{fg} of water at 300°C is 1404.9 kJ/kg (Table A-4).

Analysis The enthalpy of vaporization h_{fg} at a given T or P represents the amount of heat transfer as 1 kg of a substance is converted from saturated liquid to saturated vapor at that T or P. Therefore, the rate of heat transfer to the steam during heat addition process is

$$\dot{Q}_H = \dot{m}h_{fg@300°C} = (5\text{kg/s})(1404.9\text{kJ/kg}) = 7025\text{kJ/s}$$



Then the power output of this heat engine becomes

$$\dot{W}_{net,out} = \eta_{th}\dot{Q}_H = (0.30)(7025\text{kW}) = \mathbf{2107.5kW}$$

6-122 A heat pump with a specified COP is to heat a house. The rate of heat loss of the house and the power consumption of the heat pump are given. The time it will take for the interior temperature to rise from 3°C to 22°C is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** The house is well-sealed so that no air leaks in or out. **3** The COP of the heat pump remains constant during operation.

Properties The constant volume specific heat of air at room temperature is $C_v = 0.718$ kJ/kg.°C (Table A-2)

Analysis The house is losing heat at a rate of

$$\dot{Q}_{Loss} = 40{,}000\ \text{kJ/h} = 11.11\ \text{kJ/s}$$

The rate at which this heat pump supplies heat is

$$\dot{Q}_H = (COP_{HP})(\dot{W}_{net,in}) = (2.4)(8\text{kW}) = 19.2\text{kW}$$

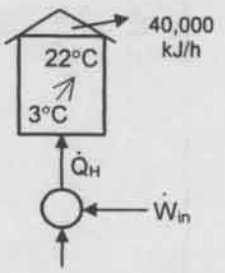
That is, this heat pump can supply heat at a rate of 19.2 kJ/s. Taking the house as the system (a closed system), the energy balance can be written as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{in} - Q_{out} = \Delta U = m(u_2 - u_1)$$

$$Q_{in} - Q_{out} = mC_v(T_2 - T_1)$$

$$(\dot{Q}_{in} - \dot{Q}_{out})\Delta t = mC_v(T_2 - T_1)$$



Substituting, $(19.2 - 11.11\text{kJ/s})\Delta t = (2000\text{kg})(0.718\text{kJ/kg}\cdot^\circ\text{C})(22 - 3)^\circ\text{C}$

Solving for Δt, it will take

$$\Delta t = 3373\ \text{s} = \mathbf{0.937\ h}$$

for the temperature in the house to rise to 22°C.

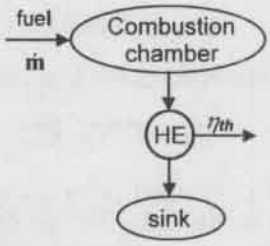
6-123 The thermal efficiency and power output of a gas turbine are given. The rate of fuel consumption of the gas turbine is to be determined.

Assumptions Steady operating conditions exist.

Properties The density and heating value of the fuel are given to be 0.8 g/cm^3 and 46,000 kJ/kg, respectively.

Analysis This gas turbine is converting 17% of the chemical energy released during the combustion process into work. The amount of energy input required to produce a power output of 6,000 kW is determined from the definition of thermal efficiency to be

$$\dot{Q}_H = \frac{\dot{W}_{net,out}}{\eta_{th}} = \frac{6{,}000 \text{ kJ,s}}{0.17} = 35{,}294 \text{ kJ/s}$$



To supply energy at this rate, the engine must burn fuel at a rate of

$$\dot{m} = \frac{35{,}294 \text{ kJ/s}}{46{,}000 \text{ kJ/kg}} = 0.767 \text{ kg/s}$$

since 46,000 kJ of thermal energy is released for each kg of fuel burned. Then the volume flow rate of the fuel becomes

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.767 \text{ kg/s}}{0.8 \text{ kg/L}} = \mathbf{0.959 \text{ L/s}}$$

6-124 It is to be shown that $COP_{HP} = COP_R + 1$ for the same temperature and heat transfer terms.

Analysis Using the definitions of COPs, the desired relation is obtained to be

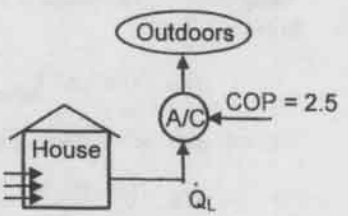
$$COP_{HP} = \frac{Q_H}{W_{net,in}} = \frac{Q_L + W_{net,in}}{W_{net,in}} = \frac{Q_L}{W_{net,in}} + 1 = \mathbf{COP_R + 1}$$

6-125 An air-conditioning system maintains a house at a specified temperature. The rate of heat gain of the house, the rate of internal heat generation, and the COP are given. The required power input is to be determined.

Assumptions Steady operating conditions exist.

Analysis The cooling load of this air-conditioning system is the sum of the heat gain from the outdoors and the heat generated in the house from the people, lights, and appliances:

$$\dot{Q}_L = 20{,}000 + 8{,}000 = 28{,}000 \text{ kJ/h}$$



Using the definition of the coefficient of performance, the power input to the air-conditioning system is determined to be

$$\dot{W}_{net,in} = \frac{\dot{Q}_L}{COP_R} = \frac{28{,}000 \text{ kJ/h}}{2.5}\left(\frac{1 \text{ kW}}{3600 \text{ kJ/h}}\right) = \mathbf{3.11 \text{ kW}}$$

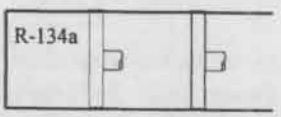
6-126 A Carnot heat engine cycle is executed in a closed system with a fixed mass of R-134a. The thermal efficiency of the cycle is given. The net work output of the engine is to be determined. √

Assumptions All components operate steadily.

Properties The enthalpy of vaporization of R-134a at 70°C is h_{fg} = 124.08 kJ/kg (Table A-11).

Analysis The enthalpy of vaporization h_{fg} at a given T or P represents the amount of heat transfer as 1 kg of a substance is converted from saturated liquid to saturated vapor at that T or P. Therefore, the amount of heat transfer to R-134a during the heat addition process of the cycle is

$$Q_H = mh_{fg@70°C} = (0.01 \text{ kg})(124.08 \text{ kJ/kg}) = 1.24 \text{ kJ}$$



Carnot HE

Then the work output of this heat engine becomes

$$W_{net,out} = \eta_{th} Q_H = (0.15)(1.24 \text{ kJ}) = \mathbf{0.186 \text{ kJ}}$$

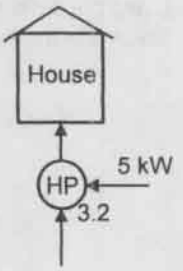
6-127 A heat pump with a specified COP and power consumption is used to heat a house. The time it takes for this heat pump to raise the temperature of a cold house to the desired level is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** The heat loss of the house during the warp-up period is negligible. **3** The house is well-sealed so that no air leaks in or out.

Properties The constant volume specific heat of air at room temperature is $C_v = 0.72$ kJ/kg.°C.

Analysis Since the house is well-sealed (constant volume), the total amount of heat that needs to be supplied to the house is

$$Q_H = (mC_v\Delta T)_{house} = (1500\text{kg})(0.72\text{kJ/kg}\cdot^\circ\text{C})(22-7)^\circ\text{C} = 16{,}200\text{kJ}$$



The rate at which this heat pump supplies heat is

$$\dot{Q}_H = (COP_{HP})(\dot{W}_{net,in}) = (3.2)(5\text{kW}) = 16\text{kW}$$

That is, this heat pump can supply 16 kJ of heat per second. Thus the time required to supply 16,200 kJ of heat is

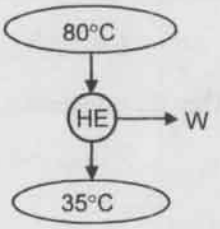
$$\Delta t = \frac{Q_H}{\dot{Q}_H} = \frac{16{,}200\text{ kJ}}{16\text{ kJ/s}} = 1{,}012.5\text{ s} = \mathbf{16.9\ min}$$

6-128 A solar pond power plant operates by absorbing heat from the hot region near the bottom, and rejecting waste heat to the cold region near the top. The maximum thermal efficiency that the power plant can have is to be determined.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{th,\max} = \eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{308\text{ K}}{353\text{ K}} = 0.127 \qquad \text{or} \qquad \mathbf{12.7\%}$$

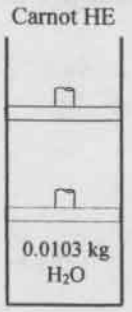
In reality, the temperature of the working fluid must be above 35°C in the condenser, and below 80°C in the boiler to allow for any effective heat transfer. Therefore, the maximum efficiency of the actual heat engine will be lower than the value calculated above.



6-129 A Carnot heat engine cycle is executed in a closed system with a fixed mass of steam. The net work output of the cycle and the ratio of sink and source temperatures are given. The low temperature in the cycle is to be determined.

Assumptions The engine is said to operate on the Carnot cycle, which is totally reversible.

Analysis The thermal efficiency of the cycle is



$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \frac{1}{2} = 0.5$$

Also,

$$\eta_{th} = \frac{W}{Q_H} \longrightarrow Q_H = \frac{W}{\eta_{th}} = \frac{25\text{ kJ}}{0.5} = 50\text{ kJ}$$

$$Q_L = Q_H - W = 50 - 25 = 25\text{ kJ}$$

and

$$q_L = \frac{Q_L}{m} = \frac{25\text{ kJ}}{0.0103\text{ kg}} = 2427\text{ kJ/kg} = h_{fg@T_L}$$

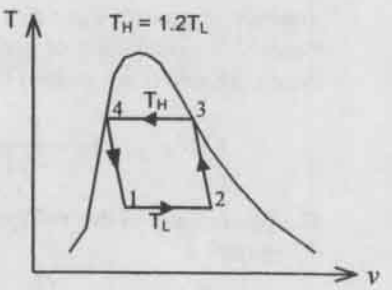
since the enthalpy of vaporization h_{fg} at a given T or P represents the amount of heat transfer as 1 kg of a substance is converted from saturated liquid to saturated vapor at that T or P. Therefore, T_L is the temperature which corresponds to the h_{fg} value of 2427 kJ/kg, and is determined from the steam tables to be

$$T_L = \mathbf{31.5°C}$$

6-130 A Carnot refrigeration cycle is executed in a closed system with a fixed mass of R-134a. The net work input and the ratio of maximum-to-minimum temperatures are given. The minimum pressure in the cycle is to be determined. √

Assumptions The refrigerator is said to operate on the reversed Carnot cycle, which is totally reversible.

Analysis The coefficient of performance of the cycle is



$$COP_R = \frac{1}{T_H/T_L - 1} = \frac{1}{1.2 - 1} = 5$$

Also,

$$COP_R = \frac{Q_L}{W_{in}} \longrightarrow Q_L = COP_R \times W_{in} = (5)(22\text{kJ}) = 110\text{kJ}$$

$$Q_H = Q_L + W = 110 + 22 = 132\text{ kJ}$$

and

$$q_H = \frac{Q_H}{m} = \frac{132\text{ kJ}}{0.96\text{ kg}} = 137.5\text{ kJ/kg} = h_{fg@T_H}$$

since the enthalpy of vaporization h_{fg} at a given T or P represents the amount of heat transfer per unit mass as a substance is converted from saturated liquid to saturated vapor at that T or P. Therefore, T_H is the temperature which corresponds to the h_{fg} value of 137.5 kJ/kg, and is determined from the R-134a tables to be

$$T_H \cong 61^\circ\text{C} = 334\text{K}$$

Then,

$$T_L = \frac{T_H}{1.2} = \frac{334\text{K}}{1.2} = 278.3\text{K} \cong 5.3^\circ\text{C}$$

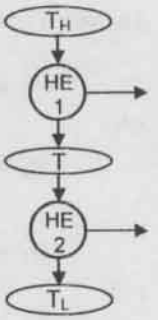
Therefore,

$$P_{\min} = P_{sat@5.3^\circ C} = \mathbf{0.354\text{ MPa}}$$

6-131 Two Carnot heat engines operate in series between specified temperature limits. If the thermal efficiencies of both engines are the same, the temperature of the intermediate medium between the two engines is to be determined.

Assumptions The engines are said to operate on the Carnot cycle, which is totally reversible.

Analysis The thermal efficiency of the two Carnot heat engines can be expressed as



$$\eta_{th,I} = 1 - \frac{T}{T_H} \qquad \text{and} \qquad \eta_{th,II} = 1 - \frac{T_L}{T}$$

Equating,
$$1 - \frac{T}{T_H} = 1 - \frac{T_L}{T}$$

Solving for T,
$$T = \sqrt{T_H T_L} = \sqrt{(1200\text{K})(300\text{K})} = \mathbf{600K}$$

6-132 A performance of a refrigerator declines as the temperature of the refrigerated space decreases. The minimum amount of work needed to remove 1 kJ of heat from liquid helium at 3 K is to the determined.

Analysis The power input to a refrigerator will be a minimum when the refrigerator operates in a reversible manner. The coefficient of performance of a reversible refrigerator depends on the temperature limits in the cycle only, and is determined from

$$COP_{R,rev} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(300\text{K})/(3\text{K}) - 1} = 0.0101$$

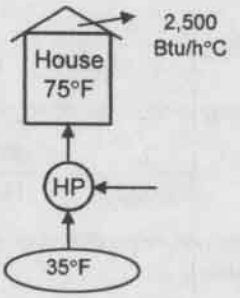
The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,

$$W_{net,in,\min} = \frac{Q_R}{COP_{R,\max}} = \frac{1\text{ kJ}}{0.0101} = \mathbf{99\ kJ}$$

6-133E A Carnot heat pump maintains a house at a specified temperature. The rate of heat loss from the house and the outdoor temperature are given. The COP and the power input are to be determined.

Analysis (*a*) The coefficient of performance of this Carnot heat pump depends on the temperature limits in the cycle only, and is determined from

$$COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(35-460\text{R})/(75+460\text{R})} = \mathbf{13.4}$$



(*b*) The heating load of the house is

$$\dot{Q}_H = (2500\text{Btu/h}\cdot{}^\circ\text{F})(75-35)^\circ\text{F} = 100{,}000\text{Btu/h}$$

Then the required power input to this Carnot heat pump is determined from the definition of the coefficient of performance to be

$$\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{100{,}000\text{Btu/h}}{13.4}\left(\frac{1\text{hp}}{2545\text{Btu/h}}\right) = \mathbf{2.93hp}$$

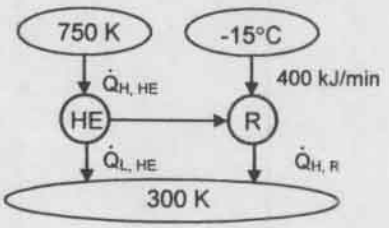
6-134 A Carnot heat engine drives a Carnot refrigerator that removes heat from a cold medium at a specified rate. The rate of heat supply to the heat engine and the total rate of heat rejection to the environment are to be determined.

Analysis (*a*) The coefficient of performance of the Carnot refrigerator is

$$COP_{R,C} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(300\text{K})/(258\text{K})-1} = 6.14$$

Then power input to the refrigerator becomes

$$\dot{W}_{net,in} = \frac{\dot{Q}_L}{COP_{R,C}} = \frac{400\text{ kJ/min}}{6.14} = 65.1\text{ kJ/min}$$



which is equal to the power output of the heat engine, $\dot{W}_{net,out}$.

The thermal efficiency of the Carnot heat engine is determined from

$$\eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{ K}}{750\text{ K}} = 0.60$$

Then the rate of heat input to this heat engine is determined from the definition of thermal efficiency to be

$$\dot{Q}_{H,HE} = \frac{\dot{W}_{net,out}}{\eta_{th,HE}} = \frac{65.1\text{ kJ/min}}{0.60} = \mathbf{108.5\text{ kJ/min}}$$

(*b*) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,HE}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,R}$),

$$\dot{Q}_{L,HE} = \dot{Q}_{H,HE} - \dot{W}_{net,out} = 108.5 - 65.1 = 43.4\text{ kJ/min}$$
$$\dot{Q}_{H,R} = \dot{Q}_{L,R} - \dot{W}_{net,in} = 400 - 65.1 = 465.1\text{ kJ/min}$$

and

$$\dot{Q}_{Ambient} = \dot{Q}_{L,HE} + \dot{Q}_{H,R} = 43.4 + 465.1 = \mathbf{508.5\text{ kJ/min}}$$

6-135 Half of the work output of a Carnot heat engine is used to drive a Carnot heat pump that is heating a house. The house is losing heat at a rate of 95,000 kJ/h. The minimum rate of heat supply to the heat engine is to be determined.

Assumptions Steady operating conditions exist.

Analysis The coefficient of performance of the Carnot heat pump is

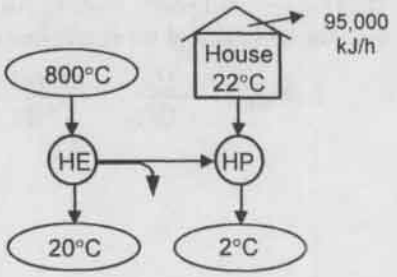
$$COP_{HP,C} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(2+273\text{K})/(22+273\text{K})} = 14.75$$

Then power input to the heat pump, which is supplying heat to the house at a rate of 80,000 kJ/h, becomes

$$\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{HP,C}} = \frac{95{,}000\text{ kJ/h}}{14.75} = 6441\text{ kJ/h}$$

which is half the power produced by the heat engine. Thus the power output of the heat engine is

$$\dot{W}_{net,out} = 2\dot{W}_{net,in} = (2)(6441\text{kJ/h}) = 12{,}882\text{kJ/h}$$



To minimize the rate of heat supply, we must use a Carnot heat engine whose thermal efficiency is determined from

$$\eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{293\text{ K}}{1073\text{ K}} = 0.727$$

Then the rate of heat supply to this heat engine is determined from the definition of thermal efficiency to be

$$\dot{Q}_{H,HE} = \frac{\dot{W}_{net,out}}{\eta_{th,HE}} = \frac{12{,}882\text{ kJ/h}}{0.727} = \mathbf{17{,}719\text{ kJ/h}}$$

6-136 A Carnot refrigeration cycle is executed in a closed system with a fixed mass of R-134a. The net work input and the maximum and minimum temperatures are given. The mass fraction of the refrigerant that vaporizes during the heat addition process, and the pressure at the end of the heat rejection process are to be determined. √

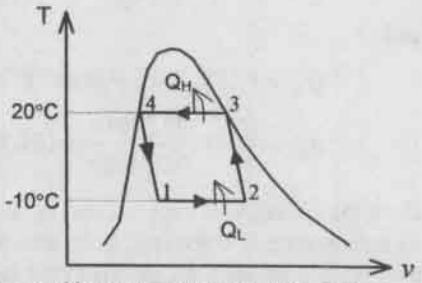
Properties The enthalpy of vaporization of R-134a at -10°C is h_{fg} = 204.46 kJ/kg (Table A-12).

Analysis The coefficient of performance of the cycle is

$$COP_R = \frac{1}{T_H / T_L - 1} = \frac{1}{293/263 - 1} = 8.77$$

and

$$Q_L = COP_R \times W_{in} = (8.77)(12\text{kJ}) = 105.2\text{kJ}$$



Then the amount of refrigerant which vaporizes during heat absorption is

$$Q_L = mh_{fg@T_L=-10°C} \longrightarrow m = \frac{105.2\text{kJ}}{204.46\text{ kJ/kg}} = 0.514\text{ kg}$$

since the enthalpy of vaporization h_{fg} at a given T or P represents the amount of heat transfer per unit mass as a substance is converted from saturated liquid to saturated vapor at that T or P. Therefore, the fraction of mass which vaporized during heat addition process is

$$\frac{0.514\text{ kg}}{0.8\text{ kg}} = 0.642 \text{ or } \mathbf{64.2\%}$$

The pressure at the end of the heat rejection process is

$$P_4 = P_{sat@20°C} = \mathbf{0.5716\ MPa}$$

6-137 A Carnot heat pump cycle is executed in a steady-flow system with R-134a flowing at a specified rate. The net power input and the ratio of the maximum-to-minimum temperatures are given. The ratio of the maximum to minimum temperatures is to be determined.

Analysis The coefficient of performance of the cycle is

$$COP_{HP} = \frac{1}{1 - T_L / T_H} = \frac{1}{1 - 1/1.15} = 8.667$$

and

$$\dot{Q}_H = COP_{HP} \times \dot{W}_{in} = (8.667)(5\text{kW}) = 43.33\text{kJ/s}$$

$$q_H = \frac{\dot{Q}_H}{\dot{m}} = \frac{43.33\text{kJ/s}}{0.264\text{kg/s}} = 164.3\text{kJ/kg} = h_{fg@T_H}$$

since the enthalpy of vaporization h_{fg} at a given T or P represents the amount of heat transfer per unit mass as a substance is converted from saturated liquid to saturated vapor at that T or P. Therefore, T_H is the temperature which corresponds to the h_{fg} value of 164.3 kJ/kg, and is determined from the R-134a tables to be

$$T_H \cong 38^\circ\text{C} = 311\text{ K}$$

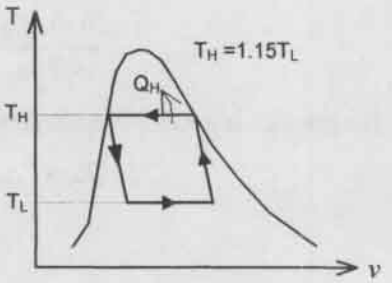
and

$$P_{\max} = P_{sat@38^\circ C} = 0.96298\text{ MPa}$$

$$T_L = \frac{T_H}{1.15} = \frac{311\text{ K}}{1.15} = 270.4\text{ K} \cong -2.6^\circ\text{C}$$

Also,

$$P_{\min} = P_{sat@-2.6^\circ C} = 0.2668\text{ MPa}$$



Then the ratio of the maximum to minimum pressures in the cycle is

$$\frac{P_{\max}}{P_{\min}} = \frac{0.96298\text{ MPa}}{0.2668\text{ MPa}} = \mathbf{3.61}$$

6-138 A Carnot heat engine is operating between specified temperature limits. The source temperature that will double the efficiency is to be determined.

Analysis Denoting the new source temperature by T_H^*, the thermal efficiency of the Carnot heat engine for both cases can be expressed as

$$\eta_{th,C} = 1 - \frac{T_L}{T_H} \qquad \text{and} \qquad \eta^*_{th,C} = 1 - \frac{T_L}{T_H^*} = 2\eta_{th,C}$$

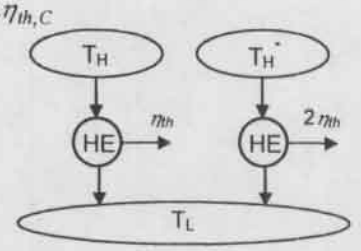
Substituting,

$$1 - \frac{T_L}{T_H^*} = 2\left(1 - \frac{T_L}{T_H}\right)$$

Solving for T_H^*,

$$T_H^* = \frac{T_H T_L}{T_H - 2T_L}$$

which is the desired relation.



6-139 A Carnot cycle is analyzed for the case of temperature differences in the boiler and condenser. The ratio of overall temperatures for which the power output will be maximum, and an expression for the maximum net power output are to be determined.

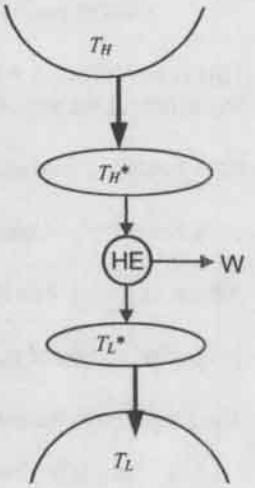
Analysis It is given that $\dot{Q}_H = (hA)_H (T_H - T_H^*)$. Therefore,

$$\dot{W} = \eta_{th}\dot{Q}_H = \left(1 - \frac{T_L^*}{T_H^*}\right)(hA)_H (T_H - T_H^*) = \left(1 - \frac{T_L^*}{T_H^*}\right)(hA)_H \left(1 - \frac{T_H^*}{T_H}\right)$$

or,

$$\frac{\dot{W}}{(hA)_H T_H} = \left(1 - \frac{T_L^*}{T_H^*}\right)\left(1 - \frac{T_H^*}{T_H}\right) = (1 - r)x \qquad (1)$$

where we defined r and x as $r = T_L^*/T_H^*$ and $x = 1 - T_H^*/T_H$.



For a reversible cycle we also have

$$\frac{T_H^*}{T_L^*} = \frac{\dot{Q}_H}{\dot{Q}_L} \longrightarrow \frac{1}{r} = \frac{(hA)_H (T_H - T_H^*)}{(hA)_L (T_L^* - T_L)} = \frac{(hA)_H T_H (1 - T_H^*/T_H)}{(hA)_L T_H (T_L^*/T_H - T_L/T_H)}$$

But $\frac{T_L^*}{T_H} = \frac{T_L^*}{T_H^*}\frac{T_H^*}{T_H} = r(1-x)$. Substituting into above relation yields

$$\frac{1}{r} = \frac{(hA)_H x}{(hA)_L [r(1-x) - T_L/T_H]}$$

Solving for x,

$$x = \frac{r - T_L/T_H}{r[(hA)_H/(hA)_L + 1]} \qquad (2)$$

Substitute (2) into (1):

$$\dot{W} = (hA)_H T_H (1 - r)\frac{r - T_L/T_H}{r[(hA)_H/(hA)_L + 1]} \qquad (3)$$

Taking the partial derivative $\frac{\partial \dot{W}}{\partial r}$ holding everything else constant and setting it equal to zero gives

$$r = \frac{T_L^*}{T_H^*} = \left(\frac{T_L}{T_H}\right)^{1/2} \qquad (4)$$

which is the desired relation. The maximum net power output in this case is determined by substituting (4) into (3). It simplifies to

$$\dot{W}_{\max} = \frac{(hA)_H T_H}{1 + (hA)_H/(hA)_L}\left\{1 - \left(\frac{T_L}{T_H}\right)^{1/2}\right\}^2$$

6-140 A decision is to be made between a cheaper but inefficient natural gas heater and an expensive but efficient natural gas heater for a house.

Assumptions The two heaters are comparable in all aspects other than the initial cost and efficiency.

Analysis Other things being equal, the logical choice is the heater that will cost less during its lifetime. The total cost of a system during its lifetime (the initial, operation, maintenance, etc.) can be determined by performing a life cycle cost analysis. A simpler alternative is to determine the simple payback period.

The annual heating cost is given to be $1100. Noting that the existing heater is 60% efficient, only 60% of that energy (and thus money) is delivered to the house, and the rest is wasted due to the inefficiency of the heater. Therefore, the monetary value of the heating load of the house is

$$\text{Cost of useful heat} = (60\%)(\text{Current annual heating cost}) = 0.60\times(\$1100/\text{yr}) = \$660/\text{yr}$$

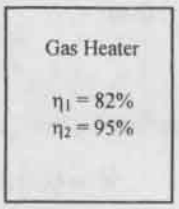
This is how much it would cost to heat this house with a heater that is 100% efficient. For heaters that are less efficient, the annual heating cost is determined by dividing $660 by the efficiency:

82% heater: Annual cost of heating = (Cost of useful heat)/Efficiency = ($660/yr)/0.82 = $805/yr

95% heater: Annual cost of heating = (Cost of useful heat)/Efficiency = ($660/yr)/0.95 = $695/yr

Annual cost savings with the efficient heater = 805 - 695 = $110

Excess initial cost of the efficient heater = 2700 - 1600 = $1100



The simple payback period becomes

$$\text{Simple payback period} = \frac{\text{Excess initial cost}}{\text{Annaul cost savings}} = \frac{\$1100}{\$110/\text{yr}} = \mathbf{10\ years}$$

Therefore, the more efficient heater will pay for the $1100 cost differential in this case in 10 years, which is more than the 8-year limit. Therefore, the purchase of the cheater and less efficient heater is a better buy in this case.

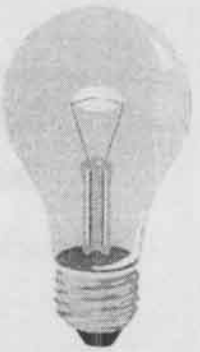
6-141 Switching to energy efficient lighting reduces the electricity consumed for lighting as well as the cooling load in summer, but increases the heating load in winter. It is to be determined if switching to efficient lighting will increase or decrease the total energy cost of a building. √

Assumptions The light escaping through the windows is negligible so that the entire lighting energy becomes part of the internal heat generation.

Analysis (*a*) Efficient lighting reduces the amount of electrical energy used for lighting year-around as well as the amount of heat generation in the house since light is eventually converted to heat. As a result, the electrical energy needed to air condition the house is also reduced. Therefore, in summer, the total cost of energy use of the household definitely decreases.

(*b*) In winter, the heating system must make up for the reduction in the heat generation due to reduced energy used for lighting. The total cost of energy used in this case will still decrease if the cost of unit heat energy supplied by the heating system is less than the cost of unit energy provided by lighting.

The cost of 1 kWh heat supplied from lighting is \$0.08 since all the energy consumed by lamps is eventually converted to thermal energy. Noting that 1 therm = 29.3 kWh and the furnace is 80% efficient, the cost of 1 kWh heat supplied by the heater is



$$\text{Cost of 1 kWh heat supplied by furnace} = (\text{Amount of useful energy}/\eta_{\text{furnace}})(\text{Price})$$
$$= [(1\,\text{kWh})/0.80](\$0.70/\text{therm})\left(\frac{1\,\text{therm}}{29.3\,\text{kWh}}\right)$$
$$= \$0.030 \ \text{(per kWh heat)}$$

which is much less than \$0.08. Thus we conclude that switching to energy efficient lighting will **reduce** the total energy cost of this building both in summer and in winter.

Discussion To determine the amount of cost savings due to switching to energy efficient lighting, consider 10 h of operation of lighting in summer and in winter for 1 kW rated power for lighting.

Current lighting:
Lighting cost: (Energy used)(Unit cost)= (1 kW)(10 h)(\$0.08/kWh) = \$0.80
Increase in air conditioning cost: (Heat from lighting/COP)(unit cost) =(10 kWh/3.5)(\$0.08/kWh) = \$0.23
Decrease in the heating cost = [Heat from lighting/Eff](unit cost)=(10/0.8 kWh)(\$0.70/29.3/kWh) = 0.30

Total cost in summer = 0.80+0.23 = \$1.03; Total cost in winter = \$0.80-0.23 = 0.57.

Energy efficient lighting:
Lighting cost: (Energy used)(Unit cost)= (0.25 kW)(10 h)(\$0.08/kWh) = \$0.20
Increase in air conditioning cost: (Heat from lighting/COP)(unit cost) =(2.5 kWh/3.5)(\$0.08/kWh) = \$0.06
Decrease in the heating cost = [Heat from lighting/Eff](unit cost)=(2.5/0.8 kWh)(\$0.70/29.3/kWh) = \$0.07

Total cost in summer = 0.20+0.06 = \$0.26; Total cost in winter = \$0.20-0.07 = 0.13.

Note that during a day with 10 h of operation, the total energy cost decreases from \$1.03 to \$0.26 in summer, and from \$0.57 to \$0.13 in winter when efficient lighting is used.

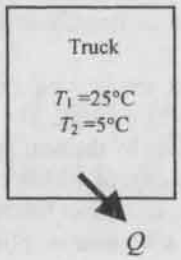
6-142 The cargo space of a refrigerated truck is to be cooled from 25°C to an average temperature of 5°C. The time it will take for an 8-kW refrigeration system to precool the truck is to be determined.

Assumptions **1** The ambient conditions remain constant during precooling. **2** The doors of the truck are tightly closed so that the infiltration heat gain is negligible. **3** The air inside is sufficiently dry so that the latent heat load on the refrigeration system is negligible. **4** Air is an ideal gas with constant specific heats.

Properties The density of air is given to be 1.2 kg/m³, and its specific heat at the average temperature of 15°C is C_p = 1.0 kJ/kg·°C (Table A-2).

Analysis The mass of air in the truck is

$$m_{air} = \rho_{air} V_{truck} = (1.2\,\text{kg/m}^3)(12\,\text{m} \times 2.3\,\text{m} \times 3.5\,\text{m}) = 116\,\text{kg}$$



The amount of heat removed as the air is cooled from 25 to 5°C

$$Q_{\text{cooling,air}} = (mC_p\Delta T)_{air} = (116\,\text{kg})(1.0\,\text{kJ/kg.°C})(25-5)°\text{C}$$
$$= 2{,}320\,\text{kJ}$$

Noting that *UA* is given to be 80 W/°C and the average air temperature in the trunk during precooling is (25+5)/2 = 15°C, the average rate of heat gain by transmission is determined to be

$$\dot{Q}_{\text{transmission,ave}} = UA\Delta T = (80\,\text{W/°C})(25-15)°\text{C} = 800\,\text{W} = 0.80\,\text{kJ/s}$$

Therefore, the time required to cool the truck from 25 to 5°C is determined to be

$$\dot{Q}_{\text{refrig.}}\Delta t = Q_{\text{cooling,air}} + \dot{Q}_{\text{transmission}}\Delta t \rightarrow \Delta t = \frac{Q_{\text{cooling,air}}}{\dot{Q}_{\text{refrig.}} - \dot{Q}_{\text{transmission}}} = \frac{2{,}320\,\text{kJ}}{(8-0.8)\,\text{kJ/s}} = 322\,\text{s} \cong \mathbf{5.4\,min}$$

6-143 A refrigeration system is to cool bread loaves at a rate of 500 per hour by refrigerated air at -30°C. The rate of heat removal from the breads, the required volume flow rate of air, and the size of the compressor of the refrigeration system are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of the bread loaves are constant. **3** The cooling section is well-insulated so that heat gain through its walls is negligible.

Properties The average specific and latent heats of bread are given to be 2.93 kJ/kg.°C and 109.3 kJ/kg, respectively. The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1), and the specific heat of air at the average temperature of (-30 + -22)/2 = -26°C ≈ 250 K is C_p =1.0 kJ/kg.°C (Table A-2).

Analysis (a) Noting that the breads are cooled at a rate of 500 loaves per hour, breads can be considered to flow steadily through the cooling section at a mass flow rate of

$$\dot{m}_{\text{bread}} = (500\ \text{breads/h})(0.45\ \text{kg/bread}) = 225\ \text{kg/h} = 0.3056\ \text{kg/s}$$

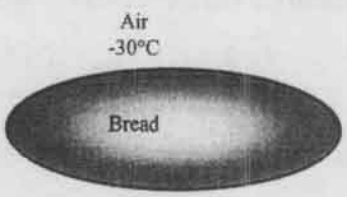
Then the rate of heat removal from the breads as they are cooled from 22°C to -10°C and frozen becomes

$$\dot{Q}_{\text{bread}} = (\dot{m}C_p\Delta T)_{\text{bread}} = (225\ \text{kg/h})(2.93\ \text{kJ/kg.°C})[(22-(-10)]\text{°C} = 21{,}096\ \text{kJ/h}$$

$$\dot{Q}_{\text{freezing}} = (\dot{m}h_{\text{latent}})_{\text{bread}} = (225\ \text{kg/h})(109.3\ \text{kJ/kg}) = 24{,}593\ \text{kJ/h}$$

and

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{bread}} + \dot{Q}_{\text{freezing}} = 21{,}096 + 24{,}593 = \mathbf{45{,}689\ kJ/h}$$



(b) All the heat released by the breads is absorbed by the refrigerated air, and the temperature rise of air is not to exceed 8°C. The minimum mass flow and volume flow rates of air are determined to be

$$m_{air} = \frac{\dot{Q}_{air}}{(C_p\Delta T)_{air}} = \frac{45{,}689\ \text{kJ/h}}{(1.0\ \text{kJ/kg.°C})(8\text{°C})} = 5{,}711\ \text{kg/h}$$

$$\rho = \frac{P}{RT} = \frac{101.3\ \text{kPa}}{(0.287\ \text{kPa.m}^3/\text{kg.K})(-30+273)\ \text{K}} = 1.45\ \text{kg/m}^3$$

$$\dot{V}_{air} = \frac{\dot{m}_{air}}{\rho_{air}} = \frac{5711\ \text{kg/h}}{1.45\ \text{kg/m}^3} = 3939\ \text{m}^3/\text{h}$$

(c) For a COP of 1.2, the size of the compressor of the refrigeration system must be

$$\dot{W}_{\text{refrig}} = \frac{\dot{Q}_{\text{refrig}}}{COP} = \frac{45{,}689\ \text{kJ/h}}{1.2} = 38{,}074\ \text{kJ/h} = 10.6\ \text{kW}$$

-144 The drinking water needs of a production facility with 20 employees is to be met by a bobbler type vater fountain. The size of compressor of the refrigeration system of this water cooler is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Water is an incompressible substance with constant properties at room temperature. **3** The cold water requirement is 0.4 L/h per person.

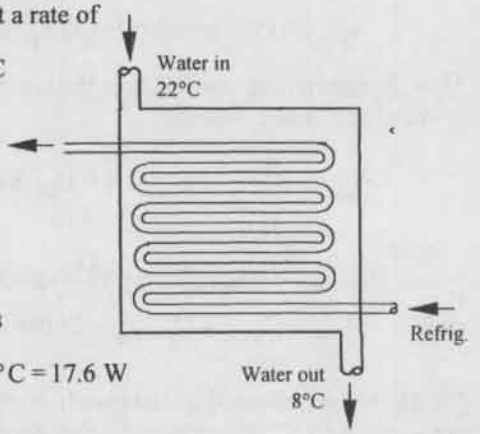
Properties The density and specific heat of water at room temperature are ρ = 1.0 kg/L and C = 4.18 kJ/kg.°C.C (Table A-3).

Analysis The refrigeration load in this case consists of the heat gain of the reservoir and the cooling of the incoming water. The water fountain must be able to provide water at a rate of

$$\dot{m}_{\text{water}} = \rho \dot{V}_{\text{water}} = (1\ \text{kg/L})(0.4\ \text{L/h}\cdot\text{person})(20\ \text{persons}) = 8.0\ \text{kg/h}$$

To cool this water from 22°C to 8°C, heat must removed from the water at a rate of

$$\dot{Q}_{\text{cooling}} = \dot{m}C_p(T_{\text{in}} - T_{\text{out}}) = (8.0\ \text{kg/h})(4.18\ \text{kJ/kg.°C})(22-8)°\text{C}$$
$$= 468\ \text{kJ/h} = 130\ \text{W} \quad (\text{since } 1\ \text{W} = 3.6\ \text{kJ/h})$$



The heat transfer surface area of the water reservoir is

$$A = 2A_{\text{base}} + A_{\text{side}} = 2(\pi D^2/4) + (\pi Dh)$$
$$= 2\pi(0.20\ \text{m})^2/4 + \pi(0.20\ \text{m})(0.25\ \text{m}) = 0.220\ \text{m}^2$$

The rate of heat gain of the water in the reservoir from the surroundings is

$$\dot{Q}_{\text{transfer}} = hA(T_{\text{surr}} - T_{\text{surface}}) = (10\ \text{W/m}^2.\text{C})(0.220\ \text{m}^2)(25-17)°\text{C} = 17.6\ \text{W}$$

Then the total refrigeration load becomes

$$\dot{Q}_{\text{refrig, total}} = \dot{Q}_{\text{cooling}} + \dot{Q}_{\text{transfer}} = 130 + 17.6 = 147.6\ \text{W}$$

Noting that the coefficient of performance of the refrigeration system is 2.9, the required power input is

$$\dot{W}_{\text{refrig}} = \frac{\dot{Q}_{\text{refrig}}}{\text{COP}} = \frac{147.6\ \text{W}}{2.9} = \mathbf{50.9\ W}$$

Therefore, the power rating of the compressor of this refrigeration system must be at least 50.9 W to meet the cold water requirements of this office.

6-145 A washing machine uses $85/year worth of hot water heated by an electric water heater. The amount of hot water an average family uses per week is to be determined. √

Assumptions **1** The electricity consumed by the motor of the washer is negligible. **2** Water is an incompressible substance with constant properties at room temperature.

Properties The density and specific heat of water at room temperature are $\rho = 1.0$ kg/L and $C = 4.18$ kJ/kg.°C (Table A-3).

Analysis The amount of electricity used to heat the water and the net amount transferred to water are

$$\text{Total energy used (electrical)} = \frac{\text{Total cost of energy}}{\text{Unit cost of energy}} = \frac{\$85/\text{year}}{\$0.082/\text{kWh}} = 1037\ \text{kWh/year}$$

$$\text{Total energy transfer to water} = \dot{E}_{\text{in}} = (\text{Efficiency})(\text{Total energy used}) = 0.9 \times 1037\ \text{kWh/year}$$

$$= 933.3\ \text{kWh/year} = (933.3\ \text{kWh/year})\left(\frac{3600\ \text{kJ}}{1\ \text{kWh}}\right)\left(\frac{1\ \text{year}}{52\ \text{weeks}}\right)$$

$$= 64{,}613\ \text{kJ/week}$$

Then the mass and the volume of hot water used per week become

$$\dot{E}_{\text{in}} = \dot{m}C(T_{\text{out}} - T_{\text{in}}) \rightarrow \dot{m} = \frac{\dot{E}_{\text{in}}}{C(T_{\text{out}} - T_{\text{in}})} = \frac{64{,}613\ \text{kJ/week}}{(4.18\ \text{kJ/kg.°C})(55-15)°\text{C}} = 386\ \text{kg/week}$$

and

$$\dot{V}_{\text{water}} = \frac{\dot{m}}{\rho} = \frac{386\ \text{kg/week}}{1\ \text{kg/L}} = \mathbf{386\ L/week}$$

Therefore, an average family uses 386 liters of hot water per week for washing clothes.

6-146E A washing machine uses $33/year worth of hot water heated by a gas water heater. The amount of hot water an average family uses per week is to be determined.

Assumptions **1** The electricity consumed by the motor of the washer is negligible. **2** Water is an incompressible substance with constant properties at room temperature.

Properties The density and specific heat of water at room temperature are $\rho = 62.1$ lbm/ft^3 and $C = 1.00$ Btu/lbm.°F (Table A-3E).

Analysis The amount of electricity used to heat the water and the net amount transferred to water are

$$\text{Total energy used (gas)} = \frac{\text{Total cost of energy}}{\text{Unit cost of energy}} = \frac{\$33/\text{year}}{\$0.605/\text{therm}} = 54.55 \text{ therms/year}$$

$$\text{Total energy transfer to water} = \dot{E}_{\text{in}} = (\text{Efficiency})(\text{Total energy used}) = 0.55 \times 54.55 \text{ therms/year}$$

$$= 30.0 \text{ therms/year} = (30.0 \text{ therms/year})\left(\frac{100{,}000 \text{ Btu}}{1 \text{ therm}}\right)\left(\frac{1 \text{ year}}{52 \text{ weeks}}\right)$$

$$= 57{,}692 \text{ Btu/week}$$

Then the mass and the volume of hot water used per week become

$$\dot{E}_{\text{in}} = \dot{m}C(T_{\text{out}} - T_{\text{in}}) \quad \rightarrow \quad \dot{m} = \frac{\dot{E}_{\text{in}}}{C(T_{\text{out}} - T_{\text{in}})} = \frac{57{,}692 \text{ Btu/week}}{(1.0 \text{ Btu/lbm.°F})(130 - 60)°\text{F}} = 824 \text{ lbm/week}$$

and

$$\dot{V}_{\text{water}} = \frac{\dot{m}}{\rho} = \frac{824 \text{ lbm/week}}{62.1 \text{ lbm/ft}^3} = 13.27 \text{ ft}^3\left(\frac{7.48 \text{ gal}}{1 \text{ ft}^3}\right) = \mathbf{99.3 \text{ gal/week}}$$

Therefore, an average family uses about 100 gallons of hot water per week for washing clothes.

6-147 A typical heat pump powered water heater costs about $800 more to install than a typical electric water heater. The number of years it will take for the heat pump water heater to pay for its cost differential from the energy it saves is to be determined.

Assumptions **1** The price of electricity remains constant. **2** Water is an incompressible substance with constant properties at room temperature. **3** Time value of money (interest, inflation) is not considered.

Properties The density and specific heat of water at room temperature are $\rho = 1.0$ kg/L and $C = 4.18$ kJ/kg.°C (Table A-3).

Analysis The amount of electricity used to heat the water and the net amount transferred to water are

$$\text{Total energy used (electrical)} = \frac{\text{Total cost of energy}}{\text{Unit cost of energy}} = \frac{\$390/\text{year}}{\$0.080/\text{kWh}} = 4875 \text{ kWh/year}$$

$$\text{Total energy transfer to water} = \dot{E}_{\text{in}} = (\text{Efficiency})(\text{Total energy used}) = 0.9 \times 4875 \text{ kWh/year}$$
$$= 4388 \text{ kWh/year}$$

The amount of electricity consumed by the heat pump and its cost are

$$\text{Energy usage (of heat pump)} = \frac{\text{Energy transfer to water}}{\text{COP}_{\text{HP}}} = \frac{4388 \text{ kWh/year}}{2.2} = 1994 \text{ kWh/year}$$

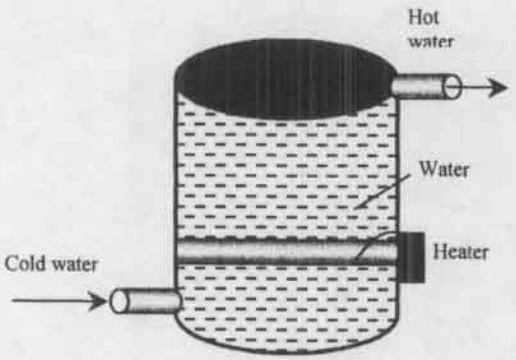
$$\text{Energy cost (of heat pump)} = (\text{Energy usage})(\text{Unit cost of energy}) = (1994 \text{ kWh/year})(\$0.08/\text{kWh})$$
$$= \$159.5/\text{year}$$

Then the money saved per year by the heat pump and the simple payback period become

$$\text{Money saved} = (\text{Energy cost of electric heater}) - (\text{Energy cost of heat pump})$$
$$= \$390 - \$159.50 = \$230.50$$

$$\text{Simple payback period} = \frac{\text{Additional installation cost}}{\text{Money saved}} = \frac{\$800}{\$230.50/\text{year}} = \mathbf{3.5 \text{ years}}$$

Discussion The economics of heat pump water heater will be even better if the air in the house is used as the heat source for the heat pump in summer, and thus also serving as an air-conditioner.



6-148E The energy contents, unit costs, and typical conversion efficiencies of various energy sources for use in water heaters are given. The lowest cost energy source is to be determined.

Assumptions The differences in installation costs of different water heaters are not considered.

Properties The energy contents, unit costs, and typical conversion efficiencies of different systems are given in the problem statement.

Analysis The unit cost of each Btu of useful energy supplied to the water heater by each system can be determined from

$$\text{Unit cost of useful energy} = \frac{\text{Unit cost of energy supplied}}{\text{Conversion efficiency}}$$

Substituting,

Natural gas heater: $$\text{Unit cost of useful energy} = \frac{\$0.0060/\text{ft}^3}{0.55}\left(\frac{1\,\text{ft}^3}{1025\,\text{Btu}}\right) = \$10.6\times10^{-6}\,/\,\text{Btu}$$

Heating oil heater: $$\text{Unit cost of useful energy} = \frac{\$1.15/\text{gal}}{0.55}\left(\frac{1\,\text{gal}}{138{,}700\,\text{kJ}}\right) = \$15.1\times10^{-6}\,/\,\text{Btu}$$

Electric heater: $$\text{Unit cost of useful energy} = \frac{\$0.084/\text{kWh})}{0.90}\left(\frac{1\,\text{kWh}}{3412\,\text{Btu}}\right) = \$27.4\times10^{-6}\,/\,\text{Btu}$$

Therefore, the lowest cost energy source for hot water heaters in this case is **natural gas**.

6-149E A home owner is considering three different heating systems for heating his house. The system with the lowest energy cost is to be determined.

Assumptions The differences in installation costs of different heating systems are not considered.

Properties The energy contents, unit costs, and typical conversion efficiencies of different systems are given in the problem statement.

Analysis The unit cost of each Btu of useful energy supplied to the house by each system can be determined from

$$\text{Unit cost of useful energy} = \frac{\text{Unit cost of energy supplied}}{\text{Conversion efficiency}}$$

Substituting,

Natural gas heater: $$\text{Unit cost of useful energy} = \frac{\$0.62/\text{therm}}{0.87}\left(\frac{1\,\text{therm}}{105{,}500\,\text{Btu}}\right) = \$6.8\times10^{-6}\,/\,\text{kJ}$$

Heating oil heater: $$\text{Unit cost of useful energy} = \frac{\$1.25/\text{gal}}{0.87}\left(\frac{1\,\text{gal}}{138{,}500\,\text{kJ}}\right) = \$10.4\times10^{-6}\,/\,\text{kJ}$$

Electric heater: $$\text{Unit cost of useful energy} = \frac{\$0.09/\text{kWh})}{1.0}\left(\frac{1\,\text{kWh}}{3600\,\text{kJ}}\right) = \$25.0\times10^{-6}\,/\,\text{kJ}$$

Therefore, the system with the lowest energy cost for heating the house is the **natural gas heater**.

6-150 A home owner is to choose between a high-efficiency natural gas furnace and a ground-source heat pump. The system with the lower energy cost is to be determined.

Assumptions The two heater are comparable in all aspects other than the cost of energy.

Analysis The unit cost of each kJ of useful energy supplied to the house by each system is

Natural gas furnace: $$\text{Unit cost of useful energy} = \frac{(\$0.71/\text{therm})}{0.97}\left(\frac{1\,\text{therm}}{105{,}500\,\text{kJ}}\right) = \$6.94\times10^{-6}\,/\,\text{kJ}$$

Heat Pump System: $$\text{Unit cost of useful energy} = \frac{(\$0.092/\text{kWh})}{3.7}\left(\frac{1\,\text{kWh}}{3600\,\text{kJ}}\right) = \$6.91\times10^{-6}\,/\,\text{kJ}$$

The two results are practically identical to each other. Therefore, the energy cost of both systems will be **about the same**.

6-151 The maximum flow rate of a standard shower head can be reduced from 13.3 to 10.5 L/min by switching to low-flow shower heads. The amount of oil and money a family of four will save per year by replacing the standard shower heads by the low-flow ones are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Showers operate at maximum flow conditions during the entire shower. **3** Each member of the household takes a 5-min shower every day.

Properties The specific heat of water is C = 4.18 kJ/kg.°C and heating value of heating oil is 146,300 kJ/gal (given). The density of water is ρ = 1 kg/L.

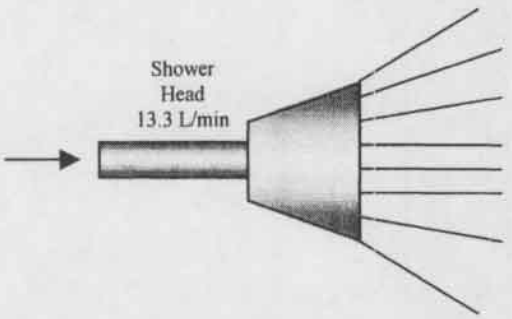
Analysis The low-flow heads will save water at a rate of

$$\dot{V}_{\text{saved}} = [(13.3-10.5)\,\text{L/min}](5\,\text{min/person.day})(4\,\text{persons})(365\,\text{days/yr}) = 20{,}440\,\text{L/year}$$
$$\dot{m}_{\text{saved}} = \rho\dot{V}_{\text{saved}} = (1\,\text{kg/L})(20{,}440\,\text{L/year}) = 20{,}440\,\text{kg/year}$$

Then the energy, fuel, and money saved per year becomes

$$\text{Energy saved} = \dot{m}_{\text{saved}}C\Delta T = (20{,}440\,\text{kg/year})(4.18\,\text{kJ/kg.°C})(42-15)°\text{C} = 2{,}307{,}000\,\text{kJ/year}$$
$$\text{Fuel saved} = \frac{\text{Energy saved}}{(\text{Efficiency})(\text{Heating value of fuel})} = \frac{2{,}307{,}000\,\text{kJ/year}}{(0.65)(146{,}300\,\text{kJ/gal})} = \mathbf{24.3\ gal/year}$$
$$\text{Money saved} = (\text{Fuel saved})(\text{Unit cost of fuel}) = (24.3\,\text{gal/year})(\$1.20/\text{gal}) = \mathbf{\$29.10/year}$$

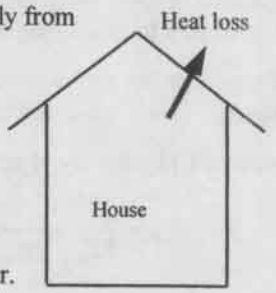
Therefore, switching to low-flow shower heads will save about $30 per year in energy costs..



6-152 The heating and cooling costs of a poorly insulated house can be reduced by up to 30 percent by adding adequate insulation. The time it will take for the added insulation to pay for itself from the energy it saves is to be determined.

Assumptions It is given that the annual energy usage of a house is $1200 a year, and 46% of it is used for heating and cooling. The cost of added insulation is given to be $200.

Analysis The amount of money that would be saved per year is determined directly from



$$\text{Money saved} = (\$1200/\text{year})(0.46)(0.30) = \$166/\text{yr}$$

Then the simple payback period becomes

$$\text{Payback period} = \frac{\text{Cost}}{\text{Money saved}} = \frac{\$200}{\$166/\text{yr}} = \mathbf{1.2\ yr}$$

Therefore, the proposed measure will pay for itself in less than one and a half year.

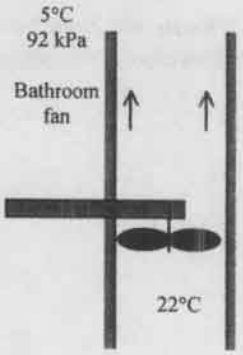
6-153 The ventilating fans of a house discharge a houseful of warmed air in one hour (ACH = 1). For an average outdoor temperature of 5°C during the heating season, the cost of energy "vented out" by the fans in 1 h is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The house is maintained at 22°C and 92 kPa at all times. **3** The infiltrating air is heated to 22°C before it is vented out. **4** Air is an ideal gas with constant specific heats at room temperature. **5** The volume occupied by the people, furniture, etc. is negligible.

Properties The gas constant of air is $R = 0.287\ \text{kPa.m}^3/\text{kg·K}$ (Table A-1). The specific heat of air at room temperature is $C_p = 1.0\ \text{kJ/kg·°C}$ (Table A-2a).

Analysis The density of air at the indoor conditions of 92 kPa and 22°C is

$$\rho_o = \frac{P_o}{RT_o} = \frac{92\ \text{kPa}}{(0.287\ \text{kPa.m}^3/\text{kg.K})(22+273\ \text{K})} = 1.087\ \text{kg/m}^3$$



Noting that the interior volume of the house is $200 \times 2.4 = 480\ \text{m}^3$, the mass flow rate of air vented out becomes

$$\dot{m}_{\text{air}} = \rho \dot{V}_{\text{air}} = (1.087\ \text{kg/m}^3)(480\ \text{m}^3/\text{h}) = 522\ \text{kg/h} = 0.145\ \text{kg/s}$$

Noting that the indoor air vented out at 22°C is replaced by infiltrating outdoor air at 5°C, this corresponds to energy loss at a rate of

$$\dot{Q}_{\text{loss,fan}} = \dot{m}_{\text{air}} C_p (T_{\text{indoors}} - T_{\text{outdoors}})$$
$$= (0.145\ \text{kg/s})(1.0\ \text{kJ/kg.°C})(22-5)°\text{C} = 2.465\ \text{kJ/s} = 2.465\ \text{kW}$$

Then the amount and cost of the heat "vented out" per hour becomes

$$\text{Fuel energy loss} = \dot{Q}_{\text{loss,fan}} \Delta t / \eta_{\text{furnace}} = (2.465\ \text{kW})(720\ \text{h/month})/0.96 = \mathbf{1849\ kWh/month}$$

$$\text{Money loss} = (\text{Fuel energy loss})(\text{Unit cost of energy})$$

$$= (1849\ \text{kWh/month})(\$0.60/\text{therm})\frac{1\ \text{therm}}{29.3\ \text{kWh}} = \mathbf{\$37.86/month}$$

Discussion Note that the energy and money loss associated with ventilating fans can be very significant. Therefore, ventilating fans should be used sparingly.

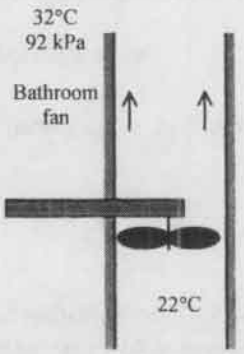
6-154 The ventilating fans of a house discharge a houseful of air-conditioned air in one hour (ACH = 1). For an average outdoor temperature of 32°C during the cooling season, the cost of energy "vented out" by the fans in 1 h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The house is maintained at 22°C and 92 kPa at all times. **3** The infiltrating air is cooled to 22°C before it is vented out. **4** Air is an ideal gas with constant specific heats at room temperature. **5** The volume occupied by the people, furniture, etc. is negligible. **6** Latent heat load is negligible.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg·K (Table A-1). The specific heat of air at room temperature is $C_p = 1.0$ kJ/kg·°C (Table A-2a).

Analysis The density of air at the indoor conditions of 92 kPa and 22°C is

$$\rho_o = \frac{P_o}{RT_o} = \frac{92\ \text{kPa}}{(0.287\ \text{kPa.m}^3/\text{kg.K})(22+273\ \text{K})} = 1.087\ \text{kg/m}^3$$



Noting that the interior volume of the house is $200 \times 2.4 = 480$ m^3, the mass flow rate of air vented out becomes

$$\dot{m}_{\text{air}} = \rho \dot{V}_{\text{air}} = (1.087\ \text{kg/m}^3)(480\ \text{m}^3/\text{h}) = 522\ \text{kg/h} = 0.145\ \text{kg/s}$$

Noting that the indoor air vented out at 22°C is replaced by infiltrating outdoor air at 32°C, this corresponds to energy loss at a rate of

$$\dot{Q}_{\text{loss,fan}} = \dot{m}_{\text{air}} C_p (T_{\text{outdoors}} - T_{\text{indoors}})$$
$$= (0.145\ \text{kg/s})(1.0\ \text{kJ/kg.°C})(32-22)\text{°C} = 1.45\ \text{kJ/s} = 1.45\ \text{kW}$$

Then the amount and cost of the electric energy "vented out" per hour becomes

Electric energy loss = $\dot{Q}_{\text{loss,fan}} \Delta t / COP = (1.45\ \text{kW})(720\ \text{h/month})/3.2 =$ **326 kWh/month**

Money loss = (Fuel energy loss)(Unit cost of energy)

= (326 kWh/month)($0.10 / kWh) = **$32.6/month**

Discussion Note that the energy and money loss associated with ventilating fans can be very significant. Therefore, ventilating fans should be used sparingly.

6-155 Caulking and weather-stripping doors and windows to reduce air leaks can reduce the energy use of a house by up to 10 percent. The time it will take for the caulking and weather-stripping to pay for itself from the energy it saves is to be determined.

Assumptions It is given that the annual energy usage of a house is $1100 a year, and the cost of caulking and weather-stripping a house is $50..

Analysis The amount of money that would be saved per year is determined directly from

$$\text{Money saved} = (\$1100/\text{year})(0.10) = \$110/\text{yr}$$

Then the simple payback period becomes

$$\text{Payback period} = \frac{\text{Cost}}{\text{Money saved}} = \frac{\$50}{\$110/\text{yr}} = \mathbf{0.45\ yr}$$

Therefore, the proposed measure will pay for itself in less than half a year.

6-156 It is estimated that 570,000 barrels of oil would be saved per day if the thermostat setting in residences in winter were lowered by 6°F (3.3°C). The amount of money that would be saved per year is to be determined.

Assumptions The average heating season is given to be 180 days, and the cost of oil to be $20/barrel.

Analysis The amount of money that would be saved per year is determined directly from

$$(570{,}000\ \text{barrel/day})(180\ \text{days/year})(\$20/\text{barrel}) = \mathbf{\$2{,}052{,}000{,}000}$$

Therefore, the proposed measure will save more than 2-billion dollars a year in energy costs.

6-157 ··· 6-166 Computer, Design, and Essay Problems

6-157 The maximum work will be obtained if a Carnot heat pump is used. The sink temperature of this heat engine will remain constant at 300 K but the source temperature will be decreasing from 350 K to 300 K. Then the thermal efficiency of the Carnot heat engine operating between pond and the ambient air can be expressed as

$$\eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{T_H}$$

where T_H is a variable. The conservation of energy relation for the pond can be written in the differential form as

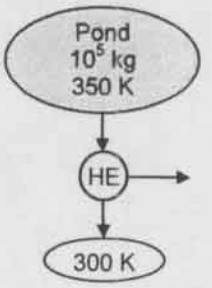
$$\partial Q_{pond} = mC\,dT_H$$

and

$$\partial Q_H = -\partial Q_{pond} = -mC\,dT_H = -(10^5\,\text{kg})(4.18\text{kJ/kg}\cdot\text{K})\,dT_H$$

Also,

$$\partial W_{net} = \eta_{th,C}\,\partial Q_H = -\left(1 - \frac{300}{T_H}\right)(10^5\,\text{kg})(4.18\text{kJ/kg}\cdot\text{K})\,\partial T_H$$



The total work output is obtained by integration,

$$W_{net} = \int_{300}^{350} \eta_{th,C}\,\partial Q_H = \int_{300}^{350} \left(1 - \frac{300}{T_H}\right)(10^5\,\text{kg})(4.18\text{kJ/kg}\cdot\text{K})\,dT_H$$

$$= 4.18\times10^5 \int_{300}^{350} \left(1 - \frac{300}{T_H}\right) dT_H = \mathbf{15.7\times10^5\,kJ}$$

which is the exact result. The values obtained by computer solution will approach this value as the temperature interval is decreased.

Chapter 7
ENTROPY – A MEASURE OF DISORDER

Entropy and the Increase of Entropy Principle

7-1C Yes. Because we used the relation $(Q_H/T_H) = (Q_L/T_L)$ in the proof, which is the defining relation of absolute temperature.

7-2C No. The $\oint \delta Q$ represents the net heat transfer during a cycle, which could be positive.

7-3C Yes.

7-4C No. A system may reject more (or less) heat than it receives during a cycle. The steam in a steam power plant, for example, receives more heat than it rejects during a cycle.

7-5C No. A system may produce more (or less) work than it receives during a cycle. A steam power plant, for example, produces more work than it receives during a cycle, the difference being the net work output.

7-6C The entropy change will be the same for both cases since entropy is a property and it has a fixed value at a fixed state.

7-7C No. In general, that integral will have a different value for different processes. However, it will have the same value for all reversible processes.

7-8C Yes.

7-9C That integral should be performed along a reversible path to determine the entropy change.

7-10C No. An isothermal process can be irreversible. Example: A system that involves paddle-wheel work while losing an equivalent amount of heat.

7-11C The value of this integral is always larger for reversible processes.

7-12C No. Because the entropy of the surrounding air increases even more during that process, making the total entropy change positive.

7-13C It is possible to create entropy, but it is not possible to destroy it.

7-14C Sometimes.

7-15C Never.

7-16C Always.

7-17C Increase.

7-18C Increases.

7-19C Decreases.

7-20C Sometimes.

7-21C Yes. This will happen when the system is losing heat, and the decrease in entropy as a result of this heat loss is equal to the increase in entropy as a result of irreversibilities.

7-22C They are heat transfer, irreversibilities, and entropy transport with mass.

7-23C Greater than.

7-24C No, because more entropy is generated during the cleaning process.

7-25C No, because more entropy is generated during that process. For example, the generation of light in the sun involves entropy generation.

7-26C No, because more entropy is generated during that process such as the entropy generated during the metabolism in the cells of the body.

7-27 A rigid tank contains an ideal gas that is being stirred by a paddle wheel. The temperature of the gas remains constant as a result of heat transfer out. The entropy change of the gas is to be determined.

Assumptions The gas in the tank is given to be an ideal gas.

Analysis The temperature and the specific volume of the gas remain constant during this process. Therefore, the initial and the final states of the gas are the same. Then $s_2 = s_1$ since entropy is a property. Therefore,

$$\Delta S_{sys} = \mathbf{0}$$



7-28 Air is compressed steadily by a compressor. The air temperature is maintained constant by heat rejection to the surroundings. The rate of entropy change of air is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas. **4** The process involves no internal irreversibilities such as friction, and thus it is an isothermal, internally reversible process.

Properties Noting that $h = h(T)$ for ideal gases, we have $h_1 = h_2$ since $T_1 = T_2 = 25°C$.

Analysis We take the compressor as the system. Noting that the enthalpy of air remains constant, the energy balance for this steady-flow system can be expressed in the rate form as

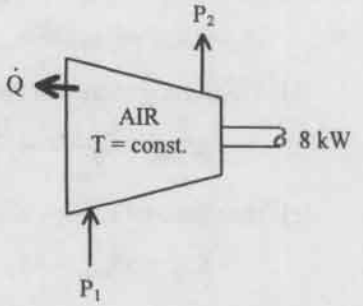
$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} = \dot{Q}_{out}$$

Therefore,

$$\dot{Q}_{out} = \dot{W}_{in} = 8 \text{ kW}$$



Noting that the process is assumed to be an isothermal and internally reversible process, the rate of entropy change of air is determined to be

$$\Delta \dot{S}_{air} = -\frac{\dot{Q}_{out,air}}{T_{sys}} = -\frac{8 \text{ kW}}{298 \text{ K}} = \mathbf{-0.0268 \text{ kW/K}}$$

7-29 Heat is transferred isothermally from a source to the working fluid of a Carnot engine. The entropy change of the working fluid, the entropy change of the source, and the total entropy generation during this process are to be determined. √

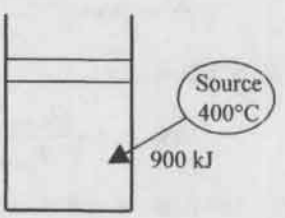
Analysis (*a*) This is a reversible isothermal process, and the entropy change during such a process is given by

$$\Delta S = \frac{Q}{T}$$

Noting that heat transferred from the source is equal to the heat transferred to the working fluid, the entropy changes of the fluid and of the source become

$$\Delta S_{fluid} = \frac{Q_{fluid}}{T_{fluid}} = \frac{Q_{in,fluid}}{T_{fluid}} = \frac{900 \text{ kJ}}{673 \text{ K}} = \mathbf{1.337 \text{ kJ/K}}$$

(*b*) $$\Delta S_{source} = \frac{Q_{source}}{T_{source}} = -\frac{Q_{\text{out, source}}}{T_{source}} = -\frac{900 \text{ kJ}}{673 \text{ K}} = \mathbf{-1.337 \text{ kJ/K}}$$



(*c*) Thus the total entropy change or entropy generation of the process is

$$S_{gen} = \Delta S_{\text{total}} = \Delta S_{fluid} + \Delta S_{source} = 1.337 - 1.337 = \mathbf{0}$$

7-30E Heat is transferred isothermally from the working fluid of a Carnot engine to a heat sink. The entropy change of the working fluid is given. The amount of heat transfer, the entropy change of the sink, and the total entropy change during the process are to be determined.

Analysis (*a*) This is a reversible isothermal process, and the entropy change during such a process is given by

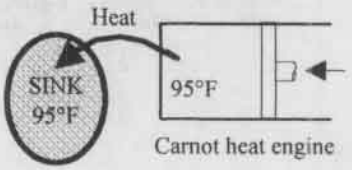
$$\Delta S = \frac{Q}{T}$$

Noting that heat transferred from the working fluid is equal to the heat transferred to the sink, the heat transfer and the entropy change of the heat sink become

$$Q_{fluid} = T_{fluid}\Delta S_{fluid} = (555\text{R})(-0.7\text{Btu/R}) = -388.5\text{Btu} \rightarrow Q_{fluid,out} = \mathbf{388.5\ Btu}$$

(*b*) The entropy change of the sink is determined from

$$\Delta S_{sink} = \frac{Q_{sink,in}}{T_{sink}} = \frac{388.5\ \text{Btu}}{555\ \text{R}} = \mathbf{0.7\ Btu/R}$$



(*c*) Thus the total entropy change of the process is

$$S_{gen} = \Delta S_{total} = \Delta S_{fluid} + \Delta S_{sink} = -0.7 + 0.7 = 0$$

This is expected since all processes of the Carnot cycle are reversible processes, and no entropy is generated during a reversible process.

7-31 R-134a enters an evaporator as a saturated liquid-vapor at a specified pressure. Heat is transferred to the refrigerant from the cooled space, and the liquid is vaporized. The entropy change of the refrigerant, the entropy change of the cooled space, and the total entropy generation for this process are to be determined.

Assumptions **1** Both the refrigerant and the cooled space involve no internal irreversibilities such as friction. **2** Any temperature change occurs within the wall of the tube, and thus both the refrigerant and the cooled space remain isothermal during this process. Thus it is an isothermal, internally reversible process.

Properties Noting that $h = h(T)$ for ideal gases, we have $h_1 = h_2$ since $T_1 = T_2 = 25°\text{C}$.

Analysis Noting that both the refrigerant and the cooled space undergo reversible isothermal processes, the entropy change for them can be determined from

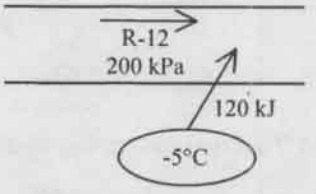
$$\Delta S = \frac{Q}{T}$$

(*a*) The pressure of the refrigerant is maintained constant. Therefore, the temperature of the refrigerant also remains constant at the saturation value,

$$T = T_{sat@200\ \text{kPa}} = -10.09°\text{C} = 263\ \text{K}$$

Then,

$$\Delta S_{refrigerant} = \frac{Q_{refrigerant,in}}{T_{refrigerant}} = \frac{120\ \text{kJ}}{263\ \text{K}} = \mathbf{0.456\ kJ/K}$$



(*b*) Similarly,

$$\Delta S_{space} = -\frac{Q_{space,out}}{T_{space}} = -\frac{120\ \text{kJ}}{268\ \text{K}} = \mathbf{-0.448\ kJ/K}$$

(*c*) The total entropy change of the process is

$$S_{gen} = S_{total} = \Delta S_{refrigerant} + \Delta S_{space} = 0.456 - 0.448 = \mathbf{0.008\ kJ/K}$$

Entropy Changes of Pure Substances

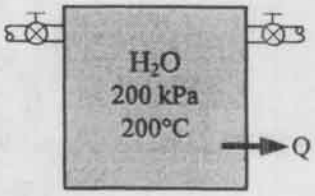
7-32C Yes, because an internally reversible, adiabatic process involves no irreversibilities or heat transfer.

7-33 The radiator of a steam heating system is initially filled with superheated steam. The valves are closed, and steam is allowed to cool until the temperature drops to a specified value by transferring heat to the room. The entropy change of the steam during this process is to be determined. √

Analysis From the steam tables,

$$\left.\begin{array}{l} P_1 = 200\text{kPa} \\ T_1 = 200^\circ\text{C} \end{array}\right\} \begin{array}{l} v_1 = 1.0803\text{m}^3/\text{kg} \\ s_1 = 7.5066\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_1 = 80^\circ\text{C} \\ v_2 = v_1 \end{array}\right\} x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{1.0803 - 0.001029}{3.407 - 0.001029} = 0.317$$



$$\begin{aligned} s_2 &= s_f + x_2 s_{fg} = 1.0753 + (0.317)(6.5369) \\ &= 3.1475\text{kJ/kg}\cdot\text{K} \end{aligned}$$

The mass of the steam is

$$m = \frac{V}{v_1} = \frac{0.020\text{m}^3}{1.0803\text{m}^3/\text{kg}} = 0.0185\text{kg}$$

Then the entropy change of the steam during this process becomes

$$\Delta S = m(s_2 - s_1) = (0.0185\text{ kg})(3.1475 - 7.5066)\text{ kJ/kg}\cdot\text{K} = \mathbf{-0.0806\ kJ/K}$$

7-34 A rigid tank is initially filled with a saturated mixture of R-134a. Heat is transferred to the tank from a source until the pressure inside rises to a specified value. The entropy change of the refrigerant, entropy change of the source, and the total entropy change for this process are to be determined. √

Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions.

Analysis (*a*) From the refrigerant tables (Tables A-11 through A-13),

$$\left.\begin{array}{l} P_1 = 200\text{kPa} \\ x_1 = 0.4 \end{array}\right\} \begin{array}{l} u_1 = u_f + x_1 u_{fg} = 36.69 + (0.4)(221.43 - 36.69) = 110.59\text{kJ/kg} \\ s_1 = s_f + x_1 s_{fg} = 0.1481 + (0.4)(0.9253 - 0.1481) = 0.4590\text{kJ/kg} \cdot \text{K} \\ v_1 = v_f + x_1 v_{fg} = 0.0007532 + (0.4)(0.0993 - 0.0007532) = 0.04017\text{m}^3/\text{kg} \end{array}$$

$$\left.\begin{array}{l} P_2 = 400\text{kPa} \\ v_2 = v_1 \end{array}\right\} \begin{array}{l} x_2 = \dfrac{v_2 - v_f}{v_{fg}} = \dfrac{0.04017 - 0.0007904}{0.0509 - 0.0007904} = 0.7859 \\ u_2 = u_f + x_2 u_{fg} = 61.69 + (0.7859)(231.97 - 61.69) = 195.51\text{kJ/kg} \\ s_2 = s_f + x_2 s_{fg} = 0.2399 + (0.7859)(0.9145 - 0.2399) = 0.7701\text{kJ/kg} \cdot \text{K} \end{array}$$

The mass of the refrigerant is

$$m = \frac{V}{v_1} = \frac{0.5\text{m}^3}{0.04017\text{m}^3/\text{kg}} = 12.45\text{ kg}$$

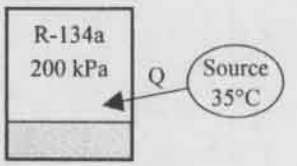
Then the entropy change of the refrigerant becomes

$$\Delta S_{system} = m(s_2 - s_1) = (12.45\text{kg})(0.7701 - 0.4590)\text{kJ/kg} \cdot \text{K} = \mathbf{3.873kJ/K}$$

(*b*) We take the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} = \Delta U = m(u_2 - u_1)$$



Substituting,

$$Q_{in} = m(u_2 - u_1) = (12.45\text{kg})(195.51 - 110.59) = 1057\text{ kJ}$$

The heat transfer for the source is equal in magnitude but opposite in direction. Therefore, $Q_{\text{source, out}} = -Q_{\text{tank, in}} = -1057$ kJ, and

$$\Delta S_{\text{source}} = -\frac{Q_{\text{source,out}}}{T_{\text{source}}} = -\frac{1057\text{ kJ}}{308\text{ K}} = \mathbf{-3.432kJ/K}$$

(*c*) The total entropy change for this process is

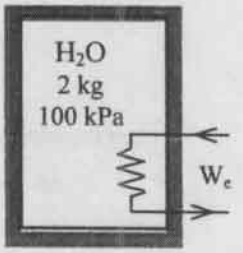
$$\Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{source}} = 3.873 + (-3.432) = \mathbf{0.441\ kJ/K}$$

7-35 An insulated rigid tank contains a saturated liquid-vapor mixture of water at a specified pressure. An electric heater inside is turned on and kept on until all the liquid vaporized. The entropy change of the water during this process is to be determined.

Analysis From the steam tables (Tables A-4 through A-6)

$$\left.\begin{array}{l} P_1 = 100\text{kPa} \\ x_1 = 0.25 \end{array}\right\} \begin{array}{l} v_1 = v_f + x_1 v_{fg} = 0.001 + (0.25)(1.694 - 0.001) = 0.4243\text{m}^3/\text{kg} \\ s_1 = s_f + x_1 s_{fg} = 1.3026 + (0.25)(6.0568) = 2.8168\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} v_2 = v_1 \\ sat.vapor \end{array}\right\} s_2 = 6.8649\text{kJ/kg}\cdot\text{K}$$



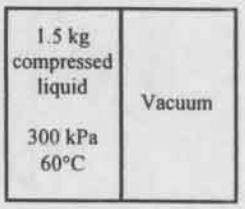
Then the entropy change of the steam becomes

$$\Delta S = m(s_2 - s_1) = (2\text{kg})(6.8649 - 2.8168)\,\text{kJ/kg}\cdot\text{K} = \mathbf{8.0962\ kJ/K}$$

7-36 A rigid tank is divided into two equal parts by a partition. One part is filled with compressed liquid water while the other side is evacuated. The partition is removed and water expands into the entire tank. The entropy change of the water during this process is to be determined. √

Analysis The properties of the water are

$$\left.\begin{array}{l} P_1 = 300\text{kPa} \\ T_1 = 60^\circ\text{C} \end{array}\right\} \begin{array}{l} v_1 \cong v_{f@60^\circ\text{C}} = 0.001017\text{m}^3/\text{kg} \\ s_1 = s_{f@60^\circ\text{C}} = 0.8312\text{kJ/kg}\cdot\text{K} \end{array}$$



Noting that $v_2 = 2v_1 = (2)(0.001017) = 0.002034\text{m}^3/\text{kg}$

$$\left.\begin{array}{l} P_2 = 15\text{kPa} \\ v_2 = 0.002034 \end{array}\right\} \begin{array}{l} x_2 = \dfrac{v_2 - v_f}{v_{fg}} = \dfrac{0.002034 - 0.001014}{10.02 - 0.001014} = 0.0001018 \\ s_2 = s_f + x_2 s_{fg} = 0.7549 + (0.0001018)(7.2536) = 0.7556\text{kJ/kg}\cdot\text{K} \end{array}$$

Then the entropy change of the water becomes

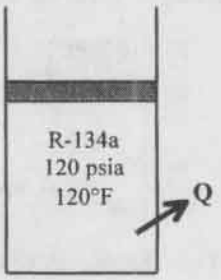
$$\Delta S = m(s_2 - s_1) = (1.5\text{kg})(0.7556 - 0.8312)\text{kJ/kg}\cdot\text{K} = \mathbf{-0.1134kJ/K}$$

7-37E A cylinder is initially filled with R-134a at a specified state. The refrigerant is cooled and condensed at constant pressure. The entropy change of refrigerant during this process is to be determined

Analysis From the refrigerant tables,

$$\left.\begin{array}{l} P_1 = 120\text{psia} \\ T_1 = 120^\circ\text{F} \end{array}\right\} \begin{array}{l} h_1 = 121.52\text{Btu/lbm} \\ s_1 = 0.2301\text{Btu/lbm} \cdot \text{R} \end{array}$$

$$\left.\begin{array}{l} T_2 = 90^\circ\text{F} \\ P_2 = 120\text{psia} \end{array}\right\} \begin{array}{l} h_2 \cong h_{f@90^\circ\text{F}} = 40.72\text{Btu/lbm} \\ s_2 \cong s_{f@90^\circ\text{F}} = 0.0836\text{Btu/lbm} \cdot \text{R} \end{array}$$



Then the entropy change of the refrigerant becomes

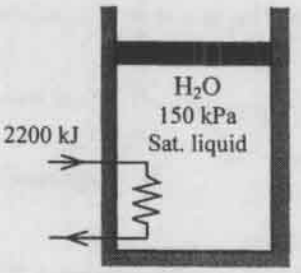
$$\Delta S = m(s_2 - s_1) = (3\text{lbm})(0.0836 - 0.2301)\text{Btu/lbm} \cdot \text{R} = \mathbf{-0.4395\text{Btu/R}}$$

7-38 An insulated cylinder is initially filled with saturated liquid water at a specified pressure. The water is heated electrically at constant pressure. The entropy change of the water during this process is to be determined.

Assumptions **1** The kinetic and potential energy changes are negligible. **2** The cylinder is well-insulated and thus heat transfer is negligible. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

Analysis From the steam tables,

$$\left.\begin{array}{l} P_1 = 150\text{kPa} \\ sat\,liquid \end{array}\right\} \begin{array}{l} v_1 = v_{f@150\text{kPa}} = 0.001053\text{m}^3/\text{kg} \\ h_1 = h_{f@150\text{kPa}} = 467.11\text{kJ/kg} \\ s_1 = s_{f@150\text{kPa}} = 1.4336\text{kJ/kg}\cdot\text{K} \end{array}$$



Also,

$$m = \frac{V}{v_1} = \frac{0.005\ \text{m}^3}{0.001053\ \text{m}^3/\text{kg}} = 4.748\ \text{kg}$$

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$W_{e,in} - W_{b,out} = \Delta U$$

$$W_{e,in} = m(h_2 - h_1)$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process. Solving for h_2,

$$h_2 = h_1 + \frac{W_{e,in}}{m} = 467.11 + \frac{2200\ \text{kJ}}{4.748\ \text{kg}} = 930.5\ \text{kJ/kg}$$

Thus,

$$\left.\begin{array}{l} P_2 = 150\text{kPa} \\ h_2 = 930.5\text{kJ/kg} \end{array}\right\} \begin{array}{l} x_2 = \dfrac{h_2 - h_f}{h_{fg}} = \dfrac{930.5 - 467.11}{2226.5} = 0.208 \\ s_2 = s_f + x_2 s_{fg} = 1.4336 + (0.208)(5.7897) = 2.638\text{kJ/kg}\cdot\text{K} \end{array}$$

Then the entropy change of the water becomes

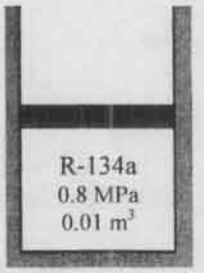
$$\Delta S = m(s_2 - s_1) = (4.748\text{kg})(2.638 - 1.4336)\text{kJ/kg}\cdot\text{K} = \mathbf{5.72kJ/K}$$

7-39 An insulated cylinder is initially filled with saturated R-134a vapor at a specified pressure. The refrigerant expands in a reversible manner until the pressure drops to a specified value. The final temperature in the cylinder and the work done by the refrigerant are to be determined. √

Assumptions **1** The kinetic and potential energy changes are negligible. **2** The cylinder is well-insulated and thus heat transfer is negligible. **3** The thermal energy stored in the cylinder itself is negligible. **4** The process is stated to be reversible.

Analysis (*a*) This is a reversible adiabatic (i.e., isentropic) process, and thus $s_2 = s_1$. From the refrigerant tables,

$$\left.\begin{array}{l} P_1 = 0.8\text{MPa} \\ sat.vapor \end{array}\right\} \begin{array}{l} v_1 = v_{g@0.8\text{MPa}} = 0.0255\text{m}^3/\text{kg} \\ u_1 = u_{g@0.8\text{MPa}} = 243.78\text{kJ/kg} \\ s_1 = s_{g@0.8\text{MPa}} = 0.9066\text{kJ/kg}\cdot\text{K} \end{array}$$



Also,

$$m = \frac{V}{v_1} = \frac{0.01\,\text{m}^3}{0.0255\,\text{m}^3/\text{kg}} = 0.392\,\text{kg}$$

and

$$\left.\begin{array}{l} P_2 = 0.4\text{MPa} \\ s_2 = s_1 \end{array}\right\} \quad x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{0.9066 - 0.2399}{0.9145 - 0.2399} = 0.988$$

$$u_2 = u_f + x_2 u_{fg} = 61.69 + (0.988)(231.97 - 61.69) = 229.93\text{kJ/kg}$$

$$T_2 = T_{sat@0.4\text{MPa}} = \mathbf{8.93^\circ C}$$

(*b*) We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this adiabatic closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$-W_{b,out} = \Delta U$$

$$W_{b,out} = m(u_1 - u_2)$$

Substituting, the work done during this isentropic process is determined to be

$$W_{b,out} = m(u_1 - u_2) = (0.392\text{kg})(243.78 - 229.93)\,\text{kJ/kg} = \mathbf{5.43kJ}$$

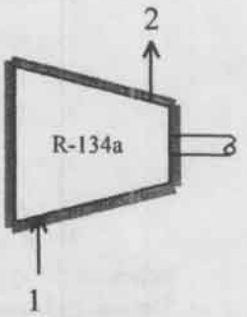
7-40 Saturated Refrigerant-134a vapor at -20°C is compressed steadily by an adiabatic compressor. The minimum power input to the compressor is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Analysis The power input to an adiabatic compressor will be a minimum when the compression process is reversible. For the reversible adiabatic process we have $s_2 = s_1$. From the refrigerant tables,

$$\left.\begin{array}{l} P_1 = 140\text{kPa} \\ sat.vapor \end{array}\right\} \begin{array}{l} v_1 = v_{g@140\text{kPa}} = 0.1395\text{m}^3/\text{kg} \\ h_1 = h_{g@140\text{kPa}} = 236.04\text{kJ/kg} \\ s_1 = s_{g@140kPa} = 0.9322\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_1 = 700\text{kPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 269.23\text{kJ/kg}$$



Also,

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{2\text{ m}^3/\text{min}}{0.1395\text{ m}^3/\text{kg}} = 14.34\text{ kg/min} = 0.239\text{ kg/s}$$

There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1)$$

Substituting, the minimum power supplied to the compressor is determined to be

$$\dot{W}_{in} = (0.239\text{kg/s})(269.23 - 236.04)\text{kJ/kg} = \mathbf{7.93kW}$$

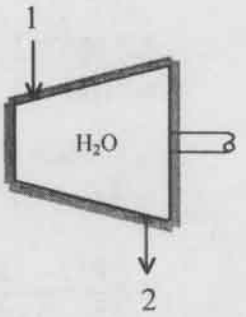
7-41E Steam expands in an adiabatic turbine. The maximum amount of work that can be done by the turbine is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Analysis The work output of an adiabatic turbine is maximum when the expansion process is reversible. For the reversible adiabatic process we have $s_2 = s_1$. From the steam tables,

$$\left.\begin{array}{l} P_1 = 800\text{psia} \\ T_1 = 900^\circ\text{F} \end{array}\right\} \begin{array}{l} h_1 = 1455.6\text{Btu/lbm} \\ s_1 = 1.6408\text{Btu/lbm}\cdot\text{R} \end{array}$$

$$\left.\begin{array}{l} P_2 = 40\text{psia} \\ s_2 = s_1 \end{array}\right\} \begin{array}{l} x_2 = \dfrac{s_2 - s_f}{s_{fg}} = \dfrac{1.6408 - 0.39214}{1.2845} = 0.972 \\ h_2 = h_f + x_2 h_{fg} = 236.16 + (0.972)(933.8) = 1143.8\text{Btu/lbm} \end{array}$$



There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{\text{out}} + \dot{m}h_2$$

$$\dot{W}_{\text{out}} = \dot{m}(h_1 - h_2)$$

Dividing by mass flow rate and substituting,

$$w_{out} = h_1 - h_2 = 1455.6 - 1143.8 = \mathbf{311.8\ Btu/lbm}$$

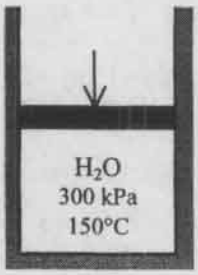
7-42 An insulated cylinder is initially filled with superheated steam at a specified state. The steam is compressed in a reversible manner until the pressure drops to a specified value. The work input during this process is to be determined. √

Assumptions **1** The kinetic and potential energy changes are negligible. **2** The cylinder is well-insulated and thus heat transfer is negligible. **3** The thermal energy stored in the cylinder itself is negligible. **4** The process is stated to be reversible.

Analysis This is a reversible adiabatic (i.e., isentropic) process, and thus $s_2 = s_1$. From the steam tables,

$$\left.\begin{array}{l} P_1 = 300\ \text{kPa} \\ T_1 = 150^\circ\text{C} \end{array}\right\} \begin{array}{l} v_1 = 0.6339\ \text{m}^3/\text{kg} \\ u_1 = 2570.8\ \text{kJ/kg} \\ s_1 = 7.0778\ \text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 1\ \text{MPa} \\ s_2 = s_1 \end{array}\right\} u_2 = 2774.2\ \text{kJ/kg}$$



Also,

$$m = \frac{V}{v_1} = \frac{0.05\ \text{m}^3}{0.6339\ \text{m}^3/\text{kg}} = 0.0789\ \text{kg}$$

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this adiabatic closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$W_{b,in} = \Delta U = m(u_2 - u_1)$$

Substituting, the work input during this adiabatic process is determined to be

$$W_{b,in} = m(u_2 - u_1) = (0.0789\text{kg})(2774.2 - 2570.8)\text{kJ/kg} = \mathbf{16.05kJ}$$

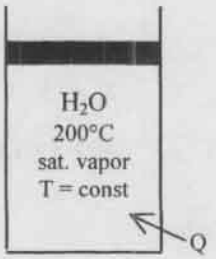
7-43 A cylinder is initially filled with saturated water vapor at a specified temperature. Heat is transferred to the steam, and it expands in a reversible and isothermal manner until the pressure drops to a specified value. The heat transfer and the work output for this process are to be determined.

Assumptions **1** The kinetic and potential energy changes are negligible. **2** The cylinder is well-insulated and thus heat transfer is negligible. **3** The thermal energy stored in the cylinder itself is negligible. **4** The process is stated to be reversible and isothermal.

Analysis From the steam tables,

$$\left.\begin{array}{l} T_1 = 200^\circ\text{C} \\ sat.vapor \end{array}\right\} \begin{array}{l} u_1 = u_{g@200^\circ\text{C}} = 2595.3\text{kJ/kg} \\ s_1 = s_{g@200^\circ\text{C}} = 6.4323\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 800\text{kPa} \\ T_2 = T_1 \end{array}\right\} \begin{array}{l} u_2 = 2630.6\text{kJ/kg} \\ s_2 = 6.8158\text{kJ/kg}\cdot\text{K} \end{array}$$



The heat transfer for this reversible isothermal process can be determined from

$$Q = T\Delta S = Tm(s_2 - s_1) = (473\text{K})(0.5\text{kg})(6.8158 - 6.4323)\text{kJ/kg}\cdot\text{K} = \mathbf{90.7kJ}$$

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} - W_{b,out} = \Delta U = m(u_2 - u_1)$$

$$W_{b,out} = Q_{in} - m(u_2 - u_1)$$

Substituting, the work done during this process is determined to be

$$W_{b,out} = 90.7\text{ kJ} - (0.5\text{kg})(2630.6 - 2595.3)\text{kJ/kg} = \mathbf{73.1kJ}$$

Entropy Change of Incompressible Substances

7-44C No, because entropy is not a conserved property.

7-45 A hot copper block is dropped into water in an insulated tank. The final equilibrium temperature of the tank and the total entropy change are to be determined. √

Assumptions **1** Both the water and the copper block are incompressible substances with constant specific heats at room temperature. **2** The system is stationary and thus the kinetic and potential energies are negligible. **3** The tank is well-insulated and thus there is no heat transfer.

Properties The density and specific heat of water at 25°C are $\rho = 997$ kg/m^3 and $C_p = 4.18$ kJ/kg.°C. The specific heat of copper at 27°C is $C_p = 0.386$ kJ/kg.°C (Table A-3).

Analysis We take the entire contents of the tank, water + copper block, as the *system*. This is a *closed system* since no mass crosses the system boundary during the process. The energy balance for this system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$0 = \Delta U$$

or,

$$\Delta U_{Cu} + \Delta U_{water} = 0$$

$$[mC(T_2 - T_1)]_{Cu} + [mC(T_2 - T_1)]_{water} = 0$$

where

$$m_{water} = \rho V = (997\text{kg/m}^3)(0.120\text{m}^3) = 120\text{kg}$$

Using specific heat values for copper and liquid water at room temperature and substituting,

$$(50\text{kg})(0.386\text{kJ/kg}\cdot^\circ\text{C})(T_2 - 80)^\circ\text{C} + (120\text{kg})(4.18\text{kJ/kg}\cdot^\circ\text{C})(T_2 - 25)^\circ\text{C} = 0$$

$$T_2 = \mathbf{27.0°C}$$

The entropy generated during this process is determined from

$$\Delta S_{\text{copper}} = mC_{ave}\ln\left(\frac{T_2}{T_1}\right) = (50\text{ kg})(0.386\text{ kJ/kg}\cdot\text{K})\ln\left(\frac{300.0\text{ K}}{353\text{ K}}\right) = -3.140\text{ kJ/K}$$

$$\Delta S_{\text{water}} = mC_{ave}\ln\left(\frac{T_2}{T_1}\right) = (120\text{ kg})(4.18\text{ kJ/kg}\cdot\text{K})\ln\left(\frac{300.0\text{ K}}{298\text{ K}}\right) = 3.355\text{ kJ/K}$$

Thus,

$$\Delta S_{total} = \Delta S_{copper} + \Delta S_{water} = -3.140 + 3.355 = \mathbf{0.215\text{ kJ/K}}$$

7-46 A hot iron block is dropped into water in an insulated tank. The amount of entropy generated during this process is to be determined.

Assumptions **1** Both the water and the iron block are incompressible substances with constant specific heats at room temperature. **2** The system is stationary and thus the kinetic and potential energies are negligible. **3** The tank is well-insulated and thus there is no heat transfer. **4** The water that evaporates, condenses back.

Properties The specific heat of water at 25°C is C_p = 4.18 kJ/kg.°C. The specific heat of iron at room temperature is C_p = 0.45 kJ/kg.°C (Table A-3).

Analysis We take the entire contents of the tank, water + iron block, as the *system*. This is a *closed system* since no mass crosses the system boundary during the process. The energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$0 = \Delta U$$



or,

$$\Delta U_{iron} + \Delta U_{water} = 0$$

$$[mC(T_2 - T_1)]_{iron} + [mC(T_2 - T_1)]_{water} = 0$$

Substituting,

$$(5\text{kg})(0.45\text{kJ/kg}\cdot\text{K})(T_2 - 350^\circ\text{C}) + (100\text{kg})(4.18\text{kJ/kg}\cdot\text{K})(T_2 - 22^\circ\text{C}) = 0$$

$$T_2 = \mathbf{23.75^\circ C}$$

The entropy generated during this process is determined from

$$\Delta S_{iron} = mC_{ave}\ln\left(\frac{T_2}{T_1}\right) = (5\text{kg})(0.45\text{kJ/kg}\cdot\text{K})\ln\left(\frac{296.7\text{K}}{623\text{K}}\right) = -1.67\text{kJ/K}$$

$$\Delta S_{water} = mC_{ave}\ln\left(\frac{T_2}{T_1}\right) = (100\text{kg})(4.184\text{kJ/kg}\cdot\text{K})\ln\left(\frac{296.7\text{K}}{295\text{K}}\right) = 2.47\text{kJ/K}$$

Thus,

$$S_{gen} = \Delta S_{total} = \Delta S_{iron} + \Delta S_{water} = -1.67 + 2.47 = \mathbf{0.80\ kJ/K}$$

Discussion The results can be improved somewhat by using specific heats at average temperature.

7-47 An aluminum block is brought into contact with an iron block in an insulated enclosure. The final equilibrium temperature and the total entropy change for this process are to be determined.

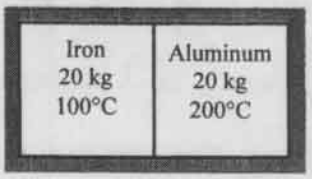
Assumptions **1** Both the aluminum and the iron block are incompressible substances with constant specific heats. **2** The system is stationary and thus the kinetic and potential energies are negligible. **3** The system is well-insulated and thus there is no heat transfer.

Properties The specific heat of aluminum at the anticipated average temperature of 450 K is C_p = 0.973 kJ/kg.°C. The specific heat of iron at room temperature (the only value available in the tables) is C_p = 0.45 kJ/kg.°C (Table A-3).

Analysis We take the iron+aluminum blocks as the system, which is a closed system. The energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$0 = \Delta U$$



or,

$$\Delta U_{\text{alum}} + \Delta U_{iron} = 0$$

$$[mC(T_2 - T_1)]_{\text{alum}} + [mC(T_2 - T_1)]_{iron} = 0$$

Substituting,

$$(20\text{kg})(0.45\text{kJ/kg}\cdot\text{K})(T_2 - 100^\circ\text{C}) + (20\text{kg})(0.973\text{kJ/kg}\cdot\text{K})(T_2 - 200^\circ\text{C}) = 0$$

$$T_2 = \mathbf{168.4^\circ C} = 441.4\text{K}$$

The entropy generated during this process is determined from

$$\Delta S_{iron} = mC_{ave}\ln\left(\frac{T_2}{T_1}\right) = (20\text{kg})(0.45\text{kJ/kg}\cdot\text{K})\ln\left(\frac{441.4\text{K}}{373\text{K}}\right) = 1.515\text{kJ/K}$$

$$\Delta S_{alum} = mC_{ave}\ln\left(\frac{T_2}{T_1}\right) = (20\text{kg})(0.973\text{kJ/kg}\cdot\text{K})\ln\left(\frac{441.4\text{K}}{473\text{K}}\right) = -1.346\text{kJ/K}$$

Thus,

$$\Delta S_{total} = \Delta S_{iron} + \Delta S_{alum} = 1.515 - 1.346 = \mathbf{0.169\ kJ/K}$$

7-48 An iron block and a copper block are dropped into a large lake. The total amount of entropy generated when both blocks cool to the lake temperature is to be determined.

Assumptions **1** Both the water and the iron block are incompressible substances with constant specific heats at room temperature. **2** Kinetic and potential energies are negligible.

Properties The specific heats of iron and copper at room temperature are $C_{p,\,iron} = 0.45$ kJ/kg.°C and $C_{p,copper} = 0.386$ kJ/kg.°C (Table A-3).

Analysis The thermal-energy capacity of the lake is very large, and thus the temperatures of both the iron and the copper blocks will drop to the lake temperature (15°C) when the thermal equilibrium is established. Then the entropy changes of the blocks become

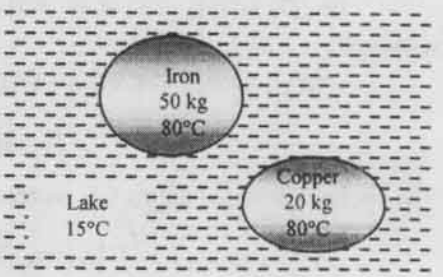
$$\Delta S_{iron} = mC_{ave}\ln\left(\frac{T_2}{T_1}\right) = (50\text{kg})(0.45\text{kJ/kg}\cdot\text{K})\ln\left(\frac{288\text{K}}{353\text{K}}\right) = -4.579\text{kJ/K}$$

$$\Delta S_{copper} = mC_{ave}\ln\left(\frac{T_2}{T_1}\right) = (20\text{kg})(0.386\text{kJ/kg}\cdot\text{K})\ln\left(\frac{288\text{K}}{353\text{K}}\right) = -1.571\text{kJ/K}$$

We take both the iron and the copper blocks, as the *system*. This is a *closed system* since no mass crosses the system boundary during the process. The energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{out} = \Delta U = \Delta U_{iron} + \Delta U_{copper}$$



or,

$$Q_{out} = [mC(T_1 - T_2)]_{iron} + [mC(T_1 - T_2)]_{copper} = 0$$

Substituting,

$$Q_{out} = (50\text{kg})(0.45\text{kJ/kg}\cdot\text{K})(353-288)\text{K} + (20\text{kg})(0.386\text{kJ/kg}\cdot\text{K})(353-288)\text{K}$$
$$= 1964\text{kJ}$$

Thus,

$$\Delta S_{lake} = \frac{Q_{lake,in}}{T_{lake}} = \frac{1964\text{kJ}}{288\text{K}} = 6.820\text{kJ/K}$$

Then the total entropy change for this process is

$$\Delta S_{total} = \Delta S_{iron} + \Delta S_{copper} + \Delta S_{lake} = -4.579 - 1.571 + 6.820 = \mathbf{0.670\text{kJ/K}}$$

Entropy Changes of Ideal Gases

7-49C For ideal gases, $C_p = C_v + R$ and

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} \longrightarrow \frac{V_2}{V_1} = \frac{T_2P_1}{T_1P_2}$$

Thus,

$$\begin{aligned} s_2 - s_1 &= C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right) \\ &= C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{T_2P_1}{T_1P_2}\right) \\ &= C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right) \\ &= C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right) \end{aligned}$$

7-50C For an ideal gas, $dh = C_p dT$ and $v = RT/P$. From the second Tds relation,

$$ds = \frac{dh}{T} - \frac{vdP}{T} = \frac{C_p dP}{T} - \frac{RT}{P}\frac{dP}{T} = C_p\frac{dT}{T} - R\frac{dP}{P}$$

Integrating,

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

since C_p is assumed to be constant.

7-51C It represents the temperature dependent part of the entropy at a specified temperature.

7-52C No. The entropy of an ideal gas depends on the pressure as well as the temperature.

7-53C Setting $\Delta s = 0$ in Eq. 7-38,

$$C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right) = 0 \longrightarrow \ln\left(\frac{T_2}{T_1}\right) = \frac{R}{C_p}\ln\left(\frac{P_2}{P_1}\right) \longrightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{R/C_p}$$

But

$$\frac{R}{C_p} = \frac{C_p - C_v}{C_p} = 1 - \frac{1}{k} = \frac{k-1}{k} \text{ since } k = C_p/C_v. \text{ Thus, } \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

7-54C The P_r and v_r are called relative pressure and relative specific volume, respectively. They are derived for isentropic processes of ideal gases, and thus their use is limited to isentropic processes only.

7-55C The entropy of a gas *can* change during an isentropic process since entropy of an ideal gas depends on the pressure as well as the temperature.

7-56C The entropy change relations of an ideal gas (Eqs. 7-37 and 7-38) simplify to

$\Delta s = C_p \ln(T_2/T_1)$ for a constant pressure process

and $\Delta s = C_v \ln(T_2/T_1)$ for a constant volume process.

Noting that $C_p > C_v$, the entropy change will be larger for a constant pressure process.

7-57 Oxygen gas is compressed from a specified initial state to a specified final state. The entropy change of oxygen during this process is to be determined for the cases of constant and variable specific heats. √

Assumptions At specified conditions, oxygen can be treated as an ideal gas.

Properties The gas constant and molar mass of oxygen are $R = 0.2598$ kJ/kg.K and $M = 32$ kg/kmol (Table A-1).

Analysis The constant volume specific heat of oxygen at the average temperature is (Table A-2)

$$T_{ave} = \frac{298 + 560}{2} = 429 \text{ K} \quad \longrightarrow \quad C_{v,ave} = 0.690 \text{ kJ/kg} \cdot \text{K}$$

Thus,

$$\begin{aligned} s_2 - s_1 &= C_{v,ave} \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \\ &= (0.690\text{kJ/kg} \cdot \text{K}) \ln \frac{560\text{K}}{298\text{K}} + (0.2598\text{kJ/kg} \cdot \text{K}) \ln \frac{0.1\text{m}^3/\text{kg}}{0.8\text{m}^3/\text{kg}} \\ &= \mathbf{-0.105\text{kJ/kg} \cdot \text{K}} \end{aligned}$$



7-58 An insulated tank contains CO_2 gas at a specified pressure and volume. A paddle-wheel in the tank stirs the gas, and the pressure and temperature of CO_2 rises. The entropy change of CO_2 during this process is to be determined using constant specific heats.

Assumptions At specified conditions, CO_2 can be treated as an ideal gas with constant specific heats at room temperature.

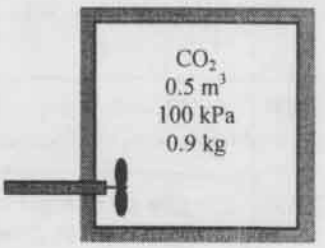
Properties The specific heat of CO_2 is $C_v = 0.657$ kJ/kg.K (Table A-2).

Analysis Using the ideal gas relation, the entropy change is determined to be

$$\frac{P_2 V}{T_2} = \frac{P_1 V}{T_1} \longrightarrow \frac{T_2}{T_1} = \frac{P_2}{P_1} = \frac{120kPa}{100kPa} = 1.2$$

Thus,

$$\begin{aligned} \Delta S = m(s_2 - s_1) &= m\left(C_{v,ave} \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}^{\nearrow 0} \right) = mC_{v,ave} \ln \frac{T_2}{T_1} \\ &= (0.9\text{kg})(0.657\text{kJ/kg} \cdot \text{K}) \ln(1.2) \\ &= \mathbf{0.108\text{kJ/K}} \end{aligned}$$



7-59 An insulated cylinder initially contains air at a specified state. A resistance heater inside the cylinder is turned on, and air is heated for 15 min at constant pressure. The entropy change of air during this process is to be determined for the cases of constant and variable specific heats. √

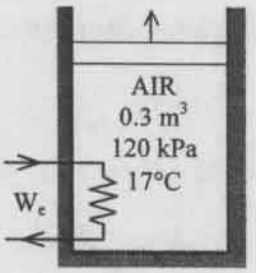
Assumptions At specified conditions, air can be treated as an ideal gas.

Properties The gas constant of air is $R = 0.287$ kJ/kg.K (Table A-1).

Analysis The mass of the air and the electrical work done during this process are

$$m = \frac{P_1 V_1}{RT_1} = \frac{(120\text{kPa})(0.3\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\text{K})} = 0.4325\text{kg}$$

$$W_{e,in} = \dot{W}_{e,in}\Delta t = (0.2\text{kJ/s})(15\times 60\text{s}) = 180\text{kJ}$$



The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{pw,in} - W_{b,out} = \Delta U$$

$$W_{pw,in} = m(h_2 - h_1) \cong C_p(T_2 - T_1)$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process.

(*a*) Using a constant C_p value at the anticipated average temperature of 450 K, the final temperature becomes

Thus, $$T_2 = T_1 + \frac{W_{e,in}}{mC_p} = 290\text{K} + \frac{180\text{kJ}}{(0.4325\text{kg})(1.02\text{kJ/kg}\cdot\text{K})} = 698\text{K}$$

Then the entropy change becomes

$$\Delta S_{sys} = m(s_2 - s_1) = m\left(C_{p,ave}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}^{\nearrow 0}\right) = mC_{p,ave}\ln\frac{T_2}{T_1}$$

$$= (0.4325\text{kg})(1.020\text{kJ/kg}\cdot\text{K})\ln\left(\frac{698\text{K}}{290\text{K}}\right)$$

$$= \mathbf{0.387\text{kJ/K}}$$

(*b*) Assuming variable specific heats,

$$W_{e,in} = m(h_2 - h_1) \longrightarrow h_2 = h_1 + \frac{W_{e,in}}{m} = 290.16\text{kJ/kg} + \frac{180\text{kJ}}{0.4325\text{kg}} = 706.34\text{kJ/kg}$$

From the air table (Table A-17, we read $s_2^\circ = 2.5628$ kJ/kg·K corresponding to this h_2 value. Then,

$$\Delta S_{sys} = m\left(s_2^\circ - s_1^\circ + R\ln\frac{P_2}{P_1}^{\nearrow 0}\right) = m(s_2^\circ - s_1^\circ)$$

$$= (0.4325\text{kg})(2.5628 - 1.66802)\text{kJ/kg}\cdot\text{K}$$

$$= \mathbf{0.387\text{kJ/K}}$$

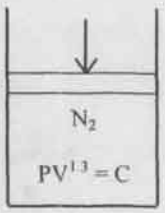
7-60 A cylinder contains N_2 gas at a specified pressure and temperature. A gas is compressed polytropically until the volume is reduced by half. The entropy change of nitrogen during this process is to be determined. √

Assumptions **1** At specified conditions, N_2 can be treated as an ideal gas. **2** Nitrogen has constant specific heats at room temperature.

Properties The gas constant of nitrogen is $R = 0.297$ kJ/kg.K (Table A-1). The constant volume specific heat of nitrogen at room temperature is $C_v = 0.743$ kJ/kg.K (Table A-2).

Analysis From the polytropic relation,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \longrightarrow T_2 = T_1\left(\frac{v_1}{v_2}\right)^{n-1} = (300K)(2)^{1.3-1} = 369.3K$$



Then the entropy change of nitrogen becomes

$$\Delta S_{N_2} = m\left(C_{v,ave}\ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}\right)$$

$$= (1.2\text{ kg})\left((0.743\text{ kJ/kg}\cdot\text{K})\ln\frac{369.3\text{ K}}{300\text{ K}} + (0.297\text{ kJ/kg}\cdot\text{K})\ln(0.5)\right)$$

$$= \mathbf{-0.0617\ kJ/K}$$

7-61E A fixed mass of helium undergoes a process from one specified state to another specified state. The entropy change of helium is to be determined for the cases of reversible and irreversible processes.

Assumptions **1** At specified conditions, helium can be treated as an ideal gas. **2** Helium has constant specific heats at room temperature.

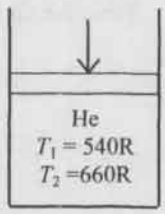
Properties The gas constant of helium is $R = 0.4961$ Btu/lbm.R (Table A-1E). The constant volume specific heat of helium is $C_v = 0.753$ Btu/lbm.R (Table A-2E).

Analysis From the ideal-gas entropy change relation,

$$\Delta S_{He} = m\left(C_{v,ave}\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}\right)$$

$$= (8\text{lbm})\left((0.753\text{Btu/lbm}\cdot\text{R})\ln\frac{660\text{R}}{540\text{R}} + (0.4961\text{Btu/lbm}\cdot\text{R})\ln\left(\frac{10\text{ft}^3/\text{lbm}}{50\text{ft}^3/\text{lbm}}\right)\right)$$

$$= \mathbf{-5.18Btu/R}$$



The entropy change will be the same for both cases.

7-62 Air is compressed in a piston-cylinder device in a reversible and isothermal manner. The entropy change of air and the work done are to be determined.

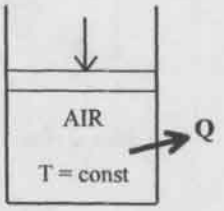
Assumptions **1** At specified conditions, air can be treated as an ideal gas. **2** The process is specified to be reversible.

Properties The gas constant of air is $R = 0.287$ kJ/kg.K (Table A-1).

Analysis (*a*) Noting that the temperature remains constant, the entropy change of air is determined from

$$\Delta S_{air} = C_{p,ave} \ln \frac{T_2}{T_1}^{\nearrow 0} - R \ln \frac{P_2}{P_1} = -R \ln \frac{P_2}{P_1}$$

$$= -(0.287\text{kJ/kg} \cdot \text{K}) \ln\left(\frac{400\text{kPa}}{90\text{kPa}}\right) = \mathbf{-0.428\,kJ/kg \cdot K}$$



Also, for a reversible isothermal process,

$$q = T\Delta s = (293\text{K})(-0.428\text{kJ/kg} \cdot \text{K}) = -125.4\text{kJ/kg} \rightarrow q_{out} = 125.4\ \text{kJ/kg}$$

(*b*) The work done during this process is determined from the closed system energy balance,

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$W_{in} - Q_{out} = \Delta U = mC_v(T_2 - T_1) = 0$$

$$w_{in} = q_{out} = \mathbf{125.4\,kJ/kg}$$

7-63 Air is compressed steadily by a 5-kW compressor from one specified state to another specified state. The rate of entropy change of air is to be determined. √

Assumptions At specified conditions, air can be treated as an ideal gas. **2** Air has variable specific heats.

Properties The gas constant of air is $R = 0.287$ kJ/kg.K (Table A-1).

Analysis From the air table (Table A-17),

$$\left.\begin{matrix} T_1 = 290\text{ K} \\ P_1 = 100\text{ kPa} \end{matrix}\right\} s_1^\circ = 1.66802\text{ kJ/kg} \cdot \text{K}$$

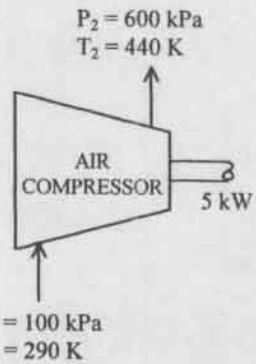
$$\left.\begin{matrix} T_2 = 440\text{ K} \\ P_2 = 600\text{ kPa} \end{matrix}\right\} s_2^\circ = 2.0887\text{ kJ/kg} \cdot \text{K}$$

Then the rate of entropy change of air becomes

$$\Delta \dot{S}_{sys} = \dot{m}\left(s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}\right)$$

$$= (1.6/60\text{kg/s})\left(2.0887 - 1.66802 - (0.287\text{kJ/kg} \cdot \text{K}) \ln\left(\frac{600\text{kPa}}{100\text{kPa}}\right)\right)$$

$$= \mathbf{-0.00250\ kW/K}$$



7-64 One side of a partitioned insulated rigid tank contains an ideal gas at a specified temperature and pressure while the other side is evacuated. The partition is removed, and the gas fills the entire tank. The total entropy change during this process is to be determined.

Assumptions The gas in the tank is given to be an ideal gas, and thus ideal gas relations apply.

Analysis Taking the entire rigid tank as the system, the energy balance can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$0 = \Delta U = m(u_2 - u_1)$$

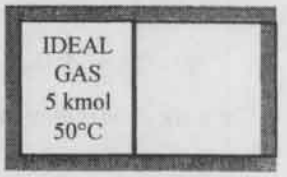
$$u_2 = u_1 \rightarrow T_2 = T_1$$

since $u = u(T)$ for an ideal gas. Then the entropy change of the gas becomes

$$\Delta S = N\left(\overline{C}_{v,ave} \ln \frac{T_2}{T_1}^{\nearrow 0} + R_u \ln \frac{V_2}{V_1}\right) = NR_u \ln \frac{V_2}{V_1}$$

$$= (5\text{kmol})(8.314\text{kJ/kmol}\cdot\text{K})\ln(2)$$

$$= \mathbf{28.81kJ/K}$$



This also represents the **total entropy change** since the tank does not contain anything else, and there are no interactions with the surroundings.

7-65 Air is compressed in a piston-cylinder device in a reversible and adiabatic manner. The final temperature and the work are to be determined for the cases of constant and variable specific heats.

Assumptions **1** At specified conditions, air can be treated as an ideal gas. **2** The process is given to be reversible and adiabatic, and thus isentropic. Therefore, isentropic relations of ideal gases apply.

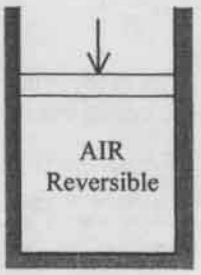
Properties The gas constant of air is $R = 0.287$ kJ/kg.K (Table A-1). The specific heat ratio of air at low to moderately high temperatures is $k = 1.4$ (Table A-2).

Analysis (*a*) Assuming constant specific heats, the ideal gas isentropic relations give

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (290\text{K})\left(\frac{800\text{kPa}}{100\text{kPa}}\right)^{0.4/1.4} = \mathbf{525.3K}$$

Then,

$$T_{ave} = (290 + 525.3)/2 = 407.7\text{K} \longrightarrow C_{v,ave} = 0.727\text{kJ/kg}\cdot\text{K}$$



We take the air in the cylinder as the system. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{in} = \Delta U = m(u_2 - u_1) \equiv mC_v(T_2 - T_1)$$

Thus,

$$w_{in} = C_{v,ave}(T_2 - T_1) = (0.727\text{kJ/kg}\cdot\text{K})(525.3 - 290)\text{K} = \mathbf{171.1kJ/kg}$$

(*b*) Assuming variable specific heats, the final temperature can be determined using the relative pressure data (Table A-17),

$$T_1 = 290\text{K} \longrightarrow \begin{matrix} P_{r_1} = 1.2311 \\ u_1 = 206.91\text{kJ/kg} \end{matrix}$$

and

$$P_{r_2} = \frac{P_2}{P_1}P_{r_1} = \frac{800\text{kPa}}{100\text{kPa}}(1.2311) = 9.849 \longrightarrow \begin{matrix} T_2 = \mathbf{522.4K} \\ u_2 = 376.16\text{kJ/kg} \end{matrix}$$

Then the work input becomes

$$w_{in} = u_2 - u_1 = (376.16 - 206.91)\text{kJ/kg} = \mathbf{169.25kJ/kg}$$

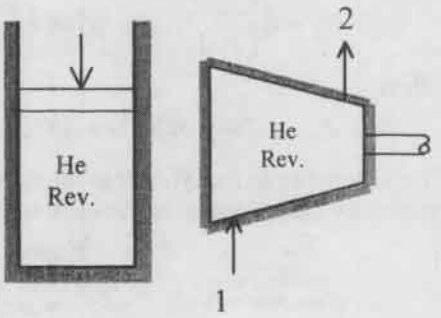
7-66 Helium gas is compressed in a piston-cylinder device in a reversible and adiabatic manner. The final temperature and the work are to be determined for the cases of the process taking place in a piston-cylinder device and a steady-flow compressor.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** The process is given to be reversible and adiabatic, and thus isentropic. Therefore, isentropic relations of ideal gases apply.

Properties The specific heats and the specific heat ratio of helium are C_v = 3.1156 kJ/kg.K, C_p = 5.1926 kJ/kg.K, and k = 1.667 (Table A-2).

Analysis (*a*) From the ideal gas isentropic relations,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (303\text{K})\left(\frac{500\text{kPa}}{100\text{kPa}}\right)^{0.667/1.667} = \mathbf{576.9K}$$



(*a*) We take the air in the cylinder as the system. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{in} = \Delta U = m(u_2 - u_1) \cong mC_v(T_2 - T_1)$$

Thus,

$$w_{in} = C_v(T_2 - T_1) = (3.1156\text{kJ/kg}\cdot\text{K})(576.9 - 303)\text{K} = \mathbf{853.4kJ/kg}$$

(*b*) If the process takes place in a steady-flow device, the final temperature will remain the same but the work done should be determined from an energy balance on this steady-flow device,

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) \cong \dot{m}C_p(T_2 - T_1)$$

Thus,

$$w_{in} = C_p(T_2 - T_1) = (5.1926\text{kJ/kg}\cdot\text{K})(576.9 - 303)\text{K} = \mathbf{1422.3kJ/kg}$$

7-67 An insulated rigid tank contains argon gas at a specified pressure and temperature. A valve is opened, and argon escapes until the pressure drops to a specified value. The final mass in the tank is to be determined.

Assumptions **1** At specified conditions, argon can be treated as an ideal gas. **2** The process is given to be reversible and adiabatic, and thus isentropic. Therefore, isentropic relations of ideal gases apply.

Properties The specific heat ratio of argon is $k = 1.667$ (Table A-2).

Analysis From the ideal gas isentropic relations,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (303\text{K})\left(\frac{150\text{kPa}}{450\text{kPa}}\right)^{0.667/1.667} = 195\text{K}$$



The final mass in the tank is determined from the ideal gas relation,

$$\frac{P_1V}{P_2V} = \frac{m_1RT_1}{m_2RT_2} \longrightarrow m_2 = \frac{P_2T_1}{P_1T_2}m_1 = \frac{(150\text{kPa})(303\text{K})}{(450\text{kPa})(195\text{K})}(4\text{kg}) = \mathbf{2.07kg}$$

7-68E Air is accelerated in an adiabatic nozzle. Disregarding irreversibilities, the exit velocity of air is to be determined.

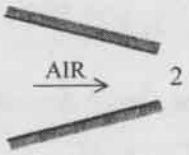
Assumptions **1** Air is an ideal gas with variable specific heats. **2** The process is given to be reversible and adiabatic, and thus isentropic. Therefore, isentropic relations of ideal gases apply. **2** The nozzle operates steadily.

Analysis Assuming variable specific heats, the exit properties are determined to be

$$T_1 = 1000\text{R} \longrightarrow \begin{matrix} P_{r_1} = 12.30 \\ h_1 = 240.98\text{Btu/lbm} \end{matrix}$$

and

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = \frac{12\text{psia}}{60\text{psia}}(12.30) = 2.46 \longrightarrow \begin{matrix} T_2 = 635.9\text{R} \\ h_2 = 152.11\text{Btu/lbm} \end{matrix}$$



We take the nozzle as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\begin{matrix}\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}\end{matrix}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\begin{matrix}\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}\end{matrix}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_2^2/2)$$

$$h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} = 0$$

Therefore,

$$\mathbf{V}_2 = \sqrt{2(h_2 - h_1) + \mathbf{V}_1^2} = \sqrt{2(240.98 - 152.11)\text{Btu/lbm}\left(\frac{25{,}037\text{ft}^2/\text{s}^2}{1\text{Btu/lbm}}\right) + (200\text{ft/s})^2}$$

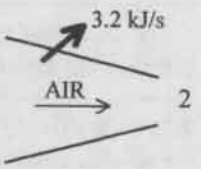
$$= \mathbf{2119\text{ft/s}}$$

7-69 Air is accelerated in an nozzle, and some heat is lost in the process. The exit temperature of air and the total entropy change during the process are to be determined. √

Assumptions **1** Air is an ideal gas with variable specific heats. **2** The nozzle operates steadily.

Analysis (*a*) Assuming variable specific heats, the inlet properties are determined to be,

$$T_1 = 350\ \text{K} \longrightarrow \begin{matrix} h_1 = 350.49\ \text{kJ/kg} \\ s_1^\circ = 1.85708\ \text{/ kJ/kg·K} \end{matrix} \quad \text{(Table A-17)}$$



We take the nozzle as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_2^2/2) + \dot{Q}_{out}$$

$$0 = q_{out} + h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}$$

Therefore,

$$h_2 = h_1 - q_{out} - \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} = 350.49 - 3.2 - \frac{(320\text{m/s})^2 - (50\text{m/s})^2}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right)$$

$$= 297.34\text{kJ/kg}$$

At this h_2 value we read, from Table A-17, $T_2 = \mathbf{297.2\ K}$, $s_2^\circ = 1.6924\ \text{kJ/kg·K}$

(*b*) The total entropy change is the sum of the entropy changes of the air and of the surroundings, and is determined from

$$\Delta s_{total} = \Delta s_{air} + \Delta s_{surr}$$

where

$$\Delta s_{air} = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1} = 1.6924 - 1.85708 - (0.287\text{kJ/kg·K})\ln\frac{85\text{kPa}}{280\text{kPa}} = 0.1775\text{kJ/kg·K}$$

and

$$\Delta s_{surr} = \frac{q_{surr,in}}{T_{surr}} = \frac{3.2\text{kJ/kg}}{293\text{K}} = 0.0109\text{kJ/kg·K}$$

Thus,

$$\Delta s_{total} = 0.1775 + 0.0109 = \mathbf{0.1884kJ/kg·K}$$

Reversible Steady-Flow Work

7-70C The work associated with steady-flow devices is proportional to the specific volume of the gas. Cooling a gas during compression will reduce its specific volume, and thus the power consumed by the compressor.

7-71C Cooling the steam as it expands in a turbine will reduce its specific volume, and thus the work output of the turbine. Therefore, this is not a good proposal.

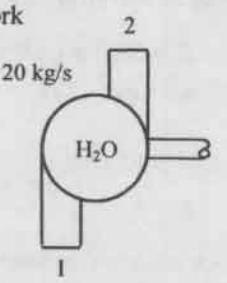
7-72C We would not support this proposal since the steady-flow work input to the pump is proportional to the specific volume of the liquid, and cooling will not affect the specific volume of a liquid significantly.

7-73 Liquid water is pumped reversibly to a specified pressure at a specified rate. The power input to the pump is to be determined. √

Assumptions **1** Liquid water is an incompressible substance. **2** Kinetic and potential energy changes are negligible. **3** The process is reversible.

Properties The specific volume of saturated liquid water at 20 kPa is $v_1 = v_{f\,@\,20\text{ kPa}} = 0.001017\ \text{m}^3/\text{kg}$ (Table A-5).

Analysis The power input to the pump can be determined directly from the steady-flow work relation for a liquid,



$$\dot{W}_{in} = \dot{m}\left(\int_1^2 vdP + \Delta ke^{\nearrow 0} + \Delta pe^{\nearrow 0} \right) = \dot{m}v_1(P_2 - P_1)$$

Substituting,

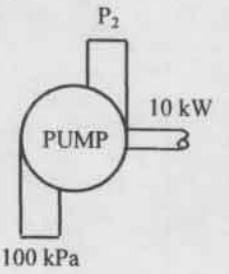
$$\dot{W}_{in} = (20\text{kg/s})(0.001017\text{m}^3/\text{kg})(6000-20)\text{kPa}\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = \mathbf{121.6kW}$$

7-74 Liquid water is to be pumped by a 10-kW pump at a specified rate. The highest pressure the water can be pumped to is to be determined.

Assumptions **1** Liquid water is an incompressible substance. **2** Kinetic and potential energy changes are negligible. **3** The process is assumed to be reversible since we will determine the limiting case.

Properties The specific volume of saturated liquid water at 20 kPa is $v_1 = v_{f\,@\,20\text{ kPa}} = 0.001017\ \text{m}^3/\text{kg}$ (Table A-5).

Analysis The highest pressure the liquid can have at the pump exit can be determined from the reversible steady-flow work relation for a liquid,



$$\dot{W}_{in} = \dot{m}\left(\int_1^2 vdP - \Delta ke^{\nearrow 0} - \Delta pe^{\nearrow 0} \right) = \dot{m}v_1(P_2 - P_1)$$

Thus,

$$10\text{kJ/s} = (5\text{kg/s})(0.001\text{m}^3/\text{kg})(P_2 - 100)\text{k Pa}\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$

It yields

$$P_2 = \mathbf{2100\ kPa}$$

-75E Saturated refrigerant-134a vapor is to be compressed reversibly to a specified pressure at a specified ıte. The power input to the compressor is to be determined, and it is also to be compared to the work input ›r the liquid case. √

ssumptions **1** Liquid refrigerant is an incompressible substance. **2** Kinetic and potential energy changes re negligible. **3** The process is reversible. **4** The compressor is adiabatic.

nalysis The compression process is reversible and adiabatic, and thus isentropic, $s_1 = s_2$. Then the roperties of the refrigerant are (Tables A-11E through A-13E)

$$\left.\begin{array}{l} P_1 = 20\text{psia} \\ sat.vapor \end{array}\right\} \begin{array}{l} h_1 = 101.39\text{Btu/lbm} \\ s_1 = 0.2227\text{Btu/lbm}\cdot\text{R} \end{array}$$

$$\left.\begin{array}{l} P_1 = 120\text{psia} \\ s_2 = s_1 \end{array}\right\} h_2 = 117.29\text{Btu/lbm}$$

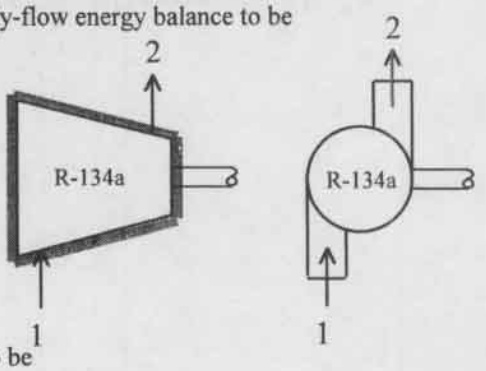
he work input to this isentropic compressor is determined from the steady-flow energy balance to be

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1)$$



hus, $w_{in} = h_2 - h_1 = 117.29 - 101.39 = \mathbf{15.9\ Btu/lbm}$

he pump work input is determined from the steady-flow work relation to be

$$w_{in} = \int_1^2 v\,dP + \Delta ke^{\nearrow 0} + \Delta pe^{\nearrow 0} = v_1(P_2 - P_1)$$

here $v_3 = v_{f@\,20\text{ psia}} = 0.01181\ \text{ft}^3/\text{kg}$. Substituting,

$$w_{in} = (0.01181\ \text{ft}^3/\text{lbm})(120 - 20)\ \text{psia}\left(\frac{1\ \text{Btu}}{5.4039\ \text{psia}\cdot\text{ft}^3}\right) = \mathbf{0.2185\ Btu/lbm}$$

7-76 A steam power plant operates between the pressure limits of 10 MPa and 20 kPa. The ratio of the turbine work to the pump work is to be determined.

Assumptions **1** Liquid water is an incompressible substance. **2** Kinetic and potential energy changes are negligible. **3** The process is reversible. **4** The pump and the turbine are adiabatic.

Properties The specific volume of saturated liquid water at 20 kPa is $v_1 = v_{f\,@\,20\text{ kPa}} = 0.001017\ \text{m}^3/\text{kg}$ (Table A-5).

Analysis Both the compression and expansion processes are reversible and adiabatic, and thus isentropic, $s_1 = s_2$ and $s_3 = s_4$. Then the properties of the steam are

$$\left.\begin{array}{l} P_4 = 20\text{kPa} \\ sat.vapor \end{array}\right\} \begin{array}{l} h_4 = h_{g@20\text{kPa}} = 2609.7\text{kJ/kg} \\ s_1 = s_{g@20\text{kPa}} = 7.9085\text{kJ/kg}\cdot\text{K} \end{array}$$

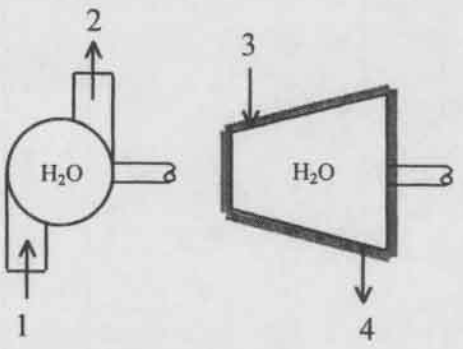
$$\left.\begin{array}{l} P_3 = 10\text{MPa} \\ s_3 = s_4 \end{array}\right\} h_3 = 4712.8\text{kJ/kg}$$

Also, $v_1 = v_{f\,@\,20\text{ kPa}} = 0.001017\ \text{m}^3/\text{kg}$.

The work output to this isentropic turbine is determined from the steady-flow energy balance to be

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m}h_3 = \dot{m}h_4 + \dot{W}_{\text{out}}$$
$$\dot{W}_{\text{out}} = \dot{m}(h_3 - h_4)$$



Substituting,

$$w_{turb,out} = h_3 - h_4 = 4712.8 - 2609.7 = 2103.1\ \text{kJ/kg}$$

The pump work input is determined from the steady-flow work relation to be

$$w_{pump,in} = \int_1^2 v\,dP - \Delta ke^{\nearrow 0} - \Delta pe^{\nearrow 0} = v_1(P_2 - P_1)$$

$$= (0.001017\text{m}^3/\text{kg})(10{,}000 - 20)\text{kPa}\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right) = 10.15\text{kJ/kg}$$

Thus,

$$\frac{w_{turb,out}}{w_{pump,in}} = \frac{2103.1\ \text{kJ/kg}}{10.15\ \text{kJ/kg}} = \mathbf{207.2}$$

7-77 Liquid water is pumped by a 10-kW pump to a specified pressure at a specified level. The highest possible mass flow rate of water is to be determined. √

Assumptions **1** Liquid water is an incompressible substance. **2** Kinetic energy changes are negligible, but potential energy changes may be significant. **3** The process is assumed to be reversible since we will determine the limiting case.

Properties The specific volume of liquid water at about atmospheric pressure can be taken to be $v_1 = 0.001$ m^3/kg (Table A-3).

Analysis The highest mass flow rate will be realized when the entire process is reversible. Thus it is determined from the reversible steady-flow work relation for a liquid,

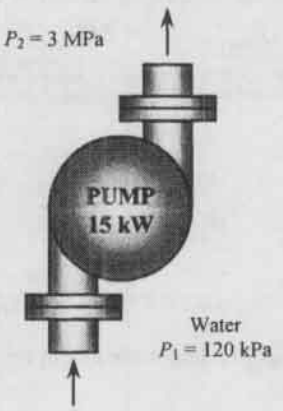
$$\dot{W}_{in} = \dot{m}\left(\int_1^2 vdP - \Delta ke^{\nearrow 0} - \Delta pe \right) = \dot{m}\{v(P_2 - P_1) + g(z_2 - z_1)\}$$

Thus,

$$15\text{kJ/s} = \dot{m}\left\{ (0.001\text{m}^3/\text{kg})(3000-120)\text{kPa}\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3} \right) + (9.8\text{m/s}^2)(10\text{m})\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2} \right) \right\}$$

It yields

$$\dot{m} = \mathbf{5.04\ kg/s}$$



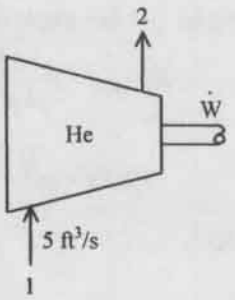
78E Helium gas is compressed from a specified state to a specified pressure at a specified rate. The power put to the compressor is to be determined for the cases of isentropic, polytropic, isothermal, and two-stage mpression.

ssumptions **1** Helium is an ideal gas with constant specific heats. **2** The process is reversible. **3** Kinetic d potential energy changes are negligible.

roperties The gas constant of helium is $R = 2.6805$ psia.ft^3/lbm.R = 0.4961 Btu/lbm.R (Table A-1). The ecific heat ratio of helium is $k = 1.667$.

nalysis The mass flow rate of helium is

$$\dot{m} = \frac{P_1\dot{V}_1}{RT_1} = \frac{(14\text{psia})(5\text{ft}^3/\text{s})}{(2.6805\text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(530\text{R})} = 0.0493\text{lbm/s}$$



) Isentropic compression with $k = 1.667$:

$$\dot{W}_{comp,in} = \dot{m}\frac{kRT_1}{k-1}\left\{\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right\}$$

$$= (0.0493\text{lbm/s})\frac{(1.667)(0.4961\text{Btu/lbm}\cdot\text{R})(530\text{R})}{1.667-1}\left\{\left(\frac{120\text{psia}}{14\text{psia}}\right)^{0.667/1.667} - 1\right\}$$

$$= 44.11\text{Btu/s}$$

$$= \mathbf{62.4hp} \quad \text{since 1 hp} = 0.7068\ \text{Btu/s}$$

) Polytropic compression with $n = 1.2$:

$$\dot{W}_{comp,in} = \dot{m}\frac{nRT_1}{n-1}\left\{\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right\}$$

$$= (0.0493\text{lbm/s})\frac{(1.2)(0.4961\text{Btu/lbm}\cdot\text{R})(530\text{R})}{1.2-1}\left\{\left(\frac{120\text{psia}}{14\text{psia}}\right)^{0.2/1.2} - 1\right\}$$

$$= 33.47\text{Btu/s}$$

$$= \mathbf{47.3hp} \quad \text{since 1 hp} = 0.7068\ \text{Btu/s}$$

) Isothermal compression:

$$\dot{W}_{comp,in} = \dot{m}RT\ln\frac{P_2}{P_1} = (0.0493\text{lbm/s})(0.4961\text{Btu/lbm}\cdot\text{R})(530\text{R})\ln\frac{120\text{psia}}{14\text{psia}} = 27.83\text{Btu/s} = \mathbf{39.4hp}$$

) Ideal two-stage compression with intercooling ($n = 1.2$): In this case, the pressure ratio across each age is the same, and its value is determined from

$$P_x = \sqrt{P_1P_2} = \sqrt{(14\text{psia})(120\text{psia})} = 41.0\text{psia}$$

he compressor work across each stage is also the same, thus total compressor work is twice the mpression work for a single stage:

$$\dot{W}_{comp,in} = 2\dot{m}w_{comp,I} = 2\dot{m}\frac{nRT_1}{n-1}\left\{\left(\frac{P_x}{P_1}\right)^{(n-1)/n} - 1\right\}$$

$$= 2(0.0493\text{lbm/s})\frac{(1.2)(0.4961\text{Btu/lbm}\cdot\text{R})(530\text{R})}{1.2-1}\left\{\left(\frac{41\text{psia}}{14\text{psia}}\right)^{0.2/1.2} - 1\right\}$$

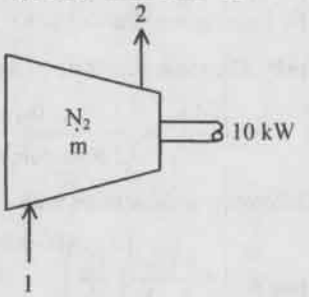
$$= 30.52\text{Btu/s}$$

$$= \mathbf{43.2hp} \quad \text{since 1 hp} = 0.7068\ \text{Btu/s}$$

7-79 Nitrogen gas is compressed by a 10-kW compressor from a specified state to a specified pressure at a specified rate. The mass flow rate of nitrogen through the compressor is to be determined for the cases of isentropic, polytropic, isothermal, and two-stage compression.

Assumptions **1** Nitrogen is an ideal gas with constant specific heats. **2** The process is reversible. **3** Kinetic and potential energy changes are negligible.

Properties The gas constant of nitrogen is R = 0.297 kJ/kg.K (Table A-1). The specific heat ratio of Nitrogen is k = 1.4.



Analysis (*a*) Isentropic compression:

$$\dot{W}_{comp,in} = \dot{m}\frac{kRT_1}{k-1}\left\{\left(P_2/P_1\right)^{(k-1)/k} - 1\right\}$$

or,

$$10\text{kJ/s} = \dot{m}\frac{(1.4)(0.297\text{kJ/kg}\cdot\text{K})(300\text{K})}{1.4-1}\left\{(480\text{kPa}/80\text{kPa})^{0.4/1.4} - 1\right\}$$

It yields

$$\dot{m} = \mathbf{0.048\ kg/s}$$

(*b*) Polytropic compression with n = 1.3:

$$\dot{W}_{comp,in} = \dot{m}\frac{nRT_1}{n-1}\left\{\left(P_2/P_1\right)^{(n-1)/n} - 1\right\}$$

or,

$$10\text{kJ/s} = \dot{m}\frac{(1.3)(0.297\text{kJ/kg}\cdot\text{K})(300\text{K})}{1.3-1}\left\{(480\text{kPa}/80\text{kPa})^{0.3/1.3} - 1\right\}$$

It yields

$$\dot{m} = \mathbf{0.051\ kg/s}$$

(*c*) Isothermal compression:

$$\dot{W}_{comp,in} = \dot{m}RT\ln\frac{P_1}{P_2} \longrightarrow 10\text{kJ/s} = \dot{m}(0.297\text{kJ/kg}\cdot\text{K})(300\text{K})\ln\left(\frac{480\text{kPa}}{80\text{kPa}}\right)$$

It yields

$$\dot{m} = \mathbf{0.063\ kg/s}$$

(*d*) Ideal two-stage compression with intercooling (n = 1.3): In this case, the pressure ratio across each stage is the same, and its value is determined to be

$$P_x = \sqrt{P_1P_2} = \sqrt{(80\text{kPa})(480\text{kPa})} = 196\text{kPa}$$

The compressor work across each stage is also the same, thus total compressor work is twice the compression work for a single stage:

$$\dot{W}_{comp,in} = 2\dot{m}w_{comp,I} = 2\dot{m}\frac{nRT_1}{n-1}\left\{\left(P_x/P_1\right)^{(n-1)/n} - 1\right\}$$

or,

$$10\text{kJ/s} = 2\dot{m}\frac{(1.3)(0.297\text{kJ/kg}\cdot\text{K})(300\text{K})}{1.3-1}\left\{(196\text{kPa}/80\text{kPa})^{0.3/1.3} - 1\right\}$$

It yields

$$\dot{m} = \mathbf{0.058\ kg/s}$$

7-80 Water mist is to be sprayed into the air stream in the compressor to cool the air as the water evaporates and to reduce the compression power. The reduction in the exit temperature of the compressed air and the compressor power saved are to be determined.

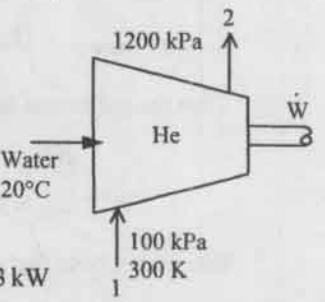
Assumptions **1** Air is an ideal gas with constant specific heats. **2** The process is reversible. **3** Kinetic and potential energy changes are negligible. **3** Air is compressed isentropically. **4** Water vaporizes completely before leaving the compressor. **4** Air properties can be used for the air-vapor mixture.

Properties The gas constant of air is $R = 0.287$ kJ/kg.K (Table A-1). The specific heat ratio of air is $k = 1.4$. The inlet enthalpies of water and air are (Tables A-4 and A-17)

$$h_{w1} = h_{f@20°C} = 83.96 \text{ kJ/kg},\ h_{fg@20°C} = 2454.1 \text{ kJ/kg and } h_{a1} = h_{@300\text{ K}} = 300.19 \text{ kJ/kg}$$

Analysis In the case of isentropic operation (thus no cooling or water spray), the exit temperature and the power input to the compressor are

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \rightarrow T_2 = (300\text{ K})\left(\frac{1200\text{ kPa}}{100\text{ kPa}}\right)^{(1.4-1)/1.4} = 610.2\text{ K}$$



$$\dot{W}_{comp,in} = \dot{m}\frac{kRT_1}{k-1}\left\{(P_2/P_1)^{(k-1)/k} - 1\right\}$$

$$= (2.1\text{ kg/s})\frac{(1.4)(0.287\text{kJ/kg}\cdot\text{K})(300\text{K})}{1.4-1}\left\{(1200\text{ kPa}/100\text{ kPa})^{0.4/1.4} - 1\right\} = 654.3\text{ kW}$$

When water is sprayed, we first need to check the accuracy of the assumption that the water vaporizes completely in the compressor. In the limiting case, the compression will be isothermal at the compressor inlet temperature, and the water will be a saturated vapor. To avoid the complexity of dealing with two fluid streams and a gas mixture, we disregard water in the air stream (other than the mass flow rate), and assume air is cooled by an amount equal to the enthalpy change of water.

The rate of heat absorption of water as it evaporates at the inlet temperature completely is

$$\dot{Q}_{cooling,\max} = \dot{m}_w h_{fg@20°C} = (0.2\text{ kg/s})(2454.1\text{ kJ/kg}) = 490.8\text{ kW}$$

The minimum power input to the compressor is

$$\dot{W}_{comp,in,\min} = \dot{m}RT\ln\frac{P_1}{P_2} = (2.1\text{ kg/a})(0.287\text{kJ/kg}\cdot\text{K})(300\text{K})\ln\left(\frac{1200\text{kPa}}{100\text{kPa}}\right) = 214.0\text{ kW}$$

This corresponds to maximum cooling from the air since, at constant temperature, $\Delta h = 0$ and thus $\dot{Q}_{out} = \dot{W}_{in} = 214.0$ kW which is less than 490.8 kW. Therefore, the assumption that all the water vaporizes is invalid. Then we can assume the compression process to be polytropic and the water to be a saturated vapor at the compressor exit temperature, and disregard the remaining liquid. But in this case there is not a unique solution, and we will have to select either the amount of water or the exit temperature or the polytropic exponent to obtain a solution. Of course we can also tabulate the results for different cases, and then make a selection.

Sample Analysis: We take the compressor exit temperature to be $T_2 = 200°C = 473$ K. Then,

$$h_{w2} = h_{g@200°C} = 2793.2 \text{ kJ/kg and } h_{a2} = h_{@473\text{ K}} = 473.2 \text{ kJ/kg}$$

Then,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n} \rightarrow \frac{473\text{ K}}{300\text{ K}} = \left(\frac{1200\text{ kPa}}{100\text{ kPa}}\right)^{(n-1)/n} \rightarrow n = 1.224$$

$$\dot{W}_{comp,in} = \dot{m}\frac{nRT_1}{n-1}\left\{(P_2/P_1)^{(n-1)/n} - 1\right\} = \dot{m}\frac{nR}{n-1}(T_2 - T_1)$$
$$= (2.1\,\text{kg/s})\frac{(1.224)(0.287\text{kJ/kg}\cdot\text{K})}{1.224-1}(473-300)\text{K} = 570\,\text{kW}$$

Energy balance:

$$\dot{W}_{comp,in} - \dot{Q}_{out} = \dot{m}(h_2 - h_1) \;\rightarrow\; \dot{Q}_{out} = \dot{W}_{comp,in} - \dot{m}(h_2 - h_1)$$
$$= 569.7\,\text{kW} - (2.1\,\text{kg/s})(475.3 - 300.19) = 202.0\,\text{kW}$$

Noting that this heat is absorbed by water, the rate at which water evaporates in the compressor becomes

$$\dot{Q}_{out,air} = \dot{Q}_{in,water} = \dot{m}_w(h_{w2} - h_{w1}) \;\rightarrow\; \dot{m}_w = \frac{\dot{Q}_{in,water}}{h_{w2} - h_{w1}} = \frac{202.0\,\text{kJ/s}}{(2793.2 - 83.96)\,\text{kJ/kg}} = 0.0746\,\text{kg/s}$$

Then the reductions in the exit temperature and compressor power input become

$$\Delta T_2 = T_{2,\text{isentropic}} - T_{2,\text{water cooled}} = 610.2 - 473 = \mathbf{137.2°C}$$
$$\Delta \dot{W}_{comp,in} = \dot{W}_{comp,\text{isentropic}} - \dot{W}_{comp,\text{water cooled}} = 654.3 - 570 = \mathbf{84.3\,kW}$$

Discussion Note that selecting a different compressor exit temperature T_2 will result in different values.

7-81 A water-injected compressor is used in a gas turbine power plant. It is claimed that the power output of a gas turbine will increase when water is injected into the compressor because of the increase in the mass flow rate of the gas (air + water vapor) through the turbine. This, however, is **not necessarily right** since the compressed air in this case enters the combustor at a low temperature, and thus it absorbs much more heat. In fact, the cooling effect will most likely dominate and cause the cyclic efficiency to drop.

Adiabatic Efficiencies of Steady-Flow Devices

7-82C The ideal process for all three devices is the reversible adiabatic (i.e., isentropic) process. The adiabatic efficiencies of these devices are defined as

$$\eta_T = \frac{\text{actual work output}}{\text{insentropic work output}},\quad \eta_C = \frac{\text{insentropic work output}}{\text{actual work output}},\quad \text{and}\quad \eta_N = \frac{\text{actual exit kinetic energy}}{\text{insentropic exit kinetic energy}}$$

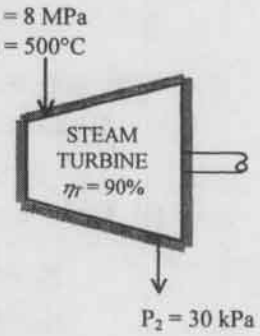
7-83C No, because the isentropic process is not the model or ideal process for compressors that are cooled intentionally.

7-84C Yes. Because the entropy of the fluid must increase during an actual adiabatic process as a result of irreversibilities. Therefore, the actual exit state has to be on the right-hand side of the isentropic exit state

7-85 Steam enters an adiabatic turbine with an isentropic efficiency of 0.90 at a specified state with a specified mass flow rate, and leaves at a specified pressure. The turbine exit temperature and power output of the turbine are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Analysis (*a*) From the steam tables (Tables A-4 and A-6),



$$\left.\begin{aligned} P_1 &= 8\text{MPa} \\ T_1 &= 500^\circ\text{C} \end{aligned}\right\} \begin{aligned} h_1 &= 3398.3\text{kJ/kg} \\ s_1 &= 6.7240\text{kJ/kg}\cdot\text{K} \end{aligned}$$

$$\left.\begin{aligned} P_{2s} &= 30\text{kPa} \\ s_{2s} &= s_1 \end{aligned}\right\} \quad x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.7240 - 0.9439}{6.8247} = 0.847$$

$$h_{2s} = h_f + x_{2s}h_{fg} = 289.23 + (0.847)(2336.1) = 2267.9\text{kJ/kg}$$

From the isentropic efficiency relation,

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \longrightarrow h_{2a} = h_1 - \eta_T(h_1 - h_{2s}) = 3398.3 - (0.9)(3398.3 - 2267.9) = 2380.9\text{kJ/kg}$$

Thus,

$$\left.\begin{aligned} P_{2a} &= 30\text{kPa} \\ h_{2a} &= 2380.9\text{kJ/kg} \end{aligned}\right\} T_{2a} = T_{sat@30\text{kPa}} = \mathbf{69.10^\circ C}$$

(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the actual turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{a,\text{out}} + \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{a,\text{out}} = \dot{m}(h_1 - h_2)$$

Substituting,

$$\dot{W}_{a,out} = (3\text{kg/s})(3398.3 - 2380.9)\text{kJ/kg} = \mathbf{3052kW}$$

7-86 Steam enters a 5-MW adiabatic turbine at a specified state, and leaves at a specified state. The mass flow rate of the steam and the isentropic efficiency are to be determined.

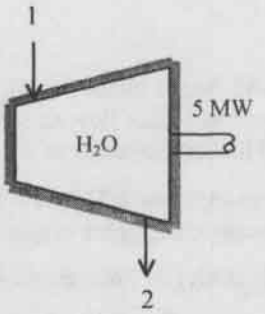
Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Analysis (*a*) From the steam tables (Tables A-4 and A-6),

$$\left.\begin{array}{l} P_1 = 6\text{MPa} \\ T_1 = 600^\circ\text{C} \end{array}\right\} \begin{array}{l} h_1 = 3658.4\text{kJ/kg} \\ s_1 = 7.1677\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 50\text{kPa} \\ T_2 = 100^\circ\text{C} \end{array}\right\} h_{2a} = 2682.5\text{kJ/kg}$$

There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the actual turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{\not=0 \text{ (steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{W}_{a,\text{out}} + \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{a,\text{out}} = -\dot{m}\left(h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right)$$

Substituting, the mass flow rate of the steam is determined to be

$$5000\text{kJ/s} = -\dot{m}\left(2682.5 - 3658.4 + \left\{(140\text{m/s})^2 - (80\text{m/s})^2/(2)\right\}\left\{1\text{kJ/kg}\right)/\left(1000\text{m}^2/\text{s}^2\right)\right\}$$

$$\dot{m} = \mathbf{5.16kg/s}$$

(*b*) The isentropic exit enthalpy of the steam and the power output of the isentropic turbine are

$$\left.\begin{array}{l} P_{2s} = 50\text{kPa} \\ s_{2s} = s_1 \end{array}\right\} \begin{array}{l} x_{2s} = \dfrac{s_{2s} - s_f}{s_{fg}} = \dfrac{7.1677 - 1.0910}{6.5029} = 0.934 \\ h_{2s} = h_f + x_{2s}h_{fg} = 340.49 + (0.934)(2305.4) = 2493.7\text{kJ/kg} \end{array}$$

and

$$\dot{W}_{s,out} = -\dot{m}\left(h_{2s} - h_1 + \left\{\left(\mathbf{V}_2^2 - \mathbf{V}_1^2\right)/2\right\}\right)$$

$$\dot{W}_{s,out} = -(5.16\text{kg/s})\left(2493.7 - 3658.4 + \left\{\left((140\text{m/s})^2 - (80\text{m/s})^2\right)/(2)\right\}\left\{1\text{kJ/kg}\right)/\left(1000\text{m}^2/\text{s}^2\right)\right\}$$

$$= 5976\text{kW}$$

Then the isentropic efficiency of the turbine becomes

$$\eta_T = \frac{\dot{W}_a}{\dot{W}_s} = \frac{5000\text{ kW}}{5976\text{ kW}} = 0.837 = \mathbf{83.7\%}$$

7-87 Argon enters an adiabatic turbine at a specified state with a specified mass flow rate, and leaves at a specified pressure. The isentropic efficiency of the turbine is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Argon is an ideal gas with constant specific heats.

Properties The specific heat ratio of argon is $k = 1.667$. The constant pressure specific heat of argon is $C_p = 0.5203$ kJ/kg.K (Table A-2).

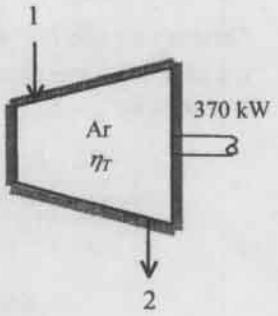
Analysis There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the isentropic turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{s,\text{out}} + \dot{m}h_{2s} \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{s,\text{out}} = \dot{m}(h_1 - h_{2s})$$



From the isentropic relations,

$$T_{2s} = T_1\left(\frac{P_{2s}}{P_1}\right)^{(k-1)/k} = (1073\text{K})\left(\frac{200\text{kPa}}{1500\text{kPa}}\right)^{0.667/1.667} = 479\text{K}$$

Then the power output of the isentropic turbine becomes

$$\dot{W}_{s,out} = \dot{m}C_p(T_1 - T_{2s}) = (80/60\text{kg/min})(0.5203\text{kJ/kg}\cdot\text{K})(1073 - 479) = 412.1\text{kW}$$

Then the isentropic efficiency of the turbine is determined from

$$\eta_T = \frac{\dot{W}_a}{\dot{W}_s} = \frac{370\text{ kW}}{412.1\text{ kW}} = 0.898 = \mathbf{89.8\%}$$

7-88E Combustion gases enter an adiabatic gas turbine with an isentropic efficiency of 86% at a specified state, and leave at a specified pressure. The work output of the turbine is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Combustion gases can be treated as air which is an ideal gas with variable specific heats.

Analysis From the air table and isentropic relations,

$$T_1 = 2000\ \text{R} \longrightarrow \begin{matrix} h_1 = 504.71\ \text{Btu/lbm} \\ P_{r_1} = 174.0 \end{matrix}$$

$$P_{r_2} = \left(\frac{P_2}{P_1}\right)P_{r_1} = \left(\frac{60\text{psia}}{120\text{psia}}\right)(174.0) = 87.0 \longrightarrow h_{2s} = 417.3\text{Btu/lbm}$$

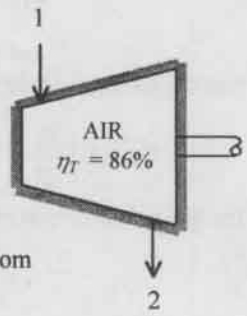
There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the actual turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{a,\text{out}} + \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{a,\text{out}} = \dot{m}(h_1 - h_2)$$



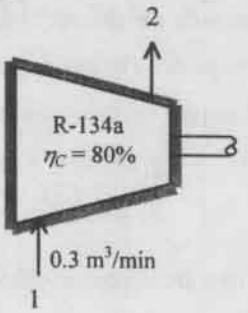
Noting that $w_a = \eta_T w_s$, the work output of the turbine per unit mass is determined from

$$w_a = (0.86)(504.71 - 417.3)\text{Btu/lbm} = \mathbf{75.2Btu/lbm}$$

7-89 Refrigerant-134a enters an adiabatic compressor with an isentropic efficiency of 0.80 at a specified state with a specified volume flow rate, and leaves at a specified pressure. The compressor exit temperature and power input to the compressor are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Analysis (a) From the refrigerant tables,



$$\left.\begin{array}{l} P_1 = 120\text{kPa} \\ sat.vapor \end{array}\right\} \begin{array}{l} h_1 = h_{g@120\text{kPa}} = 233.86\text{kJ/kg} \\ s_1 = s_{g@120\text{kPa}} = 0.9354\text{kJ/kg}\cdot\text{K} \\ v_1 = v_{g@120\text{kPa}} = 0.1614\text{m}^3\text{/kg} \end{array}$$

$$\left.\begin{array}{l} P_2 = 1\text{MPa} \\ s_{2s} = s_1 \end{array}\right\} h_{2s} = 277.84\text{kJ/kg}$$

From the isentropic efficiency relation,

$$\eta_C = \frac{h_{2s} - h_1}{h_{2a} - h_1} \longrightarrow h_{2a} = h_1 + (h_{2s} - h_1)/\eta_C = 233.86 + (277.84 - 233.86)/0.80 = 288.84\text{kJ/kg}$$

Thus,

$$\left.\begin{array}{l} P_{2a} = 1\text{MPa} \\ h_{2a} = 288.84\text{kJ/kg} \end{array}\right\} T_{2a} = \mathbf{57.7^\circ C}$$

(b) The mass flow rate of the refrigerant is determined from

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{0.3/60\ \text{m}^3/\text{s}}{0.1614\ \text{m}^3/\text{kg}} = 0.031\ \text{kg/s}$$

There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the actual compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{\nearrow 0 \text{ (steady)}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{a,\text{in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{a,\text{in}} = \dot{m}(h_2 - h_1)$$

Substituting, the power input to the compressor becomes,

$$\dot{W}_{a,in} = (0.031\text{kg/s})(288.84 - 233.86)\text{kJ/kg} = \mathbf{1.70kW}$$

7-90 Air enters an adiabatic compressor with an isentropic efficiency of 84% at a specified state, and leaves at a specified temperature. The exit pressure of air and the power input to the compressor are to be determined. √

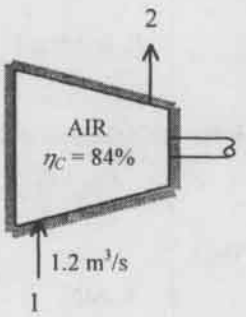
Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Air is an ideal gas with variable specific heats.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1)

Analysis (*a*) From the air table (Table A-17),

$$T_1 = 290\text{ K} \longrightarrow h_1 = 290.16\text{ kJ/kg}, \qquad P_{r1} = 1.2311$$

$$T_2 = 530\text{ K} \longrightarrow h_{2a} = 533.98\text{ kJ/kg}$$



From the isentropic efficiency relation $\eta_C = \dfrac{(h_{2s} - h_1)}{(h_{2a} - h_1)}$,

$$h_{2s} = h_1 + \eta_C (h_{2a} - h_1)$$

$$= 290.16 + (0.84)(533.98 - 290.16) = 495.0\text{kJ/kg} \longrightarrow P_{r_2} = 7.951$$

Then from the isentropic relation ,

$$\frac{P_2}{P_1} = \frac{P_{r_2}}{P_{r_1}} \longrightarrow P_2 = \left(\frac{P_{r_2}}{P_{r_1}}\right) P_1 = \left(\frac{7.951}{1.2311}\right)(100\text{kPa}) = \mathbf{646kPa}$$

(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the actual compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{a,\text{in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{a,\text{in}} = \dot{m}(h_2 - h_1)$$

where $$\dot{m} = \frac{P_1 \dot{V}_1}{RT_1} = \frac{(100\text{kPa})(1.2\text{m}^3/\text{s})}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\text{K})} = 1.442\text{kg/s}$$

Then the power input to the compressor is determined to be

$$\dot{W}_{a,out} = (1.442\text{kg/s})(533.98 - 290.16)\text{kJ/kg} = \mathbf{352kW}$$

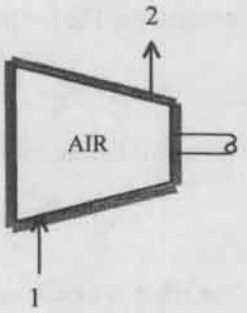
7-91 Air is compressed by an adiabatic compressor from a specified state to another specified state. The isentropic efficiency of the compressor and the exit temperature of air for the isentropic case are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Air is an ideal gas with variable specific heats.

Analysis From the air table (Table A-17),

$$T_1 = 300\ \text{K} \longrightarrow h_1 = 300.19\ \text{kJ/kg}, \qquad P_{r_1} = 1.386$$

$$T_2 = 550\ \text{K} \longrightarrow h_{2a} = 554.74\ \text{kJ/kg}$$



From the isentropic relation,

$$P_{r_2} = \left(\frac{P_2}{P_1}\right)P_{r_1} = \left(\frac{600\text{kPa}}{95\text{kPa}}\right)(1.386) = 8.754 \longrightarrow h_{2s} = 508.72\text{kJ/kg}$$

Then the isentropic efficiency becomes

$$\eta_C = \frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{508.72 - 300.19}{554.74 - 300.19} = 0.819 = \mathbf{81.9\%}$$

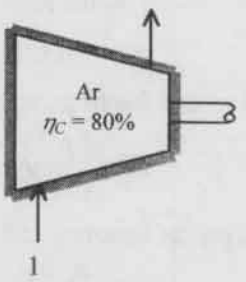
(b) If the process were isentropic, the exit temperature would be

$$h_{2s} = 508.72\ \text{kJ/kg} \longrightarrow T_{2s} = \mathbf{505.5\ K}$$

7-92E Argon enters an adiabatic compressor with an isentropic efficiency of 80% at a specified state, and leaves at a specified state. The exit temperature of argon and the work input to the compressor are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Argon is an ideal gas with constant specific heats.

Properties The specific heat ratio of argon is $k = 1.667$. The constant pressure specific heat of argon is $C_p = 0.1253$ Btu/lbm.R (Table A-2E).



Analysis (*a*) The isentropic exit temperature T_{2s} is determined from

$$T_{2s} = T_1\left(\frac{P_{2s}}{P_1}\right)^{(k-1)/k} = (550\text{R})\left(\frac{200\text{psia}}{20\text{psia}}\right)^{0.667/1.667} = 1381.9\text{R}$$

The actual kinetic energy change during this process is

$$\Delta ke_a = \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} = \frac{(240\text{ft/s})^2 - (60\text{ft/s})^2}{2}\left(\frac{1\text{Btu/lbm}}{25{,}037\text{ft}^2/\text{s}^2}\right) = 1.08\text{Btu/lbm}$$

The effect of kinetic energy on isentropic efficiency is very small. Therefore, we can take the kinetic energy changes for the actual and isentropic cases to be same in efficiency calculations. From the isentropic efficiency relation, including the effect of kinetic energy,

$$\eta_C = \frac{w_s}{w_a} = \frac{(h_{2s} - h_1) + \Delta ke}{(h_{2a} - h_1) + \Delta ke} = \frac{C_p(T_{2s} - T_1) + \Delta ke_s}{C_p(T_{2a} - T_1) + \Delta ke_a} \longrightarrow 0.8 = \frac{0.1253(1381.9 - 550) + 1.08}{0.1253(T_{2a} - 550) + 1.08}$$

It yields $T_{2a} =$ **1592 R**

(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the actual compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{a,\text{in}} + \dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{a,\text{in}} = \dot{m}\left(h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}\right) \rightarrow w_{a,\text{in}} = h_2 - h_1 + \Delta ke$$

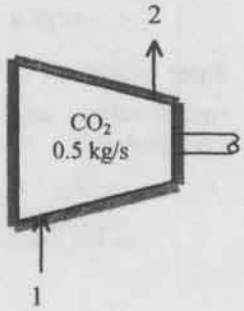
Substituting, the work input to the compressor is determined to be

$$w_{a,in} = (0.1253\text{Btu/lbm} \cdot \text{R})(1592 - 550)\text{R} + 1.08\text{Btu/lbm} = \mathbf{131.6Btu/lbm}$$

7-93 CO_2 gas is compressed by an adiabatic compressor from a specified state to another specified state. The isentropic efficiency of the compressor and the rate of entropy generation are to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** CO_2 is an ideal gas with constant specific heats.

Properties At the average temperature of (300 + 450)/2 = 375 K, the constant pressure specific heat and the specific heat ratio of CO_2 are $k = 1.260$ and $C_p = 0.917$ kJ/kg.K (Table A-2).



Analysis The isentropic exit temperature T_{2s} is

$$T_{2s} = T_1\left(\frac{P_{2s}}{P_1}\right)^{(k-1)/k} = (300\ \text{K})\left(\frac{600\text{kPa}}{100\text{kPa}}\right)^{0.260/1.260} = 434.2\ \text{K}$$

From the isentropic efficiency relation,

$$\eta_C = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{C_p(T_{2s} - T_1)}{C_p(T_{2a} - T_1)} = \frac{T_{2s} - T_1}{T_{2a} - T_1} = \frac{434.2 - 300}{450 - 300} = 0.895 = \mathbf{89.5\%}$$

7-94E Air is accelerated in a 90% efficient adiabatic nozzle from low velocity to a specified velocity. The exit temperature and pressure of the nozzle are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Air is an ideal gas with variable specific heats.

Analysis From the air table (Table A-17),

$$T_1 = 1480 \text{ R} \longrightarrow h_1 = 363.89 \text{ Btu/lbm}, \qquad P_{r_1} = 53.04$$

There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

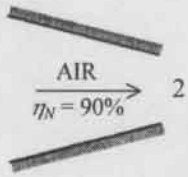
$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{W} = \dot{Q} \cong \Delta \text{pe} \cong 0)$$

$$h_2 = h_1 - \frac{\mathbf{V}_2^2 - \mathbf{V}_1^{2\,\nearrow 0}}{2}$$

Substituting, the exit temperature of air is determined to be

$$h_2 = 363.89 \text{kJ/kg} - \frac{(800 \text{ft/s})^2 - 0}{2}\left(\frac{1 \text{Btu/lbm}}{25{,}037 \text{ft}^2/\text{s}^2}\right) = 351.11 \text{Btu/lbm}$$



From the air table we read T_{2a} = **1431.4 R**

From the isentropic efficiency relation $\eta_N = \dfrac{(h_{2a} - h_1)}{(h_{2s} - h_1)}$,

$$h_{2s} = h_1 + (h_{2a} - h_1)/\eta_N = 363.89 + (351.11 - 363.89)/(0.90) = 349.69 \text{Btu/lbm} \longrightarrow P_{r_2} = 46.04$$

Then the exit pressure is determined from the isentropic relation to be

$$\frac{P_2}{P_1} = \frac{P_{r_2}}{P_{r_1}} \longrightarrow P_2 = \left(\frac{P_{r_2}}{P_{r_1}}\right)P_1 = \left(\frac{46.09}{53.04}\right)(60 \text{psia}) = \mathbf{52.1 psia}$$

7-95 Hot combustion gases are accelerated in a 92% efficient adiabatic nozzle from low velocity to a specified velocity. The exit velocity and the exit temperature are to be determined.

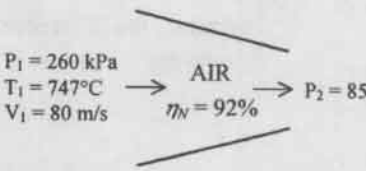
Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Combustion gases can be treated as air which is an ideal gas with variable specific heats.

Analysis From the air table (Table A-17),

$$T_1 = 1020\ \text{K} \longrightarrow h_1 = 1068.89\ \text{kJ/kg}, \qquad P_{r_1} = 123.4$$

From the isentropic relation ,

$$P_{r_2} = \left(\frac{P_2}{P_1}\right)P_{r_1} = \left(\frac{85\text{kPa}}{260\text{kPa}}\right)(123.4) = 40.34 \longrightarrow h_{2s} = 783.92\text{kJ/kg}$$



There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system for the isentropic process can be expressed as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_{2s} + \mathbf{V}_{2s}^2/2) \quad (\text{since } \dot{W} = \dot{Q} \cong \Delta\text{pe} \cong 0)$$

$$h_{2s} = h_1 - \frac{\mathbf{V}_{2s}^2 - \mathbf{V}_1^2}{2}$$

Then the isentropic exit velocity becomes

$$\mathbf{V}_{2s} = \sqrt{\mathbf{V}_1^2 + 2(h_1 - h_{2s})} = \sqrt{(80\text{m/s})^2 + 2(1068.89 - 763.92)\text{kJ/kg}\left(\frac{1000\text{m}^2/\text{s}^2}{1\text{kJ/kg}}\right)} = 759.2\text{m/s}$$

Therefore,

$$\mathbf{V}_{2a} = \sqrt{\eta_N}\,\mathbf{V}_{2s} = \sqrt{0.92}(759.2\text{m/s}) = \mathbf{728.2\ m/s}$$

The exit temperature of air is determined from the steady-flow energy equation,

$$h_{2a} = 1068.89\text{kJ/kg} - \frac{(728.2\text{m/s})^2 - (80\text{m/s})^2}{2}\left(\frac{1\text{kJ/kg}}{1000\text{m}^2/\text{s}^2}\right) = 806.95\text{kJ/kg}$$

From the air table we read $T_{2a} =$ **786.3 K**

Review Problems

7-96 It is to be shown that the difference between the steady-flow and boundary works is the flow energy.

Analysis The total differential of flow energy Pv can be expressed as

$$d(Pv) = Pdv + vdP = \delta w_b - \delta w_{flow} = \delta(w_b - w_{flow})$$

Therefore, the difference between the reversible steady-flow work and the reversible boundary work is the flow energy.

7-97E An insulated rigid can initially contains R-134a at a specified state. A crack develops, and refrigerant escapes slowly. The final pressure in the can is to be determined when the pressure inside drops to a specified value.

Assumptions **1** The can is well-insulated and thus heat transfer is negligible. **2** The refrigerant that remains in the can underwent a reversible adiabatic process.

Analysis Noting that for a reversible adiabatic (i.e., isentropic) process, $s_1 = s_1$, the properties of the refrigerant in the can are

$$\left.\begin{array}{l} P_1 = 120\text{psia} \\ T_1 = 80^\circ\text{F} \end{array}\right\} s_1 \cong s_{f@80^\circ\text{F}} = 0.0548\text{Btu/lbm}\cdot\text{R}$$



$$\left.\begin{array}{l} P_1 = 30\text{psia} \\ s_2 = s_1 \end{array}\right\} \quad x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{0.0548 - 0.0245}{0.1679 - 0.0245} = 0.2113$$

$$v_2 = v_f + x_2 v_{fg} = 0.0112 + (0.2113)(1.2964 - 0.0112) = 0.2828\text{ft}^3/\text{lbm}$$

Thus the final mass of the refrigerant in the can is

$$m = \frac{V}{v_1} = \frac{0.8\text{ft}^3}{0.2828\text{ft}^3/\text{lbm}} = \mathbf{2.829lbm}$$

7-98 An insulated rigid tank is connected to a piston-cylinder device with zero clearance that is maintained at constant pressure. A valve is opened, and some steam in the tank is allowed to flow into the cylinder. The final temperatures in the tank and the cylinder are to be determined.

Assumptions **1** Both the tank and cylinder are well-insulated and thus heat transfer is negligible. **2** The water that remains in the tank underwent a reversible adiabatic process. **3** The thermal energy stored in the tank and cylinder themselves is negligible. **4** The system is stationary and thus kinetic and potential energy changes are negligible.

Analysis (a) The steam in tank A undergoes a reversible, adiabatic process, and thus $s_2 = s_1$. From the steam tables,

$$\left.\begin{array}{l} P_1 = 500\text{kPa} \\ sat.vapor \end{array}\right\} \begin{array}{l} v_1 = v_{g@500\text{kPa}} = 0.3749\text{m}^3/\text{kg} \\ u_1 = u_{g@500\text{kPa}} = 2561.2\text{kJ/kg} \\ s_1 = s_{g@500\text{kPa}} = 6.8213\text{kJ/kg}\cdot\text{K} \end{array}$$

$$T_{2,A} = T_{sat@150\text{kPa}} = \mathbf{111.37^\circ C}$$

$$\left.\begin{array}{l} P_1 = 150\text{kPa} \\ s_2 = s_1 \\ (sat.mixture) \end{array}\right\} \begin{array}{l} x_{2,A} = \dfrac{s_{2,A} - s_f}{s_{fg}} = \dfrac{6.8213 - 1.4336}{5.7897} = 0.9306 \\ v_{2,A} = v_f + x_{2,A} v_{fg} = 0.001053 + (0.9306)(1.1593 - 0.001053) = 1.079\text{m}^3/\text{kg} \\ u_{2,A} = u_f + x_{2,A} u_{fg} = 466.94 + (0.9306)(2052.7\text{kJ/kg}) = 2377.2\text{kJ/kg} \end{array}$$

The initial and the final masses in tank A are

$$m_{1,A} = \frac{V_A}{v_{1,A}} = \frac{0.4\text{m}^3}{0.3749\text{m}^3/\text{kg}} = 1.067\text{kg} \quad \textit{and} \quad m_{2,A} = \frac{V_A}{v_{2,A}} = \frac{0.4\text{m}^3}{1.079\text{m}^3/\text{kg}} = 0.371\text{kg}$$

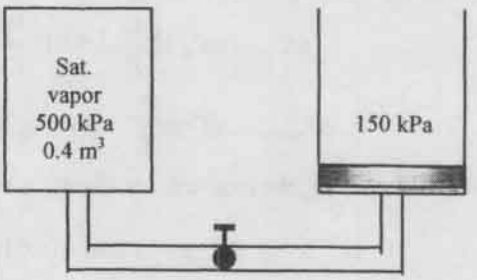
Thus,

$$m_{2,B} = m_{1,A} - m_{2,B} = 1.067 - 0.371 = 0.697\text{kg}$$

The boundary work done during this process is

$$W_{b,out} = \int_1^2 P\,dV = P_B(V_{2,B} - 0) = P_B m_{2,B} v_{2,B}$$

Taking the contents of both the tank and the cylinder to be the system, the energy balance for this closed system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$-W_{b,\text{out}} = \Delta U = (\Delta U)_A + (\Delta U)_B$$

$$W_{b,out} + (\Delta U)_A + (\Delta U)_B = 0$$

or,
$$P_B m_{2,B} v_{2,B} + (m_2 u_2 - m_1 u_1)_A + (m_2 u_2)_B = 0$$
$$m_{2,B} h_{2,B} + (m_2 u_2 - m_1 u_1)_A = 0$$

Thus,

$$h_{2,B} = \frac{(m_2 u_2 - m_1 u_1)_A}{m_{2,B}} = \frac{(1.067)(2561.2) - (0.371)(2377.2)}{0.696} = 2659.3 kJ/kg$$

At 150 kPa, $h_f = 467.11$ and $h_g = 2693.6$ kJ/kg. Thus at the final state, the cylinder will contain a saturated liquid-vapor mixture since $h_f < h_2 < h_g$. Therefore,

$$T_{2,B} = T_{sat@150\,kPa} = \mathbf{111.37^\circ C}$$

7-99 One ton of liquid water at 80°C is brought into a room. The final equilibrium temperature in the room and the entropy generated are to be determined. √

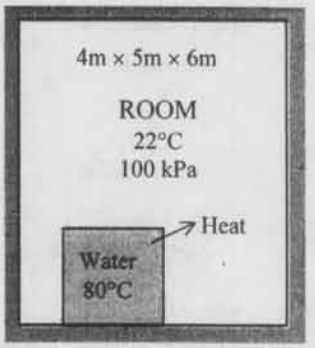
Assumptions **1** The room is well insulated and well sealed. **2** The thermal properties of water and air are constant at room temperature. **3** The system is stationary and thus the kinetic and potential energy changes are zero. **4** There are no work interactions involved.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1). The specific heat of water at room temperature is $C = 4.18$ kJ/kg·°C (Table A-3). For air is $C_v = 0.718$ kJ/kg·°C at room temperature.

Analysis The volume and the mass of the air in the room are

$$V = 4x5x6 = 120 \text{ m}^3$$

$$m_{air} = \frac{P_1 V_1}{RT_1} = \frac{(100\text{kPa})(120\text{m}^3)}{(0.2870\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(295\text{K})} = 141.7\text{kg}$$



Taking the contents of the room, including the water, as our system, the energy balance can be written as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}} \rightarrow 0 = \Delta U = (\Delta U)_{\text{water}} + (\Delta U)_{\text{air}}$$

or $$[mC(T_2 - T_1)]_{\text{water}} + [mC_v(T_2 - T_1)]_{\text{air}} = 0$$

Substituting,

$$(1000\text{kg})(4.184\text{kJ/kg}\cdot^\circ\text{C})(T_f - 80)^\circ\text{C} + (141.7\text{kg})(0.718\text{kJ/kg}\cdot^\circ\text{C})(T_f - 22)^\circ\text{C} = 0$$

It gives the final equilibrium temperature in the room to be

$$T_f = \mathbf{78.6°C}$$

(*b*) Considering that the system is well-insulated and no mass is entering and leaving, the total entropy change during this process is the sum of the entropy changes of water and the room air,

$$\Delta S_{\text{total}} = S_{\text{gen}} = \Delta S_{\text{air}} + \Delta S_{\text{water}}$$

where

$$\Delta S_{air} = mC_v \ln\frac{T_2}{T_1} + mR \ln\frac{V_2}{V_1}^{\nearrow 0} = (141.74\text{kg})(0.718\text{kJ/kg}\cdot\text{K})\ln\frac{351.6\text{K}}{295\text{K}} = 17.86\text{kJ/K}$$

$$\Delta S_{water} = mC \ln\frac{T_2}{T_1} = (1000\text{kg})(4.184\text{kJ/kg}\cdot\text{K})\ln\frac{351.6\text{K}}{353\text{K}} = -16.63\text{kJ/K}$$

Substituting, the total entropy change is determined to be

$$\Delta S_{\text{total}} = 17.86 - 16.63 = \mathbf{1.23\ kJ/K}$$

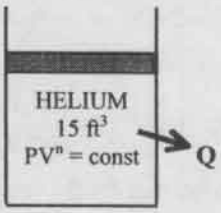
7-100E A cylinder initially filled with helium gas at a specified state is compressed polytropically to a specified temperature and pressure. The entropy changes of the helium and the surroundings are to be determined, and it is to be assessed if the process is reversible, irreversible, or impossible.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** The cylinder is stationary and thus the kinetic and potential energy changes are negligible. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

Properties The gas constant of helium is $R = 2.6805$ psia.ft^3/lbm.R = 0.4961 Btu/lbm.R (Table A-1E). The specific heats of helium are $C_v = 0.753$ and $C_v = 1.25$ Btu/lbm.R (Table A-2E).

Analysis (*a*) Helium at specified conditions can be treated as an ideal gas. The mass of helium is

$$m = \frac{P_1V_1}{RT_1} = \frac{(25\text{psia})(15\text{ft}^3)}{(2.6805\text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(530\text{R})} = 0.264\text{lbm}$$



Then the entropy change of helium becomes

$$\Delta S_{sys} = \Delta S_{helium} = m\left(C_{p,ave}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right)$$

$$= (0.264\text{lbm})\left[(1.25\text{Btu/lbm}\cdot\text{R})\ln\frac{760\text{R}}{530\text{R}} - (0.4961\text{Btu/lbm}\cdot\text{R})\ln\frac{70\text{psia}}{25\text{psia}}\right] = \mathbf{-0.016Btu/R}$$

(*b*) The exponent n and the boundary work for this polytropic process are determined to be

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \longrightarrow V_2 = \frac{T_2}{T_1}\frac{P_1}{P_2}V_1 = \frac{(760\text{R})(25\text{psia})}{(530\text{R})(70\text{psia})}(15\text{ft}^3) = 7.682\text{ft}^3$$

$$P_2V_2^n = P_1V_1^n \longrightarrow \left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^n \longrightarrow \left(\frac{70}{25}\right) = \left(\frac{15}{7.682}\right)^n \longrightarrow n = 1.539$$

Then the boundary work for this polytropic process can be determined from

$$W_{b,in} = -\int_1^2 P\,dV = -\frac{P_2V_2 - P_1V_1}{1-n} = -\frac{mR(T_2 - T_1)}{1-n}$$

$$= -\frac{(0.264\text{lbm})(0.4961\text{Btu/lbm}\cdot\text{R})(760-530)\text{R}}{1-1.539} = 55.9\text{Btu}$$

We take the helium in the cylinder as the system, which is a closed system. Taking the direction of heat transfer to be from the cylinder, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{out} + W_{b,in} = \Delta U = m(u_2 - u_1)$$

$$-Q_{out} = m(u_2 - u_1) - W_{b,in}$$

$$Q_{out} = W_{b,in} - mC_v(T_2 - T_1)$$

Substituting,

$$Q_{out} = 55.9\text{Btu} - (0.263\text{lbm})(0.753\text{Btu/lbm}\cdot\text{R})(760-530)\text{R} = 10.2\text{Btu}$$

Noting that the surroundings undergo a reversible isothermal process, its entropy change becomes

$$\Delta S_{surr} = \frac{Q_{surr,in}}{T_{surr}} = \frac{10.2\text{ Btu}}{530\text{ R}} = \mathbf{0.019\ Btu/R}$$

(*c*) Noting that the system+surroundings combination can be treated as an isolated system,

$$\Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} = -0.016 + 0.019 = 0.003\text{Btu/R} > 0$$

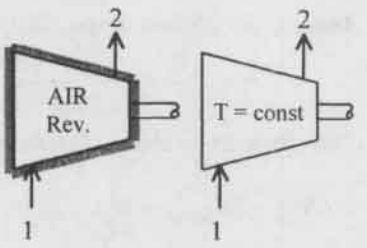
Therefore, the process is **irreversible**.

101 Air is compressed steadily by a compressor from a specified state to a specified pressure. The nimum power input required is to be determined for the cases of adiabatic and isothermal operation.

sumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential ergy changes are negligible. **3** Air is an ideal gas with variable specific heats. **4** The process is reversible ce the work input to the compressor will be minimum when the compression process is reversible.

operties The gas constant of air is $R = 0.287$ kJ/kg.K (Table A-1).

alysis (*a*) For the adiabatic case, the process will be reversible and adiabatic (i.e., isentropic), s the isentropic relations are applicable. Using variable specific heats,



$$T_1 = 290\text{K} \longrightarrow P_{r_1} = 1.2311 \text{ and } h_1 = 290.16\text{kJ/kg}$$

d

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = \frac{700\text{kPa}}{100\text{kPa}}(1.2311) = 8.6177 \rightarrow \begin{matrix} T_2 = 503.3\text{K} \\ h_2 = 506.57\text{kJ/kg} \end{matrix}$$

e energy balance for the compressor, which is a steady-flow system, n be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2 \rightarrow \dot{W}_{\text{in}} = \dot{m}(h_2 - h_1)$$

bstituting, the power input to the compressor is determined to be

$$\dot{W}_{in} = (2/60\text{kg/s})(506.57 - 290.16)\text{kJ/kg} = \mathbf{7.21kW}$$

) In the case of the reversible isothermal process, the steady-flow energy balance becomes

$$\dot{E}_{in} = \dot{E}_{out} \rightarrow \dot{W}_{in} + \dot{m}h_1 - \dot{Q}_{out} = \dot{m}h_2 \rightarrow \dot{W}_{\text{in}} = \dot{Q}_{out} + \dot{m}(h_2 - h_1)^{\nearrow 0} = \dot{Q}_{out}$$

ce $h = h(T)$ for ideal gases, and thus the enthalpy change in this case is zero. Also, for a reversible thermal process,

$$\dot{Q}_{out} = \dot{m}T(s_1 - s_2) = -\dot{m}T(s_2 - s_1)$$

ere

$$s_2 - s_1 = \left(s_2^\circ - s_1^\circ\right)^{\nearrow 0} - R\ln\frac{P_2}{P_1} = -R\ln\frac{P_2}{P_1} = -(0.287\text{kJ/kg}\cdot\text{K})\ln\frac{700\text{kPa}}{100\text{kPa}} = -0.5585\text{kJ/kg}\cdot\text{K}$$

bstituting, the power input for the reversible isothermal case becomes

$$\dot{W}_{in} = -(2/60\text{kg/s})(290\text{K})(-0.5585\text{kJ/kg}\cdot\text{K}) = \mathbf{5.40kW}$$

7-102 Air is compressed in a two-stage ideal compressor with intercooling. For a specified mass flow rate of air, the power input to the compressor is to be determined, and it is to be compared to the power input to a single-stage compressor. √

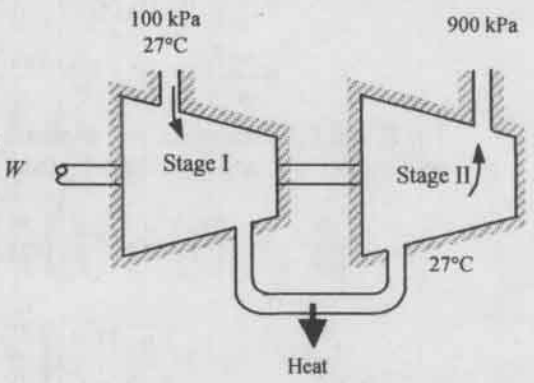
Assumptions **1** The compressor operates steadily. **2** Kinetic and potential energies are negligible. **3** The compression process is reversible adiabatic, and thus isentropic. **4** Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R =0.287 kPa.m^3/kg.K (Table A-1). The specific heat ratio of air is k = 1.4 (Table A-2)

Analysis The intermediate pressure between the two stages is

$$P_x = \sqrt{P_1 P_2} = \sqrt{(100\text{kPa})(900\text{kPa})} = 300\text{kPa}$$

The compressor work across each stage is the same, thus total compressor work is twice the compression work for a single stage:



$$w_{comp,in} = (2)(w_{comp,in,I}) = 2\frac{kRT_1}{k-1}\left((P_x/P_1)^{(k-1)/k} - 1\right)$$

$$= 2\frac{(1.4)(0.287\text{kJ/kg}\cdot\text{K})(300\text{K})}{1.4-1}\left(\left(\frac{300\text{kPa}}{100\text{kPa}}\right)^{0.4/1.4} - 1\right)$$

$$= 222.2\text{kJ/kg}$$

and

$$\dot{W}_{in} = \dot{m} w_{comp,in} = (0.02\text{kg/s})(222.2\text{kJ/kg}) = \mathbf{4.44kW}$$

The work input to a single-stage compressor operating between the same pressure limits would be

$$w_{comp,in} = \frac{kRT_1}{k-1}\left((P_2/P_1)^{(k-1)/k} - 1\right) = \frac{(1.4)(0.287\text{kJ/kg}\cdot\text{K})(300\text{K})}{1.4-1}\left(\left(\frac{900\text{kPa}}{100\text{kPa}}\right)^{0.4/1.4} - 1\right) = 263.2\text{kJ/kg}$$

and

$$\dot{W}_{in} = \dot{m} w_{comp,in} = (0.02\text{kg/s})(263.2\text{kJ/kg}) = \mathbf{5.26\ kW}$$

Discussion Note that the power consumption of the compressor decreases significantly by using 2-stage compression with intercooling.

7-103 A three-stage compressor with two stages of intercooling is considered. The two intermediate pressures that will minimize the work input are to be determined in terms of the inlet and exit pressures.

Analysis The work input to this three-stage compressor with intermediate pressures P_x and P_y and two intercoolers can be expressed as

$$\begin{aligned} w_{comp} &= w_{comp,\mathrm{I}} + w_{comp,\mathrm{II}} + w_{comp,\mathrm{III}} \\ &= \frac{nRT_1}{n-1}\left(1-\left(P_x/P_1\right)^{(n-1)/n}\right)+\frac{nRT_1}{n-1}\left(1-\left(P_y/P_x\right)^{(n-1)/n}\right)+\frac{nRT_1}{n-1}\left(1-\left(P_x/P_1\right)^{(n-1)/n}\right) \\ &= \frac{nRT_1}{n-1}\left(1-\left(P_x/P_1\right)^{(n-1)/n}+1-\left(P_y/P_x\right)^{(n-1)/n}+1-\left(P_x/P_1\right)^{(n-1)/n}\right) \\ &= \frac{nRT_1}{n-1}\left(3-\left(P_x/P_1\right)^{(n-1)/n}-\left(P_y/P_x\right)^{(n-1)/n}-\left(P_x/P_1\right)^{(n-1)/n}\right) \end{aligned}$$

The P_x and P_y values which will minimize the work input are obtained by taking the partial differential of w with respect to P_x and P_y, and setting them equal to zero:

$$\frac{\partial w}{\partial P_x}=0 \longrightarrow -\frac{n-1}{n}\left(\frac{1}{P_1}\right)\left(\frac{P_x}{P_1}\right)^{\frac{n-1}{n}-1}+\frac{n-1}{n}\left(\frac{1}{P_y}\right)\left(\frac{P_x}{P_y}\right)^{-\frac{n-1}{n}-1}=0$$

$$\frac{\partial w}{\partial P_y}=0 \longrightarrow -\frac{n-1}{n}\left(\frac{1}{P_x}\right)\left(\frac{P_y}{P_x}\right)^{\frac{n-1}{n}-1}+\frac{n-1}{n}\left(\frac{1}{P_2}\right)\left(\frac{P_y}{P_2}\right)^{-\frac{n-1}{n}-1}=0$$

Simplifying,

$$\frac{1}{P_1}\left(\frac{P_x}{P_1}\right)^{-\frac{1}{n}}=\frac{1}{P_y}\left(\frac{P_x}{P_y}\right)^{-\frac{2n-1}{n}} \longrightarrow \frac{1}{P_1^n}\left(\frac{P_1}{P_x}\right)=\frac{1}{P_y^n}\left(\frac{P_x}{P_y}\right)^{1-2n} \longrightarrow P_x^{2(1-n)}=\left(P_1P_y\right)^{1-n}$$

$$\frac{1}{P_x}\left(\frac{P_y}{P_x}\right)^{-\frac{1}{n}}=\frac{1}{P_2}\left(\frac{P_y}{P_2}\right)^{-\frac{2n-1}{n}} \longrightarrow \frac{1}{P_x^n}\left(\frac{P_x}{P_y}\right)=\frac{1}{P_2^n}\left(\frac{P_y}{P_2}\right)^{1-2n} \longrightarrow P_y^{2(1-n)}=\left(P_xP_2\right)^{1-n}$$

which yield

$$\mathbf{P_x^2 = P_1\sqrt{P_xP_2} \longrightarrow P_x = \left(P_1^2P_2\right)^{1/3}}$$

$$\mathbf{P_y^2 = P_2\sqrt{P_1P_y} \longrightarrow P_y = \left(P_1P_2^2\right)^{1/3}}$$

7-104 Steam expands in a two-stage adiabatic turbine from a specified state to specified pressure. Some steam is extracted at the end of the first stage. The power output of the turbine is to be determined for the cases of 100% and 88% isentropic efficiencies.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The turbine is adiabatic and thus heat transfer is negligible.

Properties From the steam tables (Tables A-4 through 6)

$$\left.\begin{matrix} P_1 = 7\text{MPa} \\ T_1 = 500^\circ\text{C} \end{matrix}\right\} \begin{matrix} h_1 = 3410.3\text{kJ/kg} \\ s_1 = 6.7965\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_2 = 1\text{MPa} \\ s_2 = s_1 \end{matrix}\right\} h_2 = 2879.4\,\text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 50\text{kPa} \\ s_3 = s_1 \end{matrix}\right\} \begin{matrix} x_{3s} = \dfrac{s_{3s} - s_f}{s_{fg}} = \dfrac{6.7975 - 1.0910}{6.5029} = 0.8775 \\ h_{3s} = h_f + x_{3s} h_{fg} = 340.49 + (0.8775)(2305.4) = 2363.5\text{kJ/kg} \end{matrix}$$

Analysis (a) The mass flow rate through the second stage is

$$\dot{m}_3 = 0.9\dot{m}_1 = (0.9)(15\text{kg/s}) = 13.5\text{kg/s}$$

We take the entire turbine, including the connection part between the two stages, as the system, which is a control volume since mass crosses the boundary. Noting that one fluid stream enters the turbine and two fluid streams leave, the energy balance for this steady-flow system can be expressed in the rate form as

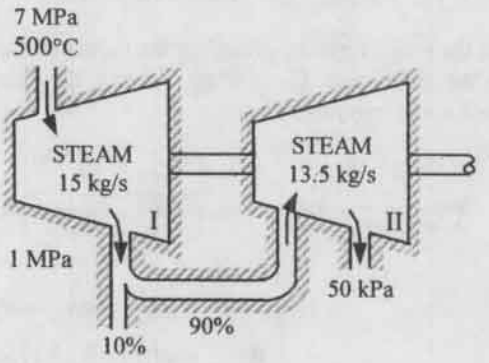
$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{\not{0}\ \text{(steady)}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 = (\dot{m}_1 - \dot{m}_3) h_2 + \dot{W}_{\text{out}} + \dot{m}_3 h_3$$

$$\dot{W}_{\text{out}} = \dot{m}_1 h_1 - (\dot{m}_1 - \dot{m}_3) h_2 - \dot{m}_3 h_3$$

$$= \dot{m}_1 (h_1 - h_2) + \dot{m}_3 (h_2 - h_3)$$



Substituting, the power output of the turbine is

$$\dot{W}_{out} = (15\text{kg/s})(3410.3 - 2879.4)\text{kJ/kg} + (13.5\text{kg})(2879.4 - 2363.5)\text{kJ/kg} = \mathbf{14{,}929\,kW}$$

(*b*) If the turbine has an adiabatic efficiency of 88%, then the power output becomes

$$\dot{W}_a = \eta_T \dot{W}_s = (0.88)(14{,}929kW) = \mathbf{13{,}138\,kW}$$

-105 Steam expands in an 84% efficient two-stage adiabatic turbine from a specified state to a specified ressure. Steam is reheated between the stages. For a given power output, the mass flow rate of steam hrough the turbine is to be determined. √

lssumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential nergy changes are negligible. **3** The turbine is adiabatic and thus heat transfer is negligible.

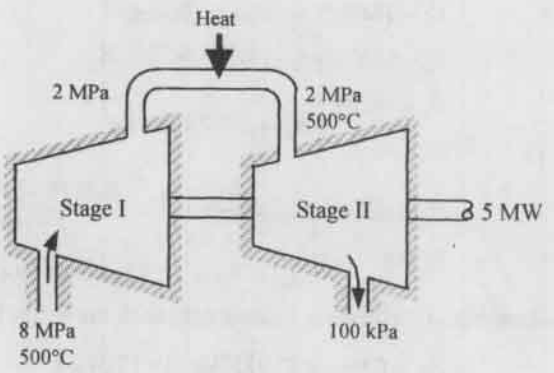
'roperties From the steam tables (Tables A-4 through 6)

$$\left.\begin{matrix} P_1 = 8\ \text{MPa} \\ T_1 = 500^\circ\text{C} \end{matrix}\right\} \begin{matrix} h_1 = 3398.3\ \text{kJ/kg} \\ s_1 = 6.7240\ \text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_{2s} = 2\ \text{MPa} \\ s_{2s} = s_1 \end{matrix}\right\} h_{2s} = 3000.3\ \text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 2\ \text{MPa} \\ T_3 = 500^\circ\text{C} \end{matrix}\right\} \begin{matrix} h_3 = 3467.6\ \text{kJ/kg} \\ s_3 = 7.4317\ \text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_{4s} = 100\ \text{kPa} \\ s_{4s} = s_3 \end{matrix}\right\} h_{4s} = 2704.2\ \text{kJ/kg}$$



nalysis The power output of the actual turbine is given to be 40 MW. hen the power output for the isentropic operation becomes

$$\dot{W}_{s,out} = \dot{W}_{a,out} / \eta_T = (40{,}000\ \text{kW}) / 0.84 = 47{,}619\text{kW}$$

/e take the entire turbine, excluding the reheat section, as the system, which is a control volume since mass rosses the boundary. The energy balance for this steady-flow system in isentropic operation can be xpressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 + \dot{m}h_3 = \dot{m}h_{2s} + \dot{m}h_{4s} + \dot{W}_{\text{s,out}}$$

$$\dot{W}_{\text{s,out}} = \dot{m}[(h_1 - h_{2s}) + (h_3 - h_{4s})]$$

ubstituting,

$$47{,}619\ \text{kJ/s} = \dot{m}\left[(3398.3 - 3000.3) + (3467.6 - 2704.2)\text{kJ/kg}\right]\text{kJ/kg}$$

hich gives

$$\dot{m} = \mathbf{41.0\ kg/s}$$

7-106 Refrigerant-134a is compressed by a 0.5-kW adiabatic compressor from a specified state to another specified state. The isentropic efficiency, the volume flow rate at the inlet, and the maximum flow rate at the compressor inlet are to be determined.

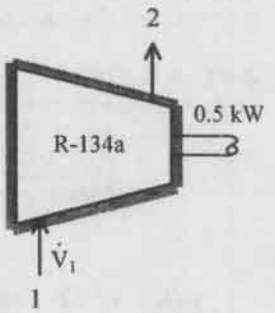
Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Properties From the R-134a tables (Tables A-11 through A-13)

$$\left.\begin{matrix} P_1 = 140\text{kPa} \\ T_1 = -10^\circ\text{C} \end{matrix}\right\} \begin{matrix} v_1 = 0.14549\text{m}^3/\text{kg} \\ h_1 = 243.40\text{kJ/kg} \\ s_1 = 0.9606\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_2 = 700\text{kPa} \\ T_2 = 60^\circ\text{C} \end{matrix}\right\} h_2 = 296.69\text{kJ/kg}$$

$$\left.\begin{matrix} P_2 = 700\text{kPa} \\ s_{2s} = s_1 \end{matrix}\right\} h_{2s} = 278.06\text{kJ/kg}$$



Analysis (a) The isentropic efficiency is determined from its definition,

$$\eta_C = \frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{278.06 - 243.40}{296.69 - 243.40} = 0.650 = \mathbf{65.0\%}$$

(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the actual compressor as the system, which is a control volume. The energy balance for this steady-flow system can be expressed as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\begin{matrix}\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}\end{matrix}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\begin{matrix}\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}\end{matrix}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{a,\text{in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{a,\text{in}} = \dot{m}(h_2 - h_1)$$

Then the mass and volume flow rates of the refrigerant are determined to be

$$\dot{m} = \frac{\dot{W}_{a,in}}{h_1 - h_{2a}} = \frac{0.5\text{kJ/s}}{(296.69 - 243.40)\text{kJ/kg}} = 0.0094\text{kg/s}$$

$$\dot{V}_1 = \dot{m}v_1 = (0.0094\text{kg/s})(0.14549\text{m}^3/\text{kg}) = 0.00137\text{m}^3/\text{s} = \mathbf{82L/min}$$

(*c*) The volume flow rate will be a maximum when the process is isentropic, and it is determined similarly from the steady-flow energy equation applied to the isentropic process. It gives

$$\dot{m}_{\max} = \frac{\dot{W}_{s,in}}{h_1 - h_{2s}} = \frac{-0.5\text{kJ/s}}{(243.40 - 278.06)\text{kJ/kg}} = 0.0144\text{kg/s}$$

$$\dot{V}_{1,\max} = \dot{m}_{\max} v_1 = (0.0144\text{kg/s})(0.14549\text{m}^3/\text{kg}) = 0.00210\text{m}^3/\text{s} = \mathbf{126L/min}$$

Discussion Note that the raising the isentropic efficiency of the compressor to 100% would increase the volumetric flow rate by more than 50%.

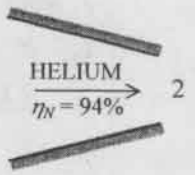
7-107E Helium is accelerated by a 94% efficient nozzle from a low velocity to 1000 ft/s. The pressure and temperature at the nozzle inlet are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Helium is an ideal gas with constant specific heats. **3** Potential energy changes are negligible. **4** The device is adiabatic and thus heat transfer is negligible.

Properties The specific heat ratio of helium is $k = 1.667$. The constant pressure specific heat of helium is 1.25 Btu/lbm.R (Table A-2E).

Analysis We take nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}{}^{\nearrow 0 \text{ (steady)}} = 0$$



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + \mathbf{V}_1^2/2) = \dot{m}(h_2 + \mathbf{V}_1^2/2) \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta pe \cong 0)$$

$$0 = h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} \longrightarrow 0 = C_{p,ave}(T_2 - T_1) + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}$$

Solving for T_1 and substituting,

$$T_1 = T_{2a} + \frac{\mathbf{V}_{2s}^2 - \mathbf{V}_1^2}{2C_p}{}^{\nearrow 0} = 180^\circ\text{F} + \frac{(1000\text{ft/s})^2}{2(1.25\text{Btu/lbm}\cdot\text{R})}\left(\frac{1\text{Btu/lbm}}{25{,}037\text{ft}^2/\text{s}^2}\right) = \mathbf{196.0^\circ F = 656R}$$

From the isentropic efficiency relation,

$$\eta_N = \frac{h_{2a} - h_1}{h_{2s} - h_1} = \frac{C_P(T_{2a} - T_1)}{C_P(T_{2s} - T_1)}$$

or,

$$T_{2s} = T_1 + (T_{2a} - T_1)/\eta_N = 656 + (640 - 656)/(0.94) = 639\text{R}$$

From the isentropic relation, $\dfrac{T_{2s}}{T_1} = \left(\dfrac{P_2}{P_1}\right)^{(k-1)/k}$

$$P_1 = P_2\left(\frac{T_1}{T_{2s}}\right)^{k/(k-1)} = (14\,psia)\left(\frac{656R}{640R}\right)^{1.667/0.667} = \mathbf{14.9psia}$$

7-108 An adiabatic compressor is powered by a direct-coupled steam turbine, which also drives a generator. The net power delivered to the generator and the rate of entropy generation are to be determined. √

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The devices are adiabatic and thus heat transfer is negligible. **4** Air is an ideal gas with variable specific heats.

Properties From the steam tables (Tables A-4 through 6) and air table (Table A-17),

$$T_1 = 295\ \text{K} \longrightarrow h_1 = 295.17\text{kJ/kg},\ s_1^\circ = 1.68515\ \text{kJ/kg}\cdot\text{K}$$

$$T_2 = 620\ \text{K} \longrightarrow h_1 = 628.07\text{kJ/kg},\ s_2^\circ = 2.44356\ \text{kJ/kg}\cdot\text{K}$$

$$\left.\begin{matrix} P_3 = 12.5\text{MPa} \\ T_3 = 500^\circ\text{C} \end{matrix}\right\} \begin{matrix} h_3 = 3341.8\text{kJ/kg} \\ s_3 = 6.4618\text{kJ/kg}\cdot\text{K} \end{matrix}$$

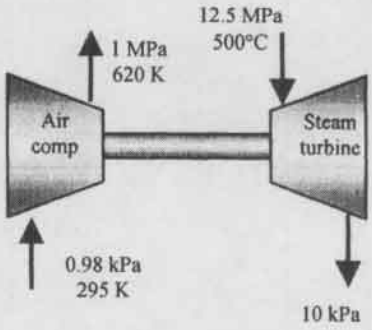
$$\left.\begin{matrix} P_4 = 10\text{kPa} \\ x_4 = 0.92 \end{matrix}\right\} \begin{matrix} h_4 = h_f + x_4 h_{fg} = 191.83 + (0.92)(2392.8) = 2393.2\text{kJ/kg} \\ s_4 = s_f + x_4 s_{fg} = 0.6493 + (0.92)(7.5009) = 7.5501\text{kJ/kg}\cdot\text{K} \end{matrix}$$

Analysis There is only one inlet and one exit for either device, and thus $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$. We take either the turbine or the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for either steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

For the turbine and the compressor it becomes



Compressor: $\dot{W}_{\text{comp, in}} + \dot{m}_{\text{air}} h_1 = \dot{m}_{\text{air}} h_2 \ \rightarrow\ \dot{W}_{\text{comp, in}} = \dot{m}_{\text{air}} (h_2 - h_1)$

Turbine: $\dot{m}_{\text{steam}} h_3 = \dot{W}_{\text{turb, out}} + \dot{m}_{\text{steam}} h_4 \ \rightarrow\ \dot{W}_{\text{turb, out}} = \dot{m}_{\text{steam}} (h_3 - h_4)$

Substituting,

$$\dot{W}_{\text{comp,in}} = (10\text{kg/s})(628.07 - 295.17)\text{kJ/kg} = 3329\ \text{kW}$$

$$\dot{W}_{\text{turb,out}} = (25\text{kg/s})(3341.8 - 2393.2)\text{kJ/kg} = 23{,}715\ \text{kW}$$

Therefore,

$$\dot{W}_{\text{net,out}} = \dot{W}_{\text{turb,out}} - \dot{W}_{\text{comp,in}} = 23{,}715 - 3329 = \mathbf{20{,}386\ kW}$$

Noting that the system is adiabatic, the total rate of entropy change (or generation) during this process is the sum of the entropy changes of both fluids,

$$\dot{S}_{\text{gen}} = \dot{m}_{air}(s_2 - s_1) + \dot{m}_{steam}(s_4 - s_3)$$

where

$$\dot{m}_{air}(s_2 - s_1) = \dot{m}\left(s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1}\right) = (10\ \text{kg/s})\left(2.44356 - 1.68515 - 0.287\ln\frac{1000\text{kPa}}{98\text{kPa}}\right)\text{kJ/kg}\cdot\text{K} = 0.92\ \text{kW/K}$$

$$\dot{m}_{steam}(s_4 - s_3) = (25\text{kg/s})(7.5501 - 6.4618)\text{kJ/kg}\cdot\text{K} = 27.2\ \text{kW/K}$$

Substituting, the total rate of entropy generation is determined to be

$$\dot{S}_{gen,total} = \dot{S}_{gen,comp} + \dot{S}_{gen,turb} = 0.92 + 27.2 = \mathbf{28.12\ kW/K}$$

7-109 Two identical bodies at different temperatures are connected to each other through a heat engine. It is to be shown that the final common temperature of the two bodies will be $T_f = \sqrt{T_1 T_2}$ when the work output of the heat engine is maximum.

Analysis For maximum power production, the entropy generation must be zero. Taking the source, the sink, and the heat engine as our system, which is adiabatic, and noting that the entropy change for cyclic devices is zero, the entropy generation for this system can be expressed as

$$S_{gen} = (\Delta S)_{\text{source}} + \cancelto{0}{(\Delta S)_{\text{engine}}} + (\Delta S)_{\text{sink}} = 0$$

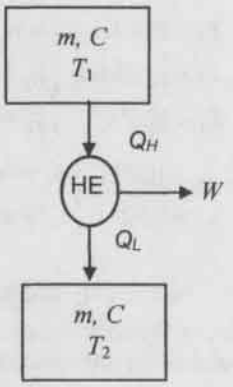
$$mC\ln\frac{T_f}{T_1} + 0 + mC\ln\frac{T_f}{T_2} = 0$$

$$\ln\frac{T_f}{T_1} + \ln\frac{T_f}{T_2} = 0 \longrightarrow \ln\frac{T_f}{T_1}\frac{T_f}{T_2} = 0 \longrightarrow T_f^2 = T_1 T_2$$

and thus

$$T_f = \sqrt{T_1 T_2}$$

for maximum power production.



7-110 The pressure in a hot water tank rises to 2 MPa, and the tank explodes. The explosion energy of the water is to be determined, and expressed in terms of its TNT equivalence.

Assumptions **1** The expansion process during explosion is isentropic. **2** Kinetic and potential energy changes are negligible. **3** Heat transfer with the surroundings during explosion is negligible.

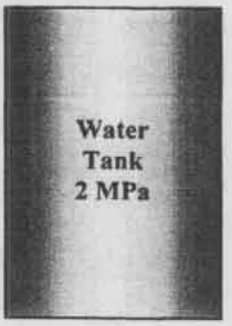
Properties The explosion energy of TNT is 3250 kJ/kg. From the steam tables (Tables A-4 through 6)

$$\left.\begin{array}{l} P_1 = 2\text{MPa} \\ Sat.liquid \end{array}\right\} \begin{array}{l} v_1 = v_{f@2\text{MPa}} = 0.001177\text{m}^3/\text{kg} \\ h_1 = h_{f@2\text{MPa}} = 906.44\text{kJ/kg} \\ s_1 = s_{f@2\text{MPa}} = 2.4474\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 100\text{kPa} \\ s_2 = s_1 \end{array}\right\} \begin{array}{l} u_f = 417.36, \quad u_{fg} = 2088.7\text{kJ/kg} \\ s_f = 1.3026, \quad s_{fg} = 6.0568\text{kJ/kg}\cdot\text{K} \end{array}$$

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{2.4474 - 1.3026}{6.0568} = 0.189$$

$$u_2 = u_f + x_2 u_{fg} = 417.36 + (0.189)(2088.7) = 812.1\text{kJ/kg}$$



Analysis We idealize the water tank as a closed system which undergoes a reversible adiabatic process with negligible changes in kinetic and potential energies. The work done during this idealized process represents the explosive energy of the tank, and is determined from the closed system energy balance to be

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$-W_{b,out} = \Delta U = m(u_2 - u_1)$$

$$E_{\text{exp}} = W_{b,out} = m(u_1 - u_2)$$

where

$$m = \frac{V}{v_1} = \frac{0.1\text{m}^3}{0.001177\text{m}^3/\text{kg}} = 85.0\text{kg}$$

Substituting,

$$E_{\text{exp}} = (85.0\text{kg})(906.44 - 812.1)\text{kJ/kg} = \mathbf{8019\,kJ}$$

which is equivalent to

$$m_{TNT} = \frac{8019\text{ kJ}}{3250\text{ kJ/kg}} = \mathbf{2.47\ kg\ TNT}$$

7-111 A 0.2-L canned drink explodes at a pressure of 1 MPa. The explosive energy of the drink is to be determined, and expressed in terms of its TNT equivalence. √

Assumptions **1** The expansion process during explosion is isentropic. **2** Kinetic and potential energy changes are negligible. **3** Heat transfer with the surroundings during explosion is negligible. **4** The drink can be treated as pure water.

Properties The explosion energy of TNT is 3250 kJ/kg. From the steam tables (Tables A-4 through 6)

$$\left.\begin{array}{l} P_1 = 1\text{MPa} \\ Comp.liquid \end{array}\right\} \begin{array}{l} v_1 = v_{f@1\text{MPa}} = 0.001127\text{m}^3/\text{kg} \\ u_1 = u_{f@1\text{MPa}} = 761.68\text{kJ/kg} \\ s_1 = s_{f@1\text{MPa}} = 2.1387\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 100kPa \\ s_2 = s_1 \end{array}\right\} \begin{array}{l} u_f = 417.36, \quad u_{fg} = 2088.7\text{kJ/kg} \\ s_f = 1.3026, \quad s_{fg} = 6.0568\text{kJ/kg}\cdot\text{K} \end{array}$$



$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{2.1387 - 1.3026}{6.0568} = 0.138$$

$$u_2 = u_f + x_2 u_{fg} = 417.36 + (0.138)(2088.7) = 705.61\text{kJ/kg}$$

Analysis We idealize the canned drink as a closed system which undergoes a reversible adiabatic process with negligible changes in kinetic and potential energies. The work done during this idealized process represents the explosive energy of the can, and is determined from the closed system energy balance to be

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$-W_{b,out} = \Delta U = m(u_2 - u_1)$$

$$E_{\text{exp}} = W_{b,out} = m(u_1 - u_2)$$

where

$$m = \frac{V}{v_1} = \frac{0.0002\text{m}^3}{0.001127\text{m}^3/\text{kg}} = 0.177\text{kg}$$

Substituting,

$$E_{\text{exp}} = (0.177\text{kg})(761.68 - 705.6)\text{kJ/kg} = \mathbf{9.9kJ}$$

which is equivalent to

$$m_{TNT} = \frac{9.9\text{kJ}}{3250\text{kJ/kg}} = \mathbf{0.00305kgTNT}$$

Chapter 8
POWER AND REFRIGERATION CYCLES

Actual and Ideal Cycles, Carnot cycle, Air-Standard Assumptions

8-1C In gas cycles, the working fluid remains in the gaseous phase throughout. In vapor cycles, however, the working fluid exists in the vapor phase during one part of the cycle, and in the liquid phase during another part.

8-2C The Carnot cycle is not suitable as an ideal cycle for all power producing devices because it cannot be approximated using the hardware of actual power producing devices.

8-3C It is less than the thermal efficiency of a Carnot cycle.

8-4C It represents the net work on both diagrams.

8-5C The cold air standard assumptions involves the additional assumption that air can be treated as an ideal gas with constant specific heats at room temperature.

8-6C They operate on an open cycle because the exhaust gases are purged at the end of each cycle and are replaced by fresh air-fuel mixture.

8-7C Under the air standard assumptions, the combustion process is modeled as a heat addition process, and the exhaust process as a heat rejection process.

8-8C The air standard assumptions are: (1) the working fluid is air which behaves as an ideal gas, (2) all the processes are internally reversible, (3) the combustion process is replaced by the heat addition process, and (4) the exhaust process is replaced by the heat rejection process which returns the working fluid to its original state.

8-9C The clearance volume is the minimum volume formed in the cylinder whereas the displacement volume is the volume displaced by the piston as the piston moves between the top dead center and the bottom dead center.

8-10C It is the ratio of the maximum to minimum volumes in the cylinder.

8-11C The MEP is the fictitious pressure which, if acted on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle.

8-12C Yes.

8-13C Assuming no accumulation of carbon deposits on the piston face, the compression ratio will remain the same (otherwise it will increase). The mean effective pressure, on the other hand, will decrease as a car gets older as a result of wear and tear.

8-14C The SI and CI engines differ from each other in the way combustion is initiated; by a spark in SI engines, and by compressing the air above the self-ignition temperature of the fuel in CI engines.

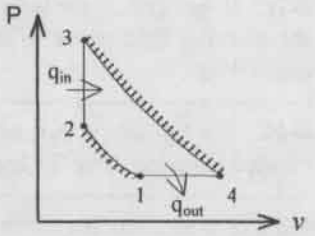
8-15C Stroke is the distance between the TDC and the BDC, bore is the diameter of the cylinder, TDC is the position of the piston when it forms the smallest volume in the cylinder, and clearance volume is the minimum volume formed in the cylinder.

8-16 The four processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the net work output and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.

Analysis (*b*) The properties of air at various states are

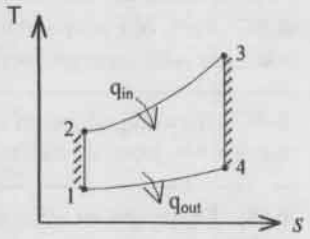


$$T_1 = 300\text{K} \longrightarrow \begin{matrix} h_1 = 300.19\text{kJ/kg} \\ P_{r_1} = 1.386 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = \frac{800\text{kPa}}{100\text{kPa}}(1.386) = 11.088 \longrightarrow \begin{matrix} u_2 = 389.22\text{kJ/kg} \\ T_2 = 539.8\text{K} \end{matrix}$$

$$T_3 = 1800\text{K} \longrightarrow \begin{matrix} u_3 = 1487.2\text{kJ/kg} \\ P_{r_3} = 1310 \end{matrix}$$

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow P_3 = \frac{T_3}{T_2} P_2 = \frac{1800\text{K}}{539.8\text{K}}(800\text{kPa}) = 2668\text{kPa}$$



$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \frac{100\text{kPa}}{2668\text{kPa}}(1310) = 49.10 \longrightarrow h_4 = 828.1\text{kJ/kg}$$

From energy balances,

$$q_{in} = u_3 - u_2 = 1487.2 - 389.2 = 1098.0\ \text{kJ/kg}$$

$$q_{out} = h_4 - h_1 = 828.1 - 300.19 = 527.9\ \text{kJ/kg}$$

$$w_{net,out} = q_{in} - q_{out} = 1098.0 - 527.9 = \mathbf{570.1\ kJ/kg}$$

(*c*) Then the thermal efficiency becomes

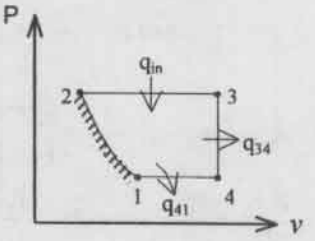
$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{570.1\ \text{kJ/kg}}{1098.0\ \text{kJ/kg}} = \mathbf{51.9\%}$$

8-17 The four processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the maximum temperature in the cycle and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg.K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (*b*) From the ideal gas isentropic relations and energy balance,



$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})\left(\frac{1000\text{kPa}}{100\text{kPa}}\right)^{0.4/1.4} = 579.2\text{K}$$

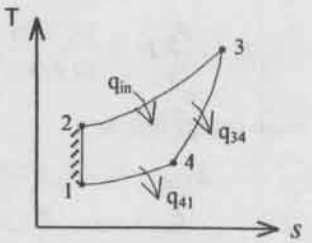
$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$

or,

$$2840\text{kJ/kg} = (1.005\text{kJ/kg}\cdot\text{K})(T_3 - 579.2)$$

$$T_{\max} = T_3 = \mathbf{3405.1\ K}$$

(*c*) $$\frac{P_3 v_3}{T_3} = \frac{P_4 v_4}{T_4} \longrightarrow T_4 = \frac{P_4}{P_3} T_3 = \frac{100\text{kPa}}{1000\text{kPa}}(3405.1\text{K}) = 340.5\text{K}$$



$$\begin{aligned} q_{out} &= q_{34,out} + q_{41,out} = (u_3 - u_4) + (h_4 - h_1) \\ &= C_v(T_3 - T_4) + C_p(T_4 - T_1) \\ &= (0.718\text{kJ/kg}\cdot\text{K})(3405.1 - 340.5)\text{K} + (1.005\text{kJ/kg}\cdot\text{K})(340.5 - 300)\text{K} \\ &= 2241.1\text{kJ/kg} \end{aligned}$$

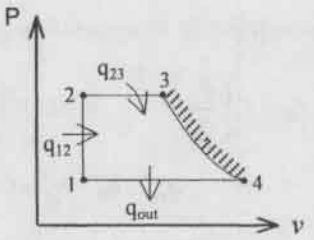
$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2241.1\text{kJ/kg}}{2840.0\text{kJ/kg}} = \mathbf{21.1\%}$$

8-18E The four processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the total heat input and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17E.

Analysis (*b*) The properties of air at various states are



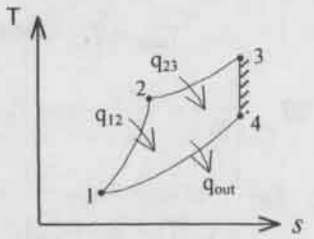
$$T_1 = 540\ \text{R} \longrightarrow \begin{matrix} u_1 = 92.04\ \text{Btu/lbm} \\ h_1 = 129.06\ \text{Btu/lbm} \end{matrix}$$

$$q_{\text{in},12} = u_2 - u_1 \longrightarrow \begin{matrix} u_2 = u_1 + q_{\text{in},12} = 92.04 + 300 = 392.04\text{Btu/lbm} \\ T_2 = 2116\text{R},\ h_2 = 537.1\text{Btu/lbm} \end{matrix}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow P_2 = \frac{T_2}{T_1} P_1 = \frac{2116\text{R}}{540\text{R}}(14.7\text{psia}) = 57.6\text{psia}$$

$$T_3 = 3200\text{R} \longrightarrow \begin{matrix} h_3 = 849.48\text{Btu/lbm} \\ P_{r_3} = 1242 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \frac{14.7\text{psia}}{57.6\text{psia}}(1242) = 317.0 \longrightarrow h_4 = 593.22\text{Btu/lbm}$$



From energy balance,

$$q_{23,in} = h_3 - h_2 = 849.48 - 537.1 = 312.38\ \text{Btu/lbm}$$

$$q_{in} = q_{12,in} + q_{23,in} = 300 + 312.38 = \mathbf{612.38\ Btu/lbm}$$

$$q_{out} = h_4 - h_1 = 593.22 - 129.06 = 464.16\ \text{Btu/lbm}$$

(*c*) Then the thermal efficiency becomes

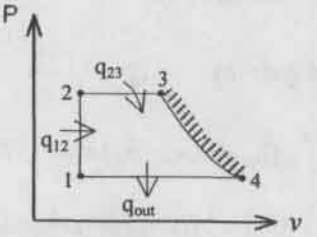
$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{464.16\ \text{Btu/lbm}}{612.38\ \text{Btu/lbm}} = \mathbf{24.2\%}$$

8-19E The four processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the total heat input and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are $C_p = 0.240$ Btu/lbm.R, $C_v = 0.171$ Btu/lbm.R, and $k = 1.4$ (Table A-2E).

Analysis (*b*)

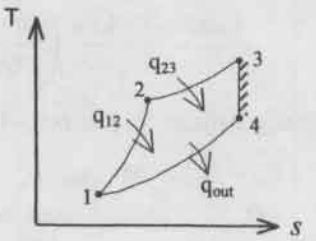


$$q_{in,12} = u_2 - u_1 = C_v(T_2 - T_1)$$
$$300\text{Btu/lbm} = (0.171\text{Btu/lbm.R})(T_2 - 540)\text{R}$$
$$T_2 = 2294\text{R}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow P_2 = \frac{T_2}{T_1} P_1 = \frac{2294\text{R}}{540\text{R}}(14.7\text{psia}) = 62.46\text{psia}$$

$$q_{in,23} = h_3 - h_2 = C_P(T_3 - T_2) = (0.24\text{Btu/lbm} \cdot \text{R})(3200 - 2294)\text{R} = 217.4\text{Btu/lbm}$$

Process 3-4 is isentropic:



$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (3200\text{R})\left(\frac{14.7\text{psia}}{62.46\text{psia}}\right)^{0.4/1.4} = 2117\text{R}$$

$$q_{in} = q_{in,12} + q_{in,23} = 300 + 217.4 = \mathbf{517.4\text{Btu/lbm}}$$

$$q_{out} = h_4 - h_1 = C_p(T_4 - T_1) = (0.240\text{Btu/lbm.R})(2117 - 540)$$
$$= 378.5\text{Btu/lbm}$$

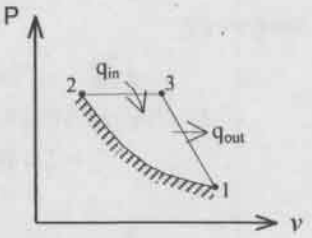
(c) $$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{378.5\ \text{Btu/lbm}}{517.4\ \text{Btu/lbm}} = \mathbf{26.8\%}$$

8-20 The three processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the heat rejected and the thermal efficiency are to be determined. √

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg.K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (*b*) $$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})\left(\frac{1000\text{kPa}}{100\text{kPa}}\right)^{0.4/1.4} = 579.2\text{K}$$



$$Q_{in} = m(h_3 - h_2) = mC_p(T_3 - T_2)$$

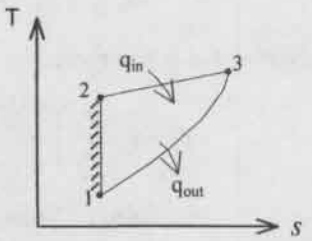
or,

$$1.84\text{kJ} = (0.001\text{kg})(1.005\text{kJ/kg}\cdot\text{K})(T_3 - 579.2) \longrightarrow T_3 = 2410\text{ K}$$

Process 3-1 is a straight line on the P-*v* diagram, thus the w_{31} is simply the area under the process curve,

$$w_{31} = area = \frac{P_3 + P_1}{2}(v_1 - v_3) = \frac{P_3 + P_1}{2}\left(\frac{RT_1}{P_1} - \frac{RT_3}{P_3}\right)$$

$$= \left(\frac{1000 + 100\text{kPa}}{2}\right)\left(\frac{300\text{K}}{100\text{kPa}} - \frac{2410\text{K}}{1000\text{kPa}}\right)(0.287\text{kJ/kg}\cdot\text{K}) = 93.1\text{kJ/kg}$$



Energy balance for process 3-1 gives

$$E_{in} - E_{out} = \Delta E_{system}$$

$$-Q_{31,out} - W_{31,out} = m(u_1 - u_3)$$

$$Q_{31,out} = -mw_{31,out} - mC_v(T_1 - T_3) = -m[w_{31,out} + C_v(T_1 - T_3)]$$

$$= -(0.001\text{kg})[93.1 + (0.718\text{kJ/kg}\cdot\text{K})(300 - 2410)\text{K}] = \mathbf{1.422kJ}$$

(*c*) $$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{1.422\text{ kJ}}{1.84\text{ kJ}} = \mathbf{22.7\%}$$

8-21 The three processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the net work per cycle and the thermal efficiency are to be determined. √

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.

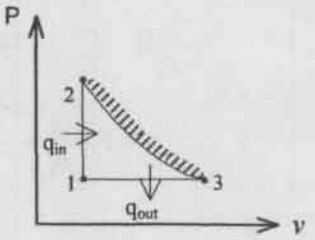
Analysis (*b*) The properties of air at various states are

$$T_1 = 290K \longrightarrow \begin{matrix} u_1 = 206.91\text{kJ/kg} \\ h_1 = 290.16\text{kJ/kg} \end{matrix}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow T_2 = \frac{P_2}{P_1} T_1 = \frac{380\text{kPa}}{95\text{kPa}}(290\text{K}) = 1160\text{K}$$

$$\longrightarrow u_2 = 897.91\text{kJ/kg},\ P_{r_2} = 207.2$$

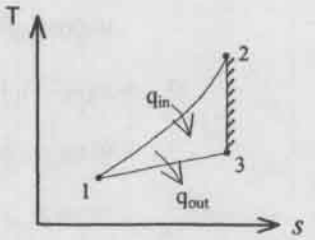
$$P_{r_3} = \frac{P_3}{P_2} P_{r_2} = \frac{95\text{kPa}}{380\text{kPa}}(207.2) = 51.8 \longrightarrow h_3 = 840.38\text{kJ/kg}$$



$$Q_{in} = m(u_2 - u_1) = (0.003\text{kg})(897.91 - 206.91)\text{kJ/kg} = 2.073\text{kJ}$$

$$Q_{out} = m(h_3 - h_1) = (0.003\text{kg})(840.38 - 290.16)\text{kJ/kg} = 1.651\text{kJ}$$

$$W_{net,out} = Q_{in} - Q_{out} = 2.073 - 1.651 = \mathbf{0.422kJ}$$



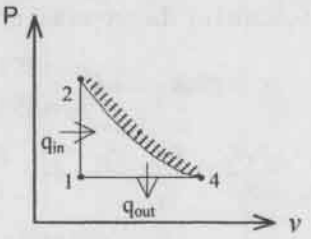
(c) $$\eta_{th} = \frac{W_{net,out}}{Q_{in}} = \frac{0.422\text{ kJ}}{2.073\text{ kJ}} = \mathbf{20.4\%}$$

8-22 The three processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the net work per cycle and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

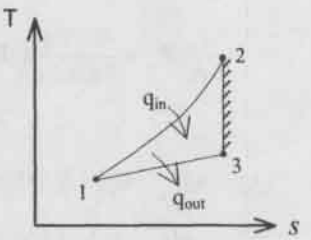
Properties The properties of air at room temperature are C_p = 1.005 kJ/kg.K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (*b*) From the isentropic relations and energy balance,



$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow T_2 = \frac{P_2}{P_1} T_1 = \frac{380\text{kPa}}{95\text{kPa}}(290\text{K}) = 1160\text{K}$$

$$T_3 = T_2\left(\frac{P_3}{P_2}\right)^{(k-1)/k} = (1160\text{K})\left(\frac{95\text{kPa}}{380\text{kPa}}\right)^{0.4/1.4} = 780.6\text{K}$$



$$Q_{in} = m(u_2 - u_1) = mC_v(T_2 - T_1)$$

$$= (0.003\text{kg})(0.718\text{kJ/kg}\cdot\text{K})(1160 - 290)\text{K} = 1.87\text{kJ}$$

$$Q_{out} = m(h_3 - h_1) = mC_p(T_3 - T_1)$$

$$= (0.003\text{kg})(1.005\text{kJ/kg}\cdot\text{K})(780.6 - 290)\text{K} = 1.48\text{kJ}$$

$$W_{net,out} = Q_{in} - Q_{out} = 1.87 - 1.48 = \mathbf{0.39kJ}$$

(*c*) $$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{0.39\text{ kJ}}{1.87\text{ kJ}} = \mathbf{20.9\%}$$

8-23 A Carnot cycle with the specified temperature limits is considered. The net work output per cycle is to be determined.

Assumptions Air is an ideal gas with constant specific heats.

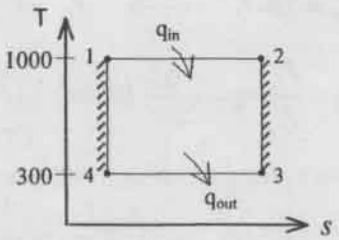
Properties The properties of air at room temperature are C_p = 1.005 kJ/kg.K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis The minimum pressure in the cycle is P_3 and the maximum pressure is P_1. Then,

$$\frac{T_2}{T_3}=\left(\frac{P_2}{P_3}\right)^{(k-1)/k}$$

or,

$$P_2 = P_3\left(\frac{T_2}{T_3}\right)^{k/(k-1)} = (20\text{kPa})\left(\frac{1000\text{K}}{300\text{K}}\right)^{1.4/0.4} = 1352\text{kPa}$$



The heat input is determined from

$$s_2 - s_1 = C_p \ln\frac{T_2}{T_1}^{\nearrow 0} - R\ln\frac{P_2}{P_1} = -(0.287\text{kJ/kg}\cdot\text{K})\ln\frac{1352\text{kPa}}{1800\text{kPa}} = 0.08205\text{kJ/kg}\cdot\text{K}$$

$$Q_{in} = mT_H(s_2 - s_1) = (0.004\text{kg})(1000\text{K})(0.08205\text{kJ/kg}\cdot\text{K}) = 0.328\text{kJ}$$

Then,

$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{K}}{1000\text{K}} = 70.0\%$$

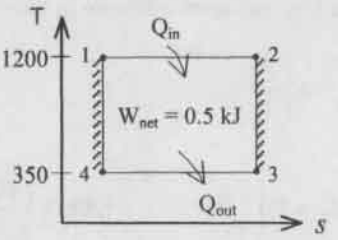
$$W_{net,out} = \eta_{th} Q_{in} = (0.70)(0.328\text{kJ}) = \mathbf{0.230kJ}$$

8-24 A Carnot cycle with specified temperature limits is considered. The maximum pressure in the cycle, the heat transfer to the working fluid, and the mass of the working fluid are to be determined. √

Assumptions Air is an ideal gas with variable specific heats.

Analysis (*a*) In a Carnot cycle, the maximum pressure occurs at the beginning of the expansion process, which is state 1.

$$T_1 = 1200\ \text{K} \longrightarrow P_{r_1} = 238$$
$$T_4 = 350\ \text{K} \longrightarrow P_{r_4} = 2.379 \qquad \text{(Table A-17)}$$



$$P_1 = \frac{P_{r_1}}{P_{r_4}} P_4 = \frac{238}{2.379}(300\text{kPa}) = \mathbf{30{,}013\,kPa} = P_{\max}$$

(*b*) The heat input is determined from

$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \frac{350\text{K}}{1200\text{K}} = 70.83\%$$
$$Q_{in} = W_{net,out} / \eta_{th} = (0.5\text{kJ})/(0.7083) = \mathbf{0.706\,kJ}$$

(*c*) The mass of air is

$$s_4 - s_3 = \left(s_4^\circ - s_3^\circ\right)^{\text{p0}} - R\ln\frac{P_4}{P_3} = -(0.287\text{kJ/kg}\cdot\text{K})\ln\frac{300\text{kPa}}{150\text{kPa}}$$
$$= -0.199\text{kJ/kg}\cdot K = s_1 - s_2$$
$$w_{net,out} = (s_2 - s_1)(T_H - T_L) = (0.199\text{kJ/kg}\cdot\text{K})(1200 - 350)K = 169.15\text{kJ/kg}$$
$$m = \frac{W_{net,out}}{w_{net,out}} = \frac{0.5\text{kJ}}{169.15\text{kJ/kg}} = \mathbf{0.00296\,kg}$$

8-25 A Carnot cycle with specified temperature limits is considered. The maximum pressure in the cycle, the heat transfer to the working fluid, and the mass of the working fluid are to be determined.

Assumptions Helium is an ideal gas with constant specific heats.

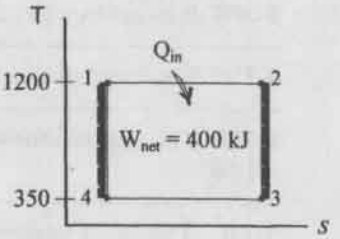
Properties The properties of helium at room temperature are $R = 0.20769$ kJ/kg.K and $k = 1.667$ (Table A-2).

Analysis (*a*) In a Carnot cycle, the maximum pressure occurs at the beginning of the expansion process, which is state 1.

$$\frac{T_1}{T_4} = \left(\frac{P_1}{P_4}\right)^{(k-1)/k}$$

or,

$$P_1 = P_4\left(\frac{T_1}{T_4}\right)^{k/(k-1)} = (300\text{kPa})\left(\frac{1200\text{K}}{350\text{K}}\right)^{1.667/0.667} = 6524\text{kPa}$$



(*b*) The heat input is determined from

$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \frac{350\text{K}}{1200\text{K}} = 70.83\%$$

$$Q_{in} = W_{net,out} / \eta_{th} = (0.5\text{kJ})/(0.7083) = \mathbf{0.706kJ}$$

(*c*) The mass of helium is determined from

$$s_4 - s_3 = C_p \ln\frac{T_4}{T_3}^{\nearrow 0} - R\ln\frac{P_4}{P_3} = -(0.20769\text{kJ/kg}\cdot\text{K})\ln\frac{300\text{kPa}}{150\text{kPa}}$$

$$= -1.4396\text{kJ/kg}\cdot K = s_1 - s_2$$

$$w_{net,out} = (s_2 - s_1)(T_H - T_L) = (1.4396\text{kJ/kg}\cdot\text{K})(1200 - 350)\text{K} = 1223.7\text{kJ/kg}$$

$$m = \frac{W_{net,out}}{w_{net,out}} = \frac{0.5\text{kJ}}{1223.7\text{kJ/kg}} = \mathbf{0.000409kg}$$

Otto Cycle

8-26C The four processes that make up the Otto cycle are (1) isentropic compression, (2) v = constant heat addition, (3) isentropic expansion, and (4) v = constant heat rejection.

8-27C The ideal Otto cycle involves external irreversibilities, and thus it has a lower thermal efficiency.

8-28C For actual four-stroke engines, the rpm is twice the number of thermodynamic cycles; for two-stroke engines, it is equal to the number of thermodynamic cycles.

8-29C They are analyzed as closed system processes because no mass crosses the system boundaries during any of the processes.

8-30C It increases with both of them.

8-31C Because high compression ratios cause engine knock.

8-32C The thermal efficiency will be the highest for argon because it has the highest specific heat ratio, k = 1.667.

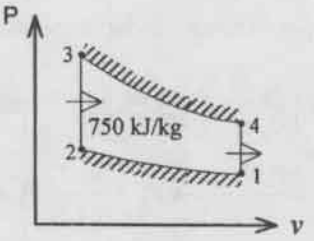
8-33C The fuel is injected into the cylinder in both engines, but it is ignited with a spark plug in gasoline engines.

8-34 An ideal Otto cycle with air as the working fluid has a compression ratio of 8. The pressure and temperature at the end of the heat addition process, the net work output, the thermal efficiency, and the mean effective pressure for the cycle are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.

Analysis (*a*) Process 1-2: isentropic compression.



$$T_1 = 300\ \text{K} \longrightarrow \begin{matrix} u_1 = 214.07\ \text{kJ/kg} \\ v_{r_1} = 621.2 \end{matrix}$$

$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \frac{1}{r} v_{r_1} = \frac{1}{8}(621.2) = 77.65$$

$$\longrightarrow \begin{matrix} T_2 = 673.1\text{K} \\ u_2 = 491.2\text{kJ/kg} \end{matrix}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow P_2 = \frac{v_1}{v_2}\frac{T_2}{T_1} P_1 = (8)\left(\frac{673.1\text{K}}{300\text{K}}\right)(95\text{kPa}) = 1705\text{kPa}$$

Process 2-3: v = constant heat addition.

$$q_{23,\text{in}} = u_3 - u_2 \longrightarrow u_3 = u_2 + q_{23,in} = 491.2 + 750 = 1241.2\ \text{kJ/kg} \longrightarrow \begin{matrix} T_3 = \mathbf{1539\ K} \\ v_{r_3} = 6.588 \end{matrix}$$

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow P_3 = \frac{T_3}{T_2} P_2 = \left(\frac{1539\text{K}}{673.1\text{K}}\right)(1705\text{kPa}) = \mathbf{3898kPa}$$

(*b*) Process 3-4: isentropic expansion.

$$v_{r_4} = \frac{v_1}{v_2} v_{r_3} = r v_{r_1} = (8)(6.588) = 52.70 \longrightarrow \begin{matrix} T_4 = 774.5\text{K} \\ u_4 = 571.69\text{kJ/kg} \end{matrix}$$

Process 4-1: v = constant heat rejection.

$$q_{\text{out}} = u_4 - u_1 = 571.69 - 214.07 = 357.62\ \text{kJ/kg}$$

$$w_{net,out} = q_{in} - q_{out} = 750 - 357.62 = \mathbf{392.38\ kJ/kg}$$

(*c*) $$\eta_{\text{th}} = \frac{w_{\text{net,out}}}{q_{\text{in}}} = \frac{392.38\text{kJ/kg}}{750\text{kJ/kg}} = \mathbf{52.3\%}$$

(*d*) $$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})}{95\text{kPa}} = 0.906\text{m}^3/\text{kg} = v_{\text{max}}$$

$$v_{\text{min}} = v_2 = \frac{v_{\text{max}}}{r}$$

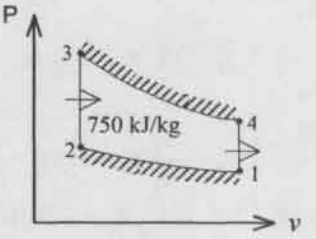
$$MEP = \frac{w_{\text{net,out}}}{v_1 - v_2} = \frac{w_{\text{net,out}}}{v_1(1 - 1/r)} = \frac{392.38\text{kJ/kg}}{(0.906\text{m}^3/\text{kg})(1 - 1/8)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{495.0kPa}$$

8-35 An ideal Otto cycle with air as the working fluid has a compression ratio of 8. The pressure and temperature at the end of the heat addition process, the net work output, the thermal efficiency, and the mean effective pressure for the cycle are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (*a*) Process 1-2: isentropic compression.



$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (300\text{K})(8)^{0.4} = 689\text{K}$$

$$\frac{P_2v_2}{T_2} = \frac{P_1v_1}{T_1} \longrightarrow P_2 = \frac{v_1}{v_2}\frac{T_2}{T_1}P_1 = (8)\left(\frac{689\text{K}}{300\text{K}}\right)(95\text{kPa}) = 1745\text{kPa}$$

Process 2-3: v = constant heat addition.

$$q_{23,\text{in}} = u_3 - u_2 = C_v(T_3 - T_2)$$
$$750\text{kJ/kg} = (0.718\text{kJ/kg}\cdot\text{K})(T_3 - 689)\text{K}$$
$$T_3 = \mathbf{1734K}$$

$$\frac{P_3v_3}{T_3} = \frac{P_2v_2}{T_2} \longrightarrow P_3 = \frac{T_3}{T_2}P_2 = \left(\frac{1734\text{K}}{689\text{K}}\right)(1745\text{kPa}) = \mathbf{4392kPa}$$

(*b*) Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = (1734\text{K})\left(\frac{1}{8}\right)^{0.4} = 755\text{K}$$

Process 4-1: v = constant heat rejection.

$$q_{\text{out}} = u_4 - u_1 = C_v(T_4 - T_1) = (0.718\text{kJ/kg}\cdot\text{K})(755 - 300)\text{K} = 327\text{kJ/kg}$$

$$w_{\text{net,out}} = q_{\text{in}} - q_{\text{out}} = 750 - 327 = \mathbf{423kJ/kg}$$

(*c*) $$\eta_{\text{th}} = \frac{w_{\text{net,out}}}{q_{\text{in}}} = \frac{423\text{ kJ/kg}}{750\text{ kJ/kg}} = \mathbf{56.4\%}$$

(*d*) $$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})}{95\text{kPa}} = 0.906\text{m}^3/\text{kg} = v_{\text{max}}$$

$$v_{\text{min}} = v_2 = \frac{v_{\text{max}}}{r}$$

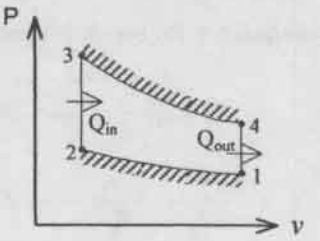
$$MEP = \frac{w_{\text{net,out}}}{v_1 - v_2} = \frac{w_{\text{net,out}}}{v_1(1 - 1/r)} = \frac{423\text{kJ/kg}}{(0.906\text{m}^3/\text{kg})(1 - 1/8)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{534kPa}$$

8-36 An ideal Otto cycle with air as the working fluid has a compression ratio of 9.5. The highest pressure and temperature in the cycle, the amount of heat transferred, the thermal efficiency, and the mean effective pressure are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (*a*) Process 1-2: isentropic compression.



$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (290\text{K})(9.5)^{0.4} = 713.7\text{K}$$

$$\frac{P_2v_2}{T_2} = \frac{P_1v_1}{T_1} \longrightarrow P_2 = \frac{v_1}{v_2}\frac{T_2}{T_1}P_1 = (9.5)\left(\frac{713.7\text{K}}{290\text{K}}\right)(100\text{kPa}) = 2338\text{kPa}$$

Process 3-4: isentropic expansion.

$$T_3 = T_4\left(\frac{v_4}{v_3}\right)^{k-1} = (800\text{K})(9.5)^{0.4} = \mathbf{1969K}$$

Process 2-3: v = constant heat addition.

$$\frac{P_3v_3}{T_3} = \frac{P_2v_2}{T_2} \longrightarrow P_3 = \frac{T_3}{T_2}P_2 = \left(\frac{1969\text{K}}{713.7\text{K}}\right)(2338\text{kPa}) = \mathbf{6449kPa}$$

(*b*) $$m = \frac{P_1V_1}{RT_1} = \frac{(100\text{kPa})(0.0006\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\text{K})} = 7.21\times10^{-4}\text{kg}$$

$$Q_{in} = m(u_3 - u_2) = mC_v(T_3 - T_2) = (7.21\times10^{-4}\text{kg})(0.718\text{kJ/kg}\cdot\text{K})(1969 - 713.7)\text{K} = \mathbf{0.650kJ}$$

(c) Process 4-1: v = constant heat rejection.

$$Q_{out} = m(u_4 - u_1) = mC_v(T_4 - T_1) = -(7.21\times10^{-4}\text{kg})(0.718\text{kJ/kg}\cdot\text{K})(800 - 290)\text{K} = \mathbf{0.264kJ}$$

$$W_{net} = Q_{in} - Q_{out} = 0.650 - 0.264 = 0.386\text{ kJ}$$

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} = \frac{0.386\text{ kJ}}{0.650\text{ kJ}} = \mathbf{59.4\%}$$

(*d*) $$V_{min} = V_2 = \frac{V_{max}}{r}$$

$$MEP = \frac{W_{net,out}}{V_1 - V_2} = \frac{W_{net,out}}{V_1(1-1/r)} = \frac{0.386\text{kJ}}{(0.0006\text{m}^3)(1-1/9.5)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{719kPa}$$

8-37 An Otto cycle with air as the working fluid has a compression ratio of 9.5. The highest pressure and temperature in the cycle, the amount of heat transferred, the thermal efficiency, and the mean effective pressure are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (*a*) Process 1-2: isentropic compression.

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (290\text{K})(9.5)^{0.4} = 713.7\text{K}$$

$$\frac{P_2v_2}{T_2} = \frac{P_1v_1}{T_1} \longrightarrow P_2 = \frac{v_1}{v_2}\frac{T_2}{T_1}P_1 = (9.5)\left(\frac{713.7\text{K}}{290\text{K}}\right)(100\text{kPa}) = 2338\text{kPa}$$

Process 3-4: polytropic expansion.

$$T_3 = T_4\left(\frac{v_4}{v_3}\right)^{n-1} = (800\text{K})(9.5)^{0.35} = \mathbf{1759K}$$

$$W_{34} = \frac{mR(T_4 - T_3)}{1-n} = \frac{(7.209\times10^{-4})(0.287\text{kJ/kg}\cdot\text{K})(800-1759)\text{K}}{1-1.35} = 0.567\text{KJ}$$

Then energy balance for process 3-4 gives

$$E_{in} - E_{out} = \Delta E_{system}$$
$$Q_{34,in} - W_{34,out} = m(u_4 - u_3)$$
$$Q_{34,in} = m(u_4 - u_3) + W_{34,out} = mC_v(T_4 - T_3) + W_{34,out}$$
$$Q_{34,in} = (7.209\times10^{-4}\text{kg})(0.718\text{kJ/kg}\cdot\text{K})(800-1759)\text{K} + 0.567\text{kJ} = 0.071\text{kJ}$$

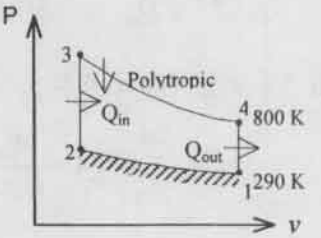
That is, 0.071 kJ of heat is added to the air during the expansion process (This is not realistic, and probably is due to assuming constant specific heats at room temperature).

Process 2-3: v = constant heat addition.

$$\frac{P_3v_3}{T_3} = \frac{P_2v_2}{T_2} \longrightarrow P_3 = \frac{T_3}{T_2}P_2 = \left(\frac{1759\text{K}}{713.7\text{K}}\right)(2338\text{kPa}) = \mathbf{5762kPa}$$

(*b*) $$m = \frac{P_1V_1}{RT_1} = \frac{(100\text{kPa})(0.0006\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\text{K})} = 7.209\times10^{-4}\text{kg}$$

$$Q_{23,in} = m(u_3 - u_2) = mC_v(T_3 - T_2)$$
$$Q_{23,in} = (7.209\times10^{-4}\text{kg})(0.718\text{kJ/kg}\cdot\text{K})(1759 - 713.7)K = \mathbf{0.541kJ}$$



Therefore,

$$Q_{in} = Q_{23,in} + Q_{34,in} = 0.541 + 0.071 = 0.612\text{ kJ}$$

(*c*) Process 4-1: v = constant heat rejection.

$$Q_{out} = m(u_4 - u_1) = mC_v(T_4 - T_1) = (7.209\times10^{-4}\text{kg})(0.718\text{kJ/kg}\cdot\text{K})(800-290)K = \mathbf{0.264kJ}$$

$$W_{net,out} = Q_{in} - Q_{out} = 0.612 - 0.264 = 0.348\text{ kJ}$$

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} = \frac{0.348\ \text{kJ}}{0.612\ \text{kJ}} = \mathbf{56.9\%}$$

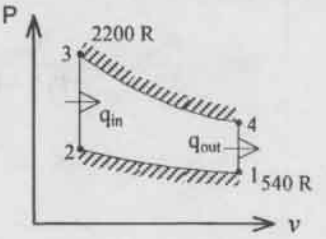
(*d*) $$V_{\min} = V_2 = \frac{V_{\max}}{r}$$

$$MEP = \frac{W_{net,out}}{V_1 - V_2} = \frac{W_{net,out}}{V_1(1-1/r)} = \frac{0.348\text{kJ}}{(0.0006\text{m}^3)(1-1/9.5)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{648kPa}$$

8-38E An ideal Otto cycle with air as the working fluid has a compression ratio of 8. The amount of heat transferred to the air during the heat addition process, the thermal efficiency, and the thermal efficiency of a Carnot cycle operating between the same temperature limits are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17E.



Analysis (a) Process 1-2: isentropic compression.

$$T_1 = 540\ \text{R} \longrightarrow \begin{matrix} u_1 = 92.04\ \text{Btu/lbm} \\ v_{r_1} = 144.32 \end{matrix}$$

$$v_{r_2} = \frac{v_2}{v_1} v_{r_2} = \frac{1}{r} v_{r_2} = \frac{1}{8}(144.32) = 18.04 \longrightarrow u_2 = 211.28\text{Btu/lbm}$$

Process 2-3: v = constant heat addition.

$$T_3 = 2200\ \text{R} \longrightarrow \begin{matrix} u_3 = 409.78\ \text{Btu/lbm} \\ v_{r_3} = 3.176 \end{matrix}$$

$$q_{in} = u_3 - u_2 = 409.78 - 211.28 = \mathbf{198.50\ Btu/lbm}$$

(b) Process 3-4: isentropic expansion.

$$v_{r_4} = \frac{v_4}{v_3} v_{r_3} = r v_{r_3} = (8)(3.176) = 25.41 \longrightarrow u_4 = 184.61\text{Btu/lbm}$$

Process 4-1: v = constant heat rejection.

$$q_{out} = u_4 - u_1 = 184.61 - 92.04 = 92.57\ \text{Btu/lbm}$$

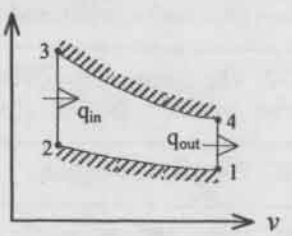
$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{92.57\ \text{Btu/lbm}}{198.50\ \text{Btu/lbm}} = \mathbf{53.4\%}$$

(c) $$\eta_{th,C} = 1 - \frac{T_H}{T_L} = 1 - \frac{540\ \text{R}}{2200\ \text{R}} = \mathbf{75.5\%}$$

8-39E An ideal Otto cycle with argon as the working fluid has a compression ratio of 8. The amount of heat transferred to the argon during the heat addition process, the thermal efficiency, and the thermal efficiency of a Carnot cycle operating between the same temperature limits are to be determined. √

Assumptions **1** The air-standard assumptions are applicable with argon as the working fluid. **2** Kinetic and potential energy changes are negligible. **3** Argon is an ideal gas with constant specific heats.

Properties The properties of argon are C_p = 0.1253 Btu/lbm.R, C_v = 0.0758 Btu/lbm.R, and k = 1.667 (Table A-2E).



Analysis (*a*) Process 1-2: isentropic compression.

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (540\ \text{R})(8)^{0.667} = 2162\ \text{R}$$

Process 2-3: v = constant heat addition.

$$q_{in} = u_3 - u_2 = C_v(T_3 - T_2) = (0.0758\ \text{Btu/lbm.R})(2200 - 2161)\ \text{R} = \mathbf{2.92\ Btu/lbm.R}$$

(*b*) Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = (2200\text{R})\left(\frac{1}{8}\right)^{0.667} = 550\text{R}$$

Process 4-1: v = constant heat rejection.

$$q_{out} = u_4 - u_1 = C_v(T_4 - T_1) = (0.0758\text{Btu/lbm.R})(550 - 540)\text{R} = 0.758\text{Btu/lbm}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{0.758\ \text{Btu/lbm}}{2.92\ \text{Btu/lbm}} = \mathbf{74.0\%}$$

(*c*) $$\eta_{th,C} = 1 - \frac{T_H}{T_L} = 1 - \frac{540\ \text{R}}{2200\ \text{K}} = \mathbf{75.5\%}$$

Diesel Cycle

8-40C Dual cycle is the ideal cycle where the combustion process is modeled as a combination of two heat transfer processes, one occurring at constant volume and the other at constant pressure.

8-41C A diesel engine differs from the gasoline engine in the way combustion is initiated. In diesel engines combustion is initiated by compressing the air above the self-ignition temperature of the fuel.

8-42C The Diesel cycle differs from the Otto cycle in the heat addition process only; it takes place at constant volume in the Otto cycle, but at constant pressure in the Diesel cycle.

8-43C The gasoline engine.

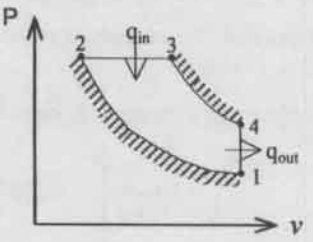
8-44C Diesel engines operate at high compression ratios because the diesel engines do not have the engine knock problem.

8-45C Cutoff ratio is the ratio of the cylinder volumes after and before the combustion process.

8-46 An air-standard Diesel cycle with a compression ratio of 16 and a cutoff ratio of 2 is considered. The temperature after the heat addition process, the thermal efficiency, and the mean effective pressure are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.



Analysis (*a*) Process 1-2: isentropic compression.

$$T_1 = 300\ \text{K} \longrightarrow \begin{matrix} u_1 = 214.07\ \text{kJ/kg} \\ v_{r_1} = 621.2 \end{matrix}$$

$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \frac{1}{r} v_{r_1} = \frac{1}{16}(621.2) = 38.325 \longrightarrow \begin{matrix} T_2 = 862.4\text{K} \\ h_2 = 890.9\text{kJ/kg} \end{matrix}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow T_3 = \frac{v_3}{v_2} T_2 = 2T_2 = (2)(862.4\text{K}) = \mathbf{1724.8K}$$

$$\longrightarrow \begin{matrix} h_3 = 1910.6\text{kJ/kg} \\ v_{r_3} = 4.546 \end{matrix}$$

(*b*) $q_{in} = h_3 - h_2 = 1910.6 - 890.9 = 1019.7\ \text{kJ/kg}$

Process 3-4: isentropic expansion.

$$v_{r_4} = \frac{v_4}{v_3} v_{r_3} = \frac{v_4}{2v_2} v_{r_3} = \frac{r}{2} v_{r_3} = \frac{16}{2}(4.546) = 36.37 \longrightarrow u_4 = 659.7\text{kJ/kg}$$

Process 4-1: *v* = constant heat rejection.

$$q_{out} = u_4 - u_1 = 659.7 - 214.07 = 445.63\ \text{kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{445.63\ \text{kJ/kg}}{1019.7\ \text{kJ/kg}} = \mathbf{56.3\%}$$

(*c*) $w_{net,out} = q_{in} - q_{out} = 1019.7 - 445.63 = 574.07\text{kJ/kg}$

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})}{95\text{kPa}} = 0.906\text{m}^3/\text{kg} = v_{max}$$

$$v_{min} = v_2 = \frac{v_{max}}{r}$$

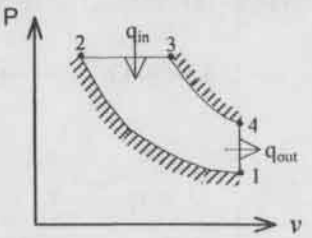
$$MEP = \frac{w_{net,out}}{v_1 - v_2} = \frac{w_{net,out}}{v_1(1 - 1/r)} = \frac{574.07\text{kJ/kg}}{(0.906\text{m}^3/\text{kg})(1 - 1/16)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{675.9kPa}$$

8-47 An air-standard Diesel cycle with a compression ratio of 16 and a cutoff ratio of 2 is considered. The temperature after the heat addition process, the thermal efficiency, and the mean effective pressure are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (*a*) Process 1-2: isentropic compression.



$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (300\text{K})(16)^{0.4} = 909.4\text{K}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow T_3 = \frac{v_3}{v_2} T_2 = 2T_2 = (2)(909.4\text{K}) = \mathbf{1818.8K}$$

(*b*) $$q_{in} = h_3 - h_2 = C_p(T_3 - T_2) = (1.005\text{kJ/kg}\cdot\text{K})(1818.8 - 909.4)\text{K} = 913.9\text{kJ/kg}$$

Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{2v_2}{v_4}\right)^{k-1} = (1818.8\text{K})\left(\frac{2}{16}\right)^{0.4} = 791.7\text{K}$$

Process 4-1: v = constant heat rejection.

$$q_{out} = u_4 - u_1 = C_v(T_4 - T_1) = (0.718\text{kJ/kg}\cdot\text{K})(791.7 - 300)\text{K} = 353\text{kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{353\text{kJ/kg}}{913.9\text{kJ/kg}} = \mathbf{61.4\%}$$

(*c*) $$w_{net,out} = q_{in} - q_{out} = 913.9 - 353 = 560.9\text{kJ/kg}$$

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})}{95\text{kPa}} = 0.906\text{m}^3/\text{kg} = v_{max}$$

$$v_{min} = v_2 = \frac{v_{max}}{r}$$

$$MEP = \frac{w_{net,out}}{v_1 - v_2} = \frac{w_{net,out}}{v_1(1 - 1/r)} = \frac{560.9\text{kJ/kg}}{(0.906\text{m}^3/\text{kg})(1 - 1/16)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{660.4kPa}$$

8-48E An air-standard Diesel cycle with a compression ratio of 18.2 is considered. The cutoff ratio, the heat rejection per unit mass, and the thermal efficiency are to be determined. √

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17E.

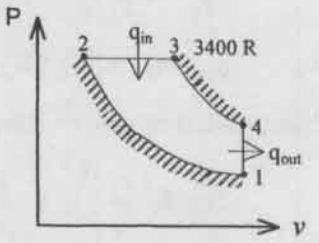
Analysis (*a*) Process 1-2: isentropic compression.

$$T_1 = 540\ \text{R} \longrightarrow \begin{matrix} u_1 = 92.04\ \text{Btu/lbm} \\ v_{r_1} = 144.32 \end{matrix}$$

$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \frac{1}{r} v_{r_1} = \frac{1}{18.2}(144.32) = 7.93 \longrightarrow \begin{matrix} T_2 = 1623.6\text{R} \\ h_2 = 402.05\text{Btu/lbm} \end{matrix}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{3400\ \text{R}}{1623.6\ \text{R}} = \mathbf{2.094}$$



(*b*) $$T_3 = 3400\ \text{R} \longrightarrow \begin{matrix} h_3 = 908.66\ \text{Btu/lbm} \\ v_{r_3} = 0.7807 \end{matrix}$$

$$q_{in} = h_3 - h_2 = 908.66 - 402.05 = 506.6\ \text{Btu/lbm}$$

Process 3-4: isentropic expansion.

$$v_{r_4} = \frac{v_4}{v_3} v_{r_3} = \frac{v_4}{2.094 v_2} v_{r_3} = \frac{r}{2.094} v_{r_3} = \frac{18.2}{2.094}(0.7807) = 6.785 \longrightarrow u_4 = 308.53\text{Btu/lbm}$$

Process 4-1: *v* = constant heat rejection.

$$q_{out} = u_4 - u_1 = 308.53 - 92.04 = \mathbf{216.5\ Btu/lbm}$$

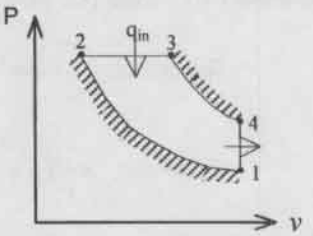
(*c*) $$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{216.5\ \text{Btu/lbm}}{506.6\ \text{Btu/lbm}} = \mathbf{57.3\%}$$

8-49E An air-standard Diesel cycle with a compression ratio of 18.2 is considered. The cutoff ratio, the heat rejection per unit mass, and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 0.240 Btu/lbm.R, C_v = 0.171 Btu/lbm.R, and k = 1.4 (Table A-2E).

Analysis (*a*) Process 1-2: isentropic compression.



$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (540\text{R})(18.2)^{0.4} = 1724\text{R}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{3400\text{R}}{1724\text{R}} = \mathbf{1.973}$$

(*b*) $$q_{in} = h_3 - h_2 = C_p(T_3 - T_2) = (0.240\text{Btu/lbm.R})(3400 - 1724)\text{R} = 402\text{Btu/lbm}$$

Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{1.973 v_2}{v_4}\right)^{k-1} = (3400\text{R})\left(\frac{1.973}{18.2}\right)^{0.4} = 1398\text{R}$$

Process 4-1: v = constant heat rejection.

$$q_{out} = u_4 - u_1 = C_v(T_4 - T_1)$$
$$= (0.171\text{Btu/lbm.R})(1398 - 540)\text{R} = \mathbf{147\text{Btu/lbm}}$$

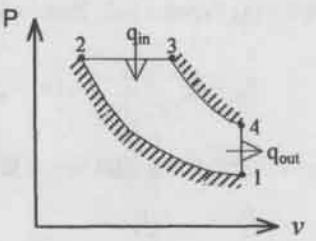
(*c*) $$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{147\text{Btu/lbm}}{402\text{Btu/lbm}} = \mathbf{63.5\%}$$

8-50 An ideal diesel engine with air as the working fluid has a compression ratio of 20. The thermal efficiency and the mean effective pressure are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are $C_p = 1.005$ kJ/kg·K, $C_v = 0.718$ kJ/kg·K, and $k = 1.4$ (Table A-2).

Analysis (*a*) Process 1-2: isentropic compression.



$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = (293\text{K})(20)^{0.4} = 971.1\text{K}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2} \longrightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2200\text{ K}}{971.1\text{ K}} = 2.265$$

Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{k-1} = T_3\left(\frac{2.265V_2}{V_4}\right)^{k-1} = T_3\left(\frac{2.265}{r}\right)^{k-1} = (2200\text{K})\left(\frac{2.2.65}{20}\right)^{0.4} = 920.6\text{K}$$

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2) = (1.005\text{kJ/kg}\cdot\text{K})(2200 - 971.1)\text{K} = 1235\text{kJ/kg}$$

$$q_{out} = u_4 - u_1 = C_v(T_4 - T_1) = (0.718\text{kJ/kg}\cdot\text{K})(920.6 - 293)\text{K} = 450.6\text{kJ/kg}$$

$$w_{net,out} = q_{in} - q_{out} = 1235 - 450.6 = 784.4\text{kJ/kg}$$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{784.4\text{kJ/kg}}{1235\text{kJ/kg}} = \mathbf{63.5\%}$$

(*b*) $$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{K})}{95\text{kPa}} = 0.885\text{m}^3/\text{kg} = v_{\max}$$

$$v_{\min} = v_2 = \frac{v_{\max}}{r}$$

$$MEP = \frac{w_{net,out}}{v_1 - v_2} = \frac{w_{net,out}}{v_1(1 - 1/r)} = \frac{784.4\text{kJ/kg}}{(0.885\text{m}^3/\text{kg})(1 - 1/20)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{933kPa}$$

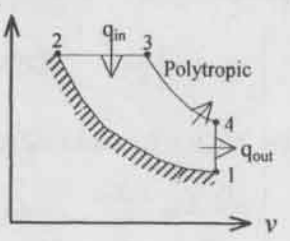
8-51 A diesel engine with air as the working fluid has a compression ratio of 20. The thermal efficiency and the mean effective pressure are to be determined. √

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (a) Process 1-2: isentropic compression.

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = (293\text{K})(20)^{0.4} = 971.1\text{ K}$$



Process 2-3: P = constant heat addition.

$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2} \longrightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2200\text{ K}}{971.1\text{ K}} = 2.265$$

Process 3-4: polytropic expansion.

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{n-1} = T_3\left(\frac{2.265V_2}{V_4}\right)^{n-1} = T_3\left(\frac{2.265}{r}\right)^{n-1} = (2200\text{ K})\left(\frac{2.265}{20}\right)^{0.35} = 1026\text{ K}$$

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2) = (1.005\text{ kJ/kg}\cdot\text{K})(2200 - 971.1)\text{ K} = 1235\text{ kJ/kg}$$

$$q_{out} = u_4 - u_1 = C_v(T_4 - T_1) = (0.718\text{ kJ/kg}\cdot\text{K})(1026 - 293)\text{ K} = 526.3\text{ kJ/kg}$$

Note that q_{out} in this case does not represent the entire heat rejected since some heat is also rejected during the polytropic process, which is determined from an energy balance on process 3-4:

$$w_{34,out} = \frac{R(T_4 - T_3)}{1-n} = \frac{(0.287\text{kJ/kg}\cdot\text{K})(1026 - 2200)\text{ K}}{1 - 1.35} = 963\text{ kJ/kg}$$

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

$$q_{34,in} - w_{34,out} = u_4 - u_3 \longrightarrow q_{34,in} = w_{34,out} + C_v(T_4 - T_3)$$
$$= 963\text{ kJ/kg} + (0.718\text{ kJ/kg}\cdot\text{K})(1026 - 2200)\text{ K}$$
$$= 120.1\text{ kJ/kg}$$

which means that 120.1 kJ/kg of heat is transferred to the combustion gases during the expansion process. This is unrealistic since the gas is at a much higher temperature than the surroundings, and a hot gas loses heat during polytropic expansion. The cause of this unrealistic result is the constant specific heat assumption. If we were to use u data from the air table, we would obtain

$$q_{34,in} = w_{34,out} + (u_4 - u_3) = 963 + (781.3 - 1872.4) = -128.1\text{ kJ/kg}$$

which is a heat loss as expected. Then q_{out} becomes

$$q_{out} = q_{34,out} + q_{41,out} = 128.1 + 526.3 = 654.4\text{ kJ/kg}$$

and

$$w_{net,out} = q_{in} - q_{out} = 1235 - 654.4 = 580.6\text{ kJ/kg}$$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{580.6\text{ kJ/kg}}{1235\text{ kJ/kg}} = 47.0\%$$

(c) $$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\ \text{K})}{95\ \text{kPa}} = 0.885\ \text{m}^3/\text{kg} = v_{max}$$

$$v_{min} = v_2 = \frac{v_{max}}{r}$$

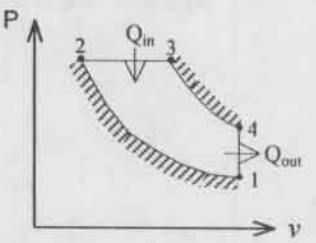
$$MEP = \frac{w_{net,out}}{v_1 - v_2} = \frac{w_{net,out}}{v_1(1-1/r)} = \frac{580.6\ \text{kJ/kg}}{(0.885\ \text{m}^3/\text{kg})(1-1/20)}\left(\frac{1\,\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{691\ kPa}$$

8-52 A four-cylinder ideal diesel engine with air as the working fluid has a compression ratio of 17 and a cutoff ratio of 2.2. The power the engine will deliver at 1500 rpm is to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis Process 1-2: isentropic compression.



$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = (300\text{K})(17)^{0.4} = 931.8\text{K}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow T_3 = \frac{v_3}{v_2} T_2 = 2.2T_2 = (2.2)(931.8\text{K}) = 2050\text{K}$$

Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{n-1} = T_3\left(\frac{2.2V_2}{V_4}\right)^{n-1} = T_3\left(\frac{2.2}{r}\right)^{n-1} = (2050\text{K})\left(\frac{2.2}{17}\right)^{0.4} = 904.8\text{K}$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{(97\text{kPa})(0.0045\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})} = 5.070\times10^{-3}\,\text{kg}$$

$$\begin{aligned} Q_{in} &= m(h_3 - h_2) = mC_p(T_3 - T_2) \\ &= (5.070\times10^{-3}\,\text{kg})(1.005\text{kJ/kg}\cdot\text{K})(2050-931.8)\text{K} = 5.698\text{kJ} \end{aligned}$$

$$\begin{aligned} Q_{out} &= m(u_4 - u_1) = mC_v(T_4 - T_1) \\ &= (5.070\times10^{-3}\,\text{kg})(0.718\text{kJ/kg}\cdot\text{K})(904.8-300)\text{K} = 2.202\text{kJ} \end{aligned}$$

$$W_{net,out} = Q_{in} - Q_{out} = 5.698 - 2.202 = 3.496\text{kJ/rev}$$

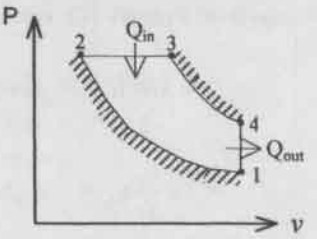
$$\dot{W}_{net,out} = \dot{n}W_{net,out} = (1500/60\text{rev/s})(3.496\text{kJ/rev}) = \mathbf{87.4kW}$$

8-53 A four-cylinder ideal diesel engine with nitrogen as the working fluid has a compression ratio of 17 and a cutoff ratio of 2.2. The power the engine will deliver at 1500 rpm is to be determined.

Assumptions **1** The air-standard assumptions are applicable with nitrogen as the working fluid. **2** Kinetic and potential energy changes are negligible. **3** Nitrogen is an ideal gas with constant specific heats.

Properties The properties of nitrogen at room temperature are C_p = 1.039 kJ/kg·K, C_v = 0.743 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis Process 1-2: isentropic compression.



$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = (300\text{K})(17)^{0.4} = 931.8\text{K}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow T_3 = \frac{v_3}{v_2} T_2 = 2.2T_2 = (2.2)(931.8\text{K}) = 2050\text{K}$$

Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{n-1} = T_3\left(\frac{2.2V_2}{V_4}\right)^{n-1} = T_3\left(\frac{2.2}{r}\right)^{n-1} = (2050\text{K})\left(\frac{2.2}{17}\right)^{0.4} = 904.8\text{K}$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{(97\text{kPa})(0.0045\text{m}^3)}{(0.2968\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})} = 4.902\times10^{-3}\,\text{kg}$$

$$\begin{aligned} Q_{in} &= m(h_3 - h_2) = mC_p(T_3 - T_2) \\ &= (4.902\times10^{-3}\,\text{kg})(1.039\text{kJ/kg}\cdot\text{K})(2050-931.8)\text{K} = 5.975\text{kJ} \end{aligned}$$

$$\begin{aligned} Q_{out} &= m(u_4 - u_1) = mC_v(T_4 - T_1) \\ &= (4.902\times10^{-3}\,\text{kg})(0.743\text{kJ/kg}\cdot\text{K})(904.8-300)\text{K} = 2.203\text{kJ} \end{aligned}$$

$$W_{net,out} = Q_{in} - Q_{out} = 5.975 - 2.203 = 3.773\text{kJ/rev}$$

$$\dot{W}_{net,out} = \dot{n}W_{net,out} = (1500/60\text{rev/s})(3.773\text{kJ/rev}) = \mathbf{94.3KW}$$

8-54 An ideal dual cycle with air as the working fluid has a compression ratio of 14. The fraction of heat transferred at constant volume and the thermal efficiency of the cycle are to be determined. √

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.

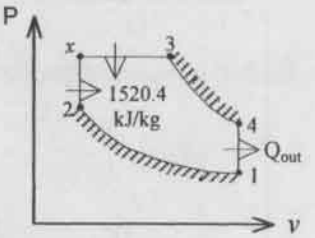
Analysis (a) Process 1-2: isentropic compression.

$$T_1 = 300\ \text{K} \longrightarrow \begin{matrix} u_1 = 214.07\ \text{kJ/kg} \\ v_{r_1} = 621.2 \end{matrix}$$

$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \frac{1}{r} v_{r_1} = \frac{1}{14}(621.2) = 44.37 \longrightarrow \begin{matrix} T_2 = 823.1\text{K} \\ u_2 = 611.2\text{kJ/kg} \end{matrix}$$

Process 2-x, x-3: heat addition,

$$T_3 = 2200\text{K} \longrightarrow \begin{matrix} h_3 = 2503.2\text{kJ/kg} \\ v_{r_3} = 2.012 \end{matrix}$$



$$q_{in} = q_{x-2,in} + q_{3-x,in} = (u_x - u_2) + (h_3 - h_x)$$
$$1520.4 = (u_x - 611.2) + (2503.2 - h_x)$$

By trial and error, we get $T_x = 1300$ K and $h_x = 1395.97$, $u_x = 1022.82$ kJ /kg.

Thus,

$$q_{2-x,in} = u_x - u_2 = 1022.82 - 611.2 = 411.62\ \text{kJ/kg}$$

and

$$ratio = \frac{q_{2-x,in}}{q_{in}} = \frac{411.62\ \text{kJ/kg}}{1520.4\ \text{kJ/kg}} = \mathbf{27.1\%}$$

(b) $$\frac{P_3 v_3}{T_3} = \frac{P_x v_x}{T_x} \longrightarrow \frac{v_3}{v_x} = \frac{T_3}{T_x} = \frac{2200\ \text{K}}{1300\ \text{K}} = 1.692 = r_c$$

$$v_{r_4} = \frac{v_4}{v_3} v_{r_3} = \frac{v_4}{1.692 v_2} v_{r_3} = \frac{r}{1.692} v_{r_3} = \frac{14}{1.692}(2.012) = 16.648 \longrightarrow u_4 = 886.3\text{kJ/kg}$$

Process 4-1: v = constant heat rejection.

$$q_{out} = u_4 - u_1 = 886.3 - 214.07 = 672.23\ \text{kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{672.23\ \text{kJ/kg}}{1520.4\ \text{kJ/kg}} = \mathbf{55.8\%}$$

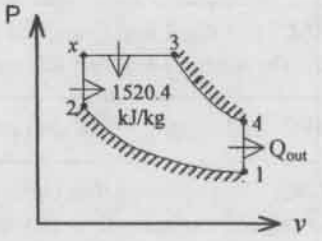
8-55 An ideal dual cycle with air as the working fluid has a compression ratio of 14. The fraction of heat transferred at constant volume and the thermal efficiency of the cycle are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (a) Process 1-2: isentropic compression.

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (300\text{K})(14)^{0.4} = 862\text{K}$$



Process 2-x, x-3: heat addition,

$$q_{in} = q_{2-x,in} + q_{3-x,in} = (u_x - u_2) + (h_3 - h_x)$$
$$= C_v(T_x - T_2) + C_p(T_3 - T_x)$$
$$1520.4\text{kJ/kg} = (0.718\text{kJ/kg}\cdot\text{K})(T_x - 862) + (1.005\text{kJ/kg}\cdot\text{K})(2200 - T_x)$$

Solving for T_x we get T_x = 250 K which is impossible. Therefore, constant specific heats at room temperature turned out to be an unreasonable assumption in this case because of the very high temperatures involved.

Ideal and Actual Gas-Turbine (Brayton) Cycles

8-56C In gas turbine engines a gas is compressed, and thus the compression work requirements are very large since the steady-flow work is proportional to the specific volume.

8-57C They are (1) isentropic compression (in a compressor), (2) P = constant heat addition, (3) isentropic expansion (in a turbine), and (4) P = constant heat rejection.

8-58C For fixed maximum and minimum temperatures, (a) the thermal efficiency increases with pressure ratio, (b) the net work first increases with pressure ratio, reaches a maximum, and then decreases.

8-59C To keep the gas temperature at safe levels.

8-60C The processes that make up the Brayton cycle are executed in steady-flow devices, and thus should be analyzed as steady-flow processes.

8-61C Back work ratio is the ratio of the compressor (or pump) work input to the turbine work output. It is usually between 0.40 and 0.6 for gas turbine engines.

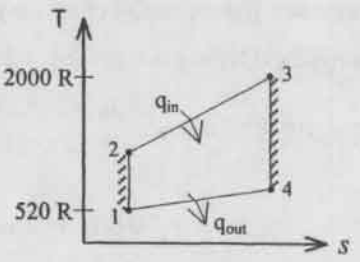
8-62C They are accounted for by incorporating adiabatic efficiencies for these devices in the analysis.

8-63C As a result of turbine and compressor inefficiencies, (a) the back work ratio increases, and (b) the thermal efficiency decreases.

8-64E A simple ideal Brayton cycle with air as the working fluid has a pressure ratio of 10. The air temperature at the compressor exit, the back work ratio, and the thermal efficiency are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17E.



Analysis (*a*) Noting that process 1-2 is isentropic,

$$T_1 = 520\ \text{R} \longrightarrow \begin{matrix} h_1 = 124.27\ \text{Btu/lbm} \\ P_{r_1} = 1.2147 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (10)(1.2147) = 12.147 \longrightarrow \begin{matrix} T_2 = \mathbf{996.5R} \\ h_2 = 240.11\text{Btu/lbm} \end{matrix}$$

(*b*) Process 3-4 is isentropic, and thus

$$T_3 = 2000\text{R} \longrightarrow \begin{matrix} h_3 = 504.71\text{Btu/lbm} \\ P_{r_3} = 174.0 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{10}\right)(174.0) = 17.4 \longrightarrow h_4 = 265.83\text{Btu/lbm}$$

$$w_{C,in} = h_2 - h_1 = 240.11 - 124.27 = 115.84\text{Btu/lbm}$$

$$w_{T,out} = h_3 - h_4 = 504.71 - 265.83 = 238.88\text{Btu/lbm}$$

Then the back-work ratio becomes

$$r_{bw} = \frac{w_{C,in}}{w_{T,out}} = \frac{115.84\ \text{Btu/lbm}}{238.88\ \text{Btu/lbm}} = \mathbf{48.5\%}$$

(*c*) $$q_{in} = h_3 - h_2 = 504.71 - 240.11 = 264.60\ \text{Btu/lbm}$$

$$w_{net,out} = w_{T,out} - w_{C,in} = 238.88 - 115.84 = 123.04\ \text{Btu/lbm}$$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{123.04\ \text{Btu/lbm}}{264.60\ \text{Btu/lbm}} = \mathbf{46.5\%}$$

8-65 A simple Brayton cycle with air as the working fluid has a pressure ratio of 8. The air temperature at the turbine exit, the net work output, and the thermal efficiency are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.

Analysis (*a*) Noting that process 1-2s is isentropic,

$$T_1 = 310\ \text{K} \longrightarrow \begin{matrix} h_1 = 310.24\ \text{kJ/kg} \\ P_{r_1} = 1.5546 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (8)(1.5546) = 12.44 \longrightarrow h_{2s} = 562.58\ \text{kJ/kg} \text{ and } T_{2s} = 557.25\ \text{K}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_C}$$

$$= 310.24 + \frac{562.58 - 310.24}{0.75} = 646.7\ \text{kJ/kg}$$

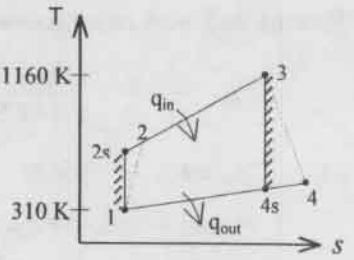
$$T_3 = 1160\text{K} \longrightarrow \begin{matrix} h_3 = 1230.92\text{kJ/kg} \\ P_{r_3} = 207.2 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(207.2) = 25.90 \longrightarrow h_{4s} = 692.19\ \text{kJ/kg} \text{ and } T_{4s} = 680.3\ \text{K}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s})$$

$$= 1230.92 - (0.82)(1230.92 - 692.19)$$

$$= 789.16\text{kJ/kg}$$



Thus, $T_4 =$ **770.1 K**

(*b*) $q_{in} = h_3 - h_2 = 1230.92 - 646.7 = 584.2\text{kJ/kg}$

$q_{out} = h_4 - h_1 = 789.16 - 310.24 = 478.92\text{kJ/kg}$

$w_{net,out} = w_{in} - w_{out} = 584.2 - 478.92 =$ **105.3kJ/kg**

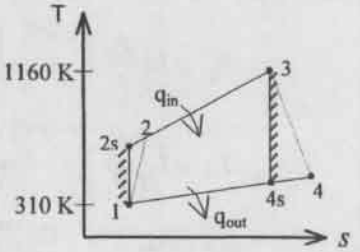
(*c*) $\eta_{th} = \dfrac{w_{net,out}}{q_{in}} = \dfrac{105.3\text{kJ/kg}}{584.2\text{kJ/kg}} =$ **18.0%**

8-66 A simple Brayton cycle with air as the working fluid has a pressure ratio of 8. The air temperature at the turbine exit, the net work output, and the thermal efficiency are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are $C_p = 1.005$ kJ/kg·K and $k = 1.4$ (Table A-2).

Analysis (*a*) Using the compressor and turbine efficiency relations,



$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (310\text{K})(8)^{0.4/1.4} = 561.5\text{K}$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1160\text{K})\left(\frac{1}{8}\right)^{0.4/1.4} = 640.4\text{K}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{C_p(T_{2s} - T_1)}{C_p(T_2 - T_1)} \longrightarrow T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_C}$$

$$= 310 + \frac{561.5 - 310}{0.75} = 645.3\text{K}$$

$$\eta_T = \frac{h_3 - h_{4s}}{h_3 - h_4} = \frac{C_p(T_3 - T_{4s})}{C_p(T_3 - T_4)} \longrightarrow T_4 = T_3 - \eta_T(T_3 - T_{4s})$$

$$= 1160 - (0.82)(1160 - 640.4)$$

$$= \mathbf{733.9K}$$

(*b*) $$q_{in} = h_3 - h_2 = C_p(T_3 - T_2) = (1.005\text{kJ/kg}\cdot\text{K})(1160 - 645.3)\text{K} = 517.3\text{kJ/kg}$$

$$q_{out} = h_4 - h_1 = C_p(T_4 - T_1) = (1.005\text{kJ/kg}\cdot\text{K})(733.9 - 310)\text{K} = 426.0\text{kJ/kg}$$

$$w_{net,out} = w_{in} - w_{out} = 517.3 - 426.0 = \mathbf{91.3kJ/kg}$$

(*c*) $$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{91.3\text{ kJ/kg}}{517.3\text{ kJ/kg}} = \mathbf{17.6\%}$$

8-67 A 30-MW gas turbine power plant that operates on the simple Brayton cycle with air as the working fluid has a pressure ratio of 12. The required mass flow rate of air is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K and k = 1.4 (Table A-2).

Analysis (*a*) Using the isentropic relations,

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})(12)^{0.4/1.4} = 610.2\text{K}$$

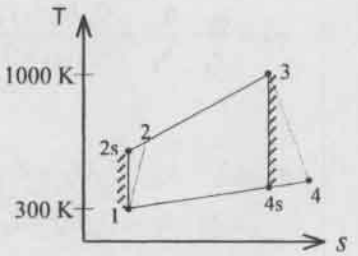
$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1000\text{K})\left(\frac{1}{12}\right)^{0.4/1.4} = 491.7\text{K}$$

$$w_{s,C,in} = h_{2s} - h_1 = C_p(T_{2s} - T_1) = (1.005\text{kJ/kg}\cdot\text{K})(610.2 - 300)\text{K} = 311.75\text{kJ/kg}$$

$$w_{s,T,out} = h_3 - h_{4s} = C_p(T_3 - T_{4s}) = (1.005\text{kJ/kg}\cdot\text{K})(100 - 491.7)\text{K} = 510.84\text{kJ/kg}$$

$$w_{s,net,out} = w_{s,T,out} - w_{s,C,in} = 510.84 - 311.75 = 199.09\text{kJ/kg}$$

$$\dot{m}_s = \frac{\dot{W}_{net,out}}{w_{s,net,out}} = \frac{30{,}000\text{kJ/s}}{199.09\text{kJ/kg}} = \mathbf{150.7kg/s}$$



(*b*) The net work output is determined to be

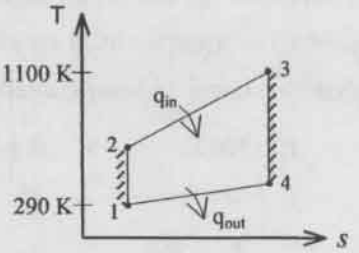
$$w_{a,net,out} = w_{a,T,out} - w_{a,C,in} = \eta_T w_{s,T,out} - w_{s,C,in}/\eta_C$$
$$= (0.80)(510.84) - 311.75/0.80 = 18.98\text{kJ/kg}$$

$$\dot{m}_a = \frac{\dot{W}_{net,out}}{w_{a,net,out}} = \frac{30{,}000\text{kJ/s}}{18.98\text{kJ/kg}} = \mathbf{1581kg/s}$$

8-68 A stationary gas-turbine power plant operates on a simple ideal Brayton cycle with air as the working fluid. The power delivered by this plant is to be determined assuming constant and variable specific heats.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas.

Analysis (*a*) Assuming constant specific heats,



$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (290\text{K})(8)^{0.4/1.4} = 525.3\text{K}$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1100\text{K})\left(\frac{1}{8}\right)^{0.4/1.4} = 607.2\text{K}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{607.2 - 290}{1100 - 525.3} = 0.448$$

$$\dot{W}_{net,out} = \eta_T \dot{Q}_{in} = (0.448)(50{,}000\text{kW}) = \mathbf{22{,}400kW}$$

(*b*) Assuming variable specific heats (Table A-17),

$$T_1 = 290K \longrightarrow \begin{matrix} h_1 = 290.16\text{kJ/kg} \\ P_{r_1} = 1.2311 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (8)(1.2311) = 9.8488 \longrightarrow h_2 = 526.11\text{kJ/kg}$$

$$T_3 = 1100\text{K} \longrightarrow \begin{matrix} h_3 = 1161.07\text{kJ/kg} \\ P_{r_3} = 167.1 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(167.1) = 20.89 \longrightarrow h_4 = 651.37\text{kJ/kg}$$

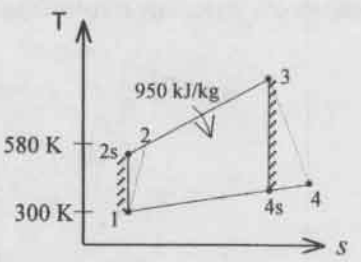
$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} == 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{651.37 - 290.16}{1161.07 - 526.11} = 0.431$$

$$\dot{W}_{net,out} = \eta_T \dot{Q}_{in} = (0.431)(50{,}000\text{kW}) = \mathbf{22{,}550kW}$$

8-69 An actual gas-turbine power plant operates at specified conditions. The fraction of the turbine work output used to drive the compressor and the thermal efficiency are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.



Analysis (a) Using the isentropic relations,

$$T_1 = 300\ \text{K} \longrightarrow h_1 = 300.19\ \text{kJ/kg}$$

$$T_2 = 580\ \text{K} \longrightarrow h_2 = 586.04\ \text{kJ/kg}$$

$$r_p = \frac{P_2}{P_1} = \frac{700}{100} = 7$$

$$q_{in} = h_3 - h_2 \longrightarrow h_3 = 950 + 586.04 = 1536.04\text{kJ/kg}$$

$$\longrightarrow P_{r_3} = 474.11$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{7}\right)(474.11) = 67.73 \longrightarrow h_{4s} = 905.83\text{kJ/kg}$$

$$w_{C,in} = h_2 - h_1 = 586.04 - 300.19 = 285.85\text{kJ/kg}$$

$$w_{T,out} = \eta_T (h_3 - h_{4s}) = (0.86)(1536.04 - 905.83) = 542.0\text{kJ/kg}$$

Thus, $$r_{bw} = \frac{w_{C,in}}{w_{T,out}} = \frac{285.85\ \text{kJ/kg}}{542.0\ \text{kJ/kg}} = \mathbf{52.7\%}$$

(b) $$w_{net,out} = w_{T,out} - w_{C,in} = 542.0 - 285.85 = 256.15\ \text{kJ/kg}$$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{256.15\ \text{kJ/kg}}{950\ \text{kJ/kg}} = \mathbf{27.0\%}$$

8-70 A gas-turbine power plant operates at specified conditions. The fraction of the turbine work output used to drive the compressor and the thermal efficiency are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

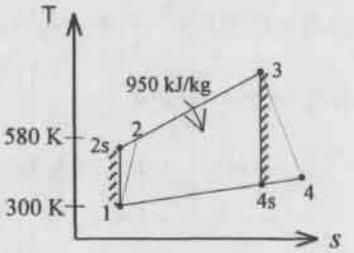
Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K and k = 1.4 (Table A-2).

Analysis (*a*) Using constant specific heats,

$$r_p = \frac{P_2}{P_1} = \frac{700}{100} = 7$$

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$

$$\longrightarrow T_3 = T_2 + q_{in}/C_p$$
$$= 580\text{K} + (950\text{kJ/kg})/(1.005\text{kJ/kg}\cdot\text{K})$$
$$= 1525.3\text{K}$$



$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1525.3\text{K})\left(\frac{1}{7}\right)^{0.4/1.4} = 874.8\text{K}$$

$$w_{C,in} = h_2 - h_1 = C_p(T_2 - T_1) = (1.005\text{kJ/kg}\cdot\text{K})(580 - 300)\text{K} = 281.4\text{kJ/kg}$$

$$w_{T,out} = \eta_T(h_3 - h_{4s}) = \eta_T C_p(T_3 - T_{4s}) = (0.86)(1.005\text{kJ/kg}\cdot\text{K})(1525.3 - 874.8)\text{K} = 562.2\text{kJ/kg}$$

Thus, $$r_{bw} = \frac{w_{C,in}}{w_{T,out}} = \frac{281.4\text{ kJ/kg}}{562.2\text{ kJ/kg}} = \mathbf{50.1\%}$$

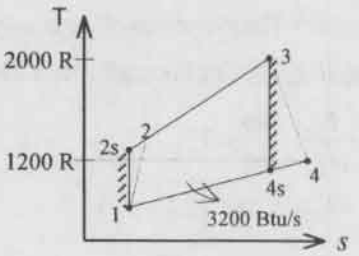
(b) $$w_{net,out} = w_{T,out} - w_{C,in} = 562.2 - 281.4 = 280.8\text{ kJ/kg}$$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{280.8\text{ kJ/kg}}{950\text{ kJ/kg}} = \mathbf{29.6\%}$$

8-71E A gas-turbine power plant operates on a simple Brayton cycle with air as the working fluid. The net power output of the plant is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17E.



Analysis Using variable specific heats for air,

$$T_3 = 2000\ \text{R} \longrightarrow h_3 = 504.71\ \text{Btu/lbm}$$

$$T_4 = 1200\ \text{R} \longrightarrow h_4 = 291.30\ \text{Btu/lbm}$$

$$r_p = \frac{P_2}{P_1} = \frac{120}{15} = 8$$

$$\dot{Q}_{out} = \dot{m}(h_4 - h_1) \longrightarrow h_1 = 291.30 + 6400/40 = 131.30\text{Btu/lbm}$$

$$\longrightarrow P_{r_1} = 1.474$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (8)(1.474) = 11.79 \longrightarrow h_{2s} = 238.07\text{Btu/lbm}$$

$$\dot{W}_{C,in} = \dot{m}(h_{2s} - h_1)/\eta_C = (40\text{lbm/s})(238.07 - 131.30)/(0.80) = 5339\text{Btu/s}$$

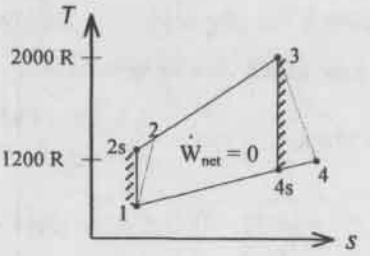
$$\dot{W}_{T,out} = \dot{m}(h_3 - h_4) = (40\text{lbm/s})(504.71 - 291.30) = 8536\text{Btu/s}$$

$$\dot{W}_{net,out} = \dot{W}_{T,out} - \dot{W}_{C,in} = 8536 - 5339 = 3197\text{Btu/s} = \mathbf{3373kW}$$

8-72E A gas-turbine power plant operates on a simple Brayton cycle with air as the working fluid. The compressor efficiency for which the power plant produces zero net work is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17E.



Analysis Using variable specific heats,

$$T_3 = 2000\ \text{R} \longrightarrow h_3 = 504.71\ \text{Btu/lbm}$$

$$T_4 = 1200\ \text{R} \longrightarrow h_4 = 291.30\ \text{Btu/lbm}$$

$$r_p = \frac{P_2}{P_1} = \frac{120}{15} = 8$$

$$\dot{Q}_{out} = \dot{m}(h_4 - h_1) \longrightarrow h_1 = 291.30 - 6400/40 = 131.30\text{Btu/lbm} \longrightarrow P_{r_1} = 1.474$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (8)(1.474) = 11.79 \longrightarrow h_{2s} = 238.07\text{Btu/lbm}$$

$$\dot{W}_{C,in} = \dot{m}(h_{2s} - h_1)/\eta_C = (40\text{lbm/s})(238.07 - 131.30)/(0.80) = 5339\text{Btu/s}$$

$$\dot{W}_{T,out} = \dot{m}(h_3 - h_4) = (40\text{lbm/s})(504.71 - 291.30) = 8536\text{Btu/s}$$

$$\dot{W}_{net,out} = \dot{W}_{T,out} - \dot{W}_{C,in} = 8536 - 5339 = 3197\text{Btu/s} = \mathbf{3373kW}$$

Then, $\dot{W}_{C,in} = \dot{W}_{T,out} \longrightarrow \dot{m}(h_{2s} - h_1)/\eta_C = \dot{m}(h_3 - h_4)$

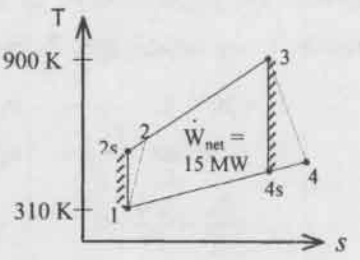
$$\eta_C = \frac{h_{2s} - h_1}{h_3 - h_4} = \frac{238.07 - 131.30}{504.71 - 291.30} = \mathbf{50.0\%}$$

8-73 A 15-MW gas-turbine power plant operates on a simple Brayton cycle with air as the working fluid. The mass flow rate of air through the cycle is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.

Analysis Using variable specific heats,



$$T_1 = 310\ \text{K} \longrightarrow \begin{matrix} h_1 = 310.24\ \text{kJ/kg} \\ P_{r_1} = 1.5546 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (8)(1.5546) = 12.44 \longrightarrow h_{2s} = 562.26\text{kJ/kg}$$

$$T_3 = 900\text{K} \longrightarrow \begin{matrix} h_3 = 932.93\text{kJ/kg} \\ P_{r_3} = 75.29 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(75.29) = 9.411 \longrightarrow h_{4s} = 519.32\text{kJ/kg}$$

$$w_{net,out} = w_{T,out} - w_{C,in} = \eta_T (h_3 - h_{4s}) - (h_{2s} - h_1)/\eta_C$$
$$= (0.86)(932.93 - 519.32) - (562.26 - 310.24)/(0.80) = 40.68\text{kJ/kg}$$

and

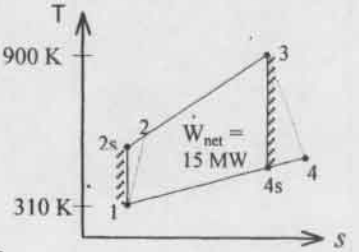
$$\dot{m} = \frac{\dot{W}_{net,out}}{w_{net,out}} = \frac{15{,}000\ \text{kJ/s}}{40.68\ \text{kJ/kg}} = \mathbf{368.7\ kg/s}$$

8-74 A 15-MW gas-turbine power plant operates on a simple Brayton cycle with air as the working fluid. The mass flow rate of air through the cycle is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg·K and k = 1.4 (Table A-2).

Analysis Using constant specific heats,



$$T_{2s} = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})(8)^{0.4/1.4} = 561.5\text{K}$$

$$T_{4s} = T_3 \left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (900\text{K})\left(\frac{1}{8}\right)^{0.4/1.4} = 496.8\text{K}$$

$$w_{net,out} = w_{T,out} - w_{C,in} = \eta_T C_p (T_3 - T_{4s}) - C_p (T_{2s} - T_1)/\eta_C$$
$$= (1.005\text{kJ/kg}\cdot\text{K})[(0.86)(900 - 496.8) - (561.5 - 310)/(0.80)]\text{K}$$
$$= 32.5\text{kJ/kg}$$

and

$$\dot{m} = \frac{\dot{W}_{net,out}}{w_{net,out}} = \frac{15{,}000\ \text{kJ/s}}{32.5\ \text{kJ/kg}} = \mathbf{461.5\ kg/s}$$

Brayton Cycle with Regeneration

8-75C Regeneration increases the thermal efficiency of a Brayton cycle by capturing some of the waste heat from the exhaust gases and preheating the air before it enters the combustion chamber.

8-76C Yes. At very high compression ratios, the gas temperature at the turbine exit may be lower than the temperature at the compressor exit. Therefore, if these two streams are brought into thermal contact in a regenerator, heat will flow to the exhaust gases instead of from the exhaust gases. As a result, the thermal efficiency will decrease.

8-77C The extent to which a regenerator approaches an ideal regenerator is called the effectiveness ε, and is defined as $\varepsilon = q_{regen,\,act}/q_{regen,\,max}$.

8-78C (b) turbine exit.

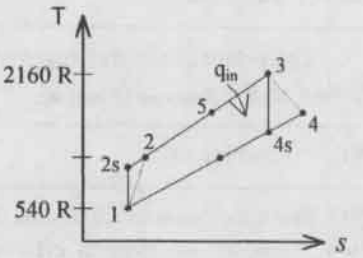
8-79C The steam injected increases the mass flow rate through the turbine and thus the power output. This, in turn, increases the thermal efficiency since $\eta = W/Q_{in}$ and W increases while Q_{in} remains constant. Steam can be obtained by utilizing the hot exhaust gases.

8-80E A car is powered by a gas turbine with a pressure ratio of 4. The thermal efficiency of the car and the mass flow rate of air for a net power output of 135 hp are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with variable specific heats. **3** The ambient air is 540 R and 14.5 psia. **4** The effectiveness of the regenerator is 0.9, and the isentropic efficiencies for both the compressor and the turbine are 80%. **5** The combustion gases can be treated as air.

Properties The properties of air at the compressor and turbine inlet temperatures can be obtained from Table A-17E.

Analysis The gas turbine cycle with regeneration can be analyzed as follows:



$$T_1 = 540\text{R} \longrightarrow \begin{matrix} h_1 = 129.06\text{Btu/lbm} \\ P_{r_1} = 1.386 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (4)(1.386) = 5.544 \longrightarrow h_{2s} = 192.0\text{Btu/lbm}$$

$$T_3 = 2160\text{R} \longrightarrow \begin{matrix} h_3 = 549.35\text{Btu/lbm} \\ P_{r_3} = 230.12 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{4}\right)(230.12) = 57.53 \longrightarrow h_{4s} = 372.2\text{Btu/lbm}$$

and

$$\eta_{\text{comp}} = \frac{h_{2s} - h_1}{h_2 - h_1} \quad \rightarrow \quad 0.80 = \frac{192.0 - 129.06}{h_2 - 129.06} \quad \rightarrow \quad h_2 = 207.74 \text{ kJ/kg}$$

$$\eta_{\text{turb}} = \frac{h_3 - h_4}{h_3 - h_{4s}} \quad \rightarrow \quad 0.80 = \frac{549.354 - h_4}{549.06 - 372.2} \quad \rightarrow \quad h_4 = 372.2 \text{ kJ/kg}$$

Then the thermal efficiency of the gas turbine cycle becomes

$$\dot{q}_{\text{regen}} = \varepsilon(h_4 - h_2) = 0.9(407.71 - 207.71) = 179.9 \text{ Btu/lbm}$$

$$q_{in} = (h_3 - h_2) - q_{regen} = (549.35 - 207.71) - 179.9 \text{ kJ/} = 161.7 \text{ Btu/s}$$

$$w_{net,out} = w_{T,out} - w_{C,in} = (h_3 - h_4) - (h_2 - h_1) = (549.35 - 407.63) - (207.71 - 129.09) = \mathbf{63.1 \text{ Btu/lbm}}$$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{63.1 \text{ Btu/lbm}}{161.7 \text{ Btu/lbm}} = \mathbf{0.39\%}$$

Finally, the mass flow rate of air through the turbine becomes

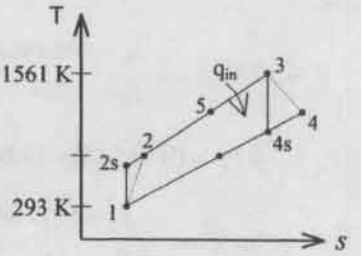
$$\dot{m}_{steam} = \frac{\dot{W}_{net}}{\dot{w}_{net}} = \frac{135 \text{ hp}}{63.1 \text{ Btu/lbm}} \left(\frac{0.7968 \text{ Btu/s}}{1 \text{ hp}}\right) = \mathbf{1.51 \text{ lbm/s}}$$

8-81 The thermal efficiency and power output of an actual gas turbine are given. The isentropic efficiency of the turbine and of the compressor, and the thermal efficiency of the gas turbine modified with a regenerator are to be determined. √

Assumptions **1** Air is an ideal gas with variable specific heats. **2** Kinetic and potential energy changes are negligible. **3** The mass flow rates of air and of the combustion gases are the same, and the properties of combustion gases are the same as those of air.

Properties The properties of air are given in Table A-17.

Analysis The properties at various states are



$$T_1 = 20°C = 293K \longrightarrow \begin{matrix} h_1 = 293.2 kJ/kg \\ P_{r_1} = 1.2765 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (14.7)(1.2765) = 18.765 \longrightarrow h_{2s} = 643.3 kJ/kg$$

$$T_3 = 1288°C = 1561\,K \longrightarrow \begin{matrix} h_3 = 1710.0 kJ/kg \\ P_{r_3} = 712.5 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{14.7}\right)(712.5) = 48.47 \longrightarrow h_{4s} = 825.23 kJ/kg$$

$$T_4 = 589°C = 862K \longrightarrow h_4 = 890.5 kJ/kg$$

The net work output and the heat input per unit mass are

$$w_{net} = \frac{\dot{W}_{net}}{\dot{m}} = \frac{159{,}000\,kW}{1{,}536{,}000\,kg/h}\left(\frac{3600\,s}{1\,h}\right) = 372.66\,kJ/kg$$

$$q_{in} = \frac{w_{net}}{\eta_{th}} = \frac{372.66\,kJ/kg}{0.359} = 1038.0\,kJ/kg$$

$$q_{in} = h_3 - h_2 \;\rightarrow\; h_2 = h_3 - q_{in} = 1710 - 1038 = 672.0\,kJ/kg$$

Then the compressor and turbine efficiencies become

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{1710 - 890.5}{1710 - 825.23} = \mathbf{0.926}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{643.3 - 293.2}{672 - 293.2} = \mathbf{0.924}$$

When a regenerator is added, the new heat input and the thermal efficiency become

$$q_{regen} = \varepsilon(h_4 - h_2) = (0.80)(890.5 - 672.0) = 174.8\,kJ/kg$$

$$q_{in,new} = q_{in} - q_{regen} = 1038 - 174.8 = 863.2\,kJ/kg$$

$$\eta_{th,new} = \frac{w_{net}}{q_{in,new}} = \frac{372.66\,kJ/kg}{863.3\,kJ/kg} = 0.432$$

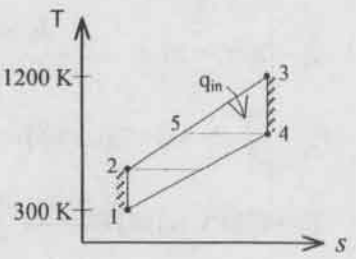
Discussion Note an 80% efficient regenerator would increase the thermal efficiency of this gas turbine from 35.9% to 43.2%.

8-82 An ideal Brayton cycle with regeneration is considered. The effectiveness of the regenerator is 100%. The net work output and the thermal efficiency of the cycle are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air are given in Table A-17.

Analysis Noting that this is an ideal cycle and thus the compression and expansion processes are isentropic, we have



$$T_1 = 300\text{K} \longrightarrow \begin{matrix} h_1 = 300.19\text{kJ/kg} \\ P_{r_1} = 1.386 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (10)(1.386) = 13.86 \longrightarrow h_2 = 579.87\text{kJ/kg}$$

$$T_3 = 1200\text{K} \longrightarrow \begin{matrix} h_3 = 1277.79\text{kJ/kg} \\ P_{r_3} = 238 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{10}\right)(238) = 23.8 \longrightarrow h_4 = 675.85\text{kJ/kg}$$

$$w_{C,in} = h_2 - h_1 = 579.87 - 300.19 = 279.68\text{kJ/kg}$$

$$w_{T,out} = h_3 - h_4 = 1277.79 - 675.85 = 601.94\text{kJ/kg}$$

Thus,

$$w_{net} = w_{T,out} - w_{C,in} = 601.97 - 279.68 = \mathbf{322.26\ kJ/kg}$$

Also, $\varepsilon = 100\% \longrightarrow h_5 = h_4 = 675.85\ \text{kJ/kg}$

$$q_{in} = h_3 - h_5 = 1277.79 - 675.85 = 601.94\ \text{kJ/kg}$$

and

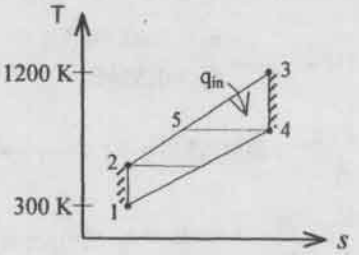
$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{322.26\ \text{kJ/kg}}{601.94\ \text{kJ/kg}} = \mathbf{53.5\%}$$

8-83 An ideal Brayton cycle with regeneration is considered. The effectiveness of the regenerator is 100%. The net work output and the thermal efficiency of the cycle are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air at room temperature are $C_p = 1.005$ kJ/kg.K and $k = 1.4$ (Table A-2a).

Analysis Noting that this is an ideal cycle and thus the compression and expansion processes are isentropic, we have



$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})(10)^{0.4/1.4} = 579.2\text{K}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1200\text{K})\left(\frac{1}{10}\right)^{0.4/1.4} = 621.5\text{K}$$

$$\varepsilon = 100\% \longrightarrow T_5 = T_4 = 621.5\text{K} \text{ and } T_6 = T_2 = 579.2\text{K}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_p(T_6 - T_1)}{C_p(T_3 - T_5)} = 1 - \frac{T_6 - T_1}{T_3 - T_5} = 1 - \frac{579.2 - 300}{1200 - 621.5} = \mathbf{0.517}$$

$$\left(\text{or, } \eta_{th} = 1 - \left(\frac{T_1}{T_3}\right) r_p^{(k-1)/k} = 1 - \left(\frac{300}{1200}\right)(10)^{(1.4-1)/1.4} = 0.517\right)$$

Then,

$$\begin{aligned} w_{net} &= w_{\text{turb, out}} - w_{\text{comp, in}} = (h_3 - h_4) - (h_2 - h_1) \\ &= C_p[(T_3 - T_4) - (T_2 - T_1)] \\ &= (1.005\text{ kJ/kg.K})[(1200 - 621.5) - (579.2 - 300)]\text{K} \\ &= \mathbf{300.6\text{ kJ/kg}} \end{aligned}$$

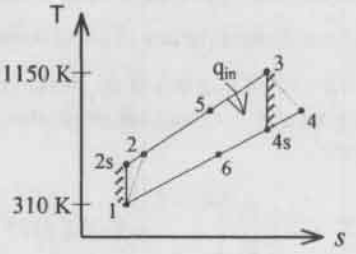
or,

$$\begin{aligned} w_{net} &= \eta_{th} q_{in} = \eta_{th}(h_3 - h_5) = \eta_{th} C_p (T_3 - T_5) \\ &= (0.517)(1.005\text{ kJ/kg.K})(1200 - 621.5) \\ &= 300.6\text{ kJ/kg} \end{aligned}$$

8-84 A Brayton cycle with regeneration using air as the working fluid is considered. The air temperature at the turbine exit, the net work output, and the thermal efficiency are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air are given in Table A-17.



Analysis (a) The properties of air at various states are

$$T_1 = 310\text{K} \longrightarrow \begin{matrix} h_1 = 310.24\text{kJ/kg} \\ P_{r_1} = 1.5546 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (8)(1.5546) = 12.44 \longrightarrow h_{2s} = 562.26\text{kJ/kg}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow h_2 = h_1 + (h_{2s} - h_1)/\eta_C = 310.24 + (562.26 - 310.24)/(0.75) = 646.32\text{kJ/kg}$$

$$T_3 = 1150\text{K} \longrightarrow \begin{matrix} h_3 = 1219.25\text{kJ/kg} \\ P_{r_3} = 200.15 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(200.15) = 25.02 \longrightarrow h_{4s} = 685.44\text{kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s}) = 1219.25 - (0.82)(1219.25 - 685.44) = 781.53\text{kJ/kg}$$

Thus, $T_4 =$ **763 K**

(*b*) $$w_{net} = w_{T,out} - w_{C,in} = (h_3 - h_4) - (h_2 - h_1)$$
$$= (1219.25 - 781.53) - (646.32 - 310.24) = \mathbf{101.64 kJ/kg}$$

(*c*) $$\varepsilon = \frac{h_5 - h_2}{h_4 - h_2} \longrightarrow h_5 = h_2 + \varepsilon(h_4 - h_2)$$
$$= 646.32 + (0.65)(781.53 - 646.32)$$
$$= 734.21\text{kJ/kg}$$

Then,

$$q_{in} = h_3 - h_5 = 1219.25 - 734.21 = 485.04\text{kJ/kg}$$

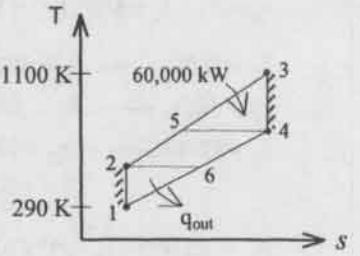
$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{101.64\text{kJ/kg}}{485.04\text{kJ/kg}} = \mathbf{21.0\%}$$

8-85 A stationary gas-turbine power plant operating on an ideal regenerative Brayton cycle with air as the working fluid is considered. The power delivered by this plant is to be determined for two cases.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas. **3** Kinetic and potential energy changes are negligible.

Properties When assuming constant specific heats, the properties of air at room temperature are C_p = 1.005 kJ/kg.K and k = 1.4 (Table A-2a). When assuming variable specific heats, the properties of air are obtained from Table A-17.

Analysis (*a*) Assuming constant specific heats,



$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (290\text{K})(8)^{0.4/1.4} = 525.3\text{K}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1100\text{K})\left(\frac{1}{8}\right)^{0.4/1.4} = 607.2\text{K}$$

$$\varepsilon = 100\% \longrightarrow T_5 = T_4 = 607.2\text{K} \text{ and } T_6 = T_2 = 525.3\text{K}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_p(T_6 - T_1)}{C_p(T_3 - T_5)} = 1 - \frac{T_6 - T_1}{T_3 - T_5} = 1 - \frac{525.3 - 290}{1100 - 607.2} = 0.5225$$

$$\dot{W}_{net} = \eta_T \dot{Q}_{in} = (0.5225)(60{,}000\text{kW}) = \mathbf{31{,}350\ kW}$$

(*b*) Assuming variable specific heats,

$$T_1 = 290\text{K} \longrightarrow \begin{matrix} h_1 = 290.16\text{kJ/kg} \\ P_{r_1} = 1.2311 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (8)(1.2311) = 9.8488 \longrightarrow h_2 = 526.11\text{kJ/kg}$$

$$T_3 = 1100\text{K} \longrightarrow \begin{matrix} h_3 = 1161.07\text{kJ/kg} \\ P_{r_3} = 167.1 \end{matrix}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(167.1) = 20.89 \longrightarrow h_4 = 651.37\text{kJ/kg}$$

$$\varepsilon = 100\% \longrightarrow h_5 = h_4 = 651.37\text{kJ/kg} \text{ and } h_6 = h_2 = 526.11\text{kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{h_6 - h_1}{h_3 - h_5} = 1 - \frac{526.11 - 290.11}{1161.07 - 651.37} = 0.5371$$

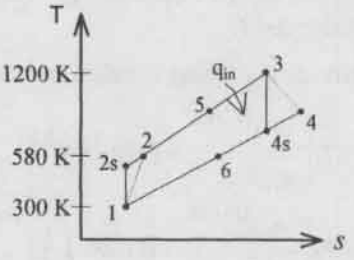
$$\dot{W}_{net} = \eta_T \dot{Q}_{in} = (0.5371)(60{,}000\text{kW}) = \mathbf{31{,}350kW}$$

8-86 A regenerative gas-turbine engine using air as the working fluid is considered. The amount of heat transfer in the regenerator and the thermal efficiency are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air are given in Table A-17.

Analysis (*a*) The properties at various states are



$$r_p = P_2 / P_1 = 800/100 = 8$$

$$T_1 = 300\text{K} \longrightarrow h_1 = 300.19\text{kJ/kg}$$
$$T_2 = 580\text{K} \longrightarrow h_2 = 586.04\text{kJ/kg}$$
$$T_3 = 1200\text{K} \longrightarrow h_3 = 1277.79\text{kJ/kg}$$
$$P_{r_3} = 238.0$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(238.0) = 29.75 \longrightarrow h_{4s} = 719.75\text{kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s})$$
$$= 1277.79 - (0.72)(1277.79 - 719.75)$$
$$= 152.52\text{kJ/kg}$$

$$q_{regen} = \varepsilon(h_3 - h_2) = (0.72)(797.87 - 586.04) = \mathbf{152.5kJ/kg}$$

(*b*)
$$w_{net} = w_{T,out} - w_{C,in} = (h_3 - h_4) - (h_2 - h_1)$$
$$= (1277.79 - 797.87) - (586.04 - 300.19) = 194.07\text{kJ/kg}$$

$$q_{in} = (h_3 - h_2) - q_{regen} = (1277.79 - 586.04) - 152.52 = 539.23\text{kJ/kg}$$

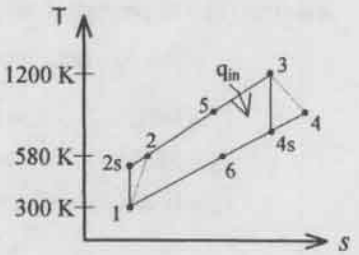
$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{194.07\text{kJ/kg}}{539.23\text{kJ/kg}} = \mathbf{36.0\%}$$

8-87 A regenerative gas-turbine engine using air as the working fluid is considered. The amount of heat transfer in the regenerator and the thermal efficiency are to be determined. √

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with constant specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg.K and k = 1.4 (Table A-2a).

Analysis (a) Using the isentropic relations and turbine efficiency,



$$r_p = P_2 / P_1 = 800/100 = 8$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1200\text{K})\left(\frac{1}{8}\right)^{0.4/1.4} = 662.5\text{K}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{C_p(T_3 - T_4)}{C_p(T_3 - T_{4s})} \longrightarrow T_4 = T_3 - \eta_T(T_3 - T_{4s})$$
$$= 1200 - (0.86)(1200 - 662.5)$$
$$= 737.8\text{K}$$

$$q_{regen} = \varepsilon(h_4 - h_2) = \varepsilon C_p(T_4 - T_2)$$
$$= (0.72)(1.005\text{kJ/kg}\cdot\text{K})(737.8 - 580)\text{K} = \mathbf{114.2kJ/kg}$$

(b)
$$w_{net} = w_{T,out} - w_{C,in} = C_p(T_3 - T_s) - C_p(T_2 - T_1)$$
$$= (1.005\text{kJ/kg}\cdot\text{K})[(1200 - 737.8) - (580 - 300)]\text{K} = 183.1\text{kJ/kg}$$

$$q_{in} = (h_3 - h_2) - q_{regen} = C_p(T_3 - T_2) - q_{regen}$$
$$= (1.005\text{kJ/kg}\cdot\text{K})(1200 - 580)\text{K} - 114.2 = 508.9\text{kJ/kg}$$

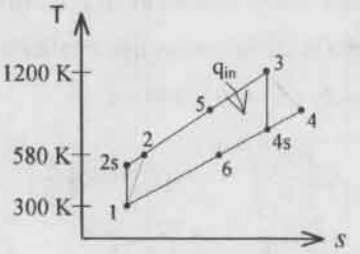
$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{183.1\text{kJ/kg}}{508.9\text{kJ/kg}} = \mathbf{36.0\%}$$

8-88 A regenerative gas-turbine engine using air as the working fluid is considered. The amount of heat transfer in the regenerator and the thermal efficiency are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air are given in Table A-17.

Analysis (*a*) The properties of air at various states are



$$r_p = P_2 / P_1 = 800/100 = 8$$

$$T_1 = 300\text{K} \longrightarrow h_1 = 300.19\text{kJ/kg}$$
$$T_2 = 580\text{K} \longrightarrow h_2 = 586.04\text{kJ/kg}$$
$$T_3 = 1200\text{K} \longrightarrow h_3 = 1277.79\text{kJ/kg}$$
$$P_{r_3} = 238.0$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(238.0) = 29.75 \longrightarrow h_{4s} = 719.75\text{kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s})$$
$$= 1277.79 - (0.72)(1277.79 - 719.75)$$
$$= 152.52\text{kJ/kg}$$

$$q_{regen} = \varepsilon (h_3 - h_2) = (0.70)(797.87 - 586.04) = \mathbf{148.3kJ/kg}$$

(*b*) $$w_{net} = w_{T,out} - w_{C,in} = (h_3 - h_4) - (h_2 - h_1)$$
$$= (1277.79 - 797.87) - (586.04 - 300.19) = 194.07\text{kJ/kg}$$

$$q_{in} = (h_3 - h_2) - q_{regen} = (1277.79 - 586.04) - 148.3 = 543.4\text{kJ/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{194.07\text{kJ/kg}}{543.4\text{kJ/kg}} = \mathbf{35.7\%}$$

Carnot Vapor Cycle

8-89C Because excessive moisture in steam causes erosion on the turbine blades. The highest moisture content allowed is about 10%.

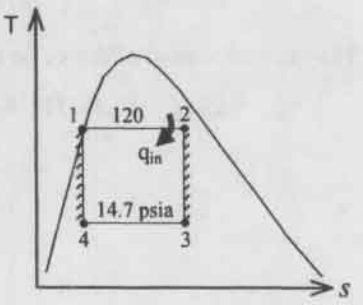
8-90C The Carnot cycle is not a realistic model for steam power plants because (1) limiting the heat transfer processes to two-phase systems to maintain isothermal conditions severely limits the maximum temperature that can be used in the cycle, (2) the turbine will have to handle steam with a high moisture content which causes erosion, and (3) it is not practical to design a compressor that will handle two phases.

8-91E A steady-flow Carnot engine with water as the working fluid operates at specified conditions. The thermal efficiency, the quality at the end of the heat rejection process, and the net work output are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) We note that

$$T_H = T_{\text{sat @120 psia}} = 341.3°\text{F} = 801.3\ \text{R}$$
$$T_L = T_{\text{sat @14.7 psia}} = 212°\text{F} = 672\ \text{R}$$



and

$$\eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{672\ \text{R}}{801.3\ \text{R}} = \mathbf{16.1\%}$$

(*b*) Noting that $s_4 = s_1 = s_{f\,@\,120\ \text{psia}} = 0.49201$ Btu/lbm·R,

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{0.49201 - 0.31212}{1.4446} = \mathbf{0.1245}$$

(*c*) The enthalpies before and after the heat addition process are

$$h_1 = h_{f@120\text{psia}} = 312.67\text{Btu/lbm}$$
$$h_2 = h_f + x_2 h_{fg} = 312.67 + (0.95)(878.5) = 1147.25\text{Btu/lbm}$$

Thus,

$$q_{\text{in}} = h_2 - h_1 = 1147.25 - 312.67 = 834.58\text{Btu/lbm}$$

and,

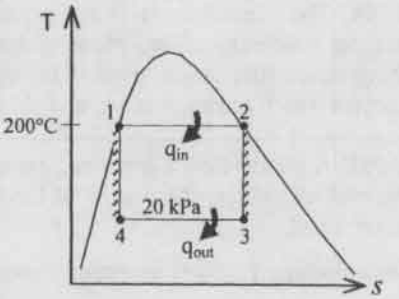
$$w_{\text{net}} = \eta_{\text{th}} q_{\text{in}} = (0.161)(834.58\text{Btu/lbm}) = \mathbf{134.4Btu/lbm}$$

8-92 A steady-flow Carnot engine with water as the working fluid operates at specified conditions. The thermal efficiency, the amount of heat rejected, and the net work input are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) Noting that T_H = 250°C = 523 K and $T_L = T_{\text{sat @ 20 kPa}}$ = 60.06°C = 333.1 K, the thermal efficiency becomes

$$\eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{333.1\text{K}}{523\text{K}} = \mathbf{36.3\%}$$



(*b*) The heat supplied during this cycle is simply the enthalpy of vaporization ,

$$q_{\text{in}} = h_{fg@250°C} = 1716.2\text{kJ/kg}$$

Thus,

$$q_{\text{out}} = q_L = \frac{T_L}{T_H} q_{\text{in}} = \left(\frac{333.1\text{K}}{523\text{K}}\right)(1716.2\text{kJ/kg}) = \mathbf{1093.1\ kJ/kg}$$

(*c*) The net work output of this cycle is

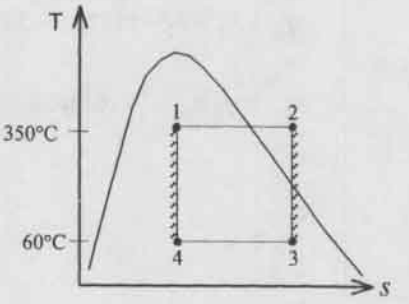
$$w_{\text{net}} = \eta_{\text{th}} q_{\text{in}} = (0.363)(1716.2\text{kJ/kg}) = \mathbf{623.0kJ/kg}$$

8-93 A steady-flow Carnot engine with water as the working fluid operates at specified conditions. The thermal efficiency, the pressure at the turbine inlet, and the net work output are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) The thermal efficiency is determined from

$$\eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{60 + 273\text{ K}}{350 + 273\text{ K}} = \mathbf{46.5\%}$$



(*b*) Note that $s_2 = s_3 = s_f + x_3 s_{fg}$

$$= 0.8312 + 0.891 \times 7.0784 = 7.138\text{ kJ/kg·K}$$

Thus ,

$$\left.\begin{aligned} T_2 &= 350°C \\ s_2 &= 7.138\text{kJ/kg}\cdot\text{K} \end{aligned}\right\} P_2 = \mathbf{1.40MPa}$$

(*c*) The net work can be determined by calculating the enclosed area on the T-s diagram,

$$s_4 = s_f + x_4 s_{fg} = 0.8312 + (0.1)(7.0784) = 1.539\text{kJ/kg}\cdot\text{K}$$

Thus,

$$w_{\text{net}} = \text{Area} = (T_H - T_L)(s_3 - s_4) = (350 - 60)(7.138 - 1.539) = \mathbf{1624kJ/kg}$$

The Simple Rankine Cycle

8-94C The four processes that make up the simple ideal cycle are (1) Isentropic compression in a pump, (2) P = constant heat addition in a boiler, (3) Isentropic expansion in a turbine, and (4) P = constant heat rejection in a condenser.

8-95C Heat rejected decreases; everything else increases.

8-96C Heat rejected decreases; everything else increases.

8-97C The pump work remains the same, the moisture content decreases, everything else increases.

8-98C The actual vapor power cycles differ from the idealized ones in that the actual cycles involve friction and pressure drops in various components and the piping, and heat loss to the surrounding medium from these components and piping.

8-99C The boiler exit pressure will be (a) lower than the boiler inlet pressure in actual cycles, and (b) the same as the boiler inlet pressure in ideal cycles.

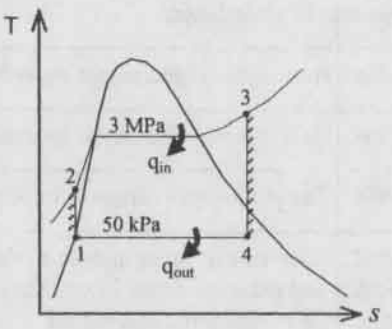
8-100C We would reject this proposal because $w_{turb} = h_1 - h_2 - q_{out}$, and any heat loss from the steam will adversely affect the turbine work output.

8-101C Yes, because the saturation temperature of steam at 10 kPa is 45.81°C, which is much higher than the temperature of the cooling water.

8-102 A steam power plant operates on a simple ideal Rankine cycle between the specified pressure limits. The thermal efficiency of the cycle and the net power output of the plant are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),



$$h_1 = h_{f@50\text{kPa}} = 340.49\text{kJ/kg}$$
$$v_1 = v_{f@50\text{kPa}} = 0.001030\text{m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.001030\text{m}^3/\text{kg})(3000 - 50\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 3.04\text{kJ/kg}$$
$$h_2 = h_1 + w_{p,\text{in}} = 340.49 + 3.04 = 343.53\text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 3\text{MPa} \\ T_3 = 400°\text{C} \end{matrix}\right\} \begin{matrix} h_3 = 3230.9\text{kJ/kg} \\ s_3 = 6.9212\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_4 = 50\text{kPa} \\ s_4 = s_3 \end{matrix}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.9212 - 1.0910}{6.5029} = 0.8966$$
$$h_4 = h_f + x_4 h_{fg} = 340.49 + (0.8966)(2305.4)$$
$$= 2407.5\text{kJ/kg}$$

Thus,

$$q_{\text{in}} = h_3 - h_2 = 3230.9 - 343.53 = 2887.37\ \text{kJ/kg}$$
$$q_{\text{out}} = h_4 - h_1 = 2407.5 - 340.49 = 2067.01\ \text{kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 2887.37 - 2067.01 = 820.36\ \text{kJ/kg}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2067.01}{2887.73} = \mathbf{28.4\%}$$

(*b*) $$\dot{W}_{\text{net}} = \dot{m} w_{\text{net}} = (25\text{kg/s})(820.36\text{kJ/kg}) = \mathbf{20.5MW}$$

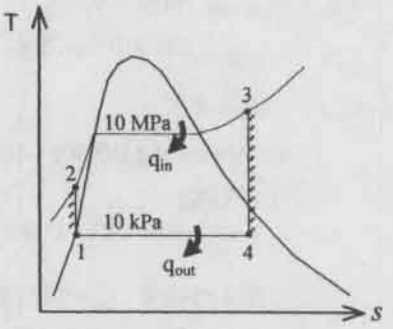
8-103 A 300-MW steam power plant that operates on a simple ideal Rankine cycle is considered. The quality of the steam at the turbine exit, the thermal efficiency of the cycle, and the mass flow rate of the steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f@10\text{kPa}} = 191.83\text{kJ/kg}$$
$$v_1 = v_{f@10\text{kPa}} = 0.00101\text{m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.00101\text{m}^3/\text{kg})(10{,}000 - 10\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 10.09\text{kJ/kg}$$
$$h_2 = h_1 + w_{p,\text{in}} = 191.83 + 10.09 = 201.92\text{kJ/kg}$$



$$\left.\begin{matrix} P_3 = 10\text{MPa} \\ T_3 = 500°\text{C} \end{matrix}\right\} \begin{matrix} h_3 = 3373.7\text{kJ/kg} \\ s_3 = 6.5966\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_4 = 10\text{kPa} \\ s_4 = s_3 \end{matrix}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.5966 - 0.6493}{7.5009} = \mathbf{0.793}$$

$$h_4 = h_f + x_4 h_{fg} = 191.83 + (0.793)(2392.8) = 2089.3\text{kJ/kg}$$

(*b*) $$q_{\text{in}} = h_3 - h_2 = 3373.7 - 201.92 = 3171.78\ \text{kJ/kg}$$
$$q_{\text{out}} = h_4 - h_1 = 2089.3 - 191.83 = 1897.47\ \text{kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 3171.78 - 1897.47 = 1274.31\ \text{kJ/kg}$$

and

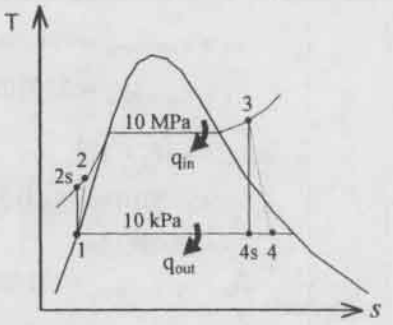
$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1274.31\ \text{kJ/kg}}{3171.78\ \text{kJ/kg}} = \mathbf{40.2\%}$$

(*c*) $$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{300{,}000\ \text{kJ/s}}{1274.31\ \text{kJ/kg}} = \mathbf{235.4\ kg/s}$$

8-104 A 300-MW steam power plant that operates on a simple nonideal Rankine cycle is considered. The quality of the steam at the turbine exit, the thermal efficiency of the cycle, and the mass flow rate of the steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),



$$h_1 = h_{f@10\text{kPa}} = 191.83\text{kJ/kg}$$
$$v_1 = v_{f@10\text{kPa}} = 0.00101\text{m}^3/\text{kg}$$
$$w_{p,\text{in}} = v_1(P_2 - P_1)/\eta_p$$
$$= (0.00101\text{m}^3/\text{kg})(10{,}000 - 10\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)/(0.85)$$
$$= 11.87\text{kJ/kg}$$
$$h_2 = h_1 + w_{p,\text{in}} = 191.83 + 11.87 = 203.70\text{kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 10\text{MPa} \\ T_3 = 500°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3373.7\text{kJ/kg} \\ s_3 = 6.5966\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_{4s} = 10\text{kPa} \\ s_{4s} = s_3 \end{array}\right\} x_4 = \frac{s_{4s} - s_f}{s_{fg}} = \frac{6.5966 - 0.6493}{7.5009} = \mathbf{0.793}$$

$$h_{4s} = h_f + x_4 h_{fg} = 191.83 + (0.793)(2392.8) = 2089.3\text{kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T(h_3 - h_{4s})$$
$$= 3373.7 - (0.85)(3373.7 - 2089.3) = 2281.96\text{kJ/kg}$$

(*b*) $$q_{\text{in}} = h_3 - h_2 = 3373.7 - 203.70 = 3170.0\ \text{kJ/kg}$$
$$q_{\text{out}} = h_4 - h_1 = 2281.96 - 191.83 = 2090.13\ \text{kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 3170.0 - 2090.13 = 1079.87\ \text{kJ/kg}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1079.87\ \text{kJ/kg}}{3170.0\ \text{kJ/kg}} = \mathbf{34.1\%}$$

(c) $$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{300{,}000\ \text{kJ/s}}{1079.87\ \text{kJ/kg}} = \mathbf{277.8\ kg/s}$$

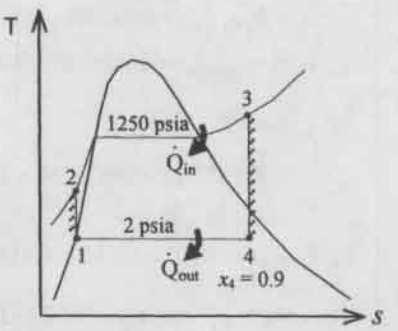
8-105E A steam power plant that operates on a simple ideal Rankine cycle between the specified pressure limits is considered. The minimum turbine inlet temperature, the rate of heat input in the boiler, and the thermal efficiency of the cycle are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4E, A-5E, and A-6E),

$$h_1 = h_{f@2\text{psia}} = 94.02\text{Btu/lbm}$$
$$v_1 = v_{f@2\text{psia}} = 0.01623\text{ft}^3\text{/lbm}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.01623\text{ft}^3\text{/lbm})(1250 - 2\text{psia})\left(\frac{1\text{Btu}}{5.4039\text{psia}\cdot\text{ft}^3}\right)$$
$$= 3.75\text{Btu/lbm}$$
$$h_2 = h_1 + w_{p,\text{in}} = 94.02 + 3.75 = 97.77\text{Btu/lbm}$$



$$h_4 = h_f + x_4 h_{fg} = 94.02 + (0.9)(1022.1) = 1013.91\text{Btu/lbm}$$
$$s_4 = s_f + x_4 s_{fg} = 0.17499 + (0.9)(1.7448) = 1.7453\text{Btu/lbm}\cdot\text{R}$$

$$\left.\begin{array}{l} P_3 = 1250\text{psia} \\ s_3 = s_4 \end{array}\right\} \begin{array}{l} h_3 = 1695.74\text{Btu/lbm} \\ T_3 = \mathbf{1340.7°F} \end{array}$$

(*b*) $$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_2) = (75\text{lbm/s})(1695.74 - 97.77) = \mathbf{119{,}848\text{Btu/s}}$$

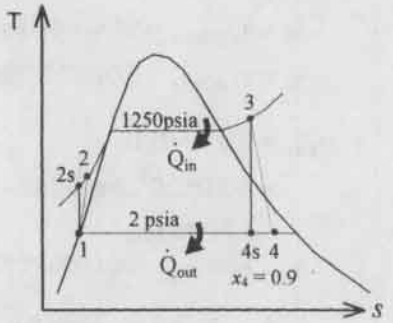
(*c*) $$\dot{Q}_{\text{out}} = \dot{m}(h_4 - h_1) = (75\text{lbm/s})(1013.91 - 94.02) = 68{,}992\text{Btu/s}$$

$$\eta_{th} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{68{,}992\text{Btu/s}}{119{,}848\text{Btu/s}} = \mathbf{42.4\%}$$

8-106E A steam power plant operates on a simple nonideal Rankine cycle between the specified pressure limits. The minimum turbine inlet temperature, the rate of heat input in the boiler, and the thermal efficiency of the cycle are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4E, A-5E, and A-6E),



$$h_1 = h_{f@2\text{psia}} = 94.02\text{Btu/lbm}$$
$$v_1 = v_{f@2\text{psia}} = 0.01623\text{ft}^3\text{/lbm}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)/\eta_P$$
$$= (0.01623\text{ft}^3\text{/lbm})(1250 - 2\text{psia})\left(\frac{1\text{Btu}}{5.4039\text{psia}\cdot\text{ft}^3}\right)/0.85$$
$$= 4.41\text{Btu/lbm}$$
$$h_2 = h_1 + w_{p,\text{in}} = 94.02 + 4.41 = 98.43\text{Btu/lbm}$$

$$h_4 = h_f + x_4 h_{fg} = 94.02 + (0.9)(1022.1) = 1013.91\text{Btu/lbm}$$
$$s_4 = s_f + x_4 s_{fg} = 0.17499 + (0.9)(1.7448) = 1.7453\text{Btu/lbm}\cdot\text{R}$$

The turbine inlet temperature is determined by trial and error ,

Try 1:
$$\left.\begin{array}{l} P_3 = 1250\text{psia} \\ T_3 = 900°\text{F} \end{array}\right\} \begin{array}{l} h_3 = 1438.4\text{Btu/lbm} \\ s_3 = 1.582\ \text{Btu/lbm.R} \end{array}$$

$$x_{4s} = \frac{s_{4s} - s_f}{s_{fg}} = \frac{s_3 - s_f}{s_{fg}} = \frac{1.582 - 0.17499}{1.7448} = 0.8064$$

$$h_{4s} = h_f + x_{4s} h_{fg} = 94.02 + (0.8064)(1022.1) = 918.2\text{Btu/lbm}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{1438.4 - 1013.91}{1438.4 - 918.2} = 0.8160$$

Try 2:
$$\left.\begin{array}{l} P_3 = 1250\text{psia} \\ T_3 = 1000°\text{F} \end{array}\right\} \begin{array}{l} h_3 = 1498.2\text{Btu/lbm} \\ s_3 = 1.6244\ \text{Btu/lbm.R} \end{array}$$

$$x_{4s} = \frac{s_{4s} - s_f}{s_{fg}} = \frac{s_3 - s_f}{s_{fg}} = \frac{1.6244 - 0.17499}{1.7448} = 0.8307$$

$$h_{4s} = h_f + x_{4s} h_{fg} = 94.02 + (0.8307)(1022.1) = 943.1\text{Btu/lbm}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{1498.2 - 1013.91}{1498.2 - 943.1} = 0.8724$$

By linear interpolation, at $\eta_T = 0.85$ we obtain $T_3 = \mathbf{960.3°F}$. Also, $h_3 = 1474.4$ Btu/lbm.

(*b*) $$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_2) = (75\text{lbm/s})(1474.4 - 98.43) = 103{,}200\ \mathbf{Btu/s}$$

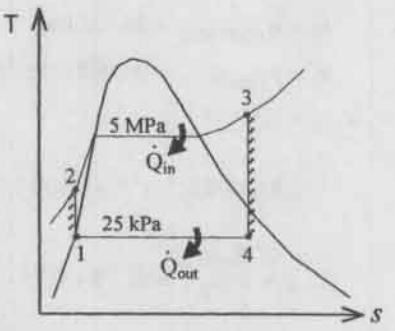
(*c*) $$\dot{Q}_{\text{out}} = \dot{m}(h_4 - h_1) = (75\text{lbm/s})(1013.91 - 94.02) = 69{,}000\text{Btu/s}$$

$$\eta_{th} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{69{,}000\text{Btu/s}}{103{,}200\text{Btu/s}} = \mathbf{33.1\%}$$

8-107 A 300-MW coal-fired steam power plant operates on a simple ideal Rankine cycle between the specified pressure limits. The overall plant efficiency and the required rate of the coal supply are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),



$$h_1 = h_{f@25\text{kPa}} = 271.93\text{kJ/kg}$$
$$v_1 = v_{f@25\text{kPa}} = 0.00102\text{m}^3\text{/kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.00102\text{m}^3\text{/kg})(5{,}000 - 25\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 5.07\text{kJ/kg}$$
$$h_2 = h_1 + w_{p,\text{in}} = 271.93 + 5.07 = 277.00\text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 5\text{MPa} \\ T_3 = 450°\text{C} \end{matrix}\right\} \begin{matrix} h_3 = 3316.2\text{kJ/kg} \\ s_3 = 6.8186\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_4 = 25\text{kPa} \\ s_4 = s_3 \end{matrix}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.8186 - 0.8931}{6.9383} = 0.8540$$

$$h_4 = h_f + x_4 h_{fg} = 271.93 + (0.8540)(2346.3) = 2275.7\text{kJ/kg}$$

The thermal efficiency is determined from

$$q_{\text{in}} = h_3 - h_2 = 3316.2 - 277.0 = 3039.2\text{ kJ/kg}$$
$$q_{\text{out}} = h_4 - h_1 = 2275.7 - 271.93 = 2003.8\text{ kJ/kg}$$

and

$$\eta_{th} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2003.8}{3039.2} = 34.1\%$$

Thus,

$$\eta_{overall} = \eta_{th} \times \eta_{\text{comb}} \times \eta_{\text{gen}} = (0.341)(0.75)(0.96) = \mathbf{24.6\%}$$

(*b*) Then the required rate of coal supply becomes

$$\dot{Q}_{\text{in}} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{overall}}} = \frac{300{,}000\text{kJ/s}}{0.246} = 1{,}219{,}500\text{kJ/s}$$

and

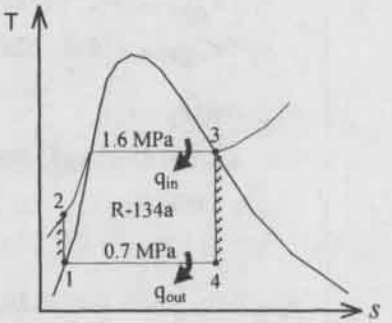
$$\dot{m}_{\text{coal}} = \frac{\dot{Q}_{\text{in}}}{C_{\text{coal}}} = \frac{1{,}219{,}500\text{kJ/s}}{29{,}300\text{kJ/kg}}\left(\frac{1\text{ton}}{1000\text{kg}}\right) = 0.04162\text{tons/s} = \mathbf{150.0tons/h}$$

8-108 A solar-pond power plant that operates on a simple ideal Rankine cycle with refrigerant-134a as the working fluid is considered. The thermal efficiency of the cycle and the power output of the plant are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the refrigerant tables (Tables A-11, A-12, and A-13),

$$h_1 = h_{f@0.7\text{MPa}} = 86.78\text{kJ/kg}$$
$$v_1 = v_{f@0.7\text{MPa}} = 0.0008328\text{m}^3/\text{kg}$$
$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.0008328\text{m}^3/\text{kg})(1600 - 700\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 0.75\text{kJ/kg}$$
$$h_2 = h_1 + w_{p,\text{in}} = 86.78 + 0.75 = 87.53\text{kJ/kg}$$



$$\left.\begin{array}{l} P_3 = 1.6\text{MPa} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_3 = h_{g@1.6\text{MPa}} = 275.33\text{kJ/kg} \\ s_3 = s_{g@1.6\text{MPa}} = 0.8982\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_4 = 0.7\text{MPa} \\ s_4 = s_3 \end{array}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{0.8982 - 0.3242}{0.9080 - 0.3242} = 0.983$$

$$h_4 = h_f + x_4 h_{fg} = 86.78 + (0.983)(175.07) = 258.87\text{kJ/kg}$$

Thus ,

$$q_{\text{in}} = h_3 - h_2 = 275.33 - 87.53 = 187.80\ \text{kJ/kg}$$
$$q_{\text{out}} = h_4 - h_1 = 258.87 - 86.78 = 172.09\ \text{kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 187.80 - 172.09 = 15.71\ \text{kJ/kg}$$

and

$$\eta_{th} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{15.71\ \text{kJ/kg}}{187.80\ \text{kJ/kg}} = \mathbf{8.4\%}$$

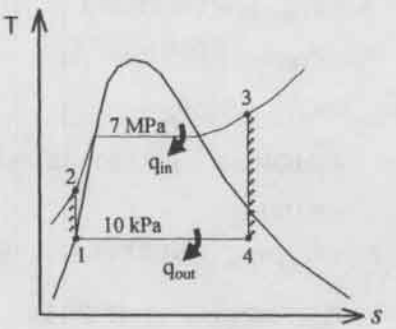
(*b*) $$\dot{W}_{\text{net}} = \dot{m} w_{\text{net}} = (6\text{kg/s})(15.71\text{kJ/kg}) = \mathbf{94.26\ kW}$$

8-109 A 30-MW steam power plant operates on a simple ideal Rankine cycle between the specified pressure limits. The thermal efficiency of the cycle, the mass flow rate of the steam, and the temperature rise of the cooling water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f@10\text{kPa}} = 191.83\text{kJ/kg}$$
$$v_1 = v_{f@10\text{kPa}} = 0.00101\text{m}^3/\text{kg}$$
$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.00101\text{m}^3/\text{kg})(7{,}000 - 10\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 7.06\text{kJ/kg}$$
$$h_2 = h_1 + w_{p,\text{in}} = 191.83 + 7.06 = 198.89\text{kJ/kg}$$



$$\left.\begin{matrix} P_3 = 7\text{MPa} \\ T_3 = 500°\text{C} \end{matrix}\right\} \begin{matrix} h_3 = 3410.5\text{kJ/kg} \\ s_3 = 6.7975\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_4 = 10\text{kPa} \\ s_4 = s_3 \end{matrix}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.7975 - 0.6493}{7.5009} = 0.820$$

$$h_4 = h_f + x_4 h_{fg} = 191.83 + (0.820)(2392.8) = 2153.93\text{kJ/kg}$$

Thus, $q_{\text{in}} = h_3 - h_2 = 3410.5 - 198.89 = 3211.61\text{ kJ/kg}$

$q_{\text{out}} = h_4 - h_1 = 2153.93 - 191.83 = 1962.10\text{ kJ/kg}$

$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 3211.61 - 1962.10 = 1249.51\text{ kJ/kg}$

and $$\eta_{th} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1249.51\text{ kJ/kg}}{3211.61\text{ kJ/kg}} = \mathbf{38.9\%}$$

(*b*) $$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{30{,}000\text{ kJ/s}}{1249.51\text{ kJ/kg}} = \mathbf{24.0}\text{ kg/s}$$

(*c*) The rate of heat rejection to the cooling water and its temperature rise are

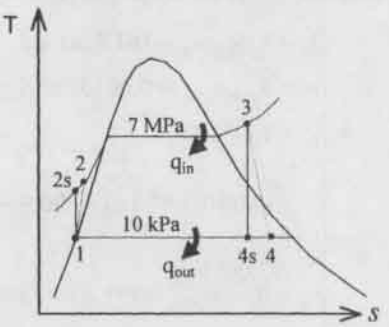
$$\dot{Q}_{\text{out}} = \dot{m} q_{\text{out}} = (24.0\text{kg/s})(1962.1\text{kJ/kg}) = 47{,}090\text{kJ/s}$$

$$\Delta T_{\text{cooling water}} = \frac{\dot{Q}_{\text{out}}}{(\dot{m}C)_{\text{cooling water}}} = \frac{47{,}090\text{kJ/s}}{(2000\text{kg/s})(4.18\text{kJ/kg}\cdot°\text{C})} = \mathbf{5.63°C}$$

8-110 A 30-MW steam power plant operates on a simple nonideal Rankine cycle between the specified pressure limits. The thermal efficiency of the cycle, the mass flow rate of the steam, and the temperature rise of the cooling water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),



$$h_1 = h_{f@10\text{kPa}} = 191.83\text{kJ/kg}$$

$$v_1 = v_{f@10\text{kPa}} = 0.00101\text{m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)/\eta_p$$
$$= (0.00101\text{m}^3/\text{kg})(7{,}000 - 10\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)/(0.87)$$
$$= 8.11\text{kJ/kg}$$

$$h_2 = h_1 + w_{p,\text{in}} = 191.83 + 8.11 = 199.94\text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 7\text{MPa} \\ T_3 = 500°\text{C} \end{matrix}\right\} \begin{matrix} h_3 = 3410.5\text{kJ/kg} \\ s_3 = 6.7975\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_4 = 10\text{kPa} \\ s_4 = s_3 \end{matrix}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.7975 - 0.6493}{7.5009} = 0.820$$

$$h_4 = h_f + x_4 h_{fg} = 191.83 + (0.820)(2392.8) = 2153.93\text{kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T(h_3 - h_{4s})$$
$$= 3410.5 - (0.87)(3410.5 - 2153.93) = 2317.28\text{kJ/kg}$$

Thus, $q_{\text{in}} = h_3 - h_2 = 3410.5 - 199.94 = 3210.56\text{ kJ/kg}$

$q_{\text{out}} = h_4 - h_1 = 2317.28 - 191.83 = 2125.45\text{ kJ/kg}$

$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 3210.56 - 2125.45 = 1085.11\text{ kJ/kg}$

and $$\eta_{th} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1085.11\text{ kJ/kg}}{3210.56\text{ kJ/kg}} = \mathbf{33.8\%}$$

(*b*) $$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{30{,}000\text{ kJ/s}}{1085.11\text{ kJ/kg}} = 27.65\text{ kg/s}$$

(*c*) The rate of heat rejection to the cooling water and its temperature rise are

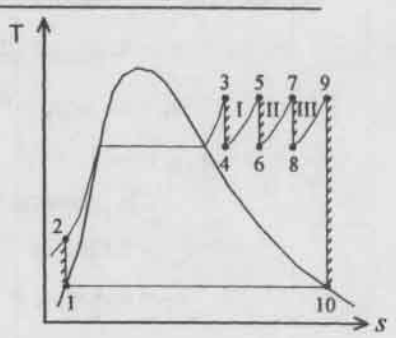
$$\dot{Q}_{\text{out}} = \dot{m}q_{\text{out}} = (27.65\text{kg/s})(2125.45\text{kJ/kg}) = 58{,}762\text{kJ/s}$$

$$\Delta T_{\text{cooling water}} = \frac{\dot{Q}_{\text{out}}}{(\dot{m}C)_{\text{cooling water}}} = \frac{52{,}762\text{kJ/s}}{(2000\text{kg/s})(4.18\text{kJ/kg}\cdot°\text{C})} = \mathbf{7.03°C}$$

The Reheat Rankine Cycle

8-111C The pump work remains the same, the moisture content decreases, everything else increases.

8-112C The T-s diagram of the ideal Rankine cycle with 3 stages of reheat is shown on the side. The cycle efficiency will increase as the number of reheating stages increases.

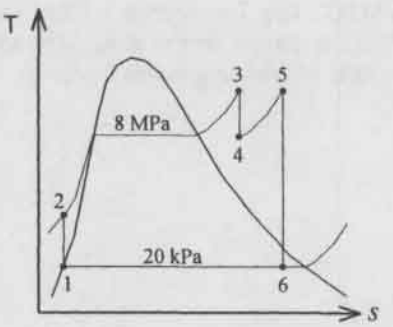


8-113C The thermal efficiency of the simple ideal Rankine cycle will probably be higher since the average temperature at which heat is added will be higher in this case.

8-114 A steam power plant that operates on the ideal reheat Rankine cycle is considered. The turbine work output and the thermal efficiency of the cycle are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),



$$h_1 = h_{f@20\text{kPa}} = 251.40\text{kJ/kg}$$

$$v_1 = v_{f@20\text{kPa}} = 0.001017\text{m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.001017\text{m}^3/\text{kg})(8{,}000 - 20\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 8.12\text{kJ/kg}$$

$$h_2 = h_1 + w_{p,\text{in}} = 251.40 + 8.12 = 259.52\text{kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 8\text{MPa} \\ T_3 = 500°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3398.3\text{kJ/kg} \\ s_3 = 6.7240\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_4 = 3\text{MPa} \\ s_4 = s_3 \end{array}\right\} h_4 = 3104.1\text{kJ/kg}$$

$$\left.\begin{array}{l} P_5 = 3\text{MPa} \\ T_5 = 500°\text{C} \end{array}\right\} \begin{array}{l} h_5 = 3456.5\text{kJ/kg} \\ s_5 = 7.2338\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_6 = 20\text{kPa} \\ s_6 = s_5 \end{array}\right\} \begin{array}{l} x_6 = \dfrac{s_6 - s_f}{s_{fg}} = \dfrac{7.2338 - 0.8320}{7.0766} = 0.9046 \\ h_6 = h_f + x_6 h_{fg} = 251.40 + (0.9046)(2358.3) = 2384.7\text{kJ/kg} \end{array}$$

The turbine work output and the thermal efficiency are determined from

$$w_{T,out} = (h_3 - h_4) + (h_5 - h_6) = 3398.3 - 3104.1 + 3456.5 - 2384.7 = \mathbf{1366\text{kJ/kg}}$$

and

$$q_{\text{in}} = (h_3 - h_2) + (h_5 - h_4) = 3398.3 - 259.52 + 3456.5 - 3104.1 = 3491.2\text{kJ/kg}$$

$$w_{\text{net}} = w_{T,out} - w_{p,\text{in}} = 1366 - 8.12 = 1357.88\ \text{kJ/kg}$$

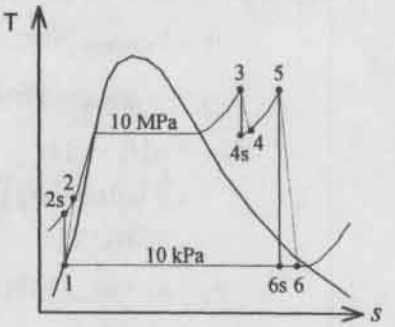
Thus,

$$\eta_{th} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1357.88\ \text{kJ/kg}}{3491.2\ \text{kJ/kg}} = \mathbf{38.9\%}$$

8-115 An 150-MW steam power plant that operates on a reheat Rankine cycle is considered. The quality (or temperature, if superheated) of the steam at the turbine exit, the thermal efficiency of the cycle, and the mass flow rate of the steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),



$$h_1 = h_{f@10\text{kPa}} = 191.83\text{kJ/kg}$$

$$v_1 = v_{f@10\text{kPa}} = 0.00101\text{m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)/\eta_p$$
$$= (0.00101\text{m}^3/\text{kg})(10{,}000 - 10\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)/(0.95)$$
$$= 10.62\text{kJ/kg}$$

$$h_2 = h_1 + w_{p,\text{in}} = 191.83 + 10.62 = 202.45\text{kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 10\text{MPa} \\ T_3 = 500°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3373.7\text{kJ/kg} \\ s_3 = 6.5966\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_{4s} = 1\text{MPa} \\ s_{4s} = s_3 \end{array}\right\} h_{4s} = 2782.5\text{kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T(h_3 - h_{4s})$$
$$= 3373.7 - (0.80)(3373.7 - 2782.5) = 2900.74\text{kJ/kg}$$

$$\left.\begin{array}{l} P_5 = 1\text{MPa} \\ T_5 = 500°\text{C} \end{array}\right\} \begin{array}{l} h_5 = 3478.5\text{kJ/kg} \\ s_5 = 7.7622\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_{6s} = 20\text{kPa} \\ s_{6s} = s_5 \end{array}\right\} \begin{array}{l} x_{6s} = \dfrac{s_{6s} - s_f}{s_{fg}} = \dfrac{7.7622 - 0.6493}{7.5009} = 0.948 \ (\text{at turbine exit}) \\ h_{6s} = h_f + x_{6s}h_{fg} = 191.83 + (0.948)(2392.8) = 2460.2\text{kJ/kg} \end{array}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_6 = h_5 - \eta_T(h_5 - h_{6s})$$
$$= 3478.5 - (0.80)(3478.5 - 2460.2)$$
$$= 2663.86\text{kJ/kg} > h_g \ (\text{superheated vapor})$$

From steam tables at 10 kPa we read $T_6 = \mathbf{87.5°C}$.

(*b*) $$w_{T,out} = (h_3 - h_4) + (h_5 - h_6) = 3373.7 - 2900.74 + 3478.5 - 2663.86 = 1287.6\text{kJ/kg}$$

$$q_{\text{in}} = (h_3 - h_2) + (h_5 - h_4) = 3373.7 - 202.45 + 3478.5 - 2900.74 = 3749.0\text{kJ/kg}$$

$$w_{\text{net}} = w_{T,out} - w_{p,\text{in}} = 1287.6 - 10.62 = 1277.0\text{kJ/kg}$$

Thus the thermal efficiency is $$\eta_{th} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1277.0\text{ kJ/kg}}{3749.0\text{ kJ/kg}} = \mathbf{34.1\%}$$

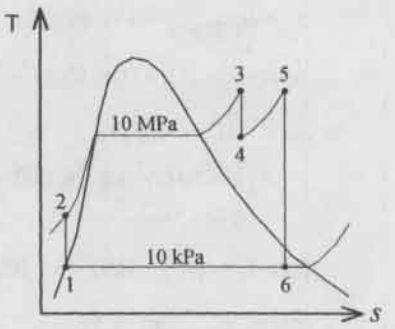
(*c*) The mass flow rate of the steam is

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{150{,}000\text{ kJ/s}}{1277.0\text{ kJ/kg}} = \mathbf{117.5\text{ kg/s}}$$

8-116 An 150-MW steam power plant that operates on the ideal reheat Rankine cycle is considered. The quality (or temperature, if superheated) of the steam at the turbine exit, the thermal efficiency of the cycle, and the mass flow rate of the steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),



$$h_1 = h_{f@10\text{kPa}} = 191.83\text{kJ/kg}$$

$$v_1 = v_{f@10\text{kPa}} = 0.0010\,1\text{m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.0010\,1\text{m}^3/\text{kg})(10{,}000 - 10\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 10.09\text{kJ/kg}$$

$$h_2 = h_1 + w_{p,\text{in}} = 191.83 + 10.09 = 201.92\text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 10\text{MPa} \\ T_3 = 500°\text{C} \end{matrix}\right\} \begin{matrix} h_3 = 3373.7\text{kJ/kg} \\ s_3 = 6.5966\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_4 = 1\text{MPa} \\ s_4 = s_3 \end{matrix}\right\} h_4 = 2782.5\text{kJ/kg}$$

$$\left.\begin{matrix} P_5 = 1\text{MPa} \\ T_5 = 500°\text{C} \end{matrix}\right\} \begin{matrix} h_5 = 3478.5\text{kJ/kg} \\ s_5 = 7.7622\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_6 = 10\text{kPa} \\ s_6 = s_5 \end{matrix}\right\} \begin{matrix} x_6 = \dfrac{s_6 - s_f}{s_{fg}} = \dfrac{7.7622 - 0.6493}{7.5009} = \mathbf{0.948}\ (\text{atturbineexit}) \\ h_6 = h_f + x_6 h_{fg} = 191.83 + (0.948)(2392.8) = 2460.2\text{kJ/kg} \end{matrix}$$

(*b*) $$w_{T,out} = (h_3 - h_4) + (h_5 - h_6) = 3373.7 - 2782.54 + 3478.5 - 2460.2 = 1609.5\text{kJ/kg}$$

$$q_{\text{in}} = (h_3 - h_2) + (h_5 - h_4) = 3373.7 - 201.92 + 3478.5 - 2782.5 = 3867.78\text{kJ/kg}$$

$$w_{\text{net}} = w_{T,out} - w_{p,\text{in}} = 1609.5 - 10.09 = 1599.41\text{kJ/kg}$$

Thus the thermal efficiency is

$$\eta_{th} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1599.41\text{ kJ/kg}}{3867.78\text{ kJ/kg}} = \mathbf{41.4\%}$$

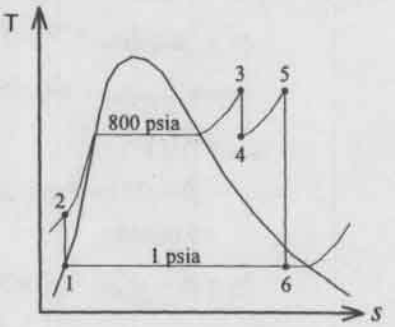
(c) The mass flow rate of the steam is

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{150{,}000\text{ kJ/s}}{1599.41\text{ kJ/kg}} = \mathbf{93.8\ kg/s}$$

8-117E A steam power plant that operates on the ideal reheat Rankine cycle is considered. The pressure at which reheating takes place, the net power output, the thermal efficiency, and the minimum mass flow rate of the cooling water required are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4E, A-5E, and A-6E),



$$h_1 = h_{\text{sat@1psia}} = 69.74 \text{ Btu/lbm}$$

$$v_1 = v_{\text{sat@1psia}} = 0.016136 \text{ft}^3/\text{lbm}$$

$$T_1 = T_{\text{sat@1psia}} = 101.70°\text{F}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.016136 \text{ft}^3/\text{lbm})(800 - 1 \text{psia})\left(\frac{1 \text{Btu}}{5.4039 \text{psia} \cdot \text{ft}^3}\right)$$
$$= 2.39 \text{Btu/lbm}$$

$$h_2 = h_1 + w_{p,\text{in}} = 69.74 + 2.39 = 72.13 \text{Btu/lbm}$$

$$\left.\begin{matrix} P_3 = 800 \text{psia} \\ T_3 = 900°\text{F} \end{matrix}\right\} \begin{matrix} h_3 = 1455.6 \text{Btu/lbm} \\ s_3 = 1.6408 \text{Btu/lbm} \cdot R \end{matrix}$$

$$\left.\begin{matrix} s_4 = s_3 \\ (\text{sat.vapor}) \end{matrix}\right\} \begin{matrix} h_4 = h_{g@s_g = s_4} = 1178.9 \text{Btu/lbm} \\ P_4 = P_{\text{sat@}s_g = s_4} = \mathbf{62.81 psia} \end{matrix}$$

$$\left.\begin{matrix} P_5 = 62.81 \text{psia} \\ T_5 = 800°\text{F} \end{matrix}\right\} \begin{matrix} h_5 = 1431.1 \text{Btu/lbm} \\ s_5 = 1.8977 \text{Btu/lbm} \cdot \text{R} \end{matrix}$$

$$\left.\begin{matrix} P_6 = 1 \text{psia} \\ s_6 = s_5 \end{matrix}\right\} \begin{matrix} x_6 = \dfrac{s_6 - s_f}{s_{fg}} = \dfrac{1.8977 - 0.13266}{1.8453} = 0.9565 \\ h_6 = h_f + x_6 h_{fg} = 69.74 + (0.9565)(1036) = 1060.7 \text{Btu/lbm} \end{matrix}$$

(*b*) $$q_{\text{in}} = (h_3 - h_2) + (h_5 - h_4) = 1455.6 - 72.13 + 1431.1 - 1178.9 = 1635.7 \text{Btu/lbm}$$

$$q_{\text{out}} = h_6 - h_1 = 1060.7 - 69.74 = 991.0 \text{Btu/lbm}$$

Thus,

$$\eta_{th} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{991.0 \text{ Btu/lbm}}{1635.7 \text{ Btu/lbm}} = \mathbf{39.4\%}$$

(*c*) The mass flow rate of the cooling water will be minimum when it is heated to the temperature of the steam in the condenser which is 101.7°F,

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{net}} = (1 - \eta_{th})\dot{Q}_{\text{in}} = (1 - 0.394)(6 \times 10^4 \text{ Btu/s}) = 3.636 \times 10^4 \text{ Btu/s}$$

$$\dot{m}_{\text{cool}} = \frac{\dot{Q}_{\text{out}}}{C\Delta T} = \frac{3.636 \times 10^4 \text{ Btu/s}}{(1.0 \text{Btu/lbm} \cdot °\text{F})(101.7 - 45)°\text{F}} = \mathbf{641.3 lbm/s}$$

8-118 A steam power plant that operates on an ideal reheat Rankine cycle between the specified pressure limits is considered. The pressure at which reheating takes place, the total rate of heat input in the boiler, and the thermal efficiency of the cycle are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

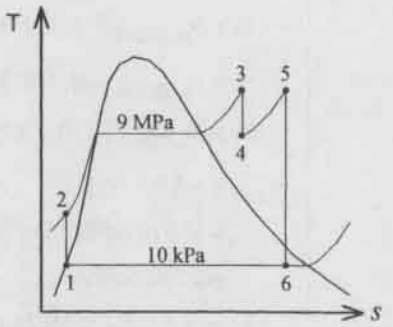
Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{\text{sat@10kPa}} = 191.83\text{kJ/kg}$$

$$v_1 = v_{\text{sat@10kPa}} = 0.00101\text{m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.00101\text{m}^3/\text{kg})(9{,}000 - 10\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 9.08\text{kJ/kg}$$

$$h_2 = h_1 + w_{p,\text{in}} = 191.83 + 9.08 = 200.91\text{kJ/kg}$$



$$\left.\begin{aligned} P_3 &= 9\text{MPa} \\ T_3 &= 500°\text{C} \end{aligned}\right\} \begin{aligned} h_3 &= 3386.1\text{kJ/kg} \\ s_3 &= 6.6576\text{kJ/kg}\cdot\text{K} \end{aligned}$$

$$\left.\begin{aligned} P_6 &= 7.5\text{kPa} \\ s_6 &= s_5 \end{aligned}\right\} \begin{aligned} h_6 &= h_f + x_6 h_{fg} = 191.83 + (0.90)(2392.9) = 2345.4\text{kJ/kg} \\ s_6 &= s_f + x_6 s_{fg} = 0.6493 + (0.90)(7.5009) = 7.4001\text{kJ/kg}\cdot\text{K} \end{aligned}$$

$$\left.\begin{aligned} T_5 &= 500°\text{C} \\ s_5 &= s_6 \end{aligned}\right\} \begin{aligned} P_5 &= \mathbf{2.146MPa}\ (\text{the reheat pressure}) \\ h_5 &= 3466.0\text{kJ/kg} \end{aligned}$$

$$\left.\begin{aligned} P_4 &= 2.146\text{MPa} \\ s_4 &= s_3 \end{aligned}\right\} h_4 = 2979.5\text{kJ/kg}$$

(*b*) The rate of heat supply is

$$\dot{Q}_{\text{in}} = \dot{m}[(h_3 - h_2) + (h_5 - h_6)]$$
$$= (25\text{kJ/s})(3386.1 - 200.91 + 3466 - 2345.4)\text{kJ/kg} = \mathbf{107{,}620\text{kJ/s}}$$

(*c*) The thermal efficiency is determined from

$$\dot{Q}_{\text{out}} = \dot{m}(h_6 - h_1) = (25\text{kJ/s})(23454 - 191.83)\text{kJ/kg} = 53{,}839\text{kJ/s}$$

Thus,

$$\eta_{th} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{53{,}839\text{kJ/s}}{107{,}620\text{kJ/s}} = \mathbf{50.0\%}$$

The Reversed Carnot Cycle

8-119C Because the compression process involves the compression of a liquid-vapor mixture which requires a compressor that will handle two phases, and the expansion process involves the expansion of high-moisture content refrigerant.

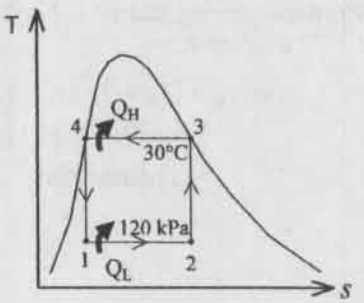
8-120C Refrigerators and heat pumps are essentially the same devices with different objectives. The objective of a heat pump is to maintain a heated space at a high temperature whereas a refrigerator is used to maintain a cooled space at a low temperature.

8-121 A steady-flow Carnot refrigeration cycle with refrigerant-134a as the working fluid is considered. The coefficient of performance, the amount of heat absorbed from the refrigerated space, and the net work input are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) Noting that $T_H = 30°C = 303$ K and $T_L = T_{sat\ @\ 120\ kPa} = -25.74°C = 247.3$ K, the COP of this Carnot refrigerator is determined from

$$COP_{R,C} = \frac{1}{T_H / T_L - 1} = \frac{1}{(300\text{K})/(247.3\text{K}) - 1} = \mathbf{4.44}$$



(b) From the refrigerant tables (Table A-11),

$$h_3 = h_{g@30°C} = 263.50\ \text{kJ/kg}$$
$$h_4 = h_{f@30°C} = 91.49\ \text{kJ/kg}$$

Thus,

$$q_H = h_3 - h_4 = 263.50 - 91.49 = 172.01\text{kJ/kg}$$

and

$$\frac{q_H}{q_L} = \frac{T_H}{T_L} \longrightarrow q_L = \frac{T_L}{T_H} q_H = \left(\frac{247.3\text{K}}{303\text{K}}\right)(172.01\text{kJ/kg})$$
$$= \mathbf{140.4kJ/kg}$$

(*c*) The net work input is determined from

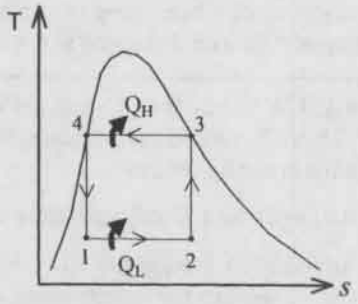
$$w_{net} = q_H - q_L = 172.01 - 140.4 = \mathbf{31.61\ kJ/kg}$$

8-122E A steady-flow Carnot refrigeration cycle with refrigerant-134a as the working fluid is considered. The coefficient of performance, the quality at the beginning of the heat-absorption process, and the net work input are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (a) Noting that $T_H = T_{sat\ @\ 90\ psia} = 73.79°F = 533.8\ R$ and $T_L = T_{sat\ @\ 30\ psia} = 11.11°F = 470.1\ R$,

$$COP_{R,C} = \frac{1}{T_H / T_L - 1} = \frac{1}{(533.8R)/(470.1R) - 1} = \mathbf{7.38}$$



(*b*) Process 4-1 is isentropic, and thus

$$s_1 = s_4 = (s_f + x_4 s_{fg})_{@90psia} = 0.0729 + (0.05)(0.2172 - 0.0729)$$
$$= 0.0801 Btu/lbm \cdot R$$

$$x_1 = \left(\frac{s_1 - s_f}{s_{fg}}\right)_{@30psia} = \frac{0.0801 - 0.0364}{0.2209 - 0.0364} = \mathbf{0.237}$$

(*c*) Remembering that on a T-s diagram the area enclosed represents the net work, and $s_3 = s_{sat\ @\ 90\ psia} = 0.2172\ Btu/lbm \cdot R$,

$$w_{net,in} = (T_H - T_L)(s_3 - s_4)$$
$$= [73.79 - (-11.11)R](0.2172 - 0.0801) Btu/lbm \cdot R$$
$$= \mathbf{11.64 Btu/lbm}$$

Ideal and Actual Vapor-Compression Cycles

8-123C Yes; the throttling process is an internally irreversible process.

8-124C To make the ideal vapor-compression refrigeration cycle more closely approximate the actual cycle.

8-125C No. Assuming the water is maintained at 10°C in the evaporator, the evaporator pressure will be the saturation pressure corresponding to this pressure, which is 1.2 kPa. It is not practical to design refrigeration or air-conditioning devices which involve such extremely low pressures.

8-126C Allowing a temperature difference of 10°C for effective heat transfer, the condensation temperature of the refrigerant should be 25°C. The saturation pressure corresponding to 25°C is 0.65 MPa. Therefore, the recommended pressure would be 0.7 MPa.

8-127C The area enclosed by the cyclic curve on a T-s diagram represents the net work input for the reversed Carnot cycle, but not so for the ideal vapor-compression refrigeration cycle. This is because the latter cycle involves an irreversible process for which the process path is not known.

8-128C The cycle that involves saturated liquid at 30°C will have a higher COP because, judging from the T-s diagram, it will require a smaller work input for the same refrigeration capacity.

8-129C The minimum temperature that the refrigerant can be cooled to before throttling is the temperature of the sink (the cooling medium) since heat is transferred from the refrigerant to the cooling medium.

8-130 An ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The rate of heat removal from the refrigerated space, the power input to the compressor, the rate of heat rejection to the environment, and the COP are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

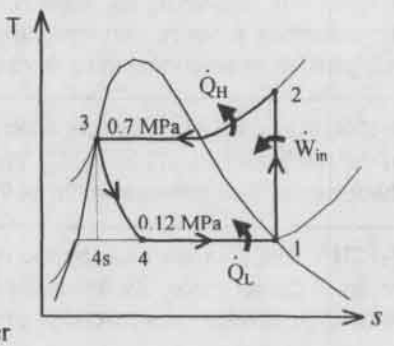
Analysis (a) In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{array}{l} P_1 = 120\text{kPa} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@120\text{kPa}} = 233.86\text{kJ/kg} \\ s_1 = s_{g@120\text{kPa}} = 0.9354\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 0.7\text{MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 270.22\text{kJ/kg}\ (T_2 = 34.6°\text{C})$$

$$\left.\begin{array}{l} P_3 = 0.7\text{MPa} \\ \text{sat.liquid} \end{array}\right\} h_3 = h_{f@0.7\text{MPa}} = 86.78\text{kJ/kg}$$

$$h_4 \cong h_3 = 86.78\text{kJ/kg}\ (\text{throttling})$$



Then the rate of heat removal from the refrigerated space and the power input to the compressor are determined from

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.05\text{kg/s})(233.86 - 86.78)\text{kJ/kg} = \mathbf{7.35kW}$$

and

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.05\text{kg/s})(270.22 - 233.86)\text{kJ/kg} = \mathbf{1.82kW}$$

(*b*) The rate of heat rejection to the environment is determined from

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 7.35 + 1.82 = \mathbf{9.17\ kW}$$

(*c*) The COP of the refrigerator is determined from its definition,

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.35\text{ kW}}{1.82\text{ kW}} = \mathbf{4.04}$$

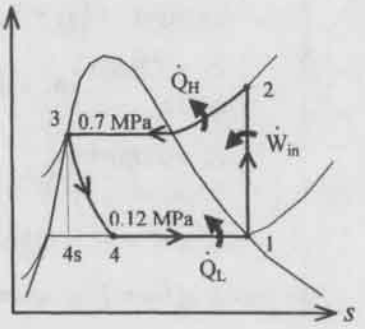
8-131 An ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The throttling valve in the cycle is replaced by an isentropic turbine. The percentage increase in the COP and in the rate of heat removal from the refrigerated space due to this replacement are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis If the throttling valve in the previous problem is replaced by an isentropic turbine, we would have $s_{4s} = s_1 = s_{f\,@\,0.7\,MPa} = 0.3242$ kJ/kg·K, and the enthalpy at the turbine exit would be

$$x_{4s} = \left(\frac{s_1 - s_f}{s_{fg}}\right)_{@120kPa} = \frac{0.3242 - 0.0879}{0.9354 - 0.0879} = 0.279$$

$$h_{4s} = \left(h_f + x_{4s} h_{fg}\right)_{@120kPa} = 21.32 + (0.279)(212.54) = 80.62\text{kJ/kg}$$



Then,

$$\dot{Q}_L = \dot{m}(h_1 - h_{4s}) = (0.05\text{kg/s})(233.86 - 80.62)\text{kJ/kg} = 7.66\text{kW}$$

and

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.66\text{kW}}{1.82\text{kW}} = 4.21$$

Then the percentage increase in $\dot{Q}$ and COP becomes

$$\text{Increase in } \dot{Q}_L = \frac{\Delta \dot{Q}_L}{\dot{Q}_L} = \frac{7.66 - 7.35}{7.35} = \mathbf{4.2\%}$$

$$\text{Increase in COP}_R = \frac{\Delta \text{COP}_R}{\text{COP}_R} = \frac{4.21 - 4.04}{4.04} = \mathbf{4.2\%}$$

8-132 An ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The quality of the refrigerant at the end of the throttling process, the COP, and the power input to the compressor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

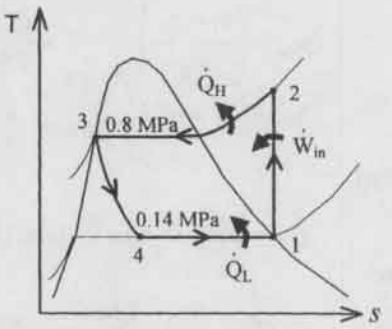
Analysis (a) In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{array}{l} P_1 = 140\text{kPa} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@140\text{kPa}} = 236.04\text{kJ/kg} \\ s_1 = s_{g@140\text{kPa}} = 0.9322\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 0.8\text{MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 272.05\text{kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 0.8\text{MPa} \\ \text{sat.liquid} \end{array}\right\} h_3 = h_{f@0.8\text{MPa}} = 93.42\text{kJ/kg}$$

$$h_4 \cong h_3 = 93.42\text{kJ/kg (throttling)}$$



The quality of the refrigerant at the end of the throttling process is

$$x_4 = \left(\frac{h_4 - h_f}{h_{fg}}\right)_{@140\text{kPa}} = \frac{93.42 - 25.77}{210.27} = \mathbf{0.322}$$

(b) The COP of the refrigerator is determined from its definition,

$$\text{COP}_R = \frac{q_L}{w_{in}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{236.04 - 93.42}{272.05 - 236.04} = \mathbf{3.96}$$

(c) The power input to the compressor is determined from

$$\dot{W}_{in} = \frac{\dot{Q}_L}{\text{COP}_R} = \frac{5\text{ kW}}{3.96} = \mathbf{1.26\text{ kW}}$$

8-133 A nonideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The quality of the refrigerant at the end of the throttling process, the COP, and the power input to the compressor are to be determined. √

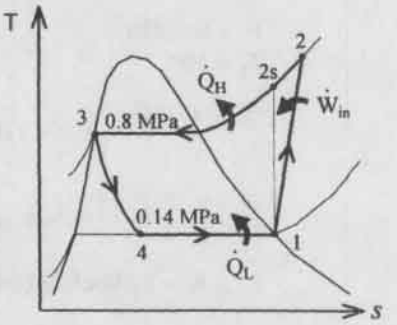
Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) The refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{array}{l} P_1 = 140\text{kPa} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@140\text{kPa}} = 236.04\text{kJ/kg} \\ s_1 = s_{g@140\text{kPa}} = 0.9322\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 0.8\text{MPa} \\ s_{2s} = s_1 \end{array}\right\} h_{2s} = 272.05\text{kJ/kg}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow h_2 = h_1 + (h_{2s} - h_1)/\eta_C$$
$$= 236.04 + (272.05 - 236.04)/(0.85)$$
$$= 278.40\ \text{kJ/kg}$$



$$\left.\begin{array}{l} P_3 = 0.8\text{MPa} \\ \text{sat.liquid} \end{array}\right\} h_3 = h_{f@0.8MPa} = 93.42\ \text{kJ/kg}$$

$$h_4 \cong h_3 = 93.42\text{kJ/kg}\ (\text{throttling})$$

The quality of the refrigerant at the end of the throttling process is

$$x_4 = \left(\frac{h_4 - h_f}{h_{fg}}\right)_{@140\text{kPa}} = \frac{93.42 - 25.77}{210.27} = \mathbf{0.322}$$

(*b*) The COP of the refrigerator is determined from its definition,

$$\text{COP}_R = \frac{q_L}{w_{in}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{236.04 - 93.42}{278.40 - 236.04} = \mathbf{3.37}$$

(*c*) The power input to the compressor is determined from

$$\dot{W}_{in} = \frac{\dot{Q}_L}{\text{COP}_R} = \frac{5\text{kW}}{3.37} = \mathbf{1.48\ kW}$$

8-134 A refrigerator with refrigerant-134a as the working fluid is considered. The rate of heat removal from the refrigerated space, the power input to the compressor, the adiabatic efficiency of the compressor, and the COP of the refrigerator are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (a) From the refrigerant tables (Tables A-12 and A-13),

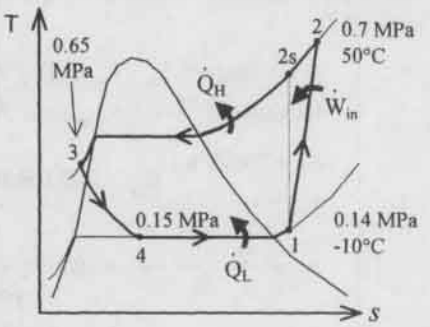
$$\left.\begin{matrix} P_1 = 0.14\text{MPa} \\ T_1 = -10°\text{C} \end{matrix}\right\} \begin{matrix} h_1 = 243.40\text{kJ/kg} \\ s_1 = 0.9606\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_2 = 0.7\text{MPa} \\ T_2 = 50°\text{C} \end{matrix}\right\} h_2 = 286.35\text{kJ/kg}$$

$$\left.\begin{matrix} P_{2s} = 0.7\text{MPa} \\ s_{2s} = s_1 \end{matrix}\right\} h_{2s} = 278.06\text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 0.65\text{MPa} \\ T_3 = 24°\text{C} \end{matrix}\right\} h_3 = h_{f@24°\text{C}} = 82.90\text{kJ/kg}$$

$$h_4 \cong h_3 = 82.90\text{kJ/kg}\ (\text{throttling})$$



Then the rate of heat removal from the refrigerated space and the power input to the compressor are determined from

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.04\text{kg/s})(243.40 - 82.90)\text{kJ/kg} = \mathbf{6.42kW}$$

and

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.04\text{kg/s})(286.35 - 243.40)\text{kJ/kg} = \mathbf{1.72kW}$$

(b) The adiabatic efficiency of the compressor is determined from

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{278.06 - 243.40}{286.35 - 243.40} = \mathbf{80.7\%}$$

(c) The COP of the refrigerator is determined from its definition,

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{6.42\text{ kW}}{1.72\text{ kW}} = \mathbf{3.73}$$

8-135E An ice-making machine operates on the ideal vapor-compression refrigeration cycle, using refrigerant-134a as the working fluid. The power input to the ice machine is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

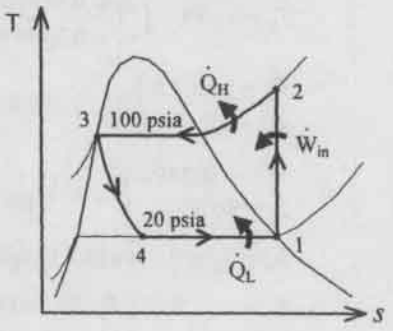
Analysis In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{array}{l} P_1 = 20\text{psia} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@20\text{psia}} = 101.39\text{Btu/lbm} \\ s_1 = s_{g@20\text{psia}} = 0.2227\text{Btu/lbm}\cdot\text{R} \end{array}$$

$$\left.\begin{array}{l} P_2 = 100\text{psia} \\ s_2 = s_1 \end{array}\right\} h_2 = 115.64\text{Btu/lbm}$$

$$\left.\begin{array}{l} P_3 = 100\text{psia} \\ \text{sat.liquid} \end{array}\right\} h_3 = h_{f@100\text{psia}} = 36.99\text{Btu/lbm}$$

$$h_4 \cong h_3 = 36.99\text{Btu/lbm (throttling)}$$



The cooling load of this refrigerator is

$$\dot{Q}_L = \dot{m}_{ice}(\Delta h)_{ice} = (20/3600\text{lbm/s})(169\text{Btu/lbm}) = 0.9389\text{Btu/s}$$

Then the mass flow rate of the refrigerant and the power input become

$$\dot{m}_R = \frac{\dot{Q}_L}{h_1 - h_4} = \frac{0.9389\text{Btu/s}}{(101.39 - 36.99)\text{Btu/lbm}} = 0.0146\text{lbm/s}$$

and

$$\dot{W}_{in} = \dot{m}_R(h_2 - h_1) = (0.0146\text{lbm/s})(115.64 - 101.39)\text{Btu/lbm}\left(\frac{1\text{hp}}{0.7068\text{Btu/s}}\right) = \mathbf{0.294hp}$$

8-136 A refrigerator with refrigerant-134a as the working fluid is considered. The power input to the compressor, the rate of heat removal from the refrigerated space, and the pressure drop and the rate of heat gain in the line between the evaporator and the compressor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the refrigerant tables (Tables A-12 and A-13),

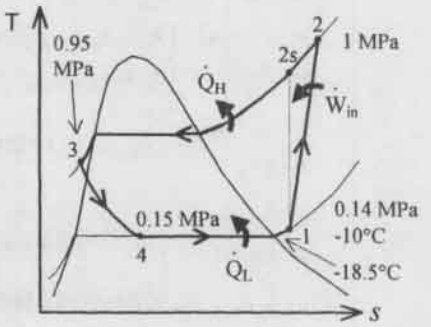
$$\left.\begin{matrix} P_1 = 140\text{kPa} \\ T_1 = -10°\text{C} \end{matrix}\right\} \begin{matrix} h_1 = 243.40\text{kJ/kg} \\ s_1 = 0.9606\text{kJ/kg}\cdot\text{K} \\ v_1 = 0.14549\text{m}^3\text{/kg} \end{matrix}$$

$$\left.\begin{matrix} P_2 = 1.0\text{MPa} \\ s_{2s} = s_1 \end{matrix}\right\} h_{2s} = 286.04\text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 0.95\text{MPa} \\ T_3 = 30°\text{C} \end{matrix}\right\} h_3 \cong h_{f@30°\text{C}} = 91.49\text{kJ/kg}$$

$$h_4 \cong h_3 = 91.49\text{kJ/kg (throttling)}$$

$$\left.\begin{matrix} T_5 = -18.5°\text{C} \\ \text{sat.vapor} \end{matrix}\right\} \begin{matrix} P_5 = 0.14287\text{MPa} \\ h_5 = 236.23\text{kJ/kg} \end{matrix}$$



Then the mass flow rate of the refrigerant and the power input becomes

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{0.2/60\text{m}^3\text{/s}}{0.14549\text{m}^3\text{/kg}} = 0.0229\text{kg/s}$$

$$\dot{W}_{\text{in}} = \dot{m}(h_{2s} - h_1)/\eta_C = (0.0229\text{kg/s})[(286.04 - 243.40)\text{kJ/kg}]/(0.78) = \mathbf{1.25kW}$$

(*b*) The rate of heat removal from the refrigerated space is

$$\dot{Q}_L = \dot{m}(h_5 - h_4) = (0.0229\text{kg/s})(236.23 - 91.49)\text{kJ/kg} = \mathbf{3.31kW}$$

(*c*) The pressure drop and the heat gain in the line between the evaporator and the compressor are

$$\Delta P = P_5 - P_1 = 142.87 - 140 = \mathbf{2.87kPa}$$

and

$$\dot{Q}_{\text{gain}} = \dot{m}(h_1 - h_5) = (0.0229\text{kg/s})(243.40 - 236.23)\text{kJ/kg} = \mathbf{0.164kW}$$

Heat Pump Systems

8-137C Heat pumps reduce the heating costs significantly, and they function as air-conditioners in summer. The disadvantage of the heat pumps is their high initial cost.

8-138C A heat pump system is more cost effective in Miami because of the low heating loads and high cooling loads at that location.

8-139C A water-source heat pump extracts heat from water instead of air. Water-source heat pumps have higher COPs than the air-source systems because the temperature of water is higher than the temperature of air in winter.

8-140E A heat pump that operates on the ideal vapor-compression cycle with refrigerant-134a is considered. The power input to the heat pump and the electric power saved by using a heat pump instead of a resistance heater are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

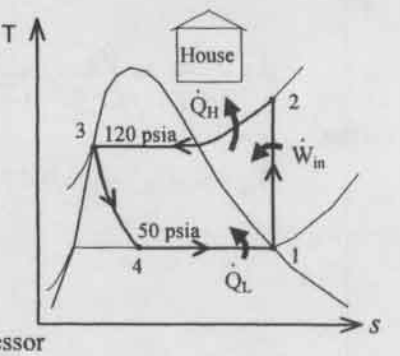
Analysis In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12E and A-13E),

$$\left.\begin{array}{l} P_1 = 50\text{psia} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@50\text{psia}} = 107.43\text{Btu/lbm} \\ s_1 = s_{g@50\text{psia}} = 0.2189\text{Btu/lbm}\cdot\text{R} \end{array}$$

$$\left.\begin{array}{l} P_2 = 120\text{psia} \\ s_2 = s_1 \end{array}\right\} h_2 = 115.15\text{Btu/lbm}$$

$$\left.\begin{array}{l} P_3 = 120\text{psia} \\ \text{sat.liquid} \end{array}\right\} h_3 = h_{f@120\text{psia}} = 40.91\text{Btu/lbm}$$

$$h_4 \cong h_3 = 40.91\text{Btu/lbm (throttling)}$$



The mass flow rate of the refrigerant and the power input to the compressor are determined from

$$\dot{m} = \frac{\dot{Q}_H}{q_H} = \frac{\dot{Q}_H}{h_2 - h_3} = \frac{90{,}000/3{,}600\text{Btu/s}}{(115.15 - 40.91)\text{Btu/lbm}} = 0.337\text{lbm/s}$$

and

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = (0.337\text{kg/s})(115.15 - 107.43)\text{Btu/lbm} = 2.60\text{Btu/s}$$
$$= \mathbf{3.68hp} \text{ since } 1\text{ hp} = 0.7068\text{Btu/s}$$

The electrical power required without the heat pump is

$$\dot{W}_e = \dot{Q}_H = (90{,}000/3{,}600\text{kJ/s})\left(\frac{1\text{hp}}{0.7068\text{Btu/s}}\right) = 35.37\text{hp}$$

Thus,

$$\dot{W}_{\text{saved}} = \dot{W}_e - \dot{W}_{\text{in}} = 35.37 - 3.68 = \mathbf{31.69hp}$$
$$= 23.6kW \text{ since } 1\text{ hp} = 0.7457\text{ kW}$$

8-141 A heat pump that operates on the ideal vapor-compression cycle with refrigerant-134a is considered. The power input to the heat pump is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

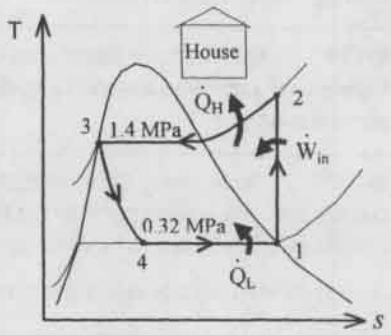
Analysis In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{array}{l} P_1 = 320\text{kPa} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@320\text{kPa}} = 248.66\text{kJ/kg} \\ s_1 = s_{g@320\text{kPa}} = 0.9177\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 1.4\text{MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 279.14\text{kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 1.4\text{MPa} \\ \text{sat.liquid} \end{array}\right\} h_3 = h_{f@1.4\text{MPa}} = 125.26\text{kJ/kg}$$

$$h_4 \cong h_3 = 125.26\text{kJ/kg}\ (\text{throttling})$$



The heating load of this heat pump is determined from

$$\dot{Q}_H = [\dot{m}C(T_2 - T_1)]_{\text{water}} = (0.18\text{kg/s})(4.18\text{kJ/kg}\cdot{}^\circ\text{C})(54-15)^\circ\text{C} = 29.34\text{kW}$$

and

$$\dot{m}_R = \frac{\dot{Q}_H}{q_H} = \frac{\dot{Q}_H}{h_2 - h_3} = \frac{29.34\text{kJ/s}}{(279.14 - 125.26)\text{kJ/kg}} = 0.191\text{kg/s}$$

Then,

$$\dot{W}_{\text{in}} = \dot{m}_R(h_2 - h_1) = (0.191\text{kg/s})(279.14 - 248.66)\text{kJ/kg} = \mathbf{5.82kW}$$

8-142 A heat pump with refrigerant-134a as the working fluid heats a house by using underground water as the heat source. The power input to the heat pump, the rate of heat absorption from the water, and the increase in electric power input if an electric resistance heater is used instead of a heat pump are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

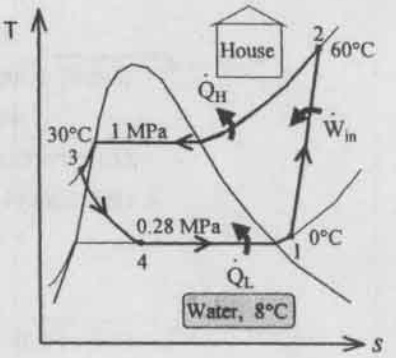
Analysis (*a*) From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{matrix} P_1 = 280\text{kPa} \\ T_1 = 0°\text{C} \end{matrix}\right\} h_1 = 247.64\text{kJ/kg}$$

$$\left.\begin{matrix} P_2 = 1.0\text{MPa} \\ T_2 = 60°\text{C} \end{matrix}\right\} h_2 = 291.36\text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 1.0\text{MPa} \\ T_3 = 30°\text{C} \end{matrix}\right\} h_3 \cong h_{f@30°\text{C}} = 91.49\text{kJ/kg}$$

$$h_4 \cong h_3 = 91.49\text{kJ/kg (throttling)}$$



The mass flow rate of the refrigerant is

$$\dot{m}_R = \frac{\dot{Q}_H}{q_H} = \frac{\dot{Q}_H}{h_2 - h_3} = \frac{60{,}000/3{,}600\text{kJ/s}}{(291.36 - 91.49)\text{kJ/kg}} = 0.0834\text{kg/s}$$

Then the power input to the compressor becomes

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = (0.0834\text{kg/s})(291.36 - 247.64)\text{kJ/kg} = \mathbf{3.65kW}$$

(*b*) The rate of hat absorption from the water is

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.0834\text{kg/s})(247.64 - 91.49)\text{kJ/kg} = \mathbf{13.02kW}$$

(*c*) The electrical power required without the heat pump is

$$\dot{W}_e = \dot{Q}_H = 60{,}000/3{,}600\text{ kJ/s} = 16.67\text{ kW}$$

Thus,

$$\dot{W}_{\text{increase}} = \dot{W}_e - \dot{W}_{\text{in}} = 16.67 - 3.65 = \mathbf{13.02\text{ kW}}$$

Review Problems

8-143 A turbocharged four-stroke V-16 diesel engine produces 4000 hp at 1050 rpm. The amount of power produced per cylinder per mechanical and per thermodynamic cycle is to be determined.

Analysis Noting that there are 16 cylinders and each thermodynamic cycle corresponds to 2 mechanical cycles, we have

$$w_{\text{mechanical}} = \frac{\text{Total power produced}}{(\text{No. of cylinders})(\text{No. of mechanical cycles})}$$

$$= \frac{4000\text{hp}}{(16\text{ cylinders})(1050\text{ rev/min})}\left(\frac{42.41\text{ Btu/min}}{1\text{ hp}}\right)$$

$$= \mathbf{10.1\ Btu/cyl \cdot mech\ cycle} \quad (= 10.7\text{ kJ/cyl} \cdot \text{mech cycle})$$

$$w_{\text{thermodynamic}} = \frac{\text{Total power produced}}{(\text{No. of cylinders})(\text{No. of thermodynamic cycles})}$$

$$= \frac{4000\text{hp}}{(16\text{ cylinders})(1050/2\text{ rev/min})}\left(\frac{42.41\text{ Btu/min}}{1\text{ hp}}\right)$$

$$= \mathbf{20.2\ Btu/cyl \cdot therm\ cycle} \quad (= 21.3\text{ kJ/cyl} \cdot \text{therm cycle})$$

8-144 A simple ideal Brayton cycle operating between the specified temperature limits is considered. The pressure ratio for which the compressor and the turbine exit temperature of air are equal is to be determined. √

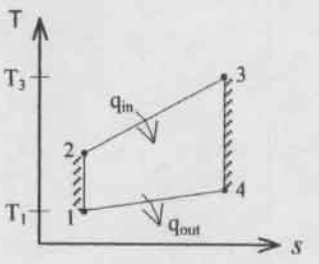
Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$ (Table A-2).

Analysis We treat air as an ideal gas with constant specific heats. Using the isentropic relations, the temperatures at the compressor and turbine exit can be expressed as

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = T_1(r_p)^{(k-1)/k}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = T_3\left(\frac{1}{r_p}\right)^{(k-1)/k}$$



Setting $T_2 = T_4$ and solving for r_p gives

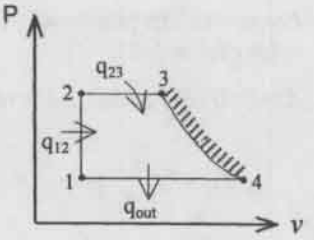
$$r_p = \left(\frac{T_3}{T_1}\right)^{k/2(k-1)} = \left(\frac{1500\text{K}}{290\text{K}}\right)^{1.4/0.8} = \mathbf{17.7}$$

Therefore, the compressor and turbine exit temperatures will be equal when the compression ratio is 17.7.

8-145 The four processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the net work output and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

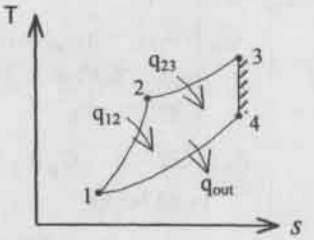
Properties The properties of air are given in Table A-17.



Analysis (b) We treat air as an ideal gas with variable specific heats,

$$T_1 = 300\text{K} \longrightarrow u_1 = 214.07\text{kJ/kg}$$
$$h_1 = 300.19\text{kJ/kg}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow T_2 = \frac{P_2}{P_1} T_1 = \left(\frac{300\text{kPa}}{100\text{kPa}}\right)(300\text{K})$$
$$= 900\text{K} \longrightarrow u_2 = 674.58\text{kJ/kg}$$
$$h_2 = 932.93\text{kJ/kg}$$



$$T_3 = 1300K \longrightarrow u_3 = 1022.82\text{kJ/kg}$$
$$h_3 = 1395.97\text{kJ/kg}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{100\text{kPa}}{300\text{kPa}}\right)(330.9) = 110.3$$

$$\longrightarrow h_4 = 1036.46\text{kJ/kg}$$

$$q_{in} = q_{12,in} + q_{23,in} = (u_2 - u_1) + (h_3 - h_2)$$
$$= (674.58 - 214.07) + (1395.97 - 932.93)$$
$$= 923.55\text{kJ/kg}$$

$$q_{out} = h_4 - h_1 = 1036.46 - 300.19 = 736.27\text{kJ/kg}$$

$$w_{net} = q_{in} - q_{out} = 923.55 - 736.27 = \mathbf{187.28kJ/kg}$$

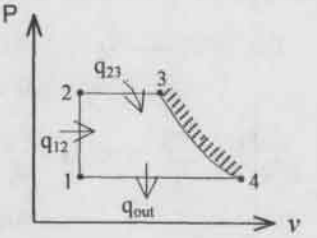
(c) $$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{187.28\text{ kJ/kg}}{923.55\text{ kJ/kg}} = \mathbf{20.3\%}$$

8-146 All four processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the net work output and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg.K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (*b*) Process 3-4 is isentropic:



$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1300\text{K})\left(\frac{1}{3}\right)^{0.4/1.4} = 949.8\text{K}$$

$$\frac{P_2v_2}{T_2} = \frac{P_1v_1}{T_1} \longrightarrow T_2 = \frac{P_2}{P_1}T_1 = \left(\frac{300\text{kPa}}{100\text{kPa}}\right)(300\text{K}) = 900\text{K}$$

$$q_{in} = q_{12,in} + q_{23,in} = (u_2 - u_1) + (h_3 - h_2) = C_v(T_2 - T_1) + C_p(T_3 - T_2)$$
$$= (0.718\text{kJ/kg}\cdot\text{K})(900 - 300)\text{K} + (1.005\text{kJ/kg}\cdot\text{K})(1300 - 900)\text{K}$$
$$= 832.8\text{kJ/kg}$$

$$q_{out} = h_4 - h_1 = C_p(T_4 - T_1) = (1.005\text{kJ/kg}\cdot\text{K})(949.8 - 300)\text{K}$$
$$= 653\text{kJ/kg}$$

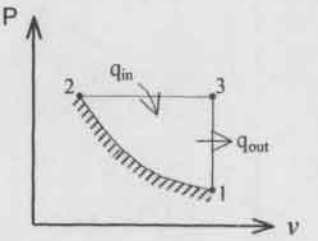
$$w_{net} = q_{in} - q_{out} = 832.8 - 653 = \mathbf{179.8kJ/kg}$$

(*c*) $$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{179.8\text{ kJ/kg}}{832.8\text{ kJ/kg}} = \mathbf{21.6\%}$$

8-147 The three processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the maximum temperature in the cycle and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.

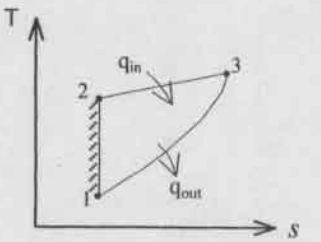


Analysis (*b*) We treat air as an ideal gas with variable specific heats,

$$T_1 = 300\text{K} \longrightarrow u_1 = 214.07\text{kJ/kg}$$
$$P_{r_1} = 1.386$$

$$P_{r_2} = \frac{P_2}{P_1}P_{r_1} = \left(\frac{700\text{kPa}}{100\text{kPa}}\right)(1.386) = 9.702 \longrightarrow h_2 = 523.90\text{kJ/kg}$$

$$\frac{P_3v_3}{T_3} = \frac{P_1v_1}{T_1} \longrightarrow T_{\max} = T_3 = \frac{P_3}{P_1}T_1 = \left(\frac{700\text{kPa}}{100\text{kPa}}\right)(300\text{K}) = \mathbf{2100K}$$



$$T_3 = 2100\text{K} \longrightarrow u_3 = 1775.3\text{kJ/kg}$$
$$h_3 = 2377.7\text{ kJ/kg}$$

(*c*) $$q_{in} = h_3 - h_2 = 2377.7 - 523.9 = 1853.8\text{kJ/kg}$$

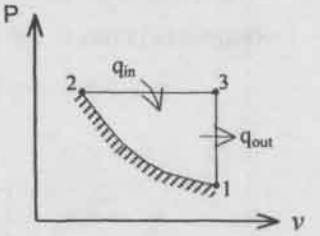
$$q_{out} = u_3 - u_1 = 1775.3 - 214.07 = 1561.23\text{kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1561.23\text{kJ/kg}}{1853.8\text{kJ/kg}} = \mathbf{15.8\%}$$

8-148 All three processes of an air-standard cycle are described. The cycle is to be shown on *P-v* and *T-s* diagrams, and the maximum temperature in the cycle and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg.K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

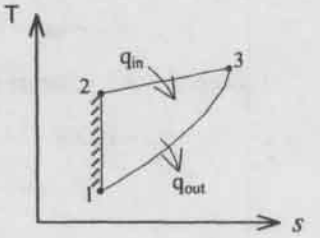


Analysis (*b*) We treat air as an ideal gas with constant specific heats.

Process 1-2 is isentropic:

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})\left(\frac{700\text{kPa}}{100\text{kPa}}\right)^{0.4/1.4} = 523.1\text{K}$$

$$\frac{P_3 v_3}{T_3} = \frac{P_1 v_1}{T_1} \longrightarrow T_{\max} = T_3 = \frac{P_3}{P_1} T_1 = \left(\frac{700\text{kPa}}{100\text{kPa}}\right)(300\text{K}) = \mathbf{2100K}$$



(*c*) $q_{in} = h_3 - h_2 = C_p(T_3 - T_2) = (1.005\text{kJ/kg}\cdot\text{K})(2100 - 523.1)\text{K} = 1584.8\text{kJ/kg}$

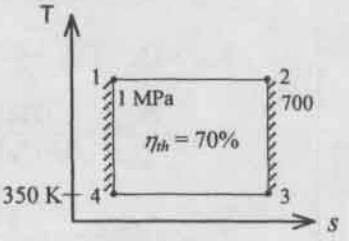
$$q_{out} = u_3 - u_1 = C_v(T_3 - T_1) = (0.718\text{kJ/kg}\cdot\text{K})(2100 - 300)\text{K} = 1292.4\text{kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1292.4\text{kJ/kg}}{1584.8\text{kJ/kg}} = \mathbf{18.5\%}$$

8-149 A Carnot cycle executed in a closed system uses air as the working fluid. The net work output per cycle is to be determined.

Assumptions **1** Air is an ideal gas with variable specific heats.

Analysis (a) The maximum temperature is determined from



$$\eta_{th} = 1 - \frac{T_L}{T_H} \longrightarrow 0.70 = 1 - \frac{350\text{K}}{T_H} \longrightarrow T_H = 1000\text{K}$$

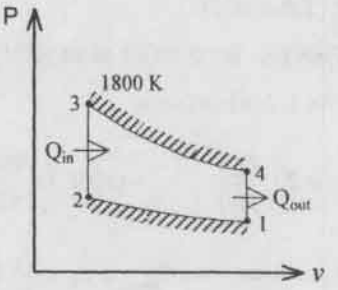
$$s_2 - s_1 = \overset{\not\!\!0}{s_2^\circ} - s_1^\circ - R\ln\frac{P_2}{P_1} = -(0.287\text{kJ/kg}\cdot\text{K})\ln\frac{700\text{kPa}}{1000\text{kPa}}$$
$$= 0.1204\text{kJ/kg}\cdot\text{K}$$

$$W_{net} = m(s_2 - s_1)(T_H - T_L)$$
$$= (0.002\text{kg})(0.1024\text{kJ/kg}\cdot\text{K})(1000 - 300)\text{K}$$
$$= \mathbf{0.143kJ}$$

8-150 A four-cylinder spark-ignition engine with a compression ratio of 8 is considered. The amount of heat supplied per cylinder, the thermal efficiency, and the rpm for a net power output of 60 kW are to be determined. √

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.



Analysis (*a*) Process 1-2: isentropic compression.

$$T_1 = 290K \longrightarrow u_1 = 206.91\text{kJ/kg}$$
$$v_{r_1} = 676.1$$

$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \frac{1}{r} v_{r_1} = \frac{1}{8}(676.1) = 84.51$$
$$\longrightarrow u_2 = 475.11\text{kJ/kg}$$

Process 2-3: v = constant heat addition.

$$T_3 = 1800\text{K} \longrightarrow u_3 = 1487.2\text{kJ/kg}$$
$$v_{r_3} = 3.994$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{(98\text{kPa})(0.0006\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\text{K})} = 7.065\times10^{-4}\text{kg}$$

$$Q_{in} = m(u_3 - u_2) = (7.065\times10^{-4}\text{kg})(1487.2 - 475.11)\text{kJ/kg} = \mathbf{0.715kJ}$$

(*b*) Process 3-4: isentropic expansion.

$$v_{r_4} = \frac{v_4}{v_3} v_{r_3} = r v_{r_3} = (8)(3.994) = 31.95 \longrightarrow u_4 = 693.23\text{ kJ/kg}$$

Process 4-1: v = constant heat rejection.

$$Q_{out} = m(u_4 - u_1) = (7.065\times10^{-4}\text{kg})(693.23 - 206.91)\text{kJ/kg} = \mathbf{0.344\text{ kJ}}$$

$$W_{net} = Q_{in} - Q_{out} = 0.715 - 0.344 = 0.371\text{ kJ}$$

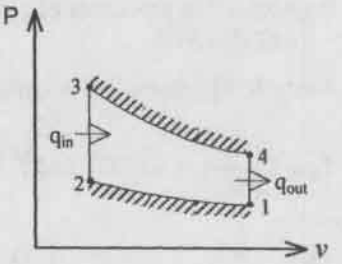
$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{0.371\text{kJ}}{0.715\text{kJ}} = \mathbf{51.9\%}$$

(*c*) $$\dot{n} = \frac{\dot{W}_{net}}{n_{cyl} W_{net,cyl}} = \frac{60\text{kJ/s}}{4\times0.371\text{ kJ}}\left(\frac{60\text{s}}{1\text{min}}\right) = \mathbf{2426\text{ rpm}}$$

8-151 An ideal Otto cycle with air as the working fluid with a compression ratio of 9.2 is considered. The amount of heat transferred to the air, the net work output, the thermal efficiency, and the mean effective pressure are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.



Analysis (*a*) Process 1-2: isentropic compression.

$$T_1 = 300\text{K} \longrightarrow u_1 = 214.07\text{kJ/kg}$$
$$v_{r_1} = 621.2$$

$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \frac{1}{r} v_{r_1} = \frac{1}{9.2}(621.2) = 67.52 \longrightarrow T_2 = 708.3K$$
$$u_2 = 518.9kJ/kg$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow P_2 = \frac{v_1}{v_2}\frac{T_2}{T_1} P_1 = (9.2)\left(\frac{708.3\text{K}}{300\text{K}}\right)(98\text{kPa}) = 2129\text{kPa}$$

Process 2-3: v = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow T_3 = \frac{P_3}{P_2} T_2 = 2T_2 = (2)(708.3) = 1416.6\text{K} \longrightarrow u_3 = 1128.7\text{kJ/kg}$$
$$v_{r_3} = 8.593$$

$$q_{in} = u_3 - u_2 = 1128.7 - 518.9 = \mathbf{609.8kJ/kg}$$

(*b*) Process 3-4: isentropic expansion.

$$v_{r_4} = \frac{v_4}{v_3} v_{r_3} = r v_{r_3} = (9.2)(8.593) = 79.06 \longrightarrow u_4 = 487.75\text{kJ/kg}$$

Process 4-1: v = constant heat rejection.

$$q_{out} = u_4 - u_1 = 487.75 - 214.07 = 273.7\ \text{kJ/kg}$$
$$w_{net} = q_{in} - q_{out} = 609.8 - 273.7 = \mathbf{336.1\ kJ/kg}$$

(*c*) $$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{336.1\ \text{kJ/kg}}{609.8\ \text{kJ/kg}} = \mathbf{55.1\%}$$

(*d*) $$v_{\max} = v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})}{98\text{kPa}} = 0.879\text{m}^3/\text{kg}$$

$$v_{\min} = v_2 = \frac{v_{\max}}{r}$$

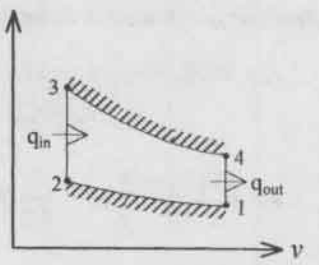
$$MEP = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1(1 - 1/r)} = \frac{336.1\text{kJ/kg}}{(0.879\text{m}^3/\text{kg})(1 - 1/9.2)}\left(\frac{1\text{kPa}\cdot\text{m}^3}{1\text{kJ}}\right) = \mathbf{429kPa}$$

8-152 An ideal Otto cycle with air as the working fluid with a compression ratio of 9.2 is considered. The amount of heat transferred to the air, the net work output, the thermal efficiency, and the mean effective pressure are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air at room temperature are C_p = 1.005 kJ/kg.K, C_v = 0.718 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis (a) Process 1-2 is isentropic compression:



$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (300\text{K})(9.2)^{0.4} = 728.8\text{K}$$

$$\frac{P_2v_2}{T_2} = \frac{P_1v_1}{T_1} \longrightarrow P_2 = \frac{v_1}{v_2}\frac{T_2}{T_1}P_1 = (9.2)\left(\frac{728.8\text{K}}{300\text{K}}\right)(98\text{kPa}) = 2190\text{kPa}$$

Process 2-3: v = constant heat addition.

$$\frac{P_3v_3}{T_3} = \frac{P_2v_2}{T_2} \longrightarrow T_3 = \frac{P_3}{P_2}T_2 = 2T_2 = (2)(728.8) = 1457.6\text{K}$$

$$q_{in} = u_3 - u_2 = C_v(T_3 - T_2) = (0.718\text{kJ/kg}\cdot\text{K})(1457.6 - 728.8)\text{K} = \mathbf{523.3kJ/kg}$$

(*b*) Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = (1457.6\text{K})\left(\frac{1}{9.2}\right)^{0.4} = 600.0\text{K}$$

Process 4-1: v = constant heat rejection.

$$q_{out} = u_4 - u_1 = C_v(T_4 - T_1) = (0.718\text{kJ/kg}\cdot\text{K})(600 - 300)\text{K} = 215.4\text{kJ/kg}$$

$$w_{net} = q_{in} - q_{out} = 523.3 - 215.4 = \mathbf{307.9kJ/kg}$$

(*c*) $$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{307.9\text{ kJ/kg}}{523.3\text{ kJ/kg}} = \mathbf{58.8\%}$$

(*d*) $$v_{max} = v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})}{98\text{kPa}} = 0.879\text{m}^3/\text{kg}$$

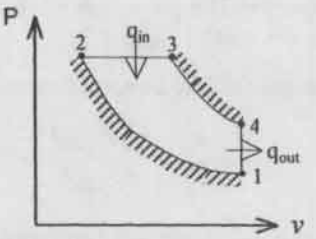
$$v_{min} = v_2 = \frac{v_{max}}{r}$$

$$MEP = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1(1 - 1/r)} = \frac{307.9\text{kJ/kg}}{(0.879\text{m}^3/\text{kg})(1 - 1/9.2)}\left(\frac{1\text{kPa}\cdot\text{m}^3}{1\text{kJ}}\right) = \mathbf{393kPa}$$

8-153 An engine operating on the ideal diesel cycle with air as the working fluid is considered. The pressure at the beginning of the heat-rejection process, the net work per cycle, and the mean effective pressure are to be determined. √

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.



Analysis (a) The compression and the cutoff ratios are

$$r = \frac{V_1}{V_2} = \frac{1200\ \text{cm}^3}{75\ \text{cm}^3} = 16 \qquad r_c = \frac{V_3}{V_2} = \frac{150\ \text{cm}^3}{75\ \text{cm}^3} = 2$$

Process 1-2: isentropic compression.

$$T_1 = 290K \longrightarrow u_1 = 206.91\text{kJ/kg}$$
$$v_{r_1} = 676.1$$

$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \frac{1}{r} v_{r_1} = \frac{1}{16}(676.1) = 42.256 \longrightarrow T_2 = 837.3\text{K}$$
$$h_2 = 863.03\text{kJ/kg}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow T_3 = \frac{v_3}{v_2} T_2 = 2T_2 = (2)(837.3) = 1674.6\ \text{K}$$
$$\longrightarrow h_3 = 1848.9\text{kJ/kg}$$
$$v_{r_3} = 5.002$$

Process 3-4: isentropic expansion.

$$v_{r_4} = \frac{v_4}{v_3} v_{r_3} = \frac{v_4}{2v_2} v_{r_3} = \frac{r}{2} v_{r_3} = \left(\frac{16}{2}\right)(5.002) = 40.016 \longrightarrow T_4 = 853.4\text{K}$$
$$u_4 = 636.00\text{kJ/kg}$$

Process 4-1: v = constant heat rejection.

$$\frac{P_4 v_4}{T_4} = \frac{P_1 v_1}{T_1} \longrightarrow P_4 = \frac{T_4}{T_1} P_1 = \left(\frac{853.4\text{K}}{290\text{K}}\right)(100\text{kPa}) = \mathbf{294.3kPa}$$

(*b*) $$m = \frac{P_1 V_1}{RT_1} = \frac{(100\text{kPa})(0.0012\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\text{K})} = 1.442\times10^{-3}\text{kg}$$

$$Q_{in} = m(h_3 - h_2) = (1.442\times10^{-3}\text{kg})(1848.9 - 863.08) = 1.422\text{kJ}$$

$$Q_{out} = m(u_4 - u_1) = (1.442\times10^{-3}\text{kg})(636.00 - 206.91)\text{kJ/kg} = 0.619\text{kJ}$$

$$W_{net} = Q_{in} - Q_{out} = 1.422 - 0.619 = \mathbf{0.803kJ}$$

(*c*) $$MEP = \frac{W_{net}}{V_1 - V_2} = \frac{W_{net}}{V_1(1 - 1/r)} = \frac{0.803\text{kJ}}{(0.0012\text{m}^3)(1 - 1/16)}\left(\frac{1\text{kPa}\cdot\text{m}^3}{1\text{kJ}}\right) = \mathbf{714kPa}$$

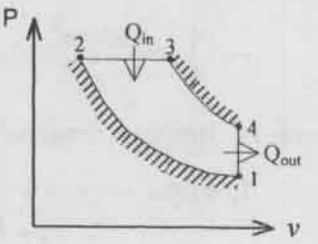
8-154 An engine operating on the ideal diesel cycle with argon as the working fluid is considered. The pressure at the beginning of the heat-rejection process, the net work per cycle, and the mean effective pressure are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Argon is an ideal gas with constant specific heats.

Properties The properties of argon at room temperature are C_p = 0.5203 kJ/kg.K, C_v = 0.3211 kJ/kg·K, and k = 1.667 (Table A-2).

Analysis (*a*) The compression and the cutoff ratios are

$$r = \frac{V_1}{V_2} = \frac{1200\ \text{cm}^3}{75\ \text{cm}^3} = 16 \qquad r_c = \frac{V_3}{V_2} = \frac{150\ \text{cm}^3}{75\ \text{cm}^3} = 2$$



Process 1-2: isentropic compression.

$$T_2 = T_1\left(\frac{V_2}{V_1}\right)^{k-1} = (290\text{K})(16)^{0.667} = 1843\text{K}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow T_3 = \frac{v_3}{v_2} T_2 = 2T_2 = (2)(1843) = 3686\text{K}$$

Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{k-1} = T_3\left(\frac{2V_2}{V_4}\right)^{k-1} = T_3\left(\frac{2}{r}\right)^{k-1} = (3686\text{K})\left(\frac{2}{16}\right)^{0.667} = 920.9\text{K}$$

Process 4-1: v = constant heat rejection.

$$\frac{P_4 v_4}{T_4} = \frac{P_1 v_1}{T_1} \longrightarrow P_4 = \frac{T_4}{T_1} P_1 = \left(\frac{920.9\text{K}}{290\text{K}}\right)(100\text{kPa}) = \mathbf{317.6kPa}$$

(*b*) $$m = \frac{P_1 V_1}{RT_1} = \frac{(100\text{kPa})(0.0012\text{m}^3)}{(0.2081\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\text{K})} = 1.988\times10^{-3}\text{kg}$$

$$Q_{in} = m(h_3 - h_2) = mC_p(T_3 - T_2) = (1.988\times10^{-3}\text{kg})(0.5203\text{kJ/kg}\cdot\text{K})(3686 - 1843)\text{K} = 1.906\text{kJ}$$

$$Q_{out} = m(u_4 - u_1) = mC_v(T_4 - T_1) = (1.988\times10^{-3}\text{kg})(0.3211\text{kJ/kg}\cdot\text{K})(920.9 - 290)\text{K} = 0.392\text{kJ}$$

$$W_{net} = Q_{in} - Q_{out} = 1.906 - 0.392 = \mathbf{1.514kJ}$$

(*c*) $$MEP = \frac{W_{net}}{V_1 - V_2} = \frac{W_{net}}{V_1(1 - 1/r)} = \frac{1.514\text{kJ}}{(0.0012\text{m}^3)(1 - 1/16)}\left(\frac{1\text{kPa}\cdot\text{m}^3}{1\text{kJ}}\right) = \mathbf{1346kPa}$$

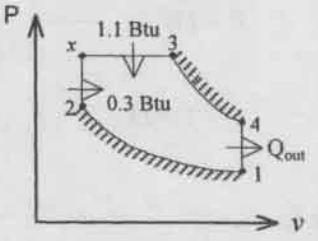
8-155E An ideal dual cycle with air as the working fluid with a compression ratio of 12 is considered. The thermal efficiency of the cycle is to be determined. √

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are C_p = 0.240 Btu/lbm.R, C_v = 0.171 Btu/lbm.R, and k = 1.4 (Table A-2E).

Analysis (*a*) The mass of air is

$$m=\frac{P_1V_1}{RT_1}=\frac{(14.7\text{psia})(75/1728\text{ft}^3)}{(0.3704\text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(550\text{R})}=3.132\times10^{-3}\text{ lbm}$$



Process 1-2: isentropic compression.

$$T_2=T_1\left(\frac{V_1}{V_2}\right)^{k-1}=(550\text{R})(12)^{0.4}=1486\text{R}$$

Process 2-x: v = *constant* heat addition,

$$Q_{2-x,in}=m(u_x-u_2)=mC_v(T_x-T_2)$$
$$0.3\text{Btu}=(3.132\times10^{-3}\text{ lbm})(0.171\text{Btu/lbm}\cdot\text{R})(T_x-1486)\text{R}\longrightarrow T_x=2046\text{R}$$

Process x-3: P = *constant* heat addition.

$$Q_{x-3,in}=m(h_3-h_x)=mC_p(T_3-T_x)$$
$$1.1\text{Btu}=(3.132\times10^{-3}\text{ lbm})(0.240\text{Btu/lbm}\cdot\text{R})(T_3-2046)\text{R}\longrightarrow T_3=3510\text{R}$$

$$\frac{P_3V_3}{T_3}=\frac{P_xV_x}{T_x}\longrightarrow r_c=\frac{V_3}{V_x}=\frac{T_3}{T_x}=\frac{3510\text{R}}{2046\text{R}}=1.715$$

Process 3-4: isentropic expansion.

$$T_4=T_3\left(\frac{V_3}{V_4}\right)^{k-1}=T_3\left(\frac{1.715V_1}{V_4}\right)^{k-1}=T_3\left(\frac{1.715}{r}\right)^{k-1}=(3510\text{R})\left(\frac{1.715}{12}\right)^{0.4}=1612\text{R}$$

Process 4-1: v = constant heat rejection.

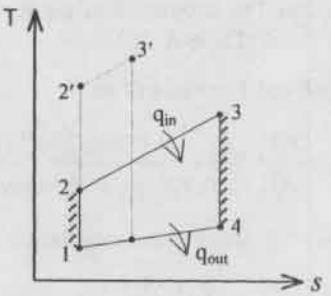
$$Q_{out}=m(u_4-u_1)=mC_v(T_4-T_1)$$
$$=(3.132\times10^{-3}\text{ lbm})(0.171\text{Btu/lbm}\cdot\text{R})(1612-550)\text{R}=0.569\text{Btu}$$

$$\eta_{th}=1-\frac{Q_{out}}{Q_{in}}=1-\frac{0.569\text{Btu}}{1.4\text{Btu}}=\mathbf{59.4\%}$$

8-156 A simple ideal Brayton cycle with air as the working fluid is considered. The net work output per unit mass and the thermal efficiency are to be determined. √

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The properties of air are given in Table A-17.



Analysis (*a*) The properties at various states are

$$T_1 = 300\ \text{K} \longrightarrow \begin{matrix} h_1 = 300.19\ \text{kJ/kg} \\ P_{r_1} = 1.386 \end{matrix}$$

$$T_3 = 1300\ \text{K} \longrightarrow \begin{matrix} h_3 = 1395.97\ \text{kJ/kg} \\ P_{r_3} = 330.9 \end{matrix}$$

For $r_p = 6$,

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (6)(1.386) = 8.316 \longrightarrow h_2 = 501.40\text{kJ/kg}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{6}\right)(330.9) = 55.15 \longrightarrow h_4 = 855.3\text{kJ/kg}$$

$$q_{in} = h_3 - h_2 = 1395.97 - 501.40 = 894.57\text{kJ/kg}$$
$$q_{out} = h_4 - h_1 = 855.3 - 300.19 = 555.11\text{kJ/kg}$$
$$w_{net} = q_{in} - q_{out} = 894.57 - 555.11 = 339.46\text{kJ/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{339.46\text{kJ/kg}}{894.57\text{kJ/kg}} = 37.9\%$$

For $r_p = 12$,

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (12)(1.386) = 16.63 \longrightarrow h_2 = 610.6\text{kJ/kg}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{12}\right)(330.9) = 27.58 \longrightarrow h_4 = 704.6\text{kJ/kg}$$

$$q_{in} = h_3 - h_2 = 1395.97 - 610.60 = 785.37\text{kJ/kg}$$
$$q_{out} = h_4 - h_1 = 704.6 - 300.19 = 404.41\text{kJ/kg}$$
$$w_{net} = q_{in} - q_{out} = 785.37 - 404.41 = 380.96\text{kJ/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{380.96\text{kJ/kg}}{785.37\text{kJ/kg}} = 48.5\%$$

Thus,

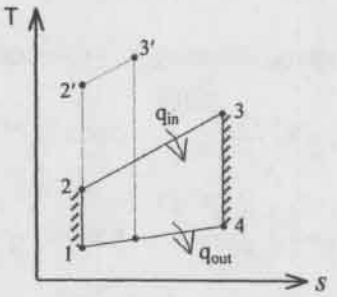
(*b*)
$$\Delta w_{net} = 380.96 - 339.46 = \mathbf{41.5kJ/kg}\ \ (\text{increase})$$
$$\Delta \eta_{th} = 48.5\% - 37.9\% = \mathbf{10.6\%}\ \ (\text{increase})$$

8-157 A simple ideal Brayton cycle with air as the working fluid is considered. The net work output per unit mass and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are $R = 0.287$ kJ/kg.K, $C_p = 1.005$ kJ/kg.K, $C_v = 0.718$ kJ/kg·K, and $k = 1.4$ (Table A-2).

Analysis Processes 1-2 and 3-4 are isentropic. Therefore, For $r_p = 6$,



$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})(6)^{0.4/1.4} = 500.6\text{K}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1300\text{K})\left(\frac{1}{6}\right)^{0.4/1.4} = 779.1\text{K}$$

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$
$$= (1.005\text{kJ/kg}\cdot\text{K})(1300 - 500.6)\text{K} = 803.4\text{kJ/kg}$$

$$q_{out} = h_4 - h_1 = C_p(T_4 - T_1)$$
$$= (1.005\text{kJ/kg}\cdot\text{K})(779.1 - 300)\text{K} = 481.5\text{kJ/kg}$$

$$w_{net} = q_{in} - q_{out} = 803.4 - 481.5 = 321.9\text{kJ/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{321.9\text{kJ/kg}}{803.4\text{kJ/kg}} = 40.1\%$$

For $r_p = 12$,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})(12)^{0.4/1.4} = 610.2\text{K}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1300\text{K})\left(\frac{1}{12}\right)^{0.4/1.4} = 639.2\text{K}$$

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$
$$= (1.005\text{kJ/kg}\cdot\text{K})(1300 - 610.2)\text{K} = 693.2\text{kJ/kg}$$

$$q_{out} = h_4 - h_1 = C_p(T_4 - T_1)$$
$$= (1.005\text{kJ/kg}\cdot\text{K})(639.2 - 300)\text{K} = 340.9\text{kJ/kg}$$

$$w_{net} = q_{in} - q_{out} = 693.2 - 340.9 = 352.3\text{kJ/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{352.3\text{kJ/kg}}{693.2\text{kJ/kg}} = 50.8\%$$

Thus,

$$\Delta w_{net} = 352.3 - 321.9 = \mathbf{30.4kJ/kg} \quad (\text{increase})$$
$$\Delta\eta_{th} = 50.8\% - 40.1\% = \mathbf{10.7\%} \quad (\text{increase})$$

8-158 A 30-MW regenerative Brayton cycle with helium as the working fluid is considered. The thermal efficiency and the required mass flow rate of helium are to be determined for 100 percent and 80 percent isentropic efficiencies for both the compressor and the turbine.

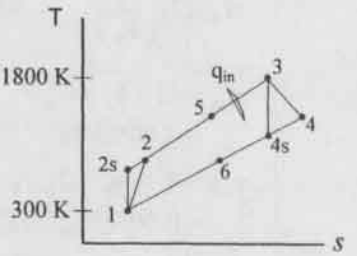
Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Helium is an ideal gas with constant specific heats.

Properties The properties of helium at room temperature are C_p = 5.1926 kJ/kg.K and k = 1.667 (Table A-2).

Analysis (a) Assuming $\eta_T = \eta_C = 100\%$,

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})(8)^{0.667/1.667} = 689.4\text{K}$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1800\text{K})\left(\frac{1}{8}\right)^{0.667/1.667} = 783.3\text{K}$$



$$\varepsilon = \frac{h_5 - h_2}{h_4 - h_2} = \frac{C_p(T_5 - T_2)}{C_p(T_4 - T_2)} \longrightarrow T_5 = T_2 + \varepsilon(T_4 - T_2)$$
$$= 689.4 + (0.75)(783.3 - 689.4)$$
$$= 759.8\text{K}$$

$$w_{net} = w_{T,out} - w_{C,in} = (h_3 - h_4) - (h_2 - h_1) = C_p[(T_3 - T_4) - (T_2 - T_1)]$$
$$= (5.1926\text{kJ/kg}\cdot\text{K})[(1800 - 783.3) - (689.4 - 300)]\text{K} = 3257.3\text{kJ/kg}$$

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{30{,}000\text{kJ/s}}{3257.3\text{kJ/kg}} = \mathbf{9.21kg/s}$$

$$q_{in} = h_3 - h_5 = C_p(T_3 - T_5) = (5.1926\text{kJ/kg}\cdot\text{K})(1800 - 759.8)\text{K} = 5401.3\text{kJ/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{3257.3\text{kJ/kg}}{5401.3\text{kJ/kg}} = \mathbf{60.3\%}$$

(*b*) Assuming $\eta_T = \eta_C = 80\%$,

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{K})(8)^{0.667/1.667} = 689.4\text{K}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{C_p(T_{2s} - T_1)}{C_p(T_2 - T_1)} \longrightarrow T_2 = T_1 + (T_{2s} - T_1)/\eta_C$$
$$= 300 + (689.4 - 300)/(0.80)$$
$$= 786.8\text{K}$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1800\text{K})\left(\frac{1}{8}\right)^{0.667/1.667} = 783.3\text{K}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{C_p(T_3 - T_4)}{C_p(T_3 - T_{4s})} \longrightarrow T_4 = T_3 - \eta_T(T_3 - T_{4s})$$
$$= 1800 - (0.80)(1800 - 783.3)$$
$$= 986.6\text{K}$$

$$\varepsilon = \frac{h_5 - h_2}{h_4 - h_2} = \frac{C_p(T_5 - T_2)}{C_p(T_4 - T_2)} \longrightarrow T_5 = T_2 + \varepsilon(T_4 - T_2)$$
$$= 786.8 + (0.75)(986.6 - 786.8)$$
$$= 936.7\text{K}$$

$$w_{net} = w_{T,out} - w_{C,in} = (h_3 - h_4) - (h_2 - h_1) = C_p[(T_3 - T_4) - (T_2 - T_1)]$$
$$= (5.1926\text{kJ/kg} \cdot \text{K})[(1800 - 986.7) - (786.8 - 300)]\text{K} = 1695.4\text{kJ/kg}$$

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{30{,}000\text{kJ/s}}{1695.4\text{kJ/kg}} = \mathbf{17.7kg/s}$$

$$q_{in} = h_3 - h_5 = C_p(T_3 - T_5) = (5.1926\text{kJ/kg} \cdot \text{K})(1800 - 936.7)\text{K} = 4482.8\text{kJ/kg}$$

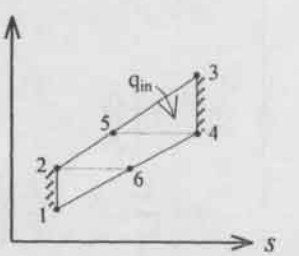
$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{1695.4\text{kJ/kg}}{4482.8\text{kJ/kg}} = \mathbf{37.8\%}$$

8-159 An ideal regenerative Brayton cycle is considered. The pressure ratio that maximizes the thermal efficiency of the cycle is to be determined, and to be compared with the pressure ratio that maximizes the cycle net work.

Analysis Using the isentropic relations, the temperatures at the compressor and turbine exit can be expressed as

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = T_1\left(r_p\right)^{(k-1)/k}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = T_3\left(\frac{1}{r_p}\right)^{(k-1)/k} = T_3 r_p^{(1-k)/k}$$



Then,

$$q_{in} = h_3 - h_5 = C_p(T_3 - T_5) = C_p(T_3 - T_4) = C_p T_3\left(1 - r_p^{(1-k)/k}\right)$$

$$q_{out} = h_6 - h_1 = C_p(T_6 - T_1) = C_p(T_2 - T_1) = C_p T_1\left(r_p^{(k-1)/k} - 1\right)$$

$$w_{net} = q_{in} - q_{out} = C_p\left(T_3 - T_3 r_p^{(1-k)/k} - T_1 r_p^{(k-1)/k} + T_1\right)$$

To maximize the net work, we must have

$$\frac{\partial w_{net}}{\partial r_p} = C_p\left(-\frac{1-k}{k} T_3 r_p^{(1-k)/k} - 1 - \frac{k-1}{k} T_1 r_p^{(k-1)/k} - 1\right) = 0$$

Solving for r_p gives

$$r_p = \left(\frac{T_1}{T_3}\right)^{k/2(1-k)}$$

Similarly,

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_p T_1\left(r_p^{(k-1)/k} - 1\right)}{C_p T_3\left(1 - r_p^{(1-k)/k}\right)}$$

which simplifies to

$$\eta_{th} = 1 - \frac{T_1}{T_3} r_p^{(k-1)/k}$$

When $r_p = 1$, the thermal efficiency becomes $\eta_{th} = 1 - T_1/T_3$, which is the Carnot efficiency. Therefore, the efficiency is a maximum when $r_p = 1$, and must decrease as r_p increases for the fixed values of T_1 and T_3. Note that the compression ratio cannot be less than 1, and the factor

$$r_p^{(k-1)/k}$$

is always greater than 1 for $r_p > 1$. Also note that the net work $w_{net} = 0$ for $r_p = 1$. This being the case, the pressure ratio for maximum thermal efficiency, which is $r_p = 1$, is always less than the pressure ratio for maximum work.

8-160 An ideal gas-turbine cycle with one stage of compression and two stages of expansion and regeneration is considered. The thermal efficiency of the cycle as a function of the compressor pressure ratio and the high-pressure turbine to compressor inlet temperature ratio is to be determined, and to be compared with the efficiency of the standard regenerative cycle.

Analysis The T-s diagram of the cycle is as shown in the figure. If the overall pressure ratio of the cycle is r_p, which is the pressure ration across the compressor, then the pressure ratio across each turbine stage in the ideal case becomes $\sqrt{r_p}$. Using the isentropic relations, the temperatures at the compressor and turbine exit can be expressed as

$$T_5 = T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = T_1\left(r_p\right)^{(k-1)/k}$$

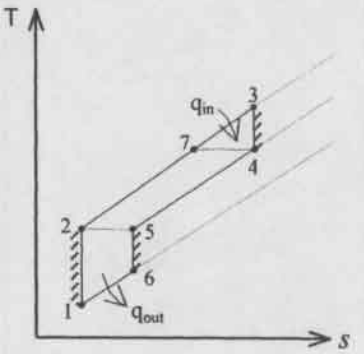
$$T_7 = T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = T_3\left(\frac{1}{\sqrt{r_p}}\right)^{(k-1)/k} = T_3 r_p^{(1-k)/2k}$$

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{(k-1)/k} = T_5\left(\frac{1}{\sqrt{r_p}}\right)^{(k-1)/k} = T_2 r_p^{(1-k)/2k} = T_1 r_p^{(k-1)/k} r_p^{(1-k)/2k} = T_1 r_p^{(k-1)/2k}$$

Then,

$$q_{in} = h_3 - h_7 = C_p\left(T_3 - T_7\right) = C_p T_3\left(1 - r_p^{(1-k)/2k}\right)$$

$$q_{out} = h_6 - h_1 = C_p\left(T_6 - T_1\right) = C_p T_1\left(r_p^{(k-1)/2k} - 1\right)$$



and thus

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_p T_1\left(r_p^{(k-1)/2k} - 1\right)}{C_p T_3\left(1 - r_p^{(1-k)/2k}\right)}$$

which simplifies to

$$\eta_{th} = 1 - \frac{T_1}{T_3} r_p^{(k-1)/2k}$$

The thermal efficiency of the single stage ideal regenerative cycle is given in Eq. 8-25 to be

$$\eta_{th} = 1 - \frac{T_1}{T_3} r_p^{(k-1)/k}$$

Therefore, the regenerative cycle with two stages of expansion has a higher thermal efficiency than the standard regenerative cycle with a single stage of expansion for any given value of the pressure ratio r_p.

8-161 A steam power plant operating on the ideal Rankine cycle with reheating is considered. The reheat pressures of the cycle are to be determined for the cases of single and double reheat.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (a) **Single Reheat:** From the steam tables (Tables A-4, A-5, and A-6),

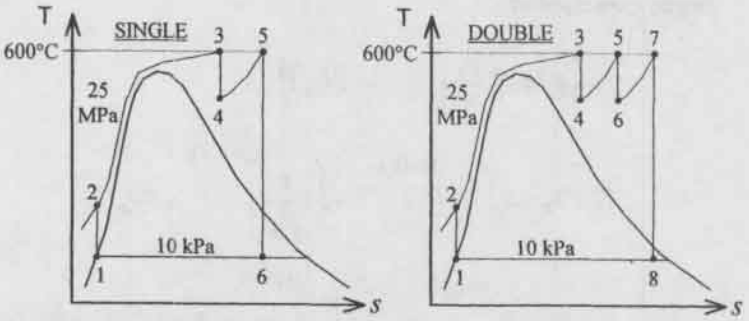
$$\left.\begin{array}{l} P_6 = 10\text{kPa} \\ x_6 = 0.88 \end{array}\right\} \begin{array}{l} h_6 = h_f + x_6 h_{fg} = 191.83 + (0.88)(2392.8) = 2297.5\text{kJ/kg} \\ s_6 = s_f + x_6 s_{fg} = 0.6493 + (0.88)(7.5009) = 7.250\text{kJ/kg} \cdot \text{K} \end{array}$$

$$\left.\begin{array}{l} T_5 = 600°\text{C} \\ s_5 = s_6 \end{array}\right\} P_5 = \mathbf{5.098MPa}$$

Double Reheat :

$$\left.\begin{array}{l} P_3 = 25\text{MPa} \\ T_3 = 600°\text{C} \end{array}\right\} s_3 = 6.3602\text{kJ/kg} \cdot \text{K}$$

$$\begin{array}{l} P_4 = P_x \\ s_4 = s_3 \end{array} \text{ and } \begin{array}{l} P_5 = P_x \\ T_5 = 600°\text{C} \end{array}$$



Any pressure P_x selected between the limits of 25 MPa and 5.098 MPa will satisfy the requirements, and can be used for the double reheat pressure.

8-162E A geothermal power plant operating on the simple Rankine cycle using an organic fluid as the working fluid is considered. The exit temperature of the geothermal water from the vaporizer, the rate of heat rejection from the working fluid in the condenser, the mass flow rate of geothermal water at the preheater, and the thermal efficiency of the Level I cycle of this plant are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) The exit temperature of geothermal water from the vaporizer is determined from the steady-flow energy balance on the geothermal water (brine),

$$\dot{Q}_{\text{brine}} = \dot{m}_{\text{brine}} C_p (T_2 - T_1)$$

$$-22{,}790{,}000\text{Btu/h} = (384{,}286\text{lbm/h})(1.03\text{Btu/lbm}\cdot{}^\circ\text{F})(T_2 - 325^\circ\text{F})$$

$$T_2 = \mathbf{267.4\,^\circ F}$$

(*b*) The rate of heat rejection from the working fluid to the air in the condenser is determined from the steady-flow energy balance on air,

$$\dot{Q}_{\text{air}} = \dot{m}_{\text{air}} C_p (T_9 - T_8)$$

$$= (4{,}195{,}100\text{lbm/h})(0.24\text{Btu/lbm}\cdot{}^\circ\text{F})(84.5 - 55^\circ\text{F})$$

$$= \mathbf{29.7\ MBtu/h}$$

(*c*) The mass flow rate of geothermal water at the preheater is determined from the steady-flow energy balance on the geothermal water,

$$\dot{Q}_{\text{geo}} = \dot{m}_{\text{geo}} C_p (T_{\text{out}} - T_{\text{in}})$$

$$-11{,}140{,}000\text{Btu/h} = \dot{m}_{\text{geo}} (1.03\text{Btu/lbm}\cdot{}^\circ\text{F})(154.0 - 221.8^\circ\text{F})$$

$$\dot{m}_{\text{geo}} = \mathbf{187{,}120lbm/h}$$

(*d*) The rate of heat input is

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{vaporizer}} + \dot{Q}_{\text{reheater}} = 22{,}790{,}000 + 11{,}140{,}000$$

$$= 33{,}930{,}000\ \text{Btu/h}$$

and

$$\dot{W}_{\text{net}} = 1271 - 200 = 1071\ \text{kW}$$

Then,

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{1071\text{kW}}{33{,}930{,}000\text{Btu/h}}\left(\frac{3412.14\text{Btu}}{1\text{kWh}}\right) = \mathbf{10.8\%}$$

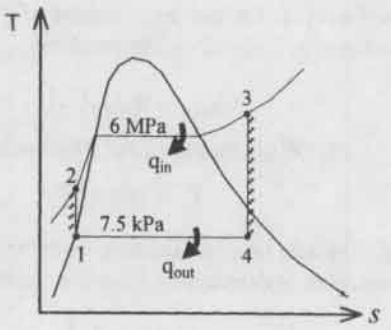
8-163 A steam power plant operates on the simple ideal Rankine cycle. The turbine inlet temperature, the net power output, the thermal efficiency, and the minimum mass flow rate of the cooling water required are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f@7.5\text{kPa}} = 168.79\text{kJ/kg}$$
$$v_1 = v_{f@7.5\text{kPa}} = 0.001008\text{m}^3/\text{kg}$$
$$T_1 = T_{\text{sat}@7.5\text{kPa}} = 40.29°\text{C}$$

$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.001008\text{m}^3/\text{kg})(6{,}000 - 7.5\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 6.04\text{kJ/kg}$$
$$h_2 = h_1 + w_{p,\text{in}} = 168.79 + 6.04 = 174.8\text{kJ/kg}$$



$$h_4 = h_{g@7.5\text{kPa}} = 2574.8\text{kJ/kg}$$
$$s_4 = s_{g@7.5\text{kPa}} = 8.2515\text{kJ/kg}$$

$$\left.\begin{matrix} P_3 = 6\text{MPa} \\ s_3 = s_4 \end{matrix}\right\} \begin{matrix} h_3 = 4856.1\text{kJ/kg} \\ T_3 = \mathbf{1092.4°C} \end{matrix}$$

(b) $$q_{\text{in}} = h_3 - h_2 = 4856.1 - 174.8 = 4681.3\text{ kJ/kg}$$
$$q_{\text{out}} = h_4 - h_1 = 2574.8 - 168.79 = 2406\text{ kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 4681.3 - 2406.0 = 2275.3\text{ kJ/kg}$$

and

$$\eta_{th} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{2275.3\text{ kJ/kg}}{4681.3\text{ kJ/kg}} = \mathbf{48.6\%}$$

Thus,

$$\dot{W}_{\text{net}} = \eta_{th}\dot{Q}_{\text{in}} = (0.486)(4\times10^4\text{ kJ/s}) = \mathbf{1.944\times10^4\,kJ/s}$$

(*c*) The mass flow rate of the cooling water will be minimum when it is heated to the temperature of the steam in the condenser which is 40.29°C,

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{net}} = 4\times10^4 - 1.944\times10^4 = 2.056\times10^4\text{ kJ/s}$$

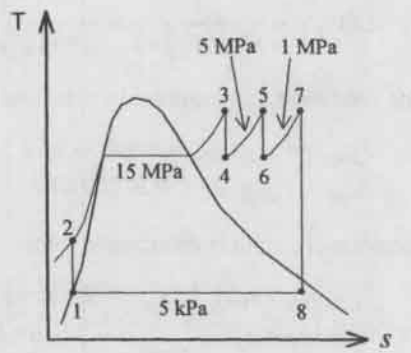
$$\dot{m}_{\text{cool}} = \frac{\dot{Q}_{\text{out}}}{C\Delta T} = \frac{2.056\times10^4\text{ kJ/s}}{(4.184\text{kJ/kg}\cdot°\text{C})(40.29 - 18°\text{C})} = \mathbf{220.5kg/s}$$

8-164 A 300-MW steam power plant operating on an ideal Rankine cycle with two stages of reheat is considered. The thermal efficiency of the cycle and the mass flow rate of the steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (a) From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f@5\text{kPa}} = 137.82\text{kJ/kg}$$
$$v_1 = v_{f@5\text{kPa}} = 0.001005\text{m}^3/\text{kg}$$
$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.001005\text{m}^3/\text{kg})(15{,}000 - 5\text{kPa})\left(\frac{1\text{kJ}}{1\text{kPa}\cdot\text{m}^3}\right)$$
$$= 15.07\text{kJ/kg}$$
$$h_2 = h_1 + w_{p,\text{in}} = 137.82 + 15.07 = 152.89\text{kJ/kg}$$



$$\left.\begin{matrix} P_3 = 15\text{MPa} \\ T_3 = 500°\text{C} \end{matrix}\right\} \begin{matrix} h_3 = 3308.6\text{kJ/kg} \\ s_3 = 6.3443\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_4 = 5\text{MPa} \\ s_4 = s_3 \end{matrix}\right\} h_4 = 3005.7\text{kJ/kg}$$

$$\left.\begin{matrix} P_5 = 5\text{MPa} \\ T_5 = 500°\text{C} \end{matrix}\right\} \begin{matrix} h_5 = 3433.8\text{kJ/kg} \\ s_5 = 6.9789\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_6 = 1\text{MPa} \\ s_6 = s_5 \end{matrix}\right\} h_6 = 2970.5\text{kJ/kg}$$

$$\left.\begin{matrix} P_7 = 1\text{MPa} \\ T_7 = 500°\text{C} \end{matrix}\right\} \begin{matrix} h_7 = 3478.5\text{kJ/kg} \\ s_7 = 7.7622\text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_8 = 5\text{kPa} \\ s_8 = s_7 \end{matrix}\right\} \begin{matrix} x_8 = \dfrac{s_8 - s_f}{s_{fg}} = \dfrac{7.7622 - 0.4764}{7.9187} = 0.920 \\ h_8 = h_f + x_8 h_{fg} = 137.82 + (0.920)(2423.7) = 2367.8\text{kJ/kg} \end{matrix}$$

Then,

$$q_{\text{in}} = (h_3 - h_2) + (h_5 - h_4) + (h_7 - h_6)$$
$$= 3308.6 - 152.89 + 3433.8 - 3005.7 + 3478.5 - 2970.5 = 4091.8\text{kJ/kg}$$
$$q_{\text{out}} = h_8 - h_1 = 2367.8 - 137.82 = 2229.0\text{kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 4091.8 - 2229.0 = 1861.8\text{kJ/kg}$$

Thus,

$$\eta_{th} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1861.8\text{ kJ/kg}}{4091.8\text{ kJ/kg}} = \mathbf{45.5\%}$$

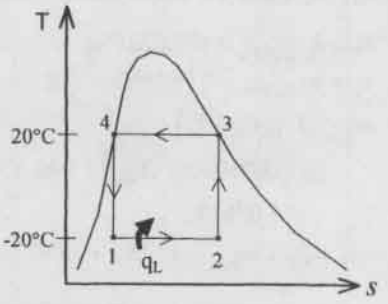
(b) $$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{300{,}000\text{ kJ/s}}{1861.8\text{ kJ/kg}} = \mathbf{161.1\text{ kg/s}}$$

8-165 A steady-flow Carnot refrigeration cycle with refrigerant-134a as the working fluid is considered. The COP, the condenser and evaporator pressures, and the net work input are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) The COP of this refrigeration cycle is determined from

$$\text{COP}_{\text{R,C}} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(293\text{K})/(253\text{K}) - 1} = \mathbf{6.33}$$



(*b*) The condenser and evaporative pressures are (Table A-11)

$$P_{\text{evap}} = P_{\text{sat @ -20°C}} = \mathbf{0.13299\ kPa}$$
$$P_{\text{cond}} = P_{\text{sat @ 20°C}} = \mathbf{0.57160\ kPa}$$

(*c*) The net work input is determined from

$$h_1 = (h_f + x_1 h_{fg})_{@-20°\text{C}} = 24.26 + (0.2)(211.05) = 66.5\text{kJ/kg}$$
$$h_2 = (h_f + x_2 h_{fg})_{@-20°\text{C}} = 24.26 + (0.85)(211.05) = 203.7\text{kJ/kg}$$

$$q_L = h_2 - h_1 = 203.7 - 66.5 = 137.2\ \text{kJ/kg}$$

and

$$w_{\text{net,in}} = \frac{q_L}{\text{COP}_R} = \frac{137.2\ \text{kJ/kg}}{6.33} = \mathbf{21.7\ kJ/kg}$$

8-166 A large refrigeration plant that operates on the ideal vapor-compression cycle with refrigerant-134a as the working fluid is considered. The mass flow rate of the refrigerant, the power input to the compressor, and the mass flow rate of the cooling water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

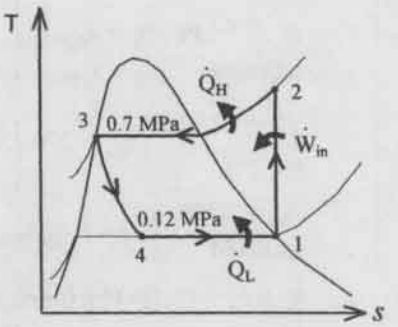
Analysis In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{array}{l} P_1 = 120\text{kPa} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@120\text{kPa}} = 233.86\text{kJ/kg} \\ s_1 = s_{g@120\text{kPa}} = 0.9354\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 0.7\text{MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 270.22\text{kJ/kg}\ (T_2 = 34.6°\text{C})$$

$$\left.\begin{array}{l} P_3 = 0.7\text{MPa} \\ \text{sat.liquid} \end{array}\right\} h_3 = h_{f@0.7\text{MPa}} = 86.78\text{kJ/kg}$$

$$h_4 \cong h_3 = 86.78\text{kJ/kg}\ (\text{throttling})$$



The mass flow rate of the refrigerant is determined from

$$\dot{m} = \frac{\dot{Q}_L}{h_1 - h_4} = \frac{100\text{kJ/s}}{(233.86 - 86.78)\text{kJ/kg}} = \mathbf{0.68kg/s}$$

(*b*) The power input to the compressor is

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = (0.68\text{kg/s})(270.22 - 233.86)\text{kJ/kg} = \mathbf{24.7kW}$$

(*c*) The mass flow rate of the cooling water is determined from

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.68\text{kg/s})(270.22 - 86.78)\text{kJ/kg} = 124.7\text{kW}$$

and

$$\dot{m}_{\text{cooling}} = \frac{\dot{Q}_H}{(C\Delta T)_{\text{water}}} = \frac{124.7\text{kJ/s}}{(4.184\text{kJ/kg}\cdot°\text{C})(8°\text{C})} = \mathbf{3.73kg/s}$$

8-167 A large refrigeration plant operates on the vapor-compression cycle with refrigerant-134a as the working fluid. The mass flow rate of the refrigerant, the power input to the compressor, the mass flow rate of the cooling water, and the rate of exergy destruction associated with the compression process are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

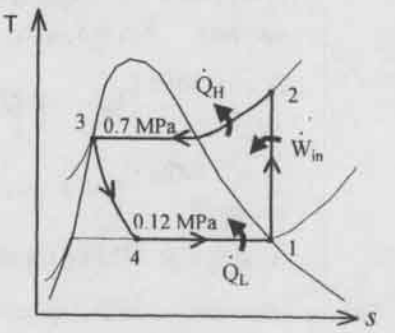
Analysis (*a*) The refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{array}{l} P_1 = 120\text{kPa} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@120\text{kPa}} = 233.86\text{kJ/kg} \\ s_1 = s_{g@120\text{kPa}} = 0.9354\text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 0.7\text{MPa} \\ s_{2s} = s_1 \end{array}\right\} h_{2s} = 270.22\text{kJ/kg}\ (T_{2s} = 34.6°\text{C})$$

$$\left.\begin{array}{l} P_3 = 0.7\text{MPa} \\ \text{sat.liquid} \end{array}\right\} h_3 = h_{f@0.7\text{MPa}} = 86.78\text{kJ/kg}$$

$$h_4 \cong h_3 = 86.78\text{kJ/kg}\ (\text{throttling})$$



The mass flow rate of the refrigerant is determined from

$$\dot{m} = \frac{\dot{Q}_L}{h_1 - h_4} = \frac{100\text{kJ/s}}{(233.86 - 86.78)\text{kJ/kg}} = \mathbf{0.68kg/s}$$

(*b*) The actual enthalpy at the compressor exit is

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow h_2 = h_1 + (h_{2s} - h_1)/\eta_C = 233.86 + (270.22 - 233.86)/(0.75)$$
$$= 282.34\text{kJ/kg}$$

Thus,

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = (0.68\text{kg/s})(282.34 - 233.86)\text{kJ/kg} = \mathbf{33.0kW}$$

(*c*) The mass flow rate of the cooling water is determined from

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.68\text{kg/s})(282.34 - 86.78)\text{kJ/kg} = 133.0\text{kW}$$

and

$$\dot{m}_{\text{cooling}} = \frac{\dot{Q}_H}{(C\Delta T)_{\text{water}}} = \frac{133.0\text{kJ/s}}{(4.184\text{kJ/kg}\cdot°\text{C})(8°\text{C})} = \mathbf{3.97kg/s}$$

The exergy destruction associated with this adiabatic compression process is determined from

$$\dot{X}_{\text{destroyed}} = T_0 \dot{S}_{\text{gen}} = T_0 \dot{m}(s_2 - s_1)$$

where

$$\left.\begin{array}{l} P_2 = 0.7\text{MPa} \\ h_2 = 282.34\text{kJ/kg} \end{array}\right\} s_2 = 0.9741\text{kJ/kg}\cdot\text{K}$$

Thus,

$$\dot{X}_{\text{destroyed}} = (298\text{K})(0.68\text{kg/s})(0.9741 - 0.9354)\text{kJ/kg}\cdot\text{K} = \mathbf{7.84kW}$$

8-168 A heat pump that operates on the ideal vapor-compression cycle with refrigerant-134a as the working fluid is used to heat a house. The rate of heat supply to the house, the volume flow rate of the refrigerant at the compressor inlet, and the COP of this heat pump are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

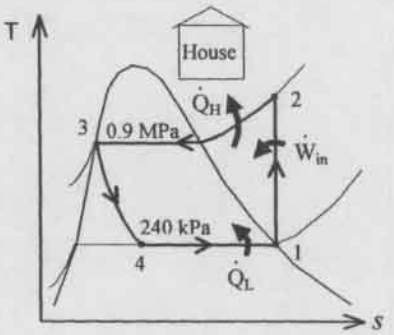
Analysis (*a*) In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

$$\left.\begin{array}{l} P_1 = 240\text{kPa} \\ \text{sat.vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@240\text{kPa}} = 244.09\text{kJ/kg} \\ s_1 = s_{g@240\text{kPa}} = 0.9222\text{kJ/kg}\cdot\text{K} \\ v_1 = v_{g@240\text{kPa}} = 0.0834\text{m}^3/\text{kg} \end{array}$$

$$\left.\begin{array}{l} P_2 = 0.9\text{MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 271.41\text{kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 0.9\text{MPa} \\ \text{sat.liquid} \end{array}\right\} h_3 = h_{f@0.9\text{MPa}} = 99.56\text{kJ/kg}$$

$$h_4 \cong h_3 = 99.56\text{kJ/kg}\ (\text{throttling})$$



The rate of heat supply to the house is determined from

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.15\text{kg/s})(271.41 - 99.56)\text{kJ/kg} = \mathbf{25.78kW}$$

(*b*) The volume flow rate of the refrigerant at the compressor inlet is

$$\dot{V}_1 = \dot{m}v_1 = (0.15\text{kg/s})(0.0834\text{m}^3/\text{kg}) = \mathbf{0.01251m^3/s}$$

(*c*) The COP of t his heat pump is determined from

$$\text{COP}_R = \frac{q_L}{w_{in}} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{271.41 - 99.56}{271.41 - 244.09} = \mathbf{6.29}$$

8-169 A house is cooled adequately by a 3.5 ton air-conditioning unit. The rate of heat gain of the house when the air-conditioner is running continuously is to be determined.

Assumptions **1** The heat gain includes heat transfer through the walls and the roof, infiltration heat gain, solar heat gain, internal heat gain, etc. **2** Steady operating conditions exist.

Analysis Noting that 1 ton of refrigeration is equivalent to a cooling rate of 211 kJ/min, the rate of heat gain of the house in steady operation is simply equal to the cooling rate of the air-conditioning system,

$$\dot{Q}_{\text{heat gain}} = \dot{Q}_{\text{cooling}} = (3.5 \text{ ton})(211 \text{ kJ/min}) = 738.5 \text{ kJ/min} = \mathbf{44{,}310 \text{ kJ/h}}$$

8-170 A room is cooled adequately by a 5000 Btu/h window air-conditioning unit. The rate of heat gain of the room when the air-conditioner is running continuously is to be determined.

Assumptions **1** The heat gain includes heat transfer through the walls and the roof, infiltration heat gain, solar heat gain, internal heat gain, etc. **2** Steady operating conditions exist.

Analysis The rate of heat gain of the room in steady operation is simply equal to the cooling rate of the air-conditioning system,

$$\dot{Q}_{\text{heat gain}} = \dot{Q}_{\text{cooling}} = \mathbf{5{,}000 \text{ Btu/h}}$$

8-171 A heat pump water heater has a COP of 2.2 and consumes 2 kW when running. It is to be determined if this heat pump can be used to meet the cooling needs of a room by absorbing heat from it.

Assumptions The COP of the heat pump remains constant whether heat is absorbed from the outdoor air or room air.

Analysis The COP of the heat pump is given to be 2.2. Then the COP of the air-conditioning system becomes

$$\text{COP}_{\text{air-cond}} = \text{COP}_{\text{heat pump}} - 1 = 2.2 - 1 = 1.2$$

Then the rate of cooling (heat absorption from the air) becomes

$$\dot{Q}_{\text{cooling}} = \text{COP}_{\text{air-cond}} \dot{W}_{in} = (1.2)(2\ \text{kW}) = 2.4\ \text{kW} = 8640\ \text{kJ/h}$$

since 1 kW = 3600 kJ/h. We conclude that this heat pump **can meet** the cooling needs of the room since its cooling rate is greater than the rate of heat gain of the room.



Chapter 9
INTRODUCTION TO FLUID MECHANICS

Classification of Fluid Flows

9-1C The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is *external flow*. The flow in a pipe or duct is *internal flow* if the fluid is completely bounded by solid surfaces. The flow of liquids in a pipe is called *open-channel flow* if the pipe is partially filled with the liquid and there is a free surface, such as the flow of water in rivers and irrigation ditches.

9-2C A fluid flow during which the density of the fluid remains nearly constant is called *incompressible flow*. A fluid whose density is practically independent of pressure (such as a liquid) is called an incompressible fluid. The flow of compressible fluid (such as air) is not necessarily compressible since the density of a compressible fluid may still remain constant during flow.

9-3C A fluid in direct contact with a solid surface sticks to the surface and there is no slip. This is known as the *no-slip condition*, and it is due to the viscosity of the fluid.

Viscosity

9-4C Viscosity is a measure of the "stickiness" or "resistance to deformation" of a fluid. It is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids, and by the molecular collisions in gases. Liquids have higher dynamic viscosities than gases.

9-5C The fluids whose shear stress is proportional to the velocity gradient are called *newtonian fluids*. Most common fluids such as water, air, gasoline, and oils are newtonian fluids.

9-6C For the same cruising speed, the submarine will consume much less power in air than it does in water because of the much lower viscosity of air relative to water.

9-7C (*a*) The dynamic viscosity of liquids decreases with temperature. (*b*) The dynamic viscosity of gases increases with temperature.

9-8C For *liquids*, the kinematic viscosity is practically independent of pressure. For *gases*, the kinematic viscosity is inversely proportional to density and thus pressure since the density of a gas is proportional to its pressure.

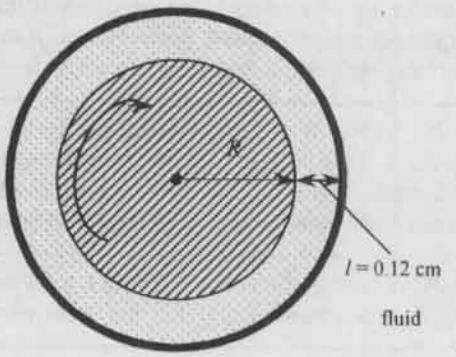
9-9 The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in oil. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is newtonian.

Analysis Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{T\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(0.8\ \text{N}\cdot\text{m})(0.012\ \text{m})}{4\pi^2 (0.075\ \text{m})^3 (200/60\ \text{s}^{-1})(0.75\ \text{m})} = \mathbf{0.0231\ N\cdot s/m^2}$$

Discussion This is the viscosity value at the temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.



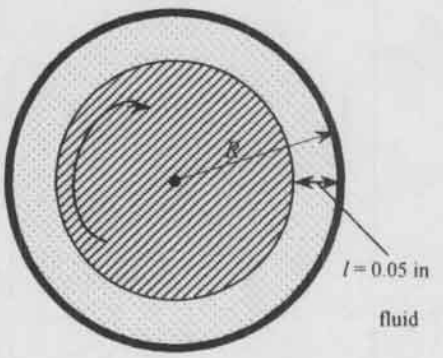
9-10E The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in the fluid. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is newtonian.

Analysis Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{T\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.2\ \text{lbf}\cdot\text{ft})(0.05/12\ \text{ft})}{4\pi^2 (5.6/12\ \text{ft})^3 (250/60\ \text{s}^{-1})(3\ \text{ft})} = \mathbf{0.000682\ lbf\cdot s/ft^2}$$

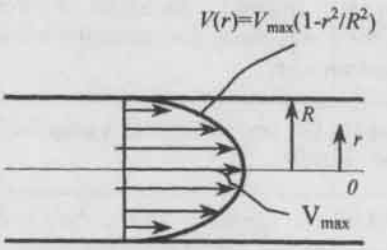
Discussion This is the viscosity value at temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.



9-11 The velocity profile for laminar one-dimensional flow through a circular pipe is given. A relation for the drag force and its numerical value for water are to be determined.

Assumptions **1** The flow through the circular pipe is one-dimensional. **2** The fluid is newtonian.

Properties The viscosity of water at 20°C is given to be 0.0010 kg/m·s.



Analysis The velocity profile for laminar flow is given by

$$\mathbf{V}(r) = \mathbf{V}_{\max}\left(1-\frac{r^2}{R^2}\right)$$

where R is the radius of the pipe, r is the radial distance from the center of the pipe, and $\mathbf{V}_{max}$ is the maximum flow velocity, which occurs at the center, $r = 0$. The shear stress at the pipe surface can be expressed as

$$\tau_w = -\mu\left.\frac{d\mathbf{V}(r)}{dr}\right|_{r=R} = -\mu\mathbf{V}_{\max}\left(-\frac{2r}{R^2}\right)_{r=R} = \frac{2\mu\mathbf{V}_{\max}}{R}$$

The negative sign is added to the viscosity relation for pipes to make shear stress in the flow direction a positive quantity. The drag (or friction) force is obtained by multiplying the wall shear by the surface area,

$$F_D = \tau_w A_s = \frac{2\mu\mathbf{V}_{\max}}{R}(2\pi RL) = 4\pi\mu L\mathbf{V}_{\max}$$

Substituting,

$$F_D = 4\pi\mu L\mathbf{V}_{\max} = 4\pi(0.0010\,\text{kg/m}\cdot\text{s})(15\,\text{m})(3\,\text{m/s})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{0.565\,N}$$

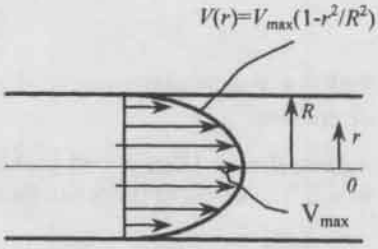
Discussion In the entrance region and during turbulent flow, the velocity gradient is greater near the wall, and thus the drag force in such cases will be greater.

9-12 The velocity profile for laminar one-dimensional flow through a circular pipe is given. A relation for the drag force and its numerical value for water are to be determined.

Assumptions **1** The flow through the circular pipe is one-dimensional. **2** The fluid is newtonian.

Properties The viscosity of water at 20°C is given to be 0.0010 kg/m·s.

Analysis The velocity profile for laminar flow is given by



$$\mathbf{V}(r) = \mathbf{V}_{\max}\left(1-\frac{r^2}{R^2}\right)$$

where R is the radius of the pipe, r is the radial distance from the center of the pipe, and $\mathbf{V}_{max}$ is the maximum flow velocity, which occurs at the center, $r = 0$. The shear stress at the pipe surface can be expressed as

$$\tau_w = -\mu\left.\frac{d\mathbf{V}(r)}{dr}\right|_{r=R} = -\mu\mathbf{V}_{\max}\left(-\frac{2r}{R^2}\right)_{r=R} = \frac{2\mu\mathbf{V}_{\max}}{R}$$

The negative sign is added to the viscosity relation for pipes to make shear stress in the flow direction a positive quantity. The drag (or friction) force is obtained by multiplying the wall shear by the surface area,

$$F_D = \tau_w A_s = \frac{2\mu\mathbf{V}_{\max}}{R}(2\pi RL) = 4\pi\mu L\mathbf{V}_{\max}$$

Substituting,

$$F_D = 4\pi\mu L\mathbf{V}_{\max} = 4\pi(0.0010\,\text{kg/m}\cdot\text{s})(15\,\text{m})(5\,\text{m/s})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{0.942\,N}$$

Discussion In the entrance region and during turbulent flow, the velocity gradient is greater near the wall, and thus the drag force in such cases will be larger.

Surface Tension and Capillary Effect

9-13C The magnitude of the pulling force at the surface of a liquid per unit length is called ***surface tension*** σ_s. It is caused by the attractive forces between the molecules. The surface tension is also surface energy since it represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.

9-14C The pressure inside a soap bubble is greater than the pressure outside, as evidenced by the stretch of the soap film.

9-15C The *capillary effect* is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. It is caused by the net effect of the *cohesive forces* (the forces between like molecules, like water) and *adhesive forces* (the forces between disalike molecules, like water and glass). The capillary effect is proportional to the cosine of the *contact angle*, which is the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.

9-16C The liquid level in the tube will drop since the contact angle is greater than 90°, and $\cos 110° < 0$.

9-17C The capillary rise is inversely proportional to the diameter of the tube, and thus it is greater in the smaller-diameter tube.

9-18E A slender glass tube is inserted into kerosene. The capillary rise of kerosene in the tube is to be determined.

Assumptions **1** There are no impurities in the kerosene, and no contamination on the surfaces of the glass tube. **2** The kerosene is open to the atmospheric air.

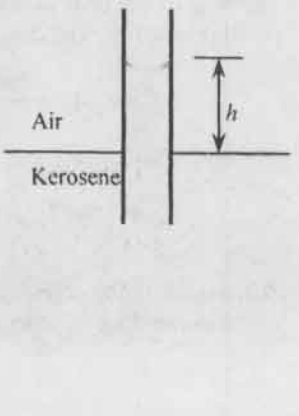
Properties The surface tension of kerosene-glass at 68°F (20°C) is $\sigma_s = 0.028 \times 0.06852 = 0.00192$ lbf/ft (Table 9-2). The density of kerosene at 68°F is $\rho = 51.2$ lbm/ft^3 (Table A-3E). The contact angle of kerosene with the glass surface is given to be 26°.

Analysis Substituting the numerical values, the capillary rise is determined to be

$$h = \frac{2\sigma_s \cos\phi}{\rho g R} = \frac{2(0.00192 \text{ lbf/ft})(\cos 26°)}{(51.2 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(0.015/12 \text{ ft})}\left(\frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}}\right)$$

$$= 0.0539 \text{ ft}$$

$$= 0.65 \text{ in}$$



Discussion The capillary rise in this case more than half of an inch, and thus it is clearly noticeable.

9-19 A glass tube is inserted into a liquid, and the capillary rise is measured. The surface tension of the liquid is to be determined.

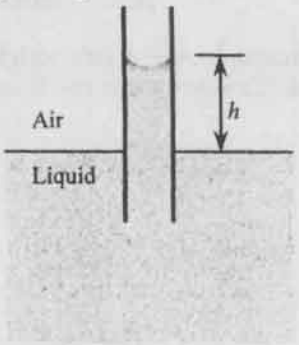
Assumptions **1** There are no impurities in the liquid, and no contamination on the surfaces of the glass tube. **2** The liquid is open to the atmospheric air.

Properties The density of the liquid is given to be 960 kg/m^3. The contact angle is given to be 15°.

Analysis Substituting the numerical values, the surface tension is determined from the capillary rise relation to be

$$\sigma_s = \frac{\rho g R h}{2\cos\phi} = \frac{(960\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.0019/2\,\text{m})(0.005\,\text{m})}{2(\cos 15°)}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{0.0232\ N/m}$$

Discussion The surface tension depends on temperature. Therefore, the value determined is valid at the temperature of the liquid.



9-20 The diameter of a soap bubble is given. The gage pressure inside the bubble is to be determined.

Assumptions The soap bubble is in atmospheric air.

Properties The surface tension of soap water at 20°C is σ_s = 0.025 N/m (Table 9-2).

Analysis The pressure difference between the inside and the outside of a bubble is given by

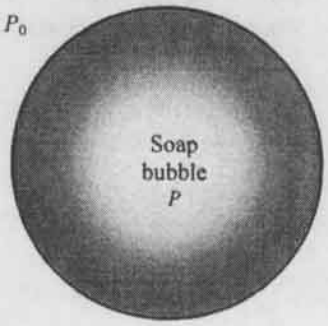
$$\Delta P_{\text{bubble}} = P_i - P_0 = \frac{4\sigma_s}{R}$$

In the open atmosphere $P_0 = P_{\text{atm}}$, and thus ΔP_{bubble} is equivalent to the gage pressure. Substituting,

$$P_{i,gage} = \Delta P_{\text{bubble}} = \frac{4(0.025\,\text{N/m})}{0.002/2\,\text{m}} = 100\,\text{N/m}^2 = 100\,\text{Pa}$$

$$P_{i,gage} = \Delta P_{\text{bubble}} = \frac{4(0.025\,\text{N/m})}{0.05/2\,\text{m}} = 4\,\text{N/m}^2 = 4\,\text{Pa}$$

Discussion Note that the gage pressure in a soap bubble is inversely proportional to the radius. Therefore, the excess pressure is larger in smaller bubbles.



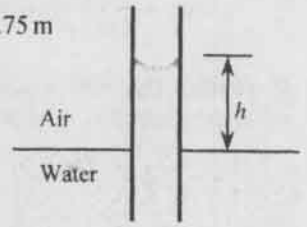
9-21 Nutrients dissolved in water are carried to upper parts of plants. The height that the water solution will rise in a tree as a result of the capillary effect is to be determined.

Assumptions **1** The solution can be treated as water with a contact angle of 15°. **2** The diameter of the tube is constant. **3** The temperature of the water solution is 20°C.

Properties The surface tension of water at 20°C is σ_s = 0.073 N/m (Table 9-2). The density of water solution can be taken to be 1000 kg/m^3. The contact angle is given to be 15°.

Analysis Substituting the numerical values, the capillary rise is determined to be

$$h = \frac{2\sigma_s \cos\phi}{\rho g R} = \frac{2(0.073\ \text{N/m})(\cos 15°)}{(100\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.5\times10^{-6}\ \text{m})}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 5.75\ \text{m}$$



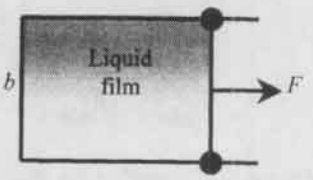
Discussion Other effects such as the chemical potential difference also cause the fluid to rise in trees.

9-22 The force acting on the movable wire of a liquid film suspended on a U-shaped wire frame is measured. The surface tension of the liquid in the air is to be determined.

Assumptions **1** There are no impurities in the liquid, and no contamination on the surfaces of the wire frame. **2** The liquid is open to the atmospheric air.

Analysis Substituting the numerical values, the surface tension is determined from the surface tension force relation to be

$$\sigma_s = \frac{F}{2b} = \frac{0.012\ \text{N}}{2(0.08\ \text{m})} = \mathbf{0.075\ N/m}$$



Discussion The surface tension depends on temperature. Therefore, the value determined is valid at the temperature of the liquid.

Review Problems

9-23E A glass tube is inserted into mercury. The capillary drop of mercury in the tube is to be determined.

Assumptions **1** There are no impurities in mercury, and no contamination on the surfaces of the glass tube. **2** The mercury is open to the atmospheric air.

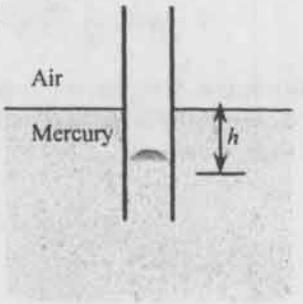
Properties The surface tension of mercury-glass in atmospheric air at 68°F (20°C) is $\sigma_s = 0.440 \times 0.06852 = 0.03015$ lbf/ft (Table 9-2). The density of mercury is $\rho = 847$ lbm/ft^3 at 77°F (Table A-3E), but we can also use this value at 68°F. The contact angle is given to be 140°.

Analysis Substituting the numerical values, the capillary drop is determined to be

$$h = \frac{2\sigma_s \cos\phi}{\rho g R} = \frac{2(0.03015\ \text{lbf/ft})(\cos 140°)}{(847\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(0.45/12\ \text{ft})}\left(\frac{32.2\ \text{lbm}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right)$$

$$= -0.00145\ \text{ft}$$

$$= -0.0175\ \text{in}$$



Discussion The negative sign indicates capillary drop instead of rise. The drop is very small in this case because of the large diameter of the tube.

9-24 A relation is to be derived for the capillary rise of a liquid between two large parallel plates a distance t apart inserted into a liquid vertically. The contact is given to be φ..

Assumptions There are no impurities in the liquid, and no contamination on the surfaces of the plates.

Analysis The magnitude of the capillary rise between two large parallel plates can be determined from a force balance on the rectangular liquid column of height h and width w between the plates. The bottom of the liquid column is at the same level as the free surface of the liquid reservoir, and thus the pressure there must be atmospheric pressure. This will balance the atmospheric pressure acting from the top surface, and thus these two effects will cancel each other. The weight of the liquid column is

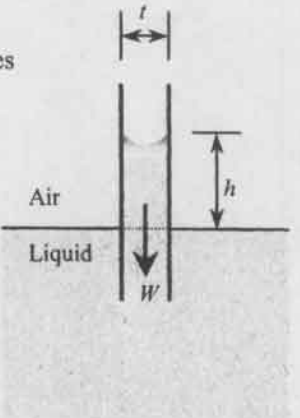
$$W = mg = \rho g V = \rho g(w \times t \times h)$$

Equating the vertical component of the surface tension force to the weight gives

$$W = F_{surface} \quad \rightarrow \quad \rho g(w \times t \times h) = 2w\sigma_s \cos\phi$$

Canceling w and solving for h gives the capillary rise to be

Capillary rise: $$h = \frac{2\sigma_s \cos\phi}{\rho g t}$$



Discussion The relation above is also valid for non-wetting liquids (such as mercury in glass), and gives the capillary drop.

9-25 A journal bearing is lubricated with oil whose viscosity is known. The torques needed to overcome the bearing friction during start-up and steady operation are to be determined.

Assumptions **1** The gap is uniform, and is completely filled with oil. **2** The end effects on the sides of the bearing are negligible. **3** The fluid is newtonian.

Properties The viscosity of oil is given to be 0.1 kg/m·s at 20°C, and 0.008 kg/m·s at 80°C.

Analysis The radius of the shaft is $R = 0.04$ m. Substituting the given values, the torque is determined to be

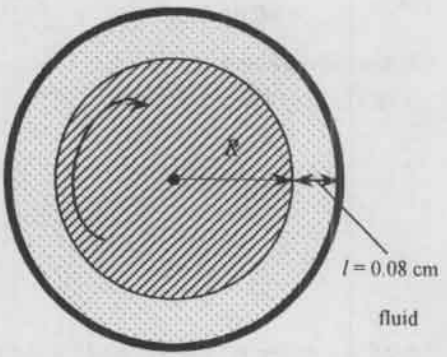
At start up at 20°C:

$$\mathbf{T} = \mu \frac{4\pi^2 R^3 \dot{n} L}{\ell} = (0.1\,\text{kg/m}\cdot\text{s}) \frac{4\pi^2 (0.04\,\text{m})^3 (500/60\,\text{s}^{-1})(0.30\,\text{m})}{0.00008\,\text{m}} = \mathbf{0.79\,N\cdot m}$$

During steady operation at 80°C:

$$\mathbf{T} = \mu \frac{4\pi^2 R^3 \dot{n} L}{\ell} = (0.008\,\text{kg/m}\cdot\text{s}) \frac{4\pi^2 (0.04\,\text{m})^3 (500/60\,\text{s}^{-1})(0.30\,\text{m})}{0.00008\,\text{m}} = \mathbf{0.063\,N\cdot m}$$

Discussion Note that the torque needed to overcome friction reduces considerably due to the decrease in the viscosity of oil at higher temperature.



Chapter 10
FLUID STATICS

Fluid Statics: Hydrostatic Forces on Plane and Curved Surfaces

10-1C The resultant hydrostatic force acting on a submerged surface is the resultant of the pressure forces acting on the surface. The point of application of this resultant force is called the center of pressure.

10-2C Yes, because the magnitude of the resultant force acting on a plane surface of a completely submerged body in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface. The pressure at the centroid of the surface is $P_C = P_0 + \rho g h_C$ where h_C is the vertical distance of the centroid from the free surface of the liquid.

10-3C There will be no change on the hydrostatic force acting on the top surface of this submerged horizontal flat plate as a result of this rotation since the magnitude of the resultant force acting on a plane surface of a completely submerged body in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface.

10-4C The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic force acting on the vertical projection of the curved surface.

10-5C The vertical component of the hydrostatic force acting on a curved surface is equal to the hydrostatic force acting on the horizontal projection of the curved surface, plus (minus, if acting in the opposite direction) the weight of the fluid block.

10-6C The resultant hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface, and all lines normal to the surface of a circle pass through the center of the circle. Thus the pressure forces form a concurrent force system at the center, which can be reduced to a single equivalent force at that point. If the magnitudes of the horizontal and vertical components of the resultant hydrostatic force are known, the tangent of the angle the resultant hydrostatic force makes with the horizontal is $\tan \alpha = F_V / F_H$.

10-7 A car is submerged in water. The hydrostatic force on the door and its line of action are to be determined for the cases of the car containing atmospheric air and the car is filled with water.

Assumptions **1** The bottom surface of the lake is horizontal. **2** The door can be approximated as a vertical rectangular plate. **3** The pressure in the car remains at atmospheric value since there is no water leaking in, and thus no compression of the air inside. Therefore, we can ignore the atmospheric pressure in calculations since it acts on both sides of the door.

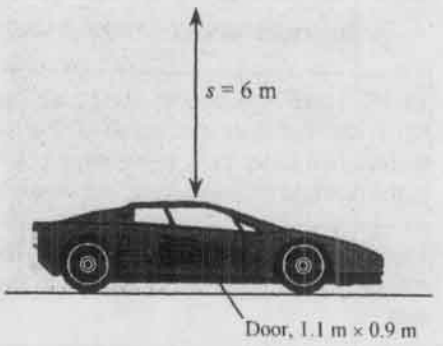
Properties We take the density of lake water to be 1000 kg/m^3 throughout.

Analysis (*a*) When the car is well-sealed and thus the pressure inside the car is the atmospheric pressure, the average pressure on the outer surface of the door is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = P_C = \rho g h_C = \rho g(s + b/2)$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(6 + 1.1/2\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 64.3\,\text{kN/m}^2$$



Then the resultant hydrostatic force on the door becomes

$$F_R = P_{ave} A = (64.3\,\text{kN/m}^2)(0.9\ \text{m} \times 1.1\,\text{m}) = \mathbf{63.7\,kN}$$

The pressure center is directly under the midpoint of the plate, and its distance from the surface of the lake is determined to be

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 6 + \frac{1.1}{2} + \frac{1.1^2}{12(6 + 1.1/2)} = \mathbf{6.56\,m}$$

(*b*) When the car is filled with water, the net force normal to the surface of the door is **zero** since the pressure on both sides of the door will be the same.

Discussion Note that it is impossible for a person to open the door of the car when it is filled with atmospheric air. But it takes no effort to open the door when car is filled with water.

10-8E The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per ft length are to be determined.

Assumptions **1** The hinge is frictionless. **2** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

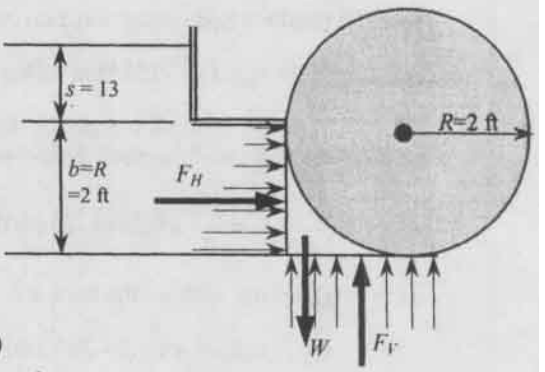
Analysis (*a*) We consider the free body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block per ft length of the cylinder are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave} A = \rho g h_C A = \rho g (s + R/2) A$$

$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(13 + 2/2 \text{ ft})(2 \text{ ft} \times 1 \text{ ft}) \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right)$$

$$= 1747 \text{ lbf}$$



Vertical force on horizontal surface (*upward*):

$$F_y = P_{ave} A = \rho g h_C \cdot A = \rho g h_{\text{bottom}} A$$

$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(15 \text{ ft})(2 \text{ ft} \times 1 \text{ ft}) \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right)$$

$$= 1872 \text{ lbf}$$

Weight of fluid block per ft length *(downward)*:

$$W = mg = \rho g V = \rho g (R^2 - \pi R^2/4)(1 \text{ ft}) = \rho g R^2 (1 - \pi/4)(1 \text{ ft})$$

$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(2 \text{ ft})^2 (1 - \pi/4)(1 \text{ ft}) \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right)$$

$$= 54 \text{ lbf}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 1872 - 54 = 1818 \text{ lbf}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{1747^2 + 1818^2} = \mathbf{2521 \text{ lbf}}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{1848 \text{ lbf}}{1747 \text{ lbf}} = 1.06 \quad \rightarrow \quad \theta = 46.6°$$

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 2521 lbf per ft length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.6° upwards from the horizontal.

(*b*) When the water level is 15-ft high, the gate opens and the reaction force at the bottom of the cylinder becomes zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about the point *A* where the hinge is and equating it to zero gives

$$F_R R \sin\theta - W_{cyl} R = 0 \quad \rightarrow \quad W_{cyl} = F_R \sin\theta = (2521 \text{ lbf}) \sin 46.6° = \mathbf{1832 \text{ lbf}} \quad \text{(per ft)}$$

Discussion The weight of the cylinder per ft length is determined to be 1832 lbf, which corresponds to a mass of 1832 lbm, and to a density of 296 lbm/ft^3 for the material of the cylinder.

10-9 An above the ground swimming pool is filled with water. The hydrostatic force on each wall and the distance of the line of action from the ground are to be determined, and the effect of doubling the wall height on the hydrostatic force is to be assessed.

Assumptions The atmospheric pressure acts on both sides of the wall of the pool, and thus it can be ignored in calculations for convenience.

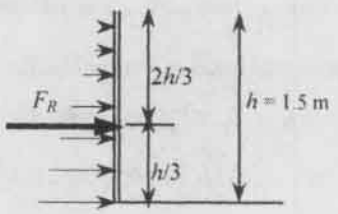
Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = P_C = \rho g h_C = \rho g(h/2)$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(1.5/2\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 7358\,\text{N/m}^2$$



Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave} A = (7358\,\text{N/m}^2)(4\,\text{m} \times 1.5\,\text{m}) = 44{,}148\,\text{N} \cong \mathbf{44.1\,kN}$$

The line of action of the force passes through the pressure center, which is 2h/3 from the free surface and h/3 from the bottom of the pool. Therefore, the distance of the line of action from the ground is

$$y_P = \frac{h}{3} = \frac{1.5}{3} = \mathbf{0.50\,m} \quad \text{(from the bottom)}$$

If the height of the walls of the pool is doubled, the hydrostatic force **quadruples** since

$$F_R = \rho g h_C A = \rho g(h/2)(h \times w) = \rho g w h^2/2$$

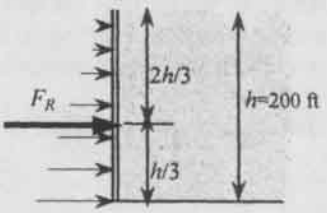
and thus the hydrostatic force is proportional to the square of the wall height, h^2.

10-10E A dam is filled to capacity. The total hydrostatic force on the dam, and the pressures at the top and the bottom are to be determined.

Assumptions The atmospheric pressure acts on both sides of the dam, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{ave} = \rho g h_C = \rho g(h/2)$$

$$= (62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(200/2\ \text{ft})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)$$

$$= 6240\ \text{lbf/ft}^2$$

Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{ave} A = (6240\ \text{lbf/ft}^2)(200\ \text{ft} \times 800\ \text{ft}) = \mathbf{9.98 \times 10^8\ lbf}$$

Resultant force per unit area is pressure, and its value at the top and the bottom of the dam becomes

$$P_{top} = \rho g h_{top} = \mathbf{0\ lbf/ft^2}$$

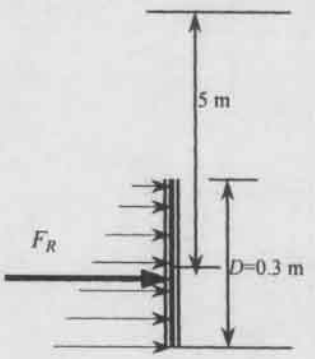
$$P_{bottom} = \rho g h_{bottom} = (62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(200\ \text{ft})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{12{,}480\ lbf/ft^2}$$

10-11 A room in the lower level of a cruise ship is considered. The hydrostatic force acting on the window and the pressure center are to be determined. .

Assumptions The atmospheric pressure acts on both sides of the window, and thus it can be ignored in calculations for convenience.

Properties The specific gravity of sea water is given to be 1.025, and thus its density is 1025 kg/m^3.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{ave} = P_C = \rho g h_C = (1025\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 50{,}276\ \text{N/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave} A = P_{ave}[\pi D^2/4] = (50{,}276\ \text{N/m}^2)[\pi(0.3\ \text{m})^2/4] = \mathbf{3554\ N}$$

The line of action of the force passes through the pressure center, whose vertical distance from the free surface is determined from

$$y_P = y_C + \frac{I_{xx,C}}{y_C A} = y_C + \frac{\pi R^4/4}{y_C \pi R^2} = y_C + \frac{R^2}{4y_C} = 5 + \frac{(0.15\ \text{m})^2}{4(5\ \text{m})} = \mathbf{5.0011\ m}$$

Discussion Note that for small surfaces deep in a liquid, the pressure center nearly coincides with the centroid of the surface.

9-12 The cross-section of a dam is a quarter-circle. The hydrostatic force on the dam and its line of action are to be determined.

Assumptions The atmospheric pressure acts on both sides of the dam, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

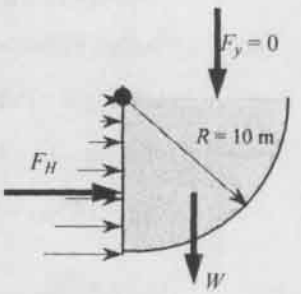
Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the dam and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave} A = \rho g h_C A = \rho g (R/2) A$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(10/2\,\text{m})(10\,\text{m} \times 100\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 4.905 \times 10^7\ \text{N}$$



Vertical force on horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per m length is

$$F_V = W = \rho g V = \rho g [w \times \pi R^2 / 4]$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)[(100\,\text{m})\pi(10\,\text{m})^2/4]\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 7.705 \times 10^7\ \text{N}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the dam become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(4.905 \times 10^7\ \text{N})^2 + (7.705 \times 10^7\ \text{N})^2} = \mathbf{9.134 \times 10^7\ N}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{7.705 \times 10^7\ \text{N}}{4.905 \times 10^7\ \text{N}} = 1.571 \quad \rightarrow \quad \theta = 57.5°$$

Therefore, the line of action of the hydrostatic force passes through the center of the curvature of the dam, making 57.5° downwards from the horizontal.

10-13 A rectangular plate hinged about a horizontal axis along its upper edge blocks a fresh water channel. The plate is restrained from opening by a fixed ridge at a point *B*. The force exerted to the plate by the ridge is to be determined.

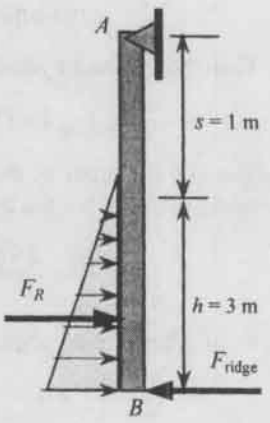
Assumptions The atmospheric pressure acts on both sides of the plate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = P_C = \rho g h_C = \rho g(h/2)$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(3/2\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) = 14.7\,\text{kN/m}^2$$



Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave} A = (14.7\,\text{kN/m}^2)(3\,\text{m}\times 5\,\text{m}) = \mathbf{221\,kN}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2\times(3\,\text{m})}{3} = 2\,\text{m}$$

Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R(s + y_P) = F_{\text{ridge}}\,\overline{AB}$$

Solving for F_{ridge} and substituting, the reaction force is determined to be

$$F_{\text{ridge}} = \frac{s + y_P}{\overline{AB}} F_R = \frac{(1+2)\,\text{m}}{4\,\text{m}}(221\,\text{kN}) = \mathbf{166\,kN}$$

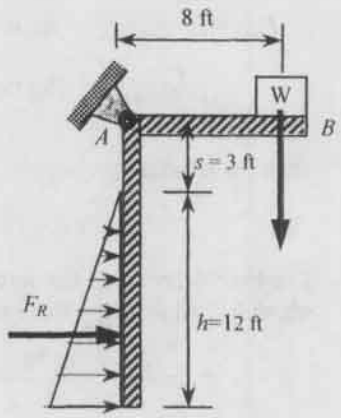
10-14E The flow of water from a reservoir is controlled by an L-shaped gate hinged at a point A. The required weight W for the gate to open at a specified water height is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** The weight of the gate is negligible.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = \rho g h_C = \rho g(h/2)$$
$$= (62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(12/2\ \text{ft})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)$$
$$= 374.4\ \text{lbf/ft}^2$$



Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{ave} A = (374.4\ \text{lbf/ft}^2)(12\ \text{ft} \times 5\ \text{ft}) = 22{,}464\ \text{lbf}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2 \times (12\ \text{ft})}{3} = 8\ \text{ft}$$

Taking the moment about point A and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R(s + y_P) = W\,\overline{AB}$$

Solving for W and substituting, the required weight is determined to be

$$W = \frac{s + y_P}{\overline{AB}} F_R = \frac{(3+8)\ \text{ft}}{8\ \text{ft}}(22{,}464\ \text{lbf}) = \mathbf{30{,}888\ lbf}$$

Discussion Note that the required weight is inversely proportional to the distance of the weight from the hinge.

10-15E The flow of water from a reservoir is controlled by an L-shaped gate hinged at a point A. The required weight W for the gate to open at a specified water height is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** The weight of the gate is negligible.

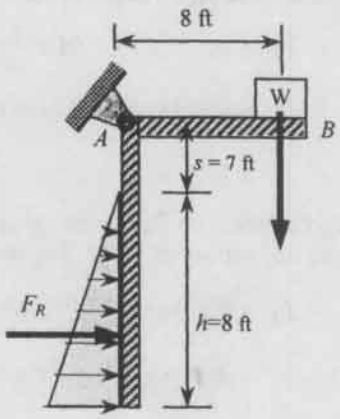
Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = \rho g h_C = \rho g (h/2)$$

$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(8/2 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)$$

$$= 249.6 \text{ lbf/ft}^2$$



Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{ave} A = (249.6 \text{ lbf/ft}^2)(8 \text{ ft} \times 5 \text{ ft}) = 9984 \text{ lbf}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2 \times (8 \text{ ft})}{3} = 5.333 \text{ ft}$$

Taking the moment about point A and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R (s + y_P) = W \overline{AB}$$

Solving for W and substituting, the required weight is determined to be

$$W = \frac{s + y_P}{\overline{AB}} F_R = \frac{(7 + 5.333) \text{ ft}}{8 \text{ ft}} (9984 \text{ lbf}) = \mathbf{15{,}390 \text{ lbf}}$$

Discussion Note that the required weight is inversely proportional to the distance of the weight from the hinge.

10-16 Two parts of a water trough of semi-circular cross-section are held together by cables placed along the length of the trough. The tension **T** in each cable when the trough is full is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

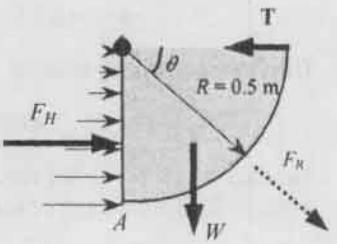
Analysis To expose the cable tension, we consider half of the trough whose cross-section is quarter-circle. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave} A = \rho g h_C A = \rho g (R/2) A$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.5/2\,\text{m})(0.5\,\text{m} \times 3\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 3679\,\text{N}$$



The vertical force on the horizontal surface is zero, since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g [w \times \pi R^2 / 4]$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)[(3\,\text{m})\pi(0.5\,\text{m})^2/4]\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 5779\,\text{N}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 3-m long section of the trough become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(3679\,\text{N})^2 + (5779\,\text{N})^2} = 6851\,\text{N}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{5779\,\text{N}}{3679\,\text{N}} = 1.571 \quad \rightarrow \quad \theta = 57.5°$$

Therefore, the line of action passes through the center of the curvature of the trough, making 57.5° downwards from the horizontal. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R R \sin(90 - 57.5)° = \mathbf{T}R$$

Solving for **T** and substituting, the tension in the cable is determined to be

$$\mathbf{T} = \frac{F_R}{\sin(90 - 57.5)°} = \frac{6851\,\text{N}}{\sin(90 - 57.5)°} = \mathbf{12{,}734\,N}$$

Discussion This problem can also be solved without finding F_R by finding the lines of action of the horizontal hydrostatic force and the weight.

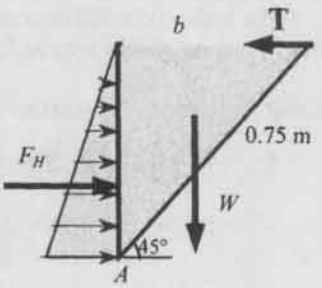
9-17 Two parts of a water trough of triangular cross-section are held together by cables placed along the length of the trough. The tension **T** in each cable when the trough is filled to the rim is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis To expose the cable tension, we consider half of the trough whose cross-section is triangular. The water height h at the midsection of the trough and width of the free surface are

$$h = L\sin\theta = (0.75\,\text{m})\sin 45° = 0.530\,\text{m}$$
$$b = L\cos\theta = (0.75\,\text{m})\cos 45° = 0.530\,\text{m}$$



The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(h/2)A$$
$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.530/2\,\text{m})(0.530\,\text{m}\times 3\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$
$$= 4133\,\text{N}$$

The vertical force on the horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g[w\times bh/2]$$
$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)[(3\,\text{m})(0.530\,\text{m})(0.530\,\text{m})/2]\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$
$$= 4133\,\text{N}$$

The distance of the centroid of a triangle from a side is 1/3 of the height of the triangle for that side. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad W\frac{b}{3} + F_H\frac{h}{3} = \mathbf{T}h$$

Solving for **T** and substituting, and noting that $h = b$, the tension in the cable is determined to be

$$\mathbf{T} = \frac{F_H + W}{3} = \frac{(4133+4133)\,\text{N}}{3} = \mathbf{2755\,N}$$

9-18 Two parts of a water trough of triangular cross-section are held together by cables placed along the length of the trough. The tension **T** in each cable when the trough is filled to the rim is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis To expose the cable tension, we consider half of the trough whose cross-section is triangular. The water height is given to be $h = 0.4$ m at the midsection of the trough, which is equivalent to the width of the free surface b since $\tan 45° = b/h = 1$.

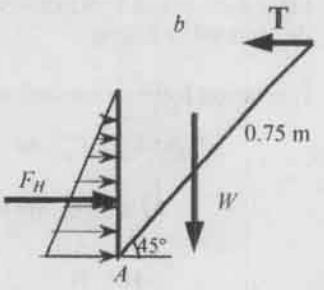
The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave} A = \rho g h_C \cdot A = \rho g (h/2) A$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.4/2\,\text{m})(0.4\,\text{m} \times 3\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 2354\,\text{N}$$



The vertical force on the horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g [w \times bh/2]$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)[(3\,\text{m})(0.4\,\text{m})(0.4\,\text{m})/2]\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 2354\,\text{N}$$

The distance of the centroid of a triangle from a side is 1/3 of the height of the triangle for that side. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad W\frac{b}{3} + F_H \frac{h}{3} = \mathbf{T}h$$

Solving for **T** and substituting, and noting that $h = b$, the tension in the cable is determined to be

$$\mathbf{T} = \frac{F_H + W}{3} = \frac{(2354 + 2354)\,\text{N}}{3} = \mathbf{1569\,N}$$

10-19 A retaining wall against mud slide is to be constructed by rectangular concrete blocks. The mud height at which the blocks will start sliding, and the blocks will tip over are to be determined.

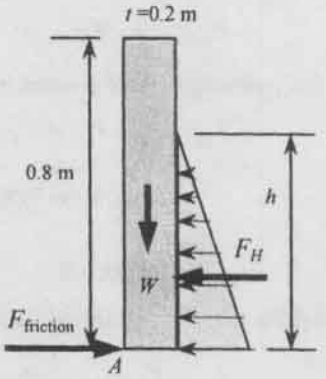
Assumptions The atmospheric pressure acts on both sides of the wall, and thus it can be ignored in calculations for convenience.

Properties The density is given to be 1800 kg/m^3 for the mud, and 2700 kg/m^3 for concrete blocks.

Analysis (*a*) The weight of the concrete wall per unit length (L = 1 m) and the friction force between the wall and the ground are

$$W_{\text{block}} = \rho g V = (2700\,\text{kg/m}^3)(9.81\,\text{m/s}^2)[0.2\times 0.8\times 1\,\text{m}^3)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 4238\,\text{N}$$

$$F_{\text{friction}} = \mu W_{\text{block}} = 0.3(4238\,\text{N}) = 1271\,\text{N}$$



The hydrostatic force exerted by the mud to the wall is

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(h/2)A$$

$$= (1800\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(h/2)(1\times h)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 8829h^2\ \text{N}$$

Setting the hydrostatic and friction forces equal to each other gives

$$F_H = F_{\text{friction}} \quad \rightarrow \quad 8829h^2 = 1271 \quad \rightarrow \quad h = \mathbf{0.38\,m}$$

(*b*) The line of action of the hydrostatic force passes through the pressure center, which is 2*h*/3 from the free surface. The line of action of the weight of the wall passes through the midplane of the wall. Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad W_{\text{block}}(t/2) = F_H(h/3) \quad \rightarrow \quad W_{\text{block}}(t/2) = 8829h^3/3$$

Solving for *h* and substituting, the mud height for tip over is determined to be

$$h = \left(\frac{3W_{block}t}{2\times 8829}\right)^{1/3} = \left(\frac{3\times 4238\times 0.2}{2\times 8829}\right)^{1/3} = \mathbf{0.52\,m}$$

Discussion Note that the concrete wall will slide before tipping. Therefore, sliding is more critical than tipping in this case.

10-20 A retaining wall against mud slide is to be constructed by rectangular concrete blocks. The mud height at which the blocks will start sliding, and the blocks will tip over are to be determined.

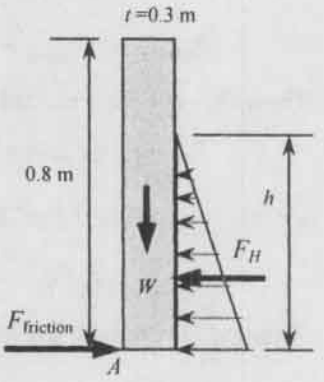
Assumptions The atmospheric pressure acts on both sides of the wall, and thus it can be ignored in calculations for convenience.

Properties The density is given to be 1800 kg/m^3 for the mud, and 2700 kg/m^3 for concrete blocks.

Analysis (*a*) The weight of the concrete wall per unit length (L = 1 m) and the friction force between the wall and the ground are

$$W_{\text{block}} = \rho g V = (2700\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[0.3\times 0.8\times 1\ \text{m}^3)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 6357\ \text{N}$$

$$F_{\text{friction}} = \mu W_{\text{block}} = 0.3(6357\ \text{N}) = 1907\ \text{N}$$



The hydrostatic force exerted by the mud to the wall is

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(h/2)A$$

$$= (1800\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h/2)(1\times h)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 8829h^2\ \text{N}$$

Setting the hydrostatic and friction forces equal to each other gives

$$F_H = F_{\text{friction}} \quad \rightarrow \quad 8829\,h^2 = 1907 \quad \rightarrow \quad h = \mathbf{0.47\ m}$$

(*b*) The line of action of the hydrostatic force passes through the pressure center, which is 2*h*/3 from the free surface. The line of action of the weight of the wall passes through the midplane of the wall. Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad W_{\text{block}}(t/2) = F_H(h/3) \quad \rightarrow \quad W_{\text{block}}(t/2) = 8829h^3/3$$

Solving for *h* and substituting, the mud height for tip over is determined to be

$$h = \left(\frac{3W_{\text{block}}t}{2\times 8829}\right)^{1/3} = \left(\frac{3\times 6357\times 0.3}{2\times 8829}\right)^{1/3} = \mathbf{0.69\ m}$$

Discussion Note that the concrete wall will slide before tipping. Therefore, sliding is more critical than tipping in this case.

10-21 A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at B where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to A at the upper edge of the gate is to be determined.

Assumptions **1** The hinge is frictionless. **2** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **3** The weight of the gate is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

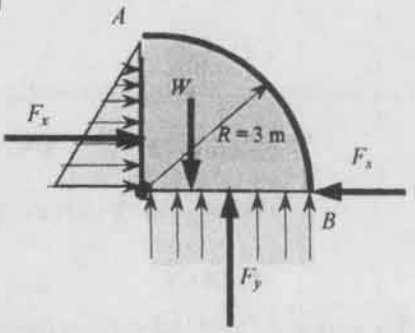
Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave} A = \rho g h_C A = \rho g (R/2) A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3/2\ \text{m})(4\ \text{m} \times 3\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}^2}\right)$$

$$= 176.6\ \text{kN}$$



Vertical force on horizontal surface (upward):

$$F_y = P_{ave} A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(4\ \text{m} \times 3\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}^2}\right)$$

$$= 353.2\ \text{kN}$$

The weight of fluid block per 4-m length (downwards):

$$W = \rho g V = \rho g [w \times \pi R^2 / 4]$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(4\ \text{m})\pi(3\ \text{m})^2/4]\left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}^2}\right)$$

$$= 277.4\ \text{kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 353.2 - 277.4 = 75.8\ \text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(176.6\ \text{kN})^2 + (75.8\ \text{kN})^2} = 192.2\ \text{kN}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{75.8\ \text{kN}}{176.6\ \text{kN}} = 0.429 \quad \rightarrow \quad \theta = 23.2°$$

Therefore, the magnitude of the hydrostatic force acting on the gate is 192.2 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point A where the hinge is, and setting it equal to zero,

$$\sum M_A = 0 \quad \rightarrow \quad F_R R \sin(90 - \theta) - F_{\text{spring}} R = 0$$

Solving for F_{spring} and substituting, the spring force is determined to be

$$F_{\text{spring}} = F_R \sin(90 - \theta) = (192.2\ \text{kN}) \sin(90° - 23.2°) = \mathbf{177\ kN}$$

10-22 A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at B where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to A at the upper edge of the gate is to be determined.

Assumptions **1** The hinge is frictionless. **2** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **3** The weight of the gate is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

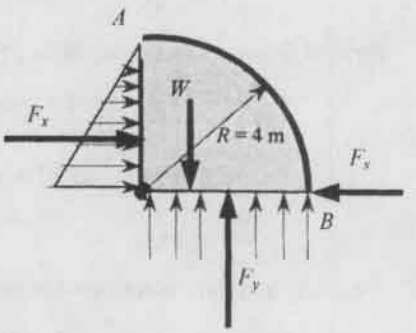
Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave} A = \rho g h_C A = \rho g (R/2) A$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(4/2\,\text{m})(4\,\text{m} \times 4\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 313.9\,\text{kN}$$



Vertical force on horizontal surface (upward):

$$F_y = P_{ave} A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(4\,\text{m})(4\,\text{m} \times 4\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 627.8\,\text{kN}$$

The weight of fluid block per 4-m length (downwards):

$$W = \rho g V = \rho g [w \times \pi R^2 / 4]$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)[(4\,\text{m})\pi(4\,\text{m})^2/4]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 493.1\,\text{kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 627.8 - 493.1 = 134.7\,\text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(313.9\,\text{kN})^2 + (134.7\,\text{kN})^2} = 341.6\ \text{kN}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{134.7\text{kN}}{313.9\,\text{kN}} = 0.429 \quad \rightarrow \quad \theta = 23.2°$$

Therefore, the magnitude of the hydrostatic force acting on the gate is 341.6 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point A where the hinge is, and setting it equal to zero,

$$\sum M_A = 0 \quad \rightarrow \quad F_R R \sin(90 - \theta) - F_{\text{spring}} R = 0$$

Solving for F_{spring} and substituting, the spring force is determined to be

$$F_{\text{spring}} = F_R \sin(90 - \theta) = (341.6\,\text{kN})\sin(90° - 23.2°) = \mathbf{314.0\,kN}$$

Buoyancy

10-23C The upward force a fluid exerts on an immersed body is called the *buoyant force*. The buoyant force is caused by the increase of pressure in a fluid with depth. The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$. The direction of the buoyant force is upwards, and its line of action passes through the centroid of the displaced volume.

10-24C The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$, which is independent of depth. Therefore, the buoyant forces acting on two identical spherical balls submerged in water at different depths will be the same.

10-25C The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$, which is independent of the density of the body (ρ_f is the fluid density). Therefore, the buoyant forces acting on the 5-cm diameter aluminum and iron balls submerged in water will be the same.

10-26C The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$, which is independent of the shape of the body. Therefore, the buoyant forces acting on the cube and sphere made of copper submerged in water will be the same since they have the same volume.

10-27C A *submerged* body whose center of gravity G is above the center of buoyancy B, which is the centroid of the displaced volume, is *unstable*. But a floating body may still be stable when G is above B since the centroid of the displaced volume shifts to the side to a point B' during a rotational disturbance while the center of gravity G of the body remains unchanged. If the point B' is sufficiently far, these two forces create a restoring moment, and return the body to the original position.

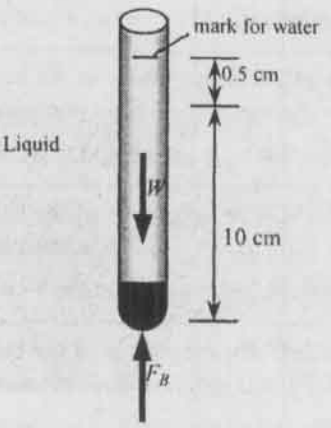
10-28 The density of a liquid is to be determined by a hydrometer by establishing division marks in water and in the liquid, and measuring the distance between these marks.

Properties We take the density of pure water to be 1000 kg/m^3.

Analysis A hydrometer floating in water is in static equilibrium, and the buoyant force F_B exerted by the liquid must always be equal to the weight W of the hydrometer, $F_B = W$.

$$F_B = \rho g V_{sub} = \rho g h A_c$$

where h is the height of the submerged portion of the hydrometer and A_c is the cross-sectional area which is constant.



In pure water: $W = \rho_w g h_w A_c$

In the liquid: $W = \rho_{liquid} g h_{liquid} A_c$

Setting the relations above equal to each other (since both equal the weight of the hydrometer) gives

$$\rho_w g h_w A_c = \rho_{liquid} g h_{liquid} A_c$$

Solving for the liquid density and substituting,

$$\rho_{liquid} = \frac{h_{water}}{h_{liquid}} \rho_{water} = \frac{10\ \text{cm}}{(10 - 0.5)\ \text{cm}} (1000\ \text{kg/m}^3) = \mathbf{1053\ kg/m^3}$$

Discussion Note that for a given cylindrical hydrometer, the product of the fluid density and the height of the submerged portion of the hydrometer is constant in any fluid.

10-29E A concrete block is lowered into the sea. The tension in the rope is to be determined before and after the block is immersed in water.

Assumptions **1** The buoyancy force in air is negligible. **2** The weight of the rope is negligible.

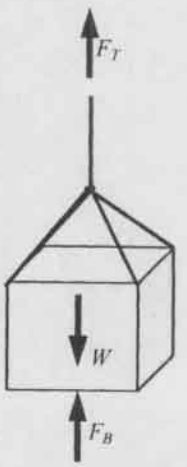
Properties The density of steel block is given to be 494 lbm/ft^3.

Analysis (*a*) The forces acting on the concrete block in air are its downward weight and the upward pull action (tension) by the rope. These two forces must balance each other, and thus the tension in the rope must be equal to the weight of the block:

$$V = 4\pi R^3/3 = 4\pi(1.5\ \text{ft})^3/3 = 14.14\ \text{ft}^3$$

$$F_T = W = \rho_{concrete} g V$$

$$= (494\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(14.14\ \text{ft}^3)\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm} \cdot \text{ft/s}^2}\right) = \mathbf{6984\ lbf}$$



(*b*) When the block is immersed in water, there is the additional force of buoyancy acting upwards. The force balance in this case gives

$$F_B = \rho_f g V = (62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(14.14\ \text{ft}^3)\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm} \cdot \text{ft/s}^2}\right) = 882\ \text{lbf}$$

$$F_{T,water} = W_c - F_B = 6984 - 882 = \mathbf{6102\ lbf}$$

Discussion Note that the weight of the concrete block and thus the tension of the rope decreases by (6984 – 6102)/6984 = 12.6% in water.

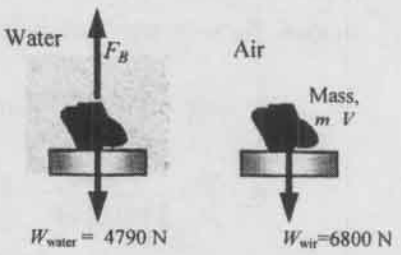
10-30 An irregularly shaped body is weighed in air and then in water with a spring scale. The volume and the average density of the body are to be determined.

Properties We take the density of water to be 1000 kg/m^3.

Assumptions **1** The buoyancy force in air is negligible. **2** The body is completely submerged in water.

Analysis The mass of the body is

$$m = \frac{W_{\text{air}}}{g} = \frac{6800\ \text{N}}{9.81\ \text{m/s}^2}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 693.2\ \text{kg}$$



The difference between the weights in air and in water is due to the buoyancy force in water,

$$F_B = W_{\text{air}} - W_{\text{water}} = 6800 - 4790 = 2010\ \text{N}$$

Noting that $F_B = \rho_{\text{water}} g V$, the volume of the body is determined to be

$$V = \frac{F_B}{\rho_{\text{water}} g} = \frac{2010\ \text{N}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = \mathbf{0.2049\ m^3}$$

Then the density of the body becomes

$$\rho = \frac{m}{V} = \frac{693.2\ \text{kg}}{0.2049\ \text{m}^3} = \mathbf{3383\ kg/m^3}$$

Discussion The volume of the body can also be measured by observing the change in the volume of the container when the body is dropped in it (assuming the body is not porous).

10-31 The height of the portion of a cubic ice block that extends above the water surface is measured. The height of the ice block below the surface is to be determined.

Assumptions **1** The buoyancy force in air is negligible. **2** The top surface of the ice block is parallel to the surface of the sea.

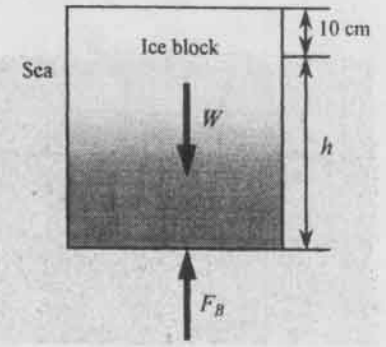
Properties The specific gravities of ice and seawater are given to be 0.92 and 1.025, respectively, and thus the corresponding densities are 920 kg/m^3 and 1025 kg/m^3.

Analysis The weight of a body floating in a fluid is equal to the buoyant force acting on it (a consequence of vertical force balance from static equilibrium). Therefore, in this case the average density of the body must be equal to the density of the fluid since

$$W = F_B$$

$$\rho_{\text{body}} g V_{\text{total}} = \rho_{\text{fluid}} g V_{\text{submerged}}$$

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}}$$



The cross-sectional of a cube is constant, and thus the "volume ratio" can be replaced by "height ratio". Then,

$$\frac{h_{\text{submerged}}}{h_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} \rightarrow \frac{h}{h+0.10} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} \rightarrow \frac{h}{h+0.10} = \frac{0.92}{1.025}$$

where h is the height of the ice block below the surface. Solving for h gives

$$h = 0.876\ \text{m} = \mathbf{87.6\ cm}$$

Discussion Note that the 0.92/1.025 = 88% of the volume of an ice block remains under water. For symmetrical ice blocks this also represents the fraction of height that remains under water.

10-32 A man dives into a lake and tries to lift a large rock. The force that the man needs to apply to lift it from the bottom of the lake is to be determined.

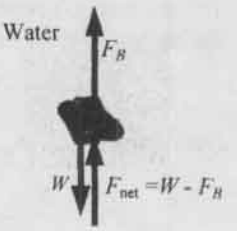
Assumptions **1** The rock is c completely submerged in water. **2** The buoyancy force in air is negligible.

Properties The density of granite rock is given to be 2700 kg/m^3. We take the density of water to be 1000 kg/m^3.

Analysis The weight and volume of the rock are

$$W = mg = (110\,\text{kg})(9.81\,\text{m/s}^2)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 1079\,\text{N}$$

$$V = \frac{m}{\rho} = \frac{110\,\text{kg}}{2700\,\text{kg/m}^3} = 0.04074\,\text{m}^3$$



The buoyancy force acting on the rock is

$$F_B = \rho_{\text{water}} gV$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.04074\,\text{m}^3)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 400\,\text{N}$$

The weight of a body submerged in water is equal to the weigh of the body in air minus the buoyancy force,

$$W_{\text{in water}} = W_{\text{in air}} - F_B = 1079 - 400 = \mathbf{679\,N}$$

Discussion This force corresponds to a mass of

$$m = \frac{W}{g} = \frac{679\,\text{N}}{9.81\,\text{m/s}^2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 69.3\,\text{kg}$$

Therefore, a person who can lift about 70 kg on earth can lift this rock in water.

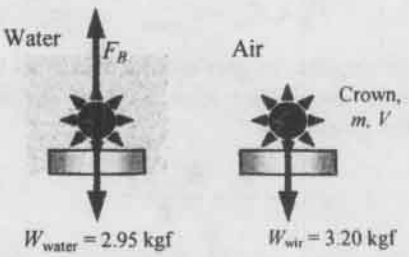
10-33 An irregularly shaped crown is weighed in air and then in water with a spring scale. It is to be determined if the crown is made of pure gold.

Assumptions **1** The buoyancy force in air is negligible. **2** The crown is completely submerged in water.

Properties We take the density of water to be 1000 kg/m^3. The density of gold is given to be 19300 kg/m^3.

Analysis The mass of the crown is

$$m = \frac{W_{\text{air}}}{g} = \frac{31.4\ \text{N}}{9.81\ \text{m/s}^2}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 3.20\ \text{kg}$$



The difference between the weights in air and in water is due to the buoyancy force in water, and thus

$$F_B = W_{\text{air}} - W_{\text{water}} = 31.4 - 28.9 = 2.50\ \text{N}$$

Noting that $F_B = \rho_{\text{water}} g V$, the volume of the crown is determined to be

$$V = \frac{F_B}{\rho_{\text{water}} g} = \frac{2.50\ \text{N}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 2.55\times10^{-4}\ \text{m}^3$$

Then the density of the crown becomes

$$\rho = \frac{m}{V} = \frac{3.20\ \text{kg}}{2.55\times10^{-4}\ \text{m}^3} = 12{,}500\ \text{kg/m}^3$$

which is considerably less than the density of gold. Therefore, the crown is **NOT** made of pure gold.

Discussion This problem can also be solved without doing any under-water weighing as follows: We would weigh a bucket half-filled with water, and drop the crown into it. After marking the new water level, we would take the crown out, and add water to the bucket until the water level rises to the mark. We would weigh the bucket again. Dividing the weight difference by the density of water and g will give the volume of the crown. Knowing both the weight and the volume of the crown, the density can easily be determined.

10-34 The average density of a person is determined by weighing the person in air and then in water. A relation is to be obtained for the volume fraction of body fat in terms of densities.

Assumptions **1** The buoyancy force in air is negligible. **2** The body is considered to consist of fat and muscle only. **3** The body is completely submerged in water, and the air volume in the lungs is negligible.

Analysis The difference between the weights of the person in air and in water is due to the buoyancy force in water. Therefore,

$$F_B = W_{\text{air}} - W_{\text{water}} \quad \rightarrow \quad \rho_{\text{water}} g V = W_{\text{air}} - W_{\text{water}}$$

Knowing the weights and the density of water, the relation above gives the volume of the person. Then the average density of the person can be determined from

$$\rho_{\text{ave}} = \frac{m}{V} = \frac{W_{\text{air}} / g}{V}$$

Under assumption #2, the total mass of a person is equal to the sum of the masses of the fat and muscle tissues, and the total volume of a person is equal to the sum of the volumes of the fat and muscle tissues. The volume fraction of body fat is the ratio of the fat volume to the total volume of the person. Therefore,

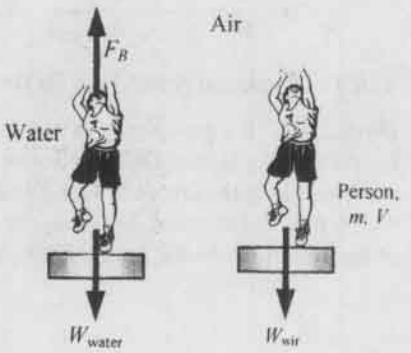
$$V = V_{\text{fat}} + V_{\text{muscle}} \quad \text{where} \quad V_{\text{fat}} = x_{\text{fat}} V \quad \text{and} \quad V_{\text{muscle}} = x_{\text{muscle}} V = (1 - x_{\text{fat}}) V$$
$$m = m_{\text{fat}} + m_{\text{muscle}}$$

Noting that mass is density times volume, the last relation can be written as

$$\rho_{\text{ave}} V = \rho_{\text{fat}} V_{\text{fat}} + \rho_{\text{muscle}} V_{\text{muscle}}$$
$$\rho_{\text{ave}} V = \rho_{\text{fat}} x_{\text{fat}} V + \rho_{\text{muscle}} (1 - x_{\text{fat}}) V$$

Canceling the V and solving for x_{fat} gives the desired relation,

$$x_{\text{fat}} = \frac{\rho_{\text{muscle}} - \rho_{\text{ave}}}{\rho_{\text{muscle}} - \rho_{\text{fat}}}$$



Discussion Weighing a person in water in order to determine its volume is not practical. A more practical way is to use a large container, and measuring the change in volume when the person is completely submerged in it.

10-35 The volume of the hull of a boat is given. The amounts of load the boat can carry in a lake and in the sea are to be determined.

Assumptions **1** The dynamic effects of the waves are disregarded. **2** The buoyancy force in air is negligible.

Properties The density of sea water is given to be $1.03\times1000 = 1030\ \text{kg/m}^3$. We take the density of water to be $1000\ \text{kg/m}^3$.

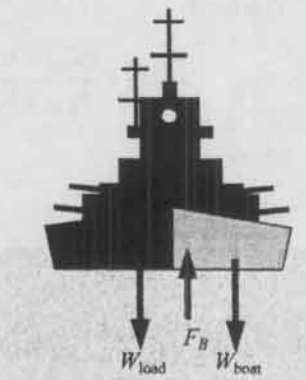
Analysis The weight of the unloaded boat is

$$W_{\text{boat}} = mg = (8560\ \text{kg})(9.81\ \text{m/s}^2)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 84.0\ \text{kN}$$

The buoyancy force becomes a maximum when the entire hull of the boat is submerged in water, and is determined to be

$$F_{B,\text{lake}} = \rho_{\text{lake}} gV = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(150\ \text{m}^3)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 1472\ \text{kN}$$

$$F_{B,\text{sea}} = \rho_{\text{sea}} gV = (1030\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(150\ \text{m}^3)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 1516\ \text{kN}$$



The total weight of a floating boat (load + boat itself) is equal to the buoyancy force. Therefore, the weight of the maximum load is

$$W_{\text{load, lake}} = F_B\text{, lake} - W_{\text{boat}} = 1472 - 84 = 1388\ \text{kN}$$
$$W_{\text{load, sea}} = F_{B,\text{sea}} - W_{\text{boat}} = 1516 - 84 = 1432\ \text{kN}$$

The corresponding masses of load are

$$m_{\text{load,lake}} = \frac{W_{\text{load,lake}}}{g} = \frac{1388\ \text{kN}}{9.81\ \text{m/s}^2}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right) = 141{,}500\ \text{kg}$$

$$m_{\text{load,sea}} = \frac{W_{\text{load,lsea}}}{g} = \frac{1432\ \text{kN}}{9.81\ \text{m/s}^2}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right) = 146.0\ \text{kg}$$

Discussion Note that this boat can carry 4500 kg more load in the sea than it can in fresh water. The fully-loaded boats in sea water should expect to sink into water deeper when they enter fresh water such a river where the port may be.

Review Problems

10-36 The density of a wood log is to be measured by tying lead weights to it until both the log and the weights are completely submerged, and then weighing them separately in air. The average density of a given log is to be determined by this approach.

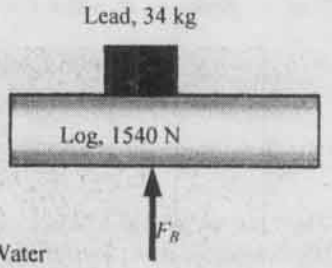
Properties The density of lead weights is given to be 11,300 kg/m^3. We take the density of water to be 1000 kg/m^3.

Analysis The weight of a body is equal to the buoyant force when the body is floating in a fluid while being completely submerged in it (a consequence of vertical force balance from static equilibrium). In this case the average density of the body must be equal to the density of the fluid since

$$W = F_B \quad \rightarrow \quad \rho_{\text{body}} g V = \rho_{\text{fluid}} g V \quad \rightarrow \quad \rho_{\text{body}} = \rho_{\text{fluid}}$$

Therefore,

$$\rho_{ave} = \frac{m_{\text{total}}}{V_{\text{total}}} = \frac{m_{\text{lead}} + m_{\text{log}}}{V_{\text{lead}} + V_{\text{log}}} = \rho_{\text{water}} \quad \rightarrow \quad V_{\text{log}} = V_{\text{lead}} + \frac{m_{\text{lead}} + m_{\text{log}}}{\rho_{\text{water}}}$$



where

$$V_{\text{lead}} = \frac{m_{\text{lead}}}{\rho_{\text{lead}}} = \frac{34\,\text{kg}}{11{,}300\,\text{kg/m}^3} = 3.01\times10^{-3}\ \text{m}^3$$

$$m_{\text{log}} = \frac{W_{\text{log}}}{g} = \frac{1540\,\text{N}}{9.81\,\text{m/s}^2}\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right) = 157.0\,\text{kg}$$

Substituting, the volume and density of the log are determined to be

$$V_{\text{log}} = V_{\text{lead}} + \frac{m_{\text{lead}} + m_{\text{log}}}{\rho_{\text{water}}} = 3.01\times10^{-3}\ \text{m}^3 + \frac{(34+157)\,\text{kg}}{1000\,\text{kg/m}^3} = 0.188\,\text{m}^3$$

$$\rho_{\text{log}} = \frac{m_{\text{log}}}{V_{\text{log}}} = \frac{157\,\text{kg}}{0.188\,\text{m}^3} = 835\,\text{kg/m}^3$$

Discussion Note that the log must be completely submerged for this analysis to be valid. Ideally, the lead weights must also be completely submerged, but this is not very critical because of the small volume of the lead weights.

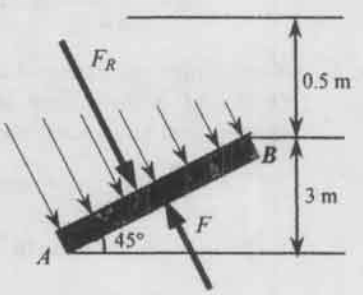
10-37 A rectangular gate that leans against the floor with an angle of 45° with the horizontal is to be opened from its lower edge by applying a normal force at its center. The minimum force F required to open the water gate is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** Friction at the hinge is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The length of the gate and the distance of the upper edge of the gate (point B) from the free surface in the plane of the gate are

$$b = \frac{3\,\text{m}}{\sin 45°} = 4.243\,\text{m} \quad \text{and} \quad s = \frac{0.5\,\text{m}}{\sin 45°} = 0.7071\,\text{m}$$



The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic on the surface,

$$F_R = P_{ave} A = \rho g h_C A$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(2\,\text{m})[5 \times 4.243\,\text{m}^2]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 416\,\text{kN}$$

The distance of the pressure center from the free surface of water along the plane of the gate is

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 0.7071 + \frac{4.243}{2} + \frac{4.243^2}{12(0.7071 + 4.243/2)} = 3.359\,\text{m}$$

The distance of the pressure center from the hinge at point B is

$$L_P = y_P - s = 3.359 - 0.7071 = 2.652\,\text{m}$$

Taking the moment about point B and setting it equal to zero gives

$$\sum M_B = 0 \quad \rightarrow \quad F_R L_P = Fb/2$$

Solving for F and substituting, the required force is determined to be

$$F = \frac{2F_R L_P}{b} = \frac{2(416\,\text{kN})(2.652\,\text{m})}{4.243\,\text{m}} = \mathbf{520\,kN}$$

Discussion The applied force is inversely proportional to the distance of the point of application from the hinge, and the required force can be reduced by applying the force at a lower point on the gate.

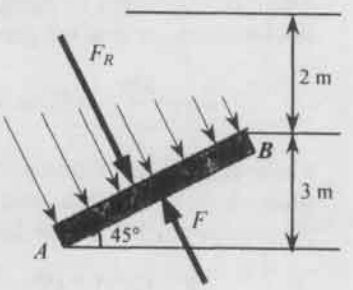
10-38 A rectangular gate that leans against the floor with an angle of 45° with the horizontal is to be opened from its lower edge by applying a normal force at its center. The minimum force F required to open the water gate is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** Friction at the hinge is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The length of the gate and the distance of the upper edge of the gate (point B) from the free surface in the plane of the gate are

$$b = \frac{3\,\text{m}}{\sin 45°} = 4.243\,\text{m} \quad \text{and} \quad s = \frac{2\,\text{m}}{\sin 45°} = 2.828\,\text{m}$$



The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic on the surface,

$$F_R = P_{ave} A = \rho g h_C A$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(3.5\,\text{m})[5 \times 4.243\,\text{m}^2]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 728\,\text{kN}$$

The distance of the pressure center from the free surface of water along the plane of the gate is

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 2.828 + \frac{4.243}{2} + \frac{4.243^2}{12(2.828 + 4.243/2)} = 5.253\,\text{m}$$

The distance of the pressure center from the hinge at point B is

$$L_P = y_P - s = 2.253 - 2.828 = 2.425\,\text{m}$$

Taking the moment about point B and setting it equal to zero gives

$$\sum M_B = 0 \quad \rightarrow \quad F_R L_P = Fb/2$$

Solving for F and substituting, the required force is determined to be

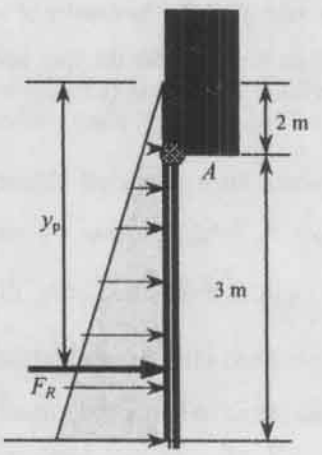
$$F = \frac{2F_R L_P}{b} = \frac{2(728\,\text{N})(2.425\,\text{m})}{4.243\,\text{m}} = \mathbf{832\,kN}$$

Discussion The applied force is inversely proportional to the distance of the point of application from the hinge, and the required force can be reduced by applying the force at a lower point on the gate.

10-39 A rectangular gate hinged about a horizontal axis along its upper edge is restrained by a fixed ridge at point *B*. The force exerted to the plate by the ridge is to be determined.

Assumptions The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.



Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the gate,

$$F_R = P_{ave} A = \rho g h_C A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.5\ \text{m})[5\times 6\ \text{m}^2]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{618\ kN}$$

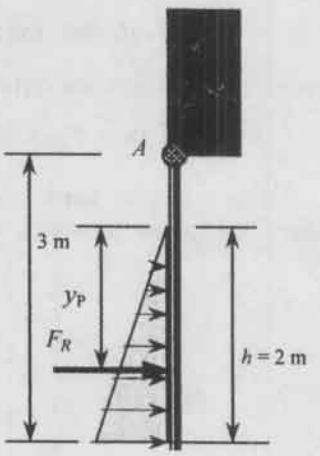
The vertical distance of the pressure center from the free surface of water is

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s+b/2)} = 2 + \frac{3}{2} + \frac{3^2}{12(2+3/2)} = \mathbf{3.71\ m}$$

10-40 A rectangular gate hinged about a horizontal axis along its upper edge is restrained by a fixed ridge at point *B*. The force exerted to the plate by the ridge is to be determined.

Assumptions The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.



Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the wetted plate area gives the resultant hydrostatic force on the gate,

$$F_R = P_{ave} A = \rho g h_C A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1\ \text{m})[2\times 6\ \text{m}^2]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{118\ kN}$$

The vertical distance of the pressure center from the free surface of water is

$$y_P = \frac{2h}{3} = \frac{2(2\ \text{m})}{3} = \mathbf{1.33\ m}$$

10-41E A semicircular tunnel is to be built under a lake. The total hydrostatic force acting on the roof of the tunnel is to be determined.

Assumptions The atmospheric pressure acts on both sides of the tunnel, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

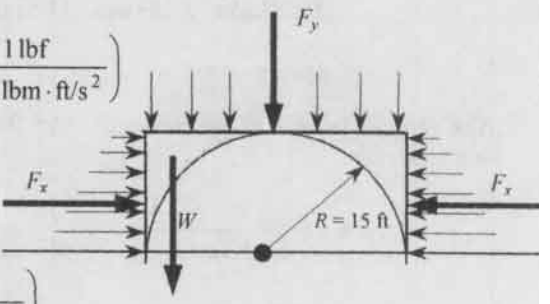
Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the tunnel and its vertical (on both sides) and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface (each side):

$$F_H = F_x = P_{ave} A = \rho g h_C A = \rho g (s + R/2) A$$

$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(135 + 15/2 \text{ ft})(15 \text{ ft} \times 2000 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)$$

$$= 2.667 \times 10^8 \text{ lbf} \quad \text{(on each side of the tunnel)}$$



Vertical force on horizontal surface (*downward*):

$$F_y = P_{ave} A = \rho g h_C A = \rho g h_{top} A$$

$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(135 \text{ ft})(30 \text{ ft} \times 2000 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)$$

$$= 5.054 \times 10^8 \text{ lbf}$$

Weight of fluid block on each side within the control volume *(downward)*:

$$W = mg = \rho g V = \rho g (R^2 - \pi R^2/4)(2000 \text{ ft})$$

$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(15 \text{ ft})^2 (1 - \pi/4)(2000 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)$$

$$= 6.026 \times 10^6 \text{ lbf} \quad \text{(on each side)}$$

Therefore, the net downward vertical force is

$$F_V = F_y + 2W = 2.667 \times 10^8 + 2 \times 0.06026 \times 10^8 = \mathbf{2.788 \times 10^8 \text{ lbf}}$$

This is also the **net force** acting on the tunnel since the horizontal forces acting on the right and left side of the tunnel cancel each other since they are equal ad opposite.

10-42 A hemispherical dome on a level surface filled with water is to be lifted by attaching a long tube to the top and filling it with water. The required height of water in the tube to lift the dome is to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the dome, and thus it can be ignored in calculations for convenience. **2** The weight of the tube and the water in it is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis We take the dome and the water in it as the system. When the dome is about to rise, the reaction force between the dome and the ground becomes zero. Then the free body diagram of this system involves the weights of the dome and the water, balanced by the hydrostatic pressure force from below. Setting these forces equal to each other gives

$$\sum F_y = 0: \qquad F_{hydro} = W_{gate} + W_{water}$$

$$\rho g(h+R)\pi R^2 = m_{gate}\, g + m_{water}\, g$$

Solving for h gives

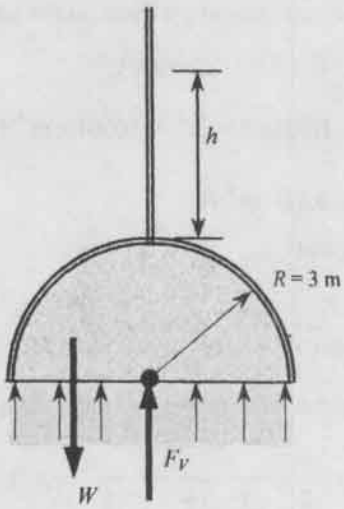
$$h = \frac{m_{dome} + m_{water}}{\rho \pi R^2} - R = \frac{m_{dome} + \rho[4\pi R^3/6]}{\rho \pi R^2} - R$$

Substituting,

$$h = \frac{(500\text{ kg}) + 4\pi(1000\text{ kg/m}^3)(3\text{ m})^3/6}{(1000\text{ kg/m}^3)\pi(3\text{ m})^2} - (3\text{ m}) = \mathbf{2.02\text{ m}}$$

Therefore, this dome can be lifted by attaching a tube which is 2.02 m long.

Discussion This problem can also be solved without finding F_R by finding the lines of action of the horizontal hydrostatic force and the weight.



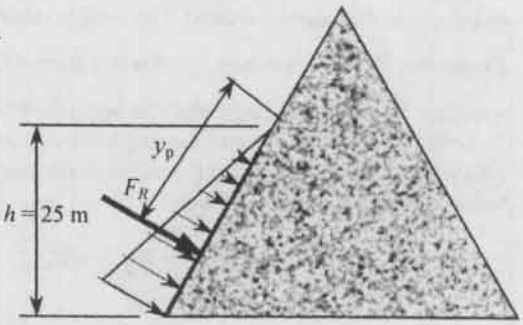
10-43 The water in a reservoir is restrained by a triangular wall. The total force (hydrostatic + atmospheric) acting on the inner surface of the wall and the horizontal component of this force are to be determined.

Assumptions **1** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** Friction at the hinge is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The length of the wall surface underwater is

$$b = \frac{25\text{ m}}{\sin 60^\circ} = 28.87\text{ m}$$



The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the surface,

$$F_R = P_{ave}A = (P_{atm} + \rho g h_C)A$$

$$= [100{,}000\text{ N/m}^2 + (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(12.5\text{ m})](150 \times 28.87\text{ m}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{9.64 \times 10^8\ N}$$

Noting that

$$\frac{P_0}{\rho g \sin 60^\circ} = \frac{100{,}000\text{ N/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\sin 60^\circ}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 11.77\text{ m}$$

the distance of the pressure center from the free surface of water along the wall surface is

$$y_p = s + \frac{b}{2} + \frac{b^2}{12\left(s + \frac{b}{2} + \frac{P_0}{\rho g \sin\theta}\right)} = 0 + \frac{28.87\text{ m}}{2} + \frac{(28.87\text{ m})^2}{12\left(0 + \frac{28.87\text{ m}}{2} + 11.77\text{ m}\right)} = \mathbf{17.1\ m}$$

The magnitude of the horizontal component of the hydrostatic force is simply $F_R \sin\theta$,

$$F_H = F_R \sin\theta = (9.64 \times 10^8\text{ N})\sin 60^\circ = \mathbf{8.35 \times 10^8\ N}$$

Discussion The atmospheric pressure is usually ignored in the analysis for convenience since it acts on both sides of the walls.

Computer, Design, and Essay Problems

10-45 The volume of a rock can be determined without using any volume measurement devices as follows: We weigh the rock in the air and then in the water. The difference between the two weights is due to the buoyancy force, which is equal to $F_B = \rho_{water} g V_{body}$. Solving this relation for V_{body} gives the volume of the rock.

Chapter 11
BERNOULLI, ENERGY, AND MOMENTUM EQUATIONS

Mechanical Energy and Pump Efficiency

11-1C The *mechanical energy* is the form of energy that can be converted to mechanical work completely and directly by a mechanical device such as a propeller. It differs from thermal energy in that thermal energy cannot be converted to work directly and completely. The forms of mechanical energy of a fluid stream are kinetic, potential, and pressure energies.

11-2C *Mechanical efficiency* is defined as the ratio of the mechanical energy output to the mechanical energy input. A mechanical efficiency of 100% for a hydraulic turbine means that the entire mechanical energy of the fluid is converted to mechanical (shaft) work.

11-3C The *combined pump-motor efficiency* of a pump/motor system is defined as the ratio of the increase in the mechanical energy of the fluid to the electrical power consumption of the motor,

$$\eta_{\text{pump-motor}} = \eta_{\text{pump}}\eta_{\text{motor}} = \frac{\dot{E}_{\text{mech,out}} - \dot{E}_{\text{mech,in}}}{\dot{W}_{\text{elect,in}}} = \frac{\Delta\dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{elect,in}}}$$

The combined pump-motor efficiency cannot be greater than either of the pump or motor efficiency since both pump and motor efficiencies are less than 1, and the product of two numbers that are less than one is less than either of the numbers.

11-4C The turbine efficiency, generator efficiency, and *combined turbine-generator efficiency* are defined as follows:

$$\eta_{\text{turbine}} = \frac{\text{Mechanical power output}}{\text{Mechanical energy extracted from the fluid}} = \frac{\dot{W}_{\text{shaft,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|}$$

$$\eta_{\text{generator}} = \frac{\text{Electrical power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{shaft,in}}}$$

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\eta_{\text{generaor}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{E}_{\text{mech,in}} - \dot{E}_{\text{mech,out}}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|}$$

11-5 A river is flowing at a specified velocity, flow rate, and elevation. The total mechanical energy of the river water per unit mass, and the power generation potential of the entire river are to be determined. √

Assumptions **1** The elevation given is the elevation of the free surface of the river. **2** The velocity given is the average velocity. **3** The mechanical energy of water at the turbine exit is negligible.

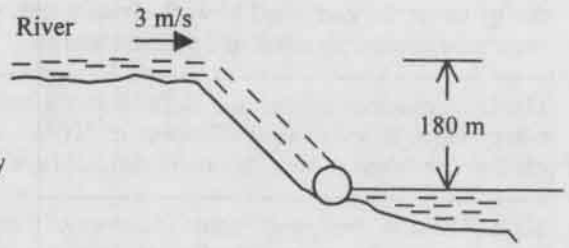
Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

Analysis Noting that the sum of the pressure energy and the potential energy is constant for a given fluid body, we can take the elevation of the entire river water to be the elevation of the free surface, and ignore the pressure energy. Then the total mechanical energy of the river water per unit mass becomes

$$e_{\text{mech}} = pe + ke = gh + \frac{V^2}{2}$$

$$= \left((9.81\ \text{m/s}^2)(180\ \text{m}) + \frac{(3\ \text{m/s})^2}{2} \right) \left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2} \right)$$

$$= 1.770\ \text{kJ/kg}$$



The power generation potential of the river water is obtained by multiplying the total mechanical energy by the mass flow rate,

$$\dot{m} = \rho \dot{V} = (1000\ \text{kg/m}^3)(500\ \text{m}^3/\text{s}) = 500{,}000\ \text{kg/s}$$

$$\dot{W}_{\max} = \dot{E}_{\text{mech}} = \dot{m} e_{\text{mech}} = (500{,}000\ \text{kg/s})(1.770\ \text{kg/s}) = 885{,}000\ \text{kW} = \mathbf{885\ MW}$$

Therefore, 885 MW of power can be generated from this river as it discharges into the lake if its power potential can be recovered completely.

Discussion Note that the kinetic energy of water is negligible compared to the potential energy, and it can be ignored in the analysis. Also, the power output of an actual turbine will be less than 885 MW because of losses and inefficiencies.

11-6 A hydraulic turbine-generator is generating electricity from the water of a large reservoir. The combined turbine-generator efficiency and the turbine efficiency are to be determined.

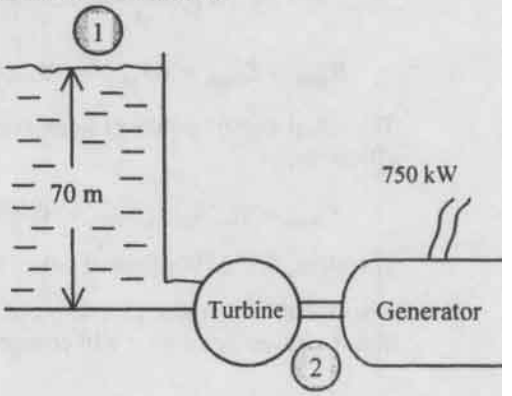
Assumptions **1** The elevation of the reservoir remains constant. **2** The mechanical energy of water at the turbine exit is negligible.

Analysis We take the free surface of the reservoir to be point 1 and the turbine exit to be point 2. We also take the turbine exit as the reference level ($z_2 = 0$), and thus the potential energy at points 1 and 2 are $pe_1 = gz_1$ and $pe_2 = 0$. The pressure energy at both points is zero since both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$). Further, the kinetic energy at both points is zero ($ke_1 = ke_2 = 0$) since the water at point 1 is essentially motionless, and the kinetic energy of water at turbine exit is assumed to be negligible. The potential energy of water at point 1 is

$$pe_1 = gz_1 = (9.81\,\text{m/s}^2)(70\,\text{m})\left(\frac{1\,\text{kJ/kg}}{1000\,\text{m}^2/\text{s}^2}\right) = 0.687\,\text{kJ/kg}$$

Then the rate at which the mechanical energy of the fluid is supplied to the turbine become

$$\begin{aligned}\left|\Delta\dot{E}_{\text{mech,fluid}}\right| &= \dot{m}(e_{\text{mech,in}} - e_{\text{mech,out}}) = \dot{m}(pe_1 - 0) = \dot{m}pe_1 \\ &= (1500\,\text{kg/s})(0.687\,\text{kJ/kg}) \\ &= 1031\,\text{kW}\end{aligned}$$



The combined turbine-generator and the turbine efficiency are determined from their definitions,

$$\eta_{\text{turbine-gen}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|} = \frac{750\,\text{kW}}{1031\,\text{kW}} = 0.727 \quad \text{or} \quad \mathbf{72.7\%}$$

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|} = \frac{800\,\text{kW}}{1031\,\text{kW}} = 0.776 \quad \text{or} \quad \mathbf{77.6\%}$$

Therefore, the reservoir supplies 1031 kW of mechanical energy to the turbine, which converts 800 kW of it to shaft work that drives the generator, which generates 750 kW of electric power.

Discussion This problem can also be solved by taking point 1 to be at the turbine inlet, and using pressure energy instead of potential energy. It would give the same result since the pressure energy at the turbine inlet is equal to the potential energy at the free surface of the reservoir.

11-7 Wind is blowing steadily at a certain velocity. The mechanical energy of air per unit mass, the power generation potential, and the actual electric power generation are to be determined. √

Assumptions **1** The wind is blowing steadily at a constant uniform velocity. **2** The efficiency of the wind turbine is independent of the wind speed.

Properties The density of air is given to be $\rho = 1.25\ \text{kg/m}^3$.

Analysis Kinetic energy is the only form of mechanical energy the wind possesses, and it can be converted to work entirely. Therefore, the power potential of the wind is its kinetic energy, which is $\mathbf{V}^2/2$ per unit mass, and $\dot{m}\mathbf{V}^2/2$ for a given mass flow rate:

$$e_{\text{mech}} = ke = \frac{\mathbf{V}^2}{2} = \frac{(25\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.3125\ \text{kJ/kg}$$

$$\dot{m} = \rho \mathbf{V} A = \rho \mathbf{V}\frac{\pi D^2}{4} = (1.25\ \text{kg/m}^3)(25\ \text{m/s})\frac{\pi(50\ \text{m})^2}{4} = 61{,}360\ \text{kg/s}$$

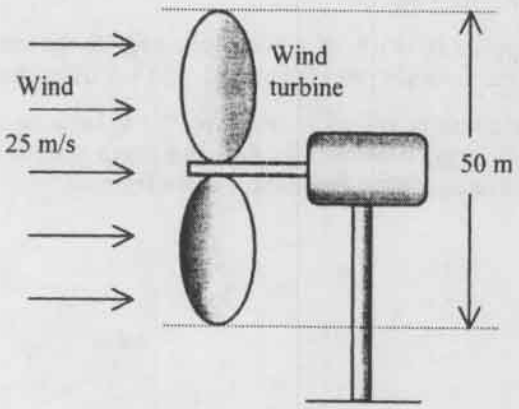
$$\dot{W}_{\text{max}} = \dot{E}_{\text{mech}} = \dot{m} e_{\text{mech}} = (61{,}360\ \text{kg/s})(0.3125\ \text{kg/s}) = 19{,}200\ \text{kW} = \mathbf{19.2\ MW}$$

The actual electric power generation is determined by multiplying the power generation potential by the efficiency,

$$\dot{W}_{\text{elect}} = \eta_{\text{wind turbine}} \dot{W}_{\text{max}} = (0.30)(19.2\ \text{MW}) = \mathbf{5.75\ MW}$$

Therefore, 5.75 MW of actual power can be generated by this wind turbine at the stated conditions.

Discussion The power generation of a wind turbine is proportional to the cube of the wind velocity, and thus the power generation will change strongly with the wind conditions.



11-8E A differential thermocouple indicates that the temperature of water rises a certain amount as it flows through a pump at a specified rate. The mechanical efficiency of the pump is to be determined.

Assumptions **1** The pump is adiabatic so that there is no heat transfer with the surroundings, and the temperature rise of water is completely due to frictional heating. **2** Water is an incompressible substance.

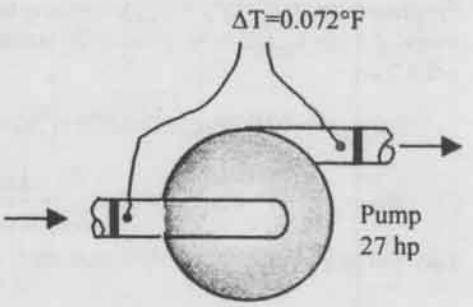
Properties We take the density of water to be $\rho = 62.4$ lbm/ft^3 and its specific heat to be $C = 1.0$ Btu/lbm·°F (Table A-3E).

Analysis The increase in the temperature of water is due to the conversion of mechanical energy to thermal energy, and the amount of mechanical energy converted to thermal energy is equal to the increase in the internal energy of water,

$$\dot{m} = \rho \dot{V} = (62.4 \text{ lbm/ft}^3)(1.5 \text{ ft}^3/\text{s}) = 93.6 \text{ lbm/s}$$

$$\dot{E}_{\text{mech, loss}} = \Delta \dot{U} = \dot{m} C \Delta T$$

$$= (93.6 \text{ lbm/s})(1.0 \text{ Btu/lbm} \cdot \text{°F})(0.072\text{°F})\left(\frac{1 \text{ hp}}{0.7068 \text{ Btu/s}}\right) = 9.53 \text{ hp}$$



The mechanical efficiency of the pump is determined from the general definition of mechanical efficiency,

$$\eta_{\text{pump}} = 1 - \frac{\dot{E}_{\text{mech,loss}}}{\dot{W}_{\text{mech, in}}} = 1 - \frac{9.53 \text{ hp}}{27 \text{ hp}} = 0.647 \quad \text{or} \quad \mathbf{64.7\%}$$

Discussion Note that despite the conversion of more than one-third of the mechanical power input into thermal energy, the temperature of water rises by only a small fraction of a degree. Therefore, the temperature rise of a fluid due to frictional heating is usually negligible in heat transfer analysis.

11-9 Water is pumped from a lake to a storage tank at a specified rate. The overall efficiency of the pump-motor unit and the pressure difference between the inlet and the exit of the pump are to be determined. √

Assumptions **1** The elevations of the tank and the lake remain constant. **2** Frictional losses in the pipes are negligible. **3** The changes in kinetic energy are negligible. **4** The elevation difference across the pump is negligible.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

Analysis (*a*) We take the free surface of the lake to be point 1 and the free surfaces of the storage tank to be point 2. We also take the lake surface as the reference level ($z_1 = 0$), and thus the potential energy at points 1 and 2 are $pe_1 = 0$ and $pe_2 = gz_2$. The pressure energy at both points is zero since both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$). Further, the kinetic energy at both points is zero ($ke_1 = ke_2 = 0$) since the water at both locations is essentially stationary. The mass flow rate of water and its potential energy at point 2 are

$$\dot{m} = \rho \dot{V} = (1000\ \text{kg/m}^3)(0.070\ \text{m}^3/\text{s}) = 70\ \text{kg/s}$$

$$pe_1 = gz_1 = (9.81\ \text{m/s}^2)(20\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.196\ \text{kJ/kg}$$

Then the rate of increase of the mechanical energy of water becomes

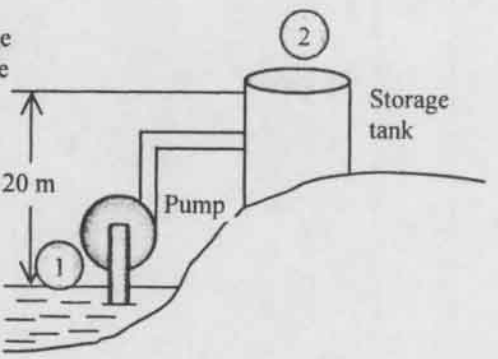
$$\Delta \dot{E}_{\text{mech,fluid}} = \dot{m}(e_{\text{mech,out}} - e_{\text{mech,in}}) = \dot{m}(pe_2 - 0) = \dot{m}pe_2 = (70\ \text{kg/s})(0.196\ \text{kJ/kg}) = 13.7\ \text{kW}$$

The overall efficiency of the combined pump-motor unit is determined from its definition,

$$\eta_{\text{pump-motor}} = \frac{\Delta \dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}} = \frac{13.7\ \text{kW}}{20.4\ \text{kW}} = 0.672 \quad \text{or} \quad \mathbf{67.2\%}$$

(*b*) Now we consider the pump. The change in the mechanical energy of water as it flows through the pump consists of the change in the pressure energy only since the elevation difference across the pump and the change in the kinetic energy are negligible. Also, this change must be equal to the useful mechanical energy supplied by the pump, which is 13.7 kW:

$$\Delta \dot{E}_{\text{mech,fluid}} = \dot{m}(e_{\text{mech,out}} - e_{\text{mech,in}}) = \dot{m}\frac{P_2 - P_1}{\rho} = \dot{V}\Delta P$$



Solving for ΔP and substituting,

$$\Delta P = \frac{\Delta \dot{E}_{\text{mech,fluid}}}{\dot{V}} = \frac{13.7\ \text{kJ/s}}{0.070\ \text{m}^3/\text{s}}\left(\frac{1\ \text{kPa}\cdot\text{m}^3}{1\ \text{kJ}}\right) = \mathbf{196\ kPa}$$

Therefore, the pump must boost the pressure of water by 196 kPa in order to raise its elevation by 20 m.

Discussion Note that only two-thirds of the electric energy consumed by the pump-motor is converted to the mechanical energy of water; the remaining one-third is wasted because of the inefficiencies of the pump and the motor.

Bernoulli Equation

11-10C The acceleration of a fluid particle along a streamline is called *streamwise acceleration*, and it is due to a change in speed along a streamline. *Normal acceleration* (or centrifugal acceleration), on the other hand, is the acceleration of a fluid particle in the direction normal to the streamline, and it is due to a change in direction.

11-11C The Bernoulli equation can be expressed in three different ways as follows:

(a) energies: $\dfrac{P}{\rho}+\dfrac{\mathbf{V}^2}{2}+gz=\text{constant}$

(b) pressures: $P+\rho\dfrac{\mathbf{V}^2}{2}+\rho gz=\text{constant}$

(c) heads: $\dfrac{P}{\rho g}+\dfrac{\mathbf{V}^2}{2g}+z=H=\text{constant}$

11-12C The three major assumptions used in the derivation of the Bernoulli equation are that the flow is steady, frictionless, and incompressible.

11-13C The *static pressure P* is the actual pressure of the fluid. The *dynamic pressure* $\rho\mathbf{V}^2/2$ is the pressure rise when the fluid in motion is brought to a stop. The *hydrostatic pressure* ρgz is not pressure in a real sense since its value depends on the reference level selected, and it accounts for the effects of fluid weight on pressure.

The sum of static, dynamic, and hydrostatic pressures is constant when flow is steady, frictionless, and incompressible.

11-14C The sum of the static and dynamic pressures is called the stagnation pressure, and it is expressed as $P_{\text{stag}}=P+\rho\mathbf{V}^2/2$. The *stagnation pressure* can be measured by a pitot tube whose inlet is normal to flow.

11-15C The *pressure head* $P/\rho g$ is the height of a fluid column that produces the static pressure P. The *velocity head* $\mathbf{V}^2/2$ is the elevation needed for a fluid to reach the velocity $\mathbf{V}$ during frictionless free fall. The *elevation head* z is the height of a fluid relative to a reference level.

11-16C The line that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the *hydraulic grade line*. The line that represents the total head of the fluid, $P/\rho g + \mathbf{V}^2/2g + z$, is called the *energy line*. For stationary bodies such as reservoirs or lakes, the EL and HGL coincide with the free surface of the liquid.

11-17C For open channel flow, the hydraulic grade line (HGL) coincides with the free surface of the liquid. At the exit of a pipe discharging to the atmosphere, it coincides with the center of the pipe.

11-18C With no losses and a 100% efficient nozzle, the water stream could reach to the water level in the tank, or 20 meters. In reality, friction losses in the hose, nozzle inefficiencies, orifice losses and air drag would prevent attainment of the maximum theoretical height.

11-19C The lower density liquid can go over a higher wall, provided that cavitation pressure is not reached. Therefore, oil can go over a higher wall.

11-20C Siphoning works because of the elevation and thus pressure difference between the inlet and exit of a tube. The pressure at the tube exit and at the free surface of a liquid is the atmospheric pressure. When the tube exit is below the free surface of the liquid, the elevation head difference drives the flow through the tube. At sea level, 1 atm pressure can support about 10.3 m of cold water (cold water has a low vapor pressure). Therefore, siphoning cold water over a 7 m wall is feasible.

11-21C At sea level, a person can siphon water over a wall as high as 10.3 m. At the top of a high mountain where the pressure is about half of the atmospheric pressure at sea level, a person can siphon water over a wall that is only half as high. An atmospheric pressure of 58.5 kPa is insufficient to support a 8.5 meter high siphon.

11-22C By Bernoulli's Equation, the smaller pipe section is consistent with higher velocity and concomitant lower pressure. Thus Manometer A is correct. The fluid levels in a manometer is independent of the flow direction, and reversing the flow direction will have no effect on the manometer.

11-23C The arrangement *B* measures the total head and static head at the same location, and thus it is more accurate. The static probe in arrangement *A* will indicate *D*/2 less water head, and thus the difference between the static and stagnation pressures (the dynamic pressure) will be larger. Consequently, arrangement *A* will indicate a higher velocity. In the case of air, the static pressure difference corresponding to the elevation head of *D*/2 is negligible, and thus both arrangements will indicate the same velocity.

11-24 A water pipe bursts as a result of freezing, and water shoots up into the air a certain height. The gage pressure of water in the pipe is to be determined. √

Assumptions **1** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The water pressure in the pipe at the burst section is equal to the water main pressure. **3** Friction between the water and air is negligible. **4** The irreversibilities that may occur at the burst section of the pipe due to abrupt expansion are negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis This problem involves the conversion of pressure, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($\mathbf{V}_1 \cong 0$) and we take the burst section of the pipe as the reference level ($z_1 = 0$). At the top of the water trajectory $\mathbf{V}_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli Equation simplifies to

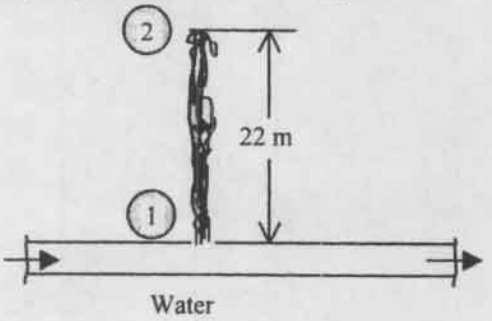
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+z_2 \quad \rightarrow \quad \frac{P_1-P_{atm}}{\rho g}=z_2 \quad \rightarrow \quad \frac{P_{1,\text{gage}}}{\rho g}=z_2$$

Solving for $P_{1,\text{gage}}$ and substituting,

$$P_{1,\text{gage}}=\rho g z_2=(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(22\,\text{m})\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right)\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)=\mathbf{216\,kPa}$$

Therefore, the pressure in the main must be at least 216 kPa above the atmospheric pressure.

Discussion The result obtained by the Bernoulli equation represents a limit, and should be interpreted accordingly. It tells us that the water pressure (gage) cannot possibly be less than 216 kPa (giving us a lower limit), and in all likelihood, the pressure will be much higher.



11-25 The velocity of an aircraft is to be measured by a pitot tube. For a given differential pressure reading, the velocity of the aircraft is to be determined. √

Assumptions **1**The air flow over the aircraft is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** Standard atmospheric conditions exist. **3** The wind effects are negligible.

Properties The density of the atmosphere at an elevation of 3000 m is $\rho = 0.909$ kg/m^3 (Table A-26).

Analysis We take point 1 at the entrance of the tube whose opening is parallel to flow, and point 2 at the entrance of the tube whose entrance is normal to flow. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

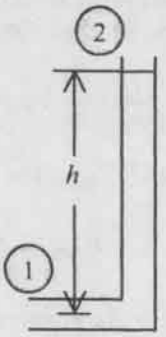
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{\mathbf{V}_1^2}{2g}=\frac{P_2-P_1}{\rho g} \quad \rightarrow \quad \frac{\mathbf{V}_1^2}{2}=\frac{P_{stag}-P_1}{\rho}$$

Solving for $\mathbf{V}_1$ and substituting,

$$\mathbf{V}_1=\sqrt{\frac{2(P_{stag}-P_1)}{\rho}}=\sqrt{\frac{2(3000\ \text{N/m}^2)}{0.909\ \text{kg/m}^3}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)}=\mathbf{81.2\ m/s = 292\ km/h}$$

since 1 Pa = 1 N/m^2 and 1 m/s = 3.6 km/h.

Discussion Note that the velocity of an aircraft can be determined by simply measuring the differential pressure on a pitot tube.

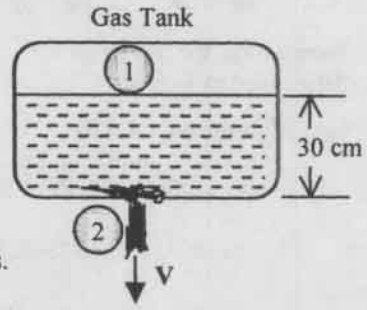


Pitot Tube

11-26 The bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. For a given height of gasoline, the initial velocity of the gasoline out of the hole is to be determined. Also, the variation of velocity with time and the effect of the tightness of the lid on flow rate are to be discussed. √

Assumptions **1** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The air space in the tank is at atmospheric pressure. **3** The splashing of the gasoline in the tank during travel is not considered.

Analysis This problem involves the conversion of pressure, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere) $\mathbf{V}_1 \cong 0$ (the tank is large relative to the outlet), and $z_1 = 0.3$ m and $z_2 = 0$ (we take the reference level at the hole. Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli Equation simplifies to



$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}$$

Solving for $\mathbf{V}_2$ and substituting,

$$\mathbf{V}_2=\sqrt{2gz_1}=\sqrt{2(9.81\,\text{m/s}^2)(0.3\,\text{m})}=\mathbf{2.43\,m/s}$$

Therefore, the gasoline will initially leave the tank with a velocity of 2.43 m/s.

Discussion The Bernoulli equation applies along a streamline, and streamlines generally do not make sharp turns. The velocity will be less than 2.43 m/s since the hole is probably sharp-edged and it will cause some head loss.

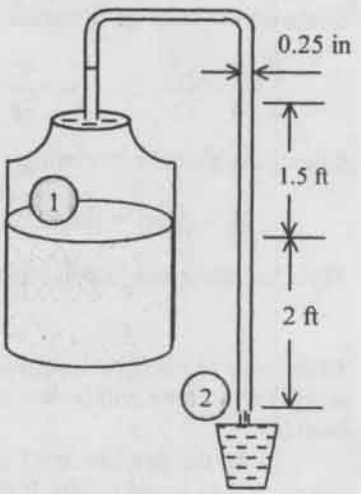
As the gasoline level is reduced, the velocity will decrease since velocity is proportional to the square root of liquid height. If the lid is tightly closed and no air can replace the lost gasoline volume, the pressure above the gasoline level will be reduced, and the velocity will be decreased.

11-27E The drinking water needs of an office are met by large water bottles with a plastic hose inserted in it. The minimum filling time of an 8-oz glass is to be determined when the bottle is full and when it is near empty. √

Assumptions **1** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** All losses are neglected to obtain the minimum filling time.

Analysis We take point 1 to be at the free surface of water in the bottle and point 2 at the exit of the tube so that $P_1 = P_2 = P_{atm}$ (the bottle is open to the atmosphere and water discharges into the atmosphere), $\mathbf{V}_1 \cong 0$ (the bottle is large relative to the tube diameter), and $z_2 = 0$ (we take point 2 as the reference level). Then the Bernoulli Equation simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{2gz_1}$$



Substituting, the discharge velocity of water and the filling time are determined as follows:

(*a*) *Full bottle* ($z_1 = 3.5$ ft):

$$\mathbf{V}_2=\sqrt{2(32.2\ \text{ft/s}^2)(3.5\ \text{ft})}=15.0\ \text{ft/s}$$

$$A=\pi D^2/4=\pi(0.25/12\ \text{ft})^2/4=3.41\times10^{-4}\ \text{ft}^2$$

$$\Delta t=\frac{V}{\dot{V}}=\frac{V}{A\mathbf{V}}=\frac{0.00835\ \text{ft}^3}{(3.41\times10^{-4}\ \text{ft}^2)(15\ \text{ft/s})}=\mathbf{1.6\ s}$$

(*b*) *Empty bottle* ($z_1 = 2$ ft):

$$\mathbf{V}_2=\sqrt{2(32.2\ \text{ft/s}^2)(2\ \text{ft})}=11.3\ \text{ft/s}$$

$$\Delta t=\frac{V}{\dot{V}}=\frac{V}{A\mathbf{V}}=\frac{0.00835\ \text{ft}^3}{(3.41\times10^{-4}\ \text{ft}^2)(11.3\ \text{ft/s})}=\mathbf{2.2\ s}$$

Discussion The siphoning time is determined assuming frictionless flow, and thus this is the *minimum time* required. In reality, the time will be longer because of friction between water and the tube surface.

11-28 The static and stagnation pressures in a horizontal pipe are measured. The velocity at the center of the pipe is to be determined. √

Assumptions The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable).

Analysis We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the entrance of the pitot tube (the stagnation point). This is a steady flow with straight and parallel streamlines, and thus the static pressure at any point is equal to the hydrostatic pressure at that point. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{\mathbf{V}_1^2}{2g}=\frac{P_2-P_1}{\rho g}$$

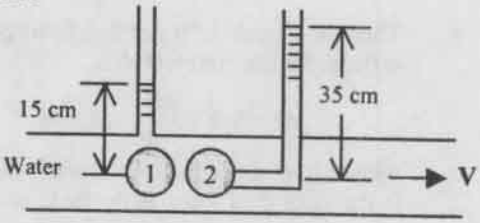
Substituting the P_1 and P_2 expressions give

$$\frac{\mathbf{V}_1^2}{2g}=\frac{P_2-P_1}{\rho g}=\frac{\rho g(h_{\text{pitot}}+R)-\rho g(h_{\text{piezo}}+R)}{\rho g}=\frac{\rho g(h_{\text{pitot}}-h_{\text{piezo}})}{\rho g}=h_{\text{pitot}}-h_{\text{piezo}}$$

Solving for $\mathbf{V}_1$ and substituting,

$$\mathbf{V}_1=\sqrt{2g(h_{\text{pitot}}-h_{\text{piezo}})}=\sqrt{2(9.81\,\text{m/s}^2)[(0.35-0.15)\,\text{m}]}=\mathbf{2.0\,m/s}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the pitot tube.



11-29 A water tank of diameter D_o and height H open to the atmosphere is initially filled with water. An orifice of diameter D with a smooth entrance (no losses) at the bottom drains to the atmosphere. Relations are to be developed for the time required for the tank to empty completely and half-way. √

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable).

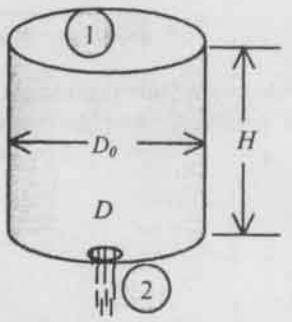
Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice. We take the reference level at the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{2gz_1}$$

For generality, we express the water height in the tank at any time t by z, and the discharge velocity by $\mathbf{V}_2 = \sqrt{2gz}$. Note that water surface in the tank moves down as the tank drains, and thus z is a variable whose value changes from H at the beginning to 0 when the tank is emptied completely.

We denote the diameter of the orifice by D, and the diameter of the tank by D_o. The flow rate of water from the tank is obtained by multiplying the discharge velocity by the orifice cross-sectional area,

$$\dot{V} = A_{\text{orifice}}\,\mathbf{V}_2 = \frac{\pi D^2}{4}\sqrt{2gz}$$



Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV = \dot{V}dt = \frac{\pi D^2}{4}\sqrt{2gz}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV = A_{\text{tank}}(-dz) = -\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{2gz}\,dt = -\frac{\pi D_0^2}{4}dz \quad \rightarrow \quad dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1}{2gz}}dz = -\frac{D_0^2}{D^2\sqrt{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_i = H$ to $t = t_f$ when $z = z_f$ gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2\sqrt{2g}}\int_{z=z_i}^{z_f} z^{-1/2}dz \quad \rightarrow \quad t_f = -\frac{D_0^2}{D^2\sqrt{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_i}^{z_f} = \frac{2D_0^2}{D^2\sqrt{2g}}\left(\sqrt{z_i}-\sqrt{z_f}\right) = \frac{D_0^2}{D^2}\left(\sqrt{\frac{2z_i}{g}}-\sqrt{\frac{2z_f}{g}}\right)$$

Then the discharging time for the two cases becomes as follows:

(*a*) *The tank empties halfway*: $z_i = H$ and $z_f = H/2$: $$t_f = \frac{D_0^2}{D^2}\left(\sqrt{\frac{2H}{g}}-\sqrt{\frac{H}{g}}\right)$$

(*b*) *The tank empties completely*: $z_i = H$ and $z_f = 0$: $$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2H}{g}}$$

Discussion Note that the discharging time is inversely proportional to the square of the orifice diameter. Therefore, the discharging time can be reduced to one-fourth by doubling the diameter of the orifice.

11-30 Water discharges to the atmosphere from the orifice at the bottom of a pressurized tank. Assuming frictionless flow, the discharge rate of water from the tank is to be determined.

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable).

Properties We take the density of water to be 1000 kg/m^3 (Table A-3).

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$) and water discharges into the atmosphere (and thus $P_2 = P_{atm}$), the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{\mathbf{V}_2^2}{2g}=\frac{P_1-P_2}{\rho g}+z_1$$

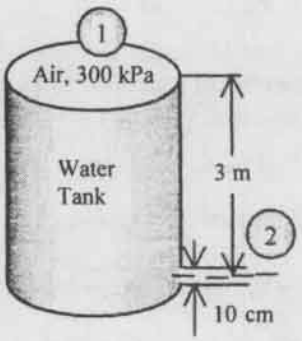
Solving for $\mathbf{V}_2$ and substituting, the discharge velocity is determined to

$$\mathbf{V}_2=\sqrt{\frac{2(P_1-P_2)}{\rho}+2gz_1}=\sqrt{\frac{2(300-100)\,\text{kPa}}{1000\,\text{kg/m}^3}\left(\frac{1000\,\text{N/m}^2}{1\,\text{kPa}}\right)\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right)+2(9.81\,\text{m/s}^2)(3\,\text{m})}$$

$$=21.4\,\text{m/s}$$

Then the initial rate of discharge of water becomes

$$\dot{V}=A_{\text{orifice}}\mathbf{V}_2=\frac{\pi D^2}{4}\mathbf{V}_2=\frac{\pi(0.10\,\text{m})^2}{4}(21.4\,\text{m/s})=\mathbf{0.168\,m^3/s}$$



Discussion Note that this is the maximum flow rate since the frictional effects are ignored. Also, the velocity and the flow rate will decrease as the water level in the tank decreases.

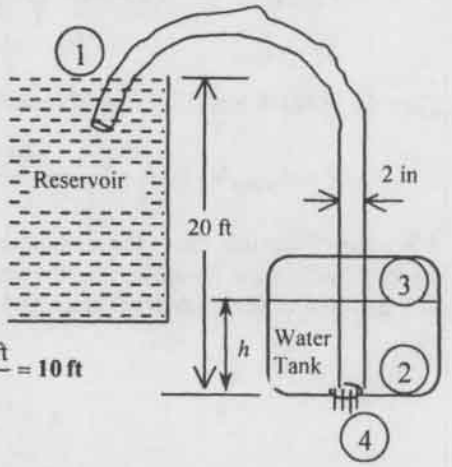
11-31E A siphon pumps water from a large reservoir to a lower tank which is initially empty. Water leaves the tank through an orifice. The height the water will rise in the tank at equilibrium is to be determined. √

Assumptions **1** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** Both the tank and the reservoir are open to the atmosphere. **3** The water level of the reservoir remains constant.

Analysis We take the reference level to be at the bottom of the tank, and the water height in the tank at any time to be h. We take point 1 to be at the free surface of reservoir, point 2 at the exit of the siphon, which is placed at the bottom of the tank, and point 3 at the free surface of the tank, and point 4 at the exit of the orifice at the bottom of the tank. Then $z_1 = 20$ ft, $z_2 = z_4 = 0$, $z_3 = h$, $P_1 = P_3 = P_4 = P_{atm}$ (the reservoir is open to the atmosphere and water discharges into the atmosphere) $P_2 = P_{atm}+\rho gh$ (the hydrostatic pressure at the bottom of the tank where the siphon discharges), and $\mathbf{V}_1 \cong \mathbf{V}_3 \cong 0$ (the free surfaces of reservoir and the tank are large relative to the tube diameter). Then the Bernoulli Equation between 1-2 and 3-4 simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_{atm}}{\rho g}+z_1=\frac{P_{atm}+\rho gh}{\rho g}+\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{2gz_1-2gh}=\sqrt{2g(z_1-h)}$$

$$\frac{P_3}{\rho g}+\frac{\mathbf{V}_3^2}{2g}+z_3=\frac{P_4}{\rho g}+\frac{\mathbf{V}_4^2}{2g}+z_4 \quad \rightarrow \quad h=\frac{\mathbf{V}_4^2}{2g} \quad \rightarrow \quad \mathbf{V}_4=\sqrt{2gh}$$



Noting that the diameters of the tube and the orifice are the same, the flow rates of water into and out of the tank will be the same when the water velocities in the tube and the orifice are equal since

$$\dot{V}_2=\dot{V}_4 \quad \rightarrow \quad A\mathbf{V}_2=A\mathbf{V}_4 \quad \rightarrow \quad \mathbf{V}_2=\mathbf{V}_4$$

Setting the two velocities equal to each other gives

$$\mathbf{V}_2=\mathbf{V}_4 \quad \rightarrow \quad \sqrt{2g(z_1-h)}=\sqrt{2gh} \quad \rightarrow \quad z_1-h=h \quad \rightarrow \quad h=\frac{z_1}{2}=\frac{20\text{ ft}}{2}=\mathbf{10\text{ ft}}$$

Therefore, the water level in the tank will stabilize when the water level rises to 10 ft.

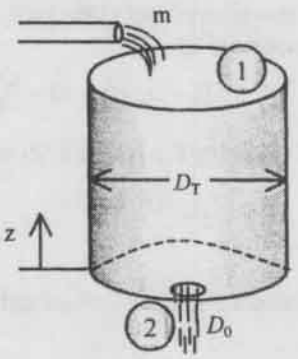
Discussion This result is obtained assuming frictionless flow. The result would be somewhat different if the friction in the pipe and orifice were considered.

11-32 Water enters an empty tank steadily at a specified rate. An orifice at the bottom allows water to escape. The maximum water level in the tank is to be determined, and a relation for water height z as a function of time is to be obtained.

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow through the orifice is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable).

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of orifice. We take the reference level at the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$) (it becomes zero when the water in the tank reaches its maximum level), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{2gz_1}$$



Then the mass flow rate through the orifice for a water height of z becomes

$$\dot{m}_{out}=\rho\dot{V}_{out}=\rho A_{orifice}\mathbf{V}_2=\rho\frac{\pi D_0^2}{4}\sqrt{2gz} \quad \rightarrow \quad z=\frac{1}{2g}\left(\frac{4\dot{m}_{out}}{\rho\pi D_0^2}\right)^2$$

Setting $z = h_{max}$ and $\dot{m}_{out} = \dot{m}_{in}$ (the incoming flow rate) gives the desired relation for the maximum height the water will reach in the tank,

$$h_{max}=\frac{1}{2g}\left(\frac{4\dot{m}_{in}}{\rho\pi D_0^2}\right)^2$$

(*b*) The amount of water that flows through the orifice and the increase in the amount of water in the tank during a differential time interval dt are

$$dm_{out}=\dot{m}_{out}\,dt=\rho\frac{\pi D_0^2}{4}\sqrt{2gz}\,dt$$

$$dm_{tank}=\rho A_{tank}\,dz=\rho\frac{\pi D_T^2}{4}\,dz$$

The amount of water that enters the tank during dt is $dm_{in} = \dot{m}_{in}\,dt$ (Recall that $\dot{m}_{in}$ = constant). Substituting them into the conservation of mass relation $dm_{tank} = dm_{in} - dm_{out}$ gives

$$dm_{tank}=\dot{m}_{in}\,dt-\dot{m}_{out}\,dt \quad \rightarrow \quad \rho\frac{\pi D_T^2}{4}\,dz=\left(\dot{m}_{in}-\rho\frac{\pi D_0^2}{4}\sqrt{2gz}\right)dt$$

Separating the variables, and integrating it from $z = 0$ at $t = 0$ to $z = z$ at time $t = t$ gives

$$\frac{\frac{1}{4}\rho\pi D_T^2\,dz}{\dot{m}_{in}-\frac{1}{4}\rho\pi D_0^2\sqrt{2gz}}=dt \quad \rightarrow \quad \int_{z=0}^{z}\frac{\frac{1}{4}\rho\pi D_T^2\,dz}{\dot{m}_{in}-\frac{1}{4}\rho\pi D_0^2\sqrt{2gz}}=\int_0^t dt=t$$

Performing the integration, the desired relation between the water height z and time t is obtained to be

$$\frac{\frac{1}{2}\rho\pi D_T^2}{(\frac{1}{4}\rho\pi D_0^2\sqrt{2g})^2}\left(\frac{1}{4}\rho\pi D_0^2\sqrt{2gz}-\dot{m}_{in}\ln\frac{\dot{m}_{in}-\frac{1}{4}\rho\pi D_0^2\sqrt{2gz}}{\dot{m}_{in}}\right)=t$$

Discussion Note that this relation is implicit in z, and thus we can't obtain a relation in the form $z = f(t)$. Substituting a z value in the left side gives the time it takes for the fluid level in the tank to reach that level.

11-33E Water flows through a horizontal pipe that consists of two sections at a specified rate. The differential height of a mercury manometer placed between the two pipe sections is to be determined. √

Assumptions **1**The flow through the pipe is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The losses in the reducing section are negligible.

Properties The densities of mercury and water are $\rho_{Hg} = 847$ lbm/ft^3 and $\rho_w = 62.4$ lbm/ft^3 (Table A-3E).

Analysis We take points 1 and 2 along the centerline of the pipe over the two tubes of the manometer. Noting that $z_1 = z_2$, the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad P_1-P_2=\frac{\rho_w(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2} \qquad (1)$$

We let the differential height of the mercury manometer be h and the distance between the centerline and the mercury level in the tube where mercury is raised be s. Then the pressure difference $P_2 - P_1$ can also be expressed as

$$P_1+\rho_w g(s+h)=P_2+\rho_w gs+\rho_{Hg}gh \quad \rightarrow \quad P_1-P_2=(\rho_{Hg}-\rho_w)gh \qquad (2)$$

Combining Eqs. (1) and (2) and solving for h,

$$\frac{\rho_w(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2}=(\rho_{Hg}-\rho_w)gh \quad \rightarrow \quad h=\frac{\rho_w(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2g(\rho_{Hg}-\rho_w)}=\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2g(\rho_{Hg}/\rho_w-1)}$$

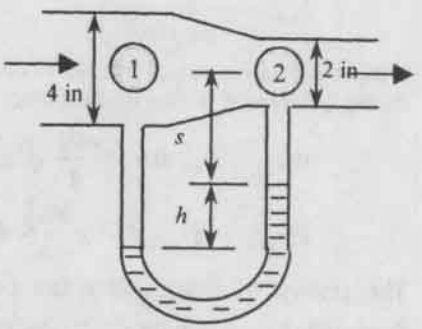
Calculating the velocities and substituting,

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{1\text{ gal/s}}{\pi(4/12\text{ ft})^2/4}\left(\frac{0.13368\text{ ft}^3}{1\text{ gal}}\right)=1.53\text{ ft/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{1\text{ gal/s}}{\pi(2/12\text{ ft})^2/4}\left(\frac{0.13368\text{ ft}^3}{1\text{ gal}}\right)=6.13\text{ ft/s}$$

$$h=\frac{(6.13\text{ ft/s})^2-(1.53\text{ ft/s})}{2(32.2\text{ ft/s}^2)(847/62.4-1)}=0.0435\text{ ft}=\mathbf{0.52\text{ in}}$$

Therefore, the differential height of the mercury column will be 0.52 in.



11-34 An airplane is flying at a certain altitude at a given speed. The pressure on the stagnation point on the nose of the plane is to be determined, and the approach to be used at high velocities is to be discussed. √

Assumptions **1**The air flow over the aircraft is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** Standard atmospheric conditions exist. **3** The wind effects are negligible.

Properties The density of the atmospheric air at an elevation of 12,000 m is $\rho = 0.312$ kg/m^3 (Table A-26).

Analysis We take point 1 well ahead of the plane at the level of the nose, and point 2 at the nose where the flow comes to a stop. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

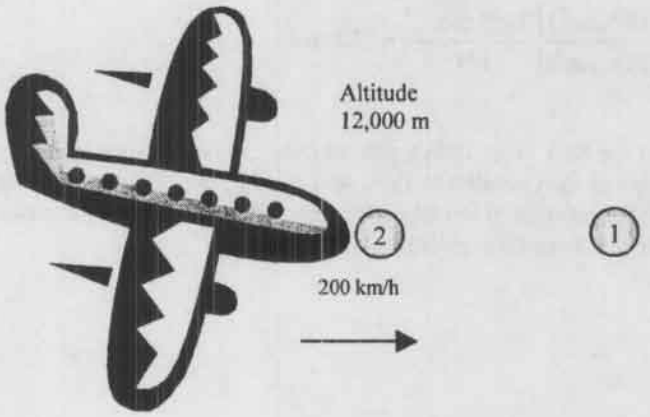
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{\mathbf{V}_1^2}{2g}=\frac{P_2-P_1}{\rho g} \quad \rightarrow \quad \frac{\mathbf{V}_1^2}{2}=\frac{P_{stag}-P_{atm}}{\rho}=\frac{P_{stag,\,gage}}{\rho}$$

Solving for $P_{stag,\,gage}$ and substituting,

$$P_{stag,\,gage}=\frac{\rho \mathbf{V}_1^2}{2}=\frac{(0.312\,\text{kg/m}^3)(200/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)=482\,\text{N/m}^2=\mathbf{482\ Pa}$$

since 1 Pa = 1 N/m^2 and 1 m/s = 3.6 km/h.

Discussion A flight velocity of 900 km/h = 250 m/s corresponds to a Mach number much greater than 0.3 (the speed of sound is about 340 m/s at room conditions, and lower at higher altitudes, and thus a Mach number of 250/340 = 0.74). Therefore, the flow can no longer be assumed to be incompressible, and the Bernoulli equation given above cannot be used. This problem can be solved using the modified Bernoulli equation that accounts for the effects of compressibility, assuming isentropic flow.



11-35 A pitot tube is inserted into the duct of an air heating system parallel to flow, and the differential height of the water column is measured. The flow velocity and the pressure rise at the tip of the pitot tube are to be determined. √

Assumptions **1**The flow through the duct is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** Air is an ideal gas.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3). The gas constant of air is $R = 0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ (Table A-1).

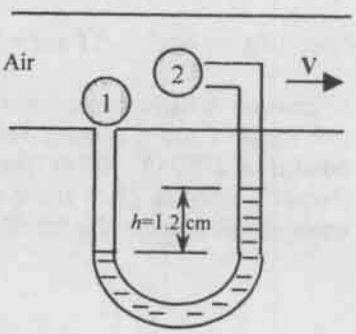
Analysis We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the pitot tube, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the pitot tube. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}=\frac{P_2}{\rho g} \quad\rightarrow\quad \mathbf{V}_1=\sqrt{\frac{2(P_2-P_1)}{\rho_{air}}}$$

where the pressure rise at the tip of the pitot tube is

$$P_2 - P_1 = \rho_w g h = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.012\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 118\ \text{N/m}^2 = 118\ \text{Pa}$$



Also, $$\rho_{air} = \frac{P}{RT} = \frac{98\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(45+273\ \text{K})} = 1.074\ \text{kg/m}^3$$

Substituting,

$$\mathbf{V}_1 = \sqrt{\frac{2(118\ \text{N/m}^2)}{1.074\ \text{kg/m}^3}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = 14.8\ \text{m/s}$$

Discussion Note that the flow velocity in a pipe or duct can be measured easily by a pitot tube by inserting the probe into the pipe or duct parallel to flow, and reading the differential pressure height. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.

11-36 The water in an above the ground swimming pool is to be emptied by unplugging the orifice of a horizontal pipe attached to the bottom of the pool. The maximum discharge rate of water is to be determined. √

Assumptions **1** The orifice has a smooth entrance, and all frictional losses are negligible. **2** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable).

Analysis We take point 1 at the free surface of the pool, and point 2 at the exit of pipe. We take the reference level at the pipe exit ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the Bernoulli equation between these two points simplifies to

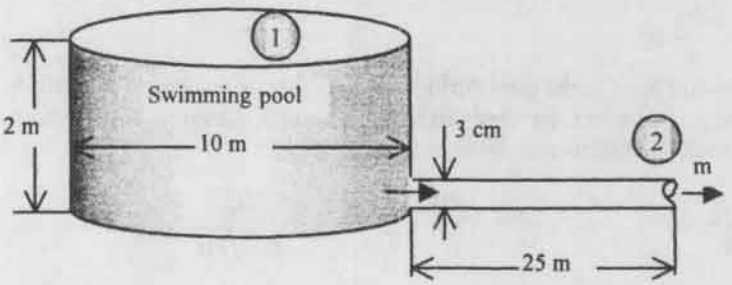
$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{2gz_1}$$

The maximum discharge rate occurs when the water height in the pool is a maximum, which is the case at the beginning and thus $z_1 = h$. Substituting, the maximum flow velocity and discharge rate become

$$\mathbf{V}_{2,\max} = \sqrt{2gh} = \sqrt{2(9.81\,\text{m/s}^2)(2\,\text{m})} = 6.26\,\text{m/s}$$

$$\dot{V}_{\max} = A_{\text{pipe}}\mathbf{V}_{2,\max} = \frac{\pi D^2}{4}\mathbf{V}_{2,\max} = \frac{\pi(0.03\,\text{m})^2}{4}(6.26\,\text{m/s}) = 0.00443\,\text{m}^3/\text{s} = 4.43\,\text{L/s}$$

Discussion The result above is obtained by disregarding all frictional effects. The actual flow rate will be less because of frictional effects during flow and the resulting pressure drop. Also, the flow rate will gradually decrease as the water level in the pipe decreases.



37 The water in an above the ground swimming pool is to be emptied by unplugging the orifice of a izontal pipe attached to the bottom of the pool. The time it will take to empty the tank is to be ermined. √

umptions **1** The orifice has a smooth entrance, and all frictional losses are negligible. **2** The flow is dy, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable).

lysis We take point 1 at the free surface of water in the pool, and point 2 at the exit of pipe. We take the rence level at the pipe exit ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the Bernoulli ation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{2gz_1}$$

generality, we express the water height in the pool at any time t by z, and the discharge velocity by $= \sqrt{2gz}$. Note that water surface in the pool moves down as the pool drains, and thus z is a variable se value changes from h at the beginning to 0 when the pool is emptied completely.

We denote the diameter of the orifice by D, and the diameter of the pool by D_o. The flow rate of r from the pool is obtained by multiplying the discharge velocity by the orifice cross-sectional area,

$$\dot{V} = A_{\text{orifice}}\,\mathbf{V}_2 = \frac{\pi D^2}{4}\sqrt{2gz}$$

n the amount of water that flows through the orifice during a differential time interval dt is

$$dV = \dot{V}dt = \frac{\pi D^2}{4}\sqrt{2gz}\,dt \qquad (1)$$

:h, from conservation of mass, must be equal to the decrease in the volume of water in the pool,

$$dV = A_{\text{tank}}(-dz) = -\frac{\pi D_0^2}{4}dz \qquad (2)$$

re dz is the change in the water level in the pool during dt. (Note that dz is a negative quantity since the tive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water harged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{2gz}\,dt = -\frac{\pi D_0^2}{4}dz \quad \rightarrow \quad dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1}{2gz}}dz = -\frac{D_0^2}{D^2\sqrt{2g}}z^{-\frac{1}{2}}dz$$

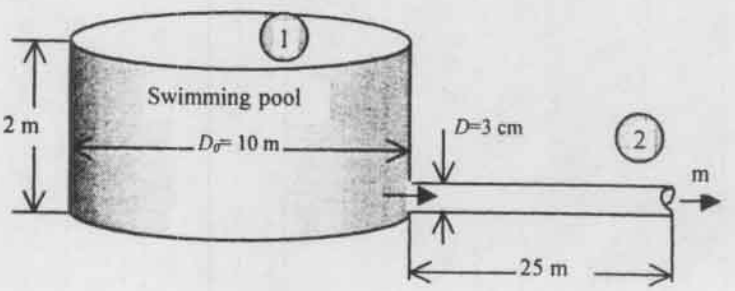
last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time integrating it from $t = 0$ when $z = h$ to $t = t_f$ when $z = 0$ (completely drained pool) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2\sqrt{2g}}\int_{z=z_1}^{0} z^{-1/2}dz \quad \rightarrow \quad t_f = -\frac{D_0^2}{D^2\sqrt{2g}}\left|\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2\sqrt{2g}}\sqrt{h} = \frac{D_0^2}{D^2}\sqrt{\frac{2h}{g}}$$

tituting, the draining time of the pool will be

$$t_f = \frac{(10\,\text{m})^2}{(0.03\,\text{m})^2}\sqrt{\frac{2(2\,\text{m})}{9.81\,\text{m/s}^2}} = 70{,}900\,\text{s} = 19.7\,\text{h}$$

ussion This is the minimum discharging time it is obtained by neglecting all friction; the l discharging time will be longer. Note that the harging time is inversely proportional to the re of the orifice diameter. Therefore, the harging time can be reduced to one-fourth by ling the diameter of the orifice.



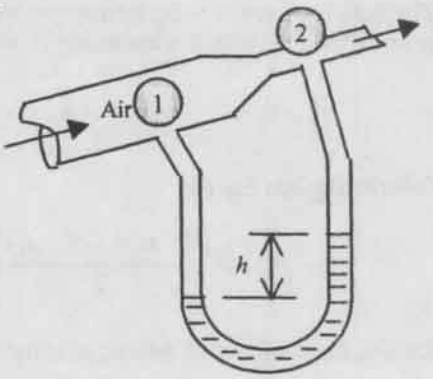
11-38 Air flows upward at a specified rate through an inclined pipe whose diameter is reduced through a reducer. The differential height between fluid levels of the two arms of a water manometer attached across the reducer is to be determined. √

Assumptions **1**The flow through the duct is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** Air is an ideal gas. **3** The effect of air column on the pressure change is negligible because of its low density. **3** The air flow is parallel to the entrance of each arm of the manometer, and thus no dynamic effects are involved.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3). The gas constant of air is $R = 0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ (Table A-1).

Analysis We take points 1 and 2 at the lower and upper connection points, respectively, of the two arms of the manometer, and take the lower connection point as the reference level. Noting that the effect of elevation on the pressure change of a gas is negligible, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad\rightarrow\quad P_1-P_2=\rho_{\text{air}}\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2}$$



where $$\rho_{air}=\frac{P}{RT}=\frac{110\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(50+273\ \text{K})}=1.19\ \text{kg/m}^3$$

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.030\ \text{m}^3/\text{s}}{\pi(0.06\ \text{m})^2/4}=10.6\ \text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.030\ \text{m}^3/\text{s}}{\pi(0.04\ \text{m})^2/4}=23.9\ \text{m/s}$$

Substituting,

$$P_2-P_1=(1.19\ \text{kg/m}^3)\frac{(23.9\ \text{m/s})^2-(10.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)=275\ \text{N/m}^2=275\ \text{Pa}$$

The differential height of water in the manometer corresponding to this pressure change is determined from $\Delta P=\rho_{water}gh$ to be

$$h=\frac{P_1-P_2}{\rho_{water}g}=\frac{275\ \text{N/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)=0.028\ \text{m}=\mathbf{2.8\ cm}$$

Discussion When the effect of air column on pressure change is considered, the pressure chance becomes

$$\begin{aligned}P_1-P_2&=\frac{\rho_{air}(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2}+g(z_2-z_1)\\&=(1.19\ \text{kg/m}^3)\left[\frac{(23.9\ \text{m/s})^2-(10.6\ \text{m/s})^2}{2}+(9.81\ \text{m/s}^2)(0.2\ \text{m})\right]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\\&=(273+2)\ \text{N/m}^2=275\ \text{N/m}^2=275\ \text{Pa}\end{aligned}$$

This difference between the two results (273 and 275 Pa) is less than 1%. Therefore, the effect of air column on pressure change is, indeed, negligible as assumed. In other words, the pressure change of air in the duct is almost entirely due to velocity change, and the effect of elevation change is negligible.

11-39E Air is flowing through a venturi meter with known diameters and measured pressures. A relation for the flow rate is to be obtained, and its numerical value is to be determined.

Assumptions 1The flow through the venturi is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The effect of air column on the pressure change is negligible because of its low density, and thus the pressure can be assumed to be uniform at a given cross-section of the venturi meter (independent of elevation change). **3** The flow is horizontal (this assumption is usually unnecessary for gas flow. But the gravity effects may be important for liquid flow, and the elevation change $z_1 - z_2$ must be considered in analysis when its effect on pressure change is significant).

Properties The density of air is given to be $\rho = 0.075$ lbm/ft^3.

Analysis We take point 1 at the main flow section and point 2 at the throat along the centerline of the venturi meter. Noting that $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

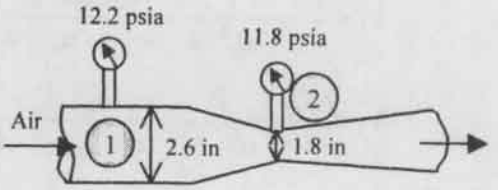
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad P_1-P_2=\rho\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2} \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad \rightarrow \quad A_1\mathbf{V}_1=A_2\mathbf{V}_2=\dot{V} \quad \rightarrow \quad \mathbf{V}_1=\frac{\dot{V}}{A_1} \quad \text{and} \quad \mathbf{V}_2=\frac{\dot{V}}{A_2} \qquad (2)$$

Substituting into Eq. (1),

$$P_1-P_2=\rho\frac{(\dot{V}/A_2)^2-(\dot{V}/A_1)^2}{2}=\frac{\rho\dot{V}^2}{2A_2^2}\left(1-\frac{A_2^2}{A_1^2}\right)$$



Solving for $\dot{V}$ gives the desired relation for the flow rate,

$$\dot{V}=A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}} \qquad (3)$$

The flow rate for the given case can be determined by substituting the given values into this relation to be

$$\dot{V}=\frac{\pi D_2^2}{4}\sqrt{\frac{2(P_1-P_2)}{\rho[1-(D_2/D_1)^4]}}=\frac{\pi(1.8/12\text{ ft})^2}{4}\sqrt{\frac{2(12.2-11.8)\text{ psi}}{(0.075\text{ lbm/ft}^3)[1-(1.8/2.6)^4]}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)}$$

$$= \mathbf{4.48\ ft^3/s}$$

Discussion Venturi meters are commonly used as flow meters to measure the flow rate of gases and liquids by simply measuring the pressure difference $P_1 - P_2$ by a manometer or pressure transducers. The actual flow rate will be less than the value obtained from Eq. (3) because of the friction losses along the wall surfaces in actual flow. But this difference can be as little as 1% in a well-designed venturi meter. The effects of deviation from the idealized Bernoulli flow can be accounted for by expressing Eq. (3) as

$$\dot{V}=C_c A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}}$$

where C_c is the *venturi discharge coefficient* whose value is less than 1 (it is as large as 0.99 for well-designed venturi meters in certain ranges of flow). For Re > 10^5, the value of venturi discharge coefficient is usually greater than 0.96.

11-40 The gage pressure in the water mains of a city at a particular location is given. It is to be determined if this main can serve water to neighborhoods that are at a given elevation relative to this location. √

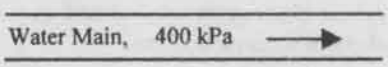
Assumptions Water is incompressible and thus its density is constant.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3).

Analysis Noting that the gage pressure at a dept of h in a fluid is given by $P_{gage} = \rho_{water} gh$, the height of a fluid column corresponding to a gage pressure of 400 kPa is determined to be

$$h = \frac{P_{gage}}{\rho_{water} g} = \frac{400{,}000\ \text{N/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 40.8\ \text{m}$$

which is less than 50 m. Therefore, this main **cannot** serve water to neighborhoods that are 50 m above this location.



Discussion Note that h must be much greater than 50 m for water to have enough pressure to serve the water needs of the neighborhood.

11-41 A hand-held bicycle pump with a liquid reservoir is used as an atomizer by forcing air at a high velocity through a small hole. The minimum speed that the piston must be moved in the cylinder to initiate the atomizing effect is to be determined. √

Assumptions **1**The flows of air and water are steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** Air is an ideal gas. **3** The liquid reservoir is open to the atmosphere. **4** The device is held horizontal. **5** The water velocity through the tube is low.

Properties We take the density of water to be $\rho = 1000$ kg/m^3 (Table A-3). The gas constant of air is $R = 0.287$ kPa·m^3/kg·K (Table A-1).

Analysis We take point 1 at the exit of the hole, point 2 in air far from the hole on a horizontal line, point 3 at the exit of the tube in air stream (so that points 1 and 3 coincide), and point 4 at the free surface of the liquid in the reservoir ($P_2 = P_4 = P_{atm}$ and $P_1 = P_3$). We also take the level of the hole to be the reference level (so that $z_1 = z_2 = z_3 = 0$ and $z_4 = -h$). Noting that $\mathbf{V}_2 \cong \mathbf{V}_3 \cong \mathbf{V}_4 \cong 0$, the Bernoulli equation for the air and water streams becomes

$$\textit{Water (3-4):}\quad \frac{P_3}{\rho g}+\frac{\mathbf{V}_3^2}{2g}+z_3=\frac{P_4}{\rho g}+\frac{\mathbf{V}_4^2}{2g}+z_4 \;\rightarrow\; \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+(-h) \;\rightarrow\; P_1-P_{atm}=-\rho_{\text{water}}gh \qquad (1)$$

$$\textit{Air (1-2):}\quad \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \;\rightarrow\; \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}=\frac{P_{atm}}{\rho g} \;\rightarrow\; \mathbf{V}_1=\sqrt{\frac{2(P_{atm}-P_1)}{\rho_{\text{air}}}} \qquad (2)$$

where

$$\rho_{air}=\frac{P}{RT}=\frac{95\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273\,\text{K})}=1.13\,\text{kg/m}^3$$

Combining Eqs. (1) and (2) and substituting the numerical values,

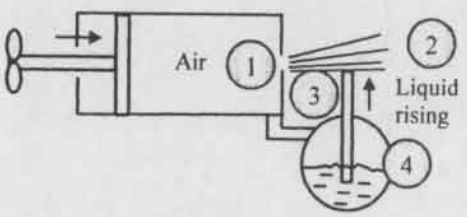
$$\mathbf{V}_1=\sqrt{\frac{2(P_{atm}-P_1)}{\rho_{\text{air}}}}=\mathbf{V}_1=\sqrt{\frac{2\rho_{\text{water}}gh}{\rho_{\text{air}}}}=\sqrt{\frac{2(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.1\,\text{m})}{1.13\,\text{kg/m}^3}}=41.7\,\text{m/s}$$

Taking the flow of air to be steady and incompressible, the conservation of mass for air can be expressed as

$$\dot{V}_{\text{piston}}=\dot{V}_{\text{hole}} \;\rightarrow\; \mathbf{V}_{\text{piston}}A_{\text{piston}}=\mathbf{V}_{\text{hole}}A_{\text{hole}} \;\rightarrow\; \mathbf{V}_{\text{piston}}=\frac{A_{\text{hole}}}{A_{\text{piston}}}\mathbf{V}_{\text{hole}}=\frac{\pi D_{\text{hole}}^2/4}{\pi D_{\text{piston}}^2/4}\mathbf{V}_1$$

Simplifying and substituting, the piston velocity is determined to be

$$\mathbf{V}_{\text{piston}}=\left(\frac{D_{\text{hole}}}{D_{\text{piston}}}\right)^2\mathbf{V}_1=\left(\frac{0.3\,\text{cm}}{5\,\text{cm}}\right)^2(41.7\,\text{m/s})=\mathbf{0.15\,m/s}$$



Discussion In reality, the piston velocity must be higher to overcome the losses. Also, a lower piston velocity will do the job if the diameter of the hole is reduced.

11-42 The water height in an airtight pressurized tank is given. A hose pointing straight up is connected to the bottom of the tank. The maximum height to which the water stream could rise is to be determined. √

Assumptions **1** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The friction between the water and air is negligible.

Properties We take the density of water to be 1000 kg/m^3.

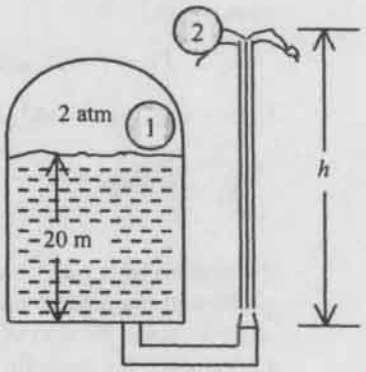
Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory. Also, we take the reference level at the bottom of the tank. At the top of the water trajectory $\mathbf{V}_2 = 0$, and atmospheric pressure pertains. Noting that $z_1 = 20$ m, $P_{1,\text{gage}} = 2$ atm, $P_2 = P_{\text{atm}}$, and that the fluid velocity at the free surface of the tank is very low ($\mathbf{V}_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_1}{\rho g}+z_1=\frac{P_{atm}}{\rho g}+z_2 \quad \rightarrow \quad z_2=\frac{P_1-P_{atm}}{\rho g}+z_1=\frac{P_{1,\text{gage}}}{\rho g}+z_1$$

Substituting,

$$z_2=\frac{2\,\text{atm}}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}\left(\frac{101{,}325\,\text{N/m}^2}{1\,\text{atm}}\right)\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right)+20=\mathbf{40.7\ m}$$

Therefore, the water jet can rise as high as 40.7 m into the sky from the ground.



Discussion The result obtained by the Bernoulli equation represents the upper limit, and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.7 m (giving us an upper limit), and in all likelihood, the rise will be much less because of frictional losses.

11-43 A pitot tube equipped with a water manometer is held parallel to air flow, and the differential height of the water column is measured. The flow velocity of air is to be determined. √

Assumptions **1**The flow of air is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The effect of air column on the pressure change is negligible because of its low density, and thus the air column in the manometer can be ignored.

Properties We take the density of water to be $\rho = 1000$ kg/m^3. The density of air is given to be 1.25 kg/m^3.

Analysis We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the pitot tube, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the pitot tube. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}=\frac{P_2}{\rho g} \quad \rightarrow \quad \mathbf{V}_1=\sqrt{\frac{2(P_2-P_1)}{\rho_{air}}} \qquad (1)$$

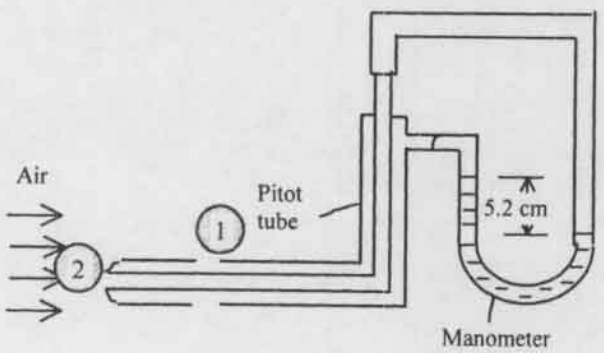
The pressure rise at the tip of the pitot tube is simply the pressure change indicated by the differential water column of the manometer,

$$P_2 - P_1 = \rho_{\text{water}}\, g h \qquad (2)$$

Combining Eqs. (1) and (2) and substituting, the flow velocity is determined to be

$$\mathbf{V}_1=\sqrt{\frac{2\rho_{\text{water}}\, g h}{\rho_{\text{air}}}}=\sqrt{\frac{2(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.052\,\text{m})}{1.25\,\text{kg/m}^3}}=\mathbf{28.6\ m/s}$$

Discussion Note that flow velocity in a pipe or duct can be measured easily by a pitot tube by inserting the probe into the pipe or duct parallel to flow, and reading the differential height. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.



11-44E A pitot tube equipped with a differential pressure gage is used to measure the air velocity in a duct. For a given differential pressure reading, the flow velocity of air is to be determined. √

Assumptions The flow of air is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable).

Properties The gas constant of air is $R = 0.3704$ psia·ft^3/lbm·R.

Analysis We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the pitot tube, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the pitot tube. Noting that point 2 is a stagnation point and thus $\mathbf{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}=\frac{P_2}{\rho g} \quad \rightarrow \quad \mathbf{V}_1=\sqrt{\frac{2(P_2-P_1)}{\rho}}$$

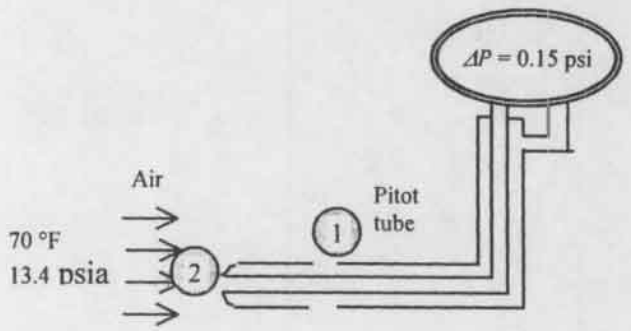
where

$$\rho=\frac{P}{RT}=\frac{13.4\ \text{psia}}{(0.3704\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(70+460\ \text{R})}=0.0683\ \text{lbm/ft}^3$$

Substituting the given values, the flow velocity is determined to be

$$\mathbf{V}_1=\sqrt{\frac{2(0.15\ \text{psi})}{0.0683\ \text{lbm/ft}^3}\left(\frac{144\ \text{lbf/ft}^2}{1\ \text{psi}}\right)\left(\frac{32.2\ \text{lbm}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right)}=\mathbf{143\ ft/s}$$

Discussion Note that flow velocity in a pipe or duct can be measured easily by a pitot tube by inserting the probe into the pipe or duct parallel to flow, and reading the pressure differential. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.



11-45 In a power plant, water enters the nozzles of a hydraulic turbine at a specified pressure. The maximum velocity water can be accelerated to by the nozzles is to be determined. √

Assumptions **1**The flow of water is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** Water enters the nozzle with a low velocity.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

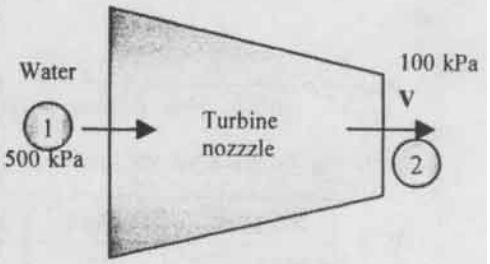
Analysis We take points 1 and 2 at the inlet and exit of the nozzle, respectively. Noting that $\mathbf{V}_1 \cong 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2(P_1-P_{atm})}{\rho}}$$

Substituting the given values, the nozzle exit velocity is determined to be

$$\mathbf{V}_1=\sqrt{\frac{2(500-100)\,\text{kPa}}{1000\,\text{kg/m}^3}\left(\frac{1000\,\text{N/m}^2}{1\,\text{kPa}}\right)\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right)}=\mathbf{28.3\ m/s}$$

Discussion This is the maximum nozzle exit velocity, and the actual velocity will be less because of friction between water and the walls of the nozzle.



Energy Equation

11-46C It is *impossible* for the fluid temperature to decrease during steady, incompressible, adiabatic flow since this would require the entropy of an adiabatic system to decrease, which would be a violation of the 2nd law of thermodynamics.

11-47C Yes, the flow is *frictionless* if the fluid temperature remains constant during steady, incompressible flow since any irreversibility such as friction would cause the entropy and thus temperature of the fluid to increase during adiabatic flow.

11-48C *Head loss* is the loss of mechanical energy expressed as an equivalent column height of fluid, i.e., head. It is related to the mechanical energy loss by $h_L = \dfrac{e_{\text{mech, loss}}}{g} = \dfrac{\dot{E}_{\text{mech, loss}}}{\dot{m}g}$.

11-49C Pump head is the useful power input to the pump expressed as an equivalent column height of fluid. It is related to the useful pumping power input by $h_{\text{pump, u}} = \dfrac{w_{\text{pump, u}}}{g} = \dfrac{\dot{W}_{\text{pump, u}}}{\dot{m}g}$

11-50C The *kinetic energy correction factor* is a correction factor to account for the fact that kinetic energy using average velocity is not the same as the actual kinetic energy using the actual velocity profile. Its effect is usually negligible (the square of a sum is not equal to the sum of the squares of its components).

11-51C By Bernoulli's Equation, the maximum theoretical height to which the water stream could rise is the tank water level, which is 20 meters above the ground. Since the water rises above the tank level, the tank cover must be airtight, containing pressurized air above the water surface. Otherwise, a pump would have to pressurize the water somewhere in the hose.

11-52 Underground water is pumped to a pool at a given elevation. The maximum flow rate and the pressures at the inlet and outlet of the pump are to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the inlet and the outlet of the pump is negligible. **3** We assume the flow to be *frictionless* since the *maximum* flow rate is to be determined, $\dot{E}_{\text{mech,loss}} = 0$.

Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 (Table A-3).

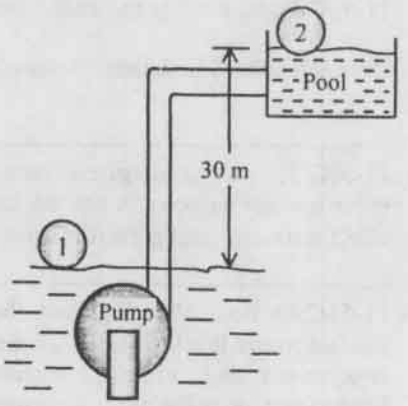
Analysis (*a*) The pump-motor draws 3-kW of power, and is 70% efficient. Then the useful mechanical (shaft) power it delivers to the fluid is

$$\dot{W}_{\text{pump,u}} = \eta_{\text{pump-motor}} \dot{W}_{\text{electric}} = (0.70)(3\,\text{kW}) = 2.1\,\text{kW}$$

We take point 1 at the free surface of underground water, which is also taken as the reference level ($z_1 = 0$), and point 2 at the free surface of the pool. Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$), the velocities are negligible at both points ($\mathbf{V}_1 = \mathbf{V}_2 = 0$), and frictional losses are disregarded. Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump,u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}}$$

$$\rightarrow \quad \dot{W}_{\text{pump,u}} = \dot{m} g z_2$$



Then the mass and volume flow rates of water become

$$\dot{m} = \frac{\dot{W}_{\text{pump,u}}}{g z_2} = \frac{2.1\,\text{kJ/s}}{(9.81\,\text{m/s}^2)(30\,\text{m})}\left(\frac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ}}\right) = 7.14\,\text{kg/s}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{7.14\,\text{kg/s}}{1000\,\text{kg/m}^3} = \mathbf{7.14\times10^{-3}\ m^3/s}$$

(*b*) We take points 3 and 4 at the inlet and the exit of the pump, respectively, where the flow velocities are

$$\mathbf{V}_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi D_3^2/4} = \frac{7.14\times10^{-3}\ \text{m}^3/\text{s}}{\pi(0.05\,\text{m})^2/4} = 3.64\,\text{m/s}, \quad \mathbf{V}_4 = \frac{\dot{V}}{A_4} = \frac{\dot{V}}{\pi D_4^2/4} = \frac{7.14\times10^{-3}\ \text{m}^3/\text{s}}{\pi(0.07\,\text{m})^2/4} = 1.86\,\text{m/s}$$

We take the pump as the control volume. Noting that $z_3 = z_4$ and $\dot{E}_{\text{mech,loss}} = 0$ (since we consider the useful pump power only), the energy equation for this control volume reduces to

$$\dot{m}\left(\frac{P_3}{\rho} + \frac{\mathbf{V}_3^2}{2} + gz_3\right) + \dot{W}_{\text{pump,u}} = \dot{m}\left(\frac{P_4}{\rho} + \frac{\mathbf{V}_4^2}{2} + gz_4\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}}$$

$$\rightarrow P_4 - P_3 = \frac{\rho(\mathbf{V}_3^2 - \mathbf{V}_4^2)}{2} + \frac{\dot{W}_{\text{pump,u}}}{\dot{V}}$$

Substituting,

$$P_4 - P_3 = \frac{(1000\,\text{kg/m}^3)[(3.64\,\text{m/s})^2 - (1.86\,\text{m/s})^2]}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) + \frac{2.1\,\text{kJ/s}}{7.14\times10^{-3}\ \text{m}^3/\text{s}}\left(\frac{1\,\text{kN}\cdot\text{m}}{1\,\text{kJ}}\right)$$

$$= (4.9 + 294.1)\,\text{kN/m}^2 = \mathbf{299\,kPa}$$

Discussion In an actual system, the flow rate of water will be less because of friction in pipes. Also, the effect of flow velocities on the pressure change across the pump is negligible in this case (under 2%) and can be ignored.

11-53 Underground water is pumped to a pool at a given elevation. For a given head loss, the flow rate and the pressures at the inlet and outlet of the pump are to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the inlet and the outlet of the pump is negligible.

Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 (Table A-3).

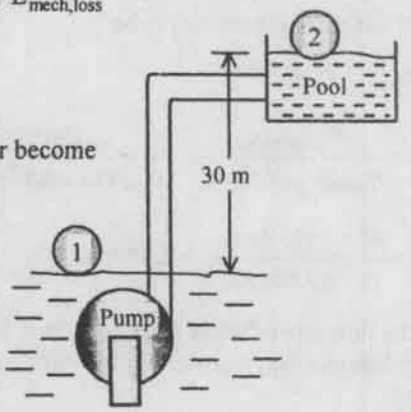
Analysis (*a*) The pump-motor draws 3-kW of power, and is 70% efficient. Then the useful mechanical (shaft) power it delivers to the fluid is

$$\dot{W}_{\text{pump,u}} = \eta_{\text{pump-motor}} \dot{W}_{\text{electric}} = (0.70)(3\,\text{kW}) = 2.1\,\text{kW}$$

We take point 1 at the free surface of underground water, which is also taken as the reference level ($z_1 = 0$), and point 2 at the free surface of the pool. Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$), and the velocities are negligible at both points ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump,u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}}$$

$$\rightarrow \quad \dot{W}_{\text{pump,u}} = \dot{m}gz_2 + \dot{E}_{\text{mech,loss}}$$



Noting that $\dot{E}_{\text{mech, loss}} = \dot{m}gh_L$, the mass and volume flow rates of water become

$$\dot{m} = \frac{\dot{W}_{\text{pump,u}}}{gz_2 + gh_L} = \frac{\dot{W}_{\text{pump,u}}}{g(z_2 + h_L)}$$

$$= \frac{2.1\,\text{kJ/s}}{(9.81\,\text{m/s}^2)(30+5\,\text{m})}\left(\frac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ}}\right) = 6.12\,\text{kg/s}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{6.12\,\text{kg/s}}{1000\,\text{kg/m}^3} = \mathbf{6.12\times10^{-3}\ m^3/s}$$

(*b*) We take points 3 and 4 at the inlet and the exit of the pump, respectively, where the flow velocities are

$$\mathbf{V}_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi D_3^2/4} = \frac{6.12\times10^{-3}\ \text{m}^3/\text{s}}{\pi(0.05\,\text{m})^2/4} = 3.14\,\text{m/s}, \quad \mathbf{V}_4 = \frac{\dot{V}}{A_4} = \frac{\dot{V}}{\pi D_4^2/4} = \frac{6.12\times10^{-3}\ \text{m}^3/\text{s}}{\pi(0.07\,\text{m})^2/4} = 1.60\,\text{m/s}$$

We take the pump as the control volume. Noting that $z_3 = z_4$ and $\dot{E}_{\text{mech,loss}} = 0$ (since we consider the pump and useful pump power only), the energy equation for this control volume reduces to

$$\dot{m}\left(\frac{P_3}{\rho} + \frac{\mathbf{V}_3^2}{2} + gz_3\right) + \dot{W}_{\text{pump,u}} = \dot{m}\left(\frac{P_4}{\rho} + \frac{\mathbf{V}_4^2}{2} + gz_4\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}}$$

$$\rightarrow P_4 - P_3 = \frac{\rho(\mathbf{V}_3^2 - \mathbf{V}_4^2)}{2} + \frac{\dot{W}_{\text{pump,u}}}{\dot{V}}$$

Substituting,

$$P_4 - P_3 = \frac{(1000\,\text{kg/m}^3)[(3.14\,\text{m/s})^2 - (1.60\,\text{m/s})^2]}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) + \frac{2.1\,\text{kJ/s}}{6.12\times10^{-3}\ \text{m}^3/\text{s}}\left(\frac{1\,\text{kN}\cdot\text{m}}{1\,\text{kJ}}\right)$$

$$= (4+343)\,\text{kN/m}^2 = \mathbf{347\,kPa}$$

Discussion Note that frictional losses in pipes causes the flow rate of water to decrease. Also, the effect of flow velocities on the pressure change across the pump is negligible in this case (about 1%) and can be ignored.

54E In a hydroelectric power plant, the elevation difference, the power generation, and the overall bine-generator efficiency are given. The minimum flow rate required is to be determined.√

umptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water levels at the reservoir the discharge site remain constant. **3** We assume the flow to be *frictionless* since the *minimum* flow rate be determined, $\dot{E}_{\text{mech,loss}} = 0$.

perties We take the density of water to be $\rho = 62.4$ lbm/ft^3.

lysis We take point 1 at the free surface of the reservoir and point 2 at the free surface of the discharge er stream, which is also taken as the reference level ($z_2 = 0$). Also, both 1 and 2 are open to the osphere ($P_1 = P_2 = P_{\text{atm}}$), the velocities are negligible at both points ($\mathbf{V}_1 = \mathbf{V}_2 = 0$), and frictional losses disregarded. Then the energy equation in terms of heads for steady incompressible flow through a trol volume between these two points that includes the turbine and the pipes reduces to

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 + h_{\text{turbine}} + h_L \quad \rightarrow \quad h_{\text{turbine}} = z_1$$

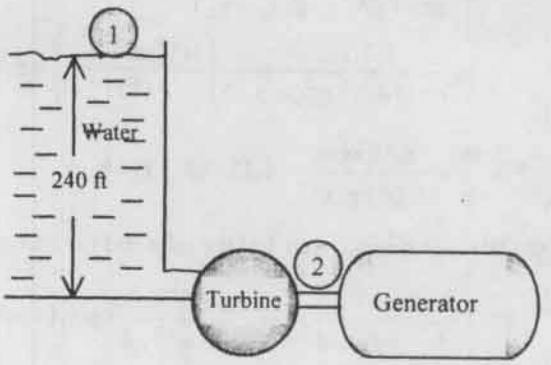
stituting and noting that $\dot{W}_{\text{turbine, elect}} = \eta_{\text{turbine-gen}} \dot{m} g h_{\text{turbine}}$, the turbine head and the mass and volume rates of water are determined to be

$$h_{\text{turbine}} = z_1 = 240 \text{ ft}$$

$$\dot{m} = \frac{\dot{W}_{\text{turbine,elect}}}{\eta_{\text{turbine-gen}} g h_{\text{turbine}}} = \frac{100 \text{ kW}}{0.83(32.2 \text{ ft/s}^2)(240 \text{ ft})} \left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}} \right) \left(\frac{0.9478 \text{ Btu/s}}{1 \text{ kW}} \right) = \mathbf{370 \text{ lbm/s}}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{370 \text{ lbm/s}}{62.4 \text{ lbm/ft}^3} = \mathbf{5.93 \text{ ft}^3/\text{s}}$$

refore, the flow rate of water must be at least 5.93 ft^3/s to erate the desired electric power while overcoming friction es in pipes.



ussion In an actual system, the flow rate of water will be e because of frictional losses in pipes.

11-55E In a hydroelectric power plant, the elevation difference, the head loss, the power generation, and the overall turbine-generator efficiency are given. The flow rate required is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water levels at the reservoir and the discharge site remain constant.

Properties We take the density of water to be $\rho = 62.4$ lbm/ft^3.

Analysis We take point 1 at the free surface of the reservoir and point 2 at the free surface of the discharge water stream, which is also taken as the reference level ($z_2 = 0$). Also, both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$), the velocities are negligible at both points ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation in terms of heads for steady incompressible flow through a control volume between these two points that includes the turbine and the pipes reduces to

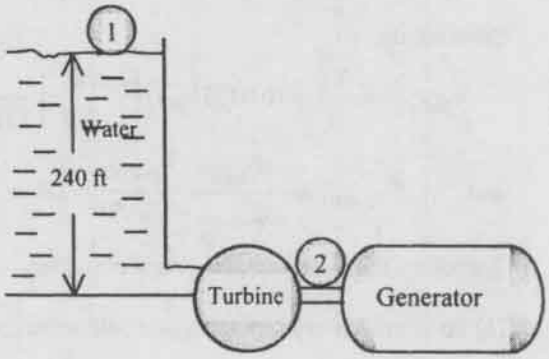
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \;\rightarrow\; h_{\text{turbine}}=z_1-h_L$$

Substituting and noting that $\dot{W}_{\text{turbine, elect}} = \eta_{\text{turbine-gen}}\dot{m}gh_{\text{turbine}}$, the turbine head and the mass and volume flow rates of water are determined to be

$$h_{\text{turbine}} = z_1 - h_L = 240 - 28 = 212 \text{ ft}$$

$$\dot{m}=\frac{\dot{W}_{\text{turbine,elect}}}{\eta_{\text{turbine-gen}}\,gh_{\text{turbine}}}=\frac{100\text{ kW}}{0.83(32.2\text{ ft/s}^2)(212\text{ ft})}\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/lbm}}\right)\left(\frac{0.9478\text{ Btu/s}}{1\text{ kW}}\right)=\mathbf{419\text{ lbm/s}}$$

$$\dot{V}=\frac{\dot{m}}{\rho}=\frac{419\text{ lbm/s}}{62.4\text{ lbm/ft}^3}=\mathbf{6.71\text{ ft}^3/\text{s}}$$



Therefore, the flow rate of water must be at least 6.71 ft^3/s to generate the desired electric power while overcoming friction losses in pipes.

Discussion Note that the effect of frictional losses in pipes is to increase the required flow rate of water to generate a specified amount of electric power.

11-56 A fan is to ventilate a bathroom by replacing the entire volume of air once every 10 minutes while air velocity remains below a specified value. The wattage of the fan-motor unit, the diameter of the fan casing, and the pressure difference across the fan are to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** Frictional losses along the flow (other than those due to the fan-motor inefficiency) are negligible. **3** The fan unit is horizontal so that z = constant along the flow (or, the elevation effects are negligible because of the low density of air).

Properties The density of air is given to be 1.25 kg/m^3.

Analysis (*a*) The volume of air in the bathroom is $V = 2\text{ m} \times 3\text{ m} \times 3\text{ m} = 18\text{ m}^3$. Then the volume and mass flow rates of air through the casing must be

$$\dot{V} = \frac{V}{\Delta t} = \frac{18\text{ m}^3}{10 \times 60\text{ s}} = 0.03\text{ m}^3/\text{s}$$

$$\dot{m} = \rho \dot{V} = (1.25\text{ kg/m}^3)(0.03\text{ m}^3/\text{s}) = 0.0375\text{ kg/s}$$

We take points 1 and 2 on the inlet and exit sides of the fan, respectively. Point 1 is sufficiently far from the fan so that $P_1 = P_{atm}$ and the flow velocity is negligible ($\mathbf{V}_1 = 0$). Also, $P_2 = P_{atm}$. Then the energy equation for this control volume between the points 1 and 2 reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2} = (0.0375\text{ kg/s})\frac{(8\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = 1.2\text{ W}$$

and $$\dot{W}_{\text{fan, elect}} = \frac{\dot{W}_{\text{fan, u}}}{\eta_{\text{fan-motor}}} = \frac{1.2\text{ W}}{0.5} = \mathbf{2.4\text{ W}}$$

Therefore, the electric power rating of the fan/motor unit must be 2.4 W.

(*b*) For air mean velocity to remain below the specified value, the diameter of the fan casing should be

$$\dot{V} = A_2\mathbf{V}_2 = (\pi D_2^2/4)\mathbf{V}_2 \quad \rightarrow \quad D_2 = \sqrt{\frac{4\dot{V}}{\pi \mathbf{V}_2}} = \sqrt{\frac{4(0.03\text{ m}^3/\text{s})}{\pi(8\text{ m/s})}} = 0.069\text{ m} = \mathbf{6.9\text{ cm}}$$

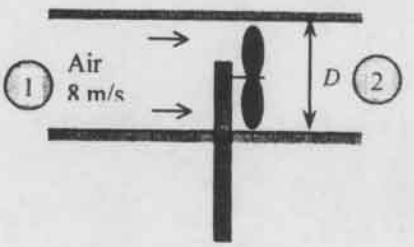
(*c*) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. Noting that $z_3 = z_4$ and $\mathbf{V}_3 = \mathbf{V}_4$ since the fan is a narrow cross-section and neglecting flow loses (other than the loses of the fan unit, which is accounted for by the efficiency), the energy equation for the fan section reduces to

$$\dot{m}\frac{P_3}{\rho} + \dot{W}_{\text{fan, u}} = \dot{m}\frac{P_4}{\rho} \quad \rightarrow \quad P_4 - P_3 = \frac{\dot{W}_{\text{fan, u}}}{\dot{m}/\rho} = \frac{\dot{W}_{\text{fan, u}}}{\dot{V}}$$

Substituting ,

$$P_4 - P_3 = \frac{1.2\text{ W}}{0.03\text{ m}^3/\text{s}}\left(\frac{1\text{ N}\cdot\text{m/s}}{1\text{ W}}\right) = 40\text{ N/m}^2 = \mathbf{40\text{ Pa}}$$

Therefore, the fan will raise the pressure of air by 40 Pa before discharging it.



Discussion Note that only half of the electric energy y consumed by the fan-motor unit is converted to the mechanical energy of air while the remaining half is converted to heat because of imperfections.

11-57 Water is pumped from a large lake to a higher reservoir. The head loss of the piping system is given. The mechanical efficiency of the pump is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the lake and the reservoir is constant.

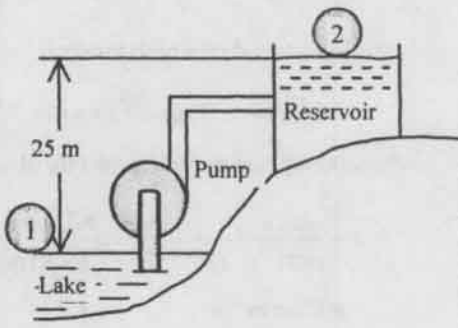
Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3).

Analysis We choose points 1 and 2 at the free surfaces of the lake and the reservoir, respectively, and take the surface of the lake as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$) and the velocities at both locations are negligible ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho}+\frac{\mathbf{V}_1^2}{2}+gz_1\right)+\dot{W}_{\text{pump,u}}=\dot{m}\left(\frac{P_2}{\rho}+\frac{\mathbf{V}_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \rightarrow \dot{W}_{\text{pump,u}}=\dot{m}gz_2+\dot{E}_{\text{mech,loss}}$$

Noting that $\dot{E}_{\text{mech, loss}} = \dot{m}gh_L$, the useful pump power is

$$\begin{aligned}\dot{W}_{\text{pump,u}} &= \dot{m}gz_2+\dot{m}gh_L=\rho\dot{V}g(z_2+h_L)\\ &= (1000\ \text{kg/m}^3)(0.025\ \text{m}^3/\text{s})[(25+7)\ \text{m}]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\\ &= 7.85\ \text{kNm/s} = 7.85\ \text{kW}\end{aligned}$$



Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}}=\frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{shaft}}}=\frac{7.85\ \text{kW}}{10\ \text{kW}}=0.785=\mathbf{78.5\%}$$

Discussion A more practical measure of performance of the pump is the overall efficiency, which can be obtained by multiplying the pump efficiency by the motor efficiency.

11-58 A pump with a specified shaft power and efficiency is used to raise water to a higher elevation. The maximum flow rate of water is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the reservoirs is constant. **3** We assume the flow to be *frictionless* since the *maximum* flow rate is to be determined, $\dot{E}_{\text{mech,loss}} = 0$.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3).

Analysis We choose points 1 and 2 at the free surfaces of the lower and upper reservoirs, respectively, and take the surface of the lower reservoir as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$) and the velocities at both locations are negligible ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m}\left(\frac{P_1}{\rho}+\frac{\mathbf{V}_1^2}{2}+gz_1\right)+\dot{W}_{\text{pump,u}}=\dot{m}\left(\frac{P_2}{\rho}+\frac{\mathbf{V}_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \rightarrow \dot{W}_{\text{pump,u}}=\dot{m}gz_2=\rho\dot{V}gz_2$$

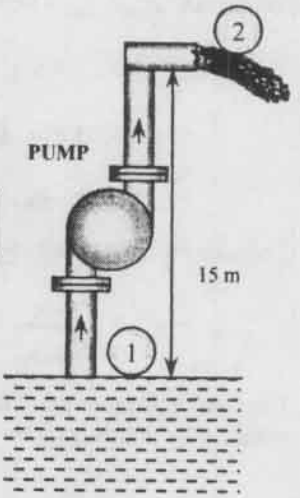
where the useful pumping power is

$$\dot{W}_{\text{pump,u}} = \eta_{\text{pump}}\dot{W}_{\text{pump, shaft}} = (0.75)(7\ \text{hp}) = 5.25\ \text{hp}$$

Substituting, the volume flow rate of water is determined to be

$$\dot{V}=\frac{\dot{W}_{\text{pump,u}}}{\rho g z_2}=\frac{5.25\ \text{hp}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(15\ \text{m})}\left(\frac{745.7\ \text{W}}{1\ \text{hp}}\right)\left(\frac{1\ \text{N}\cdot\text{m/s}}{1\ \text{W}}\right)\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)$$

$$= \mathbf{0.0266\ m^3/s}$$



Discussion This is the maximum flow rate since the frictional effects are ignored. In an actual system, the flow rate of water will be less because of friction in pipes.

11-59 Water flows at a specified rate in a horizontal pipe whose diameter is decreased by a reducer. The pressures are measured before and after the reducer. The head loss in the reducer is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The pipe is horizontal.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3).

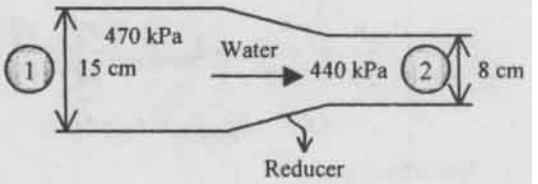
Analysis We take points 1 and 2 along the centerline of the pipe before and after the reducer, respectively. Noting that $z_1 = z_2$, the energy equation for steady incompressible flow through a control volume between these two points reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_L=\frac{P_1-P_2}{\rho g}+\frac{\mathbf{V}_1^2-\mathbf{V}_2^2}{2g}$$

where

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.035\,\text{m}^3/\text{s}}{\pi(0.15\,\text{m})^2/4}=1.98\,\text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.035\,\text{m}^3/\text{s}}{\pi(0.08\,\text{m})^2/4}=6.96\,\text{m/s}$$



Substituting, the head loss in the reducer is determined to be

$$h_L=\frac{(470-440)\,\text{kPa}}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}\left(\frac{1\,\text{kN/m}^2}{1\,\text{kPa}}\right)\left(\frac{1000\,\text{kg}\cdot\text{m/s}^2}{1\,\text{kN}}\right)+\frac{(1.98\,\text{m/s})^2-(6.96\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}$$

$$=3.06-2.27=\mathbf{0.79\ m}$$

Discussion Note that the 0.79 m of the head loss is due to frictional effects and 2.27 m is due to the increase in velocity. This head loss corresponds to a power potential loss of

$$\dot{E}_{\text{mech,loss}}=\rho\dot{V}gh_L=(1000\,\text{kg/m}^3)(0.035\,\text{m}^3/\text{s})(9.81\,\text{m/s}^2)(0.79\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{W}}{1\,\text{N}\cdot\text{m/s}}\right)=\mathbf{271\ W}$$

11-60 A hose connected to the bottom of a tank is equipped with a nozzle at the end pointing straight up. The water is pressurized by a pump, and the height of the water jet is measured. The minimum pressure rise supplied by the pump is to be determined.

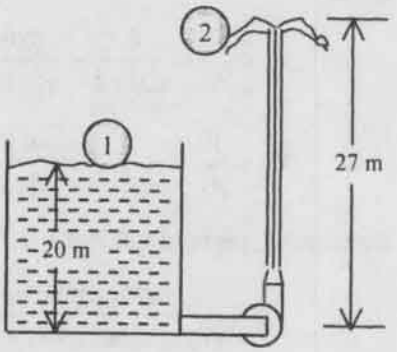
Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** Friction between the water and air as well as friction in the hose is negligible. **3** The water surface is open to the atmosphere.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3).

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory where $\mathbf{V}_2 = 0$ and $P_1 = P_2 = P_{\text{atm}}$. Also, we take the reference level at the bottom of the tank. Noting that $z_1 = 20$ m and $z_2 = 27$ m, $h_L = 0$ (to get the minimum value for required pressure rise), and that the fluid velocity at the free surface of the tank is very low ($\mathbf{V}_1 \cong 0$), the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the water stream reduces to

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 + h_{\text{turbine}} + h_L$$

$$\rightarrow \quad h_{\text{pump,u}} = z_2 - z_1$$



Substituting,

$$h_{\text{pump,u}} = 27 - 20 = 7\ \text{m}$$

A water column height of 7 m corresponds to a pressure rise of

$$\Delta P_{\text{pump, min}} = \rho g h_{\text{pump,u}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(7\ \text{m})\left(\frac{1\ \text{N}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 68.7\ \text{kN/m}^2 = \mathbf{68.7\ kPa}$$

Therefore, the pump must supply a minimum pressure rise of 68.7 kPa.

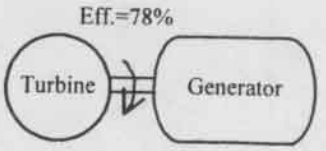
Discussion The result obtained above represents the minimum value, and should be interpreted accordingly. In reality, a larger pressure rise will need to be supplied to overcome friction.

11-61 The available head of a hydraulic turbine and its overall efficiency are given. The electric power output of this turbine is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The available head remains constant.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3).

Analysis When the turbine head is available, the corresponding power output is determined from



$$\dot{W}_{\text{turbine}} = \eta_{\text{turbine}}\dot{m}gh_{\text{turbine}} = \eta_{\text{turbine}}\rho\dot{V}gh_{\text{turbine}}$$

Substituting,

$$\dot{W}_{\text{turbine}} = 0.78(1000\ \text{kg/m}^3)(0.25\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(60\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right) = \mathbf{115\ kW}$$

Discussion Note that the power output of a hydraulic turbine is proportional to the available turbine head and the flow rate.

11-62 An entrepreneur is to build a large reservoir above the lake level, and pump water from the lake to the reservoir at night using cheap power, and let the water flow from the reservoir back to the lake during the day, producing power. The potential revenue this system can generate per year is to be determined.

Assumptions **1** The flow in each direction is steady, one-dimensional, and incompressible. **2** The elevation difference between the lake and the reservoir can be taken to be constant, and the elevation change of reservoir during charging and discharging is disregarded. **3** The given unit prices remain constant. **4** The system operates every day of the year for 10 hours in each mode.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3).

Analysis We choose points 1 and 2 at the free surfaces of the lake and the reservoir, respectively, and take the surface of the lake as the reference level. Both points are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$) and the velocities at both locations are negligible ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation in terms of heads for steady incompressible flow through a control volume between these two points that includes the pump (or the turbine) and the pipes reduces to

$$\textit{Pump mode:}\ \frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \rightarrow\ h_{\text{pump, u}}=z_2+h_L=40+4=44\ \text{m}$$

Turbine mode: (switch points 1 and 2 so that 1 is on inlet side) $\rightarrow\quad h_{\text{turbine}}=z_1-h_L=40-4=36\ \text{m}$

The pump and turbine power corresponding to these heads are

$$\dot{W}_{\text{pump}}=\frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\rho\dot{V}gh_{\text{pump, u}}}{\eta_{\text{pump-motor}}}$$

$$=\frac{(1000\ \text{kg/m}^3)(2\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(44\ \text{m})}{0.75}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right)=1151\ \text{kW}$$

$$\dot{W}_{\text{turbine}}=\eta_{\text{turbine-gen}}\dot{m}gh_{\text{turbine}}=\eta_{\text{turbine-gen}}\rho\dot{V}gh_{\text{turbine}}$$

$$=0.75(1000\ \text{kg/m}^3)(2\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(36\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right)=530\ \text{kW}$$

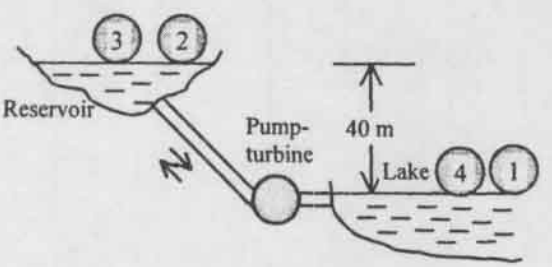
Then the power cost of the pump, the revenue generated by the turbine, and the net income (revenue minus cost) per year become

$$\text{Cost}=\dot{W}_{\text{pump}}\Delta t\times\text{Unit price}=(1151\ \text{kW})(365\times 24\ \text{h/year})(\$0.03/\text{kWh})=\$302{,}500/\text{year}$$

$$\text{Reveue}=\dot{W}_{\text{turbine}}\Delta t\times\text{Unit price}=(530\ \text{kW})(365\times 24\ \text{h/year})(\$0.08/\text{kWh})=\$371{,}400/\text{year}$$

Net income = Revenue – Cost = 371,400 – 302,500 = **$68,900/year**

Discussion It appears that this pump-turbine system has a potential annual income of about $70,000. A decision on such a system will depend on the initial cost of the system, its life, the operating and maintenance costs, the interest rate, and the length of the contract period, among other things.



11-63 Water flows through a horizontal pipe at a specified rate. The pressure drop across a valve in the pipe is measured. The corresponding head loss and the power needed to overcome it are to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The pipe is given to be horizontal (otherwise the elevation difference across the valve is negligible). **3** The mean flow velocities at the inlet and the exit of the valve are equal since the pipe diameter is constant.

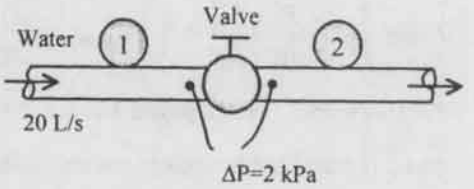
Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$ (Table A-3).

Analysis We take the valve as the control volume, and points 1 and 2 at the inlet and exit of the valve, respectively. Noting that $z_1 = z_2$ and $\mathbf{V}_1 = \mathbf{V}_2$, the energy equation for steady incompressible flow through this control volume reduces to

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 + h_{\text{turbine}} + h_L \quad \rightarrow \quad h_L = \frac{P_1 - P_2}{\rho g}$$

Substituting,

$$h_L = \frac{2\ \text{kN/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right) = \mathbf{0.204\ m}$$



The pumping power needed to overcome this head loss is

$$\dot{W}_{\text{pump, u}} = \dot{E}_{\text{mech, loss}} = \dot{m}gh_L = \rho\dot{V}gh_L$$

$$= (1000\ \text{kg/m}^3)(0.020\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(0.204\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{40\ W}$$

Therefore, this valve would cause a head loss of 0.204 m, and it would take 40 W of useful pumping power to overcome it.

Discussion The required useful pumping power could also be determined from

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = (0.020\ \text{m}^3/\text{s})(2000\ \text{Pa})\left(\frac{1\ \text{W}}{1\ \text{Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{40\ W}$$

11-64E A hose connected to the bottom of a pressurized tank is equipped with a nozzle at the end pointing straight up. The minimum tank air pressure (gage) corresponding to a given height of water jet is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** Friction between water and air as well as friction in the hose is negligible. **3** The water surface is open to the atmosphere.

Properties We take the density of water to be $\rho = 62.4$ lbm/ft^3 (Table A-3).

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory where $\mathbf{V}_2 = 0$ and $P_1 = P_2 = P_{atm}$. Also, we take the reference level at the bottom of the tank. Noting that $z_1 = 66$ ft and $z_2 = 90$ ft, $h_L = 0$ (to get the minimum value for the required air pressure), and that the fluid velocity at the free surface of the tank is very low ($\mathbf{V}_1 \cong 0$), the energy equation for steady incompressible flow through a control volume between these two points reduces to

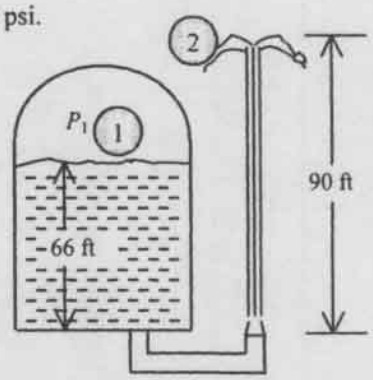
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad \frac{P_1-P_{atm}}{\rho g}=z_2-z_1 \quad \rightarrow \quad \frac{P_{1,gage}}{\rho g}=z_2-z_1$$

Rearranging and substituting, the gage pressure of pressurized air in the tank is determined to be

$$P_{1,\text{gage}}=\rho g(z_2-z_1)=(62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(90-66\ \text{ft})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\ \text{psi}}{144\ \text{lbf/ft}^2}\right)=10.4\ \text{psi}$$

Therefore, the gage air pressure on top of the water tank must be at least 10.4 psi.

Discussion The result obtained above represents the minimum value, and should be interpreted accordingly. In reality, a larger pressure will be needed to overcome friction.



11-65 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere. The initial discharge velocity from the tank is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The tank is open to the atmosphere.

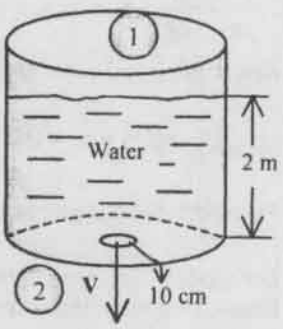
Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the orifice. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the tank is very low ($\mathbf{V}_1 \cong 0$), the energy equation between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1+\frac{\mathbf{V}_2^2}{2g}=z_2+h_L$$

Solving for $\mathbf{V}_2$ and substituting,

$$\mathbf{V}_2=\sqrt{2g(z_1-z_2-h_L)}=\sqrt{2(9.81\,\text{m/s}^2)(2-0.3\,\text{m})}=\mathbf{5.78\ m/s}$$

Discussion This is the velocity that will prevail at the beginning. The mean flow velocity will decrease as the water level in the tank decreases.



11-66 Water enters a hydraulic turbine-generator system with a known flow rate, pressure drop, and efficiency. The net electric power output is to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** All the losses in the turbine are accounted for by turbine efficiency and thus $h_L = 0$. **3** The elevation difference across the turbine is negligible.

Properties We take the density of water to be 1000 kg/m^3 and the density of mercury to be 13,560 kg/m^3 (Table A-3).

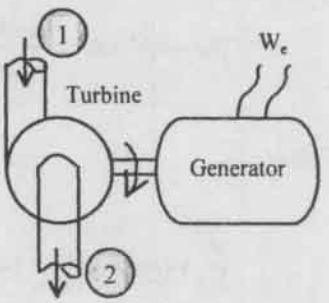
Analysis We choose points 1 and 2 at the inlet and the exit of the turbine, respectively. Noting that the elevation effects are negligible, the energy equation in terms of heads for the turbine reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_{\text{turbine}}=\frac{P_1-P_2}{\rho_{\text{water}}g}+\frac{\mathbf{V}_1^2-\mathbf{V}_2^2}{2g} \qquad (1)$$

where

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.8\,\text{m}^3/\text{s}}{\pi(0.30\,\text{m})^2/4}=11.3\,\text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.8\,\text{m}^3/\text{s}}{\pi(0.25\,\text{m})^2/4}=16.3\,\text{m/s}$$



The pressure drop corresponding to a differential height of 1.2 m in the mercury manometer is

$$\begin{aligned}P_1-P_2&=(\rho_{\text{Hg}}-\rho_{\text{water}})gh\\&=[(13{,}560-1000)\,\text{kg/m}^3](9.81\,\text{m/s}^2)(1.2\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\\&=148\,\text{kN/m}^2=148\,\text{kPa}\end{aligned}$$

Substituting into Eq. (1), the turbine head is determined to be

$$h_{\text{turbine}}=\frac{148\,\text{kN/m}^2}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}\left(\frac{1000\,\text{kg}\cdot\text{m/s}^2}{1\,\text{kN}}\right)+\frac{(11.3\,\text{m/s})^2-(16.3\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=15.1-7.0=8.1\,\text{m}$$

Then the net electric power output of this hydroelectric turbine becomes

$$\begin{aligned}\dot{W}_{\text{turbine}}&=\eta_{\text{turbine-gen}}\dot{m}gh_{\text{turbine}}=\eta_{\text{turbine-gen}}\rho\dot{V}gh_{\text{turbine}}\\&=0.83(1000\,\text{kg/m}^3)(0.8\,\text{m}^3/\text{s})(9.81\,\text{m/s}^2)(8.1\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kW}}{1000\,\text{N}\cdot\text{m/s}}\right)=\mathbf{53\,kW}\end{aligned}$$

Discussion It appears that this hydroelectric turbine will generate 53 kW of electric power under given conditions. Note that almost half of the available pressure head is discarded as kinetic energy. This demonstrates the need for a larger turbine exit area and better recovery. For example, the power output can almost be doubled to 98 kW by redesigning the turbine and making the exit diameter of the pipe equal to the inlet diameter, $D_2 = D_1$. Further, if a much larger exit diameter is used and the exit velocity is reduced to a very low level, the power generation can increase to as much as 140 kW.

11-67 The velocity profile for turbulent flow in a circular pipe is given. The kinetic energy correction factor for this flow is to be determined.

Analysis The velocity profile is given by $\mathbf{V}(r)=\mathbf{V}_{\max}(1-r/R)^{1/n}$ with $n = 7$ The kinetic energy correction factor is then expressed as

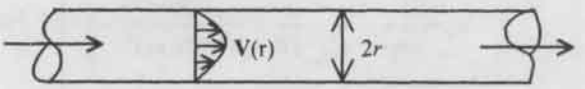
$$\alpha=\frac{1}{A}\int_A\left(\frac{\mathbf{V}(r)}{\mathbf{V}_m}\right)^3 dA=\frac{1}{A\mathbf{V}_m^3}\int_A \mathbf{V}(r)^3\,dA=\frac{1}{\pi R^2\mathbf{V}_m^3}\int_{r=0}^{R}\mathbf{V}_{\max}^3\left(1-\frac{r}{R}\right)^{\frac{3}{n}}(2\pi r)dr=\frac{2\mathbf{V}_{\max}^3}{R^2\mathbf{V}_m^3}\int_{r=0}^{R}\left(1-\frac{r}{R}\right)^{\frac{3}{n}}rdr$$

where the mean velocity is

$$\mathbf{V}_m=\frac{1}{A}\int_A \mathbf{V}(r)dA=\frac{1}{\pi R^2}\int_{r=0}^{R}\mathbf{V}_{\max}\left(1-\frac{r}{R}\right)^{1/n}(2\pi r)dr=\frac{2\mathbf{V}_{\max}}{R^2}\int_{r=0}^{R}\left(1-\frac{r}{R}\right)^{1/n}rdr$$

From integral tables,

$$\int(a+bx)^n x\,dx=\frac{(a+bx)^{n+2}}{b^2(n+2)}-\frac{a(a+bx)^{n+1}}{b^2(n+1)}$$



Then,

$$\int_{r=0}^{R}\mathbf{V}(r)rdr=\int_{r=0}^{R}\left(1-\frac{r}{R}\right)^{1/n}rdr=\left.\frac{(1-r/R)^{\frac{1}{n}+2}}{\frac{1}{R^2}(\frac{1}{n}+2)}-\frac{(1-r/R)^{\frac{1}{n}+1}}{\frac{1}{R^2}(\frac{1}{n}+1)}\right|_{r=0}^{R}=\frac{n^2R^2}{(n+1)(2n+1)}$$

$$\int_{r=0}^{R}\mathbf{V}(r)^3rdr=\int_{r=0}^{R}\left(1-\frac{r}{R}\right)^{3/n}rdr=\left.\frac{(1-r/R)^{\frac{3}{n}+2}}{\frac{1}{R^2}(\frac{3}{n}+2)}-\frac{(1-r/R)^{\frac{3}{n}+1}}{\frac{1}{R^2}(\frac{3}{n}+1)}\right|_{r=0}^{R}=\frac{n^2R^2}{(n+3)(2n+3)}$$

Substituting,

$$\mathbf{V}_m=\frac{2\mathbf{V}_{\max}}{R^2}\frac{n^2R^2}{(n+1)(2n+1)}=\frac{2n^2\mathbf{V}_{\max}}{(n+1)(2n+1)}=0.8167\mathbf{V}_{\max}$$

and

$$\alpha=\frac{2\mathbf{V}_{\max}^3}{R^2}\left(\frac{2n^2\mathbf{V}_{\max}}{(n+1)(2n+1)}\right)^{-3}\frac{n^2R^2}{(n+3)(2n+3)}=\frac{(n+1)^3(2n+1)^3}{4n^4(n+3)(2n+3)}=\frac{(7+1)^3(2\times7+1)^3}{4\times7^4(7+3)(2\times7+3)}=\mathbf{1.06}$$

Discussion Note that ignoring the kinetic energy correction factor results in an error of just 6% in this case in the kinetic energy term (which may be small itself). Considering that the uncertainties in some terms are usually more that 6%, we can usually ignore this correction factor in analysis.

11-68 A pump is pumping oil at a specified rate. The pressure rise of oil in the pump is measured, and the motor efficiency is specified. The mechanical efficiency of the pump is to be determined.

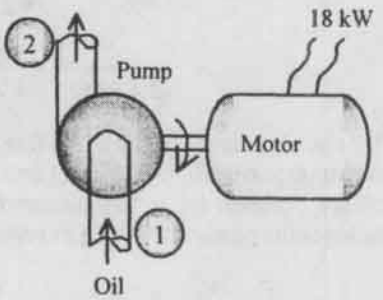
Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference across the pump is negligible. **3** All the losses in the pump are accounted for by the pump efficiency and thus $h_L = 0$.

Properties The density of oil is given to be $\rho = 860\ \text{kg/m}^3$.

Analysis We take points 1 and 2 at the inlet and the exit of the pump, respectively. Noting that $z_1 = z_2$, the energy equation for the pump reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_{\text{pump,u}}=\frac{P_2-P_1}{\rho g}+\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2g}$$

where



$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.1\,\text{m}^3/\text{s}}{\pi(0.08\,\text{m})^2/4}=19.9\,\text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.1\,\text{m}^3/\text{s}}{\pi(0.12\,\text{m})^2/4}=8.84\,\text{m/s}$$

Substituting, the useful pump head and the corresponding useful pumping power are determined to be

$$h_{\text{pump,u}}=\frac{400{,}000\ \text{N/m}^2}{(860\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)+\frac{(8.84\ \text{m/s})^2-(19.9\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=47.4-16.2=31.2\ \text{m}$$

$$\dot{W}_{\text{pump,u}}=\rho\dot{V}gh_{\text{pump,u}}=(860\ \text{kg/m}^3)(0.1\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(31.2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right)=26.3\ \text{kW}$$

Then the shaft pumping power and the mechanical efficiency of the pump become

$$\dot{W}_{\text{pump, shaft}}=\eta_{\text{motor}}\dot{W}_{\text{electric}}=(0.90)(35\ \text{kW})=31.5\ \text{kW}$$

$$\eta_{\text{pump}}=\frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump, shaft}}}=\frac{26.3\ \text{kW}}{31.5\ \text{kW}}=0.835=83.5\%$$

Discussion The overall efficiency of this pump/motor unit is the product of the mechanical and motor efficiencies, which is $0.9\times0.835 = 0.75$.

11-69E Water is pumped from a lake to a nearby pool by a pump with specified power and efficiency. The head loss of the piping system and the mechanical power used to overcome it are to be determined. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The elevation difference between the lake and the free surface of the pool is constant. **3** All the losses in the pump are accounted for by the pump efficiency and thus h_L represents the losses in piping.

Properties We take the density of water to be $\rho = 62.4$ lbm/ft^3 (Table A-3E).

Analysis The useful pumping power and the corresponding useful pumping head are

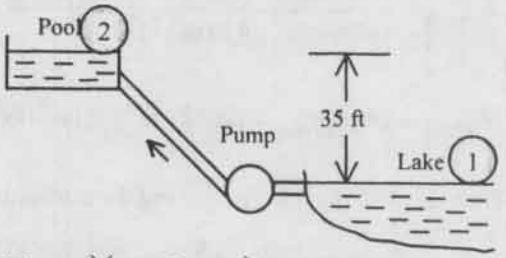
$$\dot{W}_{\text{pump, u}} = \eta_{\text{pump}} \dot{W}_{\text{pump}} = (0.73)(8\text{ hp}) = 5.84\text{ hp}$$

$$h_{\text{pump, u}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{m}g} = \frac{\dot{W}_{\text{pump, u}}}{\rho \dot{V} g}$$

$$= \frac{5.84\text{ hp}}{(62.4\text{ lbm/ft}^3)(1.2\text{ ft}^3\text{/s})(32.2\text{ ft/s}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)\left(\frac{550\text{ lbf}\cdot\text{ft/s}}{1\text{ hp}}\right) = 42.9\text{ ft}$$

We choose points 1 and 2 at the free surfaces of the lake and the pool, respectively. Both points are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$) and the velocities at both locations are negligible ($\mathbf{V}_1 = \mathbf{V}_2 = 0$). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 + h_{\text{turbine}} + h_L \quad \rightarrow \quad h_L = h_{\text{pump,u}} + z_1 - z_2$$



Substituting, the head loss is determined to be

$$h_L = h_{\text{pump, u}} - (z_2 - z_1) = 42.9 - 35 = \mathbf{7.9\text{ ft}}$$

Then the power used to overcome it becomes

$$\dot{E}_{\text{mech, loss}} = \rho \dot{V} g h_L$$

$$= (62.4\text{ lbm/ft}^3)(1.2\text{ ft}^3\text{/s})(32.2\text{ ft/s}^2)(7.9\text{ ft})\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ hp}}{550\text{ lbf}\cdot\text{ft/s}}\right)$$

$$= \mathbf{1.08\text{ hp}}$$

Discussion Note that the pump must raise the water an additional height of 7.9 ft to overcome the frictional losses in pipes, which requires an additional useful pumping power of about 1 hp.

11-70 A fireboat is fighting fires by drawing sea water and discharging it through a nozzle. The head loss of the system and the elevation of the nozzle are given. The shaft power input to the pump and the water discharge velocity are to be determined. √

Assumptions The flow is steady, one-dimensional, and incompressible.

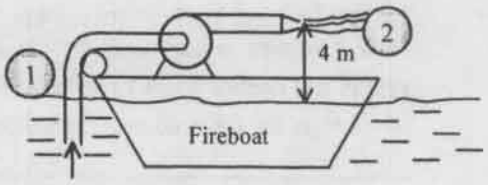
Properties The density of sea water is given to be $\rho = 1030$ kg/m^3.

Analysis We take point 1 at the free surface of the sea and point 2 at the nozzle exit. Noting that $P_1 = P_2 = P_{atm}$ and $\mathbf{V}_1 \cong 0$ (point 1 is at the free surface; not at the pipe inlet), the energy equation for the control volume between 1 and 2 that includes the pump and the piping system reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_{\text{pump,u}}=z_2-z_1+\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the water discharge velocity is

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.1\,\text{m}^3/\text{s}}{\pi(0.05\,\text{m})^2/4}=\mathbf{50.9\ m/s}$$



Substituting, the useful pump head and the corresponding useful pump power are determined to be

$$h_{\text{pump,u}}=(4\,\text{m})+\frac{(50.9\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}+(3\,\text{m})=138.5\,\text{m}$$

$$\dot{W}_{\text{pump,u}}=\rho\dot{V}gh_{\text{pump,u}}=(1030\,\text{kg/m}^3)(0.1\,\text{m}^3/\text{s})(9.81\,\text{m/s}^2)(138.5\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kW}}{1\,\text{kN}\cdot\text{m/s}}\right)$$

$$=140\,\text{kW}$$

Then the required shaft power input to the pump becomes

$$\dot{W}_{\text{pump, shaft}}=\frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump}}}=\frac{140\,\text{kW}}{0.70}=\mathbf{200\,kW}$$

Discussion Note that the pump power is used primarily to increase the kinetic energy of water.

Momentum Equation

11-71C The *Lagrangian approach* identifies a fixed quantity of fluid, and follows it through as it moves. The Lagrangian boundary can change in shape and size, but no mass crosses its boundary. In the *Eulerian approach*, a fixed volume and its boundary surface are identified, and the stationary observer analyzes movement of fluid entering and leaving this volume.

11-72C A control volume that comprises an operating jet engine bolted to a slab is a classical Eulerian control volume.

11-73C Since momentum ($m\mathbf{V}$) is the product of a vector (velocity) and a scalar (mass), momentum must be a vector that points in the same direction as the velocity vector.

11-74C The relationship between the time rates of change of an extensive property for a system and for a control volume is expressed by the Reynolds transport theorem, which provides the link between the system and control volume concepts. The linear momentum equation is obtained by setting $b = \vec{\mathbf{V}}$ and thus $B = m\vec{\mathbf{V}}$ in the Reynolds transport theorem.

11-75C The momentum equation for steady one-dimensional flow for the case of no external forces is

$$\frac{d(m\vec{\mathbf{V}})_{CV}}{dt} = \sum_{\text{in}} \dot{m}_i \vec{\mathbf{V}}_i - \sum_{\text{out}} \dot{m}_e \vec{\mathbf{V}}_e$$

where the left hand side is the rate of change of the momentum of a control volume, and first term on the right hand side is the incoming momentum flux and the second term is the outgoing momentum flux by mass.

11-76C In the application of the momentum equation, we can ignore the atmospheric pressure and work with gage pressures only since the atmospheric pressure acts in all directions, and its effect cancels out in every direction.

11-77C The fireman who holds the hose backwards so that the water makes a U-turn before being discharged will experience a greater reaction force since the numerical values of momentum fluxes across the nozzle are added in this case instead of being subtracted.

11-78C No, $\mathbf{V}$ is not the upper limit to the rocket's ultimate velocity. Without friction the rocket velocity will continue to increase as more gas exits the nozzle.

11-79C A helicopter hovers because the strong downdraft of air, caused by the overhead propeller blades, manifests a momentum in the air stream. This momentum must be countered by the helicopter lift force.

11-80C As the air density decreases, it requires more energy for a helicopter to hover, because more air must be forced into the downdraft by the helicopter blades to provide the same lift force. Therefore, it takes more power for a helicopter to hover on the top of a high mountain than it does at sea level.

11-81C In winter the air is generally colder, and thus denser. Therefore, less air must be driven by the blades to provide the same helicopter lift, requiring less power.

11-82C The force required to hold the plate against the water stream will increase by a factor of 4 when the velocity is doubled since

$$F = \dot{m}\mathbf{V} = (\rho A\mathbf{V})\mathbf{V} = \rho A\mathbf{V}^2$$

and thus the force is proportional to the square of the velocity.

11-83C The acceleration will not be constant since the force is not constant. The impulse force exerted by water on the plate is $F = \dot{m}\mathbf{V} = (\rho A\mathbf{V})\mathbf{V} = \rho A\mathbf{V}^2$, where **V** is the relative velocity between the water and the plate, which is moving. The plate acceleration will be **a = F/m.** But as the plate begins to move, **V** decreases, so the acceleration must also decrease.

11-84C The maximum velocity possible for the plate is the velocity of the water jet. As long as the plate is moving slower than the jet, the water will exert a force on the plate, which will cause it to accelerate, until terminal jet velocity is reached.

11-85 It is to be shown that the force exerted by a liquid jet of velocity **V** on a stationary nozzle is proportional to $\mathbf{V}^2$, or alternatively, to $\dot{m}^2$. √

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The nozzle is given to be stationary. **3** The nozzle involves a 90° turn and thus the incoming and outgoing flow streams are normal to each other. **4** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

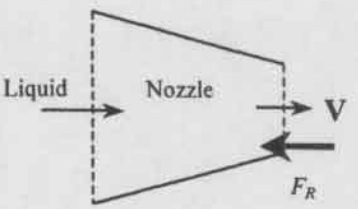
Analysis We take the nozzle as the control volume, and the flow direction at the exit as the x axis. Note that the nozzle makes a 90° turn, and thus the flow direction at the inlet is in the normal direction to exit flow, and thus it does not contribute to any pressure force or momentum flux term at the inlet in the x direction. Noting that $\dot{m} = \rho A\mathbf{V}$ where A is the nozzle exit area and **V** is the mean nozzle exit velocity, the momentum equation for steady one-dimensional flow in the x direction reduces to

$$\sum \vec{F} = \sum_{\text{out}} \dot{m}_e \vec{\mathbf{V}}_e - \sum_{\text{in}} \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad F_{Rx} = \dot{m}_e \mathbf{V}_e = \dot{m}\mathbf{V}$$

where F_{Rx} is the reaction force on the nozzle due to liquid jet at the nozzle exit. Then,

$$\dot{m} = \rho A\mathbf{V} \quad \rightarrow \quad F_{Rx} = \dot{m}\mathbf{V} = \rho A\mathbf{V}\mathbf{V} = \rho A\mathbf{V}^2 \quad \text{or} \quad F_{Rx} = \dot{m}\mathbf{V} = \dot{m}\frac{\dot{m}}{\rho A} = \frac{\dot{m}^2}{\rho A}$$

Therefore, the force exerted by a liquid jet of velocity **V** on this stationary nozzle is proportional to $\mathbf{V}^2$, or alternatively, to $\dot{m}^2$.



11-86 A water jet of velocity **V** impinges on a plate moving toward the water jet with velocity ½**V**. The force required to move the plate towards the jet is to be determined in terms of F acting on the stationary plate.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The plate is vertical and the jet is stationary and normal to plate. **3** The pressure on both sides of the plate is atmospheric pressure (and thus its effect can be disregarded). **4** Fiction during motion is negligible. **5** There is no acceleration of the plate. **6** The water splashes off the sides of the plate in a plane normal to the jet.

Analysis We take the plate as the control volume. The relative velocity between the plate and the jet is **V** when the plate is stationary, and 1.5**V** when the plate is moving with a velocity ½**V** towards the plate. Then the momentum equation for steady one-dimensional flow in the horizontal direction reduces to

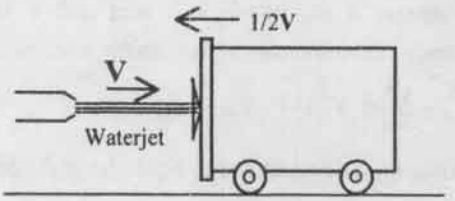
$$\sum \vec{F} = \sum_{\text{out}} \dot{m}_e \vec{\mathbf{V}}_e - \sum_{\text{in}} \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad -F_R = -\dot{m}_i \mathbf{V}_i \quad \rightarrow \quad F_R = \dot{m}_i \mathbf{V}_i$$

Stationary plate: $(\mathbf{V}_i = \mathbf{V} \text{ and } \dot{m}_i = \rho A \mathbf{V}_i = \rho A \mathbf{V}) \rightarrow F_R = \rho A \mathbf{V}^2 = F$

Moving plate: $(\mathbf{V}_i = 1.5\mathbf{V} \text{ and } \dot{m}_i = \rho A \mathbf{V}_i = \rho A (1.5\mathbf{V})) \rightarrow F_R = \rho A (1.5\mathbf{V})^2 = 2.25 \rho A \mathbf{V}^2 = 2.25F$

Therefore, the force required to hold the plate stationary against the oncoming water jet becomes **2.25 times** when the jet velocity becomes 1.5 times.

Discusion Note that when the plate is stationary, **V** is also the jet velocity. But if the plate moves toward the stream with velocity ½**V**, then the relative velocity is 1.5**V**, and the amount of mass striking the plate (and falling off its sides) per unit time also increases by 50%.



11-87 A 90° elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The gage pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place are to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by y. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho A\mathbf{V}$, the mean inlet and exit velocities of water are

$$\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V} = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.1\,\text{m})^2/4]} = 3.18\ \text{m/s}$$

Noting that $\mathbf{V}_1 = \mathbf{V}_2$ and $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \ \rightarrow\ P_1 - P_2 = \rho g(y_2 - y_1) \ \rightarrow\ P_{1,\text{gage}} = \rho g(y_2 - y_1)$$

Substituting,

$$P_{1,\text{gage}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.35\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 3.434\ \text{kN/m}^2 = \mathbf{3.434\ kPa}$$

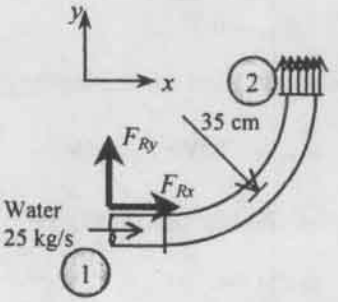
(*b*) The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. We let the x- and y- components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and y axes become

$$F_{Rx} + P_{1,\text{gage}}A_1 = 0 - \dot{m}(+\mathbf{V}_1) = -\dot{m}\mathbf{V}$$
$$F_{Ry} = \dot{m}(+\mathbf{V}_2) = \dot{m}\mathbf{V}$$

Solving for F_{Rx} and F_{Ry}, and substituting the given values,

$$F_{Rx} = -\dot{m}\mathbf{V} - P_{1,\text{gage}}A_1$$
$$= -(25\ \text{kg/s})(3.18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) - (3434\ \text{N/m}^2)[\pi(0.1\,\text{m})^2/4]$$
$$= -107\ \text{N}$$

$$F_{Ry} = \dot{m}\mathbf{V} = (25\ \text{kg/s})(3.18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 79.5\ \text{N}$$



and $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-107)^2 + 79.5^2} = \mathbf{133\ N}$, $\theta = \tan^{-1}\dfrac{F_{Ry}}{F_{Rx}} = \tan^{-1}\dfrac{79.5}{-107} = -36.6° = \mathbf{143.4°}$

Discussion Note that the magnitude of the anchoring force is 133 N, and its line of action makes 143.4° from the positive x direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

11-88 An 180° elbow forces the flow to make a U-turn and discharges it to the atmosphere at a specified rate. The gage pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place are to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by y. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho A\mathbf{V}$, the mean inlet and exit velocities of water are

$$\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V} = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.1\ \text{m})^2/4]} = 3.18\ \text{m/s}$$

Noting that $\mathbf{V}_1 = \mathbf{V}_2$ and $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \ \rightarrow\ P_1 - P_2 = \rho g(y_2 - y_1) \ \rightarrow\ P_{1,\text{gage}} = \rho g(y_2 - y_1)$$

Substituting,

$$P_{1,\text{gage}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.70\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 6.867\ \text{kN/m}^2 = \mathbf{6.867\ kPa}$$

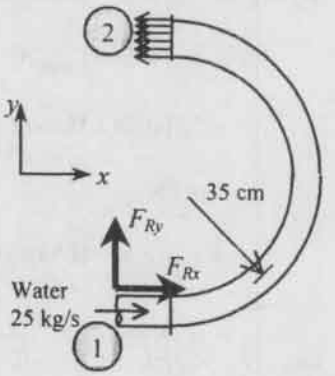
(*b*) The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. We let the x- and y- components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and y axes become

$$F_{Rx} + P_{1,\text{gage}}A_1 = \dot{m}(-\mathbf{V}_2) - \dot{m}(+\mathbf{V}_1) = -2\dot{m}\mathbf{V}$$
$$F_{Ry} = 0$$

Solving for F_{Rx} and substituting the given values,

$$F_{Rx} = -2\dot{m}\mathbf{V} - P_{1,\text{gage}}A_1$$
$$= -2(25\ \text{kg/s})(3.18\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) - (6867\ \text{N/m}^2)[\pi(0.1\ \text{m})^2/4]$$
$$= -213\ \text{N}$$

and $F_R = F_{Rx} = \mathbf{-213\ N}$ since the y-component of the anchoring force is zero. Therefore, the anchoring force has a magnitude of 213 N and it acts in the negative x direction.



Discussion Note that a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

11-89E A horizontal water jet strikes a vertical stationary plate normally at a specified velocity. For a given anchoring force needed to hold the plate in place, the flow rate of water is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water splatters off the sides of the plate in a plane normal to the jet. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on the entire control surface. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the plate as the control volume such that it contains the entire plate and cuts through the water jet and the support bar normally, and the direction of flow as the positive direction of x axis. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\Sigma \vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad -F_{Rx} = -\dot{m}\mathbf{V}_1 \quad \rightarrow \quad F_R = \dot{m}\mathbf{V}_1$$

We note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Solving for $\dot{m}$ and substituting the given values,

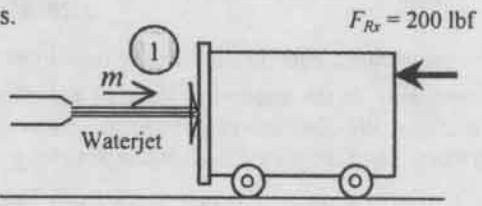
$$\dot{m} = \frac{F_{Rx}}{\mathbf{V}_1} = \frac{200 \text{ lbf}}{30 \text{ ft/s}} \left(\frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right) = 215 \text{ lbm/s}$$

Then the volume flow rate becomes

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{215 \text{ lbm/s}}{62.4 \text{ lbm/ft}^3} = \mathbf{3.45 \text{ ft}^3/\text{s}}$$

Therefore, the volume flow rate of water under stated assumptions must be 3.45 ft^3/s.

Discussion In reality, some water will be scattered back, and this will add to the reaction force of water. The flow rate in that case will be less.



11-90 A reducing elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The anchoring force needed to hold the elbow in place is to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N} = 0.4905\text{ kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by y. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho A\mathbf{V}$, the inlet and exit velocities of water are

$$\mathbf{V}_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0150\text{ m}^2)} = 2.0\text{ m/s}$$

$$\mathbf{V}_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0025\text{ m}^2)} = 12\text{ m/s}$$

Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

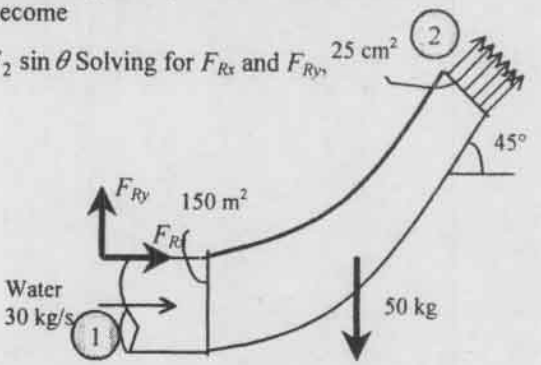
$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \;\rightarrow\; P_1 - P_2 = \rho g\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g} + y_2 - y_1\right) \;\rightarrow\; P_{1,\text{gage}} = \rho g\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g} + y_2\right)$$

Substituting,

$$P_{1,\text{gage}} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(12\text{ m/s})^2 - (2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0.4\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 73.9\text{ kN/m}^2 = 73.9\text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. We let the x- and y-components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and y axes become

$$F_{Rx} + P_{1,\text{gage}}A_1 = \dot{m}\mathbf{V}_2\cos\theta - \dot{m}\mathbf{V}_1 \quad\text{and}\quad F_{Ry} - W = \dot{m}\mathbf{V}_2\sin\theta$$ Solving for F_{Rx} and F_{Ry}, substituting the given values,



$$F_{Rx} = \dot{m}(\mathbf{V}_2\cos\theta - \mathbf{V}_1) - P_{1,\text{gage}}A_1$$
$$= (30\text{ kg/s})[(12\cos 45° - 2)\text{ m/s}]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$
$$-(73.9\text{ kN/m}^2)(0.0150\text{ m}^2)$$
$$= -0.914\text{ kN}$$

$$F_{Ry} = \dot{m}\mathbf{V}_2\sin\theta - W = (30\text{ kg/s})(12\sin 30°\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) - 0.4905\text{ kN} = 0.745\text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-0.914)^2 + 0.745^2} = \mathbf{1.18\text{ kN}}, \quad \theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{0.745}{-0.914} = -39.2° = \mathbf{140.8°}$$

Discussion Note that the magnitude of the anchoring force is 1.18 kN, and its line of action makes 140.8° from the positive x direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

11-91 A reducing elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The anchoring force needed to hold the elbow in place is to be determined. √

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the exit is zero.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\ \text{kg})(9.81\ \text{m/s}^2) = 490.5\ \text{N} = 0.4905\ \text{kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by y. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho A \mathbf{V}$, the inlet and exit velocities of water are

$$\mathbf{V}_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0150\ \text{m}^2)} = 2.0\ \text{m/s}$$

$$\mathbf{V}_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0025\ \text{m}^2)} = 12\ \text{m/s}$$

Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \ \rightarrow\ P_1 - P_2 = \rho g\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g} + y_2 - y_1\right) \ \rightarrow\ P_{1,\text{gage}} = \rho g\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g} + y_2\right)$$

Substituting,

$$P_{1,\text{gage}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(\frac{(12\ \text{m/s})^2 - (2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.4\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 73.9\ \text{kN/m}^2 = 73.9\ \text{kPa}$$

The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. We let the x- and y-components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and y axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = \dot{m}\mathbf{V}_2\cos\theta - \dot{m}\mathbf{V}_1 \quad \text{and} \quad F_{Ry} - W = \dot{m}\mathbf{V}_2\sin\theta$$

Solving for F_{Rx} and F_{Ry}, and substituting the given values,

$$F_{Rx} = \dot{m}(\mathbf{V}_2\cos\theta - \mathbf{V}_1) - P_{1,\text{gage}} A_1$$

$$= (30\ \text{kg/s})[(12\cos 135° - 2)\ \text{m/s}]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) - (73.9\ \text{kN/m}^2)(0.0150\ \text{m}^2) = -1.423\ \text{kN}$$

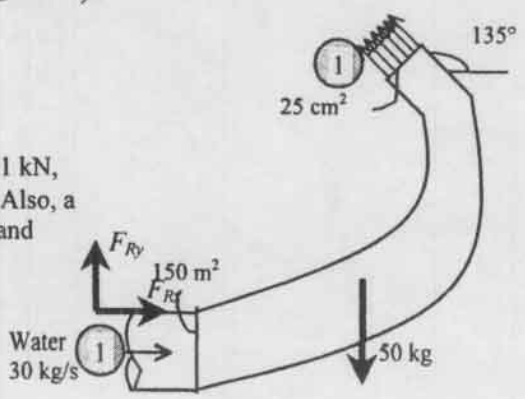
$$F_{Ry} = \dot{m}\mathbf{V}_2\sin\theta - W = (30\ \text{kg/s})(12\sin 30°\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) - 0.4905\ \text{kN} = 0.745\ \text{kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-1.423)^2 + 0.745^2} = \mathbf{1.61\ kN},$$

and

$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{0.745}{-1.423} = \mathbf{-27.6°}$$

Discussion Note that the magnitude of the anchoring force is 1.61 kN, and its line of action makes –27.6° from the positive x direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.



11-92 Water accelerated by a nozzle strikes the back surface of a cart moving horizontally at a constant velocity. The braking force and the power wasted by the brakes are to be determined. .

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water splatters off the sides of the plate in all directions in the plane of the back surface. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force. **5** Fiction during motion is negligible. **6** There is no acceleration of the cart. **7** The motions of the water jet and the cart are horizontal.

Analysis We take the cart as the control volume, and the direction of flow as the positive direction of x axis. The relative velocity between the cart and the jet is

$$\mathbf{V}_r = \mathbf{V}_{\text{jet}} - \mathbf{V}_{\text{cart}} = 15 - 10 = 10 \text{ m/s}$$

Therefore, we can assume the cart to be stationary and the jet to move with a velocity of 10 m/s. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\Sigma \vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad F_{Rx} = -\dot{m}_i \mathbf{V}_i \quad \rightarrow \quad F_{\text{brake}} = -\dot{m}\mathbf{V}_r$$

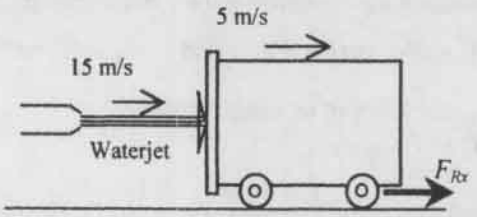
We note that the brake force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Substituting the given values,

$$F_{\text{brake}} = -\dot{m}\mathbf{V}_r = -(25 \text{ kg/s})(+10 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = \mathbf{-250 \text{ N}}$$

The negative sign indicates that the braking force acts in the opposite direction to motion, as expected. Noting that work is force times distance and the distance traveled by the cart per unit time is the cart velocity, the power wasted by the brakes is

$$\dot{W} = F_{\text{brake}} \mathbf{V}_{\text{cart}} = (250 \text{ N})(5 \text{ m/s})\left(\frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}}\right) = \mathbf{1.25 \text{ kW}}$$

Discussion Note that the power wasted is equivalent to the maximum power that can be generated as the cart velocity is maintained constant.

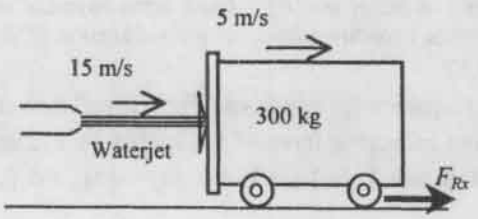


11-93 Water accelerated by a nozzle strikes the back surface of a cart moving horizontally. The acceleration of the cart if the brakes fail is to be determined.

Analysis The braking force was determined in Prob. 11-92 to be 250 N. When the brakes fail, this force will propel the cart forward, and the accelerating will be

$$a = \frac{F}{m_{\text{cart}}} = \frac{250\ \text{N}}{300\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = \mathbf{0.833\ m/s^2}$$

Discussion This is the acceleration at the moment the brakes fail. The acceleration will decrease as the relative velocity between the water jet and the cart (and thus the force) decreases.



11-94E A water jet hits a stationary splitter, such that half of the flow is diverted upward at 45°, and the other half is directed down. The force required to hold the splitter in place is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet before and after the split is the atmospheric pressure which is disregarded since it acts on all surfaces. **3** The gravitational effects are disregarded.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis The mass flow rate of water jet is

$$\dot{m} = \rho \dot{V} = (62.4 \text{ lbm/ft}^3)(100 \text{ ft}^3/\text{s}) = 6240 \text{ lbm/s}$$

We take the splitting section of water jet, including the splitter as the control volume, and designate the entrance by 1 and the exit of either arm by 2 (both arms have the same velocity and mass flow rate). We also designate the horizontal coordinate by x with the direction of flow as being the positive direction and the vertical coordinate by y.

The momentum equation for steady one-dimensional flow is $\Sigma \vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i$. We let the x- and y- components of the anchoring force of the splitter be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Noting that $\mathbf{V}_2 = \mathbf{V}_1 = \mathbf{V}$ and $\dot{m}_2 = \frac{1}{2}\dot{m}$, the momentum equations along the x and y axes become

$$F_{Rx} = 2(\tfrac{1}{2}\dot{m})\mathbf{V}_2 \cos\theta - \dot{m}\mathbf{V}_1 = \dot{m}\mathbf{V}(\cos\theta - 1)$$
$$F_{Ry} = \tfrac{1}{2}\dot{m}(+\mathbf{V}_2 \sin\theta) + \tfrac{1}{2}\dot{m}(-\mathbf{V}_2 \sin\theta) - 0 = 0$$

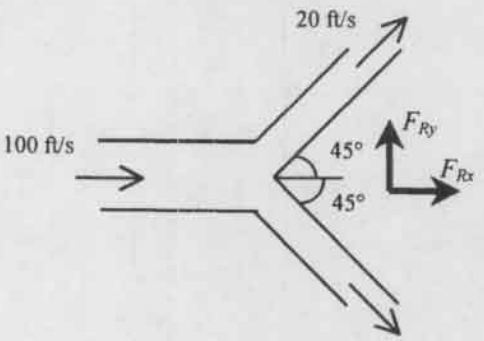
Substituting the given values,

$$F_{Rx} = (6240 \text{ lbm/s})(20 \text{ ft/s})(\cos 45° - 1)\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right) = \mathbf{-1135 \text{ lbf}}$$

$$F_{Ry} = \mathbf{0}$$

The negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed. Therefore, a force of 1135 lbf must be applied to the splitter in the opposite direction to flow to hold it in place. No holding force is necessary in the vertical direction. This can also be concluded from the symmetry.

Discussion In reality, the gravitational effects will cause the upper stream to slow down and the lower stream to speed up after the split. But for short distances, these effects are indeed negligible.



11-95 A horizontal water jet impinges normally upon a vertical plate which is held on a frictionless track and is initially stationary. The initial acceleration of the plate, the time it takes to reach a certain velocity, and the velocity at a given time are to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water always splatters in the plane of the retreating plate. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal motion. **5** The tract is frictionless, and thus fiction during motion is negligible. **6** The motions of the water jet and the cart are horizontal. **7** The velocity of the jet relative to the plate remains constant, $\mathbf{V}_r = \mathbf{V}_{jet} = \mathbf{V}$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the vertical plate on the frictionless track as the control volume, and the direction of flow as the positive direction of *x* axis. The mass flow rate of water in the jet is

$$\dot{m} = \rho VA = (1000\,\text{kg/m}^3)(25\,\text{m/s})[\pi(0.05\ \text{m})^2/4] = 49.1\,\text{kg/s}$$

The momentum equation for steady one-dimensional flow in the *x* (flow) direction reduces in this case to

$$\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i \quad \rightarrow \quad F_{Rx} = -\dot{m}_i\mathbf{V}_i \quad \rightarrow \quad F_{Rx} = -\dot{m}\mathbf{V}$$

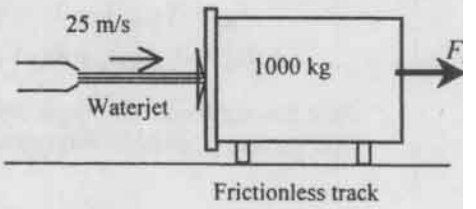
where F_{Rx} is the reaction force required to hold the plate in place. When the plate is released, an equal and opposite impulse force acts on the plate, which is determined to

$$F_{\text{plate}} = -F_{Rx} = \dot{m}\mathbf{V} = (49.1\,\text{kg/s})(25\,\text{m/s})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 1227\,\text{N}$$

Then the initial acceleration of the plate becomes

$$a = \frac{F_{\text{plate}}}{m_{\text{plate}}} = \frac{1227\,\text{N}}{1000\,\text{kg}}\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = \mathbf{1.237\ m/s^2}$$

This acceleration will remain constant during motion since the force acting on the plate remains constant.



(*b*) Noting that $a = d\mathbf{V}/dt = \Delta\mathbf{V}/\Delta t$ since the acceleration *a* is constant, the time it takes for the plate to reach a velocity of 12.5 m/s is

$$\Delta t = \frac{\Delta\mathbf{V}_{\text{plate}}}{a} = \frac{(12.5-0)\,\text{m/s}}{1.237\,\text{m/s}^2} = \mathbf{10.1\,s}$$

(*c*) Noting that $a = d\mathbf{V}/dt$ and thus $d\mathbf{V} = a\,dt$ and that the acceleration *a* is constant, the plate velocity in 20 s becomes

$$\mathbf{V}_{\text{plate}} = \mathbf{V}_{0,\text{plate}} + a\Delta t = 0 + (1.237\,\text{m/s}^2)(20\,\text{s}) = \mathbf{24.5\,m/s}$$

Discussion The assumption that the relative velocity between the water jet and the plate remains constant is valid only for the initial moments of motion when the plate velocity is low unless the water jet is moving with the plate at the same velocity as the plate.

11-96 A 90° reducer elbow deflects water downwards into a smaller diameter pipe. The resultant force exerted on the reducer by water is to be determined.

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is disregarded since the gravitational effects are negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the elbow as the control volume, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by y. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 353.4$ kg/s. Noting that $\dot{m} = \rho A\mathbf{V}$, the mass flow rate of water and its exit velocity are

$$\dot{m} = \rho \mathbf{V}_1 A_1 = \rho \mathbf{V}_1 (\pi D_1^2 / 4) = (1000 \text{ kg/m}^3)(5 \text{ m/s})[\pi (0.3 \text{ m})^2 / 4] = 353.4 \text{ kg/s}$$

$$\mathbf{V}_2 = \frac{\dot{m}}{\rho A_2} = \frac{\dot{m}}{\rho \pi D_2^2 / 4} = \frac{353.4 \text{ kg/s}}{(1000 \text{ kg/m}^3)[\pi (0.15 \text{ m})^2 / 4]} = 20 \text{ m/s}$$

The Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + y_2 \quad \rightarrow \quad P_2 = P_1 + \rho g \left(\frac{\mathbf{V}_1^2 - \mathbf{V}_2^2}{2g} + y_1 - y_2 \right)$$

Substituting, the gage pressure at the exit becomes

$$P_2 = (300 \text{ kPa}) + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left(\frac{(5 \text{ m/s})^2 - (20 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0.5 \right) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) = 117.4 \text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\Sigma \vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i$. We let the x- and y-components of the anchoring force of the elbow be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Then the momentum equations along the x and y axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = 0 - \dot{m}\mathbf{V}_1$$
$$F_{Ry} - P_{2,\text{gage}} A_2 = \dot{m}(-\mathbf{V}_2) - 0$$

Note that we should not forget the negative sign for forces and velocities in the negative x or y direction. Solving for F_{Rx} and F_{Ry}, and substituting the given values,

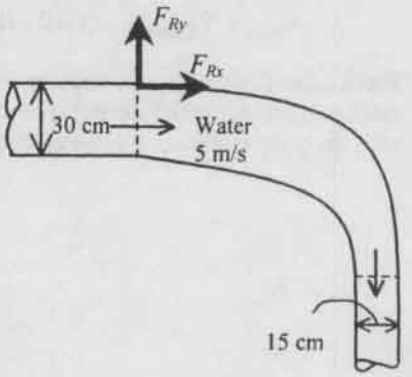
$$F_{Rx} = -\dot{m}\mathbf{V}_1 - P_{1,\text{gage}} A_1 = -(353.4 \text{ kg/s})(5 \text{ m/s}) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) - (300 \text{ kN/m}^2) \frac{\pi (0.3 \text{ m})^2}{4} = -23.0 \text{ kN}$$

$$F_{Ry} = -\dot{m}\mathbf{V}_2 + P_{2,\text{gage}} A_1 = -(353.4 \text{ kg/s})(20 \text{ m/s}) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) + (117.4 \text{ kN/m}^2) \frac{\pi (0.15 \text{ m})^2}{4} = -5.0 \text{ kN}$$

and

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-23.0)^2 + (-5.0)^2} = \mathbf{23.5 \text{ kN}},$$

$$\theta = \tan^{-1} \frac{F_{Ry}}{F_{Rx}} = \tan^{-1} \frac{-5.0}{-23.0} = \mathbf{12.3°}$$



Discussion Note that the magnitude of the anchoring force is 23.5 kN, and its line of action makes 12.3° from the positive x direction. Also, negative values for F_{Rx} and F_{Ry} indicate that the assumed directions are wrong, and should be reversed.

11-97 A wind turbine with a given span diameter and efficiency is subjected to steady winds. The power generated and the horizontal force on the supporting mast of the turbine are to be determined. √

Assumptions **1** The wind flow is steady, one-dimensional, and incompressible. **2** The efficiency of the turbine-generator is independent of wind speed. **3** The frictional effects are negligible, and thus none of the incoming kinetic energy is converted to thermal energy.

Properties The density of air is given to be 1.25 kg/m^3.

Analysis (*a*) The power potential of the wind is its kinetic energy, which is $\mathbf{V}^2/2$ per unit mass, and $\dot{m}\mathbf{V}^2/2$ for a given mass flow rate:

$$\mathbf{V}_1 = (25\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 6.94\text{ m/s}$$

$$\dot{m} = \rho_1 \mathbf{V}_1 A_1 = \rho_1 \mathbf{V}_1 \frac{\pi D^2}{4} = (1.25\text{ kg/m}^3)(6.94\text{ m/s})\frac{\pi(90\text{ m})^2}{4} = 55{,}200\text{ kg/s}$$

$$\dot{W}_{\max} = \dot{m}ke_1 = \dot{m}\frac{\mathbf{V}_1^2}{2} = (55{,}200\text{ kg/s})\frac{(6.94\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = 1330\text{ kW}$$

Then the actual power produced becomes

$$\dot{W}_{\text{act}} = \eta_{\text{wind turbine}}\dot{W}_{\max} = (0.32)(1330\text{ kW}) = \mathbf{425\text{ kW}}$$

(*b*) The frictional effects are assumed to be negligible, and thus the portion of incoming kinetic energy not converted to electric power leaves the wind turbine as outgoing kinetic energy. Therefore,

$$\dot{m}ke_2 = \dot{m}ke_1(1-\eta_{\text{wind turbine}}) \quad \rightarrow \quad \dot{m}\frac{\mathbf{V}_2^2}{2} = \dot{m}\frac{\mathbf{V}_1^2}{2}(1-\eta_{\text{wind turbine}})$$

or

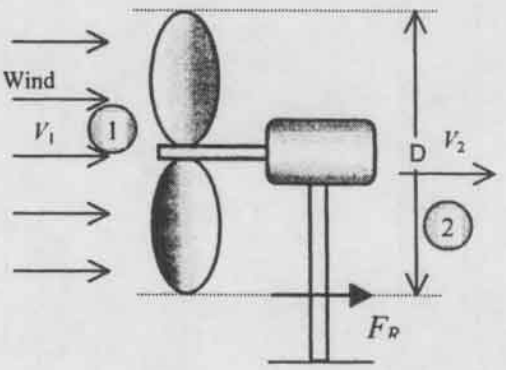
$$\mathbf{V}_2 = \mathbf{V}_1\sqrt{1-\eta_{\text{wind turbine}}} = (6.94\text{ m/s})\sqrt{1-0.32} = 5.72\text{ m/s}$$

We choose the control volume around the wind turbine such that the wind is normal to the control surface at the inlet and the exit, and the entire control surface is at the atmospheric pressure. The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. Writing it along the x-direction (without forgetting the negative sign for forces and velocities in the negative x-direction) and assuming the flow velocity through the turbine to be equal to the wind velocity give

$$F_R = \dot{m}\mathbf{V}_2 - \dot{m}\mathbf{V}_1 = \dot{m}(\mathbf{V}_2 - \mathbf{V}_1) = (55{,}200\text{ kg/s})(5.72 - 6.94\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{-67.3\text{ kN}}$$

The negative sign indicates that the reaction force acts in the negative x direction, as expected.

Discussion This force acts on top of the tower where the wind turbine is installed, and the bending moment it generates at the bottom of the tower is obtained by multiplying this force by the tower height.



11-98E A horizontal water jet strikes a curved plate, which deflects the water back to its original direction. The force required to hold the plate against the water stream is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Friction between the plate and the surface it is on is negligible (or the friction force can be included in the required force to hold the plate). **4** There is no splattering of water or the deformation of the jet, and the reversed jet leaves horizontally at the same velocity and flow rate.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the plate together with the curved water jet as the control volume, and designate the jet inlet by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of incoming flow as being the positive direction). The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$ where

$$\dot{m} = \rho VA = \rho V[\pi D^2 / 4] = (62.4 \text{ lbm/ft}^3)(100 \text{ ft/s})[\pi(3/12 \text{ ft})^2 / 4] = 306.3 \text{ lbm/s}$$

The momentum equation for steady one-dimensional flow is $\Sigma \vec{F} = \Sigma \dot{m}_e \vec{V}_e - \Sigma \dot{m}_i \vec{V}_i$. Letting the reaction force to hold the plate be F_{Rx} and assuming it to be in the positive direction, the momentum equation along the x axis becomes

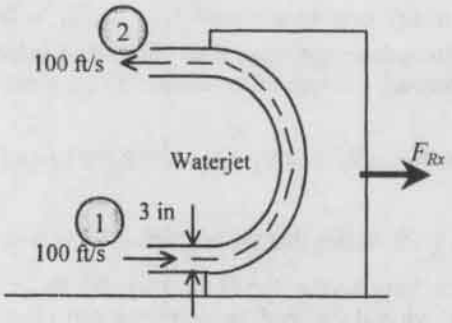
$$F_{Rx} = \dot{m}(-V_2) - \dot{m}(+V_1) = -2\dot{m}V$$

Substituting,

$$F_{Rx} = -2(306.3 \text{ lbm/s})(100 \text{ ft/s})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right) = \mathbf{-1902 \text{ lbf}}$$

Therefore, a force of 1902 lbm must be applied on the plate in the negative x direction to hold it in place.

Discussion Note that a negative value for F_{Rx} indicates the assumed direction is wrong (as expected), and should be reversed. Also, there is no need for an analysis in the vertical direction since the fluid streams are horizontal.



11-99E A horizontal water jet strikes a bent plate, which deflects the water by 135° from its original direction. The force required to hold the plate against the water stream is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Frictional and gravitational effects are negligible. **4** There is no splattering of water or the deformation of the jet, and the reversed jet leaves horizontally at the same velocity and flow rate.

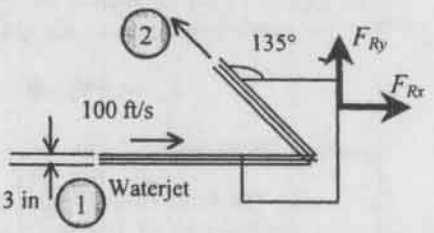
Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take the plate together with the curved water jet as the control volume, and designate the jet inlet by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of incoming flow as being the positive direction), and the vertical coordinate by y. The continuity equation for this one-inlet one-exit steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$ where

$$\dot{m} = \rho V A = \rho V[\pi D^2/4] = (62.4\ \text{lbm/ft}^3)(100\ \text{ft/s})[\pi(3/12\ \text{ft})^2/4] = 306.3\ \text{lbm/s}$$

The momentum equation for steady one-dimensional flow is $\Sigma \vec{F} = \Sigma \dot{m}_e \vec{V}_e - \Sigma \dot{m}_i \vec{V}_i$. We let the x- and y-components of the anchoring force of the plate be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Then the momentum equations along the x and y axes become

$$F_{Rx} = \dot{m}(-\mathbf{V}_2)\cos 45° - \dot{m}(+\mathbf{V}_1) = -\dot{m}\mathbf{V}(1+\cos 45°)$$
$$F_{Ry} = \dot{m}(+\mathbf{V}_2)\sin 45° = \dot{m}\mathbf{V}\sin 45°$$



Substituting the given values,

$$F_{Rx} = -2(306.3\ \text{lbm/s})(100\ \text{ft/s})(1+\cos 45°)\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)$$
$$= -1624\ \text{lbf}$$
$$F_{Ry} = (306.3\ \text{lbm/s})(100\ \text{ft/s})\sin 45°\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = 673\ \text{lbf}$$

and

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-1624)^2 + 673^2} = \mathbf{1758\ lbf}, \quad \theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{673}{-1624} = -22.7° = \mathbf{157.3°}$$

Discussion Note that the magnitude of the anchoring force is 1758 lbf, and its line of action makes 157.3° from the positive x direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

11-100 Firemen are holding a nozzle at the end of a hose while trying to extinguish a fire. The average water exit velocity and the resistance force required of the firemen to hold the nozzle are to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Gravitational effects and vertical forces are disregarded since the horizontal resistance force is to be determined.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the nozzle and the horizontal portion of the hose as the system such that water enters the control volume vertically and exits horizontally (this way the pressure force and the momentum flux at the inlet are in the vertical direction, with no contribution to the force balance in the horizontal direction), and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction). The average exit velocity and the mass flow rate of water are determined from

$$\mathbf{V} = \frac{\dot{V}}{A} = \frac{\dot{V}}{\pi D^2/4} = \frac{5\,\text{m}^3/\text{min}}{\pi(0.06\,\text{m})^2/4} = 1768\,\text{m/min} = \mathbf{29.5\,m/s}$$

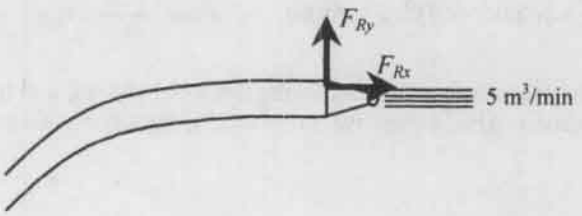
$$\dot{m} = \rho\dot{\mathbf{V}} = (1000\,\text{kg/m}^3)(5\,\text{m}^3/\text{min}) = 5000\,\text{kg/min} = 83.3\,\text{kg/s}$$

(*b*) The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. We let horizontal force applied by the firemen to the nozzle to hold it be F_{Rx}, and assume it to be in the positive x direction. Then the momentum equation along the x direction gives

$$F_{Rx} = \dot{m}\mathbf{V}_e - 0 = \dot{m}\mathbf{V} = (83.3\,\text{kg/s})(29.5\,\text{m/s})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{2457\,N}$$

Therefore, the firemen must be able to resist a force of 2457 N to hold the nozzle in place.

Discussion The force of 2457 N is equivalent to the weight of about 250 kg. That is, holding the nozzle requires the strength of holding a weight of 250 kg, which cannot be done by a single person. This demonstrates why several firemen are used to hold a hose with a high flow rate.



11-101 A horizontal jet of water with a given velocity strikes a flat plate that is moving in the same direction at a specified velocity. The force that the water stream exerts against the plate is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water splatters in all directions in the plane of the plate. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal force exerted on the plate. **5** The velocity of the plate, and the velocity of the water jet relative to the plate, are constant.

Analysis We take the plate as the control volume, and the direction of flow as the positive direction of x axis. The relative velocity between the plate and the jet is

$$\mathbf{V}_r = \mathbf{V}_{\text{jet}} - \mathbf{V}_{\text{plate}} = 30 - 10 = 20 \text{ m/s}$$

Therefore, we can assume the plate to be stationary and the jet to move with a velocity of 20 m/s. Then the mass flow rate of water striking the plate becomes

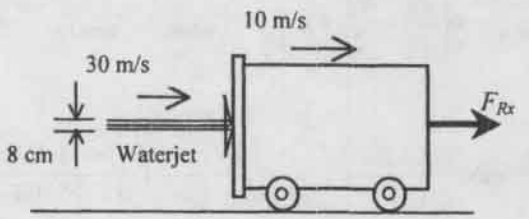
$$\dot{m} = \rho \mathbf{V}_r A = \rho \mathbf{V}_r \frac{\pi D^2}{4} = (1000 \text{ kg/m}^3)(20 \text{ m/s}) \frac{\pi (0.08 \text{ m})^2}{4} = 100.5 \text{ kg/s}$$

The momentum equation for steady one-dimensional flow is $\Sigma \vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i$. We let the horizontal reaction force applied to the plate in the negative x direction to counteract the impulse of the water jet be F_{Rx}. Then the momentum equation along the x direction gives

$$-F_{Rx} = 0 - \dot{m}\mathbf{V}_i \quad \rightarrow \quad F_{Rx} = \dot{m}\mathbf{V}_r = (100.5 \text{ kg/s})(20 \text{ m/s}) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = \mathbf{2010 \text{ N}}$$

Therefore, the water jet applies a force of 2010 N on the plate in the direction of motion, and an equal and opposite force must be applied on the plate if its velocity is to remain constant.

Discussion Note that we used the relative velocity in the determination of the mass flow rate of water in the momentum analysis since water will enter the control volume at this rate. (In the limiting case of the plate and the water jet moving at the same velocity, the mass flow rate of water relative to the plate will be zero since no water will be able to strike the plate).



11-102E A fan moves air at sea level at a specified rate. The force required to hold the fan and the minimum power input required for the fan are to be determined. √

Assumptions **1** The flow of air is steady, one-dimensional, and incompressible. **2** Standard atmospheric conditions exist so that the pressure at sea level is 1 atm. **3** Air leaves the fan at a uniform velocity at atmospheric pressure. **4** Air approaches the fan through a large area at atmospheric pressure with negligible velocity. **5** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects).

Properties The gas constant of air is $R = 0.3704$ psi·ft^3/lbm·R. The standard atmospheric pressure at sea level is 1 atm = 14.7 psi.

Analysis (*a*) We take the control volume to be a horizontal hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) and the fan located at the narrow cross-section at the end (section 2), and let its centerline be the x axis. The density, mass flow rate, and discharge velocity of air are

$$\rho = \frac{P}{RT} = \frac{14.7\ \text{psi}}{(0.3704\ \text{psi}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(530\ \text{R})} = 0.0749\ \text{lbm/ft}^3$$

$$\dot{m} = \rho\dot{V} = (0.0749\ \text{lbm/ft}^3)(2000\ \text{ft}^3/\text{min}) = 149.8\ \text{lbm/min} = 2.50\ \text{lbm/s}$$

$$\mathbf{V}_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{2000\ \text{ft}^3/\text{min}}{\pi(2\ \text{ft})^2/4} = 636.6\ \text{ft/min} = 10.6\ \text{ft/s}$$

The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. Letting the reaction force to hold the fan be F_{Rx} and assuming it to be in the positive x (i.e., the flow) direction, the momentum equation along the x axis becomes

$$F_{Rx} = \dot{m}(\mathbf{V}_2) - 0 = \dot{m}\mathbf{V} = (2.50\ \text{lbm/s})(10.6\ \text{ft/s})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{0.82\ lbf}$$

Therefore, a force of 0.82 lbf must be applied (through friction at the base, for example) to prevent the fan from moving in the horizontal direction under the influence of this force.

(*b*) Noting that $P_1 = P_2 = P_{\text{atm}}$ and $\mathbf{V}_1 \cong 0$, the energy equation for the selected control volume reduces to

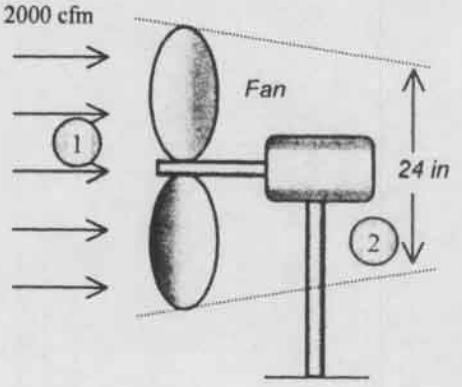
$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{fan,u}} = \dot{m}\frac{\mathbf{V}_2^2}{2} = (2.50\ \text{lbm/s})\frac{(10.6\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\ \text{W}}{0.73756\ \text{lbf}\cdot\text{ft/s}}\right) = \mathbf{5.91\ W}$$

Therefore, a useful mechanical power of 5.91 W must be supplied to air. This is the *minimum* required power input required for the fan.

Discussion The actual power input to the fan will be larger than 5.91 W because of the fan inefficiency in converting mechanical power to kinetic energy.



11-103 A helicopter hovers at sea level while being loaded. The volumetric air flow rate and the required power input during unloaded hover, and the rpm and the required power input during loaded hover are to be determined. √

Assumptions **1** The flow of air is steady, one-dimensional, and incompressible. **2** Air leaves the blades at a uniform velocity at atmospheric pressure. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects). **5** The change in air pressure with elevation is negligible because of the low density of air. **6** There is no acceleration of the helicopter, and thus the lift generated is equal to the total weight.

Properties The density of air is given to be 1.18 kg/m^3.

Analysis (*a*) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the z axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. Noting that the only force acting on the control volume is the total weight W and it acts in the negative z direction, the momentum equation along the z axis gives

$$-W = \dot{m}(-\mathbf{V}_2) - 0 \quad \rightarrow \quad W = \dot{m}\mathbf{V}_2 = (\rho A\mathbf{V}_2)\mathbf{V}_2 = \rho A\mathbf{V}_2^2 \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{\frac{W}{\rho A}}$$

where A is the blade span area,

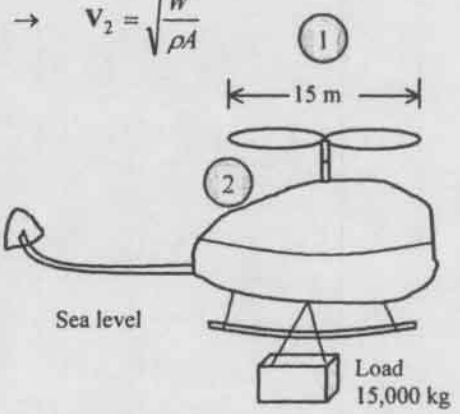
$$A = \pi D^2/4 = \pi(15\text{ m})^2/4 = 176.7\text{ m}^2$$

Then the discharge velocity, volume flow rate, and the mass flow rate of air in the unloaded mode become

$$\mathbf{V}_{2,\text{unloaded}} = \sqrt{\frac{m_{\text{unloaded}}\,g}{\rho A}} = \sqrt{\frac{(10{,}000\text{ kg})(9.81\text{ m/s}^2)}{(1.18\text{ kg/m}^3)(176.7\text{ m}^2)}} = 21.7\text{ m/s}$$

$$\dot{V}_{\text{unloaded}} = A\mathbf{V}_{2,\text{unloaded}} = (176.7\text{ m}^2)(21.7\text{ m/s}) = \mathbf{3834\text{ m}^3\text{/s}}$$

$$\dot{m}_{\text{unloaded}} = \rho\dot{V}_{\text{unloaded}} = (1.18\text{ kg/m}^3)(3834\text{ m}^3\text{/s}) = 4524\text{ kg/s}$$



Noting that $P_1 = P_2 = P_{\text{atm}}$, $\mathbf{V}_1 \cong 0$, the elevation effects are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{unloaded fan,u}} = \left(\dot{m}\frac{\mathbf{V}_2^2}{2}\right)_{\text{unloaded}} = (4524\text{ kg/s})\frac{(21.7\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{1065\text{ kW}}$$

(*b*) We now repeat the calculations for the *loaded* helicopter, whose mass is 10,000+15,000 = 25,000 kg:

$$\mathbf{V}_{2,\text{loaded}} = \sqrt{\frac{m_{\text{loaded}}\,g}{\rho A}} = \sqrt{\frac{(25{,}000\text{ kg})(9.81\text{ m/s}^2)}{(1.18\text{ kg/m}^3)(176.7\text{ m}^2)}} = 34.3\text{ m/s}$$

$$\dot{m}_{\text{loaded}} = \rho\dot{V}_{\text{loaded}} = \rho A\mathbf{V}_{2,\text{loaded}} = (1.18\text{ kg/m}^3)(176.7\text{ m}^2)(34.3\text{ m/s}) = 7152\text{ kg/s}$$

$$\dot{W}_{\text{loaded fan,u}} = \left(\dot{m}\frac{\mathbf{V}_2^2}{2}\right)_{\text{loaded}} = (7152\text{ kg/s})\frac{(34.3\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{4207\text{ kW}}$$

Noting that the average flow velocity is proportional to the overhead blade rotational velocity, the rpm of the loaded helicopter blades becomes

$$V_2 = k\dot{n} \quad \rightarrow \quad \frac{V_{2,\text{loaded}}}{V_{2,\text{unloaded}}} = \frac{\dot{n}_{\text{loaded}}}{\dot{n}_{\text{unloaded}}} \quad \rightarrow \quad \dot{n}_{\text{loaded}} = \frac{V_{2,\text{loaded}}}{V_{2,\text{unloaded}}}\dot{n}_{\text{unloaded}} = \frac{34.3}{21.7}(400 \text{ rpm}) = \mathbf{632 \text{ rpm}}$$

Discussion The actual power input to the helicopter blades will be considerably larger than the calculated power input because of the fan inefficiency in converting mechanical power to kinetic energy.

11-104 A helicopter hovers on top of a high mountain where the air density considerably lower than that at sea level. The blade rotational velocity to hover at the higher altitude and the percent increase in the required power input to hover at high altitude relative to that at sea level are to be determined. √

Assumptions **1** The flow of air is steady, one-dimensional, and incompressible. **2** The air leaves the blades at a uniform velocity at atmospheric pressure. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects). **5** The change in air pressure with elevation while hovering at a given location is negligible because of the low density of air. **6** There is no acceleration of the helicopter, and thus the lift generated is equal to the total weight.

Properties The density of air is given to be 1.18 kg/m^3 at sea level, and 0.79 kg/m^3 on top of the mountain.

Analysis (*a*) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the z axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\Sigma \vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i$. Noting that the only force acting on the control volume is the total weight W and it acts in the negative z direction, the momentum equation along the z axis gives

$$-W = \dot{m}(-\mathbf{V}_2) - 0 \quad \rightarrow \quad W = \dot{m}\mathbf{V}_2 = (\rho A \mathbf{V}_2)\mathbf{V}_2 = \rho A \mathbf{V}_2^2 \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{\frac{W}{\rho A}}$$

where A is the blade span area. Then for a given weight W, the ratio of discharge velocities becomes

$$\frac{\mathbf{V}_{2,\text{mountain}}}{\mathbf{V}_{2,\text{sea}}} = \frac{\sqrt{W/\rho_{\text{mountain}} A}}{\sqrt{W/\rho_{\text{sea}} A}} = \sqrt{\frac{\rho_{\text{sea}}}{\rho_{\text{mountain}}}} = \sqrt{\frac{1.18\ \text{kg/m}^3}{0.79\ \text{kg/m}^3}} = 1.222$$

Noting that the average flow velocity is proportional to the overhead blade rotational velocity, the rpm of the helicopter blades on top of the mountain becomes

$$\dot{n} = k\mathbf{V}_2 \quad \rightarrow \quad \frac{\dot{n}_{\text{mountain}}}{\dot{n}_{\text{sea}}} = \frac{\mathbf{V}_{2,\text{mountain}}}{\mathbf{V}_{2,\text{sea}}} \quad \rightarrow \quad \dot{n}_{\text{mountain}} = \frac{\mathbf{V}_{2,\text{mountain}}}{\mathbf{V}_{2,\text{sea}}} \dot{n}_{\text{sea}} = 1.222(400\ \text{rpm}) = \mathbf{489\ rpm}$$

Noting that $P_1 = P_2 = P_{\text{atm}}$, $\mathbf{V}_1 \cong 0$, the elevation effect are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2}$$

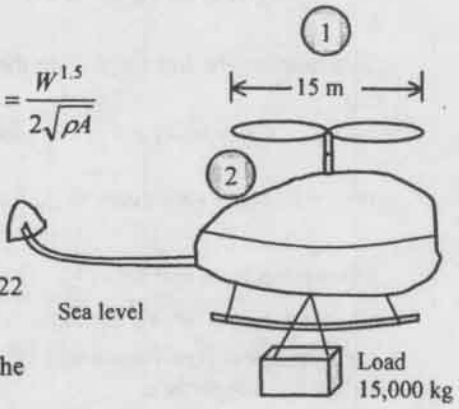
or

$$\dot{W}_{\text{fan,u}} = \dot{m}\frac{\mathbf{V}_2^2}{2} = \rho A \mathbf{V}_2 \frac{\mathbf{V}_2^2}{2} = \rho A \frac{\mathbf{V}_2^3}{2} = \tfrac{1}{2}\rho A\left(\sqrt{\frac{W}{\rho A}}\right)^3 = \tfrac{1}{2}\rho A\left(\frac{W}{\rho A}\right)^{1.5} = \frac{W^{1.5}}{2\sqrt{\rho A}}$$

Then the ratio of the required power input on top of the mountain to that at sea level becomes

$$\frac{\dot{W}_{\text{mountain fan,u}}}{\dot{W}_{\text{sea fan,u}}} \frac{0.5W^{1.5}/\sqrt{\rho_{\text{mountain}} A}}{0.5W^{1.5}/\sqrt{\rho_{\text{sea}} A}} = \sqrt{\frac{\rho_{\text{sea}}}{\rho_{\text{mountain}}}} = \sqrt{\frac{1.18\ \text{kg/m}^3}{0.79\ \text{kg/m}^3}} = 1.222$$

Therefore, the required power input will increase by **22.2%** on top of the mountain relative to the sea level.



Discussion Note that both the rpm and the required power input to the helicopter are inversely proportional to the square root of air density. Therefore, more power is required at higher elevations for the helicopter to operate because air is less dense, and more air must be forced by the blades into the downdraft.

11-105 The flow rate in a channel is controlled by a sluice gate by raising or lowering a vertical plate. A relation for the force acting on a sluice gate of width w for steady and uniform flow is to be developed.

Assumptions **1** The flow is steady, incompressible, frictionless, and uniform (and thus the Bernoulli equation is applicable.) **2** Wall shear forces at surfaces are negligible. **3** The channel is exposed to the atmosphere, and thus the pressure at free surfaces is the atmospheric pressure. **4** The flow is horizontal.

Analysis We take point 1 at the free surface of the upstream flow before the gate and point 2 at the free surface of the downstream flow after the gate. We also take the bottom surface of the channel as the reference level so that the elevations of points 1 and 2 are y_1 and y_2, respectively. The application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+y_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+y_2 \quad \rightarrow \quad \mathbf{V}_2^2-\mathbf{V}_1^2=2g(y_1-y_2) \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad \rightarrow \quad A_1\mathbf{V}_1=A_2\mathbf{V}_2=\dot{V} \quad \rightarrow \quad \mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{wy_1} \quad \text{and} \quad \mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{wy_2} \qquad (2)$$

Substituting into Eq. (1),

$$\left(\frac{\dot{V}}{wy_2}\right)^2-\left(\frac{\dot{V}}{wy_1}\right)^2=2g(y_1-y_2) \rightarrow \dot{V}=w\sqrt{\frac{2g(y_1-y_2)}{1/y_2^2-1/y_1^2}} \rightarrow \dot{V}=wy_2\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \qquad (3)$$

Substituting Eq. (3) into Eqs. (2) gives the following relations for velocities,

$$\mathbf{V}_1=\frac{y_2}{y_1}\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \quad \text{and} \quad \mathbf{V}_2=\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \qquad (4)$$

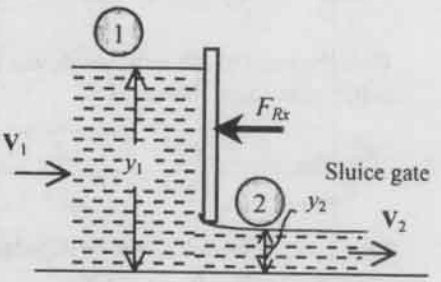
We choose the control volume as the water body surrounded by the vertical cross-sections of the upstream and downstream flows, free surfaces of water, the inner surface of the sluice gate, and the bottom surface of the channel. The momentum equation for steady one-dimensional flow is $\Sigma\vec{F}=\Sigma\dot{m}_e\vec{\mathbf{V}}_e-\Sigma\dot{m}_i\vec{\mathbf{V}}_i$. The force acting on the sluice gate F_{Rx} is horizontal since the wall shear at the surfaces is negligible, and it is equal and opposite to the force applied on water by the sluice gate. Noting that the pressure force acting on a vertical surface is equal to the product of the pressure at the centroid of the surface and the surface area, the momentum equation along the x direction gives

$$-F_{Rx}+P_1A_1-P_2A_2=\dot{m}\mathbf{V}_2-\dot{m}\mathbf{V}_1 \quad \rightarrow \quad -F_{Rx}+\left(\rho g\frac{y_1}{2}\right)(wy_1)-\left(\rho g\frac{y_2}{2}\right)(wy_2)=\dot{m}(\mathbf{V}_2-\mathbf{V}_1)$$

Rearranging, the force acting on the sluice gate is determined to be

$$F_{Rx}=\dot{m}(\mathbf{V}_1-\mathbf{V}_2)+\frac{w}{2}\rho g(y_1^2-y_2^2) \qquad (5)$$

where $\mathbf{V}_1$ and $\mathbf{V}_2$ are given in Eq. (4).



Discussion Note that for $y_1 >> y_2$, Eq. (3) simplifies to $\dot{V}=y_2w\sqrt{2gy_1}$ or $\mathbf{V}_2=\sqrt{2gy_1}$ which is the Toricelli equation for frictionless flow from a tank through a hole a distance y_1 below the free surface.

Review Problems

11-106 A stationary water tank placed on wheels on a frictionless surface is propelled by a water jet that leaves the tank through a smooth hole. Relations are to be developed for the acceleration, the velocity, and the distance traveled by the tank as a function of time as water discharges.

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **3** The surface under the wheeled tank is level and frictionless. **4** The water jet is discharged horizontally and rearward. **5** The mass of the tank and wheel assembly is negligible compared to the mass of water in the tank.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the hole, which is also taken to be the reference level ($z_2 = 0$) so that the water height above the hole at any time is z. Noting that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), it is open to the atmosphere ($P_1 = P_{atm}$), and water discharges into the atmosphere (and thus $P_2 = P_{atm}$), the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad z=\frac{\mathbf{V}_J^2}{2g}+0 \quad \rightarrow \quad \mathbf{V}_J=\sqrt{2gz}$$

The discharge rate of water from the tank through the hole is

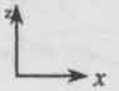
$$\dot{m}=\rho A\mathbf{V}_J=\rho\frac{\pi D_0^2}{4}\mathbf{V}_J=\rho\frac{\pi D_0^2}{4}\sqrt{2gz}$$

The momentum equation for steady one-dimensional flow is $\Sigma\vec{F}=\Sigma\dot{m}_e\vec{\mathbf{V}}_e-\Sigma\dot{m}_i\vec{\mathbf{V}}_i$. Applying it to the water tank, the horizontal force that acts on the tank is determined to be

$$F=\dot{m}\mathbf{V}_e-0=\dot{m}\mathbf{V}_J=\rho\frac{\pi D_0^2}{4}2gz=\rho gz\frac{\pi D_0^2}{2}$$

The acceleration of the water tank is determined from Newton's 2nd law of motion $F = ma$ where m is the mass of water in the tank, $m=\rho V_{tank}=\rho(\pi D^2/4)z$,

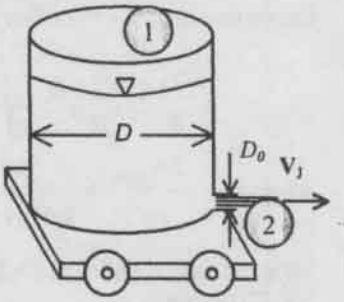
$$a=\frac{F}{m}=\frac{\rho gz(\pi D_0^2/2)}{\rho z(\pi D^2/4)} \quad \rightarrow \quad a=2g\frac{D_0^2}{D^2}$$



Note that the acceleration of the tank is constant.

(*b*) Noting that $a = d\mathbf{V}/dt$ and thus $d\mathbf{V} = adt$ and that the acceleration a is constant, the velocity is expressed as

$$\mathbf{V}=at \quad \rightarrow \quad \mathbf{V}=2g\frac{D_0^2}{D^2}t$$



(*c*) Noting that $\mathbf{V} = dx/dt$ and thus $dx = \mathbf{V}dt$, the distance traveled by the water tank is determined by integration to be

$$dx=\mathbf{V}dt \quad \rightarrow \quad dx=2g\frac{D_0^2}{D^2}tdt \quad \rightarrow \quad x=g\frac{D_0^2}{D^2}t^2$$

since $x = 0$ at $t = 0$.

Discussion In reality, the flow rate discharge velocity and thus the force acting on the water tank will be less because of the frictional losses at the hole. But these losses can be accounted for by incorporating a discharge coefficient.

11-107 Water discharges from the orifice at the bottom of a pressurized tank to the atmosphere. The time it will take for half of the water in the tank to be discharged and the water level after 10 s are to be determined.

Assumptions **1** The flow is frictionless, uniform, and incompressible. **2** The tank air pressure above the water level is maintained constant.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice. We take the reference level at the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at point 2 is open to the atmosphere (and thus $P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_1}{\rho g}+z_1=\frac{P_{atm}}{\rho g}+\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{2gz_1+2P_{1,gage}/\rho}$$

For generality, we express the water height in the tank at any time t by z, and the discharge velocity by $\mathbf{V}_2=\sqrt{2gz+2P_{1,gage}/\rho}$. Note that water surface in the tank moves down as the tank drains, and thus z is a variable whose value changes from H at the beginning to 0 when the tank is emptied completely.

We denote the diameter of the orifice by D, and the diameter of the tank by D_o. The flow rate of water from the tank is obtained by multiplying the discharge velocity by the orifice cross-sectional area,

$$\dot{V}=A_{\text{orifice}}\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{2gz+2P_{1,gage}/\rho}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{2gz+2P_{1,gage}/\rho}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV=A_{\text{tank}}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

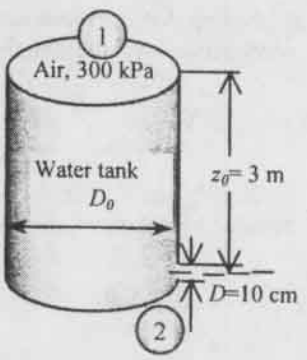
where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{2gz+2P_{1,gage}/\rho}\,dt=-\frac{\pi D_0^2}{4}dz \quad \rightarrow \quad dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1}{2gz+2P_{1,gage}/\rho}}\,dz$$

The last relation can be integrated since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_0$ to $t = t$ when $z = z$ gives

$$\sqrt{\frac{2z_0}{g}+\frac{2P_{1,gage}}{\rho g^2}}-\sqrt{\frac{2z}{g}+\frac{2P_{1,gage}}{\rho g^2}}=\frac{D_0^2}{D^2}t$$

where $$\frac{2P_{1,gage}}{\rho g^2}=\frac{2(300-100)\,\text{kN/m}^2}{(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)^2}\left(\frac{1000\,\text{kg}\cdot\text{m/s}^2}{1\,\text{kN}}\right)=4.156\,\text{s}^2$$



Then the time it will take for half of the water in the tank to be discharged ($z = z_0/2$) becomes

$$\sqrt{\frac{2(3\,\text{m})}{9.81\,\text{m/s}^2}+4.156\,\text{s}^2}-\sqrt{\frac{2(1.5\,\text{m})}{9.81\,\text{m/s}^2}+4.156\,\text{s}^2}=\frac{(0.1\,\text{m})^2}{(2\,\text{m})^2}t \quad \rightarrow \quad t=\mathbf{28.5\ s}$$

(*b*) Water level after 10s is $$\sqrt{\frac{2(3\,\text{m})}{9.81\,\text{m/s}^2}+4.156\,\text{s}^2}-\sqrt{\frac{2z}{9.81\,\text{m/s}^2}+4.156\,\text{s}^2}=\frac{(0.1\,\text{m})^2}{(2\,\text{m})^2}(10\,\text{s}) \rightarrow z=\mathbf{2.47\ m}$$

Discussion Note that the discharging time is inversely proportional to the square of the orifice diameter. Therefore, the discharging time can be reduced to one-fourth by doubling the diameter of the orifice.

11-108 Air flows through a pipe that consists of two sections at a specified rate. The differential height of a water manometer placed between the two pipe sections is to be determined. √

Assumptions **1**The flow through the pipe is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The losses in the reducing section are negligible. **3** The pressure difference across an air column is negligible because of the low density of air, and thus the air column in the manometer can be ignored.

Properties The density of air is given to be $\rho_{air} = 1.20$ kg/m^3. We take the density of water to be $\rho_w = 1000$ kg/m^3.

Analysis We take points 1 and 2 along the centerline of the pipe over the two tubes of the manometer. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases), the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad P_1-P_2=\frac{\rho_{air}(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2} \qquad (1)$$

We let the differential height of the water manometer be h. Then the pressure difference $P_2 - P_1$ can also be expressed as

$$P_1 - P_2 = \rho_w g h \qquad (2)$$

Combining Eqs. (1) and (2) and solving for h,

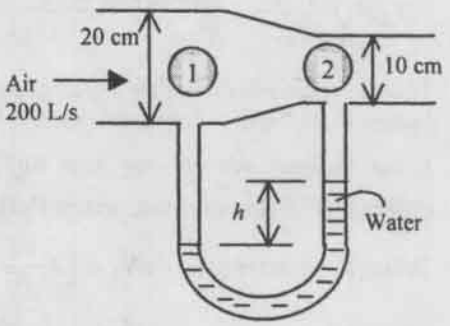
$$\frac{\rho_{air}(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2}=\rho_w g h \quad \rightarrow \quad h=\frac{\rho_{air}(\mathbf{V}_2^2-\mathbf{V}_1^2)}{2g\rho_w}=\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2g\rho_w/\rho_{air}}$$

Calculating the velocities and substituting,

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.2\,\text{m}^3/\text{s}}{\pi(0.2\,\text{m})^2/4}=6.37\,\text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.2\,\text{m}^3/\text{s}}{\pi(0.1\,\text{m})^2/4}=25.5\,\text{m/s}$$

$$h=\frac{(25.5\,\text{m/s})^2-(6.37\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(1000/1.20)}=0.037\,\text{m}=\mathbf{3.7\,cm}$$



Therefore, the differential height of the water column will be 3.7 cm.

Discussion Note that the differential height of the manometer is inversely proportional to the density of the manometer fluid. Therefore, heavy fluids such as mercury are used when measuring large pressure differences.

11-109 Air flows through a horizontal duct of variable cross-section. For a given differential height of a water manometer placed between the two pipe sections, the downstream velocity of air is to be determined, and an error analysis is to be conducted. √

Assumptions **1**The flow through the duct is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The losses in this section of the duct are negligible. **3** The pressure difference across an air column is negligible because of the low density of air, and thus the air column in the manometer can be ignored.

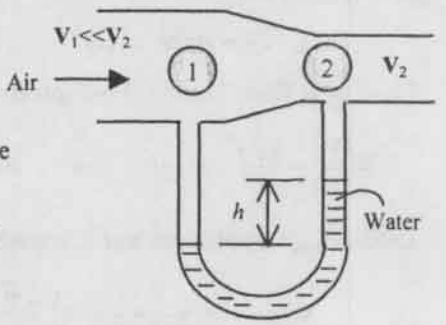
Properties The gas constant of air is $R = 0.287$ kPa·m^3/kg·K (Table A-1). We take the density of water to be $\rho_w = 1000$ kg/m^3

Analysis We take points 1 and 2 along the centerline of the duct over the two tubes of the manometer. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases) and $\mathbf{V}_1 \cong 0$, the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{P_1}{\rho g}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2(P_1-P_2)}{\rho_{air}}} \qquad (1)$$

where $P_1 - P_2 = \rho_w g h$

and $$\rho_{air}=\frac{P}{RT}=\frac{100\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\,\text{K})}=1.17\,\text{kg/m}^3$$



Substituting into (1), the downstream velocity of air $\mathbf{V}_2$ is determined to be

$$\mathbf{V}_2=\sqrt{\frac{2\rho_w g h}{\rho_{air}}}=\sqrt{\frac{2(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.05\,\text{m})}{1.17\,\text{kg/m}^3}}=\mathbf{29.0\,m/s} \qquad (2)$$

Therefore, the velocity of air increases from a low level in the first section to 29.0 m/s in the second section.

Error Analysis We observe from Eq. (2) that the velocity is proportional to the square root of the differential height of the manometer fluid. That is, $\mathbf{V}_2 = k\sqrt{h}$.

Taking the differential: $$d\mathbf{V}_2=\tfrac{1}{2}k\frac{dh}{\sqrt{h}}$$

Dividing by $\mathbf{V}_2$: $$\frac{d\mathbf{V}_2}{\mathbf{V}_2}=\tfrac{1}{2}k\frac{dh}{\sqrt{h}}\frac{1}{k\sqrt{h}} \quad \rightarrow \quad \frac{d\mathbf{V}_2}{\mathbf{V}_2}=\frac{dh}{2h}=\frac{\pm 2\,\text{mm}}{2\times 50\,\text{mm}}=\pm\mathbf{0.02}$$

Therefore, the uncertainty in the velocity corresponding to an uncertainty of 2 mm in the differential height of water is 2%, which corresponds to 0.02×(29.0 m/s) = 0.58 ≅ 0.6 m/s. Then the discharge velocity can be expressed as

$$\mathbf{V}_2 = 29.0 \pm 0.6\ \text{m/s}$$

11-110 A tap is opened on the wall of a very large tank that contains air. The maximum flow rate of air through the tap is to be determined, and the effect of a larger diameter lead section is to be assessed. √

Assumptions The flow through the tap is steady, frictionless, incompressible, and irrotational (so that the flow rate is maximum, and the Bernoulli equation is applicable).

Properties The gas constant of air is $R = 0.287$ kPa·m^3/kg·K (Table A-1).

Analysis The density of air in the tank is

$$\rho_{air} = \frac{P}{RT} = \frac{102\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\,\text{K})} = 1.21\,\text{kg/m}^3$$

We take point 1 in the tank, and point 2 at the exit of the tap along the same horizontal line. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases) and $\mathbf{V}_1 \cong 0$, the Bernoulli equation between points 1 and 2 gives

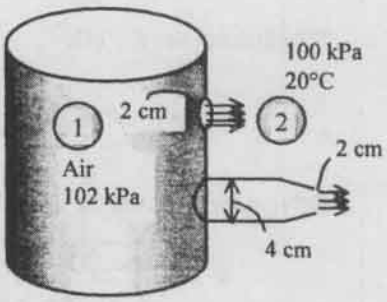
$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_{air}}}$$

Substituting, the discharge velocity and the flow rate becomes

$$\mathbf{V}_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_{air}}} = \sqrt{\frac{2(102-100)\,\text{kN/m}^2}{1.21\,\text{kg/m}^3}\left(\frac{1000\,\text{kg}\cdot\text{m/s}^2}{1\,\text{kN}}\right)} = 57.5\,\text{m/s}$$

$$\dot{V} = A\mathbf{V}_2 = \frac{\pi D_2^2}{4}\mathbf{V}_2 = \frac{\pi(0.02\,\text{m})^2}{4}(57.5\,\text{m/s}) = \mathbf{0.0181\,m^3/s}$$

This is the *maximum* flow rate since it is determined by assuming frictionless flow. The actual flow rate will be less.



Adding a 2-m long larger diameter lead section will have **no effect** on the flow rate since the flow is frictionless (by using the Bernoulli equation, it can be shown that the velocity in this section increases, but the pressure decreases, and there is a smaller pressure difference to drive the flow through the tab, with zero net effect on the discharge rate).

Discussion If the pressure in the tank were 300 kPa, the flow is no longer incompressible, and thus the problem in that case should be analyzed using compressible flow theory.

11-111 Water is flowing through a venturi meter with known diameters and measured pressures. The flow rate of water is to be determined for the case of frictionless flow.

Assumptions **1** The flow through the venturi is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The flow is horizontal so that elevation along the centerline is constant. **3** The pressure is uniform at a given cross-section of the venturi meter (or the elevation effects on pressure measurement are negligible).

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

Analysis We take point 1 at the main flow section and point 2 at the throat along the centerline of the venturi meter. Noting that $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

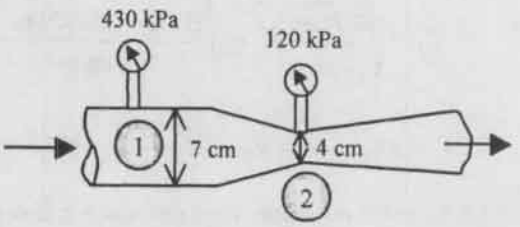
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad P_1-P_2=\rho\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2} \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad \rightarrow \quad A_1\mathbf{V}_1=A_2\mathbf{V}_2=\dot{V} \quad \rightarrow \quad \mathbf{V}_1=\frac{\dot{V}}{A_1} \quad \text{and} \quad \mathbf{V}_2=\frac{\dot{V}}{A_2} \qquad (2)$$

Substituting into Eq. (1),

$$P_1-P_2=\rho\frac{(\dot{V}/A_2)^2-(\dot{V}/A_1)^2}{2}=\frac{\rho\dot{V}^2}{2A_2^2}\left(1-\frac{A_2^2}{A_1^2}\right)$$



Solving for $\dot{V}$ gives the desired relation for the flow rate,

$$\dot{V}=A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}} \qquad (3)$$

The flow rate for the given case can be determined by substituting the given values into this relation to be

$$\dot{V}=\frac{\pi D_2^2}{4}\sqrt{\frac{2(P_1-P_2)}{\rho[1-(D_2/D_1)^4]}}=\frac{\pi(0.04\,\text{m})^2}{4}\sqrt{\frac{2(430-120)\,\text{kN/m}^2}{(1000\,\text{kg/m}^3)[1-(4/7)^4]}\left(\frac{1000\,\text{kg}\cdot\text{m/s}^2}{1\,\text{kN}}\right)}=\mathbf{0.538\ m^3/s}$$

Discussion Venturi meters are commonly used as flow meters to measure the flow rate of gases and liquids by simply measuring the pressure difference P_1 - P_2 by a manometer or pressure transducers. The actual flow rate will be less than the value obtained from Eq. (3) because of the friction losses along the wall surfaces in actual flow. But this difference can be as little as 1% in a well-designed venturi meter. The effects of deviation from the idealized Bernoulli flow can be accounted for by expressing Eq. (3) as

$$\dot{V}=C_c A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}}$$

where C_c is the *venturi discharge coefficient* whose value is less than 1 (it is as large as 0.99 for well-designed venturi meters in certain ranges of flow). For $\text{Re} > 10^5$, the value of venturi discharge coefficient is usually greater than 0.96.

11-112E A hose is connected to the bottom of a water tank open to the atmosphere. The hose is equipped with a pump and a nozzle at the end. The maximum height to which the water stream could rise is to be determined.

Assumptions **1** The flow is frictionless and incompressible. **2** The friction between the water and air is negligible. **3** We take the head loss to be zero ($h_L = 0$) to determine the maximum rise of water jet.

Properties We take the density of water to be 62.4 lbm/ft^3.

Analysis We take point 1 at the free surface of the tank, and point 2 at the top of the water trajectory where $\mathbf{V}_2 = 0$. We take the reference level at the bottom of the tank. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 + h_{\text{turbine}} + h_L \quad \rightarrow \quad z_1 + h_{\text{pump,u}} = z_2$$

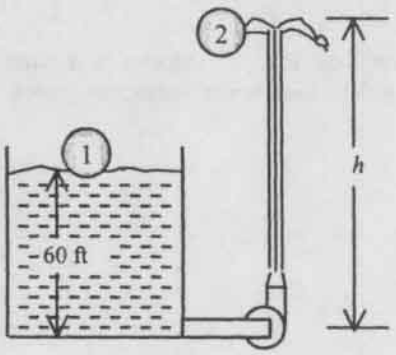
where the useful pump head is

$$h_{\text{pump, u}} = \frac{\Delta P_{\text{pump, u}}}{\rho g} = \frac{10\ \text{psi}}{(62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)}\left(\frac{144\ \text{lbf/ft}^2}{1\ \text{psi}}\right)\left(\frac{32.2\ \text{lbm}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right) = 23.1\ \text{ft}$$

Substituting, the maximum height rise of water jet from the ground level is determined to be

$$z_2 = z_1 + h_{\text{pump,u}} = 60 + 23.2 = \mathbf{83.2ft}$$

Discussion The actual rise of water will be less because of the frictional effects between the water and the hose walls and between the water jet and air.



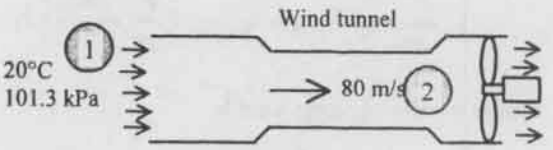
11-113 A wind tunnel draws atmospheric air by a large fan. For a given air velocity, the pressure in the tunnel is to be determined. √

Assumptions 1The flow through the pipe is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** Air is and ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa·m³/kg·K (Table A-1).

Analysis We take point 1 in atmospheric air before it enters the wind tunnel (and thus $P_1 = P_{atm}$ and $\mathbf{V}_1 \cong 0$), and point 2 in the wind tunnel. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases), the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad P_2 = P_1-\frac{\rho \mathbf{V}_2^2}{2} \qquad (1)$$



where

$$\rho=\frac{P}{RT}=\frac{101.3\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\,\text{K})}=1.205\,\text{kg/m}^3$$

Substituting, the pressure in the wind tunnel is determined to be

$$P_2=(101.3\,\text{kPa})-(1.205\,\text{kg/m}^3)\frac{(80\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1000\,\text{N/m}^2}\right)=\mathbf{97.4\,kPa}$$

Discussion Note that the velocity in a wind tunnel increases at the expense of pressure. In reality, the pressure will be even lower because of losses.

11-114 Water flows through the enlargement section of a horizontal pipe at a specified rate. For a given head loss, the pressure change across the enlargement section is to be determined. √

Assumptions **1** The flow through the pipe is steady, one-dimensional, and incompressible. **2** The pipe is horizontal.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

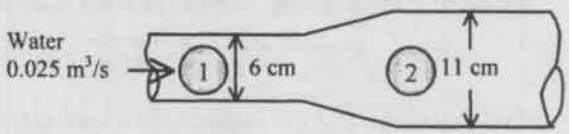
Analysis We take points 1 and 2 at the inlet and exit of the enlargement section along the centerline of the pipe. Noting that $z_1 = z_2$, the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad P_2-P_1=\rho\frac{\mathbf{V}_1^2-\mathbf{V}_2^2}{2}-\rho g h_L$$

where the inlet and exit velocities are

$$\mathbf{V}_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}=\frac{0.025\ \text{m}^3/\text{s}}{\pi(0.06\ \text{m})^2/4}=8.84\ \text{m/s}$$

$$\mathbf{V}_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.025\ \text{m}^3/\text{s}}{\pi(0.11\ \text{m})^2/4}=2.63\ \text{m/s}$$



Substituting, the pressure change across the enlargement section is determined to b e

$$P_2-P_1=(1000\ \text{kg/m}^3)\left(\frac{(8.84\ \text{m/s})^2-(2.63\ \text{m/s})^2}{2}-(9.81\ \text{m/s}^2)(0.8\ \text{m})\right)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)=\mathbf{27.8\ kPa}$$

Therefore, the water pressure increases by 27.8 kPa across the enlargement section.

Discussion Note that the pressure increases despite the head loss in the enlargement section. This is due to dynamic pressure being converted to static pressure. But the total pressure (static + dynamic) decreases by 0.8 m (or 7.85 kPa) as a result of frictional effects.

11-115 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe with a specified head loss. The initial discharge velocity is to be determined. √

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** There are no pumps or turbines in the system.

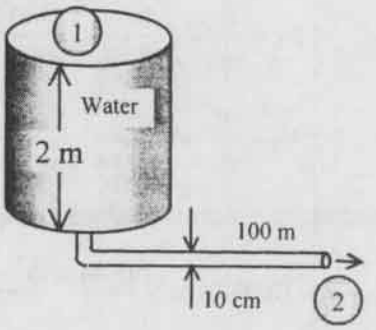
Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the orifice ($z_2 = 0$. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the head loss is given to be $h_L = 1.5$ m. Solving for $\mathbf{V}_2$ and substituting, the discharge velocity of water is determined to be

$$\mathbf{V}_2=\sqrt{2g(z_1-h_L)}=\sqrt{2(9.81\,\text{m/s}^2)(2-1.5)\,\text{m}}=\mathbf{3.13\ m/s}$$

Discussion Note that this is the discharge velocity at the beginning, and the velocity will decrease as the water level in the tank drops. The head loss in that case will change since it depends on velocity.



11-116 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe equipped with a pump with a specified head loss. The required pump head to assure a certain velocity is to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal.

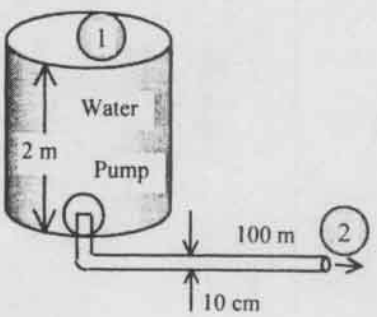
Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1+h_{\text{pump,u}}=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the head loss is given to be $h_L = 1.5$ m. Solving for $h_{\text{pump, u}}$ and substituting, the required pump head is determined to be

$$h_{\text{pump, u}}=\sqrt{\frac{\mathbf{V}_2^2}{2g}-z_1+h_L}=\sqrt{\frac{(4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}-(2\,\text{m})+(1.5\,\text{m})}=\mathbf{0.315\ m}$$

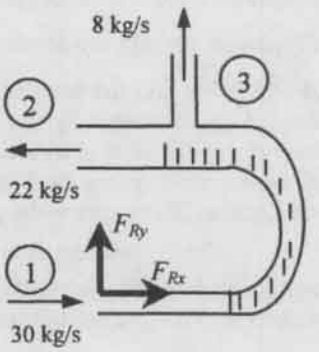
Discussion Note that this is the required useful pump head at the beginning, and it will need to be increased as the water level in the tank drops to make up for the lost elevation head to maintain the constant discharge velocity.



11-117 Water is flowing into and discharging from a pipe U-section with a secondary discharge section normal to return flow. The total x- and y- forces at the two flanges that connect the pipes are to be determined

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The weight of the U-turn and the water in it is negligible.

Properties We take the density of water to be 1000 kg/m^3.



Analysis The flow velocities of the 3 streams are

$$\mathbf{V}_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{\dot{m}_1}{\rho(\pi D_1^2/4)} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.05\ \text{m})^2/4]} = 0.51\ \text{m/s}$$

$$\mathbf{V}_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{\dot{m}_2}{\rho(\pi D_2^2/4)} = \frac{22\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.10\ \text{m})^2/4]} = 2.80\ \text{m/s}$$

$$\mathbf{V}_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{\dot{m}_3}{\rho(\pi D_3^2/4)} = \frac{8\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.03\ \text{m})^2/4]} = 11.3\ \text{m/s}$$

We take the entire U-section as the control volume. We designate the horizontal coordinate by x with the direction of incoming flow as being the positive direction and the vertical coordinate by y. The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. We let the x- and y- components of the anchoring force of the cone be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Then the momentum equations along the x and y axes become

$$F_{Rx} + P_1A_1 + P_2A_2 = \dot{m}_2(-\mathbf{V}_2) - \dot{m}_1\mathbf{V}_1 \quad \rightarrow \quad F_{Rx} = -P_1A_1 - P_2A_2 - \dot{m}_2\mathbf{V}_2 - \dot{m}_1\mathbf{V}_1$$

$$F_{Ry} + 0 = \dot{m}_3\mathbf{V}_3 - 0 \quad \rightarrow \quad F_{Ry} = \dot{m}_3\mathbf{V}_3$$

Substituting the given values,

$$F_{Rx} = -[(200-100)\ \text{kN/m}^2]\frac{\pi(0.05\ \text{m})^2}{4} - [(150-100)\ \text{kN/m}^2]\frac{\pi(0.10\ \text{m})^2}{4}$$
$$-(22\ \text{kg/s})(2.80\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) - (30\ \text{kg/s})(0.51\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= -0.666\ \text{kN} = \mathbf{-666\ N}$$

$$F_{Ry} = (8\ \text{kg/s})(11.3\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{90.4\ N}$$

The negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed. Therefore, a force of 666 N acts on the flanges in the opposite direction. A vertical force of 90.4 N acts on the flange in the vertical direction.

Discussion To assess the significance of gravity forces, we estimate the weight of the weight of water in the U-turn and compare it to the vertical force. Assuming the length of the U-turn to be 0.5 m and the average diameter to be 7.5 cm, the mass of the water becomes

$$m = \rho V = \rho AL = \rho\frac{\pi D^2}{4}L = (1000\ \text{kg/m}^3)\frac{\pi(0.075\ \text{m})^2}{4}(0.5\ \text{m}) = 2.2\ \text{kg}$$

whose weight is 2.2×9.81 = 22 N, which is much less than 90.4 N, but still significant. Therefore, disregarding the gravitational effects is a reasonable assumption if great accuracy is not required.

11-118 A fireman was hit by a nozzle held by a tripod with a rated holding force. The accident is to be investigated by calculating the water velocity, the flow rate, and the nozzle velocity.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Gravitational effects and vertical forces are disregarded since the horizontal resistance force is to be determined.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the nozzle and the horizontal portion of the hose as the system such that water enters the control volume vertically and exits horizontally (this way the pressure force and the momentum flux at the inlet are in the vertical direction, with no contribution to the force balance in the horizontal direction, and designate the entrance by 1 and the exit by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction).

The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. We let the horizontal force applied by the tripod to the nozzle to hold it be F_{Rx}, and assume it to be in the positive x direction. Then the momentum equation along the x direction becomes

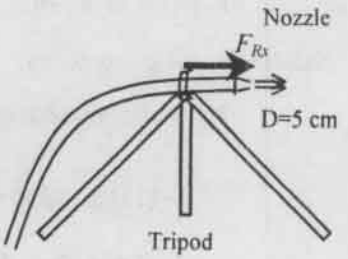
$$F_{Rx} = \dot{m}\mathbf{V}_e - 0 = \dot{m}\mathbf{V} = \rho A\mathbf{V}\mathbf{V} = \rho\frac{\pi D^2}{4}\mathbf{V}^2 \quad \rightarrow \quad (1800\text{ N})\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right) = (1000\text{ kg/m}^3)\frac{\pi(0.05\text{ m})^2}{4}\mathbf{V}^2$$

Solving for the water exit velocity gives **V = 30.2 m/s.** Then the water flow rate becomes

$$\dot{V} = A\mathbf{V} = \frac{\pi D^2}{4}\mathbf{V} = \frac{\pi(0.05\text{ m})^2}{4}(30.2\text{ m/s}) = \mathbf{0.0593\ m^3/s}$$

When the nozzle was released, its acceleration must have been

$$a_{\text{nozzle}} = \frac{F}{m_{\text{nozzle}}} = \frac{1800\text{ N}}{10\text{ kg}}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{N}}\right) = 180\text{ m/s}^2$$



Assuming the reaction force acting on the nozzle and thus its acceleration to remain constant, the time it takes for the nozzle to travel 60 cm and the nozzle velocity at that moment were (note that both the distance x and the velocity **V** are zero at time $t = 0$)

$$x = \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(0.6\text{ m})}{180\text{ m/s}^2}} = 0.0816\,s$$

$$\mathbf{V} = at = (180\text{ m/s}^2)(0.0816\text{ s}) = \mathbf{14.7\ m/s}$$

Thus we conclude that the nozzle hit the fireman with a velocity of 14.7 m/s.

Discussion Engineering analyses such as this one are frequently used in accident reconstruction cases, and they often form the basis for judgment in courts.

11-119 During landing of an airplane, the thrust reverser is lowered in the path of the exhaust jet, which deflects the exhaust and provides breaking. The thrust of the engine and the braking force produced after the thrust reverser is deployed are to be determined.

Assumptions **1** The flow of exhaust gases is steady and one-dimensional. **2** The exhaust gas stream is exposed to the atmosphere, and thus its pressure is the atmospheric pressure. **3** The velocity of exhaust gases remains constant during reversing.

Analysis (*a*) The thrust exerted on an airplane is simply the momentum flux of the combustion gases in the reverse direction,

$$\text{Thrust} = \dot{m}_{ex}\mathbf{V}_{ex} = (10\,\text{kg/s})(250\,\text{m/s})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{2500\,N}$$

(*b*) We take the thrust reverser as the control volume such that it cuts through both exhaust streams normally and the connecting bars to the airplane, and the direction of flow of exhaust gases as the positive direction of x axis. We take the reaction force (acting on the connecting bars) to be in the positive x direction. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces to

$$\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i \quad \rightarrow \quad F_{Rx} = \dot{m}(-\mathbf{V})\cos 20° - \dot{m}\mathbf{V} \quad \rightarrow \quad F_{Rx} = -(1+\cos 20°)\dot{m}\mathbf{V}_i$$

Substituting, the reaction force is determined to be

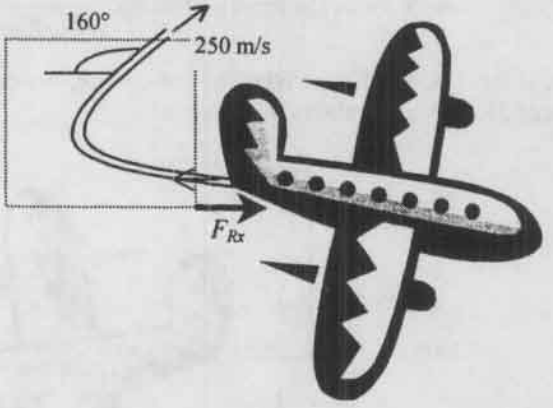
$$F_R = -(1+\cos 20°)(10\,\text{kg/s})(250\,\text{m/s}) = -4849\,\text{N}$$

The breaking force acting on the plane is equal and opposite to this force,

$$F_{\text{breaking}} = \mathbf{4849\,N}$$

Therefore, a braking force of 4849 N develops in the positive x direction. Note that this is an equal and opposite force to the reaction force acting on the thrust reverser in the negative x direction.

Discussion Noting that the exhaust gases generate a thrust (forward force) of 2500 N, the net breaking force acting on the airplane is 4849 – 2500 = 2349 N. In other words, the effect of deploying this thrust reverser is equivalent to turning off the jet engines and applying a force of 2349 N on the brakes when the airplane is on the ground.



11-120E The rocket of a spacecraft is fired in the opposite direction to motion. The acceleration, the velocity change, and the thrust are to be determined.

Assumptions **1** The flow of combustion gases is steady and one-dimensional during firing period. **2** There are no external forces acting on the spacecraft, and the effect of pressure force at the nozzle exit is negligible. **3** The mass of discharged fuel is negligible relative to the mass of the spacecraft, and thus the spacecraft may be treated as a solid body with a constant mass.

Analysis (*a*) A body moving at constant velocity can be considered to be stationary for convenience. Then the velocities of fluid steams become simply their relative velocities relative to the moving body. We take the direction of motion of the spacecraft as the positive direction along the x axis. There are no external forces acting on the spacecraft, and its mass is nearly constant. Therefore, the spacecraft can be treated as a solid body with constant mass, and the momentum equation in this case is

$$\frac{d(m\vec{\mathbf{V}})_{CV}}{dt} = \sum_{\text{in}} \dot{m}_i \vec{\mathbf{V}}_i - \sum_{\text{out}} \dot{m}_e \vec{\mathbf{V}}_e \quad \rightarrow \quad m_{\text{space}} \frac{d\vec{\mathbf{V}}_{\text{space}}}{dt} = -\dot{m}_f \vec{\mathbf{V}}_f$$

Noting that the motion is on a straight line and the discharged gases move in the negative x direction, we can write the momentum equation using magnitudes as

$$m_{\text{space}} \frac{d\mathbf{V}_{\text{space}}}{dt} = \dot{m}_f \mathbf{V}_f \quad \rightarrow \quad \frac{d\mathbf{V}_{\text{space}}}{dt} = \frac{\dot{m}_f}{m_{\text{space}}} \mathbf{V}_f$$

Substituting, the acceleration of the spacecraft during the first 5 seconds is determined to be

$$a_{\text{space}} = \frac{d\mathbf{V}_{\text{space}}}{dt} = \frac{\dot{m}_f}{m_{\text{space}}} \mathbf{V}_f = \frac{150 \text{ lbm/s}}{18{,}000 \text{ lbm}} (5000 \text{ ft/s}) = \mathbf{41.7 \text{ ft/s}^2}$$

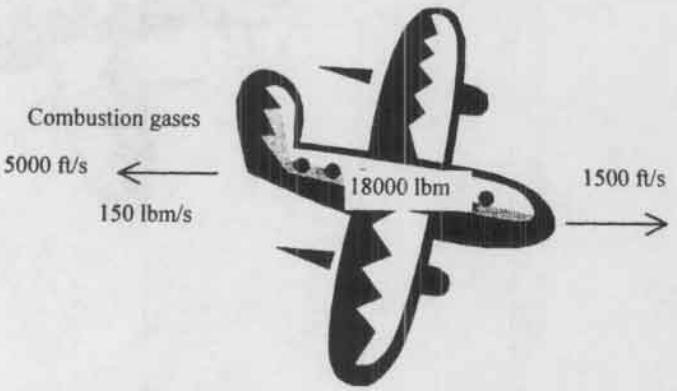
(*b*) Knowing acceleration, which is constant, the velocity change of the spacecraft during the first 5 seconds is determined from the definition of acceleration $a_{\text{space}} = d\mathbf{V}_{\text{space}} / dt$ to be

$$d\mathbf{V}_{\text{space}} = a_{\text{space}} dt \quad \rightarrow \quad \Delta\mathbf{V}_{\text{space}} = a_{\text{space}} \Delta t = (41.7 \text{ ft/s}^2)(5 \text{ s}) = \mathbf{209 \text{ ft/s}}$$

(*c*) The thrust exerted on the system is simply the momentum flux of the combustion gases in the reverse direction,

$$\text{Thrust} = F_R = -\dot{m}_f \mathbf{V}_f = -(150 \text{ lbm/s})(-5000 \text{ ft/s}) \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = \mathbf{23{,}290 \text{ lbf}}$$

Therefore, if this spacecraft were attached somewhere, it would exert a force of 23,290 lbf (equivalent to the weight of 23,290 lbm of mass) to its support.



11-121 A horizontal water jet strikes a vertical stationary flat plate normally at a specified velocity. For a given flow velocity, the anchoring force needed to hold the plate in place is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water splatters off the sides of the plate in a plane normal to the jet. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on the entire control surface. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the plate as the control volume such that it contains the entire plate and cuts through the water jet and the support bar normally, and the direction of flow as the positive direction of x axis. We take the reaction force to be in the negative x direction. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\Sigma \vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i \quad \rightarrow \quad -F_{Rx} = -\dot{m}_i \mathbf{V}_i \quad \rightarrow \quad F_{Rx} = \dot{m}\mathbf{V}$$

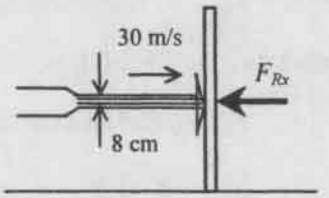
We note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. The mass flow rate of water is

$$\dot{m} = \rho \dot{V} = \rho A \mathbf{V} = \rho \frac{\pi D^2}{4} \mathbf{V} = (1000\,\text{kg/m}^3)\frac{\pi (0.08\,\text{m})^2}{4}(30\,\text{m/s}) = 150.8\,\text{kg/s}$$

Substituting, the reaction force is determined to be

$$F_{Rx} = (150.8\,\text{kg/s})(30\,\text{m/s}) = \mathbf{4524\ N}$$

Therefore, a force of 4524 N must be applied to the plate in the opposite direction to flow to hold it in place.



Discussion In reality, some water will be scattered back, and this will add to the reaction force of water.

11-122 A water jet hits a stationary cone, such that the flow is diverted equally in all directions at 45°. The force required to hold the cone in place against the water stream is to be determined.

Assumptions **1** The flow is steady, one-dimensional, and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet before and after the split is the atmospheric pressure which is disregarded since it acts on all surfaces. **3** The gravitational effects are disregarded.

Properties We take the density of water to be 1000 kg/m^3.

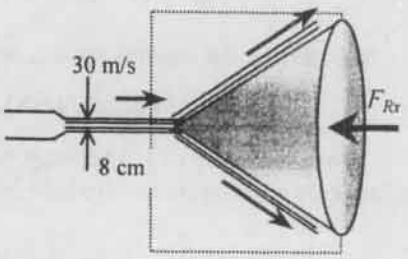
Analysis The mass flow rate of water jet is

$$\dot{m} = \rho \dot{V} = \rho A \mathbf{V} = \rho \frac{\pi D^2}{4} \mathbf{V} = (1000\,\text{kg/m}^3)\frac{\pi (0.08\,\text{m})^2}{4}(30\,\text{m/s}) = 150.8\,\text{kg/s}$$

We take the diverting section of water jet, including the cone as the control volume, and designate the entrance by 1 and the exit after divergence by 2. We also designate the horizontal coordinate by x with the direction of flow as being the positive direction and the vertical coordinate by y.

The momentum equation for steady one-dimensional flow is $\Sigma \vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i$. We let the x- and y- components of the anchoring force of the cone be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Noting that $\mathbf{V}_2 = \mathbf{V}_1 = \mathbf{V}$ and $\dot{m}_2 = \dot{m}_1 = \dot{m}$, the momentum equations along the x and y axes become

$$F_{Rx} = \dot{m}\mathbf{V}_2 \cos\theta - \dot{m}\mathbf{V}_1 = \dot{m}\mathbf{V}(\cos\theta - 1)$$
$$F_{Ry} = 0 \quad \text{(because of symmetry about x axis)}$$



Substituting the given values,

$$F_{Rx} = (150.8\,\text{kg/s})(30\,\text{m/s})(\cos 45° - 1)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$
$$= \mathbf{-1325\,N}$$
$$F_{Ry} = \mathbf{0}$$

The negative value for F_{Rx} indicates that the assumed direction is wrong, and should be reversed. Therefore, a force of 1325 N must be applied to the cone in the opposite direction to flow to hold it in place. No holding force is necessary in the vertical direction due to symmetry and neglecting gravitational effects.

Discussion In reality, the gravitational effects will cause the upper part of flow to slow down and the lower part to speed up after the split. But for short distances, these effects are negligible.

11-123 An ice skater is holding a flexible hose (essentially weightless) which directs a stream of water horizontally at a specified velocity. The velocity and the distance traveled in 5 seconds, and the time it takes to move 5 m and the velocity at that moment are to be determined.

Assumptions **1** Friction between the skates and ice is negligible. **2** The flow of water is steady and one-dimensional. **3** The ice skating arena is level, and the water jet is discharged horizontally. **4** The mass of the hose and the water in it is negligible. **5** The skater is standing still initially at $t = 0$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) The mass flow rate of water through the hose is

$$\dot{m} = \rho A \mathbf{V} = \rho \frac{\pi D^2}{4} \mathbf{V} = (1000\,\text{kg/m}^3)\frac{\pi(0.02\,\text{m})^2}{4}(10\,\text{m/s}) = 3.14\,\text{kg/s}$$

The thrust exerted on the skater by the water stream is simply the momentum flux of the water stream, and it acts in the reverse direction,

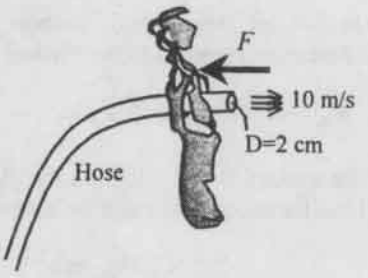
$$F = \text{Thrust} = \dot{m}\mathbf{V} = (3.14\,\text{kg/s})(10\,\text{m/s})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 31.4\ \text{N (constant)}$$

The acceleration of the skater is determined from Newton's 2$^{\text{nd}}$ law of motion $F = ma$ where m is the mass of the skater,

$$a = \frac{F}{m} = \frac{31.4\,\text{N}}{60\,\text{kg}}\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right) = 0.523\,\text{m/s}^2$$

Note that thrust and thus the acceleration of the skater is constant. The velocity of the skater and the distance traveled in 5 s are

$$\mathbf{V}_{\text{skater}} = at = (0.523\,\text{m/s}^2)(5\,\text{s}) = \mathbf{2.62\ m/s}$$
$$x = \tfrac{1}{2}at^2 = \tfrac{1}{2}(0.523\,\text{m/s}^2)(5\,\text{s})^2 = \mathbf{6.54\ m}$$



(*b*) The time it will take to move 5 m and the velocity at that moment are

$$x = \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(5\,\text{m})}{0.523\,\text{m/s}^2}} = \mathbf{4.4\ s}$$
$$\mathbf{V}_{\text{skater}} = at = (0.523\,\text{m/s}^2)(4.4\,\text{s}) = \mathbf{2.3\ m/s}$$

Discussion In reality, the velocity of the skater will be lower because of friction on ice and the resistance of the hose to follow the skater.

11-124 Indiana Jones is to ascend a building by building a platform, and mounting four water nozzles pointing down at each corner. The minimum water jet velocity needed to raise the system, the time it will take to rise to the top of the building and the velocity of the system at that moment, the additional rise when the water is shut off, and the time he has to jump from the platform to the roof are to be determined.

Assumptions **1** The air resistance is negligible. **2** The flow of water is steady and one-dimensional. **3** The platform is still initially at $t = 0$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) The total mass flow rate of water through the 4 hoses and the total weight of the platform are

$$\dot{m} = \rho A \mathbf{V} = 4\rho \frac{\pi D^2}{4} \mathbf{V} = 4(1000\,\text{kg/m}^3)\frac{\pi(0.05\,\text{m})^2}{4}(15\,\text{m/s}) = 118\,\text{kg/s}$$

$$W = mg = (150\,\text{kg})(9.81\,\text{m/s}^2)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 1472\ \text{N}$$

We take the platform as the system. The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. The minimum water jet velocity needed to raise the platform is determined by setting the net force acting on the platform equal to zero,

$$-W = \dot{m}(-\mathbf{V}_{\min}) - 0 \quad \rightarrow \quad W = \dot{m}\mathbf{V}_{\min} = \rho A \mathbf{V}_{\min}\mathbf{V}_{\min} = 4\rho\frac{\pi D^2}{4}\mathbf{V}_{\min}^2$$

Solving for $\mathbf{V}_{\min}$ and substituting,

$$\mathbf{V}_{\min} = \sqrt{\frac{W}{\rho\pi D^2}} = \sqrt{\frac{1472\,\text{N}}{(1000\,\text{kg/m}^3)\pi(0.05\,\text{m})^2}\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right)} = \mathbf{13.7\ m/s}$$

(*b*) We let the vertical reaction force (assumed upwards) acting on the platform be F_{Ry}. Then the momentum equation in the vertical direction becomes

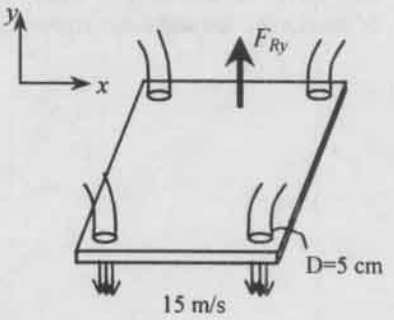
$$F_{Ry} - W = \dot{m}(-\mathbf{V}) - 0 = \dot{m}\mathbf{V} \quad \rightarrow \quad F_{Ry} = W - \dot{m}\mathbf{V} = (1472\,\text{N}) - (118\,\text{kg/s})(15\,\text{m/s})\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right) = -298\,\text{N}$$

The upward thrust acting on the platform is equal and opposite to this reaction force, and thus $F = 298$ N. Then the acceleration and the ascending time to rise 10 m and the velocity at that moment become

$$a = \frac{F}{m} = \frac{298\,\text{N}}{150\,\text{kg}}\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right) = 2.0\,\text{m/s}^2$$

$$x = \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(10\,\text{m})}{2\,\text{m/s}^2}} = \mathbf{3.2\ s}$$

$$\mathbf{V} = at = (2\,\text{m/s}^2)(3.2\,\text{s}) = \mathbf{6.4\ m/s}$$



(*c*) When water is shut off at 10 m height (where the velocity is 6.4 m/s), the platform will decelerate under the influence of gravity, and the time it takes to come to a stop and the additional rise above 10 m become

$$\mathbf{V} = \mathbf{V}_0 - gt = 0 \quad \rightarrow \quad t = \frac{\mathbf{V}_0}{g} = \frac{6.4\,\text{m/s}}{9.81\,\text{m/s}^2} = \mathbf{0.65\ s}$$

$$x = \mathbf{V}_0 t - \tfrac{1}{2}gt^2 = (6.4\,\text{m/s})(0.65\,\text{s}) - \tfrac{1}{2}(9.81\,\text{m/s}^2)(0.65\,\text{s})^2 = \mathbf{2.1\ m}$$

Therefore, Jones has 2×0.65 = 1.3 s to jump off from the platform to the roof since it takes another 0.65 s for the platform to descend to the 10 m level.

11-125E A box-enclosed fan is faced down so the air blast is directed downwards, and it is to be hovered by increasing the blade rpm. The required blade rpm, air exit velocity, the volumetric flow rate, and the minimum mechanical power are to be determined.

Assumptions **1** The flow of air is steady, one-dimensional, and incompressible. **2** The air leaves the blades at a uniform velocity at atmospheric pressure. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air (no conversion to thermal energy through frictional effects). **5** The change in air pressure with elevation is negligible because of the low density of air. **6** There is no acceleration of the fan, and thus the lift generated is equal to the total weight.

Properties The density of air is given to be 0.070 lbm/ft^3.

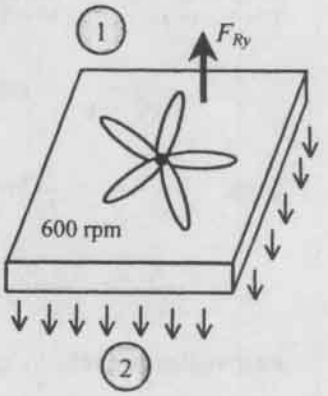
Analysis (*a*) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the z axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\Sigma\vec{F} = \Sigma\dot{m}_e\vec{\mathbf{V}}_e - \Sigma\dot{m}_i\vec{\mathbf{V}}_i$. Noting that the only force acting on the control volume is the total weight W and it acts in the negative z direction, the momentum equation along the z axis gives

$$-W = \dot{m}(-\mathbf{V}_2) - 0 \quad \rightarrow \quad W = \dot{m}\mathbf{V}_2 = (\rho A\mathbf{V}_2)\mathbf{V}_2 = \rho A\mathbf{V}_2^2 \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{\frac{W}{\rho A}}$$

where A is the blade span area,

$$A = \pi D^2/4 = \pi(3\text{ ft})^2/4 = 7.069\text{ ft}^2$$



Then the discharge velocity to produce 5 lbf of upward force becomes

$$\mathbf{V}_2 = \sqrt{\frac{5\text{ lbf}}{(0.070\text{ lbm/ft}^3)(7.069\text{ ft}^2)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)} = \mathbf{18.0\text{ ft/s}}$$

(*b*) The volume flow rate and the mass flow rate of air are determined from their definitions,

$$\dot{V} = A\mathbf{V}_2 = (7.069\text{ ft}^2)(18.0\text{ ft/s}) = \mathbf{127\text{ ft}^3\text{/s}}$$

$$\dot{m} = \rho\dot{V} = (0.070\text{ lbm/ft}^3)(127\text{ ft}^3\text{/s}) = 8.89\text{ lbm/s}$$

(*c*) Noting that $P_1 = P_2 = P_{\text{atm}}$, $\mathbf{V}_1 \cong 0$, the elevation effects are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathbf{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathbf{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{\mathbf{V}_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{fan,u}} = \dot{m}\frac{\mathbf{V}_2^2}{2} = (8.89\text{ lbm/s})\frac{(18.0\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ W}}{0.73756\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{60.6\text{ W}}$$

Therefore, the minimum mechanical power that must be supplied to the air stream is 60.6 W.

Discussion The actual power input to the fan will be considerably larger than the calculated power input because of the fan inefficiency in converting mechanical work to kinetic energy.

11-126 A parachute slows a soldier from his terminal velocity $\mathbf{V}_T$ to his landing velocity of $\mathbf{V}_F$. A relation is to be developed for the soldier's velocity after he opens the parachute at time $t = 0$.

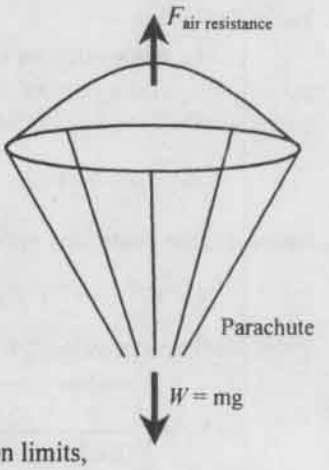
Assumptions **1** The air resistance is proportional to the velocity squared (i.e. $F = -k\mathbf{V}^2$). **2** The variation of the air properties with altitude is negligible. **3** The buoyancy force applied by air to the person (and the parachute) is negligible because of the small volume occupied and the low density of air. **4** The final velocity of the soldier is equal to its terminal velocity with his parachute open.

Analysis The terminal velocity of a free falling object is reached when the air resistance (or air drag) equals the weight of the object, less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_{\text{air resistance}} = W \quad \rightarrow \quad k\mathbf{V}_F^2 = mg \quad \rightarrow \quad k = \frac{mg}{\mathbf{V}_F^2}$$

This is the desired relation for the constant of proportionality k. When the parachute is deployed and the soldier starts to decelerate, the net downward force acting on him is his weight less the air resistance,

$$F_{\text{net}} = W - F_{\text{air resistance}} = mg - k\mathbf{V}^2 = mg - \frac{mg}{\mathbf{V}_F^2}\mathbf{V}^2 = mg\left(1 - \frac{\mathbf{V}^2}{\mathbf{V}_F^2}\right)$$



Substituting it into Newton's 2nd law relation $F_{\text{net}} = ma = m\dfrac{d\mathbf{V}}{dt}$ gives

$$mg\left(1 - \frac{\mathbf{V}^2}{\mathbf{V}_F^2}\right) = m\frac{d\mathbf{V}}{dt}$$

Canceling m and separating variables, and integrating from $t = 0$ when $\mathbf{V} = \mathbf{V}_T$ to $t = t$ when $\mathbf{V} = \mathbf{V}$ gives

$$\frac{d\mathbf{V}}{1 - \mathbf{V}^2/\mathbf{V}_F^2} = g\,dt \quad \rightarrow \quad \int_{\mathbf{V}_T}^{\mathbf{V}} \frac{d\mathbf{V}}{\mathbf{V}_F^2 - \mathbf{V}^2} = \frac{g}{\mathbf{V}_F^2}\int_0^t dt$$

Using $\displaystyle\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\frac{a+x}{a-x}$ from integral tables and applying the integration limits,

$$\frac{1}{2\mathbf{V}_F}\left(\ln\frac{\mathbf{V}_F + \mathbf{V}}{\mathbf{V}_F - \mathbf{V}} - \ln\frac{\mathbf{V}_F + \mathbf{V}_T}{\mathbf{V}_F - \mathbf{V}_T}\right) = \frac{gt}{\mathbf{V}_F^2}$$

Rearranging, the velocity can be expressed explicitly as a function of time as

$$\mathbf{V} = \mathbf{V}_F\,\frac{\mathbf{V}_T + \mathbf{V}_F + (\mathbf{V}_T - \mathbf{V}_F)e^{-2gt/\mathbf{V}_F}}{\mathbf{V}_T + \mathbf{V}_F - (\mathbf{V}_T - \mathbf{V}_F)e^{-2gt/\mathbf{V}_F}}$$

Discussion Note that as $t \rightarrow \infty$, the velocity approaches the landing velocity of $\mathbf{V}_F$, as expected.

11-127 An empty cart is to be driven by a horizontal water jet that enters from a hole at the rear of the cart. A relation is to be developed for cart velocity versus time.

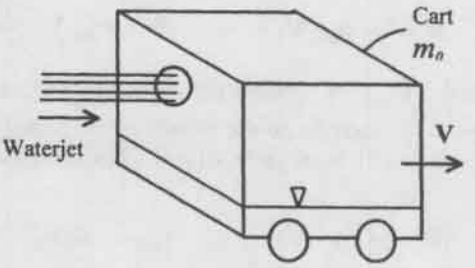
Assumptions **1** The flow of water is steady, one-dimensional, incompressible, and horizontal. **2** All the water which enters the cart is retained. **3** The path of the cart is level and frictionless. **4** The cart is initially empty and stationary, and thus $\mathbf{V} = 0$ at time $t = 0$. **5** Friction between water jet and air is negligible, and the entire momentum of water jet is used to drive the cart with no losses.

Analysis We note that the water jet velocity $\mathbf{V}_J$ is constant, but the car velocity $\mathbf{V}$ is variable. Noting that $\dot{m} = \rho A(\mathbf{V}_J - \mathbf{V})$ where A is the cross-sectional area of the water jet and $\mathbf{V}_J$ - $\mathbf{V}$ is the velocity of the water jet relative to the cart, the mass of water in the cart at any time t is

$$m_w = \int_0^t \dot{m}dt = \int_0^t \rho A(\mathbf{V}_J - \mathbf{V})dt = \rho A\mathbf{V}_J t - \rho A\int_0^t \mathbf{V}dt \quad (1)$$

Also,

$$\frac{dm_w}{dt} = \dot{m} = \rho A(\mathbf{V}_J - \mathbf{V})$$



We take the cart as the system. The net force acting on the cart in this case is equal to the momentum flux of the water jet. Newton's 2nd law $F = ma = d(m\mathbf{V})/dt$ in this case can be expressed as

$$F = \frac{d(m_{\text{total}}\mathbf{V})}{dt} \quad \text{where} \quad F = \dot{m}(\mathbf{V}_J - \mathbf{V}) = \rho A(\mathbf{V}_J - \mathbf{V})^2$$

and

$$\frac{d(m_{\text{total}}\mathbf{V})}{dt} = \frac{d[(m_c + m_w)\mathbf{V}]}{dt} = m_c\frac{d\mathbf{V}}{dt} + \frac{d(m_w\mathbf{V})}{dt} = m_c\frac{d\mathbf{V}}{dt} + m_w\frac{d\mathbf{V}}{dt} + \mathbf{V}\frac{dm_w}{dt}$$
$$= (m_c + m_w)\frac{d\mathbf{V}}{dt} + \rho A(\mathbf{V}_J - \mathbf{V})\mathbf{V}$$

Substituting,

$$\rho A(\mathbf{V}_J - \mathbf{V})^2 = (m_c + m_w)\frac{d\mathbf{V}}{dt} + \rho A(\mathbf{V}_J - \mathbf{V})\mathbf{V} \quad \rightarrow \quad \rho A(\mathbf{V}_J - \mathbf{V})(\mathbf{V}_J - 2\mathbf{V}) = (m_c + m_w)\frac{d\mathbf{V}}{dt}$$

Noting that m_w is a function of t (as given by Eq. 1) and separating variables,

$$\frac{d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})(\mathbf{V}_J - 2\mathbf{V})} = \frac{dt}{m_c + m_w} \quad \rightarrow \quad \frac{d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})(\mathbf{V}_J - 2\mathbf{V})} = \frac{dt}{m_c + \rho A\mathbf{V}_J t - \rho A\int_0^t \mathbf{V}dt}$$

Integrating from $t = 0$ when $\mathbf{V} = 0$ to $t = t$ when $\mathbf{V} = \mathbf{V}$ gives the desired integral,

$$\int_0^{\mathbf{V}} \frac{d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})(\mathbf{V}_J - 2\mathbf{V})} = \int_o^t \frac{dt}{m_c + \rho A\mathbf{V}_J t - \rho A\int_0^t \mathbf{V}dt}$$

Discussion Note that the time integral involves the integral of velocity, which complicates the solution.

1-128 A plate is maintained in a horizontal position by frictionless vertical guide rails. The underside of ıe plate is subjected to a water jet. The minimum mass flow rate $\dot{m}_{min}$ to just levitate the plate is to be ɛtermined, and a relation is to be obtained for the steady state upward velocity. Also, the integral that ·lates velocity to time when the water is first turned on is to be obtained.

ssumptions **1** The flow of water is steady and one-dimensional. **2** The water jet splatters in the plane of he late. **3** The vertical guide rails are frictionless. **4** Times are short, so the velocity of the rising jet can be ɔnsidered to remain constant with height. **5** At time t = 0, the plate is at rest.

nalysis (*a*) We take the plate as the system. The momentum equation for steady one-dimensional flow is $\vec{F} = \Sigma \dot{m}_e \vec{\mathbf{V}}_e - \Sigma \dot{m}_i \vec{\mathbf{V}}_i$. Noting that $\dot{m} = \rho A \mathbf{V}_J$ where A is the cross-sectional area of the water jet and $W = m_p g$, the minimum mass flow rate of water needed to raise the plate is determined by setting the net force ɛting on the plate equal to zero,

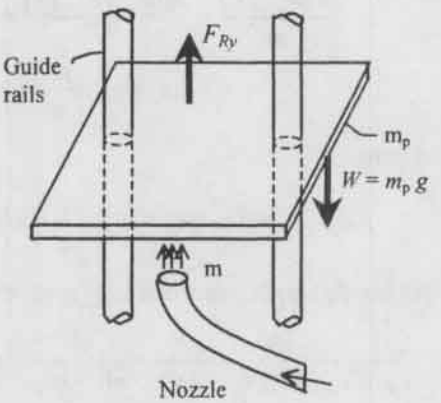
$$-W = 0 - \dot{m}_{min}\mathbf{V}_J \quad \rightarrow \quad W = \dot{m}_{min}\mathbf{V}_J \rightarrow \quad m_p g = \dot{m}_{min}(\dot{m}_{min}/A\mathbf{V}_J) \rightarrow \quad \dot{m}_{min} = \sqrt{\rho A m_p g}$$

ɔr $\dot{m} > \dot{m}_{min}$, a relation for the steady state upward velocity **V** is obtained setting the upward impulse ɔplied by water jet to the weight of the plate (during steady motion, the plate velocity **V** is constant, and ıe velocity of water jet relative to plate is $\mathbf{V}_J - \mathbf{V}$),

$$W = \dot{m}(\mathbf{V}_J - \mathbf{V}) \quad \rightarrow \quad m_p g = \rho A(\mathbf{V}_J - \mathbf{V})^2 \quad \rightarrow \quad \mathbf{V}_J - \mathbf{V} = \sqrt{\frac{m_p g}{\rho A}} \quad \rightarrow \quad \mathbf{V} = \frac{\dot{m}}{\rho A} - \sqrt{\frac{m_p g}{\rho A}}$$

ɔ) At time t = 0 the plate is at rest (**V** = 0), and it is subjected to water jet with $\dot{m} > \dot{m}_{min}$ and thus the net ɔrce acting on it is greater than the weight of the plate, and the difference between the jet impulse and the ɛight will accelerate the plate upwards. Therefore, Newton's 2nd law $F = ma = mdV/dt$ in this case can be ‹pressed as

$$\dot{m}(\mathbf{V}_J - \mathbf{V}) - W = m_p a \quad \rightarrow \quad \rho A(\mathbf{V}_J - \mathbf{V})^2 - m_p g = m_p \frac{d\mathbf{V}}{dt}$$



ɛparating the variables and integrating from t = 0 when **V** = 0 to t t when **V** = **V** gives the desired integral,

$$\int_0^{\mathbf{V}} \frac{m_p d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})^2 - m_p g} = \int_{t=0}^{t} dt \quad \rightarrow \quad t = \int_0^{\mathbf{V}} \frac{m_p d\mathbf{V}}{\rho A(\mathbf{V}_J - \mathbf{V})^2 - m_p g}$$

iscussion This integral can be performed with the help of ıtegral tables. But the relation obtained will be implicit in **V**.

Chapter 12
FLOW IN PIPES

Laminar and Turbulent Flow

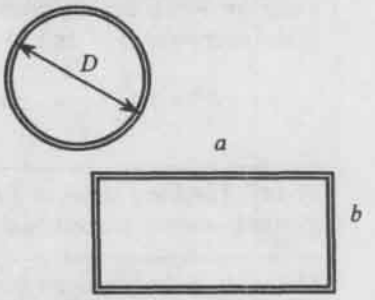
12-1C Liquids are usually transported in circular pipes because pipes with a circular cross-section can withstand large pressure differences between the inside and the outside without undergoing any distortion.

12-2C Reynolds number is the ratio of the inertial forces to viscous forces, and it serves as a criteria for determining the flow regime. At *large* Reynolds numbers, for example, the flow is turbulent since the inertia forces are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. It is defined as follows:

(*a*) For flow in a circular tube of inner diameter *D*: $\mathrm{Re} = \dfrac{\mathbf{V}_m D}{\nu}$

(*b*) For flow in a rectangular duct of cross-section $a \times b$: $\mathrm{Re} = \dfrac{\mathbf{V}_m D_h}{\nu}$

where $D_h = \dfrac{4A_c}{p} = \dfrac{4ab}{2(a+b)} = \dfrac{2ab}{(a+b)}$ is the hydraulic diameter.



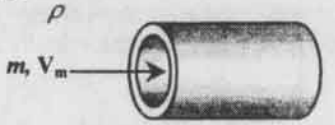
12-3C The kinematic viscosities at 20°C are 1.5×10^{-5} m^2/s for air and 1×10^{-6} m^2/s for water (due to large density). Noting that Re = V*D*/ν and the velocity and characteristics lengths are the same for both cases, the Reynolds number will be much larger for walking in water.

12-4C Reynolds number for flow in a circular tube of diameter D is expressed as

$$\mathrm{Re} = \frac{\mathbf{V}_m D}{\upsilon} \quad \text{where} \quad \mathbf{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{4\dot{m}}{\rho\pi D^2} \quad \text{and} \quad \upsilon = \frac{\mu}{\rho}$$

Substituting,

$$\mathrm{Re} = \frac{\mathbf{V}_m D}{\upsilon} = \frac{4\dot{m}D}{\rho\pi D^2(\mu/\rho)} = \frac{4\dot{m}}{\pi D\mu}$$



12-5C Engine oil requires a larger pump because of its much larger density.

12-6C The generally accepted value of the Reynolds number above which the flow in a smooth pipe is turbulent is 4000.

12-7C In the fully developed region of flow in a circular tube, the velocity profile will not change in the flow direction.

12-8C The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the *hydrodynamic entry region*, and the length of this region is called *hydrodynamic entry length*. The entry length is much longer in laminar flow than it is in turbulent flow. But at very low Reynolds numbers, L_h is very small ($L_h = 1.2D$ at Re = 20).

12-9C The friction factor is highest at the tube inlet where the thickness of the boundary layer is zero, and decreases gradually to the fully developed value. The same is true for turbulent flow.

12-10C In turbulent flow, the tubes with rough surfaces have much higher friction factors than the tubes with smooth surfaces. In the case of laminar flow, the effect of surface roughness on the friction factor is negligible.

12-11C The friction factor f remains constant along the flow direction in the fully developed region in both laminar and turbulent flow.

Fully Developed Flow In Pipes

12-12C The fluid viscosity is responsible for the development of the velocity boundary layer. For the idealized inviscid fluids (fluids with zero viscosity), there will be no velocity boundary layer.

12-13C The friction factor for flow in a tube is proportional to the pressure drop. Since the pressure drop along the flow is directly related to the power requirements of the pump to maintain flow, the friction factor is also proportional to the power requirements. The applicable relations are

$$\Delta P = f\frac{L}{D}\frac{\rho V^2}{2} \quad \text{and} \quad \dot{W}_{\text{pump}} = \frac{\dot{m}\Delta P}{\rho}$$

12-14C The shear stress at the center of a circular pipe during fully developed laminar flow is zero since the shear stress is proportional to the velocity gradient, which is zero at the pipe center.

12-15C In fully developed flow in a circular pipe with negligible entrance effects, if the length of the pipe is doubled, the pressure drop will also *double* (the pressure drop is proportional to length).

12-16C Yes, the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross-sectional area, and dividing the result by 2 since $\dot{V} = \mathbf{V}_{\text{ave}} A_c = (\mathbf{V}_{\text{max}}/2)A_c$.

12-17C No, the average velocity in a circular pipe in fully developed laminar flow **cannot** be determined by simply measuring the velocity at $R/2$ (midway between the wall surface and the centerline). The mean velocity is $\mathbf{V}_{\text{max}}/2$, but the velocity at $R/2$ is

$$\mathbf{V}(R/2) = \mathbf{V}_{\text{max}}\left(1-\frac{r^2}{R^2}\right)_{r=R/2} = \frac{3\mathbf{V}_{\text{max}}}{4}$$

12-18C In fully developed laminar flow in a circular pipe, the pressure drop is given by

$$\Delta P = \frac{8\mu L \mathbf{V}_m}{R^2} = \frac{32\mu L \mathbf{V}_m}{D^2}$$

The mean velocity can be expressed in terms of the flow rate as $\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2 / 4}$. Substituting,

$$\Delta P = \frac{8\mu L \mathbf{V}_m}{R^2} = \frac{32\mu L \mathbf{V}_m}{D^2} = \frac{32\mu L}{D^2}\frac{\dot{V}}{\pi D^2 / 4} = \frac{128\mu L \dot{V}}{\pi D^4}$$

Therefore, at constant flow rate and pipe length, the pressure drop is inversely proportional to the 4th power of diameter, and thus reducing the pipe diameter by half will increase the pressure drop **by a factor of 16**.

12-19C In fully developed laminar flow in a circular pipe, the pressure drop is given by

$$\Delta P = \frac{8\mu L \mathbf{V}_m}{R^2} = \frac{32\mu L \mathbf{V}_m}{D^2}$$

When the flow rate and thus mean velocity are held constant, the pressure drop becomes proportional to viscosity. Therefore, the pressure drop will be **reduced by half** when the viscosity of the fluid is reduced by half.

12-20C The head loss is related to pressure drop by $h_L = \Delta P / \rho g$. For a given fluid, the head loss can be converted to pressure drop by multiplying the head loss by the acceleration of gravity and the density of the fluid.

12-21C In turbulent flow, it is the *turbulent eddies* due to enhanced mixing that cause the friction factor to be larger.

12-22C At very large Reynolds numbers, the flow is fully rough and the friction factor is independent of the Reynolds number. This is because the thickness of laminar sublayer decreases with increasing Reynolds number, and it be comes so thin that the surface roughness protrudes into the flow. The viscous effects in this case are produced in the main flow primarily by the protruding roughness elements, and the contribution of the laminar sublayer becomes negligible.

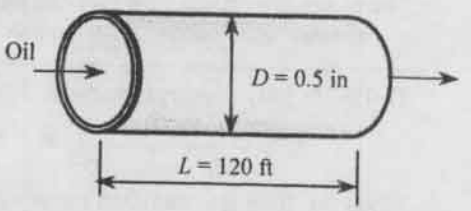
12-23E The pressure readings across a pipe are given. The flow rates are to be determined for 3 different orientations of horizontal, uphill, and downhill flow. √

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is laminar (to be verified). **4** The pipe involves no components such as bends, valves, and connectors. **5** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of oil are given to be $\rho = 56.8$ lbm/ft^3 and $\mu = 0.0278$ lbm/ft·s, respectively.

Analysis The pressure drop across the pipe and the cross-sectional area of the pipe are

$$\Delta P = P_1 - P_2 = 120 - 14 = 106 \text{ psi}$$
$$A_c = \pi D^2/4 = \pi(0.5/12 \text{ ft})^2/4 = 0.001364 \text{ ft}^2$$



(*a*) The flow rate for all three cases can be determined from

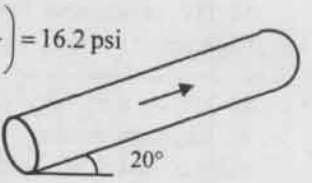
$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ = 0 and thus sin θ = 0. Therefore,

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} = \frac{(106 \text{ psi})\pi(0.5/12 \text{ ft})^4}{128(0.0278 \text{ lbm/ft}\cdot\text{s})(120 \text{ ft})}\left(\frac{144 \text{ lbf/ft}^2}{1 \text{ psi}}\right)\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right) = \mathbf{0.0109 \text{ ft}^3/\text{s}}$$

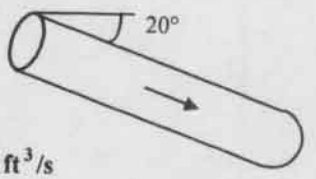
(*b*) For uphill flow with an inclination of 20°, we have θ = +20°, and

$$\rho g L \sin\theta = (56.8 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(120 \text{ ft})\sin 20°\left(\frac{1 \text{ psi}}{144 \text{ lbf/ft}^2}\right)\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = 16.2 \text{ psi}$$



$$\dot{V}_{\text{uphill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L} =$$
$$= \frac{(106 - 16.2 \text{ psi})\pi(0.5/12 \text{ ft})^4}{128(0.0278 \text{ lbm/ft}\cdot\text{s})(120 \text{ ft})}\left(\frac{144 \text{ lbf/ft}^2}{1 \text{ psi}}\right)\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right) = \mathbf{0.00923 \text{ ft}^3/\text{s}}$$

(*c*) For downhill flow with an inclination of 20°, we have θ = -20°, and



$$\dot{V}_{\text{downhill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$
$$= \frac{[106 - (-16.2) \text{ psi}]\pi(0.5/12 \text{ ft})^4}{128(0.0278 \text{ lbm/ft}\cdot\text{s})(120 \text{ ft})}\left(\frac{144 \text{ lbf/ft}^2}{1 \text{ psi}}\right)\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right) = \mathbf{0.0126 \text{ ft}^3/\text{s}}$$

The flow rate is the highest for downhill flow case, as expected. The average fluid velocity and the Reynolds number in this case are

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{0.0126 \text{ ft}^3/\text{s}}{0.001364 \text{ ft}^2} = 9.24 \text{ ft/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(56.8 \text{ lbm/ft}^3)(9.24 \text{ ft/s})(0.5/12 \text{ ft})}{0.0278 \text{ lbm/ft}\cdot\text{s}} = 787$$

which is less than 2300. Therefore, the flow is **laminar** for all three cases, and the analysis above is valid.

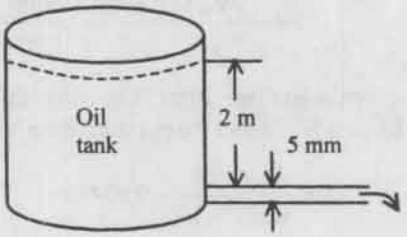
Discussion Note that the flow is driven by the combined effect of pressure difference and gravity. As can be seen from the calculated rates above, gravity opposes uphill flow, but helps downhill flow. Gravity has no effect on the flow rate in the horizontal case. Downhill flow can occur even in the absence of an applied pressure difference.

12-24 Oil is being discharged by a horizontal pipe from a storage tank open to the atmosphere. The flow rate of oil through the pipe is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The entrance and exit loses are negligible. **4** The flow is laminar (to be verified). **5** The pipe involves no components such as bends, valves, and connectors. **6** The piping section involves no work devices such as pumps and turbines.

Properties The density and kinematic viscosity of oil are given to be $\rho = 850$ kg/m^3 and $\nu = 0.00062$ m^2/s, respectively. The dynamic viscosity is calculated to be

$$\mu = \rho\nu = (850\,\text{kg/m}^3)(0.00062\,\text{m}^2/s) = 0.527\,\text{kg/m}\cdot\text{s}$$



Analysis The pressure at the entrance of the pipe is

$$P_{1,\text{gage}} = \rho g h$$

$$= (850\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(2\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 16.7\,\text{kN/m}^2$$

Then the pressure drop across the pipe becomes

$$\Delta P = P_1 - P_2 = P_1 - P_{\text{atm}} = P_{1,\text{gage}} = 16.7\,\text{kN/m}^2 = 16.7\,\text{kPa}$$

The flow rate through a horizontal pipe in laminar flow is determined from

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} = \frac{(16.7\,\text{kN/m}^2)\pi(0.005\,\text{m})^4}{128(0.527\,\text{kg/m}\cdot\text{s})(40\,\text{m})}\left(\frac{1000\,\text{kg}\cdot\text{m/s}^2}{1\,\text{kN}}\right) = \mathbf{1.22\times10^{-8}\ m^3/s}$$

The flow rate is the highest for downhill flow case, as expected. The average fluid velocity and the Reynolds number in this case are

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{1.22\times10^{-8}\ \text{m}^3/\text{s}}{\pi(0.005\,\text{m})^2/4} = 6.21\times10^{-4}\ \text{m/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(850\,\text{kg/m}^3)(6.21\times10^{-4}\ \text{m/s})(0.005\,\text{m})}{0.527\,\text{kg/m}\cdot\text{s}} = 0.05$$

which is less than 2300. Therefore, the flow is *laminar* and the analysis above is valid.

Discussion The flow rate will be even less when the entrance losses are considered, unless the entrance is well-rounded.

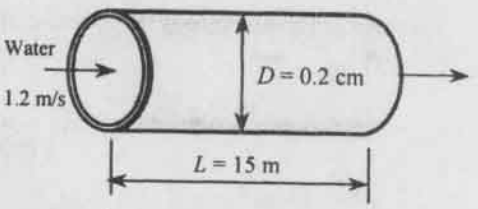
12-25 The average flow velocity in a pipe is given. The pressure drop, the head loss, and the pumping power are to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be $\rho = 999.7\ \text{kg/m}^3$ and $\mu = 1.307\times10^{-3}$ kg/m·s, respectively.

Analysis (*a*) First we need to determine the flow regime. The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V_m D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})(2\times10^{-3}\ \text{m})}{1.307\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1836$$



which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the pressure drop become

$$f = \frac{64}{\text{Re}} = \frac{64}{1836} = 0.0349$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_m^2}{2} = 0.0349\frac{15\ \text{m}}{0.002\ \text{m}}\frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = \mathbf{188\ kPa}$$

(*b*) The head loss in the pipe is determined from

$$h_L = \frac{\Delta P}{\rho g} = f\frac{L}{D}\frac{V_m^2}{2g} = 0.0349\frac{15\ \text{m}}{0.002\ \text{m}}\frac{(1.2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \mathbf{19.2\ m}$$

(*c*) The volume flow rate and the pumping power requirements are

$$\dot{V} = V_m A_c = V_m(\pi D^2/4) = (1.2\ \text{m/s})[\pi(0.002\ \text{m})^2/4] = 3.77\times10^{-6}\ \text{m}^3/s$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (3.77\times10^{-6}\ \text{m}^3/s)(188\ \text{kPa})\left(\frac{1000\ \text{W}}{1\ \text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.71\ W}$$

Therefore, power input in the amount of 0.71 W is needed to overcome the frictional losses in the flow due to viscosity.

12-26 The flow rate through a specified water pipe is given. The pressure drop, the head loss, and the pumping power requirements are to be determined. √

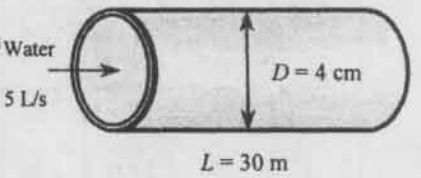
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be $\rho = 999.1$ kg/m^3 and $\mu = 1.138\times10^{-3}$ kg/m·s, respectively. The roughness of stainless steel is 0.002 mm (Fig. A-27).

Analysis First we calculate the mean velocity and the Reynolds number to determine the flow regime:

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.005\,\text{m}^3/s}{\pi(0.04\,\text{m})^2/4} = 3.98\,\text{m}/s$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(999.1\,\text{kg/m}^3)(3.98\,\text{m/s})(0.04\,\text{m})}{1.138\times10^{-3}\,\text{kg/m}\cdot\text{s}} = 1.40\times10^5$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{2\times10^{-6}\,\text{m}}{0.04\,\text{m}} = 5\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{5\times10^{-5}}{3.7} + \frac{2.51}{1.40\times10^5\sqrt{f}}\right)$$

It gives $f = 0.0171$. Then the pressure drop, head loss, and the required power input become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0171\frac{30\,\text{m}}{0.04\,\text{m}}\frac{(999.1\,\text{kg/m}^3)(3.98\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right) = 101.5\ \text{kPa}$$

$$h_L = \frac{\Delta P}{\rho g} = f\frac{L}{D}\frac{\mathbf{V}_m^2}{2g} = 0.0171\frac{30\,\text{m}}{0.04\,\text{m}}\frac{(3.98\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \mathbf{10.35\,m}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.005\,\text{m}^3/s)(101.5\,\text{kPa})\left(\frac{1\,\text{kW}}{1\,\text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.508\,kW}$$

Therefore, useful power input in the amount of 0.508 kW is needed to overcome the frictional losses in the pipe.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give $f = 0.0169$, which is sufficiently close to 0.0171. Also, the friction factor corresponding to $\varepsilon = 0$ in this case is 0.0168, which indicates that stainless steel pipes can be assumed to be smooth with an error of about 2%. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

12-27E The flow rate and the head loss in an air duct is given. The minimum diameter of the duct is to be determined. √

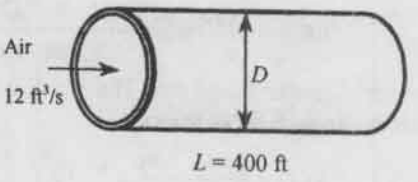
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The duct involves no components such as bends, valves, and connectors. **4** Air is an ideal gas. **5** The duct is smooth since it is made of plastic, $\varepsilon \approx 0$. **6** The flow is turbulent (to be verified).

Properties The density, dynamic viscosity, and kinematic viscosity of air at 100°F are $\rho = 0.071$ lbm/ft^3, $\mu = 1.285\times10^{-5}$ lbm/ft·s, and $\nu = 0.180\times10^{-3}$ ft^2/s (Table A-18E).

Analysis The average velocity, Reynolds number, friction factor, and the head loss relations can be expressed as (D is in ft, $\mathbf{V}$ is in ft/s, Re and f are dimensionless)

$$\mathbf{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{12\ \text{ft}^3/s}{\pi D^2/4}$$

$$\text{Re} = \frac{\mathbf{V}_m D}{\nu} = \frac{\mathbf{V}_m D}{0.180\times10^{-3}\ \text{ft}^2/s}$$



$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{\mathbf{V}_m^2}{2g} \quad \rightarrow \quad 50 = f\frac{L}{D}\frac{\mathbf{V}^2}{2g} = f\frac{400\ \text{ft}}{D}\frac{\mathbf{V}^2}{2(32.2\ \text{ft/s}^2)}$$

This is a set of 4 equations in 4 unknowns, and solving them with an equation solver gives

$$D = \mathbf{0.88\ ft}, \quad f = 0.0181, \quad \mathbf{V} = 19.8\ \text{ft/s}, \quad \text{and} \quad \text{Re} = 96{,}500$$

Therefore, the diameter of the duct should be more than 0.88 ft if the head loss is not to exceed 50 ft. Note that Re > 4000, and thus the turbulent flow assumption is verified.

The diameter can also be determined directly from the third Swamee-Jain formula to be

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

$$= 0.66\left[0 + (0.180\times10^{-3}\ \text{ft}^2/s)(12\ \text{ft}^3/s)^{9.4}\left(\frac{400\ \text{ft}}{(32.2\ \text{ft/s}^2)(50\ \text{ft})}\right)^{5.2}\right]^{0.04}$$

$$= 0.89\ \text{ft}$$

Discussion Note that the difference between the two results is less than 2%. Therefore, the simple Swamee-Jain relation can be used with confidence.

12-28 In fully developed laminar flow in a circular pipe, the velocity at $r = R/2$ is measured. The velocity at the center of the pipe ($r = 0$) is to be determined. √

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by

$$\mathbf{V}(r) = \mathbf{V}_{\max}\left(1-\frac{r^2}{R^2}\right)$$

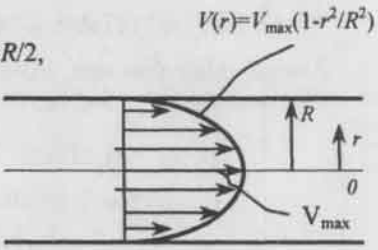
where $\mathbf{V}_{\max}$ is the maximum velocity which occurs at pipe center, $r = 0$. At $r = R/2$,

$$\mathbf{V}(R/2) = \mathbf{V}_{\max}\left(1-\frac{(R/2)^2}{R^2}\right) = \mathbf{V}_{\max}\left(1-\frac{1}{4}\right) = \frac{3\mathbf{V}_{\max}}{4}$$

Solving for $\mathbf{V}_{\max}$ and substituting,

$$\mathbf{V}_{\max} = \frac{4\mathbf{V}(R/2)}{3} = \frac{4(6\text{ m/s})}{3} = \mathbf{8\text{ m/s}}$$

which is the velocity at the pipe center.



12-29 The velocity profile in fully developed laminar flow in a circular pipe is given. The mean and maximum velocities are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by

$$\mathbf{V}(r) = \mathbf{V}_{\max}\left(1-\frac{r^2}{R^2}\right)$$

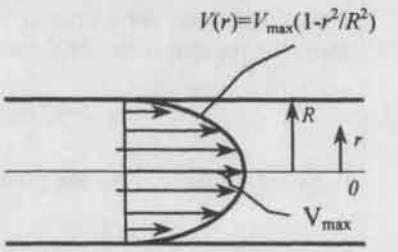
The velocity profile in this case is given by

$$\mathbf{V}(r) = 4(1-r^2/R^2)$$

Comparing the two relations above gives the maximum velocity to be $\mathbf{V}_{\max} = 4$ m/s. Then the mean velocity and volume flow rate become

$$\mathbf{V}_m = \frac{\mathbf{V}_{\max}}{2} = \frac{4\text{ m/s}}{2} = \mathbf{2\text{ m/s}}$$

$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m(\pi R^2) = (2\text{ m/s})[\pi(0.02\text{ m})^2] = \mathbf{0.00251\text{ m}^3/\text{s}}$$



12-30 The velocity profile in fully developed laminar flow in a circular pipe is given. The mean and maximum velocities are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by

$$\mathbf{V}(r) = \mathbf{V}_{\max}\left(1-\frac{r^2}{R^2}\right)$$

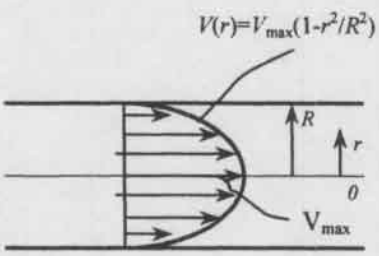
The velocity profile in this case is given by

$$\mathbf{V}(r) = 4(1-r^2/R^2)$$

Comparing the two relations above gives the maximum velocity to be $\mathbf{V}_{\max} = 4$ m/s. Then the mean velocity and volume flow rate become

$$\mathbf{V}_m = \frac{\mathbf{V}_{\max}}{2} = \frac{4\text{ m/s}}{2} = \mathbf{2\text{ m/s}}$$

$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m(\pi R^2) = (2\text{ m/s})[\pi(0.05\text{ m})^2] = \mathbf{0.0157\text{ m}^3/\text{s}}$$

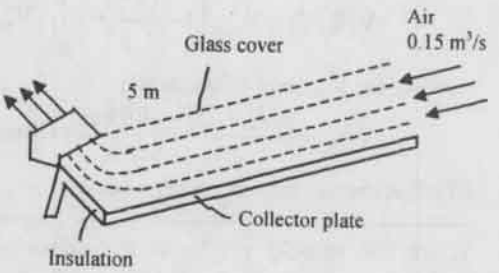


12-31 Air enters the constant spacing between the glass cover and the plate of a solar collector. The pressure drop of air in the collector is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The roughness effects are negligible, and thus the inner surfaces are considered to be smooth, $\varepsilon \approx 0$. **4** Air is an ideal gas. **5** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and 47°C = 320 K are $\rho = 1.11$ kg/m^3, $\mu = 1.94\times10^{-5}$ kg/m·s, and $\nu = 1.77\times10^{-5}$ m^2/s (Table A-18).

Analysis Mass flow rate, cross sectional area, hydraulic diameter, mean velocity of air and the Reynolds number are



$$\dot{m} = \rho \dot{V} = (1.11\,\text{kg/m}^3)(0.15\,\text{m}^3/\text{s}) = 0.1665\,\text{kg/s}$$

$$A_c = a \times b = (1\,\text{m})(0.03\,\text{m}) = 0.03\,\text{m}^2$$

$$D_h = \frac{4A_c}{p} = \frac{4(0.03\,\text{m}^2)}{2(1+0.03)\,\text{m}} = 0.05825\,\text{m}$$

$$\mathbf{V} = \frac{\dot{V}}{A_c} = \frac{0.15\,\text{m}^3/s}{0.03\,\text{m}^2} = 5\,\text{m}/s$$

$$\text{Re} = \frac{\mathbf{V}D_h}{\upsilon} = \frac{(5\,\text{m/s})(0.05825\,\text{m})}{1.77\times10^{-5}\,\text{m}^2/\text{s}} = 1.65\times10^4$$

which is greater than 4000. Therefore, the flow is turbulent. The friction factor corresponding to this Reynolds number for a smooth flow section ($\varepsilon/D = 0$) can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(0 + \frac{2.51}{16{,}500\sqrt{f}}\right)$$

It gives $f = 0.0272$. Then the pressure drop becomes

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0272\frac{5\,\text{m}}{0.05825\,\text{m}}\frac{(1.11\,\text{kg/m}^3)(5\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{Pa}}{1\,\text{N/m}^2}\right) = \mathbf{29\ Pa}$$

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give $f = 0.0270$, which is sufficiently close to 0.0272.

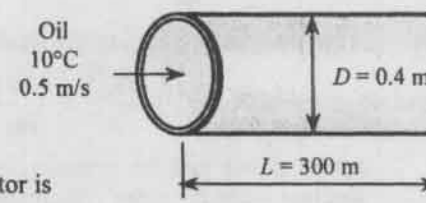
12-32 Oil flows through a pipeline that passes through icy waters of a lake. The pumping power needed to overcome pressure losses is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The flow section considered is away from the entrance, and thus the flow is fully developed. **3** The roughness effects are negligible, and thus the inner surfaces are considered to be smooth, $\varepsilon \approx 0$.

Properties The properties of oil are given to be $\rho = 894\ \text{kg/m}^3$ and $\mu = 2.33\ \text{kg/m·s}$.

Analysis The volume flow rate and the Reynolds number in this case are

(Icy lake, 0°C)



$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m \frac{\pi D^2}{4} = (0.5\ \text{m/s})\frac{\pi(0.4\ \text{m})^2}{4} = 0.0628\ \text{m}^3/\text{s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(894\ \text{kg/m}^3)(0.5\ \text{m/s})(0.4\ \text{m})}{2.33\ \text{kg/m·s}} = 76.7$$

which is less than 2300. Therefore, the flow is laminar, and the friction factor is

$$f = \frac{64}{\text{Re}} = \frac{64}{76.7} = 0.834$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^{\,2}}{2} = 0.834\frac{300\ \text{m}}{0.4\ \text{m}}\frac{(894\ \text{kg/m}^3)(0.5\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = 69.9\ \text{kPa}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.0628\ \text{m}^3/\text{s})(69.9\ \text{kPa})\left(\frac{1\ \text{kW}}{1\ \text{kPa·m}^3/\text{s}}\right) = \mathbf{4.38\ kW}$$

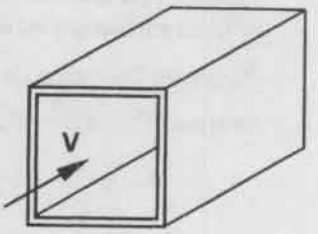
Discussion The power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be much more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

12-33 Laminar flow through a square channel is considered. The change in the pressure drop is to be determined when the mean velocity is doubled. √

Assumptions **1** The flow remains laminar at all times. **2** The entrance effects are negligible, and thus the flow is fully developed.

Analysis The friction factor for fully developed laminar flow in a square channel is (Table 12-1)

$$f = \frac{56.92}{\mathrm{Re}} \quad \text{where} \quad \mathrm{Re} = \frac{\rho V_m D}{\mu}$$



Then the pressure drop of the fluid for laminar flow can be expressed as

$$\Delta P_1 = f \frac{L}{D} \frac{\rho V_m^2}{2} = \frac{56.92}{\mathrm{Re}} \frac{L}{D} \frac{\rho V_m^2}{2} = \frac{56.92\mu}{\rho V_m D} \frac{L}{D} \frac{\rho V_m^2}{2} = 28.46 V_m \frac{\mu L}{D^2}$$

which shows that the pressure drop is proportional to the mean velocity. Therefore, the pressure drop **doubles** when the mean velocity is doubled. This can also be shown as

$$\Delta P_2 = 28.46 V_{m2} \frac{\mu L}{D^2} = 28.46(2V_m) \frac{\mu L}{D^2} = 2\left(28.46 V_{m2} \frac{\mu L}{D^2}\right) = 2\Delta P_1$$

Discussion The conclusion above is also valid for laminar flow in channels of different cross-sections.

12-34 Turbulent flow through a smooth pipe is considered. The change in the pressure drop is to be determined when the mean velocity is doubled.

Assumptions **1** The flow remains turbulent at all times. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The inner surface of the pipe is smooth.

Analysis The friction factor for the turbulent flow in smooth pipes is given as

$$f = 0.184\,\mathrm{Re}^{-0.2} \quad \text{where} \quad \mathrm{Re} = \frac{\rho V_m D}{\mu}$$

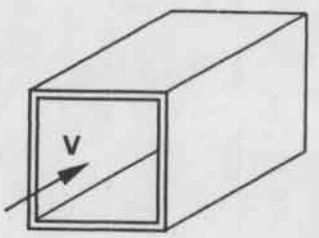
Then the pressure drop of the fluid for turbulent flow can be expressed as

$$\Delta P_1 = f \frac{L}{D} \frac{\rho V_m^2}{2} = 0.184\,\mathrm{Re}^{-0.2} \frac{L}{D} \frac{\rho V_m^2}{2} = 0.184\left(\frac{\rho V_m D}{\mu}\right)^{-0.2} \frac{L}{D} \frac{\rho V_m^2}{2} = 0.184\left(\frac{\rho D}{\mu}\right)^{-0.2} \frac{L}{D} \frac{\rho V_m^{1.8}}{2}$$

which shows that the pressure drop is proportional to the 1.8^{th} power of the mean velocity. Therefore, the pressure drop increases by a factor of $2^{1.8} = 3.48$ when the mean velocity is doubled. This can also be shown as

$$\Delta P_2 = 0.184\left(\frac{\rho D}{\mu}\right)^{-0.2} \frac{L}{D} \frac{\rho V_{m2}^{1.8}}{2} = 0.184\left(\frac{\rho D}{\mu}\right)^{-0.2} \frac{L}{D} \frac{\rho (2V_m)^{1.8}}{2}$$

$$= 2^{1.8}\left[0.184\left(\frac{\rho D}{\mu}\right)^{-0.2} \frac{L}{D} \frac{\rho V_{m2}^{1.8}}{2}\right] = 2^{1.8}\Delta P_1 = 3.48\Delta P_1$$



For fully rough flow in a rough pipe, the friction factor is independent of the Reynolds number and thus the flow velocity. Therefore, the pressure drop increases by a **factor of 4** in this case since

$$\Delta P_1 = f \frac{L}{D} \frac{\rho V_m^2}{2}$$

and thus the pressure drop is proportional to the **square** of the mean velocity when f, L, and D are constant.

Discussion Most flows in practice are in the fully rough regime, and thus the pressure drop is generally assumed to be proportional to the square of the mean velocity for all kinds of turbulent flow.

12-35 Air enters a rectangular duct. The fan power needed to overcome the pressure losses is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors. **5** The flow section involves no work devices such as fans or turbines

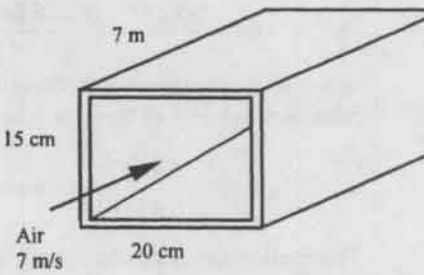
Properties The properties of air at 1 atm and 37°C = 310 K are $\rho = 1.143$ kg/m^3, $\mu = 1.90\times10^{-5}$ kg/m·s, and $\nu = 1.67\times10^{-5}$ m^2/s (Table A-18). The roughness of commercial steel surfaces is $\varepsilon = 0.000045$ m (Fig. A-27)

Analysis The hydraulic diameter, the Reynolds number, and the volume flow rate in this case are

$$D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = \frac{4(0.15\,\text{m})(0.20\,\text{m})}{2(0.15+0.20)\,\text{m}} = 0.1714\,\text{m}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D_h}{\mu} = \frac{(1.143\,\text{kg/m}^3)(7\,\text{m/s})(0.1714\,\text{m})}{1.90\times10^{-5}\,\text{kg/m}\cdot\text{s}} = 72{,}200$$

$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m (a\times b) = (7\,\text{m/s})(0.15\times0.20\,\text{m}^2) = 0.21\,\text{m}^3/\text{s}$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D_h = \frac{4.5\times10^{-5}\,\text{m}}{0.1714\,\text{m}} = 2.625\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.625\times10^{-4}}{3.7} + \frac{2.51}{72{,}200\sqrt{f}}\right)$$

It gives $f = 0.0204$. Then the pressure drop in the duct and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0204\frac{7\,\text{m}}{0.1714\,\text{m}}\frac{(1.143\,\text{kg/m}^3)(7\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{Pa}}{1\,\text{N/m}^2}\right) = 23.3\,\text{Pa}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.21\,\text{m}^3/\text{s})(23.3\,\text{Pa})\left(\frac{1\,\text{W}}{1\,\text{Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{4.9\,W}$$

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give $f = 0.0201$, which is sufficiently close to 0.0204. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be much more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency.

12-36E Water passes through copper tubes at a specified rate. The pumping power required per ft length to maintain flow is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

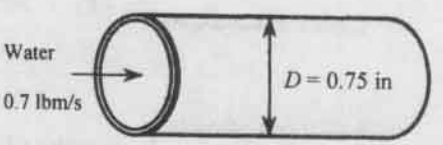
Properties The density and dynamic viscosity of water at 60°F are $\rho = 62.36$ lbm/ft^3 and $\mu = 2.713$ lbm/ft·h $= 7.536\times10^{-4}$ lbm/ft·s (Table A-15E). The roughness of copper tubing is 5×10^{-6} ft (Fig. A-27).

Analysis First we calculate the mean velocity and the Reynolds number to determine the flow regime:

$$\mathbf{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{0.7 \text{ lbm}/s}{(62.36 \text{ lbm/ft}^3)[\pi(0.75/12 \text{ ft})^2/4]} = 3.66 \text{ ft/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(62.36 \text{ lbm/ft}^3)(3.66 \text{ ft/s})(0.75/12 \text{ ft})}{7.536\times10^{-4} \text{ lbm/ft}\cdot\text{s}} = 18{,}900$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is



$$\varepsilon / D = \frac{5\times10^{-6} \text{ ft}}{0.75/12 \text{ ft}} = 8\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{8\times10^{-5}}{3.7} + \frac{2.51}{18{,}900\sqrt{f}}\right)$$

It gives $f = 0.0264$. Then the pressure drop and the required power input become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0264\frac{1 \text{ ft}}{0.75/12 \text{ ft}}\frac{(62.36 \text{ lbm/ft}^3)(3.66 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.174 \text{ lbm}\cdot\text{ft/s}}\right) = 5.48 \text{ lbf/ft}^2$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.7 \text{ lbm/s})(5.48 \text{ lbf/ft}^2)}{62.36 \text{ lbm/ft}^3}\left(\frac{1 \text{ W}}{0.737 \text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.083\ W} \text{ (per ft length)}$$

Therefore, useful power input in the amount of 0.083 W is needed per ft of tube length to overcome the frictional losses in the pipe.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give $f = 0.0262$, which is sufficiently close to 0.0264. Also, the friction factor corresponding to $\varepsilon = 0$ in this case is 0.0263, which indicates that stainless steel pipes can be assumed to be smooth with a negligible error. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

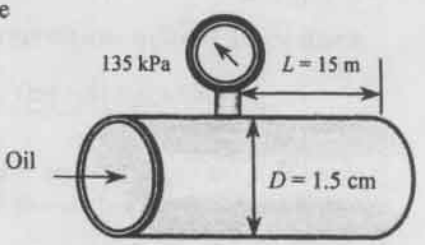
12-37 The pressure of oil in a pipe which discharges into the atmosphere is measured at a certain location. The flow rates are to be determined for 3 different orientations.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is laminar (to be verified). **4** The pipe involves no components such as bends, valves, and connectors. **5** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of oil are given to be $\rho = 876$ kg/m^3 and $\mu = 0.24$ kg/m·s, respectively.

Analysis The pressure drop across the pipe and the cross-sectional area are

$$\Delta P = P_1 - P_2 = 135 - 88 = 47 \text{ kPa}$$
$$A_c = \pi D^2/4 = \pi(0.015 \text{ m})^2/4 = 1.767\times10^{-4} \text{ m}^2$$



(*a*) The flow rate for all three cases can be determined from,

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ = 0 and thus sin θ = 0. Therefore,

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} = \frac{(47 \text{ kPa})\pi(0.015 \text{ m})^4}{128(0.24 \text{ kg/m}\cdot\text{s})(15 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right) = \mathbf{1.62\times10^{-5} \text{ m}^3/\text{s}}$$

(*b*) For uphill flow with an inclination of 8°, we have θ = +8°, and

$$\dot{V}_{\text{uphill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$
$$= \frac{[(47{,}000 \text{ Pa} - (876 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(15 \text{ m})\sin 8°]\pi(0.015 \text{ m})^4}{128(0.24 \text{ kg/m}\cdot\text{s})(15 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ Pa}\cdot\text{m}^2}\right)$$
$$= \mathbf{1.00\times10^{-5} \text{ m}^3/\text{s}}$$

(*c*) For downhill flow with an inclination of 8°, we have θ = -8°, and

$$\dot{V}_{\text{downhill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$
$$= \frac{[(47{,}000 \text{ Pa} - (876 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(15 \text{ m})\sin(-8°)]\pi(0.015 \text{ m})^4}{128(0.24 \text{ kg/m}\cdot\text{s})(15 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ Pa}\cdot\text{m}^2}\right)$$
$$= \mathbf{2.24\times10^{-5} \text{ m}^3/\text{s}}$$

The flow rate is the highest for downhill flow case, as expected. The average fluid velocity and the Reynolds number in this case are

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{2.24\times10^{-5}.\text{m}^3/\text{s}}{1.767\times10^{-4} \text{ m}^2} = 0.127 \text{ m/s}$$
$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(876 \text{ kg/m}^3)(0.127 \text{ m/s})(0.015 \text{ m})}{0.24 \text{ kg/m}\cdot\text{s}} = 7.0$$

which is less than 2300. Therefore, the flow is **laminar** for all three cases, and the analysis above is valid.

Discussion Note that the flow is driven by the combined effect of pressure difference and gravity. As can be seen from the calculated rates above, gravity opposes uphill flow, but helps downhill flow. Gravity has no effect on the flow rate in the horizontal case.

12-38 Glycerin is flowing through a horizontal pipe which discharges into the atmosphere at a specified flow rate. The absolute pressure at a specified location in the pipe, and the angle θ that the pipe must be inclined downwards for the pressure in the entire pipe to be atmospheric pressure are to be determined.

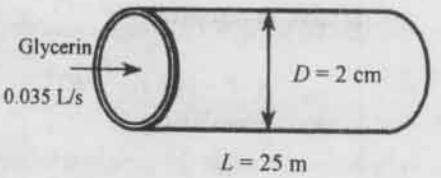
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is laminar (to be verified). **4** The pipe involves no components such as bends, valves, and connectors. **5** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of glycerin at 40°C are given to be $\rho = 1252$ kg/m^3 and $\mu = 0.27$ kg/m·s.

Analysis (*a*) The flow rate for horizontal or inclined pipe can be determined from

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L} \qquad (1)$$

where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ = 0 and thus sin θ = 0. Therefore,



$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} \qquad (2)$$

Solving for ΔP and substituting,

$$\Delta P = \frac{128\mu L \dot{V}_{\text{horiz}}}{\pi D^4} = \frac{128(0.24\ \text{kg/m}\cdot\text{s})(25\ \text{m})(0.035\times10^{-3}\ \text{m}^3/\text{s})}{\pi(0.02\ \text{m})^4}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 53.3\ \text{kN/m}^2 = 53.3\ \text{kPa}$$

Then the pressure 25 m before the pipe exit becomes

$$\Delta P = P_1 - P_2 \quad \rightarrow \quad P_1 = P_2 + \Delta P = 100 + 53.3 = \mathbf{153.3\ kPa}$$

(*b*) When the flow is gravity driven downhill with an inclination θ, and the pressure in the entire pipe is constant at the atmospheric pressure, the hydrostatic pressure rise with depth is equal to pressure drop along the pipe due to frictional effects. Setting $\Delta P = P_1 - P_2 = 0$ in Eq. (1) and substituting, θ is determined to be

$$\dot{V}_{\text{downhill}} = \frac{\rho g \sin\theta \pi D^4}{128\mu}$$

$$0.035\times10^{-3}\ \text{m}^3/\text{s} = \frac{-(1252\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\sin\theta\pi(0.02\ \text{m})^4}{128(0.27\ \text{kg/m}\cdot\text{s})} \quad \rightarrow \quad \theta = -38°$$

Therefore, the pipe must be inclined 38° downwards from the horizontal to maintain flow in the pipe at the same rate.

Checking The average fluid velocity and the Reynolds number in this case are

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.035\times10^{-3}.\text{m}^3/\text{s}}{\pi(0.02\ \text{m})^2/4} = 0.111\ \text{m/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(1252\ \text{kg/m}^3)(0.111\ \text{m/s})(0.02\ \text{m})}{0.27\ \text{kg/m}\cdot\text{s}} = 10.3$$

which is less than 2300. Therefore, the flow is **laminar**, as assumed, and the analysis above is valid.

Discussion Note that the flow is driven by the combined effect of pressure difference and gravity. Gravity has no effect on the flow rate in the horizontal case, but it governs the flow alone when there is no pressure difference across the pipe.

12-39 Air in a heating system is distributed through a rectangular duct made of commercial steel at a specified rate. The pressure drop and head loss through a section of the duct are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors. **5** The flow section involves no work devices such as fans or turbines.

Properties The roughness of commercial steel surfaces is $\varepsilon = 0.000045$ m (Fig. A-27). The dynamic viscosity of air at 43°C = 315 K is $\mu = 2.42\times10^{-5}$ kg/m·s, and it is independent of pressure (Table A-18). The density of air listed in that table is for 1 atm. The density at 105 kPa and 315 K can be determined from the ideal gas relation to be

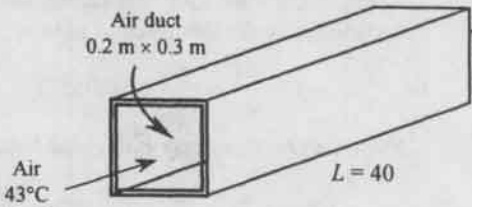
$$\rho = \frac{P}{RT} = \frac{105\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg.K})(315\,\text{K})} = 1.161\,\text{kg/m}^3$$

Analysis The hydraulic diameter, mean velocity, and Reynolds number in this case are

$$D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = \frac{4(0.3\,\text{m})(0.20\,\text{m})}{2(0.3+0.20)\,\text{m}} = 0.24\,\text{m}$$

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{a\times b} = \frac{0.5.\text{m}^3/\text{s}}{(0.3\,\text{m})(0.2\,\text{m})} = 8.33\,\text{m/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D_h}{\mu} = \frac{(1.161\,\text{kg/m}^3)(8.33\,\text{m/s})(0.24\,\text{m})}{2.42\times10^{-5}\,\text{kg/m}\cdot\text{s}} = 95{,}950$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the duct is

$$\varepsilon / D_h = \frac{4.5\times10^{-5}\,\text{m}}{0.24\,\text{m}} = 1.875\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{1.875\times10^{-4}}{3.7} + \frac{2.51}{95{,}950\sqrt{f}}\right)$$

It gives $f = 0.0191$. Then the pressure drop in the duct and the head loss become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0191\frac{40\,\text{m}}{0.24\,\text{m}}\frac{(1.161\,\text{kg/m}^3)(8.33\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 128\,\text{N/m}^2 = \mathbf{128\,Pa}$$

$$h_L = \frac{\Delta P}{\rho g} = f\frac{L}{D}\frac{\mathbf{V}_m^2}{2g} = 0.0191\frac{40\,\text{m}}{0.24\,\text{m}}\frac{(8.33\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \mathbf{93.8\,m}$$

Discussion The required pumping power in this case is

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.5\,\text{m}^3/\text{s})(128\,\text{Pa})\left(\frac{1\,\text{W}}{1\,\text{Pa}\cdot\text{m}^3/\text{s}}\right) = 64\,\text{W}$$

Therefore, 64 W of mechanical power needs to be imparted to the fluid. The shaft power will be more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency. Also, the friction factor could be determined easily from the explicit Haaland relation. It would give $f = 0.0188$, which is sufficiently close to 0.0191.

12-40 Glycerin is flowing through a smooth pipe with a specified mean velocity. The pressure drop per 10 m of the pipe is to be determined.

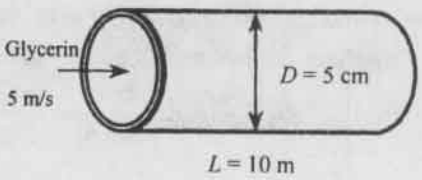
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of glycerin at 40°C are given to be $\rho = 1252$ kg/m^3 and $\mu = 0.27$ kg/m·s, respectively.

Analysis The volume flow rate and the Reynolds number are

$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m (\pi D^2 / 4) = (5\ \text{m/s})[\pi(0.05\ \text{m})^2 / 4] = 0.00982\ \text{m}^3/\text{s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D_h}{\mu} = \frac{(1252\ \text{kg/m}^3)(5\ \text{m/s})(0.05\ \text{m})}{0.27\ \text{kg/m} \cdot \text{s}} = 1159$$



which is less than 2300. Therefore, the flow is laminar, and the friction factor for this circular pipe is

$$f = \frac{64}{\text{Re}} = \frac{64}{1159} = 0.0552$$

Then the pressure drop in the pipe becomes

$$\Delta P = f \frac{L}{D} \frac{\rho \mathbf{V}_m^{\,2}}{2} = 0.0552 \frac{10\ \text{m}}{0.05\ \text{m}} \frac{(1252\ \text{kg/m}^3)(5\ \text{m/s})^2}{2} \left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}} \right) \left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2} \right) = \mathbf{173\ kPa}$$

Discussion The required pumping power in this case is

$$\dot{W}_{\text{pump,u}} = \dot{V} \Delta P = (0.00982\ \text{m}^3/\text{s})(173\ \text{kPa}) \left(\frac{1\ \text{kW}}{1\ \text{kPa} \cdot \text{m}^3/\text{s}} \right) = 1.7\ \text{kW}$$

Therefore, 1.7 kW of mechanical power needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

12-41E Air is flowing through a square duct made of commercial steel at a specified rate. The pressure drop and head loss per ft of duct are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors. **5** The flow section involves no work devices such as fans or turbines.

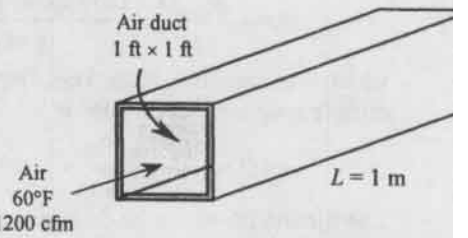
Properties The density and dynamic viscosity of air at 1 atm and 60°F are $\rho = 0.077$ lbm/ft^3 and $\mu = 1.165\times10^{-5}$ lbm/ft·s (Table A-18). The roughness of commercial steel surfaces is $\varepsilon = 0.00015$ ft (Fig. A-27).

Analysis The hydraulic diameter, the mean velocity, and the Reynolds number in this case are

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 1\text{ ft}$$

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{a^2} = \frac{1200\text{ ft}^3/\text{min}}{(1\text{ ft})^2} = 1200\text{ ft/min} = 20\text{ ft/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D_h}{\mu} = \frac{(0.077\text{ lbm/ft}^3)(20\text{ ft/s})(1\text{ ft})}{1.165\times10^{-5}\text{ lbm/ft}\cdot\text{s}} = 1.322\times10^5$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the duct is

$$\varepsilon / D_h = \frac{0.00015\text{ ft}}{1\text{ ft}} = 1.5\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{1.875\times10^{-4}}{3.7} + \frac{2.51}{95{,}950\sqrt{f}}\right)$$

It gives $f = 0.0179$. Then the pressure drop in the duct and the head loss become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0179\frac{1\text{ ft}}{1\text{ ft}}\frac{(0.077\text{ lbm/ft}^3)(20\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}}\right) = \mathbf{8.56\times10^{-3}\text{ lbf/ft}^2}$$

$$h_L = \frac{\Delta P}{\rho g} = f\frac{L}{D}\frac{\mathbf{V}_m^2}{2g} = 0.0179\frac{1\text{ ft}}{1\text{ ft}}\frac{(20\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = \mathbf{0.222\text{ ft}}$$

Discussion The required pumping power in this case is

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (1200/60\text{ ft}^3/\text{s})(8.56\times10^{-3}\text{ lbf/ft}^2)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = 0.232\text{ W (per ft length)}$$

Therefore, 0.232 W of mechanical power needs to be imparted to the fluid per ft length of the duct. The shaft power will be more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency. Also, the friction factor could be determined easily from the explicit Haaland relation. It would give $f = 0.0176$, which is sufficiently close to 0.0179.

12-42 Liquid ammonia is flowing through a copper tube at a specified mass flow rate. The pressure drop, the head loss, and the pumping power required to overcome the frictional losses in the tube are to be determined.

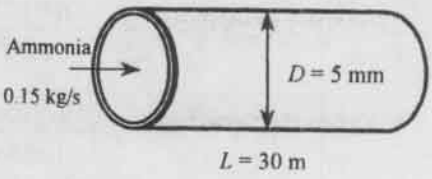
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of liquid ammonia at -20°C are $\rho = 667\ \text{kg/m}^3$ and $\mu = 2.54\times10^{-4}\ \text{kg/m·s}$ (Table A-16). The roughness of copper tubing is 1.5×10^{-6} m (Fig. A-27).

Analysis First we calculate the mean velocity and the Reynolds number to determine the flow regime:

$$\mathbf{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{0.15\ \text{kg/s}}{(667\ \text{kg/m}^3)[\pi(0.005\ \text{m})^2/4]} = 11.45\ \text{m/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(667\ \text{kg/m}^3)(11.45\ \text{m/s})(0.005\ \text{m})}{2.54\times10^{-4}\ \text{kg/m·s}} = 1.50\times10^5$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{1.5\times10^{-6}\ \text{m}}{0.005\ \text{m}} = 3\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{3\times10^{-4}}{3.7} + \frac{2.51}{1.50\times10^5\sqrt{f}}\right)$$

It gives $f = 0.0184$. Then the pressure drop, the head loss, and the useful pumping power required become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0184\frac{30\ \text{m}}{0.005\ \text{m}}\frac{(667\ \text{kg/m}^3)(11.45\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = \mathbf{4827\ kPa}$$

$$h_L = \frac{\Delta P}{\rho g} = f\frac{L}{D}\frac{\mathbf{V}_m^2}{2g} = 0.0184\frac{30\ \text{m}}{0.005\ \text{m}}\frac{(11.45\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \mathbf{738\ m}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.15\ \text{kg/s})(4827\ \text{kPa})}{667\ \text{kg/m}^3}\left(\frac{1\ \text{kW}}{1\ \text{kPa·m}^3/\text{s}}\right) = \mathbf{1.09\ kW}$$

Therefore, useful power input in the amount of 1.09 kW is needed to overcome the frictional losses in the tube.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give $f = 0.0181$, which is sufficiently close to 0.0184. The friction factor corresponding to $\varepsilon = 0$ in this case is 0.0166, which indicates that copper tubing can be assumed to be smooth with an error of about 10%.

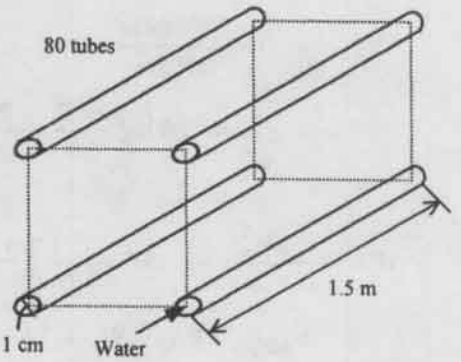
Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

12-43 Water is flowing through a brass tube bank of a heat exchanger at a specified flow rate. The pressure drop and the pumping power required are to be determined. Also, the percent reduction in the flow rate of water through the tubes is to be determined after scale build-up on the inner surfaces of the tubes. √

Assumptions **1** The flow is steady, horizontal, and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed (this is a questionable assumption since the tubes are short, and it will be verified). **3** The inlet, exit, and header losses are negligible, and the tubes involve no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water at 20°C are $\rho = 983.3$ kg/m^3 and $\mu = 0.467\times10^{-3}$ kg/m·s, respectively. The roughness of brass tubing is 1.5×10^{-6} m (Fig. A-27).

Analysis First we calculate the mean velocity and the Reynolds number to determine the flow regime:



$$V_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{N_{\text{tune}}(\pi D^2/4)} = \frac{0.015\,\text{m}^3/\text{s}}{80[\pi(0.01\,\text{m})^2/4]} = 2.387\,\text{m/s}$$

$$\text{Re} = \frac{\rho V_m D_h}{\mu} = \frac{(983.3\,\text{kg/m}^3)(2.387\,\text{m/s})(0.01\,\text{m})}{0.467\times10^{-3}\,\text{kg/m}\cdot\text{s}} = 50{,}300$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{1.5\times10^{-6}\,\text{m}}{0.01\,\text{m}} = 1.5\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{1.5\times10^{-4}}{3.7} + \frac{2.51}{50{,}300\sqrt{f}}\right)$$

It gives $f = 0.0214$. Then the pressure drop, the head loss, and the useful pumping power required become

$$\Delta P = f\frac{L}{D}\frac{\rho V_m^2}{2} = 0.0214\frac{1.5\,\text{m}}{0.01\,\text{m}}\frac{(983.3\,\text{kg/m}^3)(2.39\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right) = \mathbf{9.00\,kPa}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.015\,\text{m}^3/\text{s})(9.00\,\text{kPa})\left(\frac{1\,\text{kW}}{1\,\text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.135\,kW}$$

Therefore, useful power input in the amount of 0.135 kW is needed to overcome the frictional losses in the tube. The hydrodynamic entry length in this case is

$$L_{\text{h,turbulent}} \approx 4.4D\,\text{Re}^{1/6} = 4.4(0.01\,\text{m})(50{,}300)^{1/6} = 0.27\,\text{m}$$

which is much less than 1.5 m. Therefore, the assumption of fully developed flow is valid. (The effect of the entry region is to increase the friction factor, and thus the pressure drop and pumping power).

(12-43 Continued)

After scale buildup: When 1-mm thick scale builds up on the inner surfaces (and thus the diameter is reduced to 0.8 cm from 1 cm) with an equivalent roughness of 0.4 mm, and the useful power input is fixed at 0.135 kW, the problem can be formulated as follows (note that the flow rate and thus the mean velocity are unknown in this case):

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{N_{\text{tune}}(\pi D^2/4)} \rightarrow \mathbf{V}_m = \frac{\dot{V}}{80[\pi(0.008\,\text{m})^2/4]} \quad (1)$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} \rightarrow \text{Re} = \frac{(983.3\,\text{kg/m}^3)\mathbf{V}_m(0.008\,\text{m})}{0.467\times10^{-3}\,\text{kg/m}\cdot\text{s}} \quad (2)$$

$$\varepsilon/D = \frac{0.00004\,\text{m}}{0.008\,\text{m}} = 0.05$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.05}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad (3)$$

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} \rightarrow \Delta P = f\frac{1.5\,\text{m}}{0.008\,\text{m}}\frac{(983.3\,\text{kg/m}^3)\mathbf{V}_m^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right) \quad (4)$$

$$\dot{W}_{\text{pump,u}} = 0.135\,\text{kW} \rightarrow \dot{V}\Delta P = 0.135 \quad (5)$$

Solving this system of 5 equations in 5 unknown (f, Re, $\mathbf{V}_m$, ΔP, and $\dot{V}$) using an equation solver (or a trial-and-error approach, by assuming a velocity value) gives

$f = 0.0723$, Re = 28,870, $\mathbf{V}_m = 1.714$ m/s, $\Delta P = 19.6$ kPa, and $\dot{V} = 0.00689\ \text{m}^3/\text{s} = 6.89$ L/s

Then the percent reduction in the flow rate becomes

$$\text{Reduction ratio} = \frac{\dot{V}_{\text{clean}} - \dot{V}_{\text{dirty}}}{\dot{V}_{\text{clean}}} = \frac{15 - 6.89}{15} = 0.54 = \mathbf{54\%}$$

Therefore, for the same pump input, the flow rate will be reduced to less than half of the original flow rate when the pipes were new and clean.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

Minor Losses

12-44C The head losses associated with the flow of a fluid through fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions are called *minor losses*, and are expressed as

$$K_L = \frac{h_L}{\mathbf{V}^2/(2g)}$$

12-45C *Equivalent length* is the length of a straight pipe which would give the same head loss as the minor loss component. It is related to the minor loss coefficient by

$$L_{\text{equiv}} = \frac{D}{f} K_L$$

12-46C The effect of rounding of a pipe inlet on the loss coefficient is (*c*) very significant.

12-47C The effect of rounding of a pipe exit on the loss coefficient is (*a*) negligible.

12-48C Gradual expansion has a greater minor loss coefficient during pipe flow than gradual contraction. This is due to flow separation.

12-49C Another way of reducing the head loss associated with turns is to install turning vanes inside the elbows.

12-50C The loss coefficient is lower for flow through sharp turns in 90° miter elbows than it is for flow through smooth curved surfaces. Therefore, using miter elbows will result in a greater reduction in pumping power requirements.

12-51 Water is to be withdrawn from a water reservoir by drilling a hole at the bottom surface. The flow rate of water through the hole is to be determined for the well-rounded and sharp-edged entrance cases.

Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface.

Analysis The loss coefficient is $K_L = 0.5$ for the sharp-edged entrance, and $K_L = 0.03$ for the well-rounded entrance.

We take point 1 at the free surface of the reservoir and point 2 at the exit of the hole. We also take the reference level at the exit of the hole ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is zero ($\mathbf{V}_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

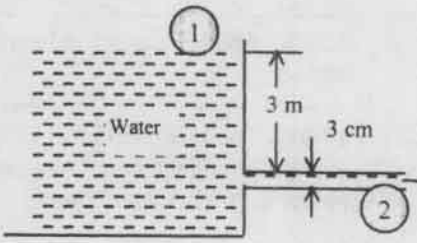
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L \dfrac{\mathbf{V}^2}{2g}$. Substituting and solving for $\mathbf{V}_2$ gives

$$z_1=\frac{\mathbf{V}_2^2}{2g}+K_L\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad 2gz_1=\mathbf{V}_2^2(1+K_L) \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2gz_1}{1+K_L}}$$

Note that in the special case of $K_L = 0$, it reduces to the Toricelli equation $\mathbf{V}_2 = \sqrt{2gz_1}$, as expected. Then the volume flow rate becomes

$$\dot{V}=A_c\mathbf{V}_2=\frac{\pi D_{hole}^2}{4}\sqrt{\frac{2gz_1}{1+K_L}}$$



Substituting the numerical values, the flow rate for both cases are determined to be

Well-rounded entrance: $\dot{V}=\dfrac{\pi D_{hole}^2}{4}\sqrt{\dfrac{2gz_1}{1+K_L}}=\dfrac{\pi(0.03\,\text{m})^2}{4}\sqrt{\dfrac{2(9.81\,\text{m/s}^2)(3\,\text{m})}{1+0.03}}=5.34\times10^{-3}\ \text{m}^3/s$

Sharp-edged entrance: $\dot{V}=\dfrac{\pi D_{hole}^2}{4}\sqrt{\dfrac{2gz_1}{1+K_L}}=\dfrac{\pi(0.03\,\text{m})^2}{4}\sqrt{\dfrac{2(9.81\,\text{m/s}^2)(3\,\text{m})}{1+0.5}}=4.42\times10^{-3}\ \text{m}^3/s$

Discussion The flow rate in the case of frictionless flow ($K_L = 0$) is 5.42×10^{-3} m^3/s. Note that the frictional losses cause the flow rate to decrease by 1.5% for well-rounded entrance, and 18.5% for the sharp-edged entrance.

12-52 Water is discharged from a water reservoir through a circular hole of diameter D at the side wall at a vertical distance H from the free surface. A relation for the "equivalent diameter" of the sharp-edged hole for use in frictionless flow relations is to be obtained.

Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface.

Analysis The loss coefficient is $K_L = 0.5$ for the sharp-edged entrance, and $K_L = 0$ for the "frictionless" flow. We take point 1 at the free surface of the reservoir and point 2 at the exit of the hole, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is zero ($\mathbf{V}_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad H=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L \dfrac{\mathbf{V}^2}{2g}$. Substituting and solving for $\mathbf{V}_2$ gives

$$H=\frac{\mathbf{V}_2^2}{2g}+K_L\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad 2gH=\mathbf{V}_2^2(1+K_L) \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2gH}{1+K_L}}$$

Then the volume flow rate becomes

$$\dot{V}=A_c\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}} \qquad (1)$$

Note that in the special case of $K_L = 0$ (frictionless flow), the velocity relation reduces to the Toricelli equation, $\mathbf{V}_{2,\text{frictionless}} = \sqrt{2gH}$. The flow rate in this case through a hole of D_e (equivalent diameter) is

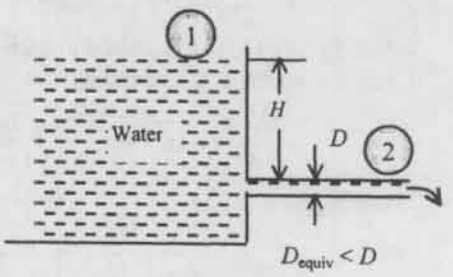
$$\dot{V}=A_{c,\text{equiv}}\mathbf{V}_{2,\text{frictionless}}=\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH} \qquad (2)$$

Setting Eqs. (1) and (2) equal to each other gives the desired relation for the equivalent diameter,

$$\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH}=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}}$$

which gives

$$D_{\text{equiv}}=\frac{D}{(1+K_L)^{1/4}}=\frac{D}{(1+0.5)^{1/4}}=0.904D$$



Discussion Note that the effect of frictional losses of a sharp-edged entrance is to reduce the diameter by about 10%. Also, noting that the flow rate is proportional to the square of the diameter, we have $\dot{V} \propto D_{\text{equiv}}^2 = (0.904D)^2 = 0.82D^2$. Therefore, the flow rate through a sharp-edged entrance is about 18% less compared to the frictionless entrance case.

12-53 Water is discharged from a water reservoir through a circular hole of diameter D at the side wall at a vertical distance H from the free surface. A relation for the "equivalent diameter" of the slightly rounded hole for use in frictionless flow relations is to be obtained.

Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface.

Analysis The loss coefficient is $K_L = 0.12$ for the slightly rounded entrance, and $K_L = 0$ for the "frictionless" flow.

We take point 1 at the free surface of the reservoir and point 2 at the exit of the hole, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is zero ($\mathbf{V}_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad H=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L \dfrac{\mathbf{V}^2}{2g}$. Substituting and solving for $\mathbf{V}_2$ gives

$$H=\frac{\mathbf{V}_2^2}{2g}+K_L\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad 2gH=\mathbf{V}_2^2(1+K_L) \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2gH}{1+K_L}}$$

Then the volume flow rate becomes

$$\dot{V}=A_c\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}} \qquad (1)$$

Note that in the special case of $K_L = 0$ (frictionless flow), the velocity relation reduces to the Toricelli equation, $\mathbf{V}_{2,\text{frictionless}} = \sqrt{2gz_1}$. The flow rate in this case through a hole of D_e (equivalent diameter) is

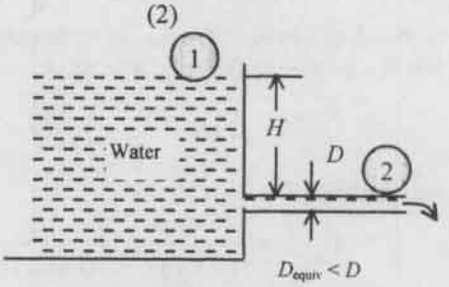
$$\dot{V}=A_{c,\text{equiv}}\mathbf{V}_{2,\text{frictionless}}=\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH} \qquad (2)$$

Setting Eqs. (1) and (2) equal to each other gives the desired relation for the equivalent diameter,

$$\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH}=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}}$$

which gives

$$D_{\text{equiv}}=\frac{D}{(1+K_L)^{1/4}}=\frac{D}{(1+0.12)^{1/4}}=0.972D$$



Discussion Note that the effect of frictional losses of a slightly rounded entrance is to reduce the diameter by about 3%. Also, noting that the flow rate is proportional to the square of the diameter, we have $\dot{V} \propto D_{\text{equiv}}^2 = (0.972D)^2 = 0.945D^2$. Therefore, the flow rate through a slightly rounded entrance is about 5% less compared to the frictionless entrance case.

12-54 A horizontal water pipe has an abrupt expansion. The water velocity and pressure in the smaller diameter pipe are given. The pressure after the expansion and the error that would have occurred if the Bernoulli Equation had been used are to be determined.

Assumptions The flow is steady, horizontal, and incompressible.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

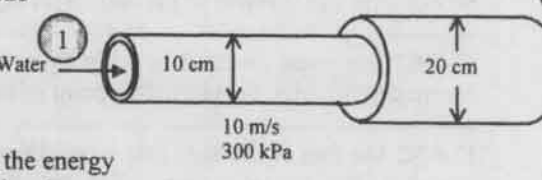
Analysis Noting that the density of water remains constant during flow (incompressible flow), the downstream velocity of water is determined to be

$$\dot{m}_1 = \dot{m}_2 \rightarrow \rho \mathbf{V}_1 A_1 = \rho \mathbf{V}_2 A_2 \rightarrow \mathbf{V}_2 = \frac{A_1}{A_2}\mathbf{V}_1 = \frac{\pi D_1^2/4}{\pi D_2^2/4}\mathbf{V}_1 = \frac{D_1^2}{D_2^2}\mathbf{V}_1 = \frac{(0.1\,\text{m})^2}{(0.2\,\text{m})^2}(10\,\text{m/s}) = 2.5\,\text{m/s}$$

The loss coefficient for sudden expansion and the head loss can be calculated from

$$K_L = \left(1 - \frac{A_{\text{small}}}{A_{\text{large}}}\right)^2 = \left(1 - \frac{D_1^2}{D_2^2}\right)^2 = \left(1 - \frac{0.1^2}{0.2^2}\right)^2 = 0.5625$$

$$h_L = K_L \frac{\mathbf{V}_1^2}{2g} = (0.5625)\frac{(10\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 2.88\,\text{m}$$



Noting that $z_1 = z_2$ and there are no pumps or turbines involved, the energy equation for the expansion section can be expressed in terms of heads as

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 + h_{\text{turbine}} + h_L \quad \rightarrow \quad \frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + h_L$$

Solving for P_2 and substituting,

$$P_2 = P_1 + \rho\left\{\frac{\mathbf{V}_1^{\,2} - \mathbf{V}_2^{\,2}}{2} - g h_L\right\}$$

$$= (300\,\text{kPa}) + (1000\,\text{kg/m}^3)\left\{\frac{(10\,\text{m/s})^2 - (2.5\,\text{m/s})^2}{2} - (9.81\,\text{m/s}^2)(2.88\,\text{m})\right\}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right)$$

$$= \mathbf{319\ kPa}$$

Therefore, despite the head (and pressure) loss, the pressure increases from 300 kPa to 319 kPa after the expansion. This is due to the conversion of dynamic pressure to static pressure when the velocity is decreased.

When the head loss is disregarded, the downstream pressure is determined from the Bernoulli equation to be

$$\frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} + \frac{\mathbf{V}_1^2}{2g} = \frac{P_2}{\rho g} + \frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad P_1 = P_1 + \rho\frac{\mathbf{V}_1^{\,2} - \mathbf{V}_2^{\,2}}{2}$$

Substituting,

$$P_2 = (300\,\text{kPa}) + (1000\,\text{kg/m}^3)\frac{(10\,\text{m/s})^2 - (2.5\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right) = 347\,\text{kPa}$$

Therefore, the error in the Bernoulli equation is

$$\text{Error} = P_{2,\,\text{Bernoulli}} - P_2 = 347 - 319 = \mathbf{28\ kPa}$$

Note that the use of the Bernoulli equation results in an error of (347 – 319)/319 = 0.088 or 8.8%.

Discussion It is common knowledge that higher pressure upstream is necessary to cause flow, and it may come as a surprise that the downstream pressure has *increased* after the abrupt expansion, despite the loss. This is because the sum of the three Bernoulli terms which comprise the total head, consisting of pressure head, velocity head, and elevation head, namely $[P/\rho g + \frac{1}{2}\mathbf{V}^2/g + z]$, drives the flow. With a geometric flow expansion, initially higher velocity head is converted to downstream pressure head, and this increase outweighs the non-convertible and non-recoverable head loss term.

Piping Systems and Pump Selection

12-55C For a piping system that involves two pipes of different diameters (but of identical length, material, and roughness) connected in *series*, (*a*) the flow rate through both pipes is the same and (*b*) the pressure drop across smaller diameter pipe is larger.

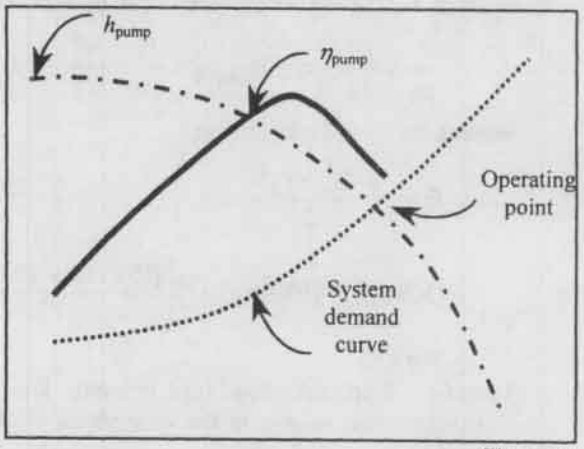
12-56C For a piping system that involves two pipes of different diameters (but of identical length, material, and roughness) connected in *parallel*, (*a*) the flow rate through the larger diameter pipe is larger and (*b*) the pressure drop through both pipes is the same.

12-57C The pressure drop through both pipes is the *same* since the pressure at a point has a single value, and the inlet and exits of these the pipes connected in parallel coincide.

12-58C Yes, when the head loss is negligible, the required pump head is equal to the elevation difference between the free surfaces of the two reservoirs.

12-59C The pump installed in a piping system will operate at the point where the *system curve* and the *characteristic curve* intersect. This point of intersection is called the *operating point*.

12-60C The plot of the head loss versus the flow rate is called the *system curve*. The experimentally determined pump head and pump efficiency versus the flow rate curves are called *characteristic curves*. The pump installed in a piping system will operate at the point where the *system curve* and the *characteristic curve* intersect. This point of intersection is called the *operating point*.



12-61 The pumping power input to a piping system with two parallel pipes between two reservoirs is given. The flow rates are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** The minor losses and the head loss in pipes other than the parallel pipes are said to be negligible. **5** The flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998\ \text{kg/m}^3$ and $\mu = 1.002\times10^{-3}$ kg/m·s (Table A-15). Plastic pipes are smooth, and their roughness is zero, $\varepsilon = 0$ (Fig. A-27).

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Therefore, we would normally use a trial-and-error approach here. However, nowadays the equation solvers such as EES are widely available, and thus below we will simply set up the equations to be solved by an equation solver. The head supplied by the pump to the fluid is determined from

$$\dot{W}_{\text{elect,in}} = \frac{\rho \dot{V} g h_{\text{pump,u}}}{\eta_{\text{pump-motor}}} \rightarrow 7000\ \text{W} = \frac{(998\ \text{kg/m}^3)\dot{V}(9.81\ \text{m/s}^2)h_{\text{pump,u}}}{0.68} \quad (1)$$

We choose points A and B at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_A = P_B = P_{\text{atm}}$) and that the fluid velocities at both points are zero ($\mathbf{V}_A = \mathbf{V}_B = 0$), the energy equation for a control volume between these two points simplifies to

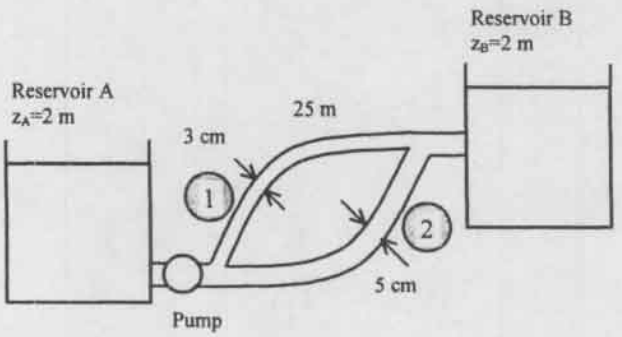
$$\frac{P_A}{\rho g} + \frac{\mathbf{V}_A^2}{2g} + z_A + h_{\text{pump,u}} = \frac{P_B}{\rho g} + \frac{\mathbf{V}_B^2}{2g} + z_B + h_L \quad \rightarrow \quad h_{\text{pump,u}} = (z_B - z_A) + h_L$$

or

$$h_{\text{pump,u}} = (9-2) + h_L \quad (2)$$

where

$$h_L = h_{L,1} = h_{L,2} \quad (3)\,(4)$$



We designate the 3-cm diameter pipe by 1 and the 5-cm diameter pipe by 2. The average velocity, Reynolds number, friction factor, and the head loss in each pipe are expressed as

$$\mathbf{V}_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \rightarrow \mathbf{V}_1 = \frac{\dot{V}_1}{\pi(0.03\text{m})^2/4} \quad (5)$$

$$\mathbf{V}_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \rightarrow \mathbf{V}_2 = \frac{\dot{V}_2}{\pi(0.05\text{m})^2/4} \quad (6)$$

$$\text{Re}_1 = \frac{\rho \mathbf{V}_1 D_1}{\mu} \rightarrow \text{Re}_1 = \frac{(998\ \text{kg/m}^3)\mathbf{V}_1(0.03\ \text{m})}{1.002\times10^{-3}\ \text{kg/m}\cdot s} \quad (7)$$

$$\text{Re}_2 = \frac{\rho \mathbf{V}_2 D_2}{\mu} \rightarrow \text{Re}_2 = \frac{(998\ \text{kg/m}^3)\mathbf{V}_2(0.05\ \text{m})}{1.002\times10^{-3}\ \text{kg/m}\cdot s} \quad (8)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \rightarrow \frac{1}{\sqrt{f_1}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \quad (9)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \rightarrow \frac{1}{\sqrt{f_2}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \quad (10$$

$$h_{L,1} = f_1 \frac{L_1}{D_1}\frac{\mathbf{V}_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1 \frac{25\,\text{m}}{0.03\,\text{m}}\frac{\mathbf{V}_1^2}{2(9.81\,\text{m/s}^2)} \qquad (11)$$

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2 \frac{25\,\text{m}}{0.05\,\text{m}}\frac{\mathbf{V}_2^2}{2(9.81\,\text{m/s}^2)} \qquad (12)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \qquad (13)$$

This is a system of 13 equations in 13 unknowns, and their simultaneous solution by an equation solver gives

$$\dot{V} = \mathbf{0.0183\,m^3/s}, \quad \dot{V}_1 = \mathbf{0.0037\,m^3/s}, \quad \dot{V}_2 = \mathbf{0.0146\,m^3/s},$$

$$\mathbf{V}_1 = 5.30\ \text{m/s}, \mathbf{V}_2 = 7.42\ \text{m/s}, \quad h_L = h_{L,1} = h_{L,2} = 19.5\,\text{m}, \quad h_{\text{pump,u}} = 26.5\ \text{m}$$

$$\text{Re}_1 = 158{,}300, \quad \text{Re}_2 = 369{,}700, \quad f_1 = 0.0164, \quad f_2 = 0.0139$$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion This problem can also be solved by using a trial-and-error approach, but it will be very time consuming. Equations solvers such as EES are invaluable for this kind of problems.

12-62E The flow rate through a piping system connecting two reservoirs is given. The elevation of the source is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** There are no pumps or turbines in the piping system.

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s (Table A-15E). The roughness of cast iron pipe is $\varepsilon = 0.00085$ ft (Fig. A-27).

Analysis The piping system involves 120 ft of 2-in diameter piping, a well-rounded entrance ($K_L = 0.03$), 4 standard flanged elbows ($K_L = 0.3$ each), a fully open gate valve ($K_L = 0.2$), and a sharp-edged exit ($K_L = 1.0$). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), the fluid velocities at both points are zero ($\mathbf{V}_1 = \mathbf{V}_2 = 0$), the free surface of the lower reservoir is the reference level ($z_2 = 0$), and that there is no pump or turbine ($h_{pump,u} = h_{turbine} = 0$), the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{pump,u}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{turbine}+h_L \quad \rightarrow \quad z_1 = h_L$$

where

$$h_L = h_{L,total} = h_{L,major} + h_{L,minor} = \left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g}$$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$\mathbf{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{10/60\ \text{ft}^3/\text{s}}{\pi(2/12\ \text{ft})^2/4} = 7.64\ \text{ft/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(62.3\ \text{lbm/ft}^3)(7.64\ \text{ft/s})(2/12\ \text{ft})}{1.307\times10^{-3}\ \text{lbm/ft}\cdot\text{s}} = 60{,}700$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.00085\ \text{ft}}{2/12\ \text{ft}} = 0.0051$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.0051}{3.7}+\frac{2.51}{60{,}700\sqrt{f}}\right)$$

It gives $f = 0.0320$. The sum of the loss coefficients is

$$\sum K_L = K_{L,entrance} + 4K_{L,elbow} + K_{L,valve} + K_{L,exit} = 0.03 + 4\times0.3 + 0.2 + 1.0 = 2.43$$

Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g} = \left((0.0320)\frac{120\ \text{ft}}{2/12\ \text{ft}}+2.43\right)\frac{(7.64\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = 23.1\ \text{ft}$$

$$z_1 = h_L = \mathbf{23.1\ ft}$$

Therefore, the free surface of the first reservoir must be 23.1 ft above the free surface of the lower reservoir to ensure water flow between the two reservoirs at the specified rate.

Discussion Note that $fL/D = 23.0$ in this case, which is almost 10 folds of the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in an error of about 10%.

12-63 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere. The initial velocity from the tank and the time required to empty the tank are to be determined. √

Assumptions **1** The flow is uniform and incompressible. **2** The flow is turbulent so that the tabulated value of the loss coefficient can be used.

Properties The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance (Table 12-4).

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the orifice. We also take the reference level at the centerline of the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

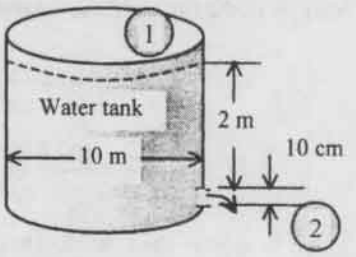
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L \dfrac{\mathbf{V}^2}{2g}$. Substituting and solving for $\mathbf{V}_2$ gives

$$z_1=\frac{\mathbf{V}_2^2}{2g}+K_L\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad 2gz_1=\mathbf{V}_2^2(1+K_L) \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2gz_1}{1+K_L}}$$

Noting that initially $z_1 = 2$ m, the initial velocity is determined to be

$$\mathbf{V}_2=\sqrt{\frac{2gz_1}{1+K_L}}=\sqrt{\frac{2(9.81\,\text{m/s}^2)(2\,\text{m})}{1+0.5}}=\mathbf{5.11\ m/s}$$



The mean discharge velocity through the orifice at any given time, in general, can be expressed as

$$\mathbf{V}_2=\sqrt{\frac{2gz}{1+K_L}}$$

where z is the water height relative to the center of the orifice at that time.

(*b*) We denote the diameter of the orifice by D, and the diameter of the tank by D_0. The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the orifice area,

$$\dot{V}=A_{\text{orifice}}\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+K_L}}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+K_L}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV=A_{\text{tank}}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+K_L}}dt = -\frac{\pi D_0^2}{4}dz \quad \rightarrow \quad dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+K_L}{2gz}}dz \quad \rightarrow \quad dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+K_L}{2g}}z^{-1/2}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+K_L}{2g}}\int_{z=z_1}^{0} z^{-1/2}dz \quad \rightarrow \quad t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1+K_L}{2g}}\left.\frac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2}\sqrt{\frac{1+K_L}{2g}}z_1^{1/2}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+K_L)}{g}} = \frac{(10\,\text{m})^2}{(0.1\,\text{m})^2}\sqrt{\frac{2(2\,\text{m})(1+0.5)}{9.81\,\text{m/s}^2}} = 7821\,\text{s} = \mathbf{2.17\,h}$$

Discussion The effect of the loss coefficient K_L on the draining time can be assessed by setting it equal to zero in the draining time relation. It gives

$$t_{f,\text{zero loss}} = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1}{g}} = \frac{(10\,\text{m})^2}{(0.1\,\text{m})^2}\sqrt{\frac{2(2\,\text{m})}{9.81\,\text{m/s}^2}} = 6396\,\text{s} = 1.77\,\text{h}$$

Note that the loss coefficient causes the draining time of the tank to increase by (2.17-1.77)/1.77 = 0.23 or 23%, which is quite **significant**. Therefore, the loss coefficient should always be considered in draining processes.

12-64 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe. The initial velocity from the tank and the time required to empty the tank are to be determined. √

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **4** The friction factor remains constant (in reality, it changes since the flow velocity and thus the Reynolds number changes).

Properties The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance (Table 12-4). The friction factor of the pipe is given to be 0.015.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the pipe ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where

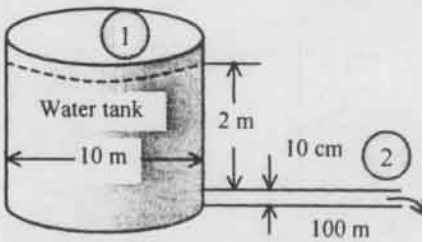
$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{\mathbf{V}^2}{2g}$$

since the diameter of the piping system is constant. Substituting and solving for $\mathbf{V}_2$ gives

$$z_1=\frac{\mathbf{V}_2^2}{2g}+\left(f\frac{L}{D}+K_L\right)\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2gz_1}{1+fL/D+K_L}}$$

Noting that initially $z_1 = 2$ m, the initial velocity is determined to be

$$\mathbf{V}_{2,i}=\sqrt{\frac{2gz_1}{1+fL/D+K_L}}=\sqrt{\frac{2(9.81\,\text{m/s}^2)(2\,\text{m})}{1+0.015(100\,\text{m})/(0.1\,\text{m})+0.5}}=\mathbf{1.54\ m/s}$$



The mean discharge velocity at any given time, in general, can be expressed as

$$\mathbf{V}_2=\sqrt{\frac{2gz}{1+fL/D+K_L}}$$

where z is the water height relative to the center of the orifice at that time.

(*b*) We denote the diameter of the pipe by D, and the diameter of the tank by D_o. The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{V}=A_{pipe}\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}$$

Then the amount of water that flows through the pipe during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV=A_{\text{tank}}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}dt = -\frac{\pi D_0^2}{4}dz \rightarrow dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2gz}}dz = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\int_{z=z_1}^{0} z^{-1/2}dz \rightarrow t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+fL/D+K_L)}{g}} = \frac{(10\,\text{m})^2}{(0.1\,\text{m})^2}\sqrt{\frac{2(2\,\text{m})[1+(0.015)(100\,\text{m})/(0.1\,\text{m})+0.5]}{9.81\,\text{m/s}^2}} = 25{,}940\,\text{s} = \mathbf{7.21\,h}$$

Discussion It can be shown by setting $L = 0$ that the draining time without the pipe is only 2.17 h. Therefore, the pipe in this case increases the draining time by more than 3 folds.

12-65 A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe equipped with a pump. For a specified initial velocity, the required useful pumping power and the time required to empty the tank are to be determined. √

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **4** The friction factor remains constant.

Properties The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance (Table 12-4). The friction factor of the pipe is given to be 0.015. The density of water at 30°C is $\rho = 996\ \text{kg/m}^3$ (Table A-15).

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{\text{atm}}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1+h_{\text{pump,u}}=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{\mathbf{V}^2}{2g}$$

since the diameter of the piping system is constant. Substituting and noting that the initial discharge velocity is 4 m/s, the required useful pumping head and power are determined to be

$$\dot{m}=\rho A_c\mathbf{V}_2=\rho(\pi d^2/4)\mathbf{V}_2=(996\,\text{kg/m}^3)[\pi(0.1\,\text{m})^2/4](4\,\text{m/s})=31.3\,\text{kg/m}^3$$

$$h_{\text{pump,u}}=\left(1+f\frac{L}{D}+K_L\right)\frac{\mathbf{V}_2^2}{2g}-z_1=\left(1+(0.015)\frac{100\,\text{m}}{0.1\,\text{m}}+0.5\right)\frac{(4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}-(2\,\text{m})=11.46\,\text{m}$$

$$\dot{W}_{\text{pump,u}}=\dot{V}\Delta P=\dot{m}gh_{\text{pump,u}}=(31.3\,\text{kg/s})(9.81\,\text{m/s}^2)(11.46\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kW}}{1\,\text{kN}\cdot\text{m/s}}\right)=3.52\,\text{kW}$$

Therefore, the pump must supply 3.52 kW of mechanical energy to water. Note that the shaft power of the pump must be greater than this to account for the pump inefficiency.

(*b*) When the discharge velocity remains constant, the flow rate of water becomes

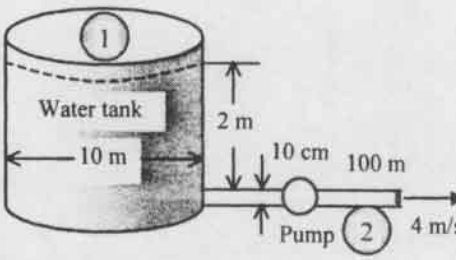
$$\dot{V}=A_c\mathbf{V}_2=(\pi D^2/4)\mathbf{V}_2=[\pi(0.1\,\text{m})^2/4](4\,\text{m/s})=0.03142\,\text{m}^3/\text{s}$$

The volume of water in the tank is

$$V=A_{\text{tank}}z_1=(\pi D_0^2/4)z_1=[\pi(10\,\text{m})^2/4](2\,\text{m})=157.1\,\text{m}^3$$

Then the discharge time becomes

$$\Delta t=\frac{V}{\dot{V}}=\frac{157.1\,\text{m}^3}{0.03142\,\text{m}^3/\text{s}}=5000\,\text{s}=1.39\,\text{h}$$



Discussion **1** Note that the pump reduces the discharging time from 7.21 h to 1.39 h. The assumption of constant discharge velocity can be justified on the basis of the pump head being much larger than the elevation head (therefore, the pump will dominate the discharging process). The answer obtained assumes that the elevation head remains constant at 2 m (rather than decreasing to zero eventually), and thus it under predicts the actual discharge time. By an exact analysis, it can be shown that when the effect of the decrease in elevation is considered, the discharge time becomes 5200 s = 1. 44 h. This is demonstrated below.

2 The required pump head (of water) is 11.46 m, which is more than 10.3 m of water column which corresponds to the atmospheric pressure at sea level. If the pump exit is at 1 atm, then the absolute pressure at pump inlet must be negative (= -1.16 m or – 11.4 kPa), which is impossible. Therefore, the system cannot work if the pump is installed near the pipe exit, and cavitation will occur long before the pipe exit where the pressure drops to 4.2 kPa and thus the pump must be installed close to the pipe entrance. A detailed analysis is given below.

Demonstration 1 for Prob. 12-65 (extra) (the effect of drop in water level on discharge time)
Noting that the water height z in the tank is variable, the mean discharge velocity through the pipe at any given time, in general, can be expressed as

$$h_{\text{pump,u}} = \left(1 + f\frac{L}{D} + K_L\right)\frac{\mathbf{V}_2^2}{2g} - z \quad \rightarrow \quad \mathbf{V}_2 = \sqrt{\frac{2g(z + h_{\text{pump,u}})}{1 + fL/D + K_L}}$$

where z is the water height relative to the center of the orifice at that time. We denote the diameter of the pipe by D, and the diameter of the tank by D_0. The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the cross-sectional area of the pipe,

$$\dot{V} = A_{pipe}\mathbf{V}_2 = \frac{\pi D^2}{4}\sqrt{\frac{2g(z + h_{\text{pump,u}})}{1 + fL/D + K_L}}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV = \dot{V}dt = \frac{\pi D^2}{4}\sqrt{\frac{2g(z + h_{\text{pump,u}})}{1 + fL/D + K_L}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV = A_{\text{tank}}(-dz) = -\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2g(z + h_{\text{pump,u}})}{1 + fL/D + K_L}}dt = -\frac{\pi D_0^2}{4}dz \quad \rightarrow dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1 + fL/D + K_L}{2g}}(z + h_{\text{pump,u}})^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1 + fL/D + K_L}{2g}}\int_{z=z_1}^{0}(z + h_{\text{pump,u}})^{-1/2}dz$$

Performing the integration gives

$$t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1 + fL/D + K_L}{2g}}\left.\frac{(z + h_{\text{pump,u}})^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0}$$

$$= \frac{D_0^2}{D^2}\left(\sqrt{\frac{2(z_1 + h_{\text{pump,u}})(1 + fL/D + K_L)}{g}} - \sqrt{\frac{2h_{\text{pump,u}}(1 + fL/D + K_L)}{g}}\right)$$

Substituting the values given, the draining time is determined to be

$$t_f = \frac{(10\,\text{m})^2}{(0.1\,\text{m})^2}\left(\sqrt{\frac{2(2 + 11.46\,\text{m})[1 + 0.015 \times 100/0.1 + 0.5]}{9.81\,\text{m/s}^2}} - \sqrt{\frac{2(11.46\,\text{m})[1 + 0.015 \times 100/0.1 + 0.5]}{9.81\,\text{m/s}^2}}\right)$$

$$= 5200\,\text{s} = \mathbf{1.44\,h}$$

Demonstration 2 for Prob. 12-65 (on cavitation)

We take the pump as the control volume, with point 1 at the inlet and point 2 at the exit. We assume the pump inlet and outlet diameters to be the same and the elevation difference between the pump inlet and the exit to be negligible. Then we have $z_1 \cong z_2$ and $\mathbf{V}_1 \cong \mathbf{V}_2$. The pump is located near the pipe exit, and thus the pump exit pressure is equal to the pressure at the pipe exit, which is the atmospheric pressure, $P_2 = P_{atm}$. Also, the can take $h_L = 0$ since the frictional effects and loses in the pump are accounted for by the pump efficiency (this is why we use the useful pump head $h_{pump,\,u}$ instead of h_{pump}). Then the energy equation for the pump (in terms of heads) reduces to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{pump,u}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{turbine}+h_L \quad \rightarrow \quad \frac{P_{1,abs}}{\rho g}+h_{pump,u}=\frac{P_{atm}}{\rho g}$$

Solving for P_1 and substituting,

$$P_{1,abs}=P_{atm}-\rho g h_{pump,u}$$

$$=(101.3\,\text{kPa})-(996\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(11.46\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right)=\mathbf{-10.7\,kPa}$$

which is impossible (absolute pressure cannot be negative). The technical answer to the question is that cavitation **will occur** since the pressure drops below the vapor pressure of 4.246 kPa. The practical answer is that the question is invalid (void) since the system will not work anyway. Therefore, we conclude that the pump must be located **near the beginning**, not the end of the pipe. Note that when doing a cavitation analysis, we must work with the absolute pressures. (If the system were installed as indicated, a water velocity of $\mathbf{V} = 4$ m/s could not be established regardless of how much pump power were applied. This is because the atmospheric and water elevation heads alone are not sufficient to drive such flow, with the pump restoring pressure after the flow.)

To determine the furthest distance from the tank the pump can be located without allowing cavitation, we assume the pump is located at a distance L^* from the exit, and choose the pump and the discharge portion of the pipe (from the pump to the exit) as the system, and write the energy equation. The energy equation this time will be as above, except that h_L (the pipe losses) must be considered and the pressure at 1 (pipe inlet) is the cavitation pressure, $P_1 = 4.246$ kPa:

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{pump,u}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{turbine}+h_L \quad \rightarrow \quad \frac{P_{1,abs}}{\rho g}+h_{pump,u}=\frac{P_{atm}}{\rho g}+f\frac{L^*}{D}\frac{\mathbf{V}^2}{2g}$$

or

$$f\frac{L^*}{D}\frac{\mathbf{V}^2}{2g}=\frac{P_{1,abs}-P_{atm}}{\rho g}+h_{pump,u}$$

Substituting the given values and solving for L^* gives

$$(0.015)\frac{L^*}{0.1\,\text{m}}\frac{(4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=\frac{(4.246-101.3)\,\text{kN/m}^2}{(996\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}\left(\frac{1000\,\text{kg}\cdot\text{m/s}^2}{1\,\text{kN}}\right)+(11.46\,\text{m}) \quad \rightarrow \quad L^*=12.5\,\text{m}$$

Therefore, the pump must be at least 12.5 m form the pipe exit to avoid cavitation at the pump inlet (this is where the lowest pressure occurs in the piping system, and where the cavitation is most likely to occur).

Cavitation onset places an upper limit to the length of the pipe on the suction side. A pipe slightly longer would become vapor bound, and the pump could not pull the suction necessary to sustain the flow. Even if the pipe on the suction side were slightly shorter than 100 – 12.5 = 87.5 m, cavitation can still occur in the pump since the liquid in the pump is usually accelerated at the expense of pressure, and cavitation in the pump could erode and destroy the pump.

Also, over time, scale and other buildup inside the pipe can and will increase the pipe roughness, increasing the friction factor f, and therefore the losses. Buildup also decreases the pipe diameter, which increases pressure drop. Therefore, flow conditions and system performance may change (generally decrease) as the system ages. A new system that marginally misses cavitation may degrade to where cavitation becomes a problem. Proper design avoids these problems, or where cavitation cannot be avoided for some reason, it can at least be anticipated.

12-66 Oil is flowing through a vertical glass funnel which is always maintained full. The flow rate of oil through the funnel and the funnel effectiveness are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed (to be verified). **3** The frictional loses in the cylindrical reservoir are negligible since its diameter is very large and thus the oil velocity is very low.

Properties The density and dynamic viscosity of oil at 20°C are $\rho = 888$ kg/m^3 and $\mu = 0.800$ kg/m·s (Table A-16).

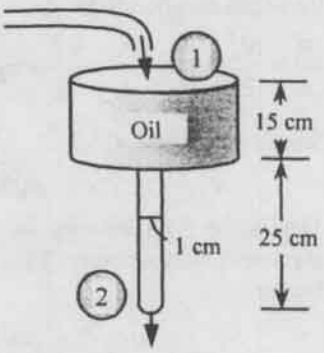
Analysis We take point 1 at the free surface of the oil in the cylindrical reservoir, and point 2 at the exit of the funnel pipe which is also taken as the reference level ($z_2 = 0$). The fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is negligible ($\mathbf{V}_1 \cong 0$). For the ideal case of "frictionless flow," the exit velocity is determined from the Bernoulli equation to be

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \quad \rightarrow \quad \mathbf{V}_2=\mathbf{V}_{2,\max}=\sqrt{2gz_1}$$

Substituting,

$$\mathbf{V}_{2,\max}=\sqrt{2gz_1}=\sqrt{2(9.81\,\text{m/s}^2)(0.40\,\text{m})}=2.80\,\text{m/s}$$

This is the flow velocity for the frictionless case, and thus it is the maximum flow velocity. Then the flow rate and the Reynolds number become



$$\dot{V}_{\max}=\mathbf{V}_{2,\max}A_2=\mathbf{V}_{2,\max}(\pi D_2^2/4)$$
$$=(2.80\,\text{m/s})[\pi(0.01\,\text{m})^2/4]=2.20\times10^{-4}\ \text{m}^3/\text{s}$$

$$\text{Re}=\frac{\rho V_m D}{\mu}=\frac{(888\,\text{kg/m}^3)(2.80\,\text{m/s})(0.01\,\text{m})}{0.800\,\text{kg/m}\cdot\text{s}}=31.1$$

which is less than 2300. Therefore, the flow is laminar, as postulated. (Note that in the actual case the velocity and thus the Reynolds number will be even smaller, verifying the flow is always laminar). The entry length in this case is

$$L_h=0.06\,\text{Re}\,D=0.06\times31.1\times(0.01\,\text{m})=0.02\,\text{m}$$

which is much less than the 0.25 m pipe length. Therefore, the entrance effects can be neglected as postulated.

Noting that the flow through the pipe is laminar and can be assumed to be fully developed, the flow rate can be determined from Eq. 12-18 with $\theta = -90°$ since the flow is downwards in the vertical direction,

$$\dot{V}=\frac{(\Delta P-\rho gL\sin\theta)\pi D^4}{128\mu L}$$

where $\Delta P = P_{\text{pipe inlet}} - P_{\text{pipe exit}} = (P_{\text{atm}} + \rho g h_{\text{cylinder}}) - P_{\text{atm}} = \rho g h_{\text{cylinder}}$ is the pressure difference across the pipe, $L = h_{\text{pipe}}$, and $\sin\theta = \sin(-90°) = -1$. Substituting, the flow rate is determined to be

$$\dot{V}=\frac{\rho g(h_{\text{cylinder}}+h_{\text{pipe}})\pi D^4}{128\mu L}=\frac{(888\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.15+0.25\,\text{m})\pi(0.01\,\text{m})^4}{128(0.800\,\text{kg/m}\cdot\text{s})(0.25\,\text{m})}=\mathbf{4.28\times10^{-6}\ m^3/s}$$

Then the "funnel effectiveness" becomes

$$\text{Eff}=\frac{\dot{V}}{\dot{V}_{\max}}=\frac{4.28\times10^{-6}\ \text{m}^3/\text{s}}{2.20\times10^{-4}\ \text{m}^3/\text{s}}=0.0195 \quad \text{or} \quad 1.95\%$$

Discussion Note that the flow is driven by gravity alone, and the actual flow rate is a small fraction of the flow rate that would have occurred if the flow were frictionless.

12-67 Oil is flowing through a vertical glass funnel which is always maintained full. The flow rate of oil through the funnel and the funnel effectiveness are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed (to be verified). **3** The frictional loses in the cylindrical reservoir are negligible since its diameter is very large and thus the oil velocity is very low.

Properties The density and dynamic viscosity of oil at 20°C are $\rho = 888$ kg/m^3 and $\mu = 0.800$ kg/m·s, respectively (Table A-16).

Analysis We take point 1 at the free surface of the oil in the cylindrical reservoir, and point 2 at the exit of the funnel pipe, which is also taken as the reference level ($z_2 = 0$). The fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is negligible ($\mathbf{V}_1 \cong 0$). For the ideal case of "frictionless flow," the exit velocity is

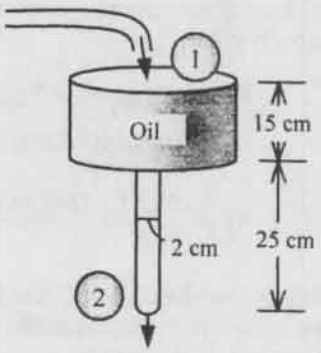
(*a*) **Case 1**: *Pipe length remains constant at 25 cm, but the pipe diameter is doubled* determined from the Bernoulli equation to be

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2 \qquad \rightarrow \qquad \mathbf{V}_2=\mathbf{V}_{2,\max}=\sqrt{2gz_1} \ \ to\ D_2=2\ cm:$$

Substitution gives

$$\mathbf{V}_{2,\max}=\sqrt{2gz_1}=\sqrt{2(9.81\,\text{m/s}^2)(0.40\,\text{m})}=2.80\,\text{m/s}$$

This is the flow velocity for the frictionless case, and thus it is the maximum flow velocity. Then the flow rate and the Reynolds number become



$$\dot{V}_{\max}=\mathbf{V}_{2,\max}A_2=\mathbf{V}_{2,\max}(\pi D_2^2/4)=(2.80\,\text{m/s})[\pi(0.02\,\text{m})^2/4]$$
$$=8.80\times10^{-4}\ \text{m}^3/\text{s}$$

$$\text{Re}=\frac{\rho \mathbf{V}_m D}{\mu}=\frac{(888\,\text{kg/m}^3)(2.80\,\text{m/s})(0.02\,\text{m})}{0.800\,\text{kg/m}\cdot\text{s}}=62.2$$

which is less than 2300. Therefore, the flow is laminar. (Note that in the actual case the velocity and thus the Reynolds number will be even smaller, verifying the flow is always laminar). The entry length is

$$L_h=0.06\,\text{Re}\,D=0.06\times62.2\times(0.02\,\text{m})=0.075\,\text{m}$$

which is considerably less than the 0.25 m pipe length. Therefore, the entrance effects can be neglected (with reservation).

Noting that the flow through the pipe is laminar and can be assumed to be fully developed, the flow rate can be determined from Eq. 12-18 with $\theta = -90°$ since the flow is downwards in the vertical direction,

$$\dot{V}=\frac{(\Delta P-\rho gL\sin\theta)\pi D^4}{128\mu L}$$

where $\Delta P = P_{\text{pipe inlet}} - P_{\text{pipe exit}} = (P_{\text{atm}} + \rho g h_{\text{cylinder}}) - P_{\text{atm}} = \rho g h_{\text{cylinder}}$ is the pressure difference across the pipe, $L = h_{\text{pipe}}$, and $\sin\theta = \sin(-90°) = -1$. Substituting, the flow rate is determined to be

$$\dot{V}=\frac{\rho g(h_{\text{cylinder}}+h_{\text{pipe}})\pi D^4}{128\mu L}=\frac{(888\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.15+0.25\,\text{m})\pi(0.02\,\text{m})^4}{128(0.800\,\text{kg/m}\cdot\text{s})(0.25\,\text{m})}=6.84\times10^{-5}\ \text{m}^3/\text{s}$$

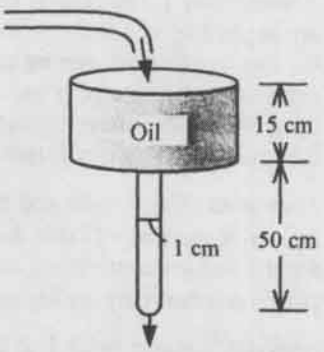
Then the "funnel effectiveness" becomes

$$\text{Eff}=\frac{\dot{V}}{\dot{V}_{\max}}=\frac{0.684\times10^{-4}\ \text{m}^3/\text{s}}{8.80\times10^{-4}\ \text{m}^3/\text{s}}=0.078 \quad \text{or} \quad 7.8\%$$

(*b*) **Case 2**: *Pipe diameter remains constant at 1 cm, but the pipe length is doubled to L = 50 cm*:

Substitution gives

$$\mathbf{V}_{2,\max} = \sqrt{2gz_1} = \sqrt{2(9.81\,\text{m/s}^2)(0.65\,\text{m})} = 3.57\,\text{m/s}$$



This is the flow velocity for the frictionless case, and thus it is the maximum flow velocity. Then the flow rate and the Reynolds number become

$$\dot{V}_{\max} = \mathbf{V}_{2,\max} A_2 = \mathbf{V}_{2,\max}(\pi D_2^2/4) = (3.57\,\text{m/s})[\pi(0.01\,\text{m})^2/4]$$
$$= 2.80\times10^{-4}\ \text{m}^3/s$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(888\,\text{kg/m}^3)(3.57\,\text{m/s})(0.01\,\text{m})}{0.800\,\text{kg/m}\cdot\text{s}} = 39.6$$

which is less than 2300. Therefore, the flow is laminar. (Note that in the actual case the velocity and thus the Reynolds number will be even smaller, verifying the flow is always laminar). The entry length is

$$L_h = 0.06\,\text{Re}\,D = 0.06\times39.6\times(0.01\,\text{m}) = 0.024\,\text{m}$$

which is much less than the 0.50 m pipe length. Therefore, the entrance effects can be neglected.

Noting that the flow through the pipe is laminar and can be assumed to be fully developed, the flow rate can be determined from Eq. 12-18 with θ = -90° since the flow is downwards in the vertical direction,

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

where $\Delta P = P_{\text{pipe inlet}} - P_{\text{pipe exit}} = (P_{\text{atm}} + \rho g h_{\text{cylinder}}) - P_{\text{atm}} = \rho g h_{\text{cylinder}}$ is the pressure difference across the pipe, $L = h_{\text{pipe}}$, and $\sin\theta = \sin(-90°) = -1$. Substituting, the flow rate is determined to be

$$\dot{V} = \frac{\rho g(h_{\text{cylinder}} + h_{\text{pipe}})\pi D^4}{128\mu L} = \frac{(888\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.15+0.50\,\text{m})\pi(0.01\,\text{m})^4}{128(0.800\,\text{kg/m}\cdot\text{s})(0.50\,\text{m})} = 3.47\times\mathbf{10}^{-6}\ \mathbf{m}^3/\mathbf{s}$$

Then the "funnel effectiveness" becomes

$$\text{Eff} = \frac{\dot{V}}{\dot{V}_{\max}} = \frac{3.47\times10^{-6}\ \text{m}^3/\text{s}}{2.80\times10^{-4}\ \text{m}^3/\text{s}} = 0.0124 \quad \text{or} \quad 1.24\%$$

Discussion Note that the funnel effectiveness increases as the pipe diameter is increased, and decreases as the pipe length is increased. This is because the frictional losses are proportional to the length but inversely proportional to the diameter of the flow sections.

12-68 Water is drained from a large reservoir through two pipes connected in series. The discharge rate of water from the reservoir is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The pipes are horizontal. **3** The entrance effects are negligible, and thus the flow is fully developed. **4** The flow is turbulent so that the tabulated value of the loss coefficients can be used. **5** The pipes involves no components such as bends, valves, and other connectors. **6** The piping section involves no work devices such as pumps and turbines. **7** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **8** The water level in the reservoir remains constant.

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1$ kg/m^3 and $\mu = 1.138\times10^{-3}$ kg/m·s, respectively (Table A-15). The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance, and it is 0.46 for the sudden contraction, corresponding to $d^2/D^2 = 4^2/10^2 = 0.16$ (Table 12-4). The pipes are made of plastic and thus they are smooth, $\varepsilon = 0$.

Analysis We take point 1 at the free surface of the reservoir, and point 2 at the exit of the pipe, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), the fluid level in the reservoir is constant ($\mathbf{V}_1 = 0$), and that there are no work devices such as pumps and turbines, the energy equation for a control volume between these two points (in terms of heads) simplifies to

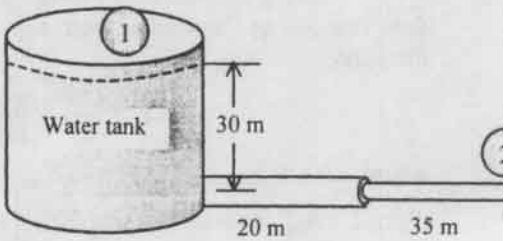
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

or

$$30\text{ m}=\frac{\mathbf{V}_2^2}{2(9.81\text{m/s}^2)}+h_L \qquad (1)$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\sum\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g}$$



Note that the diameters of the two pipes, and thus the flow velocities through them are different. Denoting the first pipe by 1 and the second pipe by 2, and using conservation of mass, the velocity in the first pipe can be expressed in terms of $\mathbf{V}_2$ as

$$\dot{m}_1=\dot{m}_2 \quad\rightarrow\quad \rho\mathbf{V}_1A_1=\rho\mathbf{V}_2A_2 \quad\rightarrow\quad \mathbf{V}_1=\frac{A_2}{A_1}\mathbf{V}_2=\frac{D_2^2}{D_1^2}\mathbf{V}_2=\frac{(4\text{ cm})^2}{(10\text{ cm})^2}\mathbf{V}_2 \quad\rightarrow\quad \mathbf{V}_1=0.16\mathbf{V}_2 \qquad (2)$$

Then the head loss can be expressed as

$$h_L=\left(f_1\frac{L_1}{D_1}+K_{L,\text{entrance}}\right)\frac{\mathbf{V}_1^2}{2g}+\left(f_2\frac{L_2}{D_2}+K_{L,\text{contraction}}\right)\frac{\mathbf{V}_2^2}{2g}$$

or

$$h_L=\left(f_1\frac{20\text{ m}}{0.10\text{ m}}+0.5\right)\frac{\mathbf{V}_1^2}{2(9.81\text{m/s}^2)}+\left(f_2\frac{35\text{ m}}{0.04\text{ m}}+0.46\right)\frac{\mathbf{V}_2^2}{2(9.81\text{m/s}^2)} \qquad (3)$$

The flow rate, the Reynolds number, and the friction factor are expressed as

$$\dot{V}=\mathbf{V}_2A_2=\mathbf{V}_2(\pi D_2^2/4) \quad\rightarrow\quad \dot{V}=\mathbf{V}_2[\pi(0.04\text{ m})^2/4] \qquad (4)$$

$$\text{Re}_1=\frac{\rho\mathbf{V}_1D_1}{\mu} \quad\rightarrow\quad \text{Re}_1=\frac{(999.1\text{ kg/m}^3)\mathbf{V}_1(0.10\text{ m})}{1.138\times10^{-3}\text{ kg/m}\cdot s} \qquad (5)$$

$$\text{Re}_2=\frac{\rho\mathbf{V}_2D_2}{\mu} \quad\rightarrow\quad \text{Re}_2=\frac{(999.1\text{ kg/m}^3)\mathbf{V}_2(0.04\text{ m})}{1.138\times10^{-3}\text{ kg/m}\cdot s} \qquad (6)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\mathrm{Re}_1\sqrt{f_1}}\right) \rightarrow \frac{1}{\sqrt{f_1}} = -2.0\log\left(0 + \frac{2.51}{\mathrm{Re}_1\sqrt{f_1}}\right) \quad (7)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\mathrm{Re}_2\sqrt{f_2}}\right) \rightarrow \frac{1}{\sqrt{f_2}} = -2.0\log\left(0 + \frac{2.51}{\mathrm{Re}_2\sqrt{f_2}}\right) \quad (8)$$

This is a system of 8 equations in 8 unknowns, and their simultaneous solution by an equation solver gives

$\dot{\mathbf{V}} = \mathbf{0.00787\ m^3/s}$, $\mathbf{V}_1 = 1.00$ m/s, $\mathbf{V}_2 = 6.27$ m/s, $h_L = h_{L1} + h_{L2} = 0.22 + 27.78 = 28.0$ m,

$\mathrm{Re}_1 = 88{,}000$, $\mathrm{Re}_2 = 220{,}000$, $f_1 = 0.0185$, $f_2 = 0.0154$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is valid.

Discussion This problem can also be solved by using a trial-and-error approach by assuming an exit velocity, but it will be very time consuming. Equation solvers such as EES are invaluable for this kind of problems.

12-69E The flow rate through a piping system between a river and a storage tank is given. The power input to the pump is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** The elevation difference between the free surfaces of the tank and the river remains constant.

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s (Table A-15E). The roughness of galvanized iron pipe is $\varepsilon = 0.0005$ ft (Fig. A-27).

Analysis The piping system involves 125 ft of 5-in diameter piping, an entrance with negligible loses, 3 standard flanged 90° smooth elbows ($K_L = 0.3$ each), and a sharp-edged exit ($K_L = 1.0$). We choose points 1 and 2 at the free surfaces of the river and the tank, respectively. We note that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), and the fluid velocity is 6 ft/s at point 1 and zero at point 2 ($\mathbf{V}_1 = 6$ ft/s and $\mathbf{V}_2 = 0$). We take the free surface of the river as the reference level ($z_1 = 0$). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad \frac{\mathbf{V}_1^2}{2g}+h_{\text{pump,u}}=z_2+h_L$$

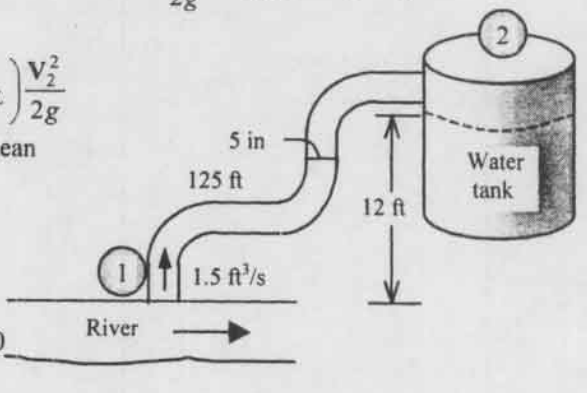
where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}$$

since the diameter of the piping system is constant. The mean velocity in the pipe and the Reynolds number are

$$\mathbf{V}_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{1.5\text{ ft}^3/\text{s}}{\pi(5/12\text{ ft})^2/4}=11.0\text{ ft/s}$$

$$\text{Re}=\frac{\rho \mathbf{V}_2 D}{\mu}=\frac{(62.3\text{ lbm/ft}^3)(11.0\text{ ft/s})(5/12\text{ ft})}{6.556\times10^{-4}\text{ lbm/ft}\cdot\text{s}}=435{,}600$$



which is greater than 4000. Therefore, the flow is turbulent.
The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.0005\text{ ft}}{5/12\text{ ft}}=0.0012$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{0.0012}{3.7}+\frac{2.51}{435{,}600\sqrt{f}}\right)$$

It gives $f = 0.0211$. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+3K_{L,\text{elbow}}+K_{L,\text{exit}}=0+3\times0.3+1.0=1.9$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g}=\left((0.0211)\frac{125\text{ ft}}{5/12\text{ ft}}+1.90\right)\frac{(11.0\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}=15.5\text{ ft}$$

The useful pump head input and the required power input to the pump are

$$h_{\text{pump,u}}=z_2+h_L-\frac{\mathbf{V}_1^2}{2g}=12+15.5-\frac{(6\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}=26.9\text{ ft}$$

$$\dot{W}_{\text{pump}} = \frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump}}} = \frac{\dot{V}\rho g h_{\text{pump,u}}}{\eta_{\text{pump}}}$$

$$= \frac{(1.5\ \text{ft}^3/\text{s})(62.30\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(26.9\ \text{ft})}{0.70}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\ \text{kW}}{737\ \text{lbf}\cdot\text{ft/s}}\right)$$

$$= \mathbf{4.87\ kW}$$

Therefore, 4.87 kW of electric power must be supplied to the pump.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give $f = 0.0211$, which is identical to the calculated value. The friction coefficient would drop to 0.0135 if smooth pipes were used. Note that $fL/D = 6.3$ in this case, which is about 3 times the total minor loss coefficient of 1.9. Therefore, the frictional losses in the pipe dominate the minor losses, but the minor losses are still significant.

12-70 A solar heated water tank is to be used for showers using gravity driven flow. For a specified flow rate, the elevation of the water level in the tank relative to shower head is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** The elevation difference between the free surface of water in the tank and the shower head remains constant. **5** There are no pumps or turbines in the piping system. **6** The losses at the entrance and at the shower head are said to be negligible. **7** The water tank is open to the atmosphere.

Properties The density and dynamic viscosity of water at 40°C are $\rho = 992.1$ kg/m^3 and $\mu = 0.653\times10^{-3}$ kg/m·s, respectively (Table A-15). The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance (Table 12-4). The roughness of galvanized iron pipe is $\varepsilon = 0.00015$ m (Fig. A-27).

Analysis The piping system involves 20 m of 1.5-cm diameter piping, an entrance with negligible loss, 4 miter bends (90°) without vanes ($K_L = 1.1$ each), and a wide open globe valve ($K_L = 10$). We choose point 1 at the free surface of water in the tank, and point 2 at the shower exit, which is also taken to be the reference level ($z_2 = 0$). Note that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), the fluid velocity at point 1 is zero ($\mathbf{V}_1 = 0$). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}$$

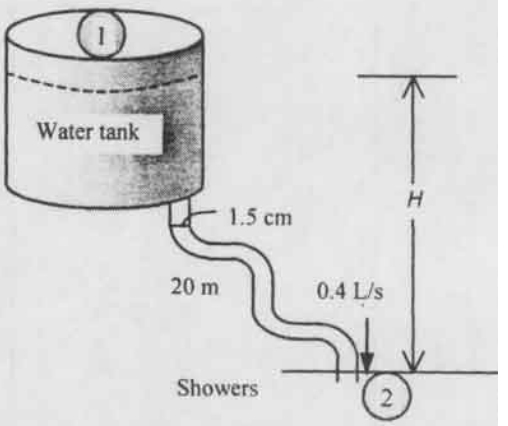
since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$\mathbf{V}_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{0.0004\,\text{m}^3/\text{s}}{\pi(0.015\,\text{m})^2/4}=2.26\,\text{m/s}$$

$$\text{Re}=\frac{\rho \mathbf{V}_2 D}{\mu}=\frac{(992.1\,\text{kg/m}^3)(2.26\,\text{m/s})(0.015\,\text{m})}{0.653\times10^{-3}\,\text{kg/m}\cdot\text{s}}=51{,}500$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00015\,\text{m}}{0.015\,\text{m}}=0.01$$



The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{0.0051}{3.7}+\frac{2.51}{60{,}700\sqrt{f}}\right)$$

It gives $f = 0.0391$. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+4K_{L,\text{elbow}}+K_{L,\text{valve}}+K_{L,\text{exit}}=0+4\times1.1+10+0=14.4$$

Note that we do not consider the exit loss unless the exit velocity is dissipated within the system considered (in this case it is not). Then the total head loss and the elevation of the source become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}=\left((0.0391)\frac{20\,\text{m}}{0.015\,\text{m}}+14.4\right)\frac{(2.26\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=17.3\,\text{m}$$

$$z_1=\frac{\mathbf{V}_2^2}{2g}+h_L=\frac{(2.26\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}+17.3\,\text{m}=17.6\,\text{m}$$

Therefore, the free surface of the tank must be 17.6 m above the shower exit to ensure water flow at the specified rate.

12-71 The flow rate through a piping system connecting two water reservoirs with the same water level is given. The absolute pressure in the pressurized reservoir is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** There are no pumps or turbines in the piping system.

Properties The density and dynamic viscosity of water at 10°C are $\rho = 999.7$ kg/m^3 and $\mu = 1.307\times10^{-3}$ kg/m·s (Table A-15). The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance, $K_L = 2$ for swing check valve, $K_L = 0.2$ for the fully open gate valve, and $K_L = 1$ for the exit (Table 12-4). The roughness of cast iron pipe is $\varepsilon = 0.00026$ m (Fig. A-27).

Analysis We choose points 1 and 2 at the free surfaces of the two reservoirs. We note that the fluid velocities at both points are zero ($\mathbf{V}_1 = \mathbf{V}_2 = 0$), the fluid at point 2 is open to the atmosphere (and thus $P_2 = P_{atm}$), both points are at the same level ($z_1 = z_2$). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \;\rightarrow\; \frac{P_1}{\rho g}=\frac{P_{\text{atm}}}{\rho g}+h_L \;\rightarrow\; P_1=P_{\text{atm}}+\rho g h_L$$

where

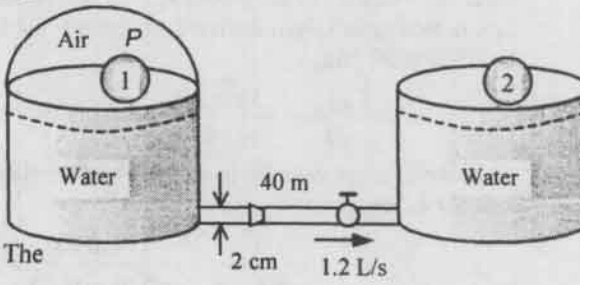
$$h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}$$

since the diameter of the piping system is constant. The average flow velocity and the Reynolds number are

$$\mathbf{V}_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.0012\,\text{m}^3/\text{s}}{\pi(0.02\,\text{m})^2/4} = 3.82\,\text{m/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_2 D}{\mu} = \frac{(999.7\,\text{kg/m}^3)(3.82\,\text{m/s})(0.02\,\text{m})}{1.307\times10^{-3}\,\text{kg/m}\cdot\text{s}} = 58{,}400$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is



$$\varepsilon/D = \frac{0.00026\,\text{m}}{0.02\,\text{m}} = 0.013$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \;\rightarrow\; \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.013}{3.7}+\frac{2.51}{58{,}400\sqrt{f}}\right)$$

It gives $f = 0.0424$. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}} + K_{L,\text{check valve}} + K_{L,\text{gate valve}} + K_{L,\text{exit}} = 0.5+2+0.2+1 = 3.7$$

Then the total head loss becomes

$$h_L = \left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g} = \left((0.0424)\frac{40\,\text{m}}{0.02\,\text{m}}+3.7\right)\frac{(3.82\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 65.8\,\text{m}$$

Substituting,

$$P_1 = P_{\text{atm}} + \rho g h_L = (88\,\text{kPa}) + (999.7\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(65.8\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right) = 733\ \text{kPa}$$

Discussion The absolute pressure above the first reservoir must be 733 kPa, which is quite high. Note that the minor losses in this case are negligible (about 4% of total losses). Also, the friction factor could be determined easily from the explicit Haaland relation (it gives the same result, 0.0424). The friction coefficient would drop to 0.0202 if smooth pipes were used.

12-72 A tanker is to be filled with fuel oil from an underground reservoir using a plastic hose. The required power input to the pump is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** The fuel oil level in the reservoir remains constant. **5** The reservoir is open to the atmosphere.

Properties The density and dynamic viscosity of fuel oil are given to be $\rho = 920$ kg/m^3 and $\mu = 0.045$ kg/m·s. The loss coefficient is $K_L = 0.12$ for a slightly-rounded entrance and $K_L = 0.3$ for a 90° smooth bend (flanged) (Table 12-4). The plastic pipe is smooth and thus $\varepsilon = 0$ (Fig. A-27).

Analysis We choose point 1 at the free surface of oil in the tanker and point 2 at the exit of the hose. We note the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and the fluid velocity at point 1 is zero ($\mathbf{V}_1 = 0$). We take the free surface of the reservoir as the reference level ($z_1 = 0$). Then the energy equation for a control volume between these two points simplifies to

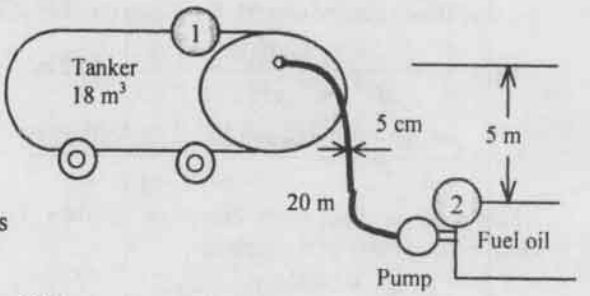
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_{\text{pump,u}}=\frac{\mathbf{V}_2^2}{2g}+z_2+h_L$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}$$

since the diameter of the piping system is constant. The flow rate is determined from the requirement that the tanker must be filled in 30 min,

$$\dot{V}=\frac{V_{\text{tanker}}}{\Delta t}=\frac{18\,\text{m}^3/\text{s}}{(30\times 60\,\text{s})}=0.01\,\text{m}^3/\text{s}$$



Then the average velocity in the pipe and the Reynolds number become

$$\mathbf{V}_2=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{0.01\,\text{m}^3/\text{s}}{\pi(0.05\,\text{m})^2/4}=5.093\,\text{m/s}$$

$$\text{Re}=\frac{\rho \mathbf{V}_2 D}{\mu}=\frac{(920\,\text{kg/m}^3)(5.093\,\text{m/s})(0.05\,\text{m})}{0.045\,\text{kg/m}\cdot\text{s}}=5206$$

which is greater than 4000. Therefore, the flow is turbulent. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{5206\sqrt{f}}\right)$$

It gives $f = 0.0370$. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+2K_{L,\text{bend}}=0.12+2\times 0.3=0.72$$

Note that we do not consider the exit loss unless the exit velocity is dissipated within the system considered (in this case it is not). Then the total head loss, the useful pump head, and the required pumping power become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}=\left((0.0370)\frac{20\,\text{m}}{0.05\,\text{m}}+0.72\right)\frac{(5.093\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=20.5\,\text{m}$$

$$h_{\text{pump,u}}=\frac{\mathbf{V}_2^2}{2g}+z_2+h_L=\frac{(5.093\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}+5\,\text{m}+20.5\,\text{m}=26.8\,\text{m}$$

$$\dot{W}_{\text{pump}} = \frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump}}} = \frac{\dot{V}\rho g h_{\text{pump,u}}}{\eta_{\text{pump}}}$$

$$= \frac{(0.01\,\text{m}^3/\text{s})(920\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(26.8\,\text{m})}{0.75}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kW}}{1\,\text{kN}\cdot\text{m/s}}\right) = \mathbf{3.23\,kW}$$

Discussion Note that the minor losses in this case are negligible (0.72/15.52 = 0.046 or about 5% of total losses). Also, the friction factor could be determined easily from the explicit Haaland relation (it gives 0.0372).

12-73 Two pipes of identical length and material are connected in parallel. The diameter of one of the pipes is twice the diameter of the other. The ratio of the flow rates in the two pipes is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The friction factor is given to be the same for both pipes. **3** The minor losses are negligible.

Analysis When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D can be expressed as

$$h_L = f\frac{L}{D}\frac{\mathbf{V}^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{A_c}\right)^2 = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2 = 8f\frac{L}{D}\frac{1}{g}\frac{\dot{V}^2}{\pi^2 D^4} = 8f\frac{L}{g\pi^2}\frac{\dot{V}^2}{D^5}$$

Solving for the flow rate gives

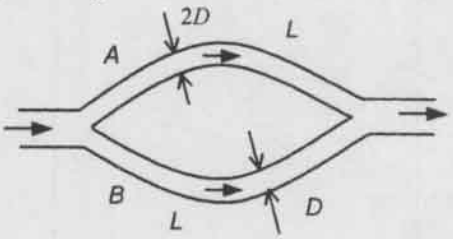
$$\dot{V} = \sqrt{\frac{\pi^2 h_L g}{8fL}}D^{2.5} = kD^{2.5} \qquad (k = \text{constant of proportionality})$$

When the pipe length, friction factor, and the head loss is constant, which is the case here for parallel connection, the flow rate becomes proportional to the 2.5^{th} power of diameter. Therefore, when the diameter is doubled, the flow rate will increase by a factor of $2^{2.5} = 5.66$ since

If $\qquad \dot{V}_A = kD_A^{2.5}$

Then $\qquad \dot{V}_B = kD_B^{2.5} = k(2D_A)^{2.5} = 2^{2.5}kD_A^{2.5} = 2^{2.5}\dot{V}_A = 5.66\dot{V}_A$

Therefore, the ratio of the flow rates in the two pipes is **5.66**.



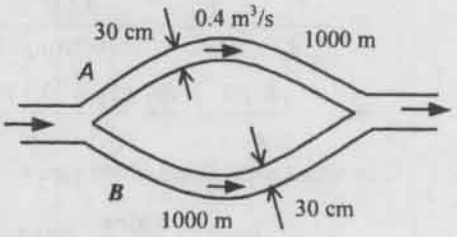
12-74 Cast iron piping of a water distribution system involves a parallel section with identical diameters but different lengths. The flow rate through one of the pipes is given, and the flow rate through the other pipe is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible. **4** The flow is fully turbulent and thus the friction factor is independent of the Reynolds number (to be verified).

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1$ kg/m^3 and $\mu = 1.138\times10^{-3}$ kg/m·s (Table A-15). The roughness of cast iron pipe is $\varepsilon = 0.00026$ m (Fig. A-27).

Analysis The mean velocity in pipe *A* is

$$\mathbf{V}_A = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.4\,\text{m}^3/\text{s}}{\pi(0.30\,\text{m})^2/4} = 5.659\,\text{m/s}$$



When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length *L* and diameter *D* is

$$h_L = f\frac{L}{D}\frac{\mathbf{V}^2}{2g}$$

Writing this for both pipes and setting them equal to each other, and noting that $D_A = D_B$ (given) and $f_A = f_B$ (to be verified) gives

$$f_A\frac{L_A}{D_A}\frac{\mathbf{V}_A^2}{2g} = f_B\frac{L_B}{D_B}\frac{\mathbf{V}_B^2}{2g} \quad\rightarrow\quad \mathbf{V}_B = V_A\sqrt{\frac{L_A}{L_B}} = (5.659\,\text{m/s})\sqrt{\frac{1000\,\text{m}}{3000\,\text{m}}} = 3.267\,\text{m/s}$$

Then the flow rate in pipe *B* becomes

$$\dot{V}_B = A_c\mathbf{V}_B = [\pi D^2/4]\mathbf{V}_B = [\pi(0.30\,\text{m})^2/4](3.267\,\text{m/s}) = \mathbf{0.231\,m^3/s}$$

Proof *that flow is fully turbulent and thus friction factor is independent of Reynolds number:*

The velocity in pipe *B* is lower. Therefore, if the flow is fully turbulent in pipe *B*, then it is also fully turbulent in pipe *A*. The Reynolds number in pipe *B* is

$$\text{Re}_B = \frac{\rho\mathbf{V}_B D}{\mu} = \frac{(999.1\,\text{kg/m}^3)(3.267\,\text{m/s})(0.30\,\text{m})}{1.138\times10^{-3}\,\text{kg/m}\cdot\text{s}} = 0.860\times10^6$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.00026\,\text{m}}{0.30\,\text{m}} = 0.00087$$

From Fig. A-27, we observe that for a relative roughness of 0.00087, the flow is fully turbulent for Reynolds number greater than about 10^6. Therefore, the flow in both pipes is fully turbulent, and thus the assumption that the friction factor is the same for both pipes is valid.

Discussion Note that the flow rate in pipe *B* is less than the flow rate in pipe *A* because of the larger losses due to the larger length.

12-75 Cast iron piping of a water distribution system involves a parallel section with identical diameters but different lengths and different valves. The flow rate through one of the pipes is given, and the flow rate through the other pipe is to be determined.

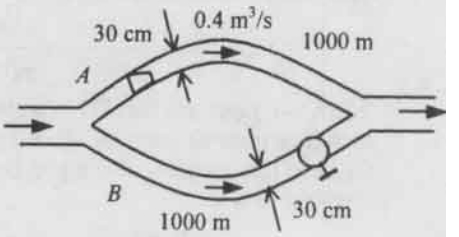
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses other than those for the valves are negligible. **4** The flow is fully turbulent and thus the friction factor is independent of the Reynolds number.

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1$ kg/m^3 and $\mu = 1.138\times10^{-3}$ kg/m·s (Table A-15). The roughness of cast iron pipe is $\varepsilon = 0.00026$ m (Fig. A-27).

Analysis For pipe A, the mean velocity and the Reynolds number are

$$\mathbf{V}_A = \frac{\dot{V}_A}{A_c} = \frac{\dot{V}_A}{\pi D^2/4} = \frac{0.4\,\text{m}^3/\text{s}}{\pi(0.30\,\text{m})^2/4} = 5.659\,\text{m/s}$$

$$\text{Re}_A = \frac{\rho \mathbf{V}_A D}{\mu} = \frac{(999.1\,\text{kg/m}^3)(5.659\,\text{m/s})(0.30\,\text{m})}{1.138\times10^{-3}\,\text{kg/m}\cdot\text{s}} = 1.49\times10^6$$



The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.00026\,\text{m}}{0.30\,\text{m}} = 8.667\times10^{-4}$$

The friction factor corresponding to this relative roughness and the Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{8.667\times10^{-4}}{3.7} + \frac{2.51}{1.49\times10^6\sqrt{f}}\right)$$

It gives $f = 0.0192$. Then the total head loss in pipe A becomes

$$h_{L,A} = \left(f\frac{L_A}{D} + K_L\right)\frac{\mathbf{V}_A^2}{2g} = \left((0.0192)\frac{1000\,\text{m}}{0.30\,\text{m}} + 2.1\right)\frac{(5.659\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 107.9\,\text{m}$$

When two pipes are parallel in a piping system, the head loss for each pipe must be same. Therefore, the head loss for pipe B must also be 107.9 m. Then the mean velocity in pipe B and the flow rate become

$$h_{L,B} = \left(f\frac{L_B}{D} + K_L\right)\frac{\mathbf{V}_B^2}{2g} \rightarrow 107.9\,\text{m} = \left((0.0192)\frac{3000\,\text{m}}{0.30\,\text{m}} + 10\right)\frac{\mathbf{V}_B^2}{2(9.81\,\text{m/s}^2)} \rightarrow \mathbf{V}_B = 3.24\,\text{m/s}$$

$$\dot{V}_B = A_c\mathbf{V}_B = [\pi D^2/4]\mathbf{V}_B = [\pi(0.30\,\text{m})^2/4](3.24\,\text{m/s}) = \mathbf{0.229\,m^3/s}$$

Discussion Note that the flow rate in pipe B decreases slightly (from 0.231 to 0.229 m^3/s) due to the larger minor loss in that pipe. Also, minor losses constitute just a few percent of the total loss, and they can be neglected if great accuracy is not required.

12-76 Geothermal water is supplied to a city through stainless steel pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

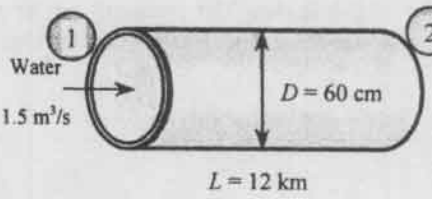
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are $\rho = 950.6$ kg/m^3, $\mu = 0.255\times10^{-3}$ kg/m·s, and $C_p = 4.229$ kJ/kg·°C (Table A-15). The roughness of stainless steel pipes is 2×10^{-6} m (Fig. A-27).

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($\mathbf{V}_1 = \mathbf{V}_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{pump,u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The mean velocity and the Reynolds number are



$$\mathbf{V}_m=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{1.5\text{ m}^3/\text{s}}{\pi(0.60\text{ m})^2/4}=5.305\text{ m/s}$$

$$\text{Re}=\frac{\rho \mathbf{V}_m D}{\mu}=\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})(0.60\text{ m})}{0.255\times10^{-3}\text{ kg/m}\cdot\text{s}}=1.187\times10^7$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{2\times10^{-6}\text{ m}}{0.60\text{ m}}=3.33\times10^{-6}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{3.33\times10^{-6}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt{f}}\right)$$

It gives $f = 0.00829$. Then the pressure drop, the head loss, and the required power input become

$$\Delta P=f\frac{L}{D}\frac{\rho\mathbf{V}_m^2}{2}=0.00829\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=2218\text{ kPa}$$

$$h_L=\frac{\Delta P}{\rho g}=f\frac{L}{D}\frac{\mathbf{V}_m^2}{2g}=(0.00829)\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(5.305\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=238\text{ m}$$

$$\dot W_{\text{elect}}=\frac{\dot W_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot V\Delta P}{\eta_{\text{pump-motor}}}=\frac{(1.5\text{ m}^3/\text{s})(2218\text{ kPa})}{0.65}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{5118\text{ kW}}$$

Therefore, the pumps will consume 5118 kW of electric power to overcome friction and maintain flow. The pumps must raise the pressure of the geothermal water by 2218 kPa. Providing a pressure rise of this magnitude at one location may create excessive stress in piping at that location. Therefore, it is more desirable to raise the pressure by smaller amounts at a several locations along the flow. This will keep the maximum pressure in the system and the stress in piping at a safer level.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount} = \dot{W}_{\text{elect,in}}\,\Delta t = (5118\,\text{kW})(24\,\text{h/day}) = 122{,}832\,\text{kWh/day}$$

$$\text{Cost} = \text{Amount} \times \text{Unit cost} = (122{,}832\,\text{kWh/day})(\$0.06/\text{kWh}) = \mathbf{\$7370/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 5118 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{elect}} = \rho \dot{V} C_p \Delta T \rightarrow \Delta T = \frac{\eta_{\text{pump-motor}} \dot{W}_{\text{elect,in}}}{\rho \dot{V} C_p} = \frac{0.65 \times (5118\,\text{kJ/s})}{(950.6\,\text{kg/m}^3)(1.5\,\text{m}^3/\text{s})(4.229\,\text{kJ/kg}\cdot°\text{C})} = \mathbf{0.55°C}$$

Therefore, the temperature of water will rise at least 0.55°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

12-77 Geothermal water is supplied to a city through cast iron pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

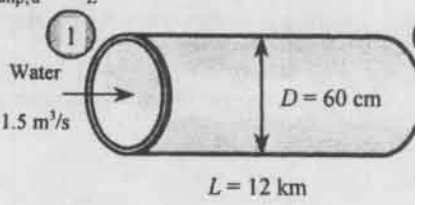
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are $\rho = 950.6$ kg/m^3, $\mu = 0.255\times10^{-3}$ kg/m·s, and $C_p = 4.229$ kJ/kg·°C (Table A-15). The roughness of cast iron pipes is 0.00026 m (Fig. A-27).

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($\mathbf{V}_1 = \mathbf{V}_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{pump,u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The mean velocity and the Reynolds number are



$$\mathbf{V}_m=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{1.5\,\text{m}^3/\text{s}}{\pi(0.60\,\text{m})^2/4}=5.305\,\text{m/s}$$

$$\text{Re}=\frac{\rho \mathbf{V}_m D}{\mu}=\frac{(950.6\,\text{kg/m}^3)(5.305\,\text{m/s})(0.60\,\text{m})}{0.255\times10^{-3}\,\text{kg/m}\cdot\text{s}}=1.187\times10^7$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\,\text{m}}{0.60\,\text{m}}=4.33\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\quad\rightarrow\quad\frac{1}{\sqrt f}=-2.0\log\left(\frac{4.33\times10^{-4}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt f}\right)$$

It gives $f = 0.0162$. Then the pressure drop, the head loss, and the required power input become

$$\Delta P=f\frac{L}{D}\frac{\rho\mathbf{V}_m^2}{2}=0.0162\frac{12{,}000\,\text{m}}{0.60\,\text{m}}\frac{(950.6\,\text{kg/m}^3)(5.305\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right)=4335\ \text{kPa}$$

$$h_L=\frac{\Delta P}{\rho g}=f\frac{L}{D}\frac{\mathbf{V}_m^2}{2g}=(0.0162)\frac{12{,}000\,\text{m}}{0.60\,\text{m}}\frac{(5.305\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=465\,\text{m}$$

$$\dot W_{\text{elect}}=\frac{\dot W_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot V\Delta P}{\eta_{\text{pump-motor}}}=\frac{(1.5\,\text{m}^3/\text{s})(4335\,\text{kPa})}{0.65}\left(\frac{1\,\text{kW}}{1\,\text{kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{10{,}004\,kW}$$

Therefore, the pumps will consume 10,004 kW of electric power to overcome friction and maintain flow. The pumps must raise the pressure of the geothermal water by 2218 kPa. Providing a pressure rise of this magnitude at one location may create excessive stress in piping at that location. Therefore, it is more desirable to raise the pressure by smaller amounts at a several locations along the flow. This will keep the maximum pressure in the system and the stress in piping at a safer level.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount} = \dot{W}_{\text{elect,in}}\,\Delta t = (10{,}004\ \text{kW})(24\ \text{h/day}) = 240{,}100\ \text{kWh/day}$$

$$\text{Cost} = \text{Amount} \times \text{Unit cost} = (240{,}100\ \text{kWh/day})(\$0.06/\text{kWh}) = \mathbf{\$14{,}400/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 10,004 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{elect}} = \rho \dot{V} C_p \Delta T \rightarrow \Delta T = \frac{\eta_{\text{pump-motor}} \dot{W}_{\text{elect,in}}}{\rho \dot{V} C_p} = \frac{0.65 \times (10{,}004\ \text{kJ/s})}{(950.6\ \text{kg/m}^3)(1.5\ \text{m}^3/\text{s})(4.229\ \text{kJ/kg} \cdot °\text{C})} = \mathbf{1.08°C}$$

Therefore, the temperature of water will rise at least 1.08°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

12-78E The air discharge rate of a clothes drier with no ducts is given. The flow rate when duct work is attached is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects in the duct are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** The losses at the vent and its proximity are negligible.

Properties The density and dynamic viscosity of air at 1 atm and 120°F are $\rho = 0.069$ lbm/ft^3 and $\mu = 1.316\times10^{-5}$ lbm/ft·s (Table A-18E). The roughness of galvanized iron pipe is $\varepsilon = 0.0005$ ft (Fig. A-27). The loss coefficient is $K_L \approx 0$ for a well-rounded entrance with negligible loss, $K_L = 0.3$ for a flanged 90° smooth bend, and $K_L = 1.0$ for an exit (Table 12-4). The friction factor of the duct is given to be 0.028.

Analysis To determine the useful fan power input, we choose point 1 inside the drier sufficiently far from the vent, and point 2 at the exit on the same horizontal level so that $z_1 = z_2$ and $P_1 = P_2$, and the flow velocity at point 1 is negligible ($\mathbf{V}_1 = 0$) since it is far from the inlet of the fan. Also, the losses between 1 and 2 are negligible. Then the energy equation for a control volume between 1 and 2 reduces to

$$\dot{m}\left(\frac{P_1}{\rho}+\frac{\mathbf{V}_1^2}{2}+gz_1\right)+\dot{W}_{\text{fan, u}}=\dot{m}\left(\frac{P_2}{\rho}+\frac{\mathbf{V}_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}}=\dot{m}\frac{\mathbf{V}_2^2}{2}$$

where the mean velocity of air is

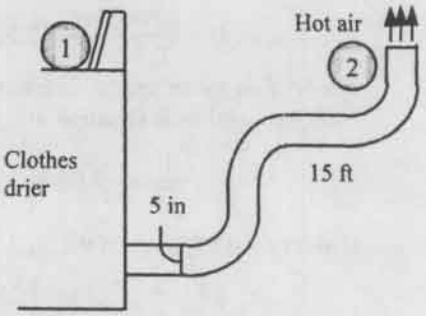
$$\mathbf{V}_2=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{1.2\text{ ft}^3/\text{s}}{\pi(5/12\text{ ft})^2/4}=8.80\text{ ft/s}$$

Now we attach the duct work, and take point 3 to be at the duct exit so that the duct is included in the control volume. The energy equation for this control volume simplifies to

$$\dot{m}\left(\frac{P_1}{\rho}+\frac{\mathbf{V}_1^2}{2}+gz_1\right)+\dot{W}_{\text{fan, u}}=\dot{m}\left(\frac{P_3}{\rho}+\frac{\mathbf{V}_3^2}{2}+gz_3\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}}=\dot{m}\frac{\mathbf{V}_3^2}{2}+\dot{m}gh_L$$

or

$$\rho\dot{V}_2\frac{\mathbf{V}_2^2}{2}=\rho\dot{V}_3\frac{\mathbf{V}_3^2}{2}+\rho\dot{V}_3gh_L \quad \rightarrow \quad \dot{V}_2\frac{\mathbf{V}_2^2}{2}=\dot{V}_3\frac{\mathbf{V}_3^2}{2}+\dot{V}_3gh_L$$



where

$$\mathbf{V}_3=\frac{\dot{V}_3}{A_c}=\frac{\dot{V}_3}{\pi D^2/4}=\frac{\dot{V}_3\text{ ft}^3/\text{s}}{\pi(5/12\text{ ft})^2/4}=7.33\dot{V}_2\text{ ft/s}$$

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_3^2}{2g}=\left(0.028\frac{15\text{ ft}}{5/12\text{ ft}}+3\times0.3+1\right)\frac{\mathbf{V}_3^2}{2g}=2.91\frac{\mathbf{V}_3^2}{2g}$$

Substituting,

$$\dot{V}_2\frac{\mathbf{V}_2^2}{2}=\dot{V}_3\frac{\mathbf{V}_3^2}{2}+\dot{V}_3g\times2.91\frac{\mathbf{V}_3^2}{2g}=\dot{V}_3\frac{(7.33\dot{V}_3)^2}{2}+\dot{V}_3\times2.91\frac{(7.33\dot{V}_3)^2}{2}=105\dot{V}_3^3$$

Solving for $\dot{V}_3$ and substituting the numerical values gives

$$\dot{V}_3=\left(\dot{V}_2\frac{\mathbf{V}_2^2}{2\times105}\right)^{1/3}=\left(1.5\frac{8.80^2}{2\times105}\right)^{1/3}=\mathbf{0.82\text{ ft}^3/\text{s}}$$

Discussion Note that the flow rate decreased considerably for the same fan power input, as expected. We could also solve this problem by solving for the useful fan power first,

$$\dot{W}_{\text{fan, u}}=\rho\dot{V}_2\frac{\mathbf{V}_2^2}{2}=(0.069\text{ lbm/ft}^3)(1.5\text{ ft}^3/\text{s})\frac{(8.80\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right)=0.17\text{ W}$$

Therefore, the fan supplies 0.17 W of useful mechanical power when the drier is running.

12-79 Hot water in a water tank is circulated through a loop made of cast iron pipes at a specified mean velocity. The required power input for the recirculating pump is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Minor losses other than those for elbows and valves are negligible.

Properties The density and dynamic viscosity of water at 60°C are $\rho = 983.3$ kg/m^3, $\mu = 0.467\times10^{-3}$ kg/m·s (Table A-15). The roughness of cast iron pipes is 0.00026 m (Fig. A-27). The loss coefficient is $K_L = 0.9$ for a threaded 90° smooth bend and $K_L = 0.2$ for a fully open gate valve (Table 12-4).

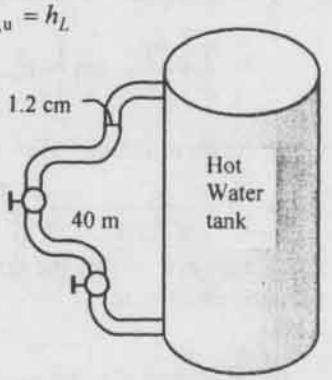
Analysis Since the water circulates continually and undergoes a cycle, we can take the entire recirculating system as the control volume, and choose points 1 and 2 at any location at the same point. Then the properties (pressure, elevation, and velocity) at 1 and 2 will be identical, and the energy equation will simplify to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{pump,u}}=h_L$$

where

$$h_L=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}$$

since the diameter of the piping system is constant. Therefore, the pumping power is to be used to overcome the head losses in the flow. The flow rate and the Reynolds number are



$$\dot{V}=\mathbf{V}_m A_c=\mathbf{V}_m(\pi D^2/4)=(2.5\text{ m/s})[\pi(0.012\text{ m})^2/4]=2.83\times10^{-4}\text{ m}^3/\text{s}$$

$$\text{Re}=\frac{\rho \mathbf{V}_m D}{\mu}=\frac{(983.3\text{ kg/m}^3)(2.5\text{ m/s})(0.012\text{ m})}{0.467\times10^{-3}\text{ kg/m}\cdot\text{s}}=63{,}200$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\text{ m}}{0.012\text{ m}}=0.0217$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{0.0217}{3.7}+\frac{2.51}{63200\sqrt{f}}\right)$$

It gives $f = 0.0508$. Then the total head loss, pressure drop, and the required pumping power input become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}=\left((0.0508)\frac{40\text{ m}}{0.012\text{ m}}+6\times0.9+2\times0.2\right)\frac{(2.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=55.8\text{ m}$$

$$\Delta P=\rho g h_L=(983.3\text{ kg/m}^3)(9.81\text{ m/s}^2)(55.8\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=538\text{ kPa}$$

$$\dot{W}_{\text{elect}}=\frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(2.83\times10^{-4}\text{ m}^3/\text{s})(538\text{ kPa})}{0.70}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{2.18\text{ kW}}$$

Therefore, the required power input of the recirculating pump is 2.18 kW

Discussion It can be shown that the required pumping power input for the recirculating pump is 2.10 kW when the minor losses are not considered. Therefore, the minor losses can be neglected in this case without a major loss in accuracy.

12-80 Hot water in a water tank is circulated through a loop made of plastic pipes at a specified mean velocity. The required power input for the recirculating pump is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Minor losses other than those for elbows and valves are negligible.

Properties The density and dynamic viscosity of water at 60°C are $\rho = 983.3$ kg/m^3, $\mu = 0.467\times10^{-3}$ kg/m·s (Table A-15). Plastic pipes are smooth, and thus their roughness is zero, $\varepsilon = 0$ (Fig. A-27). The loss coefficient is $K_L = 0.9$ for a threaded 90° smooth bend and $K_L = 0.2$ for a fully open gate valve (Table 12-4).

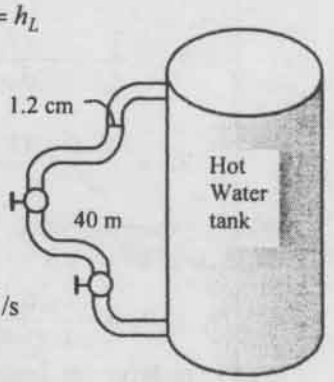
Analysis Since the water circulates continually and undergoes a cycle, we can take the entire recirculating system as the control volume, and choose points 1 and 2 at any location at the same point. Then the properties (pressure, elevation, and velocity) at 1 and 2 will be identical, and the energy equation will simplify to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_{\text{pump,u}}=h_L$$

where

$$h_L=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}$$

since the diameter of the piping system is constant. Therefore, the pumping power is to be used to overcome the head losses in the flow. The flow rate and the Reynolds number are



$$\dot{V}=\mathbf{V}_m A_c=\mathbf{V}_m(\pi D^2/4)=(2.5\text{ m/s})[\pi(0.012\text{ m})^2/4]=2.83\times10^{-4}\text{ m}^3/\text{s}$$

$$\text{Re}=\frac{\rho \mathbf{V}_m D}{\mu}=\frac{(983.3\text{ kg/m}^3)(2.5\text{ m/s})(0.012\text{ m})}{0.467\times10^{-3}\text{ kg/m}\cdot\text{s}}=63{,}200$$

which is greater than 4000. Therefore, the flow is turbulent. The friction factor corresponding to the relative roughness of zero and this Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{63200\sqrt{f}}\right)$$

It gives $f = 0.0198$. Then the total head loss, pressure drop, and the required pumping power input become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}=\left((0.0198)\frac{40\text{ m}}{0.012\text{ m}}+6\times0.9+2\times0.2\right)\frac{(2.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=22.9\text{ m}$$

$$\Delta P=\rho g h_L=(983.3\text{ kg/m}^3)(9.81\text{ m/s}^2)(22.9\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=221\text{ kPa}$$

$$\dot{W}_{\text{elect}}=\frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(2.83\times10^{-4}\text{ m}^3/\text{s})(221\text{ kPa})}{0.70}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{0.893\text{ kW}}$$

Therefore, the required power input of the recirculating pump is 0.893 kW

Discussion It can be shown that the required pumping power input for the recirculating pump is 0.821 kW when the minor losses are not considered. Therefore, the minor losses can be neglected in this case without a major loss in accuracy.

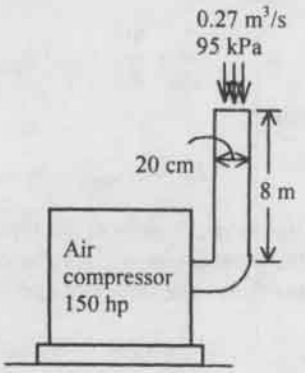
Review Problems

12-81 A compressor takes in air at a specified rate at the outdoor conditions. The useful power used by the compressor to overcome the frictional losses in the duct is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors, and thus minor losses are negligible. **5** The flow section involves no work devices such as fans or turbines.

Properties The properties of air at 1 atm = 101.3 kPa and 17°C = 290 K are ρ_0 = 1.224 kg/m^3 and μ = 1.80×10^{-5} kg/m·s (Table A-18). The roughness of galvanized iron surfaces is ε = 0.00015 m (Fig. A-27). The dynamic viscosity is independent of pressure, but density of an ideal gas is proportional to pressure. The density of air at 95 kPa is $\rho = (P/P_0)\rho_0 = (95/101.3)(1.224\,\text{kg/m}^3) = 1.148\,\text{kg/m}^3$.

Analysis The mean velocity and the Reynolds number are



$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.27\,\text{m}^3/\text{s}}{\pi(0.20\,\text{m})^2/4} = 8.594\,\text{m/s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(1.148\,\text{kg/m}^3)(8.594\,\text{m/s})(0.20\,\text{m})}{1.80\times10^{-5}\,\text{kg/m}\cdot\text{s}} = 196\times10^5$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D_h = \frac{1.5\times10^{-4}\,\text{m}}{0.20\,\text{m}} = 7.5\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{7.5\times10^{-4}}{3.7} + \frac{2.51}{1.96\times10^5\sqrt{f}}\right)$$

It gives f = 0.0200. Then the pressure drop in the duct and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0200\frac{8\,\text{m}}{0.20\,\text{m}}\frac{(1.148\,\text{kg/m}^3)(8.594\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{Pa}}{1\,\text{N/m}^2}\right) = 33.9\,\text{Pa}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.27\,\text{m}^3/\text{s})(33.9\,\text{Pa})\left(\frac{1\,\text{W}}{1\,\text{Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{9.16\,W}$$

Discussion Note hat the pressure drop in the duct and the power needed to overcome it is very small (relative to 150 hp), and can be disregarded.

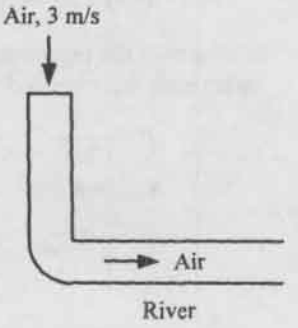
The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.0199, which is sufficiently close to 0.0200. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency (but probably no more than 20 W).

12-82 Air enters the underwater section of a circular duct. The fan power needed to overcome the flow resistance in this section of the duct is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors. **5** The flow section involves no work devices such as fans or turbines. **6** The pressure of air is 1 atm.

Properties The properties of air at 1 atm and 17°C = 290 K are $\rho = 1.224$ kg/m^3 and $\mu = 1.80\times10^{-5}$ kg/m·s (Table A-18). The roughness of stainless steel pipes is $\varepsilon = 0.000005$ m (Fig. A-27).

Analysis The volume flow rate and the Reynolds number are



$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m(\pi D^2/4) = (3\text{ m/s})[\pi(0.20\text{ m})^2/4] = 0.0942\text{ m}^3/\text{s}$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D_h}{\mu} = \frac{(1.224\text{ kg/m}^3)(3\text{ m/s})(0.20\text{ m})}{1.80\times10^{-5}\text{ kg/m}\cdot\text{s}} = 40{,}800$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D_h = \frac{5\times10^{-6}\text{ m}}{0.20\text{ m}} = 2.5\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.5\times10^{-5}}{3.7} + \frac{2.51}{40{,}800\sqrt{f}}\right)$$

It gives $f = 0.0220$. Then the pressure drop in the duct and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0220\frac{15\text{ m}}{0.2\text{ m}}\frac{(1.224\text{ kg/m}^3)(3\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ Pa}}{1\text{ N/m}^2}\right) = 9.09\text{ Pa}$$

$$\dot{W}_{\text{pump}} = \frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}} = \frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}} = \frac{(0.0942\text{ m}^3/\text{s})(9.09\text{ Pa})}{0.55} = \left(\frac{1\text{ W}}{1\text{ Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{1.6\text{ W}}$$

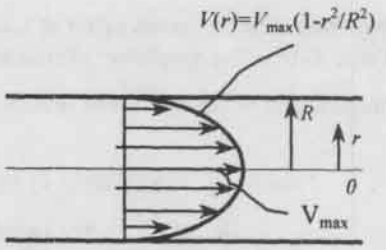
Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give $f = 0.0218$, which is sufficiently close to 0.0220. Assuming the pipe to be smooth would give 0.0219 for the friction factor, which is almost identical to the f value obtained from the Colebrook relation. Therefore, the duct can be treated as being smooth with negligible error.

12-83 The velocity profile in fully developed laminar flow in a circular pipe is given. The radius of the pipe, the mean velocity, and the maximum velocity are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$\mathbf{V}(r) = \mathbf{V}_{\max}\left(1-\frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$\mathbf{V}(r) = 6(1-0.01r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, and the mean velocity to be

$$R^2 = \frac{1}{100} \quad \rightarrow \quad R = \mathbf{0.10\ m}$$

$$\mathbf{V}_{\max} = \mathbf{6\ m/s}$$

$$\mathbf{V}_m = \frac{\mathbf{V}_{\max}}{2} = \frac{4\ \text{m/s}}{2} = \mathbf{2\ m/s}$$

12-84E The velocity profile in fully developed laminar flow in a circular pipe is given. The volume flow rate, the pressure drop, and the useful pumping power required to overcome this pressure drop are to be determined.

Assumptions **1** The flow is steady, laminar, and fully developed. **2** The pipe is horizontal.

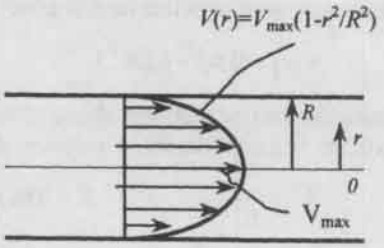
Properties The density and dynamic viscosity of water at 40°F are $\rho = 62.42$ lbm/ft^3 and $\mu = 3.74$ lbm/ft·h $= 1.039\times10^{-3}$ lbm/ft·s, respectively.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$\mathbf{V}(r) = \mathbf{V}_{\max}\left(1-\frac{r^2}{R^2}\right)$$

The velocity profile in this case is given by

$$\mathbf{V}(r) = 0.8(1-625r^2)$$



Comparing the two relations above gives the pipe radius, the maximum velocity, and the mean velocity to be

$$R^2 = \frac{1}{625} \quad \rightarrow \quad R = 0.04 \text{ ft}$$

$$\mathbf{V}_{\max} = 0.8 \text{ ft/s}$$

$$\mathbf{V}_m = \frac{\mathbf{V}_{\max}}{2} = \frac{0.8 \text{ ft/s}}{2} = 0.4 \text{ ft/s}$$

Then the volume flow rate and the pressure drop become

$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m(\pi R^2) = (0.4 \text{ ft/s})[\pi(0.04 \text{ ft})^2] = \mathbf{0.00201 \text{ ft}^3/\text{s}}$$

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128 \mu L} \quad \rightarrow \quad 0.00201 \text{ ft}^3/\text{s} = \frac{(\Delta P)\pi(0.08 \text{ ft})^4}{128(1.039\times10^{-3} \text{ lbm/ft}\cdot\text{s})(80 \text{ ft})}\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right)$$

It gives

$$\Delta P = 5.16 \text{ lbf/ft}^2 = \mathbf{0.0358 \text{ psi}}$$

Then the useful pumping power requirement becomes

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.00201 \text{ ft}^3/\text{s})(5.16 \text{ lbf/ft}^2)\left(\frac{1 \text{ W}}{0.737 \text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.014 \text{ W}}$$

Checking The flow was assumed to be laminar. To verify this assumption, we determine the Reynolds number:

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(62.42 \text{ lbm/ft}^3)(0.4 \text{ ft/s})(0.08 \text{ ft})}{1.039\times10^{-3} \text{ lbm/ft}\cdot\text{s}} = 1922$$

which is less than 2300. Therefore, the flow is laminar.

Discussion Note that the pressure drop across the water pipe and the required power input to maintain flow is negligible. This is due to the very low flow velocity. Such water flows are the exception in practice rather than the rule.

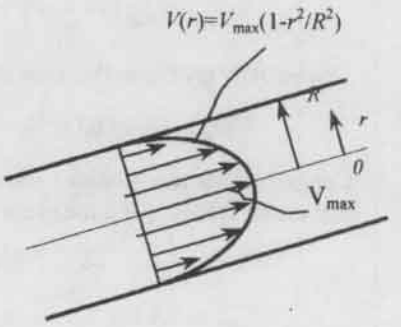
12-85E The velocity profile in fully developed laminar flow in a circular pipe is given. The volume flow rate, the pressure drop, and the useful pumping power required to overcome this pressure drop are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Properties The density and dynamic viscosity of water at 40°F are $\rho = 62.42$ lbm/ft^3 and $\mu = 3.74$ lbm/ft·h $= 1.039\times10^{-3}$ lbm/ft·s, respectively.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$\mathbf{V}(r) = \mathbf{V}_{\max}\left(1-\frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$\mathbf{V}(r) = 0.8(1-625r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, the mean velocity, and the volume flow rate to be

$$R^2 = \frac{1}{625} \quad \rightarrow \quad R = 0.04 \text{ ft}$$

$$\mathbf{V}_{\max} = 0.8 \text{ ft/s}$$

$$\mathbf{V}_m = \frac{\mathbf{V}_{\max}}{2} = \frac{0.8 \text{ ft/s}}{2} = 0.4 \text{ ft/s}$$

$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m(\pi R^2) = (0.4 \text{ ft/s})[\pi(0.04 \text{ ft})^2] = \mathbf{0.00201 \text{ ft}^3/\text{s}}$$

For uphill flow with an inclination of 12°, we have $\theta = +12°$, and

$$\rho g L \sin\theta = (62.42 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(80 \text{ ft})\sin 12°\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = 1038 \text{ lbf/ft}^2$$

$$\dot{V}_{\text{uphill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L} \rightarrow 0.00201 \text{ ft}^3/\text{s} = \frac{(\Delta P - 1038)\pi(0.08 \text{ ft})^4}{128(1.039\times10^{-3} \text{ lbm/ft}\cdot\text{s})(80 \text{ ft})}\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right)$$

It gives

$$\Delta P = 1043 \text{ lbf/ft}^2 = 7.24 \text{ psi}$$

Then the useful pumping power requirement becomes

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.00201 \text{ ft}^3/\text{s})(1043 \text{ lbf/ft}^2)\left(\frac{1 \text{ W}}{0.737 \text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{2.84 \text{ W}}$$

Checking The flow was assumed to be laminar. To verify this assumption, we determine the Reynolds number:

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(62.42 \text{ lbm/ft}^3)(0.4 \text{ ft/s})(0.08 \text{ ft})}{1.039\times10^{-3} \text{ lbm/ft}\cdot\text{s}} = 1922$$

which is less than 2300. Therefore, the flow is laminar.

Discussion Note that the pressure drop across the water pipe and the required power input to maintain flow is negligible. This is due to the very low flow velocity. Such water flows are the exception in practice rather than the rule.

12-86 Water is discharged from a water reservoir through a circular pipe of diameter D at the side wall at a vertical distance H from the free surface with a reentrant section. A relation for the "equivalent diameter" of the reentrant pipe for use in relations for frictionless flow through a hole is to be obtained.

Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **3** The water level in the reservoir remains constant. **4** The pipe is horizontal. **5** The entrance effects are negligible, and thus the flow is fully developed and the friction factor f is constant.

Properties The loss coefficient is $K_L = 0.8$ for the reentrant section, and $K_L = 0$ for the "frictionless" flow (Table 12-4).

Analysis We take point 1 at the free surface of the reservoir and point 2 at the exit of the pipe, which is also taken as the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the reservoir is zero ($\mathbf{V}_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad H=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where

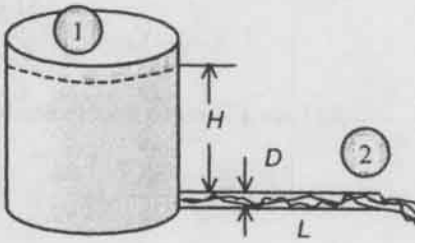
$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_2^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{\mathbf{V}_2^2}{2g}$$

since the diameter of the pipe is constant. Substituting and solving for $\mathbf{V}_2$ gives

$$H=\frac{\mathbf{V}_2^2}{2g}+\left(f\frac{L}{D}+K_L\right)\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2gH}{1+fL/D+K_L}}$$

Then the volume flow rate becomes

$$\dot{V}=A_c\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+fL/D+K_L}} \qquad (1)$$



Note that in the special case of $K_L = 0$ and $f = 0$ (frictionless flow), the velocity relation reduces to the Toricelli equation, $\mathbf{V}_{2,\text{frictionless}}=\sqrt{2gz_1}$. The flow rate in this case through a hole of D_e (equivalent diameter) is

$$\dot{V}=A_{c,\text{equiv}}\mathbf{V}_{2,\text{frictionless}}=\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH} \qquad (2)$$

Setting Eqs. (1) and (2) equal to each other gives the desired relation for the equivalent diameter,

$$\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH}=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+fL/D+K_L}}$$

which gives

$$D_{\text{equiv}}=\frac{D}{(1+fL/D+K_L)^{1/4}}=\frac{D}{(1+0.023\times 10/0.04+0.8)^{1/4}}=0.60D=0.024\ \text{m}$$

Discussion Note that the effect of frictional losses of a pipe with a reentrant section is to reduce the diameter by about 40% in this case. Also, noting that the flow rate is proportional to the square of the diameter, we have $\dot{V}\propto D_{\text{equiv}}^2=(0.60D)^2=0.36D^2$. Therefore, the flow rate through a sharp-edged entrance is about two-thirds less compared to the frictionless flow case.

12-87 A water tank open to the atmosphere is initially filled with water. The tank is drained to the atmosphere through a 90° horizontal bend of negligible length. The flow rate is to be determined for the cases of the bend being a flanged smooth bend and a miter bend without vanes.

Assumptions **1** The flow is steady and incompressible. **2** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **3** The water level in the tank remains constant. **4** The length of the bend and thus the frictional loss associated with its length is negligible. **5** The entrance is well-rounded, and the entrance loss is negligible.

Properties The loss coefficient is $K_L = 0.3$ for a flanged smooth bend and $K_L = 1.1$ for a miter bend without vanes (Table 12-4).

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the bend, which is also taken as the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \equiv 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L \dfrac{\mathbf{V}^2}{2g}$. Substituting and solving for $\mathbf{V}_2$ gives

$$z_1=\frac{\mathbf{V}_2^2}{2g}+K_L\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad 2gz_1=\mathbf{V}_2^2(1+K_L) \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2gz_1}{1+K_L}}$$

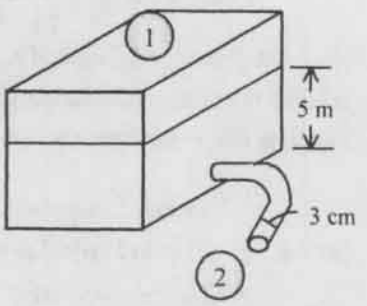
Then the flow rate becomes

$$\dot{V}=A_{\text{pipe}}\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{1+K_L}}$$

(*a*) **Case 1** Flanged smooth bend ($K_L = 0.3$):

$$\dot{V}=A_c\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{1+K_L}}=\frac{\pi(0.03\,\text{m})^2}{4}\sqrt{\frac{2(9.81\,\text{m/s}^2)(5\,\text{m})}{1+0.3}}$$

$$=0.00614\,\text{m}^3/\text{s}$$



(*b*) **Case 1** Miter bend without vanes ($K_L = 1.1$):

$$\dot{V}=A_c\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{1+K_L}}=\frac{\pi(0.03\,\text{m})^2}{4}\sqrt{\frac{2(9.81\,\text{m/s}^2)(5\,\text{m})}{1+1.1}}$$

$$=0.00483\,\text{m}^3/\text{s}$$

Discussion Note that the type of bend used has a significant effect on the flow rate, and a conscious effort should be made when selecting components in a piping system.

12-88 The piping system of a geothermal district heating system is being designed. The pipe diameter that will optimize the initial system cost and the energy cost is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses, the only significant energy loss arises from pipe friction. **4** The piping system is horizontal. **5** The properties of geothermal water are the same as fresh water. **6** The friction factor is constant at the given value. **7** The interest rate, the inflation rate, and the salvage value of the system are all zero. **8** The flow rate through the system remains constant.

Properties We take the density of water to be $\rho = 1000$ kg/m^3. The friction factor is given to be $f = 0.015$.

Analysis The system operates in a loop, and thus we can take any point in the system as points 1 and 2 (the same point), and thus $z_1 = z_2$, $\mathbf{V}_1 = \mathbf{V}_2$, and $P_1 = P_2$. Then the energy equation for this piping system simplifies to

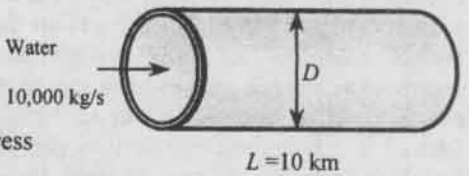
$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_{\text{pump,u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D can be expressed as

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2}$$

The flow rate of geothermal water is

$$\dot{V}=\frac{\dot{m}}{\rho}=\frac{10{,}000\ \text{kg/s}}{1000\ \text{kg/m}^3}=10\ \text{m}^3/\text{s}$$



To expose the dependence of pressure drop on diameter, we express it in terms of the flow rate as

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2}=f\frac{L}{D}\frac{\rho}{2}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2=f\frac{16L}{D}\frac{\rho \dot{V}^2}{2\pi^2 D^4}=f\frac{8L}{D^5}\frac{\rho \dot{V}^2}{\pi^2}$$

Then the required pumping power can be expressed as

$$\dot{W}_{\text{pump}}=\frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{\dot{V}}{\eta_{\text{pump-motor}}}f\frac{8L}{D^5}\frac{\rho \dot{V}^2}{\pi^2}=f\frac{8L}{D^5}\frac{\rho \dot{V}^3}{\eta_{\text{pump-motor}}\pi^2}$$

Note that the pumping power requirement is proportional to f and L, consistent with our intuitive expectation. Perhaps not so obvious is that power is proportional to the *cube* of flow rate. The fact that the *power is inversely proportional to pipe diameter D to the fifth power* means that a slight increase in pipe diameter will manifest as a tremendous reduction in power dissipation due to friction in a long pipeline. Substituting the given values and expressing the diameter D in meters,

$$\dot{W}_{\text{pump}}=(0.015)\frac{8(10{,}000\ \text{m})}{D^5\ \text{m}^5}\frac{(1000\ \text{kg/m}^3)(10\ \text{m}^3/\text{s})^3}{\pi^2(0.80)}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right)=\frac{1.52\times10^5}{D^5}\ \text{kW}$$

The number of hours in one year are 24×365 = 8760 h. Then the total amount of electric power used and its cost per year are

$$E_{\text{pump}}=\dot{W}_{\text{pump}}\Delta t=\frac{1.52\times10^5}{D^5}(8760\ \text{h})=\frac{1.332\times10^9}{D^5}\ \text{kWh/yr}$$

$$\text{Energy cost}=E_{\text{pump}}\times\text{Unit cost}=\left(\frac{1.332\times10^9}{D^5}\ \text{kWh/y}\right)(\$0.06/\text{kWh})=\frac{7.99\times10^7}{D^5}\ \$/\text{yr}$$

The installation cost of the system with a 30-year lifetime is given to be Cost = \$10^6 D^2 where D is in meters. The annual cost of the system is then 1/30$^{\text{th}}$ of it, which is

$$\text{System cost} = \frac{\text{Total cost}}{\text{Life time}} = \frac{\$10^6 D^2}{30\ \text{yr}} = \$3.33\times10^4 D^2\ \text{(per year)}$$

Then the total annual cost of the system (installation + operating) becomes

$$\text{Total cost} = \text{Energy cost} + \text{System cost} = \frac{7.99\times10^7}{D^5} + 3.33\times10^4 D^2 \quad \$/\text{yr}$$

The optimum pipe diameter is the value that minimizes this total, and it is determined by taking the derivative of the total cost with respect to D and setting it equal to zero,

$$\frac{\partial(\text{Total cost})}{\partial D} = -5\times\frac{7.99\times10^7}{D^6} + 2\times3.33\times10^4 D = 0$$

Simplifying gives $D^7 = 5998$ whose solution is

$$D = \mathbf{3.5\ m}$$

This is the **optimum** pipe diameter that minimizes the total cost of the system under stated assumptions. A larger diameter pipe will increase the system cost more than it decreases the energy cost, and a smaller diameter pipe will increase the system cost more than it decreases the energy cost.

Discussion The assumptions of zero interest and zero inflation are not realistic, and an actual economic analysis must consider these factors as they have a major effect on the pipe diameter. This is done by considering the time value of money, and expressing all the costs at the same time. Pipe purchase is a present cost, and energy expenditures are future annual costs spread out over the project lifetime. Thus, to provide consistent dollar comparisons between initial and future costs, all future energy costs must be expressed as a single present lump sum to reflect the time-value of money. Then we can compare pipe and energy costs on a consistent basis. Economists call the necessary factor the "Annuity Present Value Factor", F. If interest rate is 10% per year with n = 30 years, then F = 9.427. Thus, if power costs $1,000,000/year for the next 30 years, then the present value of those future payments is $9,427,000 (and not $30,000,000!) if money is worth 10%. Alternatively, if you must pay $1,000,000 every year for 30 years, and you can today invest $9,437,000 at 10%, then you can meet 30 years of payments at the end of each year. The energy cost in this case can be determined by dividing the energy cost expression developed above by 9.427. This will result in a pipe diameter of D = 2.5 m. In an actual design, we also need to calculate the mean flow velocity and the pressure head to make sure that they are reasonable. For a pipe diameter of 2.5 m, for example, the mean flow velocity is 1.47 m/s and the pump pressure head is 5.6 m.

12-89 Water is drained from a large reservoir through two pipes connected in series at a specified rate using a pump. The required pumping head and the minimum pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The pipes are horizontal. **3** The entrance effects are negligible, and thus the flow is fully developed. **4** The flow is turbulent so that the tabulated value of the loss coefficients can be used. **5** The pipes involve no components such as bends, valves, and other connectors that cause additional minor losses. **6** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **7** The water level in the reservoir remains constant.

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1$ kg/m^3 and $\mu = 1.138\times10^{-3}$ kg/m·s (Table A-15). The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance (Table 12-4). The roughness of cast iron pipes is $\varepsilon = 0.00026$ m (Fig. A-27).

Analysis We take point 1 at the free surface of the tank, and point 2 and the reference level at the centerline of the pipe ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the tank is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad z_1+h_{\text{pump,u}}=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\sum\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g}$$

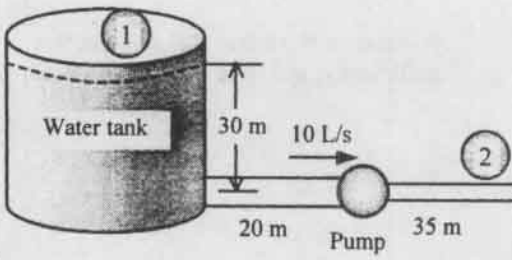
and the summation is over two pipes. Noting that the two pipes are connected in series and thus the flow rate through each of them is the same, the head loss for each pipe is determined as follows (we designate the first pipe by 1 and the second one by 2):

Pipe 1: $\mathbf{V}_1=\dfrac{\dot V}{A_{c1}}=\dfrac{\dot V}{\pi D_1^2/4}=\dfrac{0.01\,\text{m}^3/\text{s}}{\pi(0.06\,\text{m})^2/4}=3.537\,\text{m/s}$

$$\text{Re}_1=\frac{\rho \mathbf{V}_1 D_1}{\mu}=\frac{(999.1\,\text{kg/m}^3)(3.537\,\text{m/s})(0.06\,\text{m})}{1.138\times10^{-3}\,\text{kg/m}\cdot\text{s}}=186{,}300$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D_1=\frac{0.00026\,\text{m}}{0.06\,\text{m}}=0.00433$$



The friction factor corresponding to this relative roughness and the Reynolds number is, from the Colebrook equation,

$$\frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{\varepsilon/D_1}{3.7}+\frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{0.00433}{3.7}+\frac{2.51}{186{,}300\sqrt{f_1}}\right)$$

It gives $f_1 = 0.0297$. The only minor loss is the entrance loss, which is $K_L = 0.5$. Then the total head loss of the first pipe becomes

$$h_{L1}=\left(f_1\frac{L_1}{D_1}+\sum K_L\right)\frac{\mathbf{V}_1^2}{2g}=\left((0.0297)\frac{20\,\text{m}}{0.06\,\text{m}}+0.5\right)\frac{(3.537\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=6.63\,\text{m}$$

Pipe 2: $\mathbf{V}_2=\dfrac{\dot V}{A_{c2}}=\dfrac{\dot V}{\pi D_2^2/4}=\dfrac{0.01\,\text{m}^3/\text{s}}{\pi(0.04\,\text{m})^2/4}=7.958\,\text{m/s}$

$$\text{Re}_2=\frac{\rho \mathbf{V}_2 D_2}{\mu}=\frac{(999.1\,\text{kg/m}^3)(7.958\,\text{m/s})(0.04\,\text{m})}{1.138\times10^{-3}\,\text{kg/m}\cdot\text{s}}=279{,}500$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D_2 = \frac{0.00026\,\text{m}}{0.04\,\text{m}} = 0.0065$$

The friction factor corresponding to this relative roughness and the Reynolds number is, from the Colebrook equation,

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon / D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \rightarrow \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{0.0065}{3.7} + \frac{2.51}{279{,}500\sqrt{f_2}}\right)$$

It gives $f_2 = 0.0332$. The second pipe involves no minor losses. Note that we do not consider the exit loss unless the exit velocity is dissipated within the system considered (in this case it is not). Then the head loss for the second pipe becomes

$$h_{L2} = f_2\frac{L_2}{D_2}\frac{\mathbf{V}_2^2}{2g} = (0.0332)\frac{35\,\text{m}}{0.04\,\text{m}}\frac{(7.958\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 93.8\,\text{m}$$

The total head loss for two pipes connected in series is the sum of the head losses of the two pipes,

$$h_L = h_{L,\text{total}} = h_{L1} + h_{L2} = 6.63 + 93.8 = 100.43\,\text{m}$$

Then the pumping head and the minimum pumping power required (the pumping power in the absence of any inefficiencies of the pump and the motor, which is equivalent to the useful pumping power) become

$$h_{\text{pump,u}} = \frac{\mathbf{V}_2^2}{2g} + h_L - z_1 = \frac{(7.958\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} + 100.43 - 30 = 73.66\,\text{m}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = \rho\dot{V}gh_{\text{pump,u}}$$

$$= (999.1\,\text{kg/m}^3)(0.01\,\text{m}^3/\text{s})(9.81\,\text{m/s}^2)(73.66\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kW}}{1\,\text{kN}\cdot\text{m/s}}\right) = 7.22\,\text{kW}$$

Therefore, the pump must supply a minimum of 7.22 kW of mechanical energy to water.

Discussion Note that the shaft power of the pump must be greater than this to account for the pump inefficiency, and the electrical power supplied must be even greater to account for the motor inefficiency.

12-90 Two pipes of identical diameter and material are connected in parallel. The length of one of the pipes is twice the length of the other. The ratio of the flow rates in the two pipes is to be determined. √

Assumptions **1** The flow is steady and incompressible. **2** The flow is fully turbulent in both pipes and thus the friction factor is independent of the Reynolds number (it is the same for both pipes since they have the same material and diameter). **3** The minor losses are negligible.

Analysis When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D can be expressed as

$$h_L = f\frac{L}{D}\frac{\mathbf{V}^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{A_c}\right)^2 = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2 = 8f\frac{L}{D}\frac{1}{g}\frac{\dot{V}^2}{\pi^2 D^4} = 8f\frac{L}{g\pi^2}\frac{\dot{V}^2}{D^5}$$

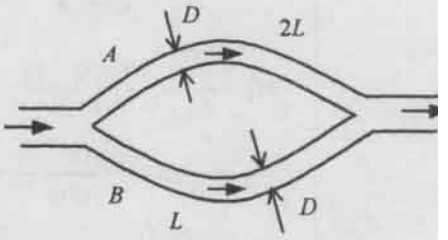
Solving for the flow rate gives

$$\dot{V} = \sqrt{\frac{\pi^2 h_L g D^5}{8fL}} = \frac{k}{\sqrt{L}} \quad (k \text{ is a constant})$$

When the pipe diameter, friction factor, and the head loss is constant, which is the case here for parallel connection, the flow rate becomes inversely proportional to the square root of length L. Therefore, when the length is doubled, the flow rate will decrease by a factor of $2^{0.5}$ =0.707 since

If $$\dot{V}_A = \frac{k}{\sqrt{L_A}}$$

Then $$\dot{V}_B = \frac{k}{\sqrt{L_B}} = \frac{k}{\sqrt{2L_A}} = \frac{k}{\sqrt{2}\sqrt{L_A}} = \frac{\dot{V}_A}{\sqrt{2}} = 0.707\dot{V}_A$$



Therefore, the ratio of the flow rates in the two pipes is **0.707**.

12-91 A pipeline that transports oil at a specified rate branches out into two parallel pipes made of commercial steel that reconnects downstream. The flow rates through each of the parallel pipes are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are given to be negligible. **4** Flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of oil at 40°C are $\rho = 876$ kg/m^3 and $\mu = 0.212$ kg/m·s (Table A-16). The roughness of commercial steel pipes is $\varepsilon = 0.000045$ m (Fig. A-27).

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Below we will set up the equations to be solved by an equation solver. The head loss in two parallel branches must be the same, and the total flow rate must the sum of the flow rates in the parallel branches. Therefore,

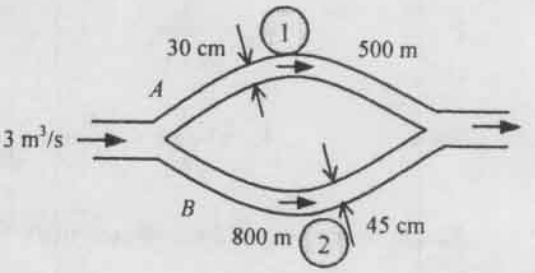
$$h_{L,1} = h_{L,2} \qquad (1)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \;\rightarrow\; \dot{V}_1 + \dot{V}_2 = 3 \qquad (2)$$

We designate the 30-cm diameter pipe by 1 and the 45-cm diameter pipe by 2. The average velocity, the relative roughness, the Reynolds number, friction factor, and the head loss in each pipe are expressed as

$$\mathbf{V}_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \;\rightarrow\; \mathbf{V}_1 = \frac{\dot{V}_1}{\pi(0.30\text{m})^2/4} \qquad (3)$$

$$\mathbf{V}_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \;\rightarrow\; \mathbf{V}_2 = \frac{\dot{V}_2}{\pi(0.45\text{m})^2/4} \qquad (4)$$



$$\text{rf}_1 = \frac{\varepsilon_1}{D_1} = \frac{4.5\times10^{-5}\text{ m}}{0.30\text{ m}} = 1.5\times10^{-4}$$

$$\text{rf}_2 = \frac{\varepsilon_2}{D_2} = \frac{4.5\times10^{-5}\text{ m}}{0.45\text{ m}} = 1\times10^{-4}$$

$$\text{Re}_1 = \frac{\rho \mathbf{V}_1 D_1}{\mu} \;\rightarrow\; \text{Re}_1 = \frac{(876\text{ kg/m}^3)\mathbf{V}_1(0.30\text{ m})}{0.212\text{ kg/m}\cdot s} \qquad (5)$$

$$\text{Re}_2 = \frac{\rho \mathbf{V}_2 D_2}{\mu} \;\rightarrow\; \text{Re}_2 = \frac{(876\text{ kg/m}^3)\mathbf{V}_2(0.45\text{ m})}{0.212\text{ kg/m}\cdot s} \qquad (6)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{1.5\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad (7)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{1\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad (8)$$

$$h_{L,1} = f_1\frac{L_1}{D_1}\frac{\mathbf{V}_1^2}{2g} \;\rightarrow\; h_{L,1} = f_1\frac{500\text{ m}}{0.30\text{ m}}\frac{\mathbf{V}_1^2}{2(9.81\text{ m/s}^2)} \qquad (9)$$

$$h_{L,2} = f_2\frac{L_2}{D_2}\frac{\mathbf{V}_2^2}{2g} \;\rightarrow\; h_{L,2} = f_2\frac{800\text{ m}}{0.45\text{ m}}\frac{\mathbf{V}_2^2}{2(9.81\text{ m/s}^2)} \qquad (10)$$

This is a system of 10 equations in 10 unknowns, and their simultaneous solution by an equation solver gives

$$\dot{V}_1 = \mathbf{0.91\text{ m}^3/\text{s}},\quad \dot{V}_2 = \mathbf{0} = \mathbf{2.09\text{ m}^3/\text{s}},$$

$$\mathbf{V}_1 = 12.9\text{ m/s},\ \mathbf{V}_2 = 13.1\text{ m/s},\quad h_{L,1} = h_{L,2} = 390\text{ m}$$

$Re_1 = 15{,}960, \quad Re_2 = 24{,}430, \quad f_1 = 0.0277, \quad f_2 = 0.0249$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion This problem can also be solved by using a trial-and-error approach, but it will be very time consuming. Equations solvers such as EES are invaluable for theses kinds of problems.

12-92 The piping of a district heating system that transports hot water at a specified rate branches out into two parallel pipes made of commercial steel that reconnects downstream. The flow rates through each of the parallel pipes are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are given to be negligible. **4** Flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of water at 80°C are $\rho = 971.8$ kg/m^3 and $\mu = 0.355\times10^{-3}$ kg/m·s (Table A-15). The roughness of commercial steel pipes is $\varepsilon = 0.000045$ m (Fig. A-27).

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Below we will set up the equations to be solved by an equation solver. The head loss in two parallel branches must be the same, and the total flow rate must the sum of the flow rates in the parallel branches. Therefore,

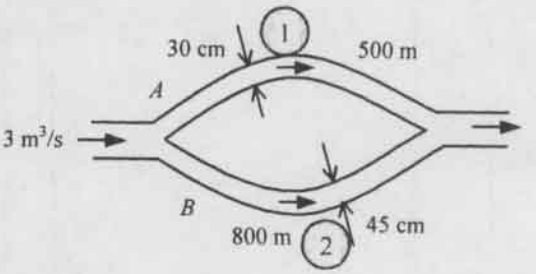
$$h_{L,1} = h_{L,2} \qquad (1)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \;\rightarrow\; \dot{V}_1 + \dot{V}_2 = 3 \qquad (2)$$

We designate the 30-cm diameter pipe by 1 and the 45-cm diameter pipe by 2. The average velocity, the relative roughness, the Reynolds number, friction factor, and the head loss in each pipe are expressed as

$$\mathbf{V}_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \;\rightarrow\; \mathbf{V}_1 = \frac{\dot{V}_1}{\pi(0.30\text{m})^2/4} \qquad (3)$$

$$\mathbf{V}_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \;\rightarrow\; \mathbf{V}_2 = \frac{\dot{V}_2}{\pi(0.45\text{m})^2/4} \qquad (4)$$



$$\text{rf}_1 = \frac{\varepsilon_1}{D_1} = \frac{4.5\times10^{-5}\text{ m}}{0.30\text{ m}} = 1.5\times10^{-4}$$

$$\text{rf}_2 = \frac{\varepsilon_2}{D_2} = \frac{4.5\times10^{-5}\text{ m}}{0.45\text{ m}} = 1\times10^{-4}$$

$$\text{Re}_1 = \frac{\rho \mathbf{V}_1 D_1}{\mu} \;\rightarrow\; \text{Re}_1 = \frac{(971.8\text{ kg/m}^3)\mathbf{V}_1(0.30\text{ m})}{0.355\times10^{-3}\text{ kg/m}\cdot s} \qquad (5)$$

$$\text{Re}_2 = \frac{\rho \mathbf{V}_2 D_2}{\mu} \;\rightarrow\; \text{Re}_2 = \frac{(971.8\text{ kg/m}^3)\mathbf{V}_2(0.45\text{ m})}{0.355\times10^{-3}\text{ kg/m}\cdot s} \qquad (6)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{1.5\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad (7)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{1\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad (8)$$

$$h_{L,1} = f_1\frac{L_1}{D_1}\frac{\mathbf{V}_1^2}{2g} \;\rightarrow\; h_{L,1} = f_1\frac{500\text{ m}}{0.30\text{ m}}\frac{\mathbf{V}_1^2}{2(9.81\text{ m/s}^2)} \qquad (9)$$

$$h_{L,2} = f_2\frac{L_2}{D_2}\frac{\mathbf{V}_2^2}{2g} \;\rightarrow\; h_{L,2} = f_2\frac{800\text{ m}}{0.45\text{ m}}\frac{\mathbf{V}_2^2}{2(9.81\text{ m/s}^2)} \qquad (10)$$

This is a system of 10 equations in 10 unknowns, and their simultaneous solution by an equation solver gives

$$\dot{V}_1 = \mathbf{0.92\text{ m}^3/\text{s}}, \quad \dot{V}_2 = \mathbf{0 = 2.08\text{ m}^3/\text{s}},$$

$V_1 = 13.0$ m/s, $V_2 = 13.1$ m/s, $h_{L,1} = h_{L,2} = 188$ m

$Re_1 = 1.067 \times 10^7$, $Re_2 = 1.612 \times 10^7$, $f_1 = 0.0131$, $f_2 = 0.0121$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion This problem can also be solved by using a trial-and-error approach, but it will be very time consuming. Equations solvers such as EES are invaluable for these kinds of problems.

12-93E A water fountain is to be installed at a remote location by attaching a cast iron pipe directly to a water main. For a specified flow rate, the minimum diameter of the piping system is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The pressure at the water main remains constant. **4** There are no dynamic pressure effects at the pipe-water main connection, and the pressure at the pipe entrance is 60 psia.

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s (Table A-15E). The roughness of cast iron pipe is $\varepsilon = 0.00085$ ft (Fig. A-27). The minor loss coefficient is $K_L = 0.5$ for a sharp-edged entrance, $K_L = 1.1$ for a 90° miter bend without vanes, $K_L = 0.2$ for a fully open gate valve, and $K_L = 5$ for an angle valve.

Analysis We choose point 1 in the water main near the entrance where the pressure is 60 psig and the velocity in the pipe direction (normal to main) is zero. We also take point 1 as the reference level. We take point 2 at the exit of the water fountain where the pressure is the atmospheric pressure ($P_2 = P_{atm}$) and the velocity is the discharge velocity. The energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad \frac{P_{1,\text{gage}}}{\rho g}=\frac{\mathbf{V}_2^2}{2g}+z_2+h_L$$

where $\mathbf{V}_2 = \mathbf{V}_m$ and

$$h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_m^2}{2g}$$

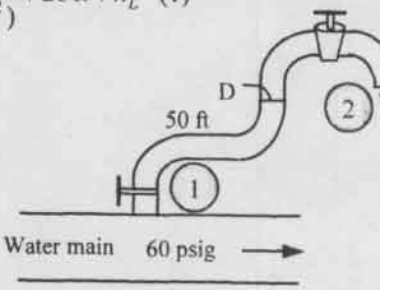
since the diameter of the piping system is constant. Then the energy equation becomes

$$\frac{60\text{ psi}}{(62.3\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)=\frac{\mathbf{V}_m^2}{2(32.2\text{ ft/s}^2)}+20\text{ ft}+h_L \quad (1)$$

The mean velocity in the pipe and the Reynolds number are

$$\mathbf{V}_m=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}\quad\rightarrow\quad \mathbf{V}_m=\frac{20/60\text{ gal/s}}{\pi D^2/4}\left(\frac{0.1337\text{ ft}^3}{1\text{ gal}}\right) \qquad (2)$$

$$\text{Re}=\frac{\rho \mathbf{V}_m D}{\mu}\quad\rightarrow\quad \text{Re}=\frac{(62.3\text{ lbm/ft}^3)\mathbf{V}_m D}{1.307\times10^{-3}\text{ lbm/ft}\cdot\text{s}} \qquad (3)$$



The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad \frac{1}{\sqrt{f}}=-2.0\log\left(\frac{0.00085/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad (4)$$

The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}} + 3K_{L,\text{elbow}} + K_{L,\text{gate valve}} + K_{L,\text{angle valve}} = 0.5+3\times1.1+0.2+5 = 9$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g}\quad\rightarrow\quad h_L=\left(f\frac{50\text{ ft}}{D}+9\right)\frac{\mathbf{V}_m^2}{2(32.2\text{ ft/s}^2)} \quad (5)$$

These are 5 equations in the 5 unknowns of $\mathbf{V}_m$, h_L, D, Re, and f, and solving them simultaneously using an equation solver such as EES gives

$$\mathbf{V}_m = 13.4\text{ ft/s},\quad h_L = 116\text{ ft},\quad D = 0.0652\text{ ft} = \mathbf{0.78\text{ in}},\quad \text{Re} = 41{,}500,\quad \text{and}\quad f = 0.0428$$

Therefore, the diameter of the pipe must be at least 0.78 in (or roughly ¾ in).

Discussion The pipe diameter can also be determined approximately by using the Swamee and Jain relation. It would give $D = 0.75$ in, which is within 5% of the result obtained above.

12-94E A water fountain is to be installed at a remote location by attaching a cast iron pipe directly to a water main. For a specified flow rate, the minimum diameter of the piping system is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The pressure at the water main remains constant. **4** There are no dynamic pressure effects at the pipe-water main connection, and the pressure at the pipe entrance is 60 psia.

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s (Table A-15E). The plastic pipes are considered to be smooth, and thus their roughness is $\varepsilon = 0$ (Fig. A-27). The minor loss coefficient is $K_L = 0.5$ for a sharp-edged entrance, $K_L = 1.1$ for a 90° miter bend without vanes, $K_L = 0.2$ for a fully open gate valve, and $K_L = 5$ for an angle valve.

Analysis We choose point 1 in the water main near the entrance where the pressure is 60 psig and the velocity in the pipe direction (normal to main) is zero. We also take point 1 as the reference level. We take point 2 at the exit of the water fountain where the pressure is the atmospheric pressure ($P_2 = P_{atm}$) and the velocity is the discharge velocity. The energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad \frac{P_{1,\text{gage}}}{\rho g}=\frac{\mathbf{V}_2^2}{2g}+z_2+h_L$$

where $\mathbf{V}_2 = \mathbf{V}_m$ and

$$h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_m^2}{2g}$$

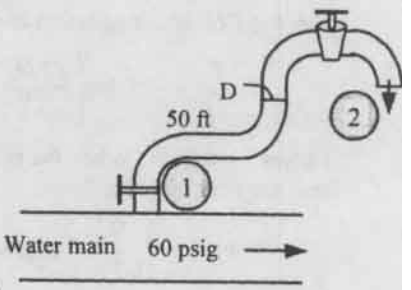
since the diameter of the piping system is constant. Then the energy equation becomes

$$\frac{60\text{ psi}}{(62.3\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)=\frac{\mathbf{V}_m^2}{2(32.2\text{ ft/s}^2)}+20\text{ ft}+h_L \quad (1)$$

The mean velocity in the pipe and the Reynolds number are

$$\mathbf{V}_m=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}\rightarrow \mathbf{V}_m=\frac{20/60\text{ gal/s}}{\pi D^2/4}\left(\frac{0.1337\text{ ft}^3}{1\text{ gal}}\right) \quad (2)$$

$$\text{Re}=\frac{\rho \mathbf{V}_m D}{\mu}\rightarrow \text{Re}=\frac{(62.3\text{ lbm/ft}^3)\mathbf{V}_m D}{1.307\times10^{-3}\text{ lbm/ft}\cdot\text{s}} \quad (3)$$



The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\rightarrow\frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad (4)$$

The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}}+3K_{L,\text{elbow}}+K_{L,\text{gate valve}}+K_{L,\text{angle valve}}=0.5+3\times1.1+0.2+5=9$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g}\rightarrow h_L=\left(f\frac{50\text{ ft}}{D}+9\right)\frac{\mathbf{V}_m^2}{2(32.2\text{ ft/s}^2)} \quad (5)$$

These are 5 equations in the 5 unknowns of $\mathbf{V}_m$, h_L, D, Re, and f, and solving them simultaneously using an equation solver such as EES gives

$$\mathbf{V}_m = 16.5\text{ ft/s},\quad h_L = 115\text{ ft},\quad D = 0.0587\text{ ft} = \mathbf{0.70\text{ in}},\quad \text{Re} = 46{,}100,\quad \text{and}\quad f = 0.0213$$

Therefore, the diameter of the pipe must be at least 0.70 in.

Discussion The pipe diameter can also be determined approximately by using the Swamee and Jain relation. It would give $D = 0.65$ in, which is within 7% of the result obtained above.

12-95 In a hydroelectric power plant, the flow rate of water, the available elevation head, and the combined turbine-generator efficiency are given. The electric power output of the plant is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The minor losses are given to be negligible. **4** The water level in the reservoir remains constant.

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998$ kg/m^3 and $\mu = 1.002\times10^{-3}$ kg/m·s (Table A-15). The roughness of cast iron pipes is $\varepsilon = 0.00026$ m (Fig. A-27).

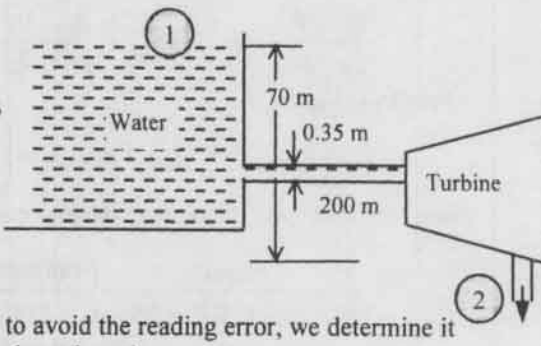
Analysis We take point 1 at the free surface of the reservoir, and point 2 and the reference level at the free surface of the water leaving the turbine site ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are very low ($\mathbf{V}_1 \cong \mathbf{V}_2 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_{\text{turbine}}=z_1-h_L$$

The mean velocity, Reynolds number, friction factor, and head loss in the pipe are

$$\mathbf{V}_m=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{0.8\,\text{m}^3/\text{s}}{\pi(0.35\,\text{m})^2/4}=8.315\,\text{m/s}$$

$$\text{Re}=\frac{\rho \mathbf{V}_m D_h}{\mu}=\frac{(998\,\text{kg/m}^3)(8.513\,\text{m/s})(0.35\,\text{m})}{1.002\times10^{-3}\,\text{kg/m}\cdot\text{s}}=2.899\times10^6$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D_h=\frac{0.00026\,\text{m}}{0.35\,\text{m}}=7.43\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right) \rightarrow \frac{1}{\sqrt f}=-2.0\log\left(\frac{7.43\times10^{-4}}{3.7}+\frac{2.51}{2.899\times10^6\sqrt f}\right)$$

It gives $f = 0.0184$. When the minor losses are negligible, the head loss in the pipe and the available turbine head are determined to be

$$h_L=f\frac{L}{D}\frac{\mathbf{V}_m^2}{2g}=0.0184\frac{200\,\text{m}}{0.35\,\text{m}}\frac{(8.315\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=37.05\,\text{m}$$

$$h_{\text{turbine}}=z_1-h_L=70-37.05=32.95\,\text{m}$$

Then the available and actual power output become

$$\dot W_{\text{turbine}}=\dot m g h_{\text{turbine}}=\rho \dot V g h_{\text{turbine}}$$

$$=(998\,\text{kg/m}^3)(0.8\,\text{m}^3/\text{s})(9.81\,\text{m/s}^2)(32.95\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kW}}{1\,\text{kN}\cdot\text{m/s}}\right)=258\,\text{kW}$$

$$\dot W_{\text{turbine-gen}}=\eta_{\text{turbine-gen}}\dot W_{\text{turbine}}=(0.75)(258\,\text{kW})=194\,\textbf{kW}$$

Discussion Note that a perfect turbine-generator would generate 258 kW of electricity from this resource. The power generated by the actual unit is only 194 kW because of the inefficiencies of the turbine and the generator. Also note that more than half of the elevation head is lost in piping due to pipe friction.

12-96 In a hydroelectric power plant, the flow rate of water, the available elevation head, and the combined turbine-generator efficiency are given. The percent increase in the electric power output of the plant is to be determined when the pipe diameter is doubled.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The minor losses are given to be negligible. **4** The water level in the reservoir remains constant.

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998$ kg/m^3 and $\mu = 1.002\times10^{-3}$ kg/m·s (Table A-15). The roughness of cast iron pipes is $\varepsilon = 0.00026$ m (Fig. A-27).

Analysis We take point 1 at the free surface of the reservoir, and point 2 and the reference level at the free surface of the water leaving the turbine site ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are very low ($\mathbf{V}_1 \cong \mathbf{V}_2 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad h_{\text{turbine}}=z_1-h_L$$

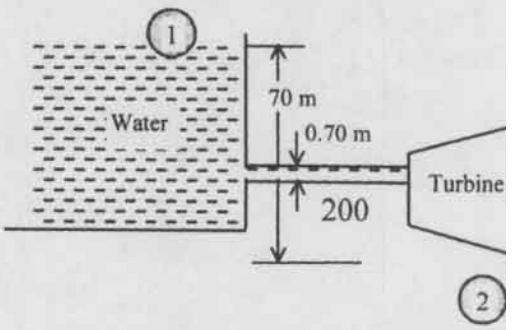
The mean velocity, Reynolds number, friction factor, and head loss in the pipe for both cases (pipe diameter being 0.35 m and 0.70 m) are

$$\mathbf{V}_{m1}=\frac{\dot V}{A_{c1}}=\frac{\dot V}{\pi D_1^2/4}=\frac{0.8\,\text{m}^3/\text{s}}{\pi(0.35\,\text{m})^2/4}=8.315\,\text{m/s},$$

$$\mathbf{V}_{m2}=\frac{\dot V}{A_{c2}}=\frac{\dot V}{\pi D_2^2/4}=\frac{0.8\,\text{m}^3/\text{s}}{\pi(0.70\,\text{m})^2/4}=2.079\,\text{m/s}$$

$$\text{Re}_1=\frac{\rho \mathbf{V}_{m1} D_1}{\mu}=\frac{(998\,\text{kg/m}^3)(8.315\,\text{m/s})(0.35\,\text{m})}{1.002\times10^{-3}\,\text{kg/m}\cdot\text{s}}=2.899\times10^6$$

$$\text{Re}_2=\frac{\rho \mathbf{V}_{m2} D_1}{\mu}=\frac{(998\,\text{kg/m}^3)(2.079\,\text{m/s})(0.70\,\text{m})}{1.002\times10^{-3}\,\text{kg/m}\cdot\text{s}}=1.449\times10^6$$



which are greater than 4000. Therefore, the flow is turbulent for both cases. The relative roughness of the pipe is

$$\varepsilon/D_1=\frac{0.00026\,\text{m}}{0.35\,\text{m}}=7.43\times10^{-4} \qquad \text{and} \qquad \varepsilon/D_2=\frac{0.00026\,\text{m}}{0.70\,\text{m}}=3.71\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{7.43\times10^{-4}}{3.7}+\frac{2.51}{2.899\times10^6\sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}}=-2.0\log\left(\frac{3.71\times10^{-4}}{3.7}+\frac{2.51}{1.449\times10^6\sqrt{f_2}}\right)$$

The friction factors are determined to be $f_1 = 0.0184$ and $f_2 = 0.0161$. When the minor losses are negligible, the head losses in the pipes and the available turbine heads are determined to be

$$h_{L1}=f_1\frac{L}{D_1}\frac{\mathbf{V}_{m1}^2}{2g}=0.0184\frac{200\,\text{m}}{0.35\,\text{m}}\frac{(8.315\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=37.05\,\text{m}$$

$$h_{L2}=f_2\frac{L}{D_2}\frac{\mathbf{V}_{m2}^2}{2g}=0.0161\frac{200\,\text{m}}{0.70\,\text{m}}\frac{(2.079\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=1.01\,\text{m}$$

$$h_{\text{turbine},1} = z_1 - h_{L1} = 70 - 37.05 = 32.95\,\text{m}$$

$$h_{\text{turbine},2} = z_2 - h_{L2} = 70 - 1.01 = 68.99\,\text{m}$$

The available or actual power output is proportional to the turbine head. Therefore, the increase in the power output when the diameter is doubled becomes

$$\text{Increase in power output} = \frac{h_{\text{turbine},2} - h_{\text{turbine},1}}{h_{\text{turbine},1}} = \frac{32.35 - 68.99}{32.95} = \mathbf{1.11 \ \ or \ 111\%}$$

Discussion Note that the power generation of the turbine more than doubles when the pipe diameter is doubled for the same flow rate and elevation.

12-97E The drinking water needs of an office are met by siphoning water through a plastic hose inserted into a large water bottle. The time it takes to fill a glass when the bottle is first opened and when it is empty are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The on/off switch is fully open during filing, and the total minor loss coefficient is as specified. **4** The water level in the bottle remains constant during filling. **5** The flow is turbulent (to be verified).

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s (Table A-15E). The plastic pipes are considered to be smooth, and thus their roughness is $\varepsilon = 0$ (Fig. A-27). The total minor loss coefficient is given to be 2.8 during filling.

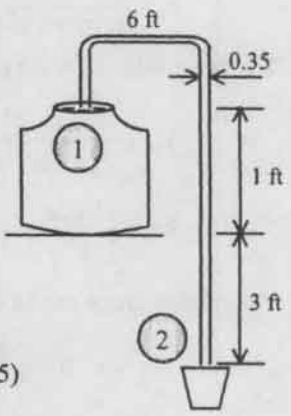
Analysis We take point 1 to be at the free surface of water in the bottle, and point 2 at the exit of the hose., which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where $\mathbf{V}_2 = \mathbf{V}_m$ and $h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_m^2}{2g} \rightarrow h_L=\left(f\frac{5\text{ ft}}{0.35/12\text{ ft}}+2.8\right)\frac{\mathbf{V}_m^2}{2(32.2\text{ ft/s}^2)}$ (1)

since the diameter of the piping system is constant. Then the energy equation becomes

$$z_1=\frac{\mathbf{V}_m^2}{2(32.2\text{ ft/s}^2)}+h_L \qquad (2)$$



The mean velocity in the pipe and the Reynolds number are

$$\mathbf{V}_m=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4} \rightarrow \mathbf{V}_m=\frac{\dot V\text{ ft}^3/\text{s}}{\pi(0.35/12\text{ ft})^2/4} \qquad (3)$$

$$\text{Re}=\frac{\rho \mathbf{V}_m D}{\mu} \rightarrow \text{Re}=\frac{(62.3\text{ lbm/ft}^3)\mathbf{V}_m(0.35/12\text{ ft})}{1.307\times10^{-3}\text{ lbm/ft}\cdot\text{s}} \qquad (4)$$

The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad (5)$$

Finally, the filling time of the glass is

$$\Delta t=\frac{V_{\text{glass}}}{\dot V}=\frac{0.00835\text{ ft}^3}{\dot V\text{ ft}^3/\text{s}} \qquad (6)$$

These are 6 equations in the 6 unknowns of $\mathbf{V}_m$, $\dot V$, h_L, Re, f, and Δt, and solving them simultaneously using an equation solver such as EES with the appropriate z_1 value gives

Case (*a*): The bottle is full and thus $z_1 = 3+1 = 4$ ft:

$\mathbf{V}_m = 4.87$ ft/s, $h_L = 3.63$ ft, $\dot V = 0.00325$ ft^3/s, Re = 6771, f=0.0343, and Δt = **2.6 s**

Case (*b*): The bottle is almost empty and thus $z_1 = 3$ ft:

$\mathbf{V}_m = 4.16$ ft/s, $h_L = 2.73$ ft, $\dot V = 0.00278$ ft^3/s, Re = 5779, f=0.0359, and Δt = **3.0 s**

Note that the flow is turbulent for both cases since Re > 4000.

Discussion The filling time of the glass increases as the water level in the bottle drops, as expected.

12-98E The drinking water needs of an office are met by siphoning water through a plastic hose inserted into a large water bottle. The time it takes to fill a glass when the bottle is first opened and when it is empty are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The on/off switch is fully open during filing, and the total minor loss coefficient is as specified. **4** The water level in the bottle remains constant during filling. **5** The flow is turbulent (to be verified).

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.360$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s (Table A-15E). The plastic pipes are considered to be smooth, and thus their roughness is $\varepsilon = 0$ (Fig. A-27). The total minor loss coefficient is given to be 2.8 during filling.

Analysis We take point 1 to be at the free surface of water in the bottle, and point 2 at the exit of the hose, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where $\mathbf{V}_2 = \mathbf{V}_m$ and $h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}_m^2}{2g} \rightarrow h_L=\left(f\frac{5\text{ ft}}{0.35/12\text{ ft}}+2.8\right)\frac{\mathbf{V}_m^2}{2(32.2\text{ ft/s}^2)}$ (1)

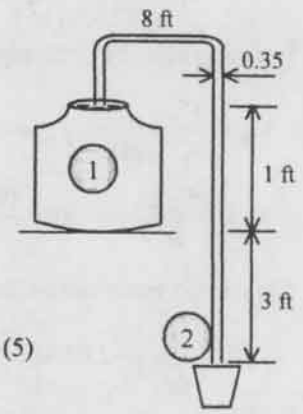
since the diameter of the piping system is constant. Then the energy equation becomes

$$z_1=\frac{\mathbf{V}_m^2}{2(32.2\text{ ft/s}^2)}+h_L \qquad (2)$$

The mean velocity in the pipe and the Reynolds number are

$$\mathbf{V}_m=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4} \rightarrow \mathbf{V}_m=\frac{\dot{V}\text{ ft}^3/\text{s}}{\pi(0.35/12\text{ ft})^2/4} \qquad (3)$$

$$\text{Re}=\frac{\rho\mathbf{V}_m D}{\mu} \rightarrow \text{Re}=\frac{(62.3\text{ lbm/ft}^3)\mathbf{V}_m(0.35/12\text{ ft})}{1.307\times10^{-3}\text{ lbm/ft}\cdot\text{s}} \qquad (4)$$



The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad (5)$$

Finally, the filling time of the glass is

$$\Delta t=\frac{V_{\text{glass}}}{\dot{V}}=\frac{0.00835\text{ ft}^3}{\dot{V}\text{ ft}^3/\text{s}} \qquad (6)$$

These are 6 equations in the 6 unknowns of $\mathbf{V}_m$, $\dot{V}$, h_L, Re, f, and Δt, and solving them simultaneously using an equation solver such as EES with the appropriate z_1 value gives

Case (*a*): The bottle is full and thus $z_1 = 3+1 = 4$ ft:

$\mathbf{V}_m = 3.67$ ft/s, $h_L = 3.79$ ft, $\dot{V} = 0.00245$ ft^3/s, Re $= 5107$, $f = 0.0371$, and $\Delta t =$ **3.4 s**

Case (*b*): The bottle is almost empty and thus $z_1 = 3$ ft:

$\mathbf{V}_m = 3.12$ ft/s, $h_L = 2.85$ ft, $\dot{V} = 0.00209$ ft^3/s, Re $= 4340$, $f = 0.0390$, and $\Delta t =$ **4.0 s**

Note that the flow is turbulent for both cases since Re > 4000.

Discussion The filling times in Prob. 12-97E were 2.6 s and 3.0 s, respectively. Therefore, doubling the tube length increases the filling time by 0.8 s when the bottle is full, and by 1.0 s when it is empty.

12-99 A water pipe has an abrupt expansion from diameter D_1 to D_2. It is to be shown that the loss coefficient is $K_L = (1 - D_1^2 / D_2^2)^2$, and K_L and P_2 are to be calculated. Also, the error that would occur if the Bernoulli Equation had been used is to be assessed.

Assumptions **1** The flow is steady and incompressible. **2** The pressure is uniform at the cross-section where expansion occurs, and is equal to the upstream pressure P_1. **3** The flow section is horizontal (or the elevation difference across the expansion section is negligible). **4** The flow is turbulent.

Properties We take the density of water to be $\rho = 1000$ kg/m^3.

Analysis We designate the cross-section where expansion occurs by x. We choose cross-section 1 in the smaller diameter pipe shortly before x, and section 2 in the larger diameter pipe shortly after x. We take the region occupied by the fluid between cross-sections 1 and 2 as the control volume, with an inlet at 1 and exit at 2. The velocity, pressure, and cross-sectional area are $\mathbf{V}_1$, P_1, and A_1 at cross-section 1, and $\mathbf{V}_2$, P_2, and A_2 at cross-section 2. We assume the pressure along the cross-section x to be P_1 so that $P_x = P_1$. Then the continuity, momentum, and energy equations applied to the control volume become

(1) *Continuity*: $\dot{m}_1 = \dot{m}_2 \rightarrow \rho \mathbf{V}_1 A_1 = \rho \mathbf{V}_2 A_2 \rightarrow \mathbf{V}_2 = \dfrac{A_1}{A_2}\mathbf{V}_1$ (1)

(2) *Momentum*: $\sum \vec{F} = \sum_{\text{out}} \dot{m}_e \vec{\mathbf{V}}_e - \sum_{\text{in}} \dot{m}_i \vec{\mathbf{V}}_i \rightarrow P_1 A_1 + P_1(A_x - A_1) - P_2 A_2 = \dot{m}(\mathbf{V}_2 - \mathbf{V}_1)$

But $P_1 A_1 + P_1(A_x - A_1) = P_1 A_x = P_1 A_2$

$$\dot{m}(\mathbf{V}_2 - \mathbf{V}_1) = \rho A_2 \mathbf{V}_2 (\mathbf{V}_2 - \mathbf{V}_1) = \rho A_2 \frac{A_1}{A_2}\mathbf{V}_1\left(\frac{A_1}{A_2}\mathbf{V}_1 - \mathbf{V}_1\right) = \rho A_2 \frac{A_1}{A_2}\left(\frac{A_1}{A_2} - 1\right)\mathbf{V}_1^2$$

Therefore, $P_1 A_2 - P_2 A_2 = \rho A_2 \dfrac{A_1}{A_2}\left(\dfrac{A_1}{A_2} - 1\right)\mathbf{V}_1^2 \rightarrow \dfrac{P_1 - P_2}{\rho} = \dfrac{A_1}{A_2}\left(\dfrac{A_1}{A_2} - 1\right)\mathbf{V}_1^2$ (2)

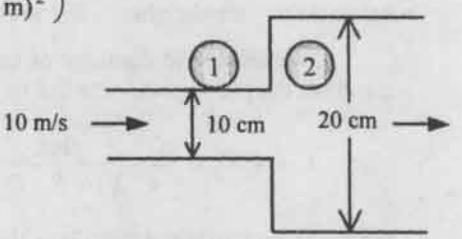
(3) *Energy*: $\dfrac{P_1}{\rho g} + \dfrac{\mathbf{V}_1^2}{2g} + z_1 + h_{\text{pump,u}} = \dfrac{P_2}{\rho g} + \dfrac{\mathbf{V}_2^2}{2g} + z_2 + h_{\text{turbine}} + h_L \rightarrow h_L = \dfrac{P_1 - P_2}{\rho g} + \dfrac{\mathbf{V}_1^2 - \mathbf{V}_2^2}{2g}$ (3)

Substituting Eqs. (1) and (2) and $h_L = K_L \dfrac{\mathbf{V}_1^2}{2g}$ into Eq. (3) gives

$$K_L \frac{\mathbf{V}_1^2}{2g} = \frac{A_1}{A_2}\left(\frac{A_1}{A_2} - 1\right)\frac{\mathbf{V}_1^2}{g} + \frac{\mathbf{V}_1^2 - (A_1^2 / A_2^2)\mathbf{V}_1^2}{2g} \rightarrow K_L = \frac{2A_1}{A_2}\left(\frac{A_1}{A_2} - 1\right) + \left(1 - \frac{A_1^2}{A_2^2}\right)$$

Simplifying and substituting $A = \pi D^2/4$ gives the desired relation and its value,

$$K_L = \left(1 - \frac{A_1}{A_1}\right)^2 = \left(1 - \frac{\pi D_1^2/4}{\pi D_2^2/4}\right)^2 = \left(1 - \frac{D_1^2}{D_2^2}\right)^2 = \left(1 - \frac{(0.10\text{ m})^2}{(0.10\text{ m})^2}\right)^2 = 0.5625$$



Also, $h_L = K_L \dfrac{\mathbf{V}_1^2}{2g} = (0.5625)\dfrac{(10\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 2.88\text{ m}$

$$\mathbf{V}_2 = \frac{A_1}{A_2}\mathbf{V}_1 = \frac{D_1^2}{D_2^2}\mathbf{V}_1 = \frac{(0.1\text{ m})^2}{(0.2\text{ m})^2}(10\text{ m/s}) = 2.5\text{ m/s}$$

Solving for P_2 from Eq. (3) and substituting,

$$P_2 = P_1 + \rho\{(\mathbf{V}_1^2 - \mathbf{V}_2^2)/2 - gh_L\}$$

$$= (120\text{ kPa}) + (1000\text{ kg/m}^3)\left\{\frac{(10\text{ m/s})^2 - (2.5\text{ m/s})^2}{2} - (9.81\text{ m/s}^2)(2.88\text{ m})\right\}\left(\frac{1\text{ kPa}\cdot\text{m}^2}{1000\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{139\text{ kPa}}$$

Note that the pressure increases by 19 kPa after the expansion due to the conversion of dynamic pressure to static pressure when the velocity is decreased. Also, $K_L = 1$ when $D_2 >> D_1$ (discharging into a reservoir).

12-100 A swimming pool is initially filled with water. A pipe with a well-rounded entrance at the bottom drains the pool to the atmosphere. The initial rate of discharge from the pool and the time required to empty the pool completely are to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The entrance effects are negligible, and thus the flow is fully developed. **4** The friction factor remains constant (in reality, it changes since the flow velocity and thus the Reynolds number changes during flow).

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998$ kg/m^3 and $\mu = 1.002\times10^{-3}$ kg/m·s (Table A-15). The friction factor of the pipe is given to be 0.022. Plastic pipes are considered to be smooth, and their surface roughness is $\varepsilon = 0$ (Fig. A-27).

Analysis We take point 1 at the free surface of the pool, and point 2 and the reference level at the exit of the pipe ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

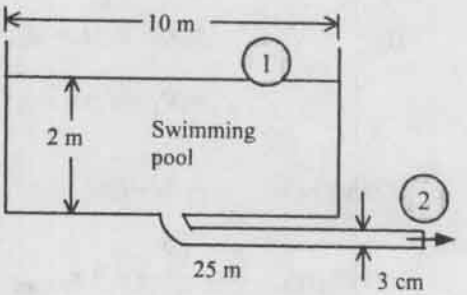
where

$$h_L = f\frac{L}{D}\frac{\mathbf{V}^2}{2g}$$

since the minor losses are negligible. Substituting and solving for $\mathbf{V}_2$ gives

$$z_1=\frac{\mathbf{V}_2^2}{2g}+\left(f\frac{L}{D}\right)\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2gz_1}{1+fL/D}}$$

Noting that initially $z_1 = 2$ m, the initial velocity and flow rate are determined to be



$$\mathbf{V}_{2,i}=\sqrt{\frac{2gz_1}{1+fL/D}}=\sqrt{\frac{2(9.81\,\text{m/s}^2)(2\,\text{m})}{1+0.022(25\,\text{m})/(0.03\,\text{m})}}=1.425\,\text{m/s}$$

$$\dot{V}_{initial}=\mathbf{V}_{2,i}A_c=\mathbf{V}_{2,i}(\pi D^2/4)=(1.423\,\text{m/s})[\pi(0.03\,\text{m})^4/4]=1.01\times10^{-3}\,\text{m}^3/\text{s}=1.01\,\text{L/s}$$

The mean discharge velocity at any given time, in general, can be expressed as

$$\mathbf{V}_2=\sqrt{\frac{2gz}{1+fL/D}}$$

where z is the water height relative to the center of the orifice at that time.

We denote the diameter of the pipe by D, and the diameter of the pool by D_o. The flow rate of water from the pool can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{V}=A_c\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D}}$$

Then the amount of water that flows through the pipe during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the pool,

$$dV = A_{c,\tan k}(-dz) = -\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the pool during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D}}dt = -\frac{\pi D_0^2}{4}dz \rightarrow dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2gz}}dz = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained pool) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}\int_{z=z_1}^{0} z^{-1/2}dz \rightarrow t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+fL/D)}{g}} = \frac{(10\,\text{m})^2}{(0.03\,\text{m})^2}\sqrt{\frac{2(2\,\text{m})[1+(0.022)(25\,\text{m})/(0.03\,\text{m})]}{9.81\,\text{m/s}^2}} = 312{,}000\,\text{s} = \mathbf{86.7\,h}$$

Checking: For plastic pipes, the surface roughness and thus the roughness factor is zero. The Reynolds number at the beginning of draining process is

$$\text{Re} = \frac{\rho V_2 D}{\mu} = \frac{(998\,\text{kg/m}^3)(1.425\,\text{m/s})(0.03\,\text{m})}{1.002\times10^{-3}\,\text{kg/m}\cdot\text{s}} = 42{,}570$$

which is greater than 4000. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(0 + \frac{2.51}{42{,}570\sqrt{f}}\right)$$

It gives $f = 0.022$. Therefore, the given value of 0.022 is accurate.

Discussion It can be shown by setting $L = 0$ that the draining time without the pipe is only about 18 h. Therefore, the pipe in this case increases the draining time by about 5 folds.

12-101 A swimming pool is initially filled with water. A pipe with a sharp-edged entrance at the bottom drains the pool to the atmosphere. The initial rate of discharge from the pool and the time required to empty the pool completely are to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **4** The friction factor remains constant (in reality, it changes since the flow velocity and thus the Reynolds number changes during flow).

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998$ kg/m^3 and $\mu = 1.002\times10^{-3}$ kg/m·s (Table A-15). The loss coefficient is $K_L = 0.5$ for the sharp-edged entrance (Table 12-4). Plastic pipes are considered to be smooth, and their surface roughness is $\varepsilon = 0$ (Fig. A-27).

Analysis We take point 1 at the free surface of the pool, and point 2 and the reference level at the exit of the pipe ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($\mathbf{V}_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\frac{\mathbf{V}_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{\mathbf{V}_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad \rightarrow \quad z_1=\frac{\mathbf{V}_2^2}{2g}+h_L$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{\mathbf{V}^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{\mathbf{V}_2^2}{2g}$$

since the diameter of the piping system is constant. Substituting and solving for $\mathbf{V}_2$ gives

$$z_1=\frac{\mathbf{V}_2^2}{2g}+\left(f\frac{L}{D}+K_L\right)\frac{\mathbf{V}_2^2}{2g} \quad \rightarrow \quad \mathbf{V}_2=\sqrt{\frac{2gz_1}{1+fL/D+K_L}}$$

Noting that initially $z_1 = 2$ m, the initial velocity and flow rate are determined to be

$$\mathbf{V}_{2,i}=\sqrt{\frac{2gz_1}{1+fL/D+K_L}}=\sqrt{\frac{2(9.81\,\text{m/s}^2)(2\,\text{m})}{1+0.022(25\,\text{m})/(0.03\,\text{m})+0.5}}=1.407\,\text{m/s}$$

$$\dot{V}_{initial}=\mathbf{V}_{2,i}A_c=\mathbf{V}_{2,i}(\pi D^2/4)=(1.407\,\text{m/s})[\pi(0.03\,\text{m})^4/4]=9.94\times10^{-4}\ \text{m}^3/\text{s}=0.994\,\text{L/s}$$

The mean discharge velocity at any given time, in general, can be expressed as

$$\mathbf{V}_2=\sqrt{\frac{2gz}{1+fL/D+K_L}}$$

where z is the water height relative to the center of the orifice at that time.

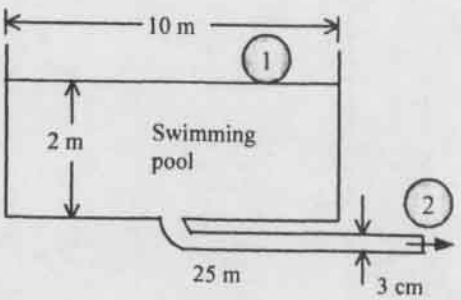
We denote the diameter of the pipe by D, and the diameter of the pool by D_o. The flow rate of water from the pool can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{V}=A_c\mathbf{V}_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}$$

Then the amount of water that flows through the pipe during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the pool,



$$dV = A_{c,\tan k}(-dz) = -\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the pool during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}dt = -\frac{\pi D_0^2}{4}dz \rightarrow dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2gz}}dz = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained pool) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\int_{z=z_1}^{0} z^{-1/2}dz \rightarrow t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+fL/D+K_L)}{g}} = \frac{(10\,\text{m})^2}{(0.03\,\text{m})^2}\sqrt{\frac{2(2\,\text{m})[1+(0.022)(25\,\text{m})/(0.03\,\text{m})+0.5]}{9.81\,\text{m/s}^2}} = 316{,}000\,\text{s} = \mathbf{87.8\,h}$$

Checking: For plastic pipes, the surface roughness and thus the roughness factor is zero. The Reynolds number at the beginning of draining process is

$$\text{Re} = \frac{\rho V_2 D}{\mu} = \frac{(998\,\text{kg/m}^3)(1.407\,\text{m/s})(0.03\,\text{m})}{1.002\times10^{-3}\,\text{kg/m}\cdot\text{s}} = 42{,}030$$

which is greater than 4000. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(0+\frac{2.51}{42{,}030\sqrt{f}}\right)$$

It gives $f = 0.022$. Therefore, the given value of 0.022 is accurate.

Discussion It can be shown by setting $L = 0$ that the draining time without the pipe is only about 24 h. Therefore, the pipe in this case increases the draining time more than 3 folds.

Chapter 13
FLOW OVER BODIES: DRAG AND LIFT

Drag, Lift, and Drag Coefficients of Common Geometries

13-1C Fluid flow is classified as *external* and *internal,* depending on whether the fluid is forced to flow over a surface or in a pipe. Internal flows (such as pipe flow) are bounded by solid walls. External flows are flows over bodies immersed in a fluid (such as the flow of air over a car or airplane).

13-2C The flow over a body is said to be *two-dimensional* when the body is too long and of constant cross-section, and the flow is normal to the body (such as the wind blowing over a long pipe perpendicular to its axis). There is no significant flow along the axis of the body. The flow along a body that possesses symmetry along an axis in the flow direction is said to be *axisymmetric* (such as a bullet piercing through air). Flow over a body that cannot be modeled as two-dimensional or axisymmetric is *three-dimensional.* The flow over a car is three-dimensional.

13-3C The velocity of the fluid relative to the immersed solid body sufficiently far away from a body is called the *free-stream velocity*, $\mathbf{V}_\infty$. The *upstream* (or *approach*) *velocity* $\mathbf{V}$ is the velocity of the approaching fluid far ahead of the body. These two velocities are equal if the flow is uniform and the body is small relative to the scale of the free-stream flow.

13-4C A body is said to be *streamlined* if a conscious effort is made to align its shape with the anticipated streamlines in the flow. Otherwise, a body tends to block the flow, and is said to be *blunt*. A tennis ball is a blunt body (unless the velocity is very low and we have "creeping flow").

13-5C The formation of vapor cavities in regions of low pressure in liquid flow is called *cavitation*. It occurs at locations such as the constrictions in a valve or the tips of impeller blades where the pressure drops below vapor pressure due to high velocities. We try to avoid cavitation since it reduces performance, generates annoying vibrations and noise, and causes erosion, surface pitting, fatigue failure, and the eventual destruction of the components or machinery.

13-6C The force a flowing fluid exerts on a body in the flow direction is called *drag*. Drag is caused by friction between the fluid and the solid surface, and the pressure difference between the front and back of the body. We try to minimize drag in order to reduce fuel consumption in vehicles, improve safety and durability of structures subjected to high winds, and to reduce noise and vibration.

13-7C The force a flowing fluid exerts on a body in the normal direction to flow that tend to move the body in that direction is called *lift*. It is caused by the components of the pressure and wall shear forces in the normal direction to flow. The wall shear also contributes to lift (unless the body is very slim), but its contribution is usually small.

13-8C When the drag force F_D, the upstream velocity $\mathbf{V}$, and the fluid density ρ are measured during flow over a body, the drag coefficient can be determined from

$$C_D = \frac{F_D}{\frac{1}{2}\rho \mathbf{V}^2 A}$$

where A is ordinarily the *frontal area* (the area projected on a plane normal to the direction of flow) of the body.

13-9C When the lift force F_L, the upstream velocity **V**, and the fluid density ρ are measured during flow over a body, the lift coefficient can be determined from

$$C_L = \frac{F_L}{\frac{1}{2}\rho \mathbf{V}^2 A}$$

where A is ordinarily the *planform area*, which is the area that would be seen by a person looking at the body from above in a direction normal to the body.

13-10C The *frontal area* of a body is the area seen by a person when looking from upstream. The frontal area is appropriate to use in drag and lift calculations for blunt bodies such as cars, cylinders, and spheres.

13-11C The *planform area* of a body is the area that would be seen by a person looking at the body from above in a direction normal to flow. The planform area is appropriate to use in drag and lift calculations for slender bodies such as flat plate and airfoils when the frontal area is very small.

13-12C The maximum velocity a free falling body can attain is called the *terminal velocity*. It is determined by setting the weight of the body equal to the drag and buoyancy forces, $W = F_D + F_B$.

13-13C The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag* $F_{D,\,\text{friction}}$ since it is caused by frictional effects, and the part that is due directly to pressure P and depends strongly on the shape of the body is called the *pressure drag* $F_{D,\,\text{pressure}}$. For slender bodies such as airfoils, the friction drag is usually more significant.

13-14C The friction drag coefficient is independent of surface roughness in *laminar flow*, but is a strong function of surface roughness in *turbulent flow* due to surface roughness elements protruding further into the highly viscous laminar sublayer.

13-15C (*a*) In general, the drag coefficient decreases with the Reynolds number at low and moderate Reynolds numbers. (*b*) The drag coefficient is nearly independent of the Reynolds number at high Reynolds numbers ($\text{Re} > 10^4$).

13-16C As a result of attaching fairings to the front and back of a cylindrical body at high Reynolds numbers, (*a*) friction drag increases, (*b*) pressure drag decreases, and (*c*) total drag decreases.

13-17C As a result of streamlining, (*a*) friction drag increases, (*b*) pressure drag decreases, and (*c*) total drag decreases at high Reynolds numbers (the general case), but increases at very low Reynolds numbers since the friction drag dominates at low Reynolds numbers.

13-18C At sufficiently high velocities, the fluid stream detaches itself from the surface of the body. This is called *separation*. It is caused by a fluid flowing over a curved surface at a high velocity (or technically, by adverse pressure gradient). Separation increases the drag coefficient drastically.

13-19C For a moving body to follow another moving body closely by staying close behind is called *drafting*. It reduces the pressure drag and thus the drag coefficient for the drafted body by taking advantage of the low pressure wake region of the moving body in front.

13-20C The car that is contoured to resemble an ellipse has a smaller drag coefficient and thus smaller air resistance, and it is more likely to be more fuel efficient than a car with sharp corners.

13-21C The bicyclist who leans down and brings his body closer to his knees will go faster since the frontal area and thus the drag force will be less in that position. The drag coefficient will also go down somewhat, but this is a secondary effect.

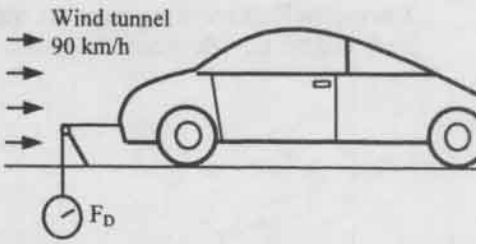
13-22 The drag force acting on a car is measured in a wind tunnel. The drag coefficient of the car at the test conditions is to be determined. √

Assumptions **1** The flow of air is steady and incompressible. **2** The cross-section of the tunnel is large enough to simulate free flow over the car. **3** The bottom of the tunnel is also moving at the speed of air to approximate actual driving conditions or this effect is negligible. **4** Air is an ideal gas.

Properties The density of air at 1 atm and 25°C = 298 K is $\rho = 1.186$ kg/m^3 (Table A-18).

Analysis The drag force acting on a body and the drag coefficient are given by

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} \quad \text{and} \quad C_D = \frac{2F_D}{\rho A \mathbf{V}^2}$$



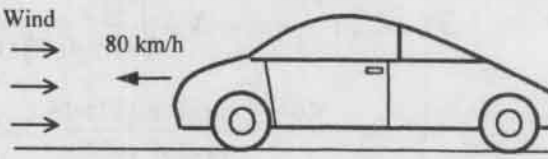
where A is the frontal area. Substituting and noting that 1 m/s = 3.6 km/h, the drag coefficient of the car is determined to be

$$C_D = \frac{2 \times (350\ \text{N})}{(1.186\ \text{kg/m}^3)(1.40 \times 1.65\ \text{m}^2)(90/3.6\ \text{m/s})^2}\left(\frac{1\ \text{kg} \cdot \text{m/s}^2}{1\ \text{N}}\right) = \mathbf{0.41}$$

Discussion Note that the drag coefficient depends on the design conditions, and its value will be different at different conditions. Therefore, the published drag coefficients of different vehicles can be compared meaningfully only if they are determined under identical conditions. This shows the importance of developing standard testing procedures in industry.

13-23 A car is moving at a constant velocity. The upstream velocity to be used in fluid flow analysis is to be determined for the cases of calm air, wind blowing against the direction of motion of the car, and wind blowing in the same direction of motion of the car. √

Analysis In fluid flow analysis, the velocity used is the relative velocity between the fluid and the solid body. Therefore:



(*a*) Calm air: $\mathbf{V} = \mathbf{V}_{car} = \mathbf{80\ km/h}$

(*b*) Wind blowing against the direction of motion:
$\mathbf{V} = \mathbf{V}_{car} + \mathbf{V}_{wind} = 80 + 30 = \mathbf{110\ km/h}$

(*c*) Wind blowing in the same direction of motion:
$\mathbf{V} = \mathbf{V}_{car} - \mathbf{V}_{wind} = 80 - 50 = \mathbf{30\ km/h}$

Discussion Note that the wind and car velocities are added when they are in opposite directions, and subtracted when they are in the same direction.

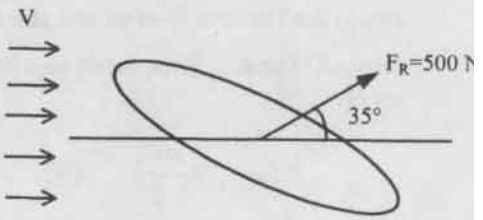
13-24 The resultant of the pressure and wall shear forces acting on a body is given. The drag and the lift forces acting on the body are to be determined. √

Analysis The drag and lift forces are determined by decomposing the resultant force into its components in the flow direction and the normal direction to flow,

Drag force: $F_D = F_R \cos\theta = (500\text{ N})\cos 35° = 410\text{ N}$

Lift force: $F_L = F_R \sin\theta = (500\text{ N})\sin 35° = 287\text{ N}$

Discussion Note that the greater the angle between the resultant force and the flow direction, the greater the lift.



13-25 The total drag force acting on a spherical body is measured, and the pressure drag acting on the body is calculated by integrating the pressure distribution. The friction drag coefficient is to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The surface of the sphere is smooth. **3** The flow over the sphere is turbulent (to be verified).

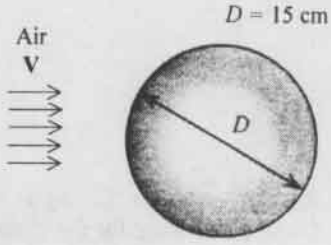
Properties The density and kinematic viscosity of air at 1 atm and 280 K are $\rho = 1.271\text{ kg/m}^3$ and $\nu = 1.40\times10^{-5}\text{ m}^2/\text{s}$ (Table A-18). The drag coefficient of sphere in turbulent flow is $C_D = 0.2$, and its frontal area is $A = \pi D^2/4$ (Table 13-2).

Analysis The total drag force is the sum of the friction and pressure drag forces. Therefore,

$$F_{D,\text{friction}} = F_D - F_{D,\text{pressure}} = 5.2 - 4.9 = 0.3\text{ N}$$

where

$$F_D = C_D A\frac{\rho V^2}{2} \quad \text{and} \quad F_{D,\text{friction}} = C_{D,\text{friction}} A\frac{\rho V^2}{2}$$



Taking the ratio of the two relations above gives

$$C_{D,\text{friction}} = \frac{F_{D,\text{friction}}}{F_D} C_D = \frac{0.3\text{ N}}{5.2\text{ N}}(0.2) = \mathbf{0.0115}$$

Now we need to verify that the flow is turbulent. This is done by calculating the flow velocity from the drag force relation, and then the Reynolds number:

$$F_D = C_D A\frac{\rho V^2}{2} \rightarrow V = \sqrt{\frac{2F_D}{\rho C_D A}} = \sqrt{\frac{2(5.2\text{ N})}{(1.271\text{ kg/m}^3)(0.2)[\pi(0.12\text{ m})^2/4]}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)} = 60.1\text{ m/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(60.1\text{ m/s})(0.12\text{ m})}{1.40\times10^{-5}\text{ m}^2/s} = 5.15\times10^5$$

which is greater than 2×10^5. Therefore, the flow is turbulent as assumed.

Discussion Note that knowing the flow regime is important in the solution of this problem since the total drag coefficient for a sphere is 0.5 in laminar flow and 0.2 in turbulent flow. This problem can be solved more accurately using a trial-and-error approach by using C_D data from Fig. 13-30.

13-26E The frontal area of a car is reduced by redesigning. The amount of fuel and money saved per year as a result are to be determined.

Assumptions **1** The car is driven 12,000 miles a year at an average speed of 55 km/h. **2** The effect of reduction of the frontal area on the drag coefficient is negligible.

Properties The densities of air and gasoline are given to be 0.075 lbm/ft^3 and 50 lbm/ft^3, respectively. The heating value of gasoline is given to be 20,000 Btu/lbm. The drag coefficient is $C_D = 0.3$ for a passenger car (Table 13-2).

Analysis The drag force acting on a body is determined from

$$F_D = C_D A \frac{\rho V^2}{2}$$

where A is the frontal area of the body. The drag force acting on the car before redesigning is

$$F_D = 0.3(18\,\text{ft}^2)\frac{(0.075\,\text{lbm/ft}^3)(55\,\text{mph})^2}{2}\left(\frac{1.4667\,\text{ft/s}}{1\,\text{mph}}\right)^2\left(\frac{1\,\text{lbf}}{32.2\,\text{lbm}\cdot\text{ft/s}^2}\right) = 40.9\,\text{lbf}$$

Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 12,000 miles are

$$W_{\text{drag}} = F_D L = (40.9\,\text{lbf})(12{,}000\,\text{miles/year})\left(\frac{5280\,\text{ft}}{1\,\text{mile}}\right)\left(\frac{1\,\text{Btu}}{778.169\,\text{lbf}\cdot\text{ft}}\right) = 3.33\times10^6\ \text{Btu/year}$$

$$E_{in} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{3.33\times10^6\ \text{Btu/year}}{0.32} = 1.041\times10^7\ \text{Btu/year}$$

Then the amount and costs of the fuel that supplies this much energy are

$$\text{Amont of fuel} = \frac{m_{\text{fuel}}}{\rho_{\text{fuel}}} = \frac{E_{\text{in}}/\text{HV}}{\rho_{\text{fuel}}} = \frac{(1.041\times10^7\ \text{Btu/year})/(20{,}000\,\text{Btu/lbm})}{50\,\text{lbm/ft}^3} = 10.41\,\text{ft}^3/\text{year}$$

$$\text{Cost} = (\text{Amount of fuel})(\text{Unit cost}) = (10.41\,\text{ft}^3/\text{year})(\$2.20/\text{gal})\left(\frac{7.4804\,\text{gal}}{1\,\text{ft}^3}\right) = \$171.3/\text{year}$$

That is, the car uses 10.41 ft^3 = 77.9 gallons of gasoline at a cost of $171.3 per year to overcome the drag.

The drag force and the work done to overcome it are directly proportional to the frontal area. Then the percent reduction in the fuel consumption due to reducing frontal area is equal to the percent reduction in the frontal area:



$$\text{Reduction ratio} = \frac{A - A_{\text{new}}}{A} = \frac{18-15}{18} = 0.167$$

$$\text{Amount reduction} = (\text{Reduction ratio})(\text{Amount})$$
$$= 0.167(77.9\ \text{gal/year}) = \mathbf{13.0\ gal/year}$$

$$\text{Cost reduction} = (\text{Reduction ratio})(\text{Cost}) = 0.167(\$171.3/\text{year}) = \mathbf{\$28.6/year}$$

Therefore, reducing the frontal area reduces the fuel consumption due to drag by 16.7%.

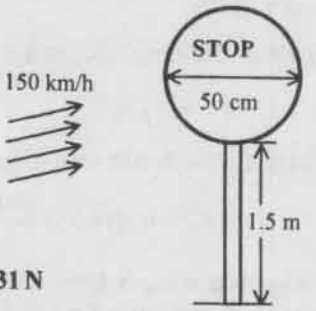
Discussion Note from this example that significant reductions in drag and fuel consumption can be achieved by reducing the frontal area of a vehicle.

13-27 A circular stop sign is subjected to high winds. The drag force acting on the sign and the bending moment at the bottom of its pole are to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The drag force on the pole is negligible. **3** The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties The drag coefficient for a thin circular disk is $C_D = 1.1$ (Table 13-2). The density of air at 100 kPa and 10°C = 283 K is

$$\rho = \frac{P}{RT} = \frac{100\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg.K})(283\,\text{K})} = 1.231\,\text{kg/m}^3$$



Analysis The frontal area of a circular plate subjected to normal flow is $A = \pi D^2/4$. Then the drag force acting on the stop sign is

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2}$$

$$= (1.1)[\pi(0.5\,\text{m})^2/4]\frac{(1.231\,\text{kg/m}^3)(150/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{231\,N}$$

Noting that the resultant force passes through the center of the stop sign, the bending moment at the bottom of the pole becomes

$$M_{bottom} = F_D \times L = (231\,\text{N})(1.5+0.25)\,\text{m} = \mathbf{404\,Nm}$$

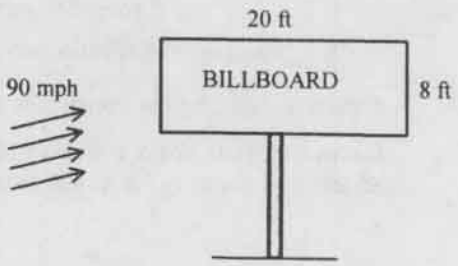
Discussion Note that the drag force is equivalent to the weight of over 23 kg of mass. Therefore, the pole must be strong enough to withstand the weight of 23 kg hanged at one of its end when it is held from the other end horizontally.

13-28E A rectangular billboard is subjected to high winds. The drag force acting on the billboard is to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The drag force on the supporting poles are negligible. **3** The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties The drag coefficient for a thin rectangular plate for normal flow is $C_D = 2.0$ (Table 13-2). The density of air at 14.3 psia and 40°F = 500 R is

$$\rho = \frac{P}{RT} = \frac{14.3\,\text{psia}}{(0.3704\,\text{psia}\cdot\text{ft}^3/\text{lbm.R})(500\,\text{R})} = 0.0772\,\text{lbm/ft}^3$$



Analysis The drag force acting on the billboard is determined from

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2}$$

$$= (2.0)(8\times 20\,\text{ft}^2)\frac{(0.0772\,\text{lbm/ft}^3)(90\times 1.4667\,\text{ft/s})^2}{2}\left(\frac{1\,\text{lbf}}{32.2\,\text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{6684\,lbf}$$

Discussion Note that the drag force is equivalent to the weight of 6684 lbm of mass. Therefore, the support bars must be strong enough to withstand the weight of 6684 lbm hanged at one of their ends when they are held from the other end horizontally.

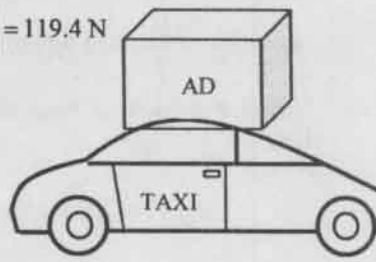
13-29 An advertisement sign in the form of a rectangular block that has the same frontal area from all four sides is mounted on top of a taxicab. The increase in the annual fuel cost due to this sign is to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The car is driven 60,000 km a year at an average speed of 50 km/h. **3** The overall efficiency of the engine is 28%. **4** The effect of the sign and the taxicab on the drag coefficient of each other is negligible (no interference), and the edge effects of the sign are negligible (a crude approximation). **5** The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties The densities of air and gasoline are given to be 1.25 kg/m^3 and 0.75 kg/L, respectively. The heating value of gasoline is given to be 42,000 kJ/kg. The drag coefficient for a square rod for normal flow is $C_D = 2.2$ (Table 13-1).

Analysis Noting that 1 m/s = 3.6 km/h, the drag force acting on the sign is

$$F_D = C_D A \frac{\rho V^2}{2} = (2.2)(1.5\times 0.3\,\text{m}^2)\frac{(1.25\,\text{kg/m}^3)(50/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 119.4\,\text{N}$$



Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 60,000 km are

$$W_{\text{drag}} = F_D \times L = (119.4\,\text{N})(60{,}000\,\text{km/year}) = 7.16\times 10^6\ \text{kJ/year}$$

$$E_{in} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{7.16\times 10^6\ \text{kJ/year}}{0.28} = 2.56\times 10^7\ \text{kJ/year}$$

Then the amount and cost of the fuel that supplies this much energy are

$$\text{Amont of fuel} = \frac{m_{\text{fuel}}}{\rho_{\text{fuel}}} = \frac{E_{\text{in}}/\text{HV}}{\rho_{\text{fuel}}} = \frac{(2.56\times 10^7\ \text{kJ/year})/(42{,}000\,\text{kJ/kg})}{0.75\,\text{kg/L}} = 813\,\text{L/year}$$

$$\text{Cost} = (\text{Amount of fuel})(\text{Unit cost}) = (813\,\text{L/year})(\$0.50/\text{L}) = \mathbf{\$406/year}$$

That is, the taxicab will use 813 L of gasoline at a cost of $406 per year to overcome the drag generated by the advertisement sign.

Discussion Note that the advertisement sign increases the fuel cost of the taxicab significantly. The taxicab operator may end up losing money by installing the sign if he/she is not aware of the major increase in the fuel cost, and negotiate accordingly.

13-30 The water needs of a recreational vehicle (RV) are to be met by installing a cylindrical tank on top of the vehicle. The additional power requirements of the RV at a specified speed for two orientations of the tank are to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The effect of the tank and the RV on the drag coefficient of each other is negligible (no interference). **3** The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties The drag coefficient for a cylinder corresponding to $L/D = 2/0.5 = 4$ is $C_D = 0.9$ when the circular surfaces of the tank face the front and back, and $C_D = 0.8$ when the circular surfaces of the tank face the sides of the RV (Table 13-2). The density of air at the specified conditions is

$$\rho = \frac{P}{RT} = \frac{87\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg.K})(295\ \text{K})} = 1.028\ \text{kg/m}^3$$

Analysis (*a*) The drag force acting on the tank when the circular surfaces face the front and back is

$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} = (0.9)[\pi(0.5\ \text{m})^2/4]\frac{(1.028\ \text{kg/m}^3)(95\ \text{km/h})^2}{2}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)^2\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 63.3\ \text{N}$$

Noting that power is force times velocity, the amount of additional power needed to overcome this drag force is

$$\dot{W}_{\text{drag}} = F_D \times \mathbf{V} = (63.3\ \text{N})(95/3.6\ \text{m/s})\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right) = \mathbf{1.67\ kW}$$

(*b*) The drag force acting on the tank when the circular surfaces face the sides of the RV is

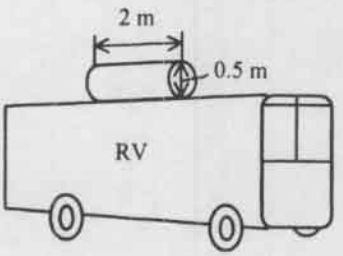
$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} = (0.8)[0.5\times 2\ \text{m}^2]\frac{(1.028\ \text{kg/m}^3)(95\ \text{km/h})^2}{2}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)^2\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 286\ \text{N}$$

Then the additional power needed to overcome this drag force is

$$\dot{W}_{\text{drag}} = F_D \times \mathbf{V} = (286\ \text{N})(95/3.6\ \text{m/s})\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right) = \mathbf{7.55\ kW}$$

Therefore, the additional power needed to overcome the drag caused by the tank is 1.67 kW and 7.55 W for the two orientations indicated.

Discussion Note that the additional power requirement is the lowest when the tank is installed such that its circular surfaces face the front and back of the RV. This is because the frontal area of the tank is minimum in this orientation.

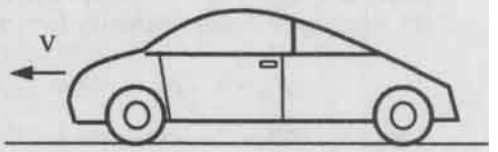


13-31E A person who normally drives at 55 mph now starts driving at 70 mph. The percentage increase in fuel consumption of the car is to be determined.

Assumptions **1** The power generated by the car engine is used entirely to overcome aerodynamic drag, and thus the fuel consumption is proportional to the drag force on a level road. **2** The drag coefficient remains the same.

Analysis The drag force is proportional to the square of the velocity, and power is force times velocity. Therefore,

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} \quad \text{and} \quad \dot{W}_{\text{drag}} = F_D \mathbf{V} = C_D A \frac{\rho \mathbf{V}^3}{2}$$



Then the ratio of the drag force at $\mathbf{V}_2$= 70 mph to the drag force at $\mathbf{V}_1$ = 55 mph becomes

$$\frac{F_{D2}}{F_{D1}} = \frac{\mathbf{V}_2^2}{\mathbf{V}_1^2} = \frac{70^3}{55^3} = \mathbf{2.06}$$

Therefore, the power to overcome the drag force and thus fuel consumption per unit time more than doubles as a result of increasing the velocity from 55 to 70 mph.

Discussion This increase appears to be large. This is because all the engine power is assumed to be used entirely to overcome drag. Still, the simple analysis above shows the strong dependence of the fuel consumption on the cruising speed of a vehicle.

A better measure of fuel consumption is the amount of fuel used per unit distance (rather than per unit time). A car cruising at 55 mph will travel a distance of 55 miles in 1 hour. But a car cruising at 70 mph will travel the same distance at 55/70 = 0.786 h or 78.5% of the time. Therefore, for a given distance, the increase in fuel consumption is 2.06×0.786 = 1.62 – an increase of **62%.**

13-32 A plastic sphere is dropped into water. The terminal velocity of the sphere in water is to be determined.

Assumptions **1** The fluid flow over the sphere is laminar (to be verified). **2** The drag coefficient remains constant.

Properties The density of plastic sphere is 1150 kg/m^3. The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and $\mu = 1.002\times10^{-3}$ kg/m·s, respectively (Table A-15). The drag coefficient for a sphere in laminar flow is $C_D = 0.5$ (Table 13-2).

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the fluid,

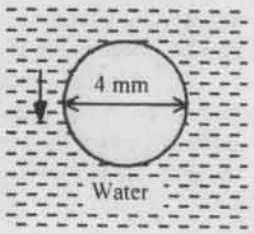
$$F_D = W - F_B \quad \text{where} \quad F_D = C_D A\frac{\rho_f \mathbf{V}^2}{2}, \quad W = \rho_s g V, \quad \text{and} \quad F_B = \rho_f g V$$

Here $A = \pi D^2/4$ is the frontal area and $V = \pi D^3/6$ is the volume of the sphere. Substituting and simplifying,

$$C_D A\frac{\rho_f \mathbf{V}^2}{2} = \rho_s g V - \rho_f g V \rightarrow C_D \frac{\pi D^2}{4}\frac{\rho_f \mathbf{V}^2}{2} = (\rho_s - \rho_f) g \frac{\pi D^3}{6} \rightarrow C_D \frac{\mathbf{V}^2}{8} = \left(\frac{\rho_s}{\rho_f} - 1\right)\frac{gD}{6}$$

Solving for **V** and substituting,

$$\mathbf{V} = \sqrt{\frac{4gD(\rho_s/\rho_f - 1)}{3C_D}} = \sqrt{\frac{4(9.81\,\text{m/s}^2)(0.004\,\text{m})(1150/998 - 1)}{3\times0.5}} = 0.126\,\text{m/s}$$



The Reynolds number is

$$\text{Re} = \frac{\rho \mathbf{V} D}{\mu} = \frac{(998\,\text{kg/m}^3)(0.126\,\text{m/s})(4\times10^{-3}\,\text{m})}{1.002\times10^{-3}\,\text{kg}\cdot\text{m/s}} = 503$$

which is less than 2×10^5. Therefore, the flow is laminar as assumed.

Discussion This problem can be solved more accurately using a trial-and-error approach by using C_D data from Fig. 13-30 (the C_D value corresponding to Re = 503 is about 0.6. Repeating the calculations for this value gives 0.115 m/s for the terminal velocity.

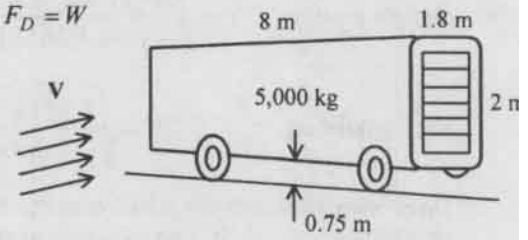
13-33 A semi truck is exposed to winds from its side surface. The wind velocity that will tip the truck over to its side is to be determined.

Assumptions **1** The flow of air in the wind is steady and incompressible. **2** The edge effects on the semi truck are negligible (a crude approximation), and the resultant drag force acts through the center of the side surface. **3** The flow is turbulent so that the tabulated value of the drag coefficient can be used. **4** The distance between the wheels on the same axle is also 2 m. **5** The semi truck is loaded uniformly so that its weight acts through its center.

Properties The density of air is given to be $\rho = 1.20$ kg/m^3. The drag coefficient for a body of rectangular cross-section corresponding to $L/D = 2/2 = 1$ is $C_D = 2.2$ when the wind is normal to the side surface (Table 13-2).

Analysis When the truck is first tipped, the wheels on the wind-loaded side of the truck will be off the ground, and thus all the reaction forces from the ground will act on wheels on the other side. Taking the moment about an axis passing through these wheels and setting it equal to zero gives the required drag force to be

$$\sum M_{\text{wheels}} = 0 \;\rightarrow\; F_D \times (1\,\text{m}) - W \times (1\,\text{m}) = 0 \quad \rightarrow \quad F_D = W$$



Substituting, the required drag force is determined to be

$$F_D = mg = (5000\,\text{kg})(9.81\,\text{m/s}^2)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 49{,}050\,\text{N}$$

The wind velocity that will cause this drag force is determined to be

$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} \;\rightarrow\; 49{,}050\,\text{N} = (2.2)(2\times 8\,\text{m}^2)\frac{(1.20\,\text{kg/m}^3)\mathbf{V}^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) \;\rightarrow\; \mathbf{V} = 48.2\,\text{m/s}$$

which is equivalent to a wind velocity of $\mathbf{V} = 48.2\times 3.6 = \mathbf{174\ km/h}$.

Discussion This is very high velocity, and it can be verified easily by calculating the Reynolds number that the flow is turbulent as assumed.

13-34 A bicyclist is riding his bicycle downhill on a road with a specified slope without pedaling or breaking. The terminal velocity of the bicyclist is to be determined for the upright and racing positions.

Assumptions **1** The rolling resistance and bearing friction are negligible. **2** The drag coefficient remains constant. **3** The buoyancy of air is negligible.

Properties The density of air is given to be $\rho = 1.25$ kg/m^3. The frontal area and the drag coefficient are given to be 0.45 m^2 and 1.1 in the upright position, and 0.4 m^2 and 0.9 on the racing position.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the component of the total weight (bicyclist + bicycle) in the flow direction,

$$F_D = W_{\text{total}} \sin\theta \quad \rightarrow \quad C_D A \frac{\rho V^2}{2} = m_{\text{total}} g \sin\theta$$



Solving for **V** gives

$$\mathbf{V} = \sqrt{\frac{2gm_{\text{total}} \sin\theta}{C_D A \rho}}$$

The terminal velocities for both positions are obtained by substituting the given values:

Upright position:
$$\mathbf{V} = \sqrt{\frac{2(9.81\,\text{m/s}^2)(80+15\,\text{kg})\sin 12^\circ}{1.1(0.45\,\text{m}^2)(1.25\,\text{kg/m}^3)}} = 25.0\,\text{m/s} = 90\,\text{km/h}$$

Racing position:
$$\mathbf{V} = \sqrt{\frac{2(9.81\,\text{m/s}^2)(80+15\,\text{kg})\sin 12^\circ}{0.9(0.4\,\text{m}^2)(1.25\,\text{kg/m}^3)}} = 29.3\,\text{m/s} = 106\,\text{km/h}$$

Discussion Note that the position of the bicyclist has a significant effect on the drag force, and thus the terminal velocity. So it is no surprise that the bicyclists maintain the racing position during a race.

13-35 The pivot of a wind turbine with two hollow hemispherical cups is stuck as a result of some malfunction. For a given wind speed, the maximum torque applied on the pivot is to be determined.

Assumptions **1** The flow of air in the wind is steady and incompressible. **2** The air flow is turbulent so that the tabulated values of the drag coefficients can be used.

Properties The density of air is given to be $\rho = 1.25$ kg/m^3. The drag coefficient for a hemispherical cup is 0.4 and 1.2 when the hemispherical and plain surfaces are exposed to wind flow, respectively.

Analysis The maximum torque occurs when the cups are normal to the wind since the length of the moment arm is maximum in this case. Noting that the frontal area is $\pi D^2/4$ for both cups, the drag force acting on each cup is determined to be

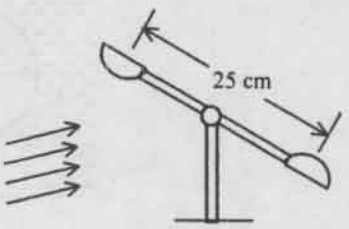
Convex side: $$F_{D1} = C_{D1} A \frac{\rho \mathbf{V}^2}{2} = (0.4)[\pi(0.05\,\text{m})^2/4]\frac{(1.25\,\text{kg/m}^3)(15\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 0.110\,\text{N}$$

Concave side: $$F_{D2} = C_{D2} A \frac{\rho \mathbf{V}^2}{2} = (1.2)[\pi(0.05\,\text{m})^2/4]\frac{(1.25\,\text{kg/m}^3)(15\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 0.331\,\text{N}$$

The moment arm for both forces is 12.5 cm since the distance between the centers of the two cups is given to be 25 cm. Taking the moment about the pivot, the net torque applied on the pivot is determined to be

$$M_{\max} = F_{D2}L - F_{D1}L = (F_{D2} - F_{D1})L = (0.331 - 0.110\,\text{N})(0.125\,\text{m}) = 0.0276\,\text{Nm}$$

Discussion Note that the torque varies between zero when both cups are aligned with the wind to the maximum value calculated above.



13-36E A spherical tank completely submerged in fresh water is being towed by a ship at a specified velocity. The required towing power is to be determined.

Assumptions **1** The flow is turbulent so that the tabulated value of the drag coefficient can be used. **2** The drag of the towing bar is negligible.

Properties The drag coefficient for a sphere is $C_D = 0.2$ in turbulent flow (it is 0.5 for laminar flow). We take the density of water to be 62.4 lbm/ft^3.

Analysis The frontal area of a sphere is $A = \pi D^2/4$. Then the drag force acting on the spherical tank is

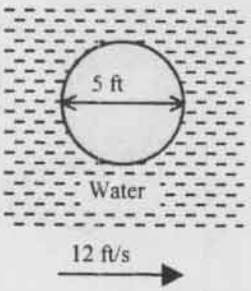
$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} = (0.2)[\pi(5\text{ ft})^2/4]\frac{(62.4\text{ lbm/ft}^3)(12\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 548\text{ lbf}$$

Noting that power is force times velocity, the power needed to overcome this drag force during towing is

$$\dot{W}_{\text{Towing}} = \dot{W}_{\text{drag}} = F_D \times \mathbf{V} = (548\text{ lbf})(12\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 8.92\text{ kW} = 12.0\text{ hp}$$

Therefore, the additional power needed to tow the tank is **12.0 hp**.

Discussion Note that the towing power is proportional the cube of the velocity. Therefore, the towing power can be reduced to one-eight (which is 1.5 hp) by reducing the towing velocity by half to 6 ft/s. But the towing time will double this time for a given distance.

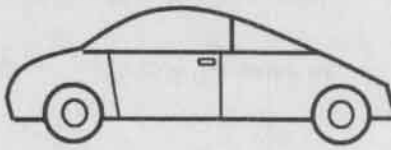


13-37 The power delivered to the wheels of a car is used to overcome aerodynamic drag and rolling resistance. For a given power, the velocity at which the rolling resistance is equal to the aerodynamic drag and the maximum velocity of the car are to be determined.

Assumptions **1** The air flow is steady and incompressible. **2** The bearing friction is negligible. **3** The drag and rolling resistance coefficients of the car are constant. **4** The car moves horizontally on a level road.

Properties The density of air is given to be $\rho = 1.20\ \text{kg/m}^3$. The drag and rolling resistance coefficients are given to be $C_D = 0.32$ and $C_{RR} = 0.04$, respectively.

Analysis (*a*) The rolling resistance of the car is



$$F_{RR} = C_{RR}W = 0.04(950\,\text{kg})(9.81\,\text{m/s}^2)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 372.8\,\text{N}$$

The velocity at which the rolling resistance equals the aerodynamic drag force is determined by setting these two forces equal to each other,

$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} \quad \rightarrow \quad 372.8\,\text{N} = (0.32)(1.8\,\text{m}^2)\frac{(1.20\,\text{kg/m}^3)\mathbf{V}^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) \quad \rightarrow \quad \mathbf{V} = 32.8\,\text{m/s}$$

(*b*) Power is force times velocity, and thus the power needed to overcome drag and rolling resistance is the product of the sum of the drag force and the rolling resistance and the velocity of the car,

$$\dot{W}_{\text{total}} = \dot{W}_{\text{drag}} + \dot{W}_{\text{RR}} = (F_D + F_{RR})\mathbf{V} = C_D A\frac{\rho \mathbf{V}^3}{2} + F_{RR}\mathbf{V}$$

Substituting the known quantities, the maximum velocity corresponding to a wheel power of 80 kW is determined to be

$$(0.32)(1.8\,\text{m}^2)\frac{(1.20\,\text{kg/m}^3)\mathbf{V}^3}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) + 372.8\mathbf{V} = 80{,}000\,\text{W}\left(\frac{1\,\text{N}\cdot\text{m/s}}{1\,\text{W}}\right)$$

or,

$$0.3456\mathbf{V}^3 + 372.8\mathbf{V} = 80{,}000$$

whose solution is $\mathbf{V} = 55.56\ \text{m/s} = \mathbf{200\ km/h}$.

Discussion Note that a net power input of 80 kW is needed to overcome the rolling resistance and the aerodynamic drag at a velocity of 200 km/h. About 75% of this power is used to overcome drag and the remaining 25% to overcome the rolling resistance. At much higher velocities, the fraction of drag becomes even higher as it is proportional to the cube of car velocity.

13-38 A submarine is treated as an ellipsoid at a specified length and diameter. The powers required for this submarine to cruise horizontally in seawater and to tow it in air are to be determined.

Assumptions **1** The submarine can be treated as an ellipsoid. **2** The flow is turbulent. **3** The drag of the towing rope is negligible. **4** The motion of submarine is steady and horizontal.

Properties The drag coefficient for an ellipsoid with $L/D = 10/5 = 2$ is $C_D = 0.1$ in turbulent flow (Table 13-2). The density of sea water is given to be 1025 kg/m^3. The density of air is given to be 1.30 kg/m^3.

Analysis Noting that 1 m/s = 3.6 km/h, the velocity of the submarine is equivalent to $\mathbf{V} = 40/3.6 = 11.1$ m/s. The frontal area of an ellipsoid is $A = \pi D^2/4$. Then the drag force acting on the submarine becomes

$$\textit{In water: } F_D = C_D A \frac{\rho \mathbf{V}^2}{2} = (0.1)[\pi(5\,\text{m})^2/4]\frac{(1025\,\text{kg/m}^3)(11.1\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) = 124.2\,\text{kN}$$

$$\textit{In air: } F_D = C_D A \frac{\rho \mathbf{V}^2}{2} = (0.1)[\pi(5\,\text{m})^2/4]\frac{(1.30\,\text{kg/m}^3)(11.1\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) = 0.158\,\text{kN}$$

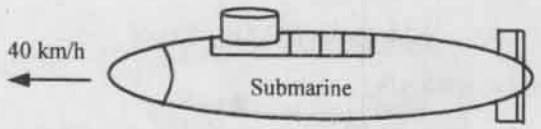
Noting that power is force times velocity, the power needed to overcome this drag force is

$$\textit{In water: } \dot{W}_{\text{drag}} = F_D \mathbf{V} = (124.2\,\text{kN})(11.1\,\text{m/s})\left(\frac{1\,\text{kW}}{1\,\text{kN}\cdot\text{m/s}}\right) = \mathbf{1380\,kW}$$

$$\textit{In air: } \dot{W}_{\text{drag}} = F_D \mathbf{V} = (0.158\,\text{kN})(11.1\,\text{m/s})\left(\frac{1\,\text{kW}}{1\,\text{kN}\cdot\text{m/s}}\right) = \mathbf{1.75\,kW}$$

Therefore, the power required for this submarine to cruise horizontally in seawater is 1380 kW and the power required to tow this submarine in air at the same velocity is 1.75 kW.

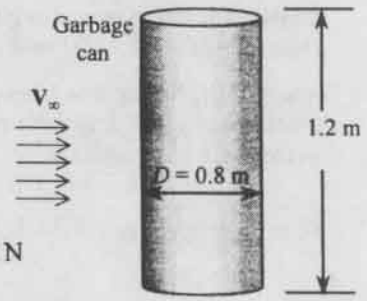
Discussion Note that the power required to move the submarine in water is about 800 times the power required to move it in air. This is due to the higher density of water compared to air (sea water is about 800 times denser than air). Also, the drag coefficient is $C_D = 0.3$ for laminar flow. Therefore, the power requirements would be 3 times larger in laminar flow.



13-39 A garbage can is found in the morning tipped over due to high winds the night before. The wind velocity during the night when the can was tipped over is to be determined.

Assumptions **1** The flow of air in the wind is steady and incompressible. **2** The ground effect on the wind and the drag coefficient is negligible (a crude approximation) so that the resultant drag force acts through the center of the side surface. **3** The garbage can is loaded uniformly so that its weight acts through its center.

Properties The density of air is given to be $\rho = 1.26$ kg/m^3, and the average density of the garbage inside the can is given to be 150 kg/m^3. The drag coefficient of the garbage can is given to be 0.7.



Analysis The volume of the garbage can and the weight of the garbage are

$$V = [\pi D^2/4]H = [\pi(0.80\,\text{m})^2/4](1.2\,\text{m}) = 0.6032\,\text{m}^2$$

$$W = mg = \rho g V = (150\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.6032\,\text{m}^3)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 887.6\,\text{N}$$

When the garbage can is first tipped, the edge on the wind-loaded side of the can will be off the ground, and thus all the reaction forces from the ground will act on the other side. Taking the moment about an axis passing through the contact point and setting it equal to zero gives the required drag force to be

$$\sum M_{\text{contact}} = 0 \;\rightarrow\; F_D \times (H/2) - W \times (D/2) = 0 \quad\rightarrow\quad F_D = \frac{WD}{H} = \frac{(887.6\,\text{N})(0.80\,\text{m})}{1.2\,\text{m}} = 591.7\,\text{N}$$

Noting that the frontal area is DH, the wind velocity that will cause this drag force is determined to be

$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} \;\rightarrow\; 591.7\,\text{N} = (0.7)[1.2\times 0.80\,\text{m}^2]\frac{(1.26\,\text{kg/m}^3)\mathbf{V}^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) \;\rightarrow\; \mathbf{V} = 42.0\,\text{m/s}$$

which is equivalent to a wind velocity of $\mathbf{V} = 42.0\times 3.6 =$ **151 km/h.**

Discussion The analysis above shows that under the stated assumptions, the wind velocity at some moment exceeded 151 km/h. But we cannot tell how high the wind velocity has been. Such analysis and predictions are commonly used in forensic engineering.

13-40E The drag coefficient of a sports car increases when the sunroof is open, and it requires more power to overcome aerodynamic drag. The additional power consumption of the car when the sunroof is opened is to be determined at two different velocities.

Assumptions **1** The car moves steadily at a constant velocity on a straight path. **2** The effect of velocity on the drag coefficient is negligible.

Properties The density of air is given to be 0.075 lbm/ft^3. The drag coefficient of the car is given to be $C_D = 0.32$ when the sunroof is closed, and $C_D = 0.41$ when it is open.



Analysis (*a*) Noting that 1 mph = 1.4667 ft/s and that power is force times velocity, the drag force acting on the car and the power needed to overcome it at 35 mph are:

$$\textit{Open sunroof:}\quad F_{D1} = 0.32(22\text{ ft}^2)\frac{(0.075\text{ lbm/ft}^3)(35\times 1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 21.6\text{ lbf}$$

$$\dot{W}_{\text{drag1}} = F_{D1}\mathbf{V} = (21.6\text{ lbf})(35\times 1.4667\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 1.50\text{ kW}$$

$$\textit{Closed sunroof:}\quad F_{D2} = 0.41(22\text{ ft}^2)\frac{(0.075\text{ lbm/ft}^3)(35\times 1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 27.7\text{ lbf}$$

$$\dot{W}_{\text{drag2}} = F_{D2}\mathbf{V} = (27.7\text{ lbf})(35\times 1.4667\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 1.93\text{ kW}$$

Therefore, the additional power required for this car when the sunroof is open is

$$\dot{W}_{\text{extra}} = \dot{W}_{\text{drag2}} - \dot{W}_{\text{drag2}} = 1.93 - 1.50 = \mathbf{0.43\text{ kW}} \quad \text{(at 35 mph)}$$

(*b*) We now repeat the calculations for 70 mph:

$$\textit{Open sunroof:}\quad F_{D1} = 0.32(22\text{ ft}^2)\frac{(0.075\text{ lbm/ft}^3)(70\times 1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 86.4\text{ lbf}$$

$$\dot{W}_{\text{drag1}} = F_{D1}\mathbf{V} = (86.4\text{ lbf})(70\times 1.4667\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 12.0\text{ kW}$$

$$\textit{Closed sunroof:}\quad F_{D2} = 0.41(22\text{ ft}^2)\frac{(0.075\text{ lbm/ft}^3)(70\times 1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 110.7\text{ lbf}$$

$$\dot{W}_{\text{drag2}} = F_{D2}\mathbf{V} = (110.7\text{ lbf})(70\times 1.4667\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = 15.4\text{ kW}$$

Therefore, the additional power required for this car when the sunroof is open is

$$\dot{W}_{\text{extra}} = \dot{W}_{\text{drag2}} - \dot{W}_{\text{drag2}} = 15.4 - 12.0 = \mathbf{3.4\text{ kW}} \quad \text{(at 70 mph)}$$

Discussion Note that the additional drag caused by open sunroof is 0.43 kW at 35 mph, and 3.4 at 70 mph, which is an increase of 8 folds when the velocity is doubled. This is expected since the power consumption to overcome drag is proportional to the cube of velocity.

Flow over Flat Plates

13-41C The fluid viscosity is responsible for the development of the velocity boundary layer. For the idealized inviscid fluids (fluids with zero viscosity), there will be no velocity boundary layer.

13-42C The friction coefficient represents the resistance to fluid flow over a flat plate. It is proportional to the drag force acting on the plate. The drag coefficient for a flat surface is equivalent to the mean friction coefficient.

13-43C The friction coefficient will change with position in laminar flow over a flat plate (it will decrease along the plate in the flow direction).

13-44C The average friction coefficient in flow over a flat plate is determined by integrating the local friction coefficient over the entire plate, and then dividing it by the length of the plate. Or, it can be determined experimentally by measuring the drag force, and dividing it by the dynamic pressure.

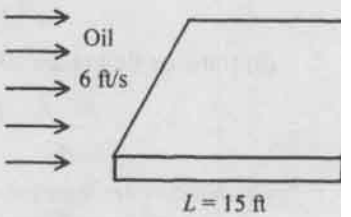
13-45E Light oil flows over a flat plate. The total drag force per unit width of the plate is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5 \times 10^5$. **3** The surface of the plate is smooth.

Properties The density and kinematic viscosity of light oil at 80°F are $\rho = 56.8$ lbm/ft^3 and $\nu = 49\times10^{-5}$ ft^2/s (Table A-16E).

Analysis Noting that $L = 15$ ft, the Reynolds number at the end of the plate is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(6\ \text{ft/s})(15\ \text{ft})}{49\times10^{-5}\ \text{ft}^2/\text{s}} = 1.837\times10^5$$



which is less than the critical Reynolds number. Thus we have *laminar flow* over the entire plate, and the average friction coefficient is determined from

$$C_f = 1.328\,\mathrm{Re}_L^{-0.5} = 1.328\times(1.837\times10^5)^{-0.5} = 0.00310$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the top surface of the plate per unit width becomes

$$F_D = C_f A\frac{\rho V^2}{2} = 0.00310\times(15\times1\ \text{ft}^2)\frac{(56.8\ \text{lbm/ft}^3)(6\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{1.5\ lbf}$$

The total drag force acting on the entire plate can be determined by multiplying the value obtained above by the width of the plate.

Discussion The force per unit width corresponds to the weight of a mass of 1.5 lbf. Therefore, a person who applies an equal and opposite force to the plate to keep it from moving will feel like he or she is using as much force as is necessary to hold a 1.5 lbf mass from dropping.

13-46 Air flows over a plane surface at high elevation. The drag force acting on the top surface of the plate is to be determined for flow along the two sides of the plate. √

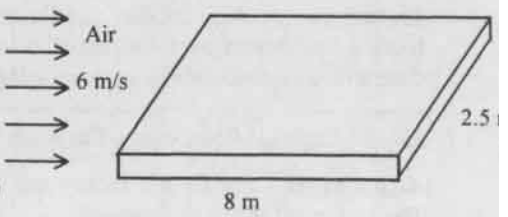
Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

Properties The dynamic viscosity is independent of pressure, and for air at 27°C = 300 K it is $\mu = 1.85\times10^{-5}$ kg/m·s (Table A-18). The air density at 300 K and 83.4 kPa is

$$\rho = \frac{P}{RT} = \frac{83.4\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\,\text{K})} = 0.967\,\text{kg/m}^3$$

Analysis (a) If the air flows parallel to the 8 m side, the Reynolds number becomes

$$\text{Re}_L = \frac{\rho V_\infty L}{\mu} = \frac{(0.967\,\text{kg/m}^3)(6\,\text{m/s})(8\,\text{m})}{1.85\times10^{-5}\,\text{kg/m}\cdot\text{s}} = 2.51\times10^6$$



which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be

$$C_f = \frac{0.074}{\text{Re}_L^{1/5}} - \frac{1742}{\text{Re}_L} = \frac{0.074}{(2.51\times10^6)^{1/5}} - \frac{1742}{2.51\times10^6} = 0.00319$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the top surface of the plate becomes

$$F_D = C_f A\frac{\rho V^2}{2} = 0.00319\times(8\times2.5\,\text{m}^2)\frac{(0.967\,\text{kg/m}^3)(6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{1.11\,N}$$

(b) If the air flows parallel to the 2.5 m side, the Reynolds number is

$$\text{Re}_L = \frac{\rho V_\infty L}{\mu} = \frac{(0.967\,\text{kg/m}^3)(6\,\text{m/s})(2.5\,\text{m})}{1.85\times10^{-5}\,\text{kg/m}\cdot\text{s}} = 7.84\times10^5$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be

$$C_f = \frac{0.074}{\text{Re}_L^{1/5}} - \frac{1742}{\text{Re}_L} = \frac{0.074}{(7.84\times10^5)^{1/5}} - \frac{1742}{7.84\times10^5} = 0.00268$$

Then the drag force acting on the top surface of the plate becomes

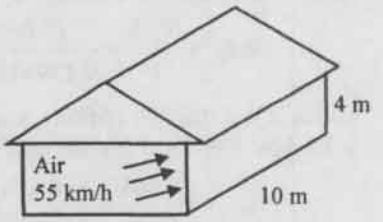
$$F_D = C_f A\frac{\rho V^2}{2} = 0.00268\times(8\times2.5\,\text{m}^2)\frac{(0.967\,\text{kg/m}^3)(6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{0.93\,N}$$

Discussion Note that the drag force is proportional to density, which is proportional to the pressure. Therefore, the altitude has a major influence on the drag force acting on a surface. Commercial airplanes take advantage of this phenomenon and cruise at high altitudes where the air density is much lower to save fuel.

13-47 Wind is blowing parallel to the side wall of a house. The drag force acting on the wall is to be determined for two different wind velocities. √

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The wall surface is smooth (the actual wall surface is usually very rough). **5** The wind blows parallel to the wall.

Properties The density and kinematic viscosity of air at 1 atm and 7°C = 280 K are $\rho = 1.271$ kg/m^3 and $\nu = 1.40\times10^{-5}$ m^2/s (Table A-18).



Analysis The Reynolds number is

$$Re_L = \frac{V_\infty L}{\upsilon} = \frac{(55/3.6\,\text{m/s})(10\,\text{m})}{1.40\times10^{-5}\,\text{m}^2/\text{s}} = 1.09\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is

$$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} = \frac{0.074}{(1.09\times10^7)^{1/5}} - \frac{1742}{1.09\times10^7} = 0.00274$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the wall surface is

$$F_D = C_f A\frac{\rho V^2}{2} = 0.00274\times(10\times4\,\text{m}^2)\frac{(1.271\,\text{kg/m}^3)(55/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{16.3\,N}$$

(b) When the wind velocity is doubled to 110 km/h, the Reynolds number becomes

$$Re_L = \frac{V_\infty L}{\upsilon} = \frac{(110/3.6\,\text{m/s})(10\,\text{m})}{1.40\times10^{-5}\,\text{m}^2/\text{s}} = 2.18\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient and the drag force become

$$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} = \frac{0.074}{(2.18\times10^7)^{1/5}} - \frac{1742}{2.18\times10^7} = 0.00244$$

$$F_D = C_f A\frac{\rho V^2}{2} = 0.00244\times(10\times4\,\text{m}^2)\frac{(1.271\,\text{kg/m}^3)(110/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{57.9\,N}$$

Discussion Note that the actual drag will probably be much higher since the wall surfaces are typically very rough. Also, we can solve this problem using the turbulent flow relation (instead of the combined laminar-turbulent flow relation) without much loss in accuracy. Finally, the drag force nearly quadruples when the velocity is doubled. This is expected since the drag force is proportional to the square of the velocity, and the effect of velocity on the friction coefficient is small.

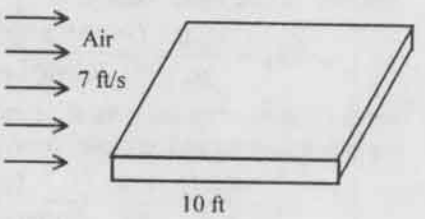
13-48E Air flows over a flat plate. The local friction coefficients at intervals of 1 ft is to be determined and plotted against the distance from the leading edge.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

Properties The density and kinematic viscosity of air at 1 atm and 65°F are $\rho = 0.077$ lbm/ft^3 and $\nu = 0.159\times10^{-3}$ ft^2/s (Table A-18E).

Analysis For the first 1 ft interval, the Reynolds number is

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(7\ \text{ft/s})(1\ \text{ft})}{0.159\times10^{-3}\ \text{ft}^2/\text{s}} = 44{,}025$$

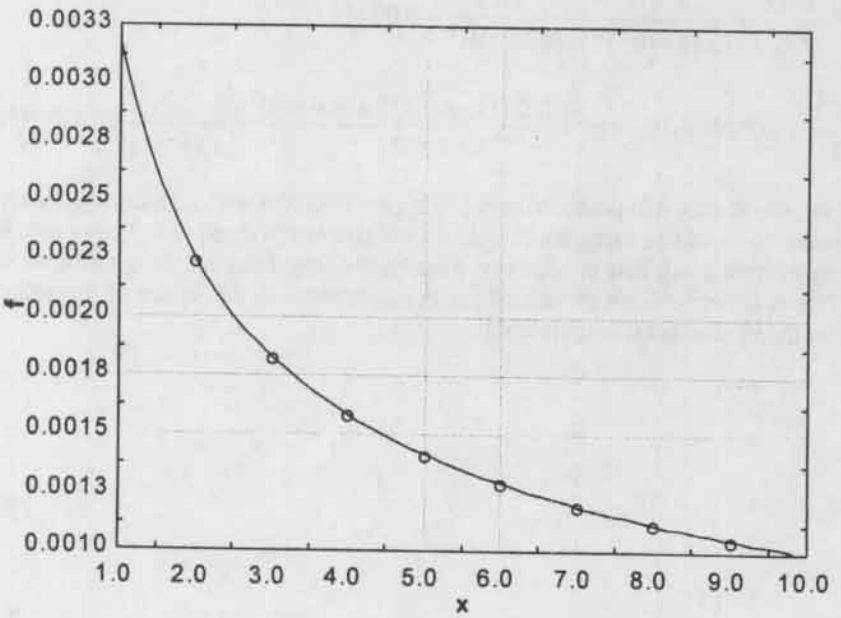


which is less than the critical value of 5×10^5. Therefore, the flow is laminar. The local friction coefficient is

$$C_{f,x} = \frac{0.664}{\mathrm{Re}^{0.5}} = \frac{0.664}{(44{,}025)^{0.5}} = 0.00316$$

We repeat calculations for all 1-ft intervals. The results are

$x = 1.0$ ft	$\mathrm{Re} = 0.440\times10^5$	$C_f = 0.00316$
$x = 2.0$ ft	$\mathrm{Re} = 0.880\times10^5$	$C_f = 0.00224$
$x = 3.0$ ft	$\mathrm{Re} = 1.32\times10^5$	$C_f = 0.00183$
$x = 4.0$ ft	$\mathrm{Re} = 1.76\times10^5$	$C_f = 0.00158$
$x = 5.0$ ft	$\mathrm{Re} = 2.20\times10^5$	$C_f = 0.00142$
$x = 6.0$ ft	$\mathrm{Re} = 2.64\times10^5$	$C_f = 0.00129$
$x = 7.0$ ft	$\mathrm{Re} = 3.08\times10^5$	$C_f = 0.00120$
$x = 8.0$ ft	$\mathrm{Re} = 3.52\times10^5$	$C_f = 0.00112$
$x = 9.0$ ft	$\mathrm{Re} = 3.96\times10^5$	$C_f = 0.00106$
$x = 10.0$ ft	$\mathrm{Re} = 4.40\times10^5$	$C_f = 0.00100$



Discussion Note that the Reynolds number remains under the critical value, and thus the flow remains laminar over the entire plate. Also note that the friction coefficient decreases along the place, as expected.

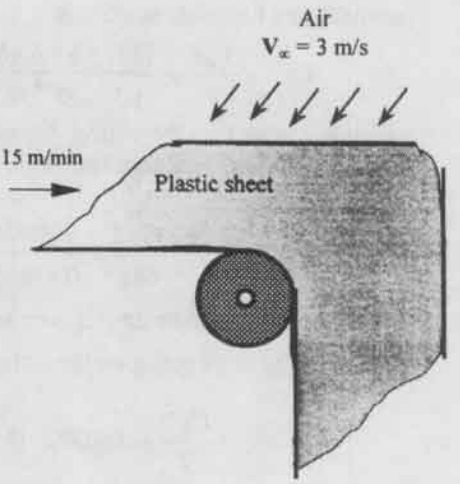
13-49 Air flows on both sides of a continuous sheet of plastic. The drag force air exerts on the plastic sheet in the direction of flow is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** Both surfaces of the plastic sheet are smooth. **5** The plastic sheet does not vibrate and thus it does not induce turbulence in air flow.

Properties The density and kinematic viscosity of air at 1 atm and 330 K are $\rho = 1.076$ kg/m^3 and $\nu = 1.86\times10^{-5}$ m^2/s (Table A-18).

Analysis The length of the cooling section is

$$L = \mathbf{V}_{\text{sheet}}\,\Delta t = [(15/60)\text{ m/s}](2\text{ s}) = 0.5\text{ m}$$



The Reynolds number is

$$\text{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(3\text{ m/s})(1.2\text{ m})}{1.86\times10^{-5}\text{ m}^2/\text{s}} = 1.935\times10^5$$

which is less than the critical Reynolds number. Thus the flow is laminar. The area on both sides of the sheet exposed to air flow is

$$A = 2wL = 2(1.2\text{ m})(0.5\text{ m}) = 1.2\text{ m}^2$$

Then the friction coefficient and the drag force become

$$C_f = \frac{1.328}{\text{Re}_L^{0.5}} = \frac{1.328}{193{,}500^{0.5}} = 0.00302$$

$$F_D = C_f A\frac{\rho\mathbf{V}_\infty^{\,2}}{2} = (0.00302)(1.2\text{ m}^2)\frac{(1.076\text{ kg/m}^3)(3\text{ m/s})^2}{2} = \mathbf{0.0175\text{ N}}$$

Discussion Note that the Reynolds number remains under the critical value, and thus the flow remains laminar over the entire plate. Also note that the friction coefficient decreases along the plate, as expected.

13-50 A train is cruising at a specified velocity. The drag force acting on the top surface of a passenger car of the train is to be determined.

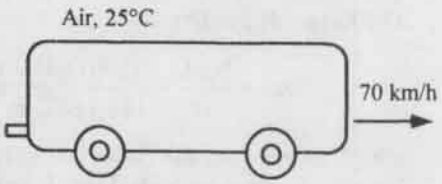
Assumptions **1** The air flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The top surface of the train is smooth (in reality it can be rough). **5** The air is calm (no significant winds).

Properties The density and kinematic viscosity of air at 1 atm and 25°C = 298 K are $\rho = 1.186$ kg/m^3 and $\nu = 1.55\times10^{-5}$ m^2/s (Table A-18).

Analysis The Reynolds number is

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(70/3.6\,\mathrm{m/s})(8\,\mathrm{m})}{1.55\times10^{-5}\ \mathrm{m^2/s}} = 1.00\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be



$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} = \frac{0.074}{(1\times10^7)^{1/5}} - \frac{1742}{1\times10^7} = 0.00277$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the surface becomes

$$F_D = C_f A\frac{\rho \mathbf{V}^2}{2} = 0.00277\times(8\times2.8\,\mathrm{m^2})\frac{(1.186\,\mathrm{kg/m^3})(70/3.6\,\mathrm{m/s})^2}{2}\left(\frac{1\,\mathrm{N}}{1\,\mathrm{kg\cdot m/s^2}}\right) = \mathbf{13.9\ N}$$

Discussion Note that we can solve this problem using the turbulent flow relation (instead of the combined laminar-turbulent flow relation) without much loss in accuracy since the Reynolds number is much greater than the critical value. Also, the actual drag force will probably be greater because of the surface roughness effects.

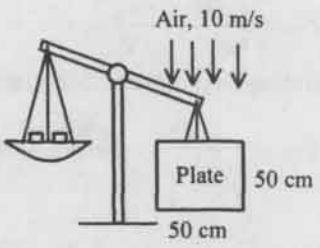
13-51 The weight of a thin flat plate exposed to air flow on both sides is balanced by a counterweight. The mass of the counterweight that needs to be added in order to balance the plate is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surfaces of the plate are smooth.

Properties The density and kinematic viscosity of air at 1 atm and 25°C = 298 K are $\rho = 1.186$ kg/m^3 and $\nu = 1.55\times10^{-5}$ m^2/s (Table A-18).

Analysis The Reynolds number is

$$Re_L = \frac{V_\infty L}{\upsilon} = \frac{(10\,\text{m/s})(0.5\,\text{m})}{1.55\times10^{-5}\,\text{m}^2/\text{s}} = 3.225\times10^5$$



which is less than the critical Reynolds number of 5×10^5. Therefore the flow is laminar. The average friction coefficient, drag force and the corresponding mass are

$$C_f = \frac{1.328}{Re_L^{0.5}} = \frac{1.328}{(3.225\times10^5)^{0.5}} = 0.00234$$

$$F_D = C_f A\frac{\rho V_\infty^2}{2} = (0.00234)[(2\times0.5\times0.5)\,\text{m}^2]\frac{(1.186\,\text{kg/m}^3)(10\,\text{m/s})^2}{2} = 0.069\,\text{kg}\cdot\text{m/s}^2 = 0.069\,\text{N}$$

The mass whose weight is 0.069 N is

$$m = \frac{F_D}{g} = \frac{0.069\,\text{kg.m/s}^2}{9.81\,\text{m/s}^2} = \mathbf{0.0070\,kg = 7.0\,g}$$

Therefore, the mass of the counterweight must be 7 g to counteract the drag force acting on the plate.

Discussion Note that the apparatus described in this problem provides a convenient mechanism to measure drag force and thus drag coefficient.

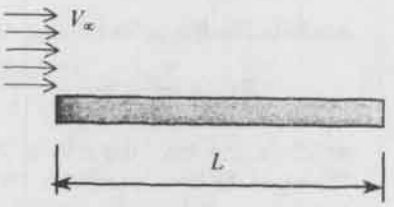
13-52 Laminar flow of a fluid over a flat plate is considered. The change in the drag force is to be determined when the free-stream velocity of the fluid is doubled. √

Analysis For the laminar flow of a fluid over a flat plate the drag force is given by

$$F_{D1} = C_f A \frac{\rho V_\infty^{\,2}}{2} \quad \text{where} \quad C_f = \frac{1.328}{\text{Re}^{0.5}}$$

Therefore

$$F_{D1} = \frac{1.328}{\text{Re}^{0.5}} A \frac{\rho V_\infty^{\,2}}{2}$$



Substituting Reynolds number relation, we get

$$F_{D1} = \frac{1.328}{\left(\dfrac{V_\infty L}{\upsilon}\right)^{0.5}} A \frac{\rho V_\infty^{\,2}}{2} = 0.664 V_\infty^{\,3/2} A \frac{\upsilon^{0.5}}{L^{0.5}}$$

When the free-stream velocity of the fluid is doubled, the new value of the drag force on the plate becomes

$$F_{D2} = \frac{1.328}{\left(\dfrac{(2V_\infty)L}{\upsilon}\right)^{0.5}} A \frac{\rho (2V_\infty)^2}{2} = 0.664 (2V_\infty)^{3/2} A \frac{\upsilon^{0.5}}{L^{0.5}}$$

The ratio of drag forces corresponding to V_∞ and $2V_\infty$ is

$$\frac{F_{D2}}{F_{D1}} = \frac{(2V_\infty)^{3/2}}{V_\infty^{\,3/2}} = 2^{3/2} = 2.83$$

Discussion Note that the drag force increases almost three times in laminar flow when the fluid velocity is doubled.

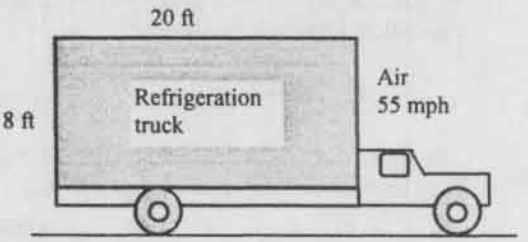
13-53E A refrigeration truck is traveling at a specified velocity. The drag force acting on the top and side surfaces of the truck and the power needed to overcome it are to be determined. √

Assumptions **1** The process is steady and incompressible. **2** The airflow over the entire outer surface is turbulent because of constant agitation. **3** Air is an ideal gas. **4** The top and side surfaces of the truck are smooth (in reality they can be rough). **5** The air is calm (no significant winds).

Properties The density and kinematic viscosity of air at 1 atm and 80°F are $\rho = 0.074$ lbm/ft^3 and $\nu = 0.17\times10^{-3}$ ft^2/s (Table A-18E).

Analysis The Reynolds number is

$$\mathrm{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[55\times1.4667\ \text{ft/s}](20\ \text{ft})}{0.17\times10^{-3}\ \text{ft}^2/\text{s}} = 9.49\times10^6$$



The air flow over the entire outer surface is assumed to be turbulent. Then the friction coefficient becomes

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(9.49\times10^6)^{1/5}} = 0.00298$$

The area of the top and side surfaces of the truck is

$$A = A_{\text{top}} + 2A_{\text{side}} = 9\times20+2\times8\times20 = 500\ \text{ft}^2$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a plane surface, the drag force acting on these surfaces becomes

$$F_D = C_f A\frac{\rho \mathbf{V}^2}{2} = 0.00298\times(500\ \text{ft}^2)\frac{(0.074\ \text{lbm/ft}^3)(55\times1.4667\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{11.1\ lbf}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

$$\dot{W}_{\text{drag}} = F_D \mathbf{V} = (11.1\ \text{lbf})(55\times1.4667\ \text{ft/s})\left(\frac{1\ \text{kW}}{737.56\ \text{lbf}\cdot\text{ft/s}}\right) = \mathbf{1.21\ kW}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

13-54 Air is flowing over a long flat plate with a specified velocity. The distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

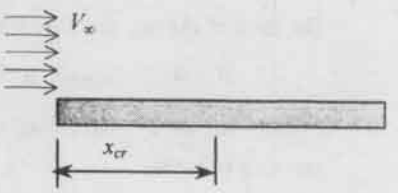
Properties The density and kinematic viscosity of air at 1 atm and 25°C = 298 K are $\rho = 1.186$ kg/m^3 and $\nu = 1.55\times10^{-5}$ m^2/s (Table A-18).

Analysis The critical Reynolds number is given to be $\mathrm{Re}_{cr} = 5\times10^5$. The distance from the leading edge of the plate where the flow becomes turbulent is the distance x_{cr} where the Reynolds number becomes equal to the critical Reynolds number,

$$\mathrm{Re}_{cr} = \frac{\mathbf{V}_\infty x_{cr}}{\upsilon} \quad \rightarrow \quad x_{cr} = \frac{\nu \mathrm{Re}_{cr}}{\mathbf{V}_\infty} = \frac{(1.55\times10^{-5}\ \mathrm{m^2/s})(5\times10^5)}{8\ \mathrm{m/s}} = \mathbf{0.969\ m}$$

The thickness of the boundary layer at that location is obtained by substituting this value of x into the laminar boundary layer thickness relation,

$$\delta_{v,x} = \frac{5x}{\mathrm{Re}_x^{1/2}} \rightarrow \delta_{v,cr} = \frac{5x_{cr}}{\mathrm{Re}_{cr}^{1/2}} = \frac{5(0.969\ \mathrm{m})}{(5\times10^5)^{1/2}} = 0.00685\ \mathrm{m} = \mathbf{0.685\ cm}$$



Discussion When the flow becomes turbulent, the boundary layer thickness starts to increase, and the value of its thickness can be determined from the boundary layer thickness relation for turbulent flow.

13-55 Water is flowing over a long flat plate with a specified velocity. The distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** The surface of the plate is smooth.

Properties The density and dynamic viscosity of water at 1 atm and 25°C are $\rho = 997$ kg/m^3 and $\mu = 0.891\times10^{-3}$ kg/m·s (Table A-16).

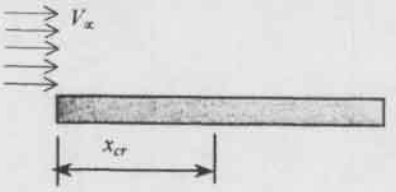
Analysis The critical Reynolds number is given to be $\mathrm{Re}_{cr} = 5\times10^5$. The distance from the leading edge of the plate where the flow becomes turbulent is the distance x_{cr} where the Reynolds number becomes equal to the critical Reynolds number,

$$\mathrm{Re}_{cr} = \frac{\rho \mathbf{V}_\infty x_{cr}}{\mu} \quad \rightarrow \quad x_{cr} = \frac{\mu \mathrm{Re}_{cr}}{\rho \mathbf{V}_\infty} = \frac{(0.891\times10^{-3}\ \mathrm{kg/m\cdot s})(5\times10^5)}{(997\ \mathrm{kg/m^3})(8\ \mathrm{m/s})} = \mathbf{0.056\ m}$$

The thickness of the boundary layer at that location is obtained by substituting this value of x into the laminar boundary layer thickness relation,

$$\delta_{v,x} = \frac{5x}{\mathrm{Re}_x^{1/2}} \rightarrow \delta_{v,cr} = \frac{5x_{cr}}{\mathrm{Re}_{cr}^{1/2}} = \frac{5(0.056\ \mathrm{m})}{(5\times10^5)^{1/2}} = 0.00040\ \mathrm{m} = \mathbf{0.4\ mm}$$

Therefore, the flow becomes turbulent after about 5 cm from the leading edge of the plate, and the thickness of the boundary layer at that location is 0.4 mm.



Discussion When the flow becomes turbulent, the boundary layer thickness starts to increase, and the value of its thickness can be determined from the boundary layer thickness relation for turbulent flow.

Flow across Cylinders and Spheres

13-56C Turbulence moves the fluid separation point further back on the rear of the body, reducing the size of the wake, and thus the magnitude of the pressure drag (which is the dominant mode of drag). As a result, the drag coefficient suddenly drops. In general, turbulence increases the drag coefficient for flat surfaces, but the drag coefficient usually remains constant at high Reynolds numbers when the flow is turbulent.

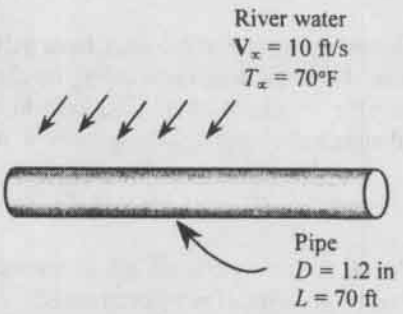
13-57C Friction drag is due to the shear stress at the surface whereas the pressure drag is due to the pressure differential between the front and back sides of the body when a wake is formed in the rear.

13-58C Flow separation in flow over a cylinder is delayed in turbulent flow because of the extra mixing due to random fluctuations and the transverse motion.

13-59E A pipe is crossing a river while remaining completely immersed in water. The drag force exerted on the pipe by the river is to be determined. √

Assumptions **1** The outer surface of the pipe is smooth so that Fig. 13-30 can be used to determine the drag coefficient. **2** Water flow in the river is steady. **3** The turbulence in water flow in the river is not considered. **4**The direction of water flow is normal to the pipe.

Properties The density and dynamic viscosity of water at 70°F are $\rho = 62.30$ lbm/ft^3 and $\mu = 2.36$ lbm/ft·h $= 6.556\times10^{-4}$ lbm/ft·s (Table A-15E).



Analysis Noting that $D = 1.2$ in $= 0.1$ ft, the Reynolds number for flow over the pipe is

$$\text{Re} = \frac{VD}{\nu} = \frac{\rho VD}{\mu} = \frac{(62.30\ \text{lbm/ft}^3)(10\ \text{ft/s})(0.1\ \text{ft})}{6.556\times10^{-4}\ \text{lbm/ft}\cdot\text{s}} = 9.50\times10^4$$

The drag coefficient corresponding to this value is, from Fig. 13-30, $C_D = 1.1$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force acting on the cylinder becomes

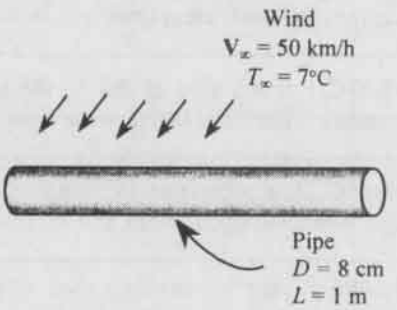
$$F_D = C_D A\frac{\rho V^2}{2} = 1.1\times(70\times0.1\ \text{ft}^2)\frac{(62.30\ \text{lbm/ft}^3)(10\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{745\ lbf}$$

Discussion Note that this force is equivalent to the weight of a 745 lbm mass. Therefore, the drag force the river exerts on the pipe is equivalent to hanging a mass of 745 lbm on the pipe supported at its ends 70 ft apart. The necessary precautions should be taken if the pipe cannot support this force. Also, the fluctuations in water flow may reduce the drag coefficients by inducing turbulence and delaying flow separation.

13-60 A pipe is exposed to high winds. The drag force exerted on the pipe by the winds is to be determined. √

Assumptions **1** The outer surface of the pipe is smooth so that Fig. 13-30 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible. **3** The turbulence in the wind is not considered. **4**The direction of wind is normal to the pipe.

Properties The density and kinematic viscosity of air at 1 atm and 7°C = 280 K are $\rho = 1.271$ kg/m^3 and $\nu = 1.40\times10^{-5}$ m^2/s (Table A-18).



Analysis Noting that $D = 0.08$ m and 1 m/s = 3.6 km/h, the Reynolds number for flow over the pipe is

$$\text{Re} = \frac{VD}{\nu} = \frac{(50/3.6\ \text{m/s})(0.08\ \text{m})}{1.40\times10^{-5}\ \text{m}^2/\text{s}} = 0.794\times10^5$$

The drag coefficient corresponding to this value is, from Fig. 13-30, $C_D = 1.0$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force becomes

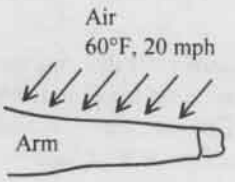
$$F_D = C_D A\frac{\rho V^2}{2} = 1.0(1\times0.08\ \text{m}^2)\frac{(1.271\ \text{kg/m}^3)(50/3.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{9.8\ N}\ \text{(per m length)}$$

Discussion Note that the drag force acting on a unit length of the pipe is equivalent to the weight of 1 kg mass. The total drag force acting on the entire pipe can be obtained by multiplying the value obtained by the pipe length. It should be kept in mind that wind turbulence may reduce the drag coefficients by inducing turbulence and delaying flow separation.

13-61E A person extends his uncovered arms into the windy air outside. The drag force exerted on both arms by the wind is to be determined. √

Assumptions **1** The surfaces of the arms are smooth so that Fig. 13-30 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible. **3** The turbulence in the wind is not considered. **4**The direction of wind is normal to the arms. **5** The arms can be treated as 2-ft-long and 3-in.-diameter cylinders with negligible end effects.

Properties The density and kinematic viscosity of air at 1 atm and 60°F are $\rho = 0.077$ lbm/ft^3 and $\nu = 0.159\times10^{-3}$ ft^2/s (Table A-18E).



Analysis Noting that $D = 3$ in = 0.25 ft and 1 mph = 1.4667 ft/s, the Reynolds number for flow over the arm is

$$\text{Re} = \frac{VD}{\nu} = \frac{(20\times1.4667\ \text{ft/s})(0.25\ \text{ft})}{0.159\times10^{-3}\ \text{ft}^2/\text{s}} = 4.61\times10^4$$

The drag coefficient corresponding to this value is, from Fig. 13-30, $C_D = 1.0$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the total drag force acting on *both* arms becomes

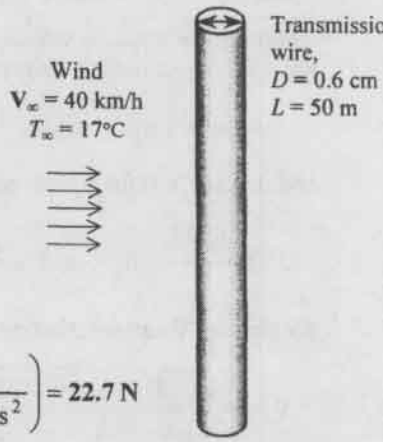
$$F_D = C_D A\frac{\rho V^2}{2} = 1.0\times(2\times2\times0.25\ \text{ft}^2)\frac{(0.077\ \text{lbm/ft}^3)(20\times1.4667\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{1.02\ lbf}$$

Discussion Note that this force is equivalent to the weight of 1 lbm mass. Therefore, the drag force the wind exerts on the arms of this person is equivalent to hanging 0.5 lbm of mass on each arm. Also, it should be kept in mind that the wind turbulence and the surface roughness may affect the calculated result significantly.

13-62 Wind is blowing across the wire of a transmission line. The drag force exerted on the wire by the wind is to be determined. √

Assumptions **1** The wire surfaces are smooth so that Fig. 13-30 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible. **3** The turbulence in the wind is not considered. **4**The direction of wind is normal to the wire.

Properties The density and kinematic viscosity of air at 1 atm and 17°C = 290 K are $\rho = 1.224$ kg/m^3 and $\nu = 1.48\times10^{-5}$ m^2/s (Table A-18).



Analysis Noting that $D = 0.006$ m and 1 m/s = 3.6 km/h, the Reynolds number for the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(40/3.6\ \text{m/s})(0.006\ \text{m})}{1.48\times10^{-5}\ \text{m}^2/\text{s}} = 4.5\times10^3$$

The drag coefficient corresponding to this value is, from Fig. 13-30, $C_D = 1.0$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force becomes

$$F_D = C_D A\frac{\rho V^2}{2} = 1.0(50\times0.006\ \text{m}^2)\frac{(1.224\ \text{kg/m}^3)(40/3.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{22.7\ N}$$

Therefore, the drag force acting on a 50-m long section of the wire is 27.4 N, which is equivalent to the weight of about 2.3 kg mass hanging on the wire.

Discussion It should be kept in mind that wind turbulence may reduce the drag coefficients by inducing turbulence and delaying flow separation.

13-63 A spherical hail is falling freely in the atmosphere. The terminal velocity of the hail in air is to be determined. . √

Assumptions **1** The surface of the hail is smooth so that Fig. 13-30 can be used to determine the drag coefficient. **2** The variation of the air properties with altitude is negligible. **3** The buoyancy force applied by air to hail is negligible since $\rho_{air} << \rho_{hail}$ (besides, the uncertainty in the density of hail is greater than the density of air). **4** Air flow over the hail is steady and incompressible when terminal velocity is established. **5** The atmosphere is calm (no winds or drafts).

Properties The density and kinematic viscosity of air at 1 atm and 7°C = 280 K are $\rho = 1.271$ kg/m^3 and $\nu = 1.40\times10^{-5}$ m^2/s (Table A-18). The density of hail is given to be 910 kg/m^3.

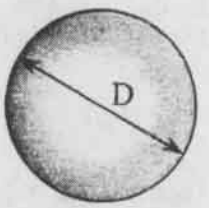
Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_D = W - F_B \quad \text{where} \quad F_D = C_D A\frac{\rho_f \mathbf{V}^2}{2}, \quad W = mg = \rho_s g V = \rho_s g(\pi D^3/6), \quad \text{and} \quad F_B \cong 0$$

and $A = \pi D^2/4$ is the frontal area. Substituting and simplifying,

$$C_D A\frac{\rho_f \mathbf{V}^2}{2} = W \rightarrow C_D\frac{\pi D^2}{4}\frac{\rho_f \mathbf{V}^2}{2} = \rho_s g\frac{\pi D^3}{6} \rightarrow C_D\rho_f \mathbf{V}^2 = \rho_s g\frac{4D}{3}$$

Hail
$D = 0.3$ cm



Air
$T_\infty = 7$°C

Solving for **V** and substituting,

$$\mathbf{V} = \sqrt{\frac{4g\rho_s D}{3C_D\rho_f}} = \sqrt{\frac{4(9.81\,\text{m/s}^2)(910\,\text{kg/m}^3)(0.008\,\text{m})}{3C_D(1.271\,\text{kg/m}^3)}} \rightarrow \mathbf{V} = \frac{8.656}{\sqrt{C_D}} \qquad (1)$$

The drag coefficient C_D is to be determined from Fig. 13-30, but it requires the Reynolds number which cannot be calculated since we do not know velocity. Therefore, the solution requires a trial-error approach. First we express the Reynolds number as

$$\text{Re} = \frac{\mathbf{V}D}{\nu} = \frac{\mathbf{V}(0.008\,\text{m})}{1.40\times10^{-5}\,\text{m}^2/\text{s}} \rightarrow \text{Re} = 571.4\mathbf{V} \qquad (2)$$

Now we choose a velocity in m/s, calculate the Re from Eq. (2), read the corresponding C_D from Fig. 13-30, and calculate **V** from Eq. (1). Repeat calculations until the assumed velocity matches the calculated velocity. With this approach the terminal velocity is determined to be

V = 13.7 m/s

The corresponding Re and C_D values are Re = 7828 and $C_D = 0.40$. Therefore, the velocity of hail will remain constant when it reaches the terminal velocity of 13.7 m/s = 49 km/h.

Discussion The simple analysis above gives us a reasonable value for the terminal velocity. A more accurate answer can be obtained by a more detailed (and complex) analysis by considering the variation of air properties with altitude, and by considering the uncertainty in the drag coefficient (a hail is not necessarily spherical and smooth).

13-64 A spherical dust particle is suspended in the air at a fixed point as a result of an updraft air motion. The magnitude of the updraft velocity is to be determined using Stokes law.

Assumptions **1** The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). **2** The updraft is steady and incompressible. **3** The buoyancy force applied by air to the dust particle is negligible since $\rho_{air} << \rho_{dust}$ (besides, the uncertainty in the density of dust is greater than the density of air). (We will solve the problem without utilizing this assumption for generality).

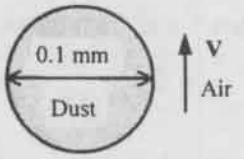
Properties The density of dust is given to be ρ_s = 2.1 g/cm^3 = 2100 kg/m^3. The density and dynamic viscosity of air at 1 atm and 25°C = 298 K are ρ_f = 1.186 kg/m^3 and $\mu = 1.85\times10^{-5}$ kg/m·s (Table A-18).

Analysis The terminal velocity of a free falling object is reached (or the suspension of an object in a flow stream is established) when the drag force equals the weight of the solid object less the buoyancy force applied by the surrounding fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu VD \text{ (Stoke's law)}, \quad W = \rho_s g V, \quad \text{and} \quad F_B = \rho_f g V$$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting,

$$3\pi\mu VD = \rho_s g V - \rho_f g V \quad \rightarrow \quad 3\pi\mu VD = (\rho_s - \rho_f) g \frac{\pi D^3}{6}$$



Solving for the velocity **V** and substituting the numerical values, the updraft velocity is determined to be

$$\mathbf{V} = \frac{gD^2(\rho_s - \rho_f)}{18\mu} = \frac{(9.81\,\text{m/s}^2)(0.0001\,\text{m})^2(2100 - 1.186)\,\text{kg/m}^3}{18(1.85\times10^{-5}\,\text{kg/m}\cdot\text{s})} = \mathbf{0.62\ m/s}$$

The Reynolds number in this case is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(1.186\,\text{kg/m}^3)(0.62\,\text{m/s})(0.0001\,\text{m})}{1.85\times10^{-5}\,\text{kg}\cdot\text{m/s}} = 4.0$$

which is in the order of 1. Therefore, the creeping flow idealization and thus Stokes law is applicable, and the value calculated is valid.

Discussion Flow separation starts at about Re = 10. Therefore, Stokes law can be used for Reynolds numbers upto this value, but this should be done with care.

13-65 Dust particles that are unsettled during high winds rise to a specified height, and start falling back when things calm down. The time it takes for the dust particles to fall back to the ground and their velocity are to be determined using Stokes law.

Assumptions **1** The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). **2** The atmosphere is calm during fall back (no winds or drafts). **3** The initial transient period during which the dust particle accelerates to its terminal velocity is negligible. **4** The buoyancy force applied by air to the dust particle is negligible since $\rho_{air} << \rho_{dust}$ (besides, the uncertainty in the density of dust is greater than the density of air). (We will solve this problem without utilizing this assumption for generality).

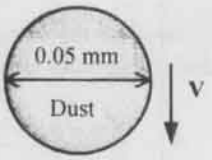
Properties The density of dust is given to be ρ_s = 1.8 g/cm^3 = 1800 kg/m^3. The density and dynamic viscosity of air at 1 atm and 17°C = 290 K are ρ_f = 1.224 kg/m^3 and $\mu = 1.80\times10^{-5}$ kg/m·s (Table A-18).

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the surrounding fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu VD \text{ (Stoke's law)}, \quad W = \rho_s g V, \quad \text{and} \quad F_B = \rho_f g V$$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting,

$$3\pi\mu VD = \rho_s g V - \rho_f g V \;\rightarrow\; 3\pi\mu VD = (\rho_s - \rho_f) g \frac{\pi D^3}{6}$$



Solving for the velocity **V** and substituting the numerical values, the terminal velocity is determined to be

$$\mathbf{V} = \frac{gD^2(\rho_s - \rho_f)}{18\mu} = \frac{(9.81\,\text{m/s}^2)(5\times10^{-5}\,\text{m})^2(1800 - 1.224)\,\text{kg/m}^3}{18(1.80\times10^{-5}\,\text{kg/m}\cdot\text{s})} = \mathbf{0.136\,m/s}$$

Then the time it takes for the dust particle to travel 500 m at this velocity becomes

$$\Delta t = \frac{L}{\mathbf{V}} = \frac{500\,\text{m}}{0.136\,\text{m/s}} = 3676\,\text{s} = \mathbf{1.02\,h}$$

The Reynolds number is

$$\text{Re} = \frac{\rho \mathbf{V} D}{\mu} = \frac{(1.224\,\text{kg/m}^3)(0.136\,\text{m/s})(5\times10^{-5}\,\text{m})}{1.80\times10^{-5}\,\text{kg}\cdot\text{m/s}} = 0.45$$

which is in the order of 1. Therefore, the creeping flow idealization and thus Stokes law is applicable.

Discussion Note that the dust particle reaches a terminal velocity of 0.136 m/s, and it takes about an hour to fall back to the ground. The presence of drafts in air may significantly increase the settling time.

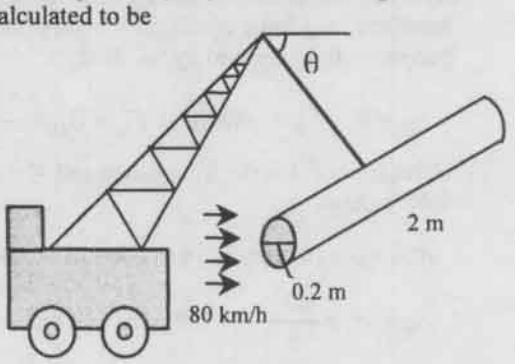
13-66 A cylindrical log suspended by a crane is subjected to normal winds. The angular displacement of the log and the tension on the cable are to be determined.

Assumptions **1** The surfaces of the log are smooth so that Fig. 13-30 can be used to determine the drag coefficient (not a realistic assumption). **2** Air flow in the wind is steady and incompressible. **3** The turbulence in the wind is not considered. **4**The direction of wind is normal to the log, which always remains horizontal. **5** The end effects of the log are negligible. **6** The weight of the cable and the drag acting on it are negligible. **7** Air is an ideal gas.

Properties The dynamic viscosity of air at 7°C = 280 K (independent of pressure) is $\mu = 1.75\times10^{-5}$ kg/m·s (Table A-18). Then the density and kinematic viscosity of air are calculated to be

$$\rho = \frac{P}{RT} = \frac{88\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(280\,\text{K})} = 1.095\,\text{kg/m}^3$$

$$\nu = \frac{\mu}{\rho} = \frac{1.75\times10^{-5}\,\text{kg/m}\cdot\text{s}}{1.095\,\text{kg/m}^3} = 1.60\times10^{-5}\,\text{m}^2/\text{s}$$



Analysis Noting that $D = 0.2$ m and 1 m/s = 3.6 km/h, the Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(40/3.6\,\text{m/s})(0.2\,\text{m})}{1.60\times10^{-5}\,\text{m}^2/\text{s}} = 1.39\times10^5$$

The drag coefficient corresponding to this value is, from Fig. 13-30, $C_D = 1.2$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the total drag force acting on the log becomes

$$F_D = C_D A\frac{\rho V^2}{2} = 1.2(2\times0.2\,\text{m}^2)\frac{(1.095\,\text{kg/m}^3)(40/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 32.4\,\text{N}$$

The weight of the log is

$$W = mg = \rho g V = \rho g\frac{\pi D^2 L}{4} = (513\,\text{kg/m}^3)(9.81\,\text{m/s}^2)\frac{\pi(0.2\,\text{m})^2(2\,\text{m})}{4}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 316\,\text{N}$$

Then the resultant force acting on the log and the angle it makes with the horizontal become

$$F_{\text{log}} = R = \sqrt{W^2 + F_D^2} = \sqrt{32.4^2 + 316^2} = \mathbf{318\,N}$$

$$\tan\theta = \frac{W}{F_D} = \frac{316}{32.4} = 9.75 \quad\rightarrow\quad \theta = \mathbf{84°}$$

Drawing a free body diagram of the log and doing a force balance will show that the magnitude of the tension on the cable must be equal to the resultant force acting on the log. Therefore, the tension on the cable is 318 N and the cable makes 84° with the horizontal.

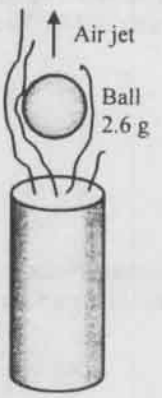
Discussion Note that the wind in this case has rotated the cable by 6° from its vertical position, and increased the tension action on it somewhat. At very high wind speeds, the increase in the cable tension can be very significant, and wind loading must always be considered in bodies exposed to high winds.

13-67 A ping-pong ball is suspended in air by an upward air jet. The velocity of the air jet is to be determined, and the phenomenon that the ball returns to the center of the air jet after a disturbance is to be explained.

Assumptions **1** The surface of the ping-pong ball is smooth so that Fig. 13-30 can be used to determine the drag coefficient. **2** Air flow over the ball is steady and incompressible.

Properties The density and kinematic viscosity of air at 1 atm and 25°C = 298 K are $\rho = 1.186$ kg/m^3 and $\nu = 1.55\times10^{-5}$ m^2/s (Table A-18).

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the fluid,



$$F_D = W - F_B \quad \text{where} \quad F_D = C_D A\frac{\rho_f \mathbf{V}^2}{2}, \quad W = mg, \quad \text{and} \quad F_B = \rho_f g V$$

Here $A = \pi D^2/4$ is the frontal area and $V = \pi D^3/6$ is the volume of the sphere. Also,

$$W = mg = (0.0026\,\text{kg})(9.81\,\text{m/s}^2) = 0.0255\,\text{kg}\cdot\text{m/s}^2 = 0.0255\,\text{N}$$

$$F_B = \rho_f g\frac{\pi D^3}{6} = (1.186\,\text{kg/m}^3)(9.81\,\text{m/s}^2)\frac{\pi(0.038\,\text{m})^3}{6} = 0.00033\,\text{kg}\cdot\text{m/s}^2 = 0.00033\,\text{N}$$

Substituting and solving for **V**,

$$C_D\frac{\pi D^2}{4}\frac{\rho_f \mathbf{V}^2}{2} = W - F_B \rightarrow \mathbf{V} = \sqrt{\frac{8(W-F_B)}{\pi D^2 C_D \rho_f}} = \sqrt{\frac{8(0.0255-0.00033)\,\text{kg}\cdot\text{m/s}^2)}{\pi(0.038\,\text{m})^2 C_D(1.186\,\text{kg/m}^3)}} \rightarrow \mathbf{V} = \frac{6.12}{\sqrt{C_D}} \quad (1)$$

The drag coefficient C_D is to be determined from Fig. 13-30, but it requires the Reynolds number which cannot be calculated since we do not know velocity. Therefore, the solution requires a trial-error approach. First we express the Reynolds number as

$$\text{Re} = \frac{\mathbf{V}D}{\nu} = \frac{\mathbf{V}(0.038\,\text{m})}{1.55\times10^{-5}\,\text{m}^2/\text{s}} \rightarrow \text{Re} = 2452\mathbf{V} \quad (2)$$

Now we choose a velocity in m/s, calculate the Re from Eq. (2), read the corresponding C_D from Fig. 13-30, and calculate **V** from Eq. (1). Repeat calculations until the assumed velocity matches the calculated velocity. With this approach the velocity of the fluid jet is determined to be

V = 9.3 m/s

The corresponding Re and C_D values are Re = 22,800 and $C_D = 0.43$. Therefore, the ping-pong ball will remain suspended in the air jet when the air velocity reaches 9.3 m/s = 33.5 km/h.

Discussion **1** If the ball is pushed to the side by a finger, the ball will come back to the center of the jet (instead of falling off) due to the Bernoulli effect. In the core of the jet the velocity is higher, and thus the pressure is lower relative to a location away from the jet.

2 Note that this simple apparatus can be used to determine the drag coefficients of certain object by simply measuring the air velocity, which is easy to do.

3 This problem can also be solved roughly by taking $C_D = 0.5$ from Table 13-3 for a sphere in laminar flow, and then verifying that the flow is laminar.

Lift

13-68C The contribution of viscous effects to lift is usually negligible for airfoils since the wall shear is parallel to the surfaces of such devices and thus nearly normal to the direction of lift.

13-69C When air flows past a symmetrical airfoil at zero angle of attack, (*a*) the lift will be zero, but (*b*) the drag acting on the airfoil will be nonzero.

13-70C When air flows past a nonsymmetrical airfoil at zero angle of attack, both the (*a*) lift and (*b*) drag acting on the airfoil will be nonzero.

13-71C When air flows past a symmetrical airfoil at an angle of attack of 5°, both the (*a*) lift and (*b*) drag acting on the airfoil will be nonzero.

13-72C The decrease of lift with an increase in the angle of attack is called *stall*. When the flow separates over nearly the entire upper half of the airfoil, the lift is reduced dramatically (the separation point is near the leading edge). Stall is caused by flow separation and the formation of a wide wake region over the top surface of the airfoil. The commercial aircraft are not allowed to fly at velocities near the stall velocity for safety reasons. Airfoils stall at high angles of attack (flow cannot negotiate the curve around the leading edge). If a plane stalls, it loses mush of its lift, and it can crash.

13-73C Both the lift and the drag of an airfoil increase with an increase in the angle of attack, but in general lift increases at a much higher rate than does the drag.

13-74C Flaps are used at the leading and trailing edges of the wings of large aircraft during takeoff and landing to alter the shape of the wings to maximize lift and to enable the aircraft to land or takeoff at low speeds. An aircraft can takeoff or land without flaps, but it can do so at very high velocities, which is undesirable during takeoff and landing.

13-75C Flaps increase both the lift and the drag of the wings. But the increase in drag during takeoff and landing is not much of a concern because of the relatively short time periods involved. This is the penalty we pay willingly to takeoff and land at safe speeds.

13-76C The effect of wing tip vortices is to increase drag (induced drag) and to decrease lift. This effect is also due to the downwash, which causes an effectively smaller angle of attack.

13-77C Induced drag is the additional drag caused by the tip vortices. The tip vortices have a lot of kinetic energy, all of which is wasted and is ultimately dissipated as heat in the air downstream. The induced drag can be reduced by using long and narrow wings.

13-78C When air is flowing past a spherical ball, the lift exerted on the ball will be zero if the ball is not spinning, and it will be nonzero if the ball is spinning about an axis normal to the free stream velocity (no lift will be generated if the ball is spinning about an axis parallel to the free stream velocity).

13-79 A tennis ball is hit with a backspin. It is to be determined if the ball will fall or rise after being hit.

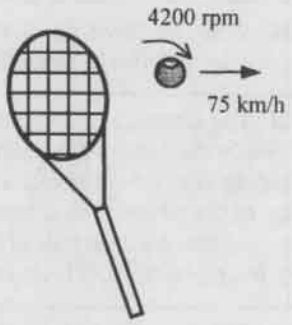
Assumptions **1** The outer surface of the ball is smooth enough for Fig. 13-46 to be applicable. **2** The ball is hit horizontally so that it starts its motion horizontally.

Properties The density and kinematic viscosity of air at 1 atm and 27°C = 300 K are $\rho = 1.177$ kg/m^3 and $\nu = 1.57\times10^{-5}$ m^2/s (Table A-18).

Analysis The ball is hit horizontally, and thus it would normally fall under the effect of gravity without the spin. The backspin will generate a lift, and the ball will rise if the lift is greater than the weight of the ball. The lift can be determined from

$$F_L = C_L A \frac{\rho \mathbf{V}^2}{2}$$

where A is the frontal area of the ball, which is $A = \pi D^2/4$. The regular and angular velocities of the ball are



$$\mathbf{V} = (75\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 20.8\text{ m/s}$$

$$\omega = (4200\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 440\text{ rad/s}$$

Then,

$$\frac{\omega D}{2\mathbf{V}} = \frac{(440\text{ rad/s})(0.064\text{ m})}{2(20.8\text{ m/s})} = 0.677\text{ rad}$$

From Fig. 13-46, the lift coefficient corresponding to this value is $C_L = 0.15$. Then the lift acting on the ball is

$$F_L = (0.15)\frac{\pi(0.064\text{ m})^2}{4}\frac{(1.177\text{ kg/m}^3)(20.8\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 0.12\text{ N}$$

The weight of the ball is

$$W = mg = (0.057\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 0.56\text{ N}$$

which is more than the lift. Therefore, the ball will **drop** under the combined effect of gravity and lift due to spinning after hitting, with a net force of 0.56 - 0.12 = 0.44 N.

Discussion The Reynolds number for this problem is

$$\text{Re}_L = \frac{\mathbf{V}D}{\upsilon} = \frac{(20.8\text{ m/s})(0.064\text{ m})}{1.57\times10^{-5}\text{ m}^2/\text{s}} = 8.5\times10^4$$

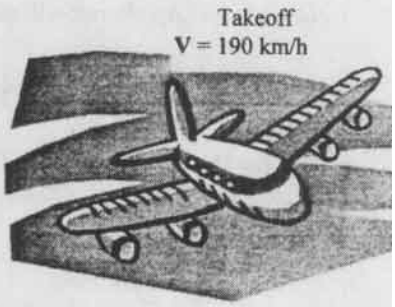
which is close enough to 6×10^4 for which Fig. 13-46 is prepared. Therefore, the result should be close enough to the actual answer.

13-80 The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the weight of the aircraft is increased by 20% as a result of overloading is to be determined. √

Assumptions **1** The atmospheric conditions (and thus the properties of air) remain the same. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$



We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and area remain constant, the ratio of the velocities of the overloaded and fully loaded aircraft becomes

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\sqrt{2W_2 / \rho C_L A}}{\sqrt{2W_1 / \rho C_L A}} = \frac{\sqrt{W_2}}{\sqrt{W_1}} \quad \rightarrow \quad \mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{W_2}{W_1}}$$

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

$$\mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{1.2W_1}{W_1}} = (190 \text{ km/h})\sqrt{1.2} = \mathbf{208 \text{ km/h}}$$

Discussion A similar analysis can be performed for the effect of the variations in density, lift coefficient, and planform area on the takeoff velocity.

13-81 The takeoff speed and takeoff time of an aircraft at sea level are given. The required takeoff speed, takeoff time, and the additional runway length required at a higher elevation are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane and the planform area remain constant. **3** The acceleration of the aircraft during takeoff remains constant.

Properties The density of standard air is $\rho_1 = 1.225$ kg/m^3 at sea level, and $\rho_2 = 1.048$ kg/m^3 at 1600 m altitude (Table A-26).

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$

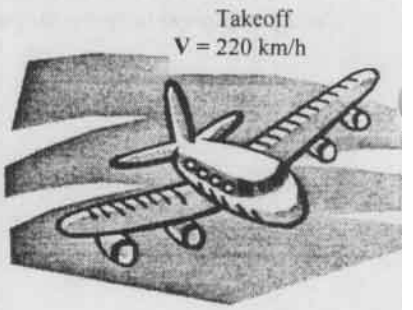
We note that the takeoff speed is inversely proportional to the square root of air density. When the weight, lift coefficient, and area remain constant, the ratio of the speeds of the aircraft at high altitude and at sea level becomes

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\sqrt{2W / \rho_2 C_L A}}{\sqrt{2W / \rho_1 C_L A}} = \frac{\sqrt{\rho_1}}{\sqrt{\rho_2}} \quad \rightarrow \quad \mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{\rho_1}{\rho_2}} = (220 \text{ km/h}) \sqrt{\frac{1.225}{1.048}} = \mathbf{238 \text{ km/h}}$$

Therefore, the takeoff velocity of the aircraft at higher altitude is 238 km/h.

(*b*) The acceleration of the aircraft at sea level is

$$a = \frac{\Delta \mathbf{V}}{\Delta t} = \frac{220 \text{ km/h} - 0}{15 \text{ s}} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 4.074 \text{ m/s}^2$$



which is assumed to be constant both at sea level and the higher altitude. Then the takeoff time at the higher altitude becomes

$$a = \frac{\Delta \mathbf{V}}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{\Delta \mathbf{V}}{a} = \frac{238 \text{ km/h} - 0}{4.074 \text{ m/s}^2} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = \mathbf{16.2 \text{ s}}$$

(*c*) The additional runway length is determined by calculating the distance traveled during takeoff for both cases, and taking their difference:

$$L_1 = \tfrac{1}{2} a t_1^2 = \tfrac{1}{2} (4.074 \text{ m/s}^2)(15 \text{ s})^2 = 458 \text{ m}$$

$$L_2 = \tfrac{1}{2} a t_2^2 = \tfrac{1}{2} (4.074 \text{ m/s}^2)(16.2 \text{ s})^2 = 535 \text{ m}$$

$$\Delta L = L_2 - L_1 = 535 - 458 = \mathbf{97 \text{ m}}$$

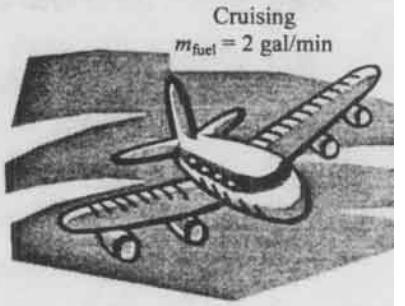
Discussion Note that altitude has a significant effect on the length of the runways, and it should be a major consideration on the design of airports. It is interesting that a 1.2 second increase in takeoff time increases the required runway length by about 100 m.

13-82E The rate of fuel consumption of an aircraft while flying at a low altitude is given. The rate of fuel consumption at a higher altitude is to be determined for the same flight velocity.

Assumptions **1** Standard atmospheric conditions exist. **2** The settings of the plane during takeoff are maintained the same so that the drag coefficient of the plane and the planform area remain constant. **3** The velocity of the aircraft and the propulsive efficiency remain constant. **4** The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties The density of standard air is $\rho_1 = 0.05648$ lbm/ft^3 at 10,000 ft, and $\rho_2 = 0.02866$ lbm/ft^3 at 30,000 ft altitude (Table A-26).

Analysis When an aircraft cruises steadily (zero acceleration) at a constant altitude, the net force acting on the aircraft is zero, and thus the thrust provided by the engines must be equal to the drag force. Also, power is force times velocity (distance per unit time), and thus the propulsive power required to overcome drag is equal to the thrust times the cruising velocity. Therefore,



$$\dot{W}_{\text{propulsive}} = \text{Thrust} \times \mathbf{V} = F_D \mathbf{V} = C_D A \frac{\rho \mathbf{V}^2}{2} \mathbf{V} = C_D A \frac{\rho \mathbf{V}^3}{2}$$

The propulsive power is also equal to the product of the rate of fuel energy supplied (which is the rate of fuel consumption times the heating value of the fuel, $\dot{m}_{\text{fuel}}\text{HV}$) and the propulsive efficiency. Then,

$$\dot{W}_{\text{prop}} = \eta_{\text{prop}} \dot{m}_{\text{fuel}} \text{HV} \quad \rightarrow \quad C_D A \frac{\rho \mathbf{V}^3}{2} = \eta_{\text{prop}} \dot{m}_{\text{fuel}} \text{HV}$$

We note that the rate of fuel consumption is proportional to the density of air. When the drag coefficient, the wing area, the velocity, and the propulsive efficiency remain constant, the ratio of the rates of fuel consumptions of the aircraft at high and low altitudes becomes

$$\frac{\dot{m}_{\text{fuel},2}}{\dot{m}_{\text{fuel},1}} = \frac{C_D A \rho_2 \mathbf{V}^3 / 2\eta_{\text{prop}} \text{HV}}{C_D A \rho_1 \mathbf{V}^3 / 2\eta_{\text{prop}} \text{HV}} = \frac{\rho_2}{\rho_1} \quad \rightarrow \quad \dot{m}_{\text{fuel},2} = \dot{m}_{\text{fuel},1} \frac{\rho_2}{\rho_1} = (2 \text{ gal/min}) \frac{0.02866}{0.05648} = \mathbf{1.01 \text{ gal/min}}$$

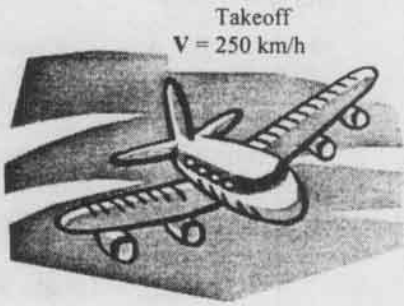
Discussion Note the fuel consumption drops by half when the aircraft flies at 30,000 ft instead of 10,000 ft altitude. Therefore, large passenger planes routinely fly at high altitudes (usually between 30,000 and 40,000 ft) to save fuel. This is especially the case for long flights.

13-83 The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the aircraft has 100 empty seats is to be determined.

Assumptions **1** The atmospheric conditions (and thus the properties of air) remain the same. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same. **3** A passenger with luggage has an average mass of 140 kg.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$



We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and wing area remain constant, the ratio of the velocities of the under-loaded and fully loaded aircraft becomes

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\sqrt{2W_2 / \rho C_L A}}{\sqrt{2W_1 / \rho C_L A}} = \frac{\sqrt{W_2}}{\sqrt{W_1}} = \frac{\sqrt{m_2 g}}{\sqrt{m_1 g}} = \frac{\sqrt{m_2}}{\sqrt{m_1}} \quad \rightarrow \quad \mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{m_2}{m_1}}$$

where

$$m_2 = m_1 - m_{\text{unused capacity}} = 400{,}00\ \text{kg} - (140\ \text{kg/passanger}) \times (100\ \text{passengers}) = 386{,}000\ \text{kg}$$

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

$$\mathbf{V}_2 = \mathbf{V}_1 \sqrt{\frac{m_2}{m_1}} = (250\ \text{km/h}) \sqrt{\frac{386{,}000}{400{,}000}} = \mathbf{246\ km/h}$$

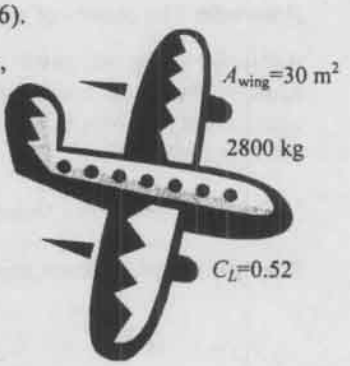
Discussion Note that the effect of empty seats on the takeoff velocity of the aircraft is small. This is because the most weight of the aircraft is due to its empty weight (the aircraft itself rather than the passengers themselves and their luggage.)

13-84 The wing area, lift coefficient at takeoff settings, the cruising drag coefficient, and total mass of a small aircraft are given. The takeoff speed, the wing loading, and the required power to maintain a constant cruising speed are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered.

Properties The density of standard air at sea level is $\rho = 1.225\ \text{kg/m}^3$ (Table A-26).

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,



$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$

Substituting, the takeoff speed is determined to be

$$\mathbf{V}_{\text{takeoff}} = \sqrt{\frac{2mg}{\rho C_{L,\text{takeoff}} A}} = \sqrt{\frac{2(2800\ \text{kg})(9.81\ \text{m/s}^2)}{(1.225\ \text{kg/m}^3)(0.52)(30\ \text{m}^2)}}$$
$$= 53.6\ \text{m/s} = \mathbf{193\ km/h}$$

(*b*) Wing loading is the average lift per unit planform area, which is equivalent to the ratio of the lift to the planform area of the wings since the lift generated during steady cruising is equal to the weight of the aircraft. Therefore,

$$F_{\text{loading}} = \frac{F_L}{A} = \frac{W}{A} = \frac{(2800\ \text{kg})(9.81\ \text{m/s}^2)}{30\ \text{m}^2} = \mathbf{916\ N/m^2}$$

(*c*) When the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force, which is

$$F_D = C_D A \frac{\rho \mathbf{V}^2}{2} = (0.0035)(30\ \text{m}^2) \frac{(1.225\ \text{kg/m}^3)(300/3.6\ \text{m/s})^2}{2} \left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}^2} \right) = 4.466\ \text{kN}$$

Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Thrust} \times \text{Velocity} = F_D \mathbf{V} = (4.466\ \text{kN})(300/3.6\ \text{m/s}) \left(\frac{1\ \text{kW}}{1\ \text{kN} \cdot \text{m/s}} \right) = \mathbf{372\ kW}$$

Therefore, the engines must supply 372 kW of propulsive power to overcome the drag during cruising.

Discussion The power determined above is the power to overcome the drag that acts on the wings only, and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc). Therefore, the total power required during cruising will be greater. The required rate of energy input can be determined by dividing the propulsive power by the propulsive efficiency.

13-85 The total mass, wing area, cruising speed, and propulsive power of a small aircraft are given. The lift and drag coefficients of this airplane while cruising are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered. **3** The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties The density of standard air at an altitude of 4000 m is $\rho = 0.819$ kg/m^3 (Table A-26).

Analysis Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity. Also, when the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. Then,

$$\dot{W}_{\text{prop}} = \text{Thrust} \times \text{Velocity} = F_D \mathbf{V} \quad \rightarrow \quad F_D = \frac{\dot{W}_{\text{prop}}}{\mathbf{V}} = \frac{190\,\text{kW}}{280/3.6\,\text{m/s}}\left(\frac{1000\,\text{N}\cdot\text{m/s}}{1\,\text{kW}}\right) = 2443\,\text{N}$$

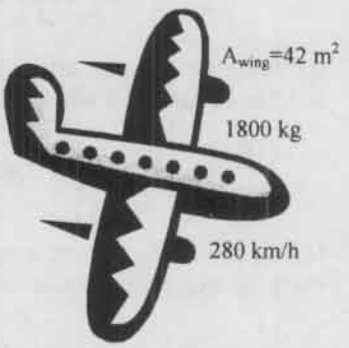
Then the drag coefficient becomes

$$F_D = C_D A\frac{\rho \mathbf{V}^2}{2} \quad \rightarrow \quad C_D = \frac{2F_D}{\rho A \mathbf{V}^2} = \frac{2(2443\,\text{N})}{(0.819\,\text{kg/m}^3)(42\,\text{m}^2)(280/3.6\,\text{m/s})^2}\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right) = \mathbf{0.0235}$$

An aircraft cruises at constant altitude when lift equals the total weight. Therefore,

$$W = F_L = \tfrac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad C_L = \frac{2W}{\rho \mathbf{V}^2 A} = \frac{2(1800\,\text{kg})(9.81\,\text{m/s}^2)}{(0.819\,\text{kg/m}^3)(42\,\text{m}^2)(280/3.6\,\text{m/s})^2} = \mathbf{0.17}$$

Therefore, the drag and lift coefficients of this aircraft during cruising are 0.0235 and 0.17, respectively, with a C_L/C_D ratio of 7.2.



Discussion The drag and lift coefficient determined are for cruising conditions. The values of these coefficient can be very different during takeoff because of the angle of attack and the wing geometry.

13-86 An airfoil has a given lift-to drag ratio at 0° angle of attack. The angle of attack that will raise this ratio to 80 is to be determined.

Analysis The ratio C_L/C_D for the given airfoil is plotted against the angle of attack in Fig. 13-38. The angle of attack corresponding to $C_L/C_D = 80$ is $\theta = \mathbf{3°}$.

Discussion Note that different airfoils have different C_L/C_D vs θ charts.

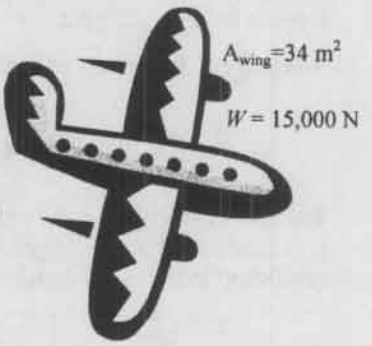
13-87 The wings of a light plane resemble the NACA 23012 airfoil with no flaps. Using data for that airfoil, the takeoff speed at a specified angle of attack and the stall speed are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered.

Properties The density of standard air at sea level is $\rho = 1.225$ kg/m^3 (Table A-26). At an angle of attack of 5°, the lift and drag coefficients are read from Fig. 13-40 to be $C_L = 0.6$ and $C_D = 0.015$. The maximum lift coefficient is $C_{L,\max} = 1.52$ and it occurs at an angle of attack of 15°.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}}$$



Substituting, the takeoff speed is determined to be

$$\mathbf{V}_{\text{takeoff}} = \sqrt{\frac{2(15{,}000\ \text{N})}{(1.225\ \text{kg/m}^3)(0.6)(34\ \text{m}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = 34.6\ \text{m/s} = \mathbf{125\ km/h}$$

since 1 m/s = 3.6 km/h. The stall velocity (the minimum takeoff velocity corresponding the stall conditions) is determined by using the maximum lift coefficient in the above equation,

$$\mathbf{V}_{\min} = \sqrt{\frac{2W}{\rho C_{L,\max} A}} = \sqrt{\frac{2(15{,}000\ \text{N})}{(1.225\ \text{kg/m}^3)(1.52)(34\ \text{m}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = 21.8\ \text{m/s} = \mathbf{78.4\ km/h}$$

Discussion The "safe" minimum velocity to avoid the stall region is obtained by multiplying the stall velocity by 1.2:

$$\mathbf{V}_{\min,\text{safe}} = 1.2\mathbf{V}_{\min} = 1.2 \times (21.8\ \text{m/s}) = 26.2\ \text{m/s} = 94.2\ \text{km/h}$$

Note that the takeoff velocity decreased from 125 km/h at an angle of attack of 5° to 94.2 km/s under stall conditions with a safety margin.

13-88 The mass, wing area, the maximum (stall) lift coefficient, the cruising speed and the cruising drag coefficient of an airplane are given. The safe takeoff speed at sea level and the thrust that the engines must deliver during cruising are to be determined.

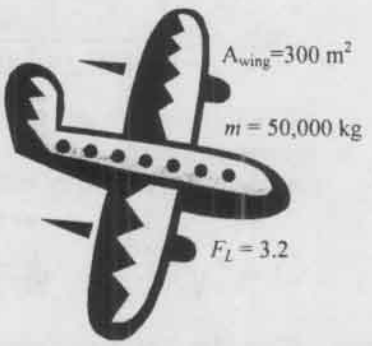
Assumptions **1** Standard atmospheric conditions exist **2** The drag and lift produced by parts of the plane other than the wings are not considered. **3** The takeoff speed is 20% over the stall speed. **4** The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties The density of standard air is $\rho_1 = 1.225$ kg/m^3 at sea level, and $\rho_2 = 0.312$ kg/m^3 at 12,000 m altitude (Table A-26). The cruising drag coefficient is given to be $C_D = 0.03$. The maximum lift coefficient is given to be $C_{L,\max} = 3.2$.

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho \mathbf{V}^2 A \quad \rightarrow \quad \mathbf{V} = \sqrt{\frac{2W}{\rho C_L A}} = \sqrt{\frac{2mg}{\rho C_L A}}$$

The stall velocity (the minimum takeoff velocity corresponding the stall conditions) is determined by using the maximum lift coefficient in the above equation,



$$\mathbf{V}_{\min} = \sqrt{\frac{2mg}{\rho_1 C_{L,\max} A}} = \sqrt{\frac{2(50{,}000\ \text{kg})(9.81\ \text{m/s}^2)}{(1.225\ \text{kg/m}^3)(3.2)(300\ \text{m}^2)}} = 28.9\ \text{m/s} = 104\ \text{km/h}$$

since 1 m/s = 3.6 km/h. Then the "safe" minimum velocity to avoid the stall region becomes

$$\mathbf{V}_{\min,\text{safe}} = 1.2\mathbf{V}_{\min} = 1.2 \times (28.9\ \text{m/s}) = 34.7\ \text{m/s} = \mathbf{125\ km/h}$$

(*b*) When the aircraft cruises steadily at a constant altitude, the net force acting on the aircraft is zero, and thus the thrust provided by the engines must be equal to the drag force, which is

$$F_D = C_D A \frac{\rho_2 \mathbf{V}^2}{2} = (0.03)(300\ \text{m}^2) \frac{(0.312\ \text{kg/m}^3)(700/3.6\ \text{m/s})^2}{2} \left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}^2} \right) = 53.08\ \text{kN}$$

Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Ttrust} \times \text{Velocity} = F_D \mathbf{V} = (53.08\ \text{kN})(700/3.6\ \text{m/s}) \left(\frac{1\ \text{kW}}{1\ \text{kN} \cdot \text{m/s}} \right) = \mathbf{10{,}300\ kW}$$

Therefore, the engines must supply 10,300 kW of propulsive power to overcome drag during cruising.

Discussion The power determined above is the power to overcome the drag that acts on the wings only, and does not include the drag that act on the remaining parts of the aircraft (the fuselage, the tail, etc). Therefore, the total power required during cruising will be greater. The required rate of energy input can be determined by dividing the propulsive power by the propulsive efficiency.

13-89E A spinning ball is dropped into a water stream. The lift and drag forces acting on the ball are to be determined.

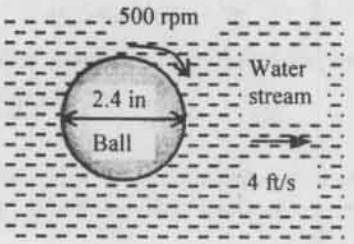
Assumptions **1** The outer surface of the ball is smooth enough for Fig. 13-46 to be applicable. **2** The ball is completely immersed in water.

Properties The density and dynamic viscosity of water at 60°F are ρ =62.36 lbm/ft^3 and μ = 2.713 lbm/ft·h =7.54×10^{-4} lbm/ft·s (Table A-18E).

Analysis The drag and lift forces can be determined from

$$F_D = C_D A \frac{\rho V^2}{2} \quad \text{and} \quad F_L = C_L A \frac{\rho V^2}{2}$$

where A is the frontal area of the ball, which is $A = \pi D^2 / 4$, and D = 2.4/12 = 0.2 ft. The Reynolds number and the angular velocity of the ball are



$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(62.36 \text{ lbm/ft}^3)(4 \text{ ft/s})(0.2 \text{ ft})}{7.54\times10^{-4} \text{ ft}^2/\text{s}} = 6.6\times10^4$$

$$\omega = (500 \text{ rev/min})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 52.4 \text{ rad/s}$$

and

$$\frac{\omega D}{2V} = \frac{(52.4 \text{ rad/s})(0.2 \text{ ft})}{2(4 \text{ ft/s})} = 1.31 \text{ rad}$$

From Fig. 13-46, the drag and lift coefficients corresponding to this value are C_D = 0.56 and C_L = 0.35. Then the drag and the lift acting on the ball are

$$F_D = (0.56)\frac{\pi(0.2 \text{ ft})^2}{4}\frac{(62.36 \text{ lbm/ft}^3)(4 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{0.27 \text{ lbf}}$$

$$F_L = (0.35)\frac{\pi(0.2 \text{ ft})^2}{4}\frac{(62.36 \text{ lbm/ft}^3)(4 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{0.17 \text{ lbf}}$$

Discussion The Reynolds number for this problem is 6.6×10^4 which is close enough to 6×10^4 for which Fig. 13-46 is prepared. Therefore, the result should be close enough to the actual answer.

Review Problems

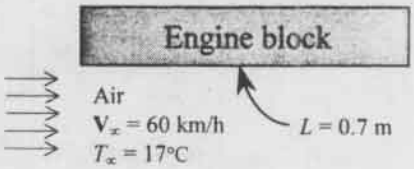
13-90 An automotive engine is approximated as a rectangular block. The drag force acting on the bottom surface of the engine is to be determined. √

Assumptions **1** The air flow is steady and incompressible. **2** Air is an ideal gas. **3** The atmospheric air is calm (no significant winds). **3** The air flow is turbulent over the entire surface because of the constant agitation of the engine block. **4** The bottom surface of the engine is a flat surface, and it is smooth (in reality it is quite rough because of the dirt collected on it).

Properties The density and kinematic viscosity of air at 1 atm and 17°C = 290 K are $\rho = 1.224$ kg/m^3 and $\nu = 1.48\times10^{-5}$ m^2/s (Table A-18).

Analysis The Reynolds number at the end of the engine block is

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(60/3.6\,\mathrm{m/s})(0.7\,\mathrm{m})}{1.48\times10^{-5}\,\mathrm{m^2/s}} = 7.88\times10^5$$



The flow is assumed to be turbulent over the entire surface. Then the average friction coefficient and the drag force acting on the surface becomes

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(7.88\times10^5)^{1/5}} = 0.00490$$

$$F_D = C_f A\frac{\rho \mathbf{V}_\infty^{\,2}}{2} = (0.00490)[0.6\times0.7\,\mathrm{m^2}]\frac{(1.224\,\mathrm{kg/m^3})(60/3.6\,\mathrm{m/s})^2}{2}\left(\frac{1\,\mathrm{N}}{1\,\mathrm{kg.m/s^2}}\right) = \mathbf{0.35\,N}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

13-91 A fluid flows over a 2.5-m long flat plate. The thickness of the boundary layer at intervals of 0.25 m is to be determined and plotted against the distance from the leading edge for air, water, and oil.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

Properties The kinematic viscosity of the three fluids at 1 atm and 20°C are: $\nu = 1.51\times10^{-5}$ m²/s for air (Table A-18), $\nu = \mu/\rho = (1.002\times10^{-3}\ \text{kg/m·s})/(998\ \text{kg/m}^3) = 1.004\times10^{-6}$ m²/s for water (Table A-15), and $\nu = 901\times10^{-6} = 0.000901$ m²/s for oil (Table A-16).

Analysis The thickness of the boundary layer along the flow for laminar and turbulent flows is given by

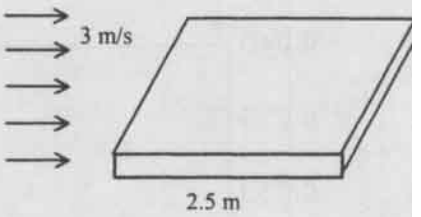
$$\text{Laminar flow:}\quad \delta_{v,x} = \frac{5x}{\text{Re}_x^{1/2}}, \qquad \text{Turbulent flow:}\quad \delta_{v,x} = \frac{0.382x}{\text{Re}_x^{1/5}}$$

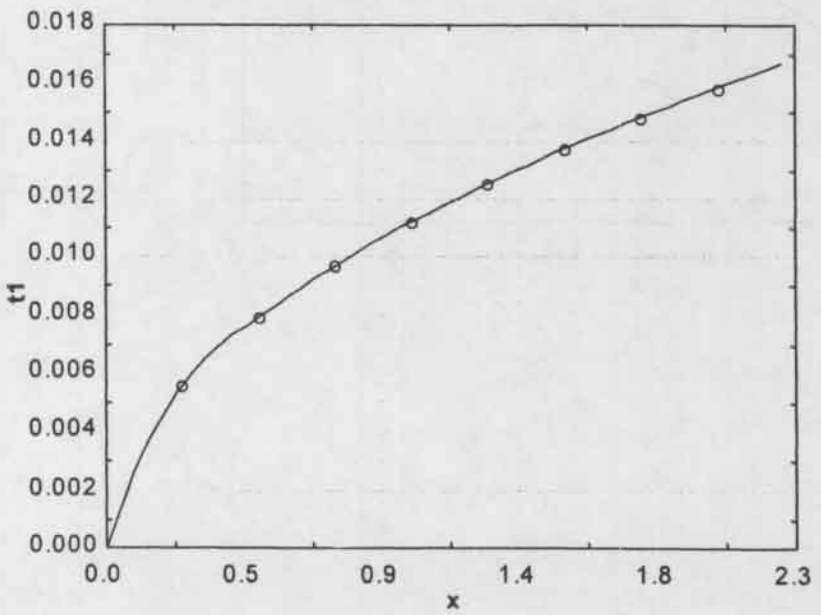
(*a*) AIR: The Reynolds number and the boundary layer thickness at the end of the first 0.25 m interval are

$$\text{Re}_x = \frac{V_\infty x}{\upsilon} = \frac{(3\ \text{m/s})(0.25\ \text{m})}{1.51\times10^{-5}\ \text{m}^2/\text{s}} = 0.497\times10^5, \quad \delta_{v,x} = \frac{5x}{\text{Re}_x^{1/2}} = \frac{5\times(0.25\ \text{m})}{(0.497\times10^5)^{0.5}} = 5.61\times10^{-3}\ \text{m}$$

We repeat calculations for all 0.25 m intervals. The results are:

$x = 0.00$ m	$\text{Re} = 0$	$\delta_{v,x} = 0$
$x = 0.25$ m	$\text{Re} = 0.497\times10^5$	$\delta_{v,x} = 5.61\times10^{-3}$ m
$x = 0.50$ m	$\text{Re} = 0.993\times10^5$	$\delta_{v,x} = 7.93\times10^{-3}$ m
$x = 0.75$ m	$\text{Re} = 1.49\times10^5$	$\delta_{v,x} = 9.72\times10^{-3}$ m
$x = 1.00$ m	$\text{Re} = 1.99\times10^5$	$\delta_{v,x} = 11.2\times10^{-3}$ m
$x = 1.25$ m	$\text{Re} = 2.48\times10^5$	$\delta_{v,x} = 12.5\times10^{-3}$ m
$x = 1.50$ m	$\text{Re} = 2.98\times10^5$	$\delta_{v,x} = 13.7\times10^{-3}$ m
$x = 1.75$ m	$\text{Re} = 3.47\times10^5$	$\delta_{v,x} = 14.8\times10^{-3}$ m
$x = 2.00$ m	$\text{Re} = 3.97\times10^5$	$\delta_{v,x} = 15.9\times10^{-3}$ m
$x = 2.25$ m	$\text{Re} = 4.47\times10^5$	$\delta_{v,x} = 16.8\times10^{-3}$ m
$x = 2.50$ m	$\text{Re} = 4.97\times10^5$	$\delta_{v,x} = 17.7\times10^{-3}$ m



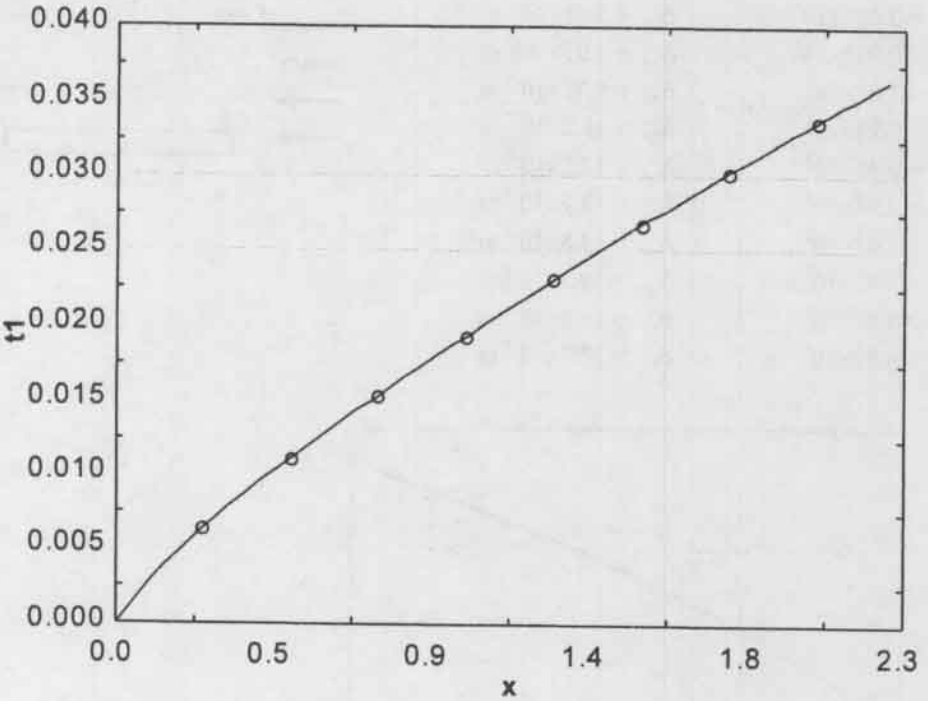


(*b*) WATER: The Reynolds number and the boundary layer thickness at the end of the first 0.25 m interval are

$$\mathrm{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(3\,\mathrm{m/s})(0.25\,\mathrm{m})}{1.004\times10^{-6}\,\mathrm{m^2/s}} = 7.47\times10^5, \quad \delta_{v,x} = \frac{0.382x}{\mathrm{Re}_x^{1/5}} = \frac{0.382\times(0.25\,\mathrm{m})}{(7.47\times10^5)^{1/5}} = 6.39\times10^{-3}\ \mathrm{m}$$

We repeat calculations for all 0.25 m intervals. The results are:

$x = 0.00$ m	$\mathrm{Re} = 0$	$\delta_{v,x} = 0$
$x = 0.25$ m	$\mathrm{Re} = 7.47\times10^5$	$\delta_{v,x} = 6.39\times10^{-3}$ m
$x = 0.50$ m	$\mathrm{Re} = 14.9\times10^5$	$\delta_{v,x} = 11.1\times10^{-3}$ m
$x = 0.75$ m	$\mathrm{Re} = 22.4\times10^5$	$\delta_{v,x} = 15.4\times10^{-3}$ m
$x = 1.00$ m	$\mathrm{Re} = 29.9\times10^5$	$\delta_{v,x} = 19.4\times10^{-3}$ m
$x = 1.25$ m	$\mathrm{Re} = 37.4\times10^5$	$\delta_{v,x} = 23.2\times10^{-3}$ m
$x = 1.50$ m	$\mathrm{Re} = 44.8\times10^5$	$\delta_{v,x} = 26.8\times10^{-3}$ m
$x = 1.75$ m	$\mathrm{Re} = 52.3\times10^5$	$\delta_{v,x} = 30.3\times10^{-3}$ m
$x = 2.00$ m	$\mathrm{Re} = 59.8\times10^5$	$\delta_{v,x} = 33.7\times10^{-3}$ m
$x = 2.25$ m	$\mathrm{Re} = 67.2\times10^5$	$\delta_{v,x} = 37.1\times10^{-3}$ m
$x = 2.50$ m	$\mathrm{Re} = 74.7\times10^5$	$\delta_{v,x} = 40.3\times10^{-3}$ m

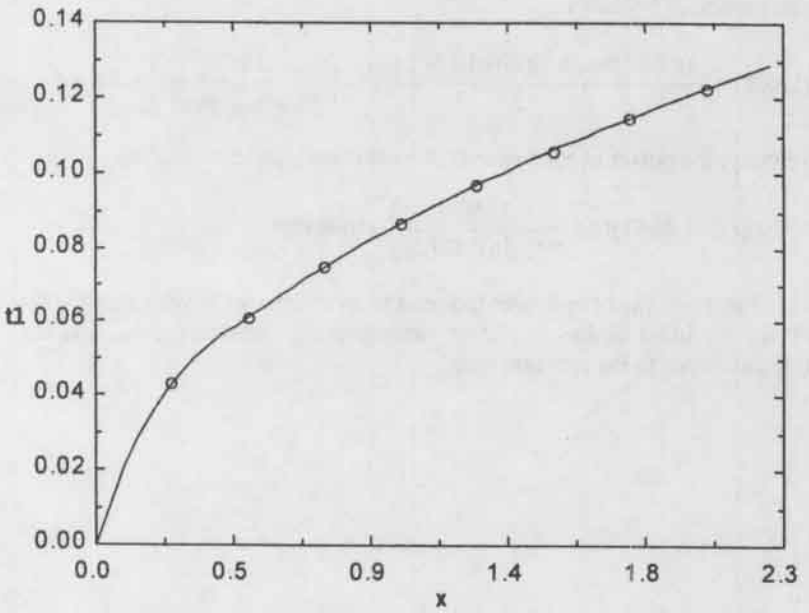


(*c*) OIL: The Reynolds number and the boundary layer thickness at the end of the first 0.25 m interval are

$$\mathrm{Re}_x = \frac{V_\infty x}{\upsilon} = \frac{(3\,\mathrm{m/s})(0.25\,\mathrm{m})}{0.000901\,\mathrm{m^2/s}} = 832, \quad \delta_{v,x} = \frac{5x}{\mathrm{Re}_x^{1/2}} = \frac{5\times(0.25\,\mathrm{m})}{(832)^{0.5}} = 0.0433\,\mathrm{m}$$

We repeat calculations for all 0.25 m intervals. The results are:

$x = 0.00$ m	Re = 0	$\delta_{v,x} = 0$
$x = 0.25$ m	Re = 832	$\delta_{v,x} = 0.043$ m
$x = 0.50$ m	Re = 1665	$\delta_{v,x} = 0.061$ m
$x = 0.75$ m	Re = 2497	$\delta_{v,x} = 0.075$ m
$x = 1.00$ m	Re = 3330	$\delta_{v,x} = 0.087$ m
$x = 1.25$ m	Re = 4162	$\delta_{v,x} = 0.097$ m
$x = 1.50$ m	Re = 4994	$\delta_{v,x} = 0.106$ m
$x = 1.75$ m	Re = 5827	$\delta_{v,x} = 0.115$ m
$x = 2.00$ m	Re = 6659	$\delta_{v,x} = 0.123$ m
$x = 2.25$ m	Re = 7492	$\delta_{v,x} = 0.130$m
$x = 2.50$ m	Re = 8324	$\delta_{v,x} = 0.137$ m



Discussion Note that the flow is laminar for (a) and (c), and turbulent for (b). Also note that the thickness of the boundary layer is very small for air and water, but it is very large for oil. This is due to the high viscosity of oil.

13-92E The passenger compartment of a minivan is modeled as a rectangular box. The drag force acting on the top and the two side surfaces and the power needed to overcome it are to be determined. √

Assumptions **1** The air flow is steady and incompressible. **2** The air flow over the exterior surfaces is turbulent because of constant agitation. **3** Air is an ideal gas. **4** The top and side surfaces of the minivan are flat and smooth (in reality they can be rough). **5** The atmospheric air is calm (no significant winds).

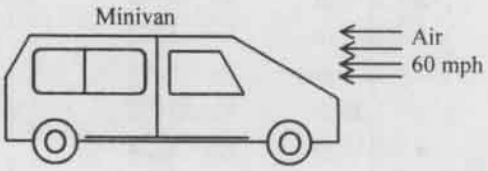
Properties The density and kinematic viscosity of air at 1 atm and 80°F are $\rho = 0.074$ lbm/ft^3 and $\nu = 0.17\times10^{-3}$ ft^2/s (Table A-18E).

Analysis The Reynolds number at the end of the top and side surfaces is

$$\mathrm{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{[60\times1.4667\ \text{ft/s}](11\ \text{ft})}{0.17\times10^{-3}\ \text{ft}^2/\text{s}} = 5.694\times10^6$$

The air flow over the entire outer surface is assumed to be turbulent. Then the friction coefficient becomes

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(5.694\times10^6)^{1/5}} = 0.00330$$



The area of the top and side surfaces of the minivan is

$$A = A_{\text{top}} + 2A_{\text{side}} = 6\times11+2\times3.2\times11 = 136.4\ \text{ft}^2$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a plane surface, the drag force acting on these surfaces becomes

$$F_D = C_f A\frac{\rho \mathbf{V}^2}{2} = 0.00330\times(136.4\ \text{ft}^2)\frac{(0.074\ \text{lbm/ft}^3)(60\times1.4667\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = \mathbf{4.0\ lbf}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

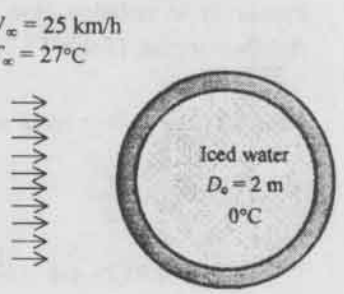
$$\dot{W}_{\text{drag}} = F_D\mathbf{V} = (4.0\ \text{lbf})(60\times1.4667\ \text{ft/s})\left(\frac{1\ \text{kW}}{737.56\ \text{lbf}\cdot\text{ft/s}}\right) = \mathbf{0.48\ kW}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

13-93 A large spherical tank located outdoors is subjected to winds. The drag force exerted on the tank by the winds is to be determined.

Assumptions **1** The outer surfaces of the tank are smooth so that Fig. 13-32 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible, and flow around the tank is uniform. **3** Turbulence in the wind is not considered. **4** The effect of any support bars on flow and drag is negligible.

Properties The density and kinematic viscosity of air at 1 atm and 27°C = 300 K are $\rho = 1.177$ kg/m^3 and $\nu = 1.57\times10^{-5}$ m^2/s (Table A-18).



Analysis Noting that $D = 2$ m and 1 m/s = 3.6 km/h, the Reynolds number for the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(25/3.6\ \text{m/s})(2\ \text{m})}{1.57\times10^{-5}\ \text{m}^2/\text{s}} = 8.85\times10^5$$

The drag coefficient for a sphere corresponding to this value is, from Fig. 13-32, $C_D = 0.13$. Also, the frontal area for flow past a sphere is $A = \pi D^2/4$. Then the drag force becomes

$$F_D = C_D A\frac{\rho V^2}{2} = 0.13[\pi(2\ \text{m}^2)/4]\frac{(1.177\ \text{kg/m}^3)(25/3.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{12\ N}$$

Discussion Note that the drag coefficient is very low in this case since the flow is turbulent ($\text{Re} > 2\times10^5$). Also, it should be kept in mind that wind turbulence may affect the drag coefficient.

13-94 A rectangular advertisement panel attached to a rectangular concrete block by two poles is to withstand high winds. For a given maximum wind speed, the maximum drag force on the panel and the poles, and the minimum length L of the concrete block for the panel to resist the winds are to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The wind is normal to the panel (to check for the worst case). **3** The flow is turbulent so that the tabulated value of the drag coefficients can be used.

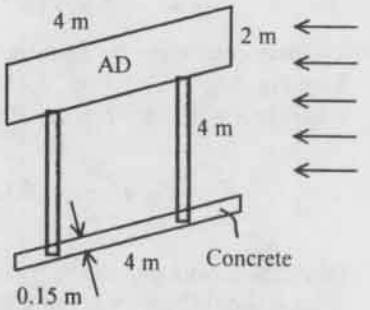
Properties In turbulent flow, the drag coefficient is $C_D = 0.3$ for a circular rod, and $C_D = 2.0$ for a thin rectangular plate (Table 13-2). The densities of air and concrete block are given to be $\rho = 1.30$ kg/m^3 and $\rho_c = 2300$ kg/m^3.

Analysis (*a*) The drag force acting on the panel is

$$F_{D,\text{panel}} = C_D A \frac{\rho V^2}{2}$$

$$= (2.0)(2\times 4\,\text{m}^2)\frac{(1.30\,\text{kg/m}^3)(150/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{18{,}000\ N}$$



(*b*) The drag force acting on each pole is

$$F_{D,\text{pole}} = C_D A \frac{\rho V^2}{2}$$

$$= (0.3)(0.05\times 4\,\text{m}^2)\frac{(1.30\,\text{kg/m}^3)(150/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{68\ N}$$

Therefore, the drag force acting on both poles is $68 \times 2 = \mathbf{136\ N}$. Note that the drag force acting on poles is negligible compared to the drag force acting on the panel.

(*c*) The weight of the concrete block is

$$W = mg = \rho g V = (2300\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(\text{L}\times 4\,\text{m}\times 0.15\,\text{m})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 13{,}540L\ \text{N}$$

Note that the resultant drag force on the panel passes through its center, the drag force on the pole passes through the center of the pole, and the weight of the panel passes through the center of the block. When the concrete block is first tipped, the wind-loaded side of the block will be lifted off the ground, and thus the entire reaction force from the ground will act on the other side. Taking the moment about this side and setting it equal to zero gives

$$\sum M = 0 \ \rightarrow\ F_{D,\text{panel}}\times(1+4+0.15) + F_{D,\text{pole}}\times(2+0.15) - W\times(L/2) = 0$$

Substituting and solving for L gives

$$18{,}000\times 5.15 + 136\times 2.15 - 13{,}540L\times L/2 = 0 \ \rightarrow\ \text{L} = 3.70\,\text{m}$$

Therefore, the minimum length of the concrete block must be $L = \mathbf{3.70}$.

Discussion This length appears to be large and impractical. It can be reduced to a more reasonable value by (*a*) increasing the height of the concrete block, (*b*) reducing the length of the poles (and thus the tipping moment), or (*c*) by attaching the concrete block to the ground (through long nails, for example).

13-95 The bottom surface of a plastic boat is approximated as a flat surface. The friction drag exerted on the bottom surface of the boat by water and the power needed to overcome it are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water is calm (no significant currents or waves). **3** The water flow is turbulent over the entire surface because of the constant agitation of the boat. **4** The bottom surface of the boat is a flat surface, and it is smooth.

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1$ kg/m^3 and $\nu = 1.138\times10^{-3}$ kg/m·s (Table A-15).

Analysis The Reynolds number at the end of the bottom surface of the boat is

$$\mathrm{Re}_L = \frac{\rho V_\infty L}{\mu} = \frac{(999.1\,\mathrm{kg/m^3})(30/3.6\,\mathrm{m/s})(2\,\mathrm{m})}{1.138\times10^{-3}\,\mathrm{kg/m\cdot s}} = 1.463\times10^7$$

The flow is assumed to be turbulent over the entire surface. Then the average friction coefficient and the drag force acting on the surface becomes



$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} = \frac{0.074}{(1.463\times10^7)^{1/5}} = 0.00273$$

$$F_D = C_f A\frac{\rho V_\infty^{\,2}}{2} = (0.00273)[1.5\times2\,\mathrm{m}^2]\frac{(999.1\,\mathrm{kg/m^3})(30/3.6\,\mathrm{m/s})^2}{2}\left(\frac{1\,\mathrm{N}}{1\,\mathrm{kg.m/s^2}}\right) = \mathbf{34.1\,N}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

$$\dot{W}_{\mathrm{drag}} = F_D V = (34.1\,\mathrm{N})(30/3.6\,\mathrm{m/s})\left(\frac{1\,\mathrm{W}}{1\,\mathrm{N\cdot m/s}}\right) = \mathbf{284\,W}$$

Discussion Note that the calculated drag force (and the power required to overcome it) is relatively small. This is not surprising since the drag force for blunt bodies (including those partially immersed in a liquid) is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

13-96E Cruising conditions of a passenger plane are given. The minimum safe landing and takeoff speeds with and without flaps, the angle of attack during cruising, and the power required are to be determined. √

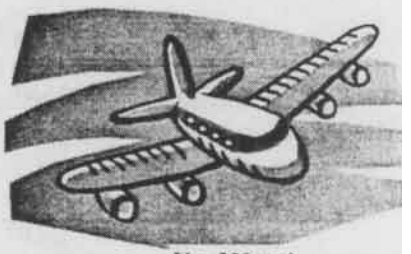
Assumptions **1** The drag and lift produced by parts of the plane other than the wings are not considered. **2** The wings are assumed to be two-dimensional airfoil sections, and the tip effects are neglected. **4** The lift and drag characteristics of the wings can be approximated by NACA 23012 so that Fig. 13-40 is applicable.

Properties The densities of air are 0.075 lbm/ft^3 on the ground and 0.0208 lbm/ft^3 at cruising altitude. The maximum lift coefficients of the wings are 3.48 and 1.52 with and without flaps, respectively (Fig. 13-40).

Analysis (*a*) The weight and cruising speed of the airplane are

$$W = mg = (150{,}000\ \text{lbm})(32.2\ \text{ft/s}^2)\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = 150{,}000\ \text{lbf}$$

$$\mathbf{V} = (550\ \text{mph})\left(\frac{1.4667\ \text{ft/s}}{1\ \text{mph}}\right) = 806.7\ \text{ft/s}$$



$\mathbf{V} = 550$ mph
$m = 150{,}000$ lbm
$A_{\text{wing}} = 1800$ m^2

The minimum velocity corresponding the stall conditions with and without flaps are

$$\mathbf{V}_{\min 1} = \sqrt{\frac{2W}{\rho C_{L,\max 1} A}} = \sqrt{\frac{2(150{,}000\ \text{lbf})}{(0.075\ \text{lbm/ft}^3)(1.52)(1800\ \text{ft}^2)}\left(\frac{32.2\ \text{lbm}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right)} = 217\ \text{ft/s}$$

$$\mathbf{V}_{\min 2} = \sqrt{\frac{2W}{\rho C_{L,\max 2} A}} == \sqrt{\frac{2(150{,}000\ \text{lbf})}{(0.075\ \text{lbm/ft}^3)(3.48)(1800\ \text{ft}^2)}\left(\frac{32.2\ \text{lbm}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right)} = 143\ \text{ft/s}$$

The "safe" minimum velocities to avoid the stall region are obtained by multiplying these values by 1.2:

Without flaps: $\mathbf{V}_{\min 1,\text{safe}} = 1.2\mathbf{V}_{\min 1} = 1.2\times(217\ \text{ft/s}) = 260\ \text{ft/s} = \mathbf{178\ mph}$

With flaps: $\mathbf{V}_{\min 2,\text{safe}} = 1.2\mathbf{V}_{\min 2} = 1.2\times(143\ \text{ft/s}) = 172\ \text{ft/s} = \mathbf{117\ mph}$

since 1 mph = 1.4667 ft/s. Note that the use of flaps allows the plane to takeoff and land at considerably lower velocities, and thus at a shorter runway.

(*b*) When an aircraft is cruising steadily at a constant altitude, the lift must be equal to the weight of the aircraft, $F_L = W$. Then the lift coefficient is determined to be

$$C_L = \frac{F_L}{\frac{1}{2}\rho\mathbf{V}^2 A} = \frac{150{,}000\ \text{lbf}}{\frac{1}{2}(0.0208\ \text{lbm/ft}^3)(806.7\ \text{ft/s})^2(1800\ \text{ft}^2)}\left(\frac{32.2\ \text{lbm}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right) = 0.40$$

For the case of no flaps, the angle of attack corresponding to this value of C_L is determined from Fig. 13-40 to be about $\alpha = \mathbf{3.5°}$.

(*c*) When the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. The drag coefficient corresponding to the cruising lift coefficient of 0.40 is determined from Fig. 13-40 to be $C_D = 0.015$. Then the drag force acting on the wings becomes

$$F_D = C_D A\frac{\rho\mathbf{V}^2}{2} = (0.015)(1800\ \text{ft}^2)\frac{(0.0208\ \text{lbm/ft}^3)(806.7\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = 5675\ \text{lbf}$$

Noting that power is force times velocity (distance per unit time), the power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Thrust}\times\text{Velocity} = F_D\mathbf{V} = (5675\ \text{lbf})(806.7\ \text{ft/s})\left(\frac{1\ \text{kW}}{737.56\ \text{lbf}\cdot\text{ft/s}}\right) = \mathbf{6200\ kW}$$

Discussion Note that the engines must supply 6200 kW of power to overcome the drag during cruising. This is the power required to overcome the drag that acts on the wings only, and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc).

13-97 A smooth ball is moving at a specified velocity. The increase in the drag coefficient when the ball spins is to be determined.

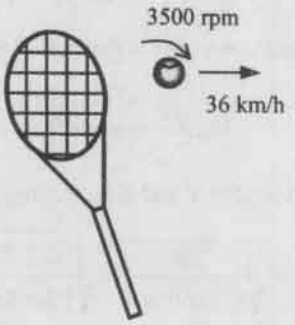
Assumptions **1** The outer surface of the ball is smooth so that Figs. 13-30 and 13-46 can be used to determine the drag coefficient. **2** The air is calm (no winds or drafts).

Properties The density and kinematic viscosity of air at 1 atm and 25°C = 298 K are $\rho = 1.186$ kg/m^3 and $\nu = 1.55\times10^{-5}$ m^2/s (Table A-18).

Analysis Noting that $D = 0.055$ m and 1 m/s = 3.6 km/h, the regular and angular velocities of the ball and the Reynolds number are

$$\mathbf{V} = (36\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 10\text{ m/s}$$

$$\omega = (3500\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 367\text{ rad/s}$$



$$\text{Re} = \frac{VD}{\nu} = \frac{(10\text{ m/s})(0.095\text{ m})}{1.55\times10^{-5}\text{ m}^2/\text{s}} = 6.13\times10^4$$

and

$$\frac{\omega D}{2V} = \frac{(367\text{ rad/s})(0.095\text{ m})}{2(10\text{ m/s})} = 1.74\text{ rad}$$

Then the drag coefficients for the ball with and without spin are determined from Figs. 13-30 and 13-46 to be:

Without spin:	$C_D = 0.48$	(Fig. 13-30)
With spin:	$C_D = 0.60$	(Fig. 13-46)

Then the increase in the drag coefficient due to spinning becomes

$$\text{Increase in } C_D = \frac{C_{D,\text{spin}} - C_{D,\text{no spin}}}{C_{D,\text{no spin}}} = \frac{0.60 - 0.48}{0.48} = 0.20 \text{ (or } \mathbf{20\%})$$

Therefore, the drag coefficient increases 20% in this case because of spinning.

Discussion Note that the Reynolds number for this problem is 6.13×10^4 which is close enough to 6×10^4 for which Fig. 13-46 is prepared. Therefore, the result obtained should be fairly accurate.

13-98 The total weight of a paratrooper and its parachute is given. The terminal velocity of the paratrooper in air is to be determined.

Assumptions **1** The air flow over the parachute is turbulent so that the tabulated value of the drag coefficient can be used. **2** The variation of the air properties with altitude is negligible. **3** The buoyancy force applied by air to the person (and the parachute) is negligible because of the small volume occupied and the low air density.

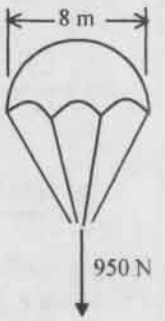
Properties The density of air is given to be 1.20 kg/m^3. The drag coefficient of a parachute is C_D = 1.3 (Table 13-2).

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_D = W - F_B \quad \text{where} \quad F_D = C_D A \frac{\rho_f \mathbf{V}^2}{2}, \quad W = mg = 950\ \text{N}, \quad \text{and} \quad F_B \cong 0$$

where $A = \pi D^2/4$ is the frontal area. Substituting and simplifying,

$$C_D A \frac{\rho_f \mathbf{V}^2}{2} = W \quad \rightarrow \quad C_D \frac{\pi D^2}{4} \frac{\rho_f \mathbf{V}^2}{2} = W$$



Solving for **V** and substituting,

$$\mathbf{V} = \sqrt{\frac{8W}{C_D \pi D^2 \rho_f}} = \sqrt{\frac{8(950\ \text{N})}{1.3\pi(8\ \text{m})^2(1.20\ \text{kg/m}^3)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = \mathbf{4.9\ m/s}$$

Therefore, the velocity of the paratrooper will remain constant when it reaches the terminal velocity of 4.9 m/s = 18 km/h.

Discussion The simple analysis above gives us a rough value for the terminal velocity. A more accurate answer can be obtained by a more detailed (and complex) analysis by considering the variation of air density with altitude, and by considering the uncertainty in the drag coefficient.

13-99 A fairing is installed to the front of a rig to reduce the drag coefficient. The maximum speed of the rig after the fairing is installed is to be determined.

Assumptions **1** The rig moves steadily at a constant velocity on a straight path in calm weather. **2** The bearing friction resistance is constant. **3** The effect of velocity on the drag and rolling resistance coefficients is negligible. **4** The buoyancy of air is negligible. **5** The power produced by the engine is used to overcome rolling resistance, bearing friction, and aerodynamic drag.

Properties The density of air is given to be 1.25 kg/m^3. The drag coefficient of the rig is given to be C_D = 0.96, and it decreases to C_D = 0.76 when a fairing is installed. The rolling resistance coefficient is C_{RR} = 0.05.

Analysis The bearing friction resistance is given to be F_{bearing} = 350 N. The rolling resistance is

$$F_{RR} = C_{RR}W = 0.05(17{,}000\,\text{kg})(9.81\,\text{m/s}^2)\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 8339\,\text{N}$$

The maximum drag occurs at maximum velocity, and its value before the fairing is installed is

$$F_{D1} = C_D A\frac{\rho \mathbf{V}_1^2}{2} = (0.96)(9.2\,\text{m}^2)\frac{(1.25\,\text{kg/m}^3)(110/3.6\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 5154\,\text{N}$$

Power is force times velocity, and thus the power needed to overcome bearing friction, drag, and rolling resistance is the product of the sum of these forces and the velocity of the rig,



$$\begin{aligned}\dot{W}_{\text{total}} &= \dot{W}_{\text{bearing}} + \dot{W}_{\text{drag}} + \dot{W}_{\text{RR}} = (F_{\text{bearing}} + F_D + F_{RR})\mathbf{V} \\ &= (350 + 8339 + 5154)(110/3.6\,\text{m/s})\left(\frac{1\,\text{kW}}{1000\,\text{N}\cdot\text{m/s}}\right) \\ &= 423\,\text{kW}\end{aligned}$$

The maximum velocity the rig can attain at the same power of 423 kW after the fairing is installed is determined by setting the sum of the bearing friction, rolling resistance, and the drag force equal to 423 kW,

$$\dot{W}_{\text{total}} = \dot{W}_{\text{bearing}} + \dot{W}_{\text{drag2}} + \dot{W}_{\text{RR}} = (F_{\text{bearing}} + F_{D2} + F_{RR})\mathbf{V} = \left(350 + C_{D2}A\frac{\rho \mathbf{V}_2^2}{2} + 5154\right)\mathbf{V}_2$$

Substituting the known quantities,

$$(423\,\text{kW})\left(\frac{1000\,\text{N}\cdot\text{m/s}}{1\,\text{kW}}\right) = \left(350\,\text{N} + (0.76)(9.2\,\text{m}^2)\frac{(1.25\,\text{kg/m}^3)\mathbf{V}_2^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) + 5154\,\text{N}\right)\mathbf{V}_2$$

or,

$$423{,}000 = 5504\mathbf{V}_2 + 4.37\mathbf{V}_2^3$$

Solving it with an equation solver gives $\mathbf{V}_2$ **= 36.9 m/s = 133 km/h**.

Discussion Note that the maximum velocity of the rig increases from 110 km/h to 133 km/h as a result of reducing its drag coefficient from 0.96 to 0.76 while holding the bearing friction and the rolling resistance constant.

13-100 A spherical object is dropped into a fluid, and its terminal velocity is measured. The viscosity of the fluid is to be determined.

Assumptions **1** The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). **2** The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body. **3** The tube is long enough to assure that the velocity measured is the terminal velocity.

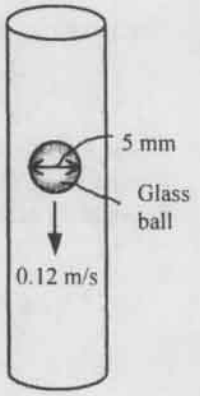
Properties The density of glass ball is given to be $\rho_s = 2500$ kg/m^3. The density of the fluid is given to be $\rho_f = 875$ kg/m^3.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

$$F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu VD \text{ (Stoke's law)}, \quad W = \rho_s g V, \quad \text{and} \quad F_B = \rho_f g V$$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting and simplifying,

$$3\pi\mu VD = \rho_s g V - \rho_f g V \quad \rightarrow \quad 3\pi\mu VD = (\rho_s - \rho_f) g \frac{\pi D^3}{6}$$



Solving for μ and substituting, the dynamic viscosity of the fluid is determined to be

$$\mu = \frac{gD^2(\rho_s - \rho_f)}{18V} = \frac{(9.81\,\text{m/s}^2)(0.005\,\text{m})^2(2500\text{-}875)\,\text{kg/m}^3}{18(0.12\,\text{m/s})} = \mathbf{0.185\,kg/m \cdot s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(875\,\text{kg/m}^3)(0.12\,\text{m/s})(0.005\,\text{m})}{0.185\,\text{kg} \cdot \text{m/s}} = 2.8$$

which is at the order of 1. Therefore, the creeping flow idealization is applicable, and the value calculated is valid.

Discussion Flow separation starts at about Re = 10. Therefore, Stokes law can be used for Reynolds numbers upto this value, but this should be done with care.

13-101 Engine oil flows over a long flat plate. The distance from the leading edge x_{cr} where the flow becomes turbulent is to be determined, and thickness of the boundary layer over a distance of $2x_{cr}$ is to be plotted.

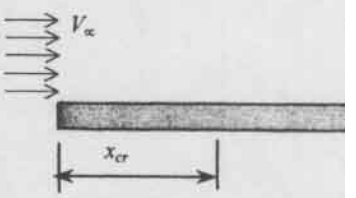
Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\text{Re}_{cr} = 5\times10^5$. **3** The surface of the plate is smooth.

Properties The kinematic viscosity of engine oil at 40°C is $\nu = 242\times10^{-6} = 0.000242\text{m}^2/\text{s}$ (Table A-16).

Analysis The thickness of the boundary layer along the flow for laminar and turbulent flows is given by

$$\text{Laminar flow:}\quad \delta_{v,x} = \frac{5x}{\text{Re}_x^{1/2}}, \qquad \text{Turbulent flow:}\quad \delta_{v,x} = \frac{0.382x}{\text{Re}_x^{1/5}}$$

The distance from the leading edge x_{cr} where the flow turns turbulent is determined by setting Reynolds number equal to the critical Reynolds number,

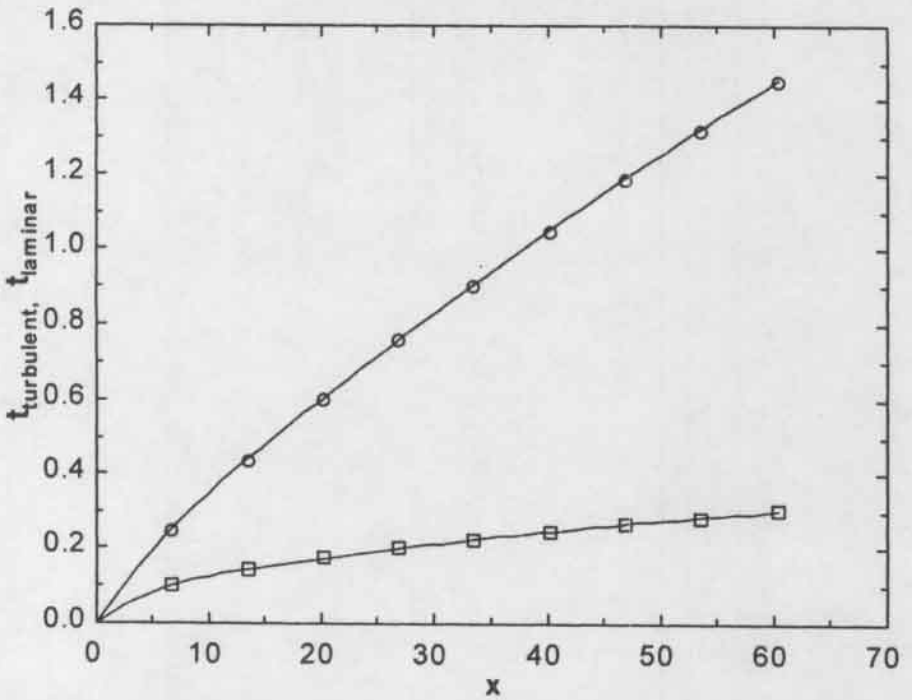


$$\text{Re}_{cr} = \frac{\text{V}_\infty x_{cr}}{\upsilon} \rightarrow \quad x_{cr} = \frac{\text{Re}_{cr}\,\upsilon}{\text{V}_\infty} = \frac{(5\times10^5)(0.000242\,\text{m}^2/\text{s})}{4\,\text{m/s}} = \mathbf{30.25\,m},$$

Therefore, we should consider flow over 2×30.25 = 60.5 m long section of the plate, and use the laminar relation for the first half, and the turbulent relation for the second part to determine the boundary layer thickness. For example, the Reynolds number and the boundary layer thickness at a distance 2 m from the leading edge of the plate are

$$\text{Re}_x = \frac{\text{V}_\infty x}{\upsilon} = \frac{(4\,\text{m/s})(2\,\text{m})}{0.000242\,\text{m}^2/\text{s}} = 33{,}060, \qquad \delta_{v,x} = \frac{5x}{\text{Re}_x^{1/2}} = \frac{5\times(2\,\text{m})}{(33{,}060)^{0.5}} = 0.055\,\text{m}$$

Calculating the boundary layer thickness and plotting give



The lower curve (laminar) upto 30.25 m and the upper curve (turbulent) afterwards constitute the boundary layer.

Chapter 14
MECHANISMS OF HEAT TRANSFER

Heat Transfer Mechanisms

14-1C The thermal conductivity of a material is the rate of heat transfer through a unit thickness of the material per unit area and per unit temperature difference. The thermal conductivity of a material is a measure of how fast heat will be conducted in that material.

14-2C The mechanisms of heat transfer are conduction, convection and radiation. Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas which is in motion, and it involves combined effects of conduction and fluid motion. Radiation is energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules.

14-3C In solids, conduction is due to the combination of the vibrations of the molecules in a lattice and the energy transport by free electrons. In gases and liquids, it is due to the collisions of the molecules during their random motion.

14-4C The parameters that effect the rate of heat conduction through a windowless wall are the geometry and surface area of wall, its thickness, the material of the wall, and the temperature difference across the wall.

14-5C Conduction is expressed by Fourier's law of conduction as $\dot{Q}_{cond} = -kA\dfrac{dT}{dx}$ where dT/dx is the temperature gradient, k is the thermal conductivity, and A is the area which is normal to the direction of heat transfer.

Convection is expressed by Newton's law of cooling as $\dot{Q}_{conv} = hA(T_s - T_\infty)$ where h is the convection heat transfer coefficient, A is the surface area through which convection heat transfer takes place, T_s is the surface temperature and T_∞ is the temperature of the fluid sufficiently far from the surface.

Radiation is expressed by Stefan-Boltzman law as $\dot{Q}_{rad} = \varepsilon\sigma A(T_s^4 - T_{surr}^4)$ where ε is the emissivity of surface, A is the surface area, T_s is the surface temperature, T_{surr} is average surrounding surface temperature and $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2.K^4}$ is the Stefan-Boltzman constant.

14-6C Convection involves fluid motion, conduction does not. In a solid we can have only conduction.

14-7C No. It is purely by radiation.

14-8C In forced convection the fluid is forced to move by external means such as a fan, pump, or the wind. The fluid motion in natural convection is due to buoyancy effects only.

14-9C Emissivity is the ratio of the radiation emitted by a surface to the radiation emitted by a blackbody at the same temperature. Absorptivity is the fraction of radiation incident on a surface that is absorbed by the surface. The Kirchhoff's law of radiation states that the emissivity and the absorptivity of a surface are equal at the same temperature and wavelength.

14-10C A blackbody is an idealized body which emits the maximum amount of radiation at a given temperature and which absorbs all the radiation incident on it. Real bodies emit and absorb less radiation than a blackbody at the same temperature.

14-11C No. Such a definition will imply that doubling the thickness will double the heat transfer rate. The equivalent but "more correct" unit of thermal conductivity is $W.m/m^2.°C$ that indicates product of heat transfer rate and thickness per unit surface area per unit temperature difference.

14-12C In a typical house, heat loss through the wall with glass window will be larger since the glass is much thinner than a wall, and its thermal conductivity is higher than the average conductivity of a wall.

14-13C Diamond is a better heat conductor.

14-14C The rate of heat transfer through both walls can be expressed as

$$\dot{Q}_{\text{wood}} = k_{\text{wood}} A \frac{T_1 - T_2}{L_{\text{wood}}} = (0.17 \text{ W/m.°C}) A \frac{T_1 - T_2}{0.1 \text{ m}} = 1.7 A (T_1 - T_2)$$

$$\dot{Q}_{\text{brick}} = k_{\text{brick}} A \frac{T_1 - T_2}{L_{\text{brick}}} = (0.72 \text{ W/m.°C}) A \frac{T_1 - T_2}{0.25 \text{ m}} = 2.88 A (T_1 - T_2)$$

Therefore, heat transfer through the brick wall will be larger despite its higher thickness.

14-15C The thermal conductivity of gases is proportional to the square root of absolute temperature. The thermal conductivity of most liquids, however, decreases with increasing temperature, with water being a notable exception.

14-16C Superinsulations are obtained by using layers of highly reflective sheets separated by glass fibers in an evacuated space. Radiation heat transfer between two surfaces is inversely proportional to the number of sheets used and thus heat loss by radiation will be very low by using this highly reflective sheets. At the same time, evacuating the space between the layers forms a vacuum under 0.000001 atm pressure which minimize conduction or convection through the air space between the layers.

14-17C Most ordinary insulations are obtained by mixing fibers, powders, or flakes of insulating materials with air. Heat transfer through such insulations is by conduction through the solid material, and conduction or convection through the air space as well as radiation. Such systems are characterized by apparent thermal conductivity instead of the ordinary thermal conductivity in order to incorporate these convection and radiation effects.

14-18C The thermal conductivity of an alloy of two metals will most likely be less than the thermal conductivities of both metals.

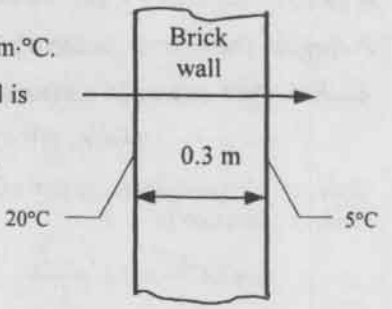
14-19 The inner and outer surfaces of a brick wall are maintained at specified temperatures. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values. **2** Thermal properties of the wall are constant.

Properties The thermal conductivity of the wall is given to be $k = 0.69$ W/m·°C.

Analysis Under steady conditions, the rate of heat transfer through the wall is

$$\dot{Q}_{cond} = kA\frac{\Delta T}{L} = (0.69\ \text{W/m·°C})(5\times 6\ \text{m}^2)\frac{(20-5)°\text{C}}{0.3\ \text{m}} = \mathbf{1035\ W}$$



14-20 The inner and outer surfaces of a window glass are maintained at specified temperatures. The amount of heat transfer through the glass in 5 h is to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the glass remain constant at the specified values. **2** Thermal properties of the glass are constant.

Properties The thermal conductivity of the glass is given to be $k = 0.78$ W/m·°C.

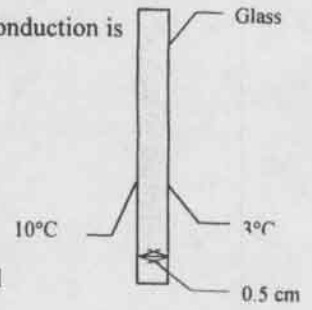
Analysis Under steady conditions, the rate of heat transfer through the glass by conduction is

$$\dot{Q}_{cond} = kA\frac{\Delta T}{L} = (0.78\ \text{W/m·°C})(2\times 2\ \text{m}^2)\frac{(10-3)°\text{C}}{0.005\ \text{m}} = 4368\ \text{W}$$

Then the amount of heat transfer over a period of 5 h becomes

$$Q = \dot{Q}_{cond}\Delta t = (4.368\text{kJ/s})(5\times 3600\text{s}) = \mathbf{78{,}620kJ}$$

If the thickness of the glass doubled to 1 cm, then the amount of heat transfer will go down by half to **39,310 kJ**.



14-21 Heat is transferred steadily to boiling water in the pan through its bottom. The inner surface of the bottom of the pan is given. The temperature of the outer surface is to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the pan remain constant at the specified values. **2** Thermal properties of the aluminum pan are constant.

Properties The thermal conductivity of the aluminum is given to be $k = 237$ W/m·°C.

Analysis The heat transfer surface area is

$$A = \pi r^2 = 3.14\text{x}(0.1\text{ m})^2 = 0.0314\text{ m}^2$$

Under steady conditions, the rate of heat transfer through the bottom of the pan by conduction is

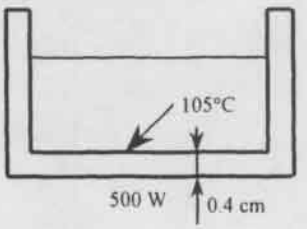
$$\dot{Q} = kA\frac{\Delta T}{L} = kA\frac{T_2 - T_1}{L}$$

Substituting,

$$500\text{ W} = (237\text{ W/m·°C})(0.0314\text{ m}^2)\frac{T_2 - 105°\text{C}}{0.004\text{ m}}$$

which gives

$$T_2 = \mathbf{105.27\ °C}$$



14-22E The inner and outer surface temperatures of the wall of an electrically heated home during a winter night are measured. The rate of heat loss through the wall that night and its cost are to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values during the entire night. **2** Thermal properties of the wall are constant.

Properties The thermal conductivity of the brick wall is given to be $k = 0.42$ Btu/h.ft.°F.

Analysis (a) Noting that the heat transfer through the wall is by conduction and the surface area of the wall is $A = 20 \text{ ft} \times 10 \text{ ft} = 200 \text{ ft}^2$, the steady rate of heat transfer through the wall can be determined from

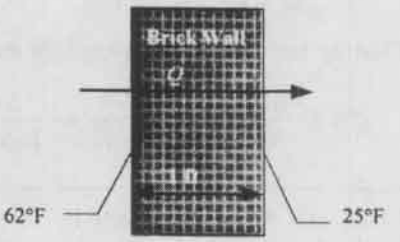
$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.42 \text{ Btu/h.ft.°F})(200 \text{ ft}^2)\frac{(62-25)°\text{F}}{1 \text{ ft}} = \mathbf{3108 \text{ Btu/h}}$$

or 0.911 kW since 1 kW = 3412 Btu/h.

(b) The amount of heat lost during an 8 hour period and its cost are

$$Q = \dot{Q}\Delta t = (0.911 \text{ kW})(8 \text{ h}) = 7.288 \text{ kWh}$$

$$\begin{aligned}\text{Cost} &= (\text{Amount of energy})(\text{Unit cost of energy}) \\ &= (7.288 \text{ kWh})(\$0.07/\text{kWh}) \\ &= \mathbf{\$0.51}\end{aligned}$$



Therefore, the cost of the heat loss through the wall to the home owner that night is $0.51.

14-23 The thermal conductivity of a material is to be determined by ensuring one-dimensional heat conduction, and by measuring temperatures when steady operating conditions are reached.

Assumptions 1 Steady operating conditions exist since the temperature readings do not change with time. **2** Heat losses through the lateral surfaces of the apparatus are negligible since those surfaces are well-insulated, and thus the entire heat generated by the heater is conducted through the samples. **3** The apparatus possesses thermal symmetry.

Analysis The electrical power consumed by the heater and converted to heat is

$$\dot{W}_e = VI = (110 \text{ V})(0.6 \text{ A}) = 66 \text{ W}$$

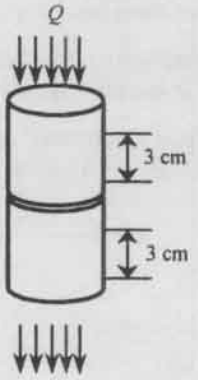
The rate of heat flow through each sample is

$$\dot{Q} = \frac{\dot{W}_e}{2} = \frac{66 \text{ W}}{2} = 33 \text{ W}$$

Then the thermal conductivity of the sample becomes

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.04 \text{ m})^2}{4} = 0.001257 \text{ m}^2$$

$$\dot{Q} = kA\frac{\Delta T}{L} \longrightarrow k = \frac{\dot{Q}L}{A\Delta T} = \frac{(33 \text{ W})(0.03 \text{ m})}{(0.001257 \text{ m}^2)(10°\text{C})} = \mathbf{78.8 \text{ W/m.°C}}$$



14-24 The thermal conductivity of a material is to be determined by ensuring one-dimensional heat conduction, and by measuring temperatures when steady operating conditions are reached.

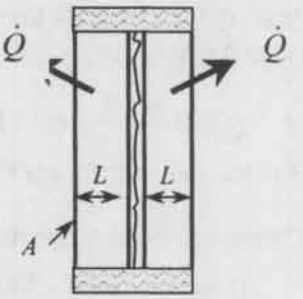
Assumptions 1 Steady operating conditions exist since the temperature readings do not change with time. **2** Heat losses through the lateral surfaces of the apparatus are negligible since those surfaces are well-insulated, and thus the entire heat generated by the heater is conducted through the samples. **3** The apparatus possesses thermal symmetry.

Analysis For each sample we have

$$\dot{Q} = 35/2 = 17.5 \text{ W}$$
$$A = (0.1 \text{ m})(0.1 \text{ m}) = 0.01 \text{ m}^2$$
$$\Delta T = 82 - 74 = 8^\circ\text{C}$$

Then the thermal conductivity of the material becomes

$$\dot{Q} = kA\frac{\Delta T}{L} \longrightarrow k = \frac{\dot{Q}L}{A\Delta T} = \frac{(17.5 \text{ W})(0.005 \text{ m})}{(0.01 \text{ m}^2)(8^\circ\text{C})} = \mathbf{1.09 \text{ W/m.}^\circ\text{C}}$$



14-25 The thermal conductivity of a material is to be determined by ensuring one-dimensional heat conduction, and by measuring temperatures when steady operating conditions are reached.

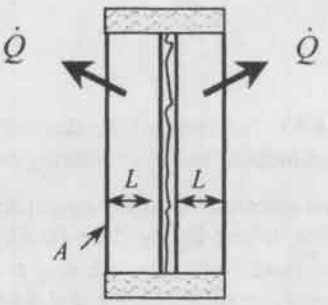
Assumptions 1 Steady operating conditions exist since the temperature readings do not change with time. **2** Heat losses through the lateral surfaces of the apparatus are negligible since those surfaces are well-insulated, and thus the entire heat generated by the heater is conducted through the samples. **3** The apparatus possesses thermal symmetry.

Analysis For each sample we have

$$\dot{Q} = 20/2 = 10 \text{ W}$$
$$A = (0.1 \text{ m})(0.1 \text{ m}) = 0.01 \text{ m}^2$$
$$\Delta T = 82 - 74 = 8^\circ\text{C}$$

Then the thermal conductivity of the material becomes

$$\dot{Q} = kA\frac{\Delta T}{L} \longrightarrow k = \frac{\dot{Q}L}{A\Delta T} = \frac{(10 \text{ W})(0.005 \text{ m})}{(0.01 \text{ m}^2)(8^\circ\text{C})} = \mathbf{0.625 \text{ W/m.}^\circ\text{C}}$$

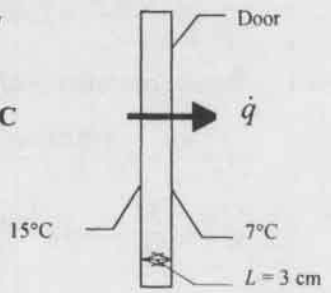


14-26 The thermal conductivity of a refrigerator door is to be determined by measuring the surface temperatures and heat flux when steady operating conditions are reached.

Assumptions 1 Steady operating conditions exist when measurements are taken. **2** Heat transfer through the door is one dimensional since the thickness of the door is small relative to other dimensions.

Analysis The thermal conductivity of the door material is determined directly from Fourier's relation to be

$$\dot{q} = k\frac{\Delta T}{L} \longrightarrow k = \frac{\dot{q}L}{\Delta T} = \frac{(25 \text{ W/m}^2)(0.03 \text{ m})}{(15-7)^\circ\text{C}} = \mathbf{0.09375 \text{ W/m.}^\circ\text{C}}$$



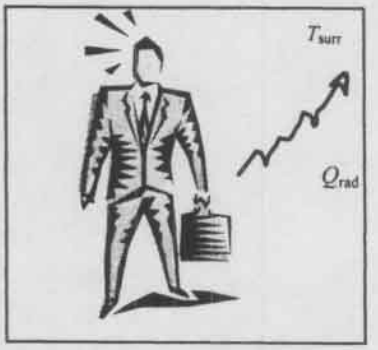
14-27 The rate of radiation heat transfer between a person and the surrounding surfaces at specified temperatur es is to be determined in summer and in winter.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by convection is not considered. **3** The person is completely surrounded by the interior surfaces of the room. **4** The surrounding surfaces are at a uniform temperature.

Properties The emissivity of a person is $\varepsilon = 0.95$ (Table A-24).

Analysis Noting that the person is completely enclosed by the surrounding surfaces, the net rates of radiation heat transfer from the body to the surrounding walls, ceiling, and the floor in both cases are:

(a) Summer: $T_{surr} = 296$ K



$$\dot{Q}_{rad} = \varepsilon\sigma A(T_s^4 - T_{surr}^4)$$
$$= (0.95)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1.6\ \text{m}^2)[(32+273)^4 - (296\ \text{K})^4]\text{K}^4$$
$$= \mathbf{84.2\ W}$$

(b) Winter: $T_{surr} = 285$ K

$$\dot{Q}_{rad} = \varepsilon\sigma A(T_s^4 - T_{surr}^4)$$
$$= (0.95)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1.6\ \text{m}^2)[(32+273)^4 - (285\ \text{K})^4]\text{K}^4$$
$$= \mathbf{177.2\ W}$$

Discussion Note that the radiation heat transfer from the person more than doubles in winter.

14-28 A person is standing in a room at a specified temperature. The rate of heat transfer between a person and the surrounding air by convection is to be determined.

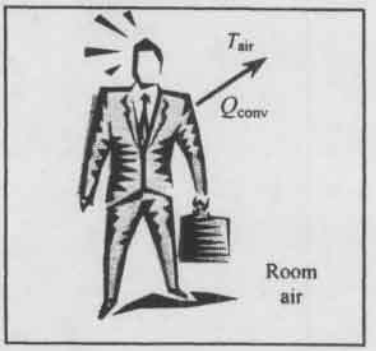
Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is not considered. **3** The environment is at a uniform temperature.

Analysis The heat transfer surface area of the person is

$$A = (\pi D)h = \pi(0.3\ \text{m})(1.70\ \text{m}) = 1.60\ \text{m}^2$$

Under steady conditions, the rate of heat transfer by convection is

$$\dot{Q}_{conv} = hA\Delta T = (15\text{W/m}^2.^\circ\text{C})(1.60\text{m}^2)(34-20)^\circ\text{C} = \mathbf{336W}$$

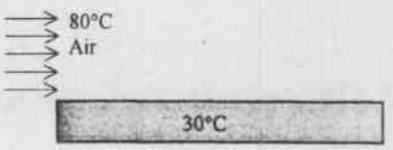


14-29 Hot air is blown over a flat surface at a specified temperature. The rate of heat transfer from the air to the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is not considered. **3** The convection heat transfer coefficient is constant and uniform over the surface.

Analysis Under steady conditions, the rate of heat transfer by convection is

$$\dot{Q}_{conv} = hA\Delta T = (55\ \text{W/m}^2\cdot^\circ\text{C})(2\times 4\ \text{m}^2)(80-30)^\circ\text{C} = \mathbf{22{,}000\ W}$$



14-30 The heat generated in the circuitry on the surface of a 3-W silicon chip is conducted to the ceramic substrate. The temperature difference across the chip in steady operation is to be determined.

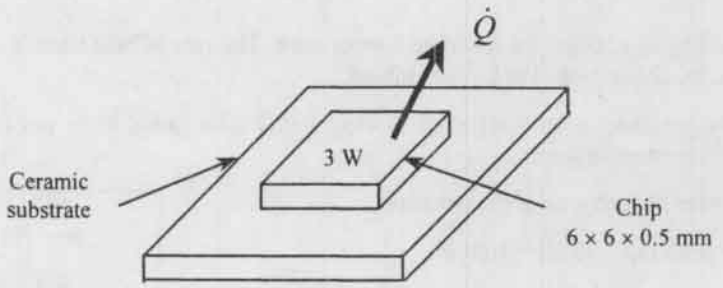
Assumptions **1** Steady operating conditions exist. **2** Thermal properties of the chip are constant.

Properties The thermal conductivity of the silicon chip is given to be $k = 130$ W/m·°C.

Analysis The temperature difference between the front and back surfaces of the chip is

$$A = (0.006\ \text{m})(0.006\ \text{m}) = 0.000036\ \text{m}^2$$

$$\dot{Q} = kA\frac{\Delta T}{L} \longrightarrow \Delta T = \frac{\dot{Q}L}{kA} = \frac{(3\ \text{W})(0.0005\ \text{m})}{(130\ \text{W/m.°C})(0.000036\ \text{m}^2)} = \mathbf{0.32°C}$$



14-31 An electric resistance heating element is immersed in water initially at 20°C. The time it will take for this heater to raise the water temperature to 80°C as well as the convection heat transfer coefficients at the beginning and at the end of the heating process are to be determined.

Assumptions **1** Steady operating conditions exist and thus the rate of heat loss from the wire equals the rate of heat generation in the wire as a result of resistance heating. **2** Thermal properties of water are constant. **3** Heat losses from the water in the tank are negligible.

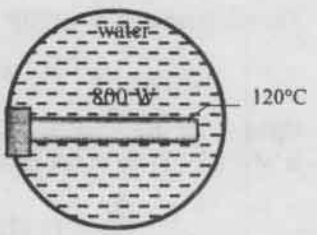
Properties The specific heat of water at room temperature is $C = 4.180$ kJ/kg·°C (Table A-15).

Analysis When steady operating conditions are reached, we have $\dot{Q} = \dot{E}_{\text{generated}} = 800$ W. This is also equal to the rate of heat gain by water. Noting that this is the only mechanism of energy transfer, the time it takes to raise the water temperature from 20°C to 80°C is determined to be

$$Q_{in} = mC(T_2 - T_1)$$

$$\dot{Q}_{in}\Delta t = mC(T_2 - T_1)$$

$$\Delta t = \frac{mC(T_2 - T_1)}{\dot{Q}_{in}} = \frac{(40\text{ kg})(4180\text{ J/kg.°C})(80-20)°\text{C}}{800\text{ J/s}} = 12{,}540\text{ s} = \mathbf{3.48\ h}$$



The surface area of the wire is

$$A = (\pi D)L = \pi(0.005\text{ m})(0.5\text{ m}) = 0.00785\text{ m}^2$$

The Newton's law of cooling for convection heat transfer is expressed as $\dot{Q} = hA(T_s - T_\infty)$. Disregarding any heat transfer by radiation and thus assuming all the heat loss from the wire to occur by convection, the convection heat transfer coefficients at the beginning and at the end of the process are determined to be

$$h_1 = \frac{\dot{Q}}{A(T_s - T_{\infty 1})} = \frac{800\text{ W}}{(0.00785\text{ m}^2)(120-20)°\text{C}} = \mathbf{1020\ W/m^2.°C}$$

$$h_2 = \frac{\dot{Q}}{A(T_s - T_{\infty 2})} = \frac{800\text{ W}}{(0.00785\text{ m}^2)(120-80)°\text{C}} = \mathbf{2550\ W/m^2.°C}$$

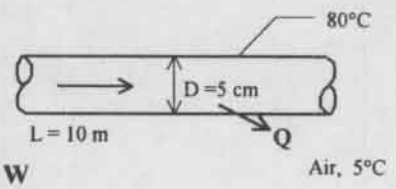
Discussion Note that a larger heat transfer coefficient is needed to dissipate heat through a smaller temperature difference for a specified heat transfer rate.

14-32 A hot water pipe at 80°C is losing heat to the surrounding air at 5°C by natural convection with a heat transfer coefficient of 25 W/ m².°C. The rate of heat loss from the pipe by convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is not considered. **3** The convection heat transfer coefficient is constant and uniform over the surface.

Analysis The heat transfer surface area is

$$A = (\pi D)L = 3.14\text{x}(0.05\text{ m})(10\text{ m}) = 1.571\text{ m}^2$$



Under steady conditions, the rate of heat transfer by convection is

$$\dot{Q}_{conv} = hA\Delta T = (25\text{ W/m}^2\text{·°C})(1.571\text{ m}^2)(80-5)°\text{C} = \mathbf{2945\ W}$$

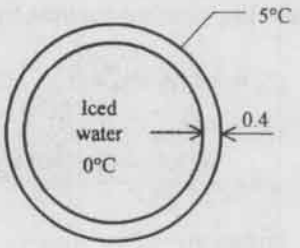
14-33 A hollow spherical iron container is filled with iced water at 0°C. The rate of heat loss from the sphere and the rate at which ice melts in the container are to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values. **2** Heat transfer through the shell is one-dimensional. **3** Thermal properties of the iron shell are constant. **4** The inner surface of the shell is at the same temperature as the iced water, 0°C.

Properties The thermal conductivity of iron is given to be $k = 80.2$ W/m·°C (Table A-19). The heat of fusion of water is given to be 333.7 kJ/kg.

Analysis This spherical shell can be approximated as a plate of thickness 0.4 cm and surface area

$$A = \pi D^2 = 3.14\text{x}(0.2\text{ m})^2 = 0.126\text{ m}^2$$



Then the rate of heat transfer through the shell by conduction is

$$\dot{Q}_{cond} = kA\frac{\Delta T}{L} = (80.2\text{ W/m}\cdot^\circ\text{C})(0.126\text{ m}^2)\frac{(5-0)^\circ\text{C}}{0.004\text{ m}} = 12{,}632\text{ W}$$

Considering that it takes 333.7 kJ of energy to melt 1 kg of ice at 0°C, the rate at which ice melts in the container can be determined from

$$\dot{m}_{ice} = \frac{\dot{Q}}{h_{if}} = \frac{12.632\text{ kJ/s}}{333.7\text{ kJ/kg}} = \mathbf{0.038\text{ kg/s}}$$

Discussion We should point out that this result is slightly in error for approximating a curved wall as a plain wall. The error in this case is very small because of the large diameter to thickness ratio. For better accuracy, we could use the inner surface area ($D = 19.2$ cm) or the mean surface area ($D = 19.6$ cm) in the calculations.

14-34E The inner and outer glasses of a double pane window with a 0.5-in air space are at specified temperatures. The rate of heat transfer through the window is to be determined

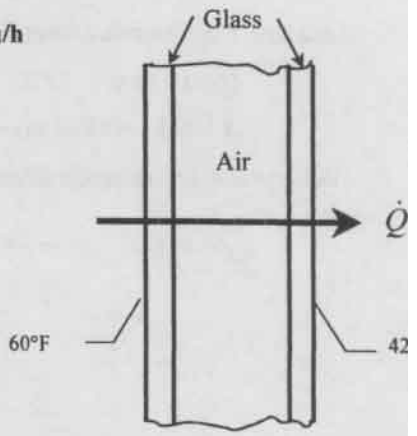
Assumptions **1** Steady operating conditions exist since the surface temperatures of the glass remain constant at the specified values. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the air are constant.

Properties The thermal conductivity of air at the average temperature of (60+42)/2 = 51°F is $k = 0.0144$ Btu/h.ft.°F.

Analysis The surface area of the window and the rate of heat loss through it are

$$A = (6\text{ ft}) \times (6\text{ ft}) = 36\text{ m}^2$$

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.0144\text{ Btu/h.ft.°F})(36\text{ ft}^2)\frac{(60-42)°\text{F}}{0.5/12\text{ ft}} = \mathbf{224\ Btu/h}$$

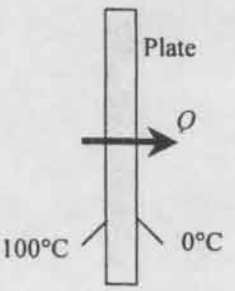


14-35 Two surfaces of a flat plate are maintained at specified temperatures, and the rate of heat transfer through the plate is measured. The thermal conductivity of the plate material is to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the plate remain constant at the specified values. **2** Heat transfer through the plate is one-dimensional. **3** Thermal properties of the plate are constant.

Analysis The thermal conductivity is determined directly from the steady one-dimensional heat conduction relation to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} \rightarrow k = \frac{\dot{Q}/A}{L(T_1 - T_2)} = \frac{500\text{ W/m}^2}{(0.02\text{ m})(100 - 0)°\text{C}} = \mathbf{250\ W/m.°C}$$



14-36 Four power transistors are mounted on a thin vertical aluminum plate that is cooled by a fan. The temperature of the aluminum plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The entire plate is nearly isothermal. **3** Thermal properties of the wall are constant. **4** The exposed surface area of the transistor can be taken to be equal to its base area. **5** Heat transfer by radiation is disregarded. **6** The convection heat transfer coefficient is constant and uniform over the surface.

Properties The thermal conductivity of the aluminum plate is given to be $k = 237$ W/m·°C.

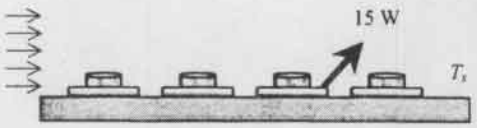
Analysis The total rate of heat dissipation from the aluminum plate and the total heat transfer area are

$$\dot{Q} = 4 \times 15 \text{ W} = 60 \text{ W}$$

$$A = (0.22 \text{ m})(0.22 \text{ m}) = 0.0484 \text{ m}^2$$

Disregarding any radiation effects, the temperature of the aluminum plate is determined to be

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 25°\text{C} + \frac{60 \text{ W}}{(25 \text{ W/m}^2.°\text{C})(0.0484 \text{ m}^2)} = \mathbf{74.6°\text{C}}$$



14-37 A styrofoam ice chest is initially filled with 40 kg of ice at 0°C. The time it takes for the ice in the chest to melt completely is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner and outer surface temperatures of the ice chest remain constant at 0°C and 8°C, respectively, at all times. **3** Thermal properties of the chest are constant. **4** Heat transfer from the base of the ice chest is negligible.

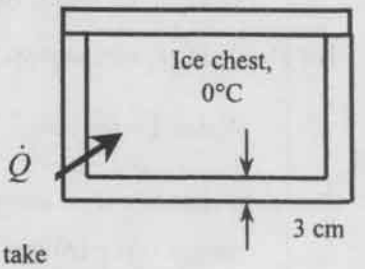
Properties The thermal conductivity of the styrofoam is given to be $k = 0.033$ W/m·°C. The heat of fusion of ice at 0°C is 333.7 kJ/kg.

Analysis Disregarding any heat loss through the bottom of the ice chest and using the average thicknesses, the total heat transfer area becomes

$$A = (40-3)(40-3) + 4 \times (40-3)(30-3) = 5365 \text{ cm}^2 = 0.5365 \text{ m}^2$$

The rate of heat transfer to the ice chest becomes

$$\dot{Q} = kA\frac{\Delta T}{L} = (0.033 \text{ W/m.°C})(0.5365 \text{ m}^2)\frac{(8-0)°\text{C}}{0.03 \text{ m}} = 4.72 \text{ W}$$



The total amount of heat needed to melt the ice completely is

$$Q = mh_{if} = (40 \text{ kg})(333.7 \text{ kJ/kg}) = 13{,}348 \text{ kJ}$$

Then transferring this much heat to the cooler to melt the ice completely will take

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{13{,}348{,}000 \text{ J}}{4.72 \text{ J/s}} = 2{,}828{,}000 \text{ s} = 785.6 \text{ h} = \mathbf{32.7 \text{ days}}$$

14-38 A transistor mounted on a circuit board is cooled by air flowing over it. The transistor case temperature is not to exceed 70°C when the air temperature is 55°C. The amount of power this transistor can dissipate safely is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is disregarded. **3** The convection heat transfer coefficient is constant and uniform over the surface. **4** Heat transfer from the base of the transistor is negligible.

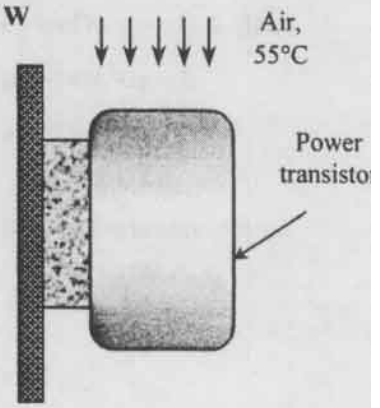
Analysis Disregarding the base area, the total heat transfer area of the transistor is

$$A = \pi DL + \pi D^2/4 = \pi(0.6 \text{ cm})(0.4 \text{ cm}) + \pi(0.6 \text{ cm})^2/4 = 1.037 \text{ cm}^2 = 1.037 \times 10^{-4} \text{ m}^2$$

Then the rate of heat transfer from the power transistor at specified conditions is

$$\dot{Q} = hA(T_s - T_\infty) = (30 \text{ W/m}^2\text{.°C})(1.037 \times 10^{-4} \text{ m}^2)(70-55)°\text{C} = \mathbf{0.047 \text{ W}}$$

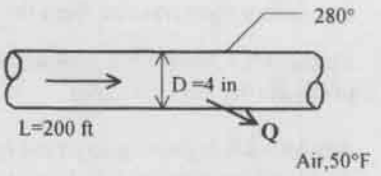
Therefore, the amount of power this transistor can dissipate safely is 0.047 W.



14-39E A 200-ft long section of a steam pipe passes through an open space at a specified temperature. The rate of heat loss from the steam pipe and the annual cost of this energy lost are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is disregarded. **3** The convection heat transfer coefficient is constant and uniform over the surface.

Analysis (a) The rate of heat loss from the steam pipe is



$$A = \pi DL = \pi(4/12 \text{ ft})(200 \text{ ft}) = 209.4 \text{ ft}^2$$

$$\dot{Q}_{\text{pipe}} = hA(T_s - T_{air}) = (6 \text{ Btu/h.ft}^2\text{.°F})(209.4 \text{ ft}^2)(280-50)\text{°F}$$

$$= \mathbf{289{,}000 \text{ Btu/h}}$$

(b) The amount of heat loss per year is

$$Q = \dot{Q}\Delta t = (289{,}000 \text{ Btu/h})(365 \times 24 \text{ h/yr}) = 2.532 \times 10^9 \text{ Btu/yr}$$

The amount of gas consumption per year in the furnace that has an efficiency of 86% is

$$\text{Annual Energy Loss} = \frac{2.532 \times 10^9 \text{ Btu/yr}}{0.86}\left(\frac{1 \text{ therm}}{100{,}000 \text{ Btu}}\right) = 29{,}438 \text{ therms/yr}$$

Then the annual cost of the energy lost becomes

$$\text{Energy cost} = (\text{Annual energy loss})(\text{Unit cost of energy})$$

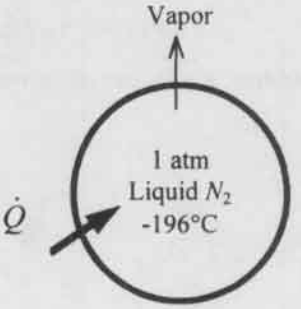
$$= (29{,}438 \text{ therms/yr})(\$0.58/\text{therm}) = \mathbf{\$17{,}074/yr}$$

14-40 A 4-m diameter spherical tank filled with liquid nitrogen at 1 atm and -196°C is exposed to convection with ambient air. The rate of evaporation of liquid nitrogen in the tank as a result of the heat transfer from the ambient air is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is disregarded. **3** The convection heat transfer coefficient is constant and uniform over the surface. **4** The temperature of the thin-shelled spherical tank is nearly equal to the temperature of the nitrogen inside.

Properties The heat of vaporization and density of liquid nitrogen at 1 atm are given to be 198 kJ/kg and 810 kg/m^3, respectively.

Analysis The rate of heat transfer to the nitrogen tank is



$$A = \pi D^2 = \pi(4 \text{ m})^2 = 50.27 \text{ m}^2$$

$$\dot{Q} = hA(T_s - T_{\text{air}}) = (25 \text{ W/m}^2\text{.°C})(50.27 \text{ m}^2)[20-(-196)]\text{°C}$$

$$= \mathbf{271{,}430 \text{ W}}$$

Then the rate of evaporation of liquid nitrogen in the tank is determined to l

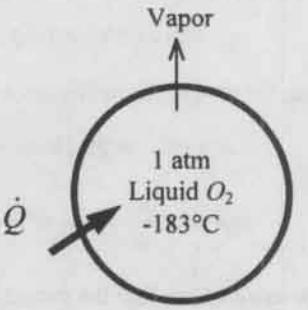
$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{271.430 \text{ kJ/s}}{198 \text{ kJ/kg}} = \mathbf{1.37 \text{ kg/s}}$$

14-41 A 4-m diameter spherical tank filled with liquid oxygen at 1 atm and -183°C is exposed to convection with ambient air. The rate of evaporation of liquid oxygen in the tank as a result of the heat transfer from the ambient air is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is disregarded. **3** The convection heat transfer coefficient is constant and uniform over the surface. **4** The temperature of the thin-shelled spherical tank is nearly equal to the temperature of the oxygen inside.

Properties The heat of vaporization and density of liquid oxygen at 1 atm are given to be 2113 kJ/kg and 1140 kg/m^3, respectively.

Analysis The rate of heat transfer to the oxygen tank is



$$A = \pi D^2 = \pi(4\text{ m})^2 = 50.27\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_{\text{air}}) = (25\text{ W/m}^2.^\circ\text{C})(50.27\text{ m}^2)[20-(-183)]^\circ\text{C}$$
$$= \mathbf{255{,}120\text{ W}}$$

Then the rate of evaporation of liquid oxygen in the tank is determined to be

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{255.120\text{ kJ/s}}{213\text{ kJ/kg}} = \mathbf{1.20\text{ kg/s}}$$

14-42 A person with a specified surface temperature is subjected to radiation heat transfer in a room at specified wall temperatures. The rate of radiation heat loss from the person is to be determined.

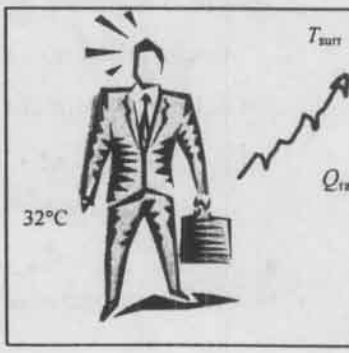
Assumptions **1** Steady operating conditions exist. **2** Heat transfer by convection is disregarded. **3** The emissivity of the person is constant and uniform over the exposed surface.

Properties The average emissivity of the person is given to be 0.7.

Analysis Noting that the person is completely enclosed by the surrounding surfaces, the net rates of radiation heat transfer from the body to the surrounding walls, ceiling, and the floor in both cases are

(*a*) $T_{\text{surr}} = 300$ K

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A(T_s^4 - T_{\text{surr}}^4)$$
$$= (0.7)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(1.7\text{ m}^2)[(32+273)^4 - (300\text{ K})^4]\text{K}^4$$
$$= \mathbf{37.4\text{ W}}$$



(*b*) $T_{\text{surr}} = 280$ K

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A(T_s^4 - T_{\text{surr}}^4)$$
$$= (0.7)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(1.7\text{ m}^2)[(32+273)^4 - (280\text{ K})^4]\text{K}^4$$
$$= \mathbf{169\text{ W}}$$

Discussion Note that the radiation heat transfer goes up by more than 4 times as the temperature of the surrounding surfaces drops from 300 K to 280 K.

14-43 A circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.04 W. All the heat generated in the chips is conducted across the circuit board. The temperature difference between the two sides of the circuit board is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Thermal properties of the board are constant. **3** All the heat generated in the chips is conducted across the circuit board.

Properties The effective thermal conductivity of the board is given to be $k = 16$ W/m·°C.

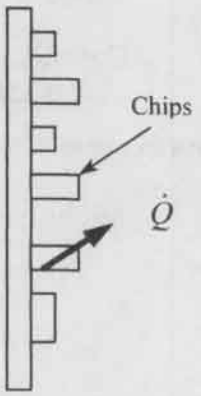
Analysis The total rate of heat dissipated by the chips is

$$\dot{Q} = 80 \times (0.04 \text{ W}) = 3.2 \text{ W}$$

Then the temperature difference between the front and back surfaces of the board is

$$A = (0.12 \text{ m})(0.18 \text{ m}) = 0.0216 \text{ m}^2$$

$$\dot{Q} = kA\frac{\Delta T}{L} \longrightarrow \Delta T = \frac{\dot{Q}L}{kA} = \frac{(3.2 \text{ W})(0.003 \text{ m})}{(16 \text{ W/m.°C})(0.0216 \text{ m}^2)} = \mathbf{0.028°C}$$



Discussion Note that the circuit board is nearly isothermal.

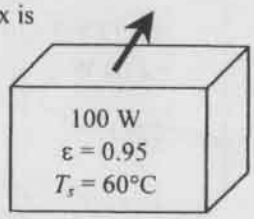
14-44 A sealed electronic box dissipating a total of 100 W of power is placed in a vacuum chamber. If this box is to be cooled by radiation alone and the outer surface temperature of the box is not to exceed 55°C, the temperature the surrounding surfaces must be kept is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by convection is disregarded. **3** The emissivity of the box is constant and uniform over the exposed surface. **4** Heat transfer from the bottom surface of the box to the stand is negligible.

Properties The emissivity of the outer surface of the box is given to be 0.95.

Analysis Disregarding the base area, the total heat transfer area of the electronic box is

$$A = (0.4 \text{ m})(0.4 \text{ m}) + 4 \times (0.2 \text{ m})(0.4 \text{ m}) = 0.48 \text{ m}^2$$



The radiation heat transfer from the box can be expressed as

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A(T_s^4 - T_{surr}^4)$$

$$100 \text{ W} = (0.95)(5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)(0.48 \text{ m}^2)\left[(55 + 273 \text{ K})^4 - T_{surr}^4\right]$$

which gives $T_{surr} = \mathbf{296.3 \text{ K} = 23.3°C}$. Therefore, the temperature of the surrounding surfaces must be less than 23.3°C.

14-45 Using the conversion factors between W and Btu/h, m and ft, and K and R, the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2.\text{K}^4$ is to be expressed in the English unit, $\text{Btu/h.ft}^2.\text{R}^4$.

Analysis The conversion factors for W, m, and K are given in conversion tables to be

$$1\ \text{W} = 3.41214\ \text{Btu/h}$$
$$1\ \text{m} = 3.2808\ \text{ft}$$
$$1\ \text{K} = 1.8\ \text{R}$$

Substituting gives the Stefan-Boltzmann constant in the desired units,

$$\sigma = 5.67\ \text{W/m}^2.\text{K}^4 = 5.67 \times \frac{3.41214\ \text{Btu/h}}{(3.2808\ \text{ft})^2 (1.8\ \text{R})^4} = \mathbf{0.171\ Btu/h.ft^2.R^4}$$

14-46 Using the conversion factors between W and Btu/h, m and ft, and °C and °F, the convection coefficient in SI units is to be expressed in $\text{Btu/h.ft}^2.°\text{F}$.

Analysis The conversion factors for W and m are straightforward, and are given in conversion tables to be

$$1\ \text{W} = 3.41214\ \text{Btu/h}$$
$$1\ \text{m} = 3.2808\ \text{ft}$$

The proper conversion factor between °C into °F in this case is

$$1°\text{C} = 1.8°\text{F}$$

since the °C in the unit $\text{W/m}^2.°\text{C}$ represents *per °C change in temperature*, and 1°C change in temperature corresponds to a change of 1.8°F. Substituting, we get

$$1\ \text{W/m}^2.°\text{C} = \frac{3.41214\ \text{Btu/h}}{(3.2808\ \text{ft})^2 (1.8\ °\text{F})} = 0.1761\ \text{Btu/h.ft}^2.°\text{F}$$

which is the desired conversion factor. Therefore, the given convection heat transfer coefficient in English units is

$$h = 20\ \text{W/m}^2.°\text{C} = 20 \times 0.1761\ \text{Btu/h.ft}^2.°\text{F} = \mathbf{3.52\ Btu/h.ft^2.°F}$$

Simultaneous Heat Transfer Mechanisms

14-47C All three modes of heat transfer can not occur simultaneously in a medium. A medium may involve two of them simultaneously.

14-48C (*a*) Conduction and convection: No. (*b*) Conduction and radiation: Yes. Example: A hot surface on the ceiling. (*c*) Convection and radiation: Yes. Example: Heat transfer from the human body.

14-49C The human body loses heat by convection, radiation, and evaporation in both summer and winter. In summer, we can keep cool by dressing lightly, staying in cooler environments, turning a fan on, avoiding humid places and direct exposure to the sun. In winter, we can keep warm by dressing heavily, staying in a warmer environment, and avoiding drafts.

14-50C The fan increases the air motion around the body and thus the convection heat transfer coefficient, which increases the rate of heat transfer from the body by convection and evaporation. In rooms with high ceilings, ceiling fans are used in winter to force the warm air at the top downward to increase the air temperature at the body level. This is usually done by forcing the air up which hits the ceiling and moves downward in a gently manner to avoid drafts.

14-51 The total rate of heat transfer from a person by both convection and radiation to the surrounding air and surfaces at specified temperatures is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The person is completely surrounded by the interior surfaces of the room. **3** The surrounding surfaces are at the same temperature as the air in the room. **4** Heat conduction to the floor through the feet is negligible. **5** The convection coefficient is constant and uniform over the entire surface of the person.

Properties The emissivity of a person is given to be $\varepsilon = 0.9$.

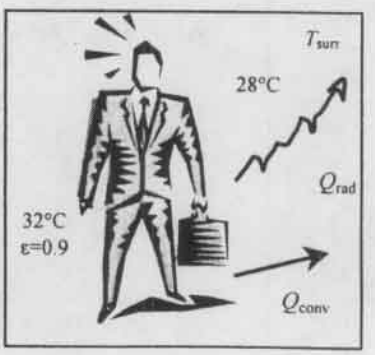
Analysis The person is completely enclosed by the surrounding surfaces, and he or she will lose heat to the surrounding air by convection, and to the surrounding surfaces by radiation. The total rate of heat loss from the person is determined from

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A(T_s^4 - T_{\text{surr}}^4) = (0.90)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1.7\ \text{m}^2)[(32+273)^4 - (23+273)^4]\text{K}^4$$
$$= 84.8\ \text{W}$$

$$\dot{Q}_{\text{conv}} = hA\Delta T = (5\ \text{W/m}^2\cdot\text{K})(1.7\ \text{m}^2)(32-23)^\circ\text{C} = 76.5\ \text{W}$$

and

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 84.8 + 76.5 = \mathbf{161.3\ W}$$



Discussion Note that heat transfer from the person by evaporation, which is of comparable magnitude, is not considered in this problem.

14-52 Two large plates at specified temperatures are held parallel to each other. The rate of heat transfer between the plates is to be determined for the cases of still air, regular insulation, and super insulation between the plates.

Assumptions **1** Steady operating conditions exist since the plate temperatures remain constant. **2** Heat transfer is one-dimensional since the plates are large. **3** The surfaces are black and thus $\varepsilon = 1$. **4** There are no convection currents in the air space between the plates.

Properties The thermal conductivities are $k = 0.00015$ W/m·°C for super insulation, $k = 0.0223$ W/m·°C for air, and $k = 0.036$ W/m·°C for fiberglass insulation.

Analysis (a) Disregarding any natural convection currents, the rates of conduction and radiation heat transfer

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.0223\ \text{W/m}^2.^\circ\text{C})(1\ \text{m}^2)\frac{(290-150)\ \text{K}}{0.02\ \text{m}} = 156\ \text{W}$$

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A(T_1^4 - T_2^4) = 1(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1\text{m}^2)\left[(290\ \text{K})^4 - (150\ \text{K})^4\right] = 372\ \text{W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} + \dot{Q}_{\text{rad}} = 156 + 372 = \mathbf{528\ W}$$

(b) When the air space between the plates is evacuated, there will be radiation heat transfer only.

Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{rad}} = \mathbf{372\ W}$$

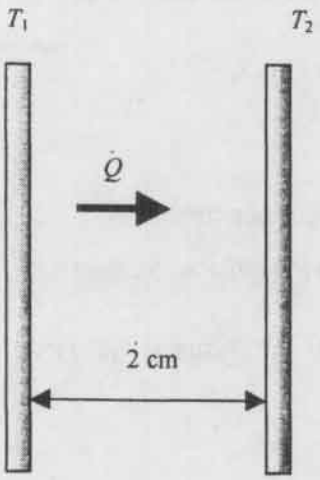
(c) In this case there will be conduction heat transfer through the fiberglass insulation only,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.036\ \text{W/m.}^\circ\text{C})(1\ \text{m}^2)\frac{(290-150)\ \text{K}}{0.02\ \text{m}} = \mathbf{252\ W}$$

(d) In the case of superinsulation, the rate of heat transfer will be

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.00015\ \text{W/m.}^\circ\text{C})(1\ \text{m}^2)\frac{(290-150)\ \text{K}}{0.02\ \text{m}} = \mathbf{1.05\ W}$$

Discussion Note that superinsulators are very effective in reducing heat transfer between to surfaces.

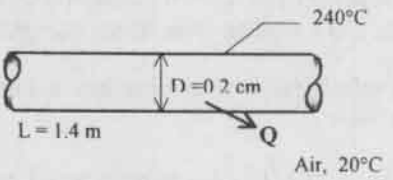


14-53 The convection heat transfer coefficient for heat transfer from an electrically heated wire to air is to be determined by measuring temperatures when steady operating conditions are reached and the electric power consumed.

Assumptions 1 Steady operating conditions exist since the temperature readings do not change with time. **2** Radiation heat transfer is negligible.

Analysis In steady operation, the rate of heat loss from the wire equals the rate of heat generation in the wire as a result of resistance heating. That is,

$$\dot{Q} = \dot{E}_{\text{generated}} = VI = (110\ \text{V})(3\ \text{A}) = 330\ \text{W}$$



The surface area of the wire is

$$A = (\pi D)L = \pi(0.002\ \text{m})(1.4\ \text{m}) = 0.00880\ \text{m}^2$$

The Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q} = hA(T_s - T_\infty)$$

Disregarding any heat transfer by radiation , the convection heat transfer coefficient is determined to be

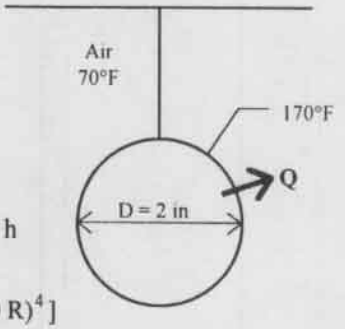
$$h = \frac{\dot{Q}}{A(T_1 - T_\infty)} = \frac{330\ \text{W}}{(0.00880\ \text{m}^2)(240-20)^\circ\text{C}} = \mathbf{170.5\ W/m^2.^\circ C}$$

Discussion If the temperature of the surrounding surfaces is equal to the air temperature in the room, the value obtained above actually represents the combined convection and radiation heat transfer coefficient.

14-54E A spherical ball whose surface is maintained at a temperature of 170°F is suspended in the middle of a room at 70°F. The total rate of heat transfer from the ball is to be determined.

Assumptions **1** Steady operating conditions exist since the ball surface and the surrounding air and surfaces remain at constant temperatures. **2** The thermal properties of the ball and the convection heat transfer coefficient are constant and uniform.

Properties The emissivity of the ball surface is given to be $\varepsilon = 0.8$.



Analysis The heat transfer surface area is

$$A = \pi D^2 = 3.14(2/12\ \text{ft})^2 = 0.08727\ \text{ft}^2$$

Under steady conditions, the rates of convection and radiation heat transfer are

$$\dot{Q}_{\text{conv}} = hA\Delta T = (3\ \text{Btu/h.ft}^2\cdot^\circ\text{F})(0.08727\ \text{ft}^2)(170-70)^\circ\text{F} = 26.2\ \text{Btu/h}$$

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A(T_s^4 - T_o^4)$$
$$= 0.8(0.08727\ \text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)[(630\ \text{R})^4 - (530\ \text{R})^4]$$
$$= 9.4\ \text{Btu/h}$$

Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 26.2 + 9.4 = \mathbf{35.6\ Btu/h}$$

Discussion Note that heat loss by convection is several times that of heat loss by radiation. The radiation heat loss can further be reduced by coating the ball with a low-emissivity material.

14-55 A 1000-W iron is left on the iron board with its base exposed to the air at 20°C. The temperature of the base of the iron is to be determined in steady operation.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of the iron base and the convection heat transfer coefficient are constant and uniform. **3** The temperature of the surrounding surfaces is the same as the temperature of the surrounding air.

Properties The emissivity of the base surface is given to be $\varepsilon = 0.6$.

Analysis At steady conditions, the 1000 W energy supplied to the iron will be dissipated to the surroundings by convection and radiation heat transfer. Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 1000 \text{ W}$$

where

$$\dot{Q}_{\text{conv}} = hA\Delta T = (35 \text{ W/m}^2 \cdot \text{K})(0.02 \text{ m}^2)(T_s - 293 \text{ K}) = 0.7(T_s - 293 \text{ K}) \text{ W}$$

and

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A(T_s^4 - T_o^4) = 0.6(0.02\text{m}^2)(5.67\times10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[T_s^4 - (293\text{K})^4]$$
$$= 0.06804\times10^{-8}[T_s^4 - (293\text{K})^4] \text{ W}$$

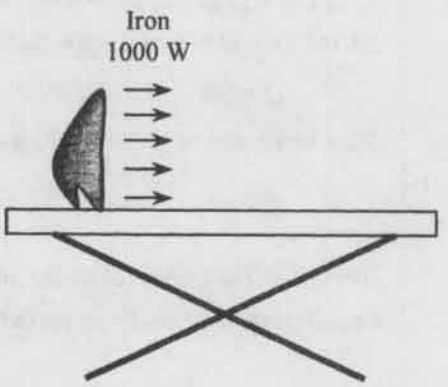
Substituting,

$$1000 \text{ W} = 0.7(T_s - 293 \text{ K}) + 0.06804\times10^{-8}[T_s^4 - (293 \text{ K})^4]$$

Solving by trial and error gives

$$T_s = \mathbf{947 \text{ K} = 674^\circ C}$$

Discussion We note that the iron will dissipate all the energy it receives by convection and radiation when its surface temperature reaches 947 K.



14-56 A spacecraft in space absorbs solar radiation while losing heat to deep space by thermal radiation. The surface temperature of the spacecraft is to be determined when steady conditions are reached..

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values. **2** Thermal properties of the wall are constant.

Properties The outer surface of a spacecraft has an emissivity of 0.8 and an absorptivity of 0.3.

Analysis When the heat loss from the outer surface of the spacecraft by radiation equals the solar radiation absorbed, the surface temperature can be determined from

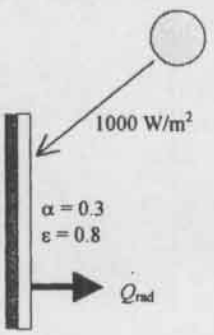
$$\dot{Q}_{\text{solar absorbed}} = \dot{Q}_{\text{rad}}$$

$$\alpha\dot{Q}_{\text{solar}} = \varepsilon\sigma A(T_s^4 - T_{\text{space}}^4)$$

$$0.3 \times A \times (1000 \text{ W/m}^2) = 0.8 \times A \times (5.67\times10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[T_s^4 - (0 \text{ K})^4]$$

Canceling the surface area A and solving for T_s gives

$$T_s = \mathbf{285 \text{ K}}$$



14-57 A spherical tank located outdoors is used to store iced water at 0°C. The rate of heat transfer to the iced water in the tank and the amount of ice at 0°C that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values. **2** Thermal properties of the tank and the convection heat transfer coefficient is constant and uniform. **3** The average surrounding surface temperature for radiation exchange is 15°C. **4** The thermal resistance of the tank is negligible, and the entire steel tank is at 0°C.

Properties The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ / kg. The emissivity of the outer surface of the tank is 0.6.

Analysis (a) The outer surface area of the spherical tank is

$$A = \pi D^2 = \pi(3.02\ \text{m})^2 = 28.65\ \text{m}^2$$

Then the rates of heat transfer to the tank by convection and radiation become

$$\dot{Q}_{\text{conv}} = hA(T_\infty - T_s) = (30\ \text{W/m}^2.^\circ\text{C})(28.65\ \text{m}^2)(25-0)^\circ\text{C} = 21{,}488\ \text{W}$$

$$\dot{Q}_{\text{rad}} = \varepsilon A\sigma(T_{surr}^4 - T_s^4) = (0.6)(28.65\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(288\ \text{K})^4 - (273\ \text{K})^4] = 1292\ \text{W}$$

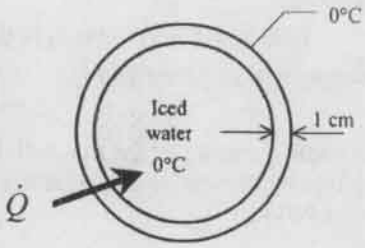
$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 21{,}488 + 1292 = 22{,}780\ \text{W}$$

(b) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (22.78\ \text{kJ/s})(24\times3600\ \text{s}) = 1{,}968{,}000\ \text{kJ}$$

Then the amount of ice that melts during this period becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{1{,}968{,}000\ \text{kJ}}{333.7\ \text{kJ/kg}} = \mathbf{5898\ kg}$$



Discussion The amount of ice that melts can be reduced to a small fraction by insulating the tank.

14-58 The roof of a house with a gas furnace consists of a 15-cm thick concrete that is losing heat to the outdoors by radiation and convection. The rate of heat transfer through the roof and the money lost through the roof that night during a 14 hour period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The emissivity and thermal conductivity of the roof are constant.

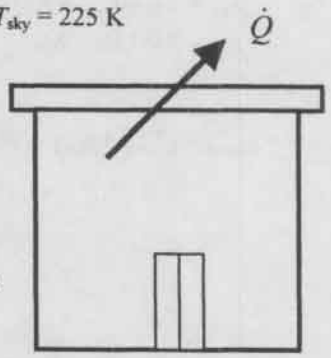
Properties The thermal conductivity of the concrete is given to be $k = 2$ W/m·°C. The emissivity of the outer surface of the roof is given to be 0.9.

Analysis In steady operation, heat transfer from the outer surface of the roof to the surroundings by convection and radiation must be equal to the heat transfer through the roof by conduction. That is,

$$\dot{Q} = \dot{Q}_{\text{roof, cond}} = \dot{Q}_{\text{roof to surroundings, conv+rad}}$$

The inner surface temperature of the roof is given to be $T_{s,in} = 15°C$. Letting $T_{s,out}$ denote the outer surface temperatures of the roof, the energy balance above can be expressed as

$$\dot{Q} = kA\frac{T_{s,in} - T_{s,out}}{L} = h_o A(T_{s,out} - T_{surr}) + \varepsilon A\sigma(T_{s,out}{}^4 - T_{surr}{}^4)$$



$$\dot{Q} = (2\ \text{W/m.°C})(300\ \text{m}^2)\frac{15°\text{C} - T_{s,out}}{0.15\ \text{m}}$$

$$= (15\ \text{W/m}^2\text{.°C})(300\ \text{m}^2)(T_{s,out} - 10)°\text{C}$$

$$+ (0.9)(300\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\text{.K}^4)\left[(T_{s,out} + 273\ \text{K})^4 - (255\ \text{K})^4\right]$$

Solving the equations above using an equation solver (or by trial and error) gives

$$\dot{Q} = \mathbf{25{,}450\ W} \text{ and } T_{s,\,out} = \mathbf{8.64°C}$$

Then the amount of natural gas consumption during a 14-hour period is

$$E_{gas} = \frac{Q_{total}}{0.85} = \frac{\dot{Q}\Delta t}{0.85} = \frac{(25.450\ \text{kJ/s})(14\times3600\ \text{s})}{0.85}\left(\frac{1\ \text{therm}}{105{,}500\ \text{kJ}}\right) = 14.3\ \text{therms}$$

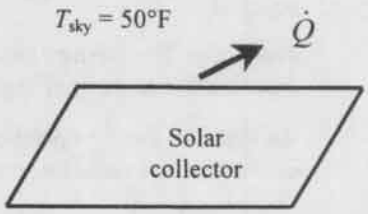
Finally, the money lost through the roof during that period is

$$\text{Money lost} = (14.3\ \text{therms})(\$0.60/\text{therm}) = \mathbf{\$8.58}$$

14-59E A flat plate solar collector is placed horizontally on the roof of a house. The rate of heat loss from the collector by convection and radiation during a calm day are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The emissivity and convection heat transfer coefficient are constant and uniform. **3** The exposed surface, ambient, and sky temperatures remain constant.

Properties The emissivity of the outer surface of the collector is given to be 0.9.



Analysis The exposed surface area of the collector is

$$A = (5 \text{ ft})(15 \text{ ft}) = 75 \text{ ft}^2$$

Noting that the exposed surface temperature of the collector is 100°F, the total rate of heat loss from the collector the environment by convection and radiation becomes

$$\dot{Q}_{conv} = hA(T_\infty - T_s) = (2.5 \text{ Btu/h.ft}^2.°\text{F})(75 \text{ ft}^2)(100-70)°\text{F} = 5625 \text{ Btu/h}$$

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_{surr}^4 - T_s^4) = (0.9)(75 \text{ ft}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2.\text{R}^4)[(100+460 \text{ R})^4 - (50+460 \text{ R})^4]$$
$$= 3551 \text{ Btu/h}$$

and

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 5625 + 3551 = \mathbf{9176 \text{ Btu/h}}$$

Review Problems

14-60 A standing man is subjected to high winds and thus high convection coefficients. The rate of heat loss from this man by convection in still air at 20°C, in windy air, and the wind-chill factor are to be determined.

Assumptions **1** A standing man can be modeled as a 30-cm diameter, 170-cm long vertical cylinder with both the top and bottom surfaces insulated. **2** The exposed surface temperature of the person and the convection heat transfer coefficient is constant and uniform. **3** Heat loss by radiation is negligible.

Analysis The heat transfer surface area of the person is

$$A = \pi Dh = 3.14\text{x}(0.3\text{ m})(1.70\text{ m}) = 1.60\text{ m}^2$$



Windy weather

The rate of heat loss from this man by convection in still air is

$$Q_{\text{still air}} = hA\Delta T = (15\text{ W/m}^2\cdot\text{°C})(1.60\text{ m}^2)(34 - 20)\text{°C} = \mathbf{336\text{ W}}$$

In windy air it would be

$$Q_{\text{windy air}} = hA\Delta T = (50\text{ W/m}^2\cdot\text{°C})(1.60\text{ m}^2)(34 - 20)\text{°C} = \mathbf{1120\text{ W}}$$

To lose heat at this rate in still air, the air temperature must be

$$1120\text{ W} = (hA\Delta T)_{\text{still air}} = (15\text{ W/m}^2\cdot\text{°C})(1.60\text{ m}^2)(34 - T_{\text{effective}})\text{°C}$$

which gives

$$T_{\text{effective}} = -12.7\text{°C}$$

That is, the windy air at 20°C feels as cold as still air at -12.7°C as a result of the wind-chill effect. Therefore, the wind-chill factor in this case is

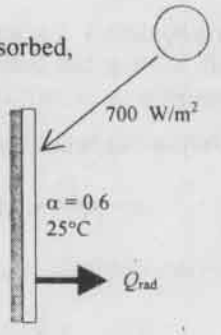
$$F_{\text{wind-chill}} = 20 - (-12.7) = \mathbf{32.7°C}$$

14-61 The backside of the thin metal plate is insulated and the front side is exposed to solar radiation. The surface temperature of the plate is to be determined when it stabilizes.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulated side of the plate is negligible. **3** The heat transfer coefficient is constant and uniform over the plate. **4** Radiation heat transfer is negligible.

Properties The solar absorptivity of the plate is given to be $\alpha = 0.6$.

Analysis When the heat loss from the plate by convection equals the solar radiation absorbed, the surface temperature of the plate can be determined from



$$\dot{Q}_{\text{solar absorbed}} = \dot{Q}_{\text{conv}}$$

$$\alpha \dot{Q}_{\text{solar}} = hA(T_s - T_o)$$

$$0.7 \times A \times 800\ \text{W/m}^2 = (30\ \text{W/m}^2 \cdot ^\circ\text{C})A(T_s - 10)$$

Canceling the surface area A and solving for T_s gives

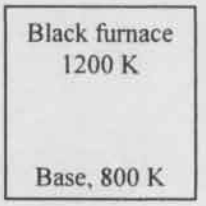
$$T_s = \mathbf{28.7^\circ C}$$

14-62 The base surface of a cubical furnace is surrounded by black surfaces at a specified temperature. The net rate of radiation heat transfer to the base surface from the top and side surfaces is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The top and side surfaces of the furnace closely approximate black surfaces. **3** The properties of the surfaces are constant.

Properties The emissivity of the base surface is $\varepsilon = 0.7$.

Analysis The base surface is completely surrounded by the top and side surfaces. Then using the radiation relation for a surface completely surrounded by another large (or black) surface, the net rate of radiation heat transfer from the top and side surfaces to the base is determined to be



$$\dot{Q}_{\text{rad,base}} = \varepsilon A \sigma (T_{\text{base}}^4 - T_{\text{surr}}^4)$$

$$= (0.7)(3 \times 3\ \text{m}^2)(5.67 \times 10^{-8}\ \text{W/m}^2.\text{K}^4)[(1200\ \text{K})^4 - (800\ \text{K})^4]$$

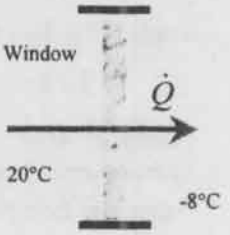
$$= \mathbf{594{,}400\ W}$$

14-63 The range of U-factors for windows are given. The range for the rate of heat loss through the window of a house is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses associated with the infiltration of air through the cracks/openings are not considered.

Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}} (T_{in} - T_{out})$$



where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Substituting,

Maximum heat loss: $\dot{Q}_{\text{window, max}} = (6.25\ \text{W/m}^2 \cdot °\text{C})(1.2 \times 1.8\ \text{m}^2)[20 - (-8)]°\text{C} = \mathbf{378\ W}$

Minimum heat loss: $\dot{Q}_{\text{window, min}} = (1.25\ \text{W/m}^2 \cdot °\text{C})(1.2 \times 1.8\ \text{m}^2)[20 - (-8)]°\text{C} = \mathbf{76\ W}$

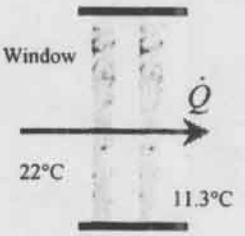
Discussion Note that the rate of heat loss through windows of identical size may differ by a factor of 5, depending on how the windows are constructed.

14-64 The windows of a house in Atlanta are of double door type with wood frames and metal spacers. The average rate of heat loss through the windows in winter is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses associated with the infiltration of air through the cracks/openings are not considered.

Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window, ave}} = U_{\text{overall}} A_{\text{window}} (T_{in} - T_{out,ave})$$



where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Substituting,

$$\dot{Q}_{\text{window, ave}} = (2.50\ \text{W/m}^2 \cdot °\text{C})(20\ \text{m}^2)(22 - 11.3)°\text{C} = \mathbf{535\ W}$$

Discussion This is the "average" rate of heat transfer through the window in winter in the absence of any infiltration.

14-65 Boiling experiments are conducted by heating water at 1 atm pressure with an electric resistance wire, and measuring the power consumed by the wire as well as temperatures. The boiling heat transfer coefficient is to be determined.

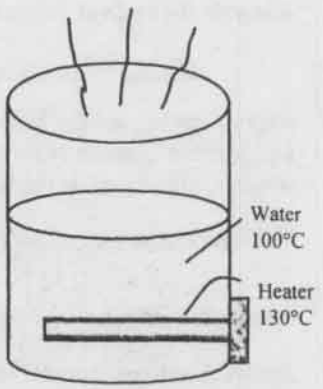
Assumptions **1** Steady operating conditions exist. **2** Heat losses from the water container are negligible.

Analysis The heat transfer area of the heater wire is

$$A = \pi DL = \pi(0.002\ \text{m})(0.50\ \text{m}) = 0.003142\ \text{m}^2$$

Noting that 4100 W of electric power is consumed when the heater surface temperature is 130°C, the boiling heat transfer coefficient is determined from Newton's law of cooling $\dot{Q} = hA(T_s - T_{sat})$ to be

$$h = \frac{\dot{Q}}{A(T_s - T_{sat})} = \frac{4100\ \text{W}}{(0.003142\,\text{m}^2)(130-100)°\text{C}} = \mathbf{43{,}500\ W/m^2 \cdot °C}$$



Chapter 15
STEADY HEAT CONDUCTION

Steady Heat Conduction In Plane Walls

15-1C (a) If the lateral surfaces of the rod are insulated, the heat transfer surface area of the cylindrical rod is the bottom or the top surface area of the rod, $A = \pi D^2 / 4$. (b) If the top and the bottom surfaces of the rod are insulated, the heat transfer area of the rod is the lateral surface area of the rod, $A = \pi DL$.

15-2C The heat transfer surface area of this glass window is $(1.5\text{ m}) \times (2\text{ m}) = 3\text{ m}^2$.

15-3C In steady heat conduction, the rate of heat transfer into the wall is equal to the rate of heat transfer out of it. Also, the temperature at any point in the wall remains constant. Therefore, the energy content of the wall does not change during steady heat conduction. However, the temperature along the wall and thus the energy content of the wall will change during transient conduction.

15-4C The temperature distribution in a plane wall will be a straight line during steady and one dimensional heat transfer with constant wall thermal conductivity.

15-5C The thermal resistance of a medium represents the resistance of that medium against heat transfer.

15-6C The combined heat transfer coefficient represents the combined effects of radiation and convection heat transfers on a surface, and is defined as $h_{\text{combined}} = h_{\text{convection}} + h_{\text{radiation}}$. It offers the convenience of incorporating the effects of radiation in the convection heat transfer coefficient, and to ignore radiation in heat transfer calculations.

15-7C Yes. The convection resistance can be defined as the inverse of the convection heat transfer coefficient per unit surface area since it is defined as $R_{conv} = 1/(hA)$.

15-8C The convection and the radiation resistances at a surface are parallel since both the convection and radiation heat transfers occur simultaneously.

15-9C For a surface of A at which the convection and radiation heat transfer coefficients are h_{conv} and h_{rad}, the single equivalent heat transfer coefficient is $h_{eqv} = h_{conv} + h_{rad}$ when the medium and the surrounding surfaces are at the same temperature. Then the equivalent thermal resistance will be $R_{eqv} = 1/(h_{eqv} A)$.

15-10C The thermal resistance network associated with a five-layer composite wall involves five single-layer resistances connected in series.

15-11C Once the rate of heat transfer $\dot{Q}$ is known, the temperature drop across any layer can be determined by multiplying heat transfer rate by the thermal resistance across that layer, $\Delta T_{layer} = \dot{Q} R_{layer}$

15-12C The temperature of each surface in this case can be determined from

$$\dot{Q} = (T_{\infty 1} - T_{s1}) / R_{\infty 1-s1} \longrightarrow T_{s1} = T_{\infty 1} - (\dot{Q} R_{\infty 1-s1})$$
$$\dot{Q} = (T_{s2} - T_{\infty 2}) / R_{s2-\infty 2} \longrightarrow T_{s2} = T_{\infty 2} + (\dot{Q} R_{s2-\infty 2})$$

where $R_{\infty-i}$ is the thermal resistance between the environment ∞ and surface i.

15-13C Yes, it is.

15-14C The window glass which consists of two 4 mm thick glass sheets pressed tightly against each other will probably have thermal contact resistance which serves as an additional thermal resistance to heat transfer through window, and thus the heat transfer rate will be smaller relative to the one which consists of a single 8 mm thick glass sheet.

15-15C Convection heat transfer through the wall is expressed as $\dot{Q} = hA(T_s - T_\infty)$. In steady heat transfer, heat transfer rate to the wall and from the wall are equal. Therefore at the outer surface which has convection heat transfer coefficient three times that of the inner surface will experience three times smaller temperature drop compared to the inner surface. Therefore, at the outer surface, the temperature will be closer to the surrounding air temperature.

15-16C The new design introduces the thermal resistance of the copper layer in addition to the thermal resistance of the aluminum which has the same value for both designs. Therefore, the new design will be a poorer conductor of heat.

15-17C The blanket will introduce additional resistance to heat transfer and slow down the heat gain of the drink wrapped in a blanket. Therefore, the drink left on a table will warm up faster.

15-18 The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

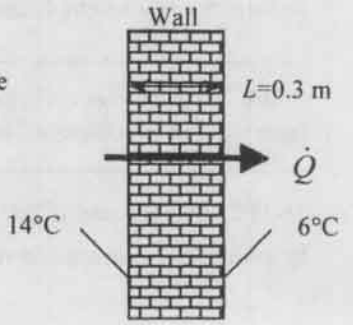
Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.8$ W/m·°C.

Analysis The surface area of the wall and the rate of heat loss through the wall are

$$A = (4 \text{ m}) \times (6 \text{ m}) = 24 \text{ m}^2$$

$$\dot{Q} = kA \frac{T_1 - T_2}{L} = (0.8 \text{ W/m.°C})(24 \text{ m}^2) \frac{(14 - 6)°\text{C}}{0.3 \text{ m}} = \mathbf{512 \text{ W}}$$



15-19 The two surfaces of a window are maintained at specified temperatures. The rate of heat loss through the window and the inner surface temperature are to be determined.

Assumptions **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass is given to be $k = 0.78$ W/m·°C.

Analysis The surface area of the window and the individual resistances are

$$A = (1.2\text{ m}) \times (2\text{ m}) = 2.4\text{ m}^2$$

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2.^\circ\text{C})(2.4\text{ m}^2)} = 0.04167\ ^\circ\text{C/W}$$

$$R_{glass} = \frac{L}{k_1 A} = \frac{0.006\text{ m}}{(0.78\text{ W/m.}^\circ\text{C})(2.4\text{ m}^2)} = 0.00321\ ^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2.^\circ\text{C})(2.4\text{ m}^2)} = 0.01667\ ^\circ\text{C/W}$$

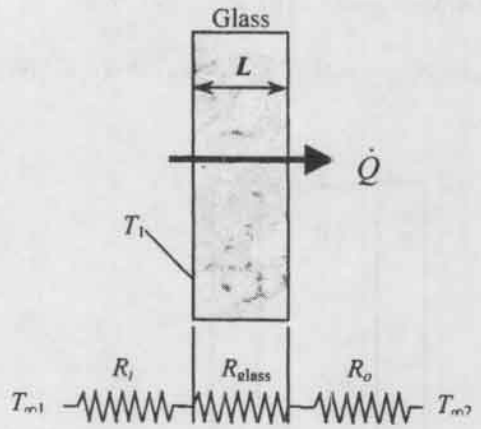
$$R_{total} = R_{conv,1} + R_{glass} + R_{conv,2} = 0.04167 + 0.00321 + 0.01667 = 0.06155\ ^\circ\text{C/W}$$

The steady rate of heat transfer through window glass is then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[24 - (-5)]^\circ\text{C}}{0.06155\ ^\circ\text{C/W}} = \mathbf{471\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 24^\circ\text{C} - (471\text{ W})(0.04167\ ^\circ\text{C/W}) = \mathbf{4.4^\circ C}$$



15-20 A double-pane window consists of two 15-mm thick layers of glass separated by a 12-mm wide stagnant air space. For specified indoors and outdoors temperatures, the rate of heat loss through the window and the inner surface temperature of the window are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass and air are given to be $k_{glass} = 0.78$ W/m·°C and $k_{air} = 0.026$ W/m·°C.

Analysis The surface area of the window and the individual resistances are

$$A = (1.2\text{ m}) \times (2\text{ m}) = 2.4\text{ m}^2$$

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2.^\circ\text{C})(2.4\text{ m}^2)} = 0.0417\ ^\circ\text{C/W}$$

$$R_1 = R_3 = R_{glass} = \frac{L_1}{k_1 A} = \frac{0.003\text{ m}}{(0.78\text{ W/m.}^\circ\text{C})(2.4\text{ m}^2)} = 0.0016\ ^\circ\text{C/W}$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.012\text{ m}}{(0.026\text{ W/m.}^\circ\text{C})(2.4\text{ m}^2)} = 0.1923\ ^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2.^\circ\text{C})(2.4\text{ m}^2)} = 0.0167\ ^\circ\text{C/W}$$

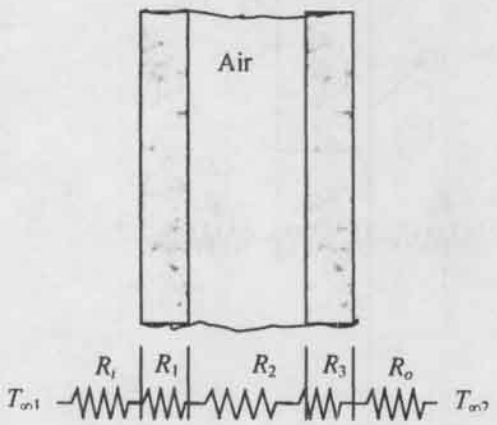
$$R_{total} = R_{conv,1} + 2R_1 + R_2 + R_{conv,2} = 0.0417 + 2(0.0016) + 0.1923 + 0.0167$$
$$= 0.2571\ ^\circ\text{C/W}$$

The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[24 - (-5)]^\circ\text{C}}{0.2571^\circ\text{C/W}} = \mathbf{113\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 24^\circ\text{C} - (113\text{ W})(0.0417^\circ\text{C/W}) = \mathbf{19.3^\circ C}$$



15-21 A double-pane window consists of two 15-mm thick layers of glass separated by an evacuated space. For specified indoors and outdoors temperatures, the rate of heat loss through the window and the inner surface temperature of the window are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass is given to be $k_{glass} = 0.78$ W/m·°C.

Analysis Heat cannot be conducted through an evacuated space since the thermal conductivity of vacuum is zero (no medium to conduct heat) and thus its thermal resistance is zero. Therefore, if radiation is disregarded, the heat transfer through the window will be zero. Then the answer of this problem is **zero** since the problem states to disregard radiation.

Discussion In reality, heat will be transferred between the glasses by radiation. We do not know the inner surface temperatures of windows. In order to determine radiation heat resistance we assume them to be 5°C and 15°C, respectively, and take the emissivity to be 1. Then individual resistances are determined to be

$$A = (1.2\text{ m}) \times (2\text{ m}) = 2.4\text{ m}^2$$

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2.^\circ\text{C})(2.4\text{ m}^2)} = 0.0417\,^\circ\text{C/W}$$

$$R_1 = R_3 = R_{glass} = \frac{L_1}{k_1 A} = \frac{0.003\text{ m}}{(0.78\text{ W/m.}^\circ\text{C})(2.4\text{ m}^2)} = 0.0016\,^\circ\text{C/W}$$

$$R_{rad} = \frac{1}{\varepsilon\sigma A(T_s^2 + T_{surr}^2)(T_s + T_{surr})}$$

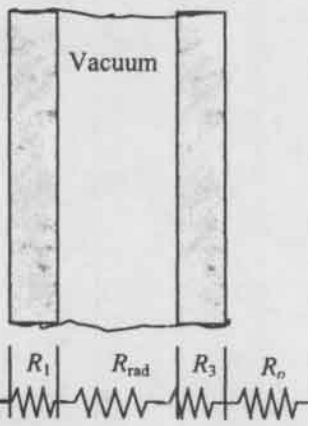
$$= \frac{1}{1(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(2.4\text{ m}^2)[288^2 + 278^2][288+278]K^3}$$

$$= 0.0810\,^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2.^\circ\text{C})(2.4\text{ m}^2)} = 0.0167\,^\circ\text{C/W}$$

$$R_{total} = R_{conv,1} + 2R_1 + R_{rad} + R_{conv,2} = 0.0417 + 2(0.0016) + 0.0810 + 0.0167$$

$$= 0.1426\,^\circ\text{C/W}$$



The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[24-(-5)]^\circ\text{C}}{0.1426^\circ\text{C/W}} = \mathbf{203\text{ W}}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 24^\circ\text{C} - (203\text{ W})(0.0417^\circ\text{C/W}) = \mathbf{15.5^\circ C}$$

Similarly, the inner surface temperatures of the glasses are calculated to be 15.2 and -1.5°C (we had assumed them to be 15 and 5°C when determining the radiation resistance). We can improve the result obtained by reevaluating the radiation resistance and repeating the calculations.

15-22E The inner and outer surfaces of the walls of an electrically heated house remain at specified temperatures during a winter day. The amount of heat lost from the house that day and its its cost are to be determined.

Assumptions **1** Heat transfer through the walls is steady since the surface temperatures of the walls remain constant at the specified values during the time period considered. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity of the walls is constant.

Properties The thermal conductivity of the brick wall is given to be $k = 0.40$ Btu/h·ft·°F.

Analysis We consider heat loss through the walls only. The total heat transfer area is

$$A = 2(40 \times 9 + 30 \times 9) = 1260 \text{ ft}^2$$

The rate of heat loss during the daytime is

$$\dot{Q}_{\text{day}} = kA\frac{T_1 - T_2}{L} = (0.40 \text{ Btu/h.ft.°F})(1260 \text{ ft}^2)\frac{(55-45)°\text{F}}{1 \text{ ft}} = 5040 \text{ Btu/h}$$

The rate of heat loss during nighttime is

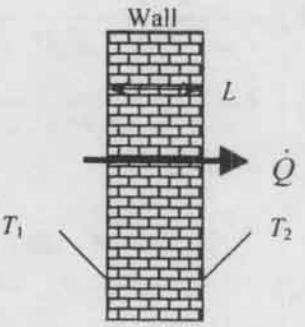
$$\dot{Q}_{\text{night}} = kA\frac{T_1 - T_2}{L} = (0.40 \text{ Btu/h.ft.°F})(1260 \text{ ft}^2)\frac{(55-35)°\text{C}}{1 \text{ ft}} = 10{,}080 \text{ Btu/h}$$

The amount of heat loss from the house that night will be

$$\dot{Q} = \frac{Q}{\Delta t} \longrightarrow Q = \dot{Q}\Delta t = 10\dot{Q}_{day} + 14\dot{Q}_{night} = (10 \text{ h})(5040 \text{ Btu/h}) + (14 \text{ h})(10{,}080 \text{ Btu/h})$$
$$= \mathbf{191{,}520 \text{ Btu}}$$

Then the cost of this heat loss for that day becomes

$$Cost = (191{,}520/3412 \text{ kWh})(\$0.075/\text{kWh}) = \mathbf{\$4.20}$$



15-23 A cylindrical resistor on a circuit board dissipates 0.15 W of power steadily in a specified environment. The amount of heat dissipated in 24 h, the surface heat flux, and the surface temperature of the resistor are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the resistor.

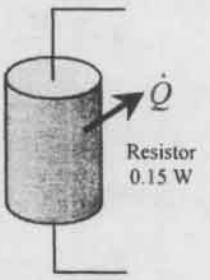
Analysis (a) The amount of heat this resistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.15\ \text{W})(24\ \text{h}) = \mathbf{3.6\ Wh}$$

(b) The heat flux on the surface of the resistor is

$$A = 2\frac{\pi D^2}{4} + \pi DL = 2\frac{\pi(0.003\ \text{m})^2}{4} + \pi(0.003\ \text{m})(0.012\ \text{m}) = 0.000127\ \text{m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{0.15\ \text{W}}{0.000127\ \text{m}^2} = \mathbf{1179\ W/m^2}$$



(c) The surface temperature of the resistor can be determined from

$$R_{conv,rad} = \frac{1}{hA} = \frac{1}{(9\ \text{W/m}^2\cdot\text{°C})(0.000127\ \text{m}^2)} = 875\ \text{°C/W}$$

$$\dot{Q} = \frac{T_s - T_\infty}{R_{conv,rad}} \longrightarrow T_s = T_\infty + \dot{Q}R_{conv,rad} = 40\text{°C} + (0.15\ \text{W})(875\ \text{°C/W}) = \mathbf{171°C}$$

15-24 A power transistor dissipates 0.2 W of power steadily in a specified environment. The amount of heat dissipated in 24 h, the surface heat flux, and the surface temperature of the resistor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the transistor.

Analysis (a) The amount of heat this transistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.2\ \text{W})(24\ \text{h}) = 4.8\ \text{Wh} = \mathbf{0.0048\ kWh}$$

(b) The heat flux on the surface of the transistor is

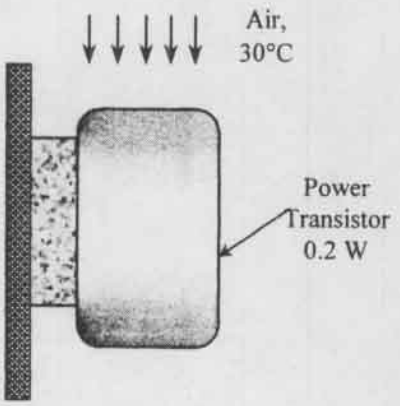
$$A = 2\frac{\pi D^2}{4} + \pi DL = 2\frac{\pi(0.005\ \text{m})^2}{4} + \pi(0.005\ \text{m})(0.004\ \text{m}) = 0.0001021\ \text{m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{0.2\ \text{W}}{0.0001021\ \text{m}^2} = \mathbf{1959\ W/m^2}$$

(c) The surface temperature of the transistor can be determined from

$$R_{conv,rad} = \frac{1}{hA} = \frac{1}{(12\ \text{W/m}^2.^\circ\text{C})(0.0001021\ \text{m}^2)} = 816\ ^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_s - T_\infty}{R_{conv,rad}} \longrightarrow T_s = T_\infty + \dot{Q}R_{conv,rad} = 30^\circ\text{C} + (0.2\ \text{W})(816^\circ\text{C/W}) = \mathbf{193^\circ C}$$



15-25 A circuit board houses 100 chips, each dissipating 0.07 W. The surface heat flux, the surface temperature of the chips, and the thermal resistance between the surface of the board and the cooling medium are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer from the back surface of the board is negligible. **2** Heat is transferred uniformly from the entire front surface.

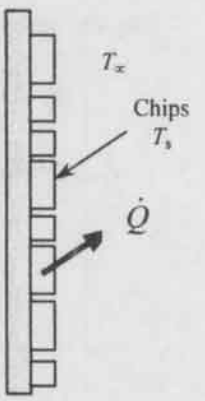
Analysis (a) The heat flux on the surface of the circuit board is

$$A = (0.12\ \text{m})(0.18\ \text{m}) = 0.0216\ \text{m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{(100 \times 0.07)\ \text{W}}{0.0216\ \text{m}^2} = \mathbf{324\ W/m^2}$$

(b) The surface temperature of the chips is

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 40°\text{C} + \frac{7\ \text{W}}{(10\ \text{W/m}^2.°\text{C})(0.0216\ \text{m}^2)} = \mathbf{72.4°C}$$



(c) The thermal resistance is

$$R_{conv} = \frac{1}{hA} = \frac{1}{(10\ \text{W/m}^2.°\text{C})(0.0216\ \text{m}^2)} = \mathbf{4.63°C/W}$$

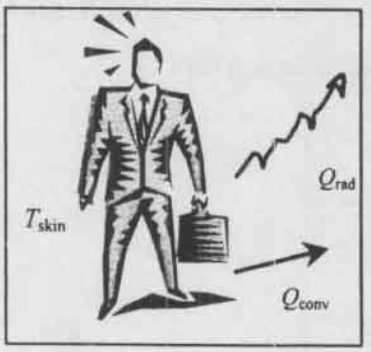
15-26 A person is dissipating heat at a rate of 150 W by natural convection and radiation to the surrounding air and surfaces. For a given deep body temperature, the outer skin temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire exposed surface of the person. **3** The surrounding surfaces are at the same temperature as the indoor air temperature. **4** Heat generation within the 0.5-cm thick outer layer of the tissue is negligible.

Properties The thermal conductivity of the tissue near the skin is given to be $k = 0.3$ W/m·°C.

Analysis The skin temperature can be determined directly from

$$\dot{Q} = kA\frac{T_1 - T_{skin}}{L} \longrightarrow T_{skin} = T_1 - \frac{\dot{Q}L}{kA} = 37°\text{C} - \frac{(150\ \text{W})(0.005\ \text{m})}{(0.3\ \text{W/m.°C})(1.7\ \text{m}^2)} = \mathbf{35.5°C}$$



15-27 Heat is transferred steadily to the boiling water in an aluminum pan. The inner surface temperature of the bottom of the pan is given. The boiling heat transfer coefficient and the outer surface temperature of the bottom of the pan are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since the thickness of the bottom of the pan is small relative to its diameter. **3** The thermal conductivity of the pan is constant.

Properties The thermal conductivity of the aluminum pan is given to be $k = 237$ W/m·°C.

Analysis (a) The boiling heat transfer coefficient is

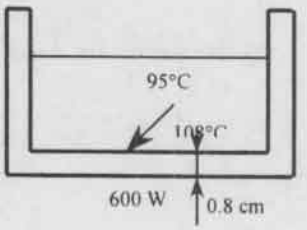
$$A = \frac{\pi D^2}{4} = \frac{\pi(0.25\text{ m})^2}{4} = 0.049\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A(T_s - T_\infty)} = \frac{600\text{ W}}{(0.049\text{ m}^2)(108-95)^\circ\text{C}} = \mathbf{942\ W/m^2.^\circ C}$$

(b) The outer surface temperature of the bottom of the pan is

$$\dot{Q} = kA\frac{T_{s,outer} - T_{s,inner}}{L}$$

$$T_{s,outer} = T_{s,inner1} + \frac{\dot{Q}L}{kA} = 108^\circ\text{C} + \frac{(600\text{ W})(0.005\text{ m})}{(237\text{ W/m.°C})(0.049\text{ m}^2)} = \mathbf{108^\circ C}$$



15-28E A wall is constructed of two layers of sheetrock with fiberglass insulation in between. The thermal resistance of the wall and its R-value of insulation are to be determined. √

Assumptions **1** Heat transfer through the wall is one-dimensional. **2** Thermal conductivities are constant.

Properties The thermal conductivities are given to be $k_{sheetrock} = 0.10$ Btu/h·ft·°F and $k_{insulation} = 0.020$ Btu/h·ft·°F.

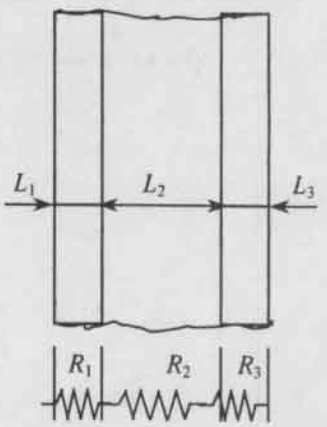
Analysis The surface area of the wall is not given and thus we consider a unit surface area ($A = 1\text{ ft}^2$). Then the *R*-value of insulation of the wall becomes equivalent to its thermal resistance, which is determined from.

$$R_{sheetrock} = R_1 = R_3 = \frac{L_1}{k_1} = \frac{0.5/12\text{ ft}}{(0.10\text{ Btu/h.ft.°F})} = 0.417\text{ ft}^2.^\circ\text{F.h/Btu}$$

$$R_{fiberglass} = R_2 = \frac{L_2}{k_2} = \frac{5/12\text{ ft}}{(0.020\text{ Btu/h.ft.°F})} = 20.83\text{ ft}^2.^\circ\text{F.h/Btu}$$

$$R_{total} = 2R_1 + R_2 = 2\times 0.417 + 20.83 = \mathbf{21.66\ ft^2.^\circ F.h/Btu}$$

Therefore, this is approximately a *R*-22 wall in English units.



15-29 The roof of a house with a gas furnace consists of 15-cm thick concrete that is losing heat to the outdoors by radiation and convection. The rate of heat transfer through the roof and the money lost through the roof that night during a 14 hour period are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The emissivity and thermal conductivity of the roof are constant.

Properties The thermal conductivity of the concrete is given to be $k = 2$ W/m·°C. The emissivity of both surfaces of the roof is given to be 0.9.

Analysis When the surrounding surface temperature is different than the ambient temperature, the thermal resistances network approach becomes cumbersome in problems that involve radiation. Therefore, we will use a different but intuitive approach.

In steady operation, heat transfer from the room to the roof (by convection and radiation) must be equal to the heat transfer from the roof to the surroundings (by convection and radiation), that must be equal to the heat transfer through the roof by conduction. That is,

$$\dot{Q} = \dot{Q}_{\text{room to roof, conv+rad}} = \dot{Q}_{\text{roof, cond}} = \dot{Q}_{\text{roof to surroundings, conv+rad}}$$

Taking the inner and outer surface temperatures of the roof to be $T_{s,in}$ and $T_{s,out}$, respectively, the quantities above can be expressed as

$$\dot{Q}_{\text{room to roof, conv+rad}} = h_i A(T_{room} - T_{s,in}) + \varepsilon A\sigma(T_{room}{}^4 - T_{s,in}{}^4) = (5\ \text{W/m}^2.^\circ\text{C})(300\ \text{m}^2)(20 - T_{s,in})^\circ\text{C} + (0.9)(300\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)\left[(20+273\ \text{K})^4 - (T_{s,in}+273\ \text{K})^4\right]$$

$$\dot{Q}_{\text{roof, cond}} = kA\frac{T_{s,in} - T_{s,out}}{L} = (2\ \text{W/m.°C})(300\ \text{m}^2)\frac{T_{s,in} - T_{s,out}}{0.15\ \text{m}}$$

$$\dot{Q}_{\text{roof to surr, conv+rad}} = h_o A(T_{s,out} - T_{surr}) + \varepsilon A\sigma(T_{s,out}{}^4 - T_{surr}{}^4) = (12\ \text{W/m}^2.^\circ\text{C})(300\ \text{m}^2)(T_{s,out} - 10)^\circ\text{C} + (0.9)(300\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)\left[(T_{s,out}+273\ \text{K})^4 - (100\ \text{K})^4\right]$$

Solving the equations above simultaneously gives

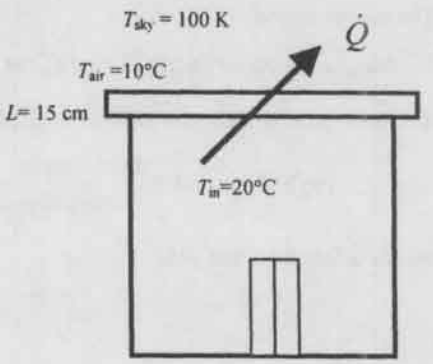
$$\dot{Q} = \mathbf{37{,}440\ W}, T_{s,in} = \mathbf{7.3°C}, \text{and } T_{s,\text{out}} = -2.1°\text{C}$$

The total amount of natural gas consumption during a 14-hour period is

$$Q_{gas} = \frac{Q_{total}}{0.80} = \frac{\dot{Q}\Delta t}{0.80} = \frac{(37.440\ \text{kJ/s})(14\times3600\ \text{s})}{0.80}\left(\frac{1\ \text{therm}}{105{,}500\ \text{kJ}}\right) = 22.36\ \text{therms}$$

Finally, the money lost through the roof during that period is

Money lost = (22.36 therms)($0.60 / therm) = **$13.4**



15-30 An exposed hot surface of an industrial natural gas furnace is to be insulated to reduce the heat loss through that section of the wall by 90 percent. The thickness of the insulation that needs to be used is to be determined. Also, the length of time it will take for the insulation to pay for itself from the energy it saves will be determined.

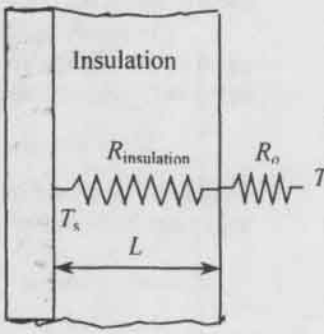
Assumptions **1** Heat transfer through the wall is steady and one-dimensional. **2** Thermal conductivities are constant. **3** The furnace operates continuously. **4** The given heat transfer coefficients accounts for the radiation effects.

Properties The thermal conductivity of the glass wool insulation is given to be $k = 0.038$ W/m·°C.

Analysis The rate of heat transfer without insulation is

$$A = (2\text{ m})(1.5\text{ m}) = 3\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) = (10\text{ W/m}^2.^\circ\text{C})(3\text{ m}^2)(80-30)^\circ\text{C} = 1500\text{ W}$$



In order to reduce heat loss by 90%, the new heat transfer rate and thermal resistance must be

$$\dot{Q} = 0.10 \times 1500\text{ W} = 150\text{ W}$$

$$\dot{Q} = \frac{\Delta T}{R_{total}} \longrightarrow R_{total} = \frac{\Delta T}{\dot{Q}} = \frac{(80-30)^\circ\text{C}}{150\text{ W}} = 0.333\ ^\circ\text{C/W}$$

and in order to have this thermal resistance, the thickness of insulation must be

$$R_{total} = R_{conv} + R_{insulation} = \frac{1}{hA} + \frac{L}{kA}$$

$$= \frac{1}{(10\text{ W/m}^2.^\circ\text{C})(3\text{ m}^2)} + \frac{L}{(0.038\text{ W/m.}^\circ\text{C})(3\text{ m}^2)} = 0.333\ ^\circ\text{C/W}$$

$$\longrightarrow L = 0.034\text{ m} = \mathbf{3.4\text{ cm}}$$

Noting that heat is saved at a rate of 0.9×1500 = 1350 W and the furnace operates continuously and thus 365×24 = 8760 h per year, and that the furnace efficiency is 78%, the amount of natural gas saved per year is

$$\text{Energy Saved} = \frac{\dot{Q}_{saved}\Delta t}{Efficiency} = \frac{(1.350\text{ kJ/s})(8760\text{ h})}{0.78}\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right) = 517.4\text{ therms}$$

The money saved is

$$\text{Money saved} = (\text{Energy Saved})(\text{Cost of energy}) = (517.4\text{ therms})(\$0.55/\text{therm}) = \$284.5\text{ (per year)}$$

The insulation will pay for its cost of $250 in

$$\text{Payback period} = \frac{\text{Money spent}}{\text{Money saved}} = \frac{\$250}{\$284.5/\text{yr}} = \mathbf{0.88\text{ yr}}$$

which is less than one year.

15-31 An exposed hot surface of an industrial natural gas furnace is to be insulated to reduce the heat loss through that section of the wall by 90 percent. The thickness of the insulation that needs to be used is to be determined. Also, the length of time it will take for the insulation to pay for itself from the energy it saves will be determined.

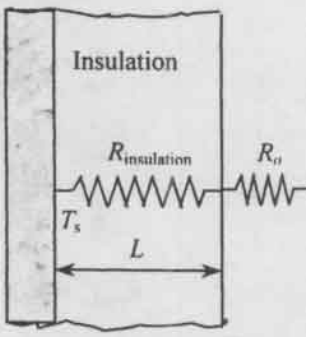
Assumptions **1** Heat transfer through the wall is steady and one-dimensional. **2** Thermal conductivities are constant. **3** The furnace operates continuously. **4** The given heat transfer coefficients accounts for the radiation effects.

Properties The thermal conductivity of the expanded perlite insulation is given to be $k = 0.052$ W/m·°C.

Analysis The rate of heat transfer without insulation is

$$A = (2\text{ m})(1.5\text{ m}) = 3\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) = (10\text{ W/m}^2.^\circ\text{C})(3\text{ m}^2)(80-30)^\circ\text{C} = 1500\text{ W}$$



In order to reduce heat loss by 90%, the new heat transfer rate and thermal resistance must be

$$\dot{Q} = 0.10 \times 1500\text{ W} = 150\text{ W}$$

$$\dot{Q} = \frac{\Delta T}{R_{total}} \longrightarrow R_{total} = \frac{\Delta T}{\dot{Q}} = \frac{(80-30)^\circ\text{C}}{150\text{ W}} = 0.333\ ^\circ\text{C/W}$$

and in order to have this thermal resistance, the thickness of insulation must be

$$R_{total} = R_{conv} + R_{insulation} = \frac{1}{hA} + \frac{L}{kA}$$

$$= \frac{1}{(10\text{ W/m}^2.^\circ\text{C})(3\text{ m}^2)} + \frac{L}{(0.052\text{ W/m.}^\circ\text{C})(3\text{ m}^2)} = 0.333\,^\circ\text{C/W}$$

$$\longrightarrow L = 0.047\text{ m} = \mathbf{4.7\text{ cm}}$$

Noting that heat is saved at a rate of 0.9×1500 = 1350 W and the furnace operates continuously and thus 365×24 = 8760 h per year, and that the furnace efficiency is 78%, the amount of natural gas saved per year is

$$\text{Energy Saved} = \frac{\dot{Q}_{saved}\Delta t}{Efficiency} = \frac{(1.350\text{ kJ/s})(8760\text{ h})}{0.78}\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right) = 517.4\text{ therms}$$

The money saved is

$$\text{Money saved} = (\text{Energy Saved})(\text{Cost of energy}) = (517.4\text{ therms})(\$0.55/\text{therm}) = \$284.5\text{ (per year)}$$

The insulation will pay for its cost of $250 in

$$\text{Payback period} = \frac{\text{Money spent}}{\text{Money saved}} = \frac{\$250}{\$284.5/\text{yr}} = \mathbf{0.88\text{ yr}}$$

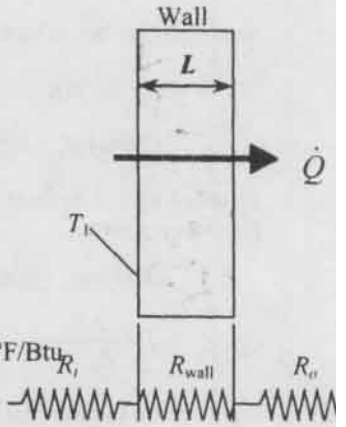
which is less than one year.

15-32E Two of the walls of a house have no windows while the other two walls have 4 windows each. The ratio of heat transfer through the walls with and without windows is to be determined.

Assumptions **1** Heat transfer through the walls and the windows is steady and one-dimensional. **2** Thermal conductivities are constant. **3** Any direct radiation gain or loss through the windows is negligible. **4** Heat transfer coefficients are constant and uniform over the entire surface.

Properties The thermal conductivity of the glass and air are given to be $k_{glass} = 0.78$ W/m·°C and $k_{air} = 0.026$ W/m·°C.

Analysis The thermal resistances through the wall without windows are



$$A = (12\text{ ft})(40\text{ ft}) = 480\text{ m}^2$$

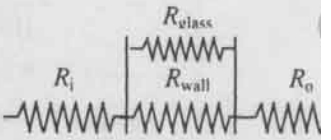
$$R_i = \frac{1}{h_i A} = \frac{1}{(2\text{ Btu/h.ft}^2\text{.°F})(480\text{ ft}^2)} = 0.0010417\text{ h}\cdot\text{°F/Btu}$$

$$R_{wall} = \frac{L}{kA} = \frac{19\text{ hft}^2\text{°F/Btu}}{(480\text{ m}^2)} = 0.03958\text{ h}\cdot\text{°F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(4\text{ Btu/h.ft}^2\text{.°F})(480\text{ ft}^2)} = 0.00052\text{ h}\cdot\text{°F/Btu}$$

$$R_{total,1} = R_i + R_{wall} + R_o = 0.0010417 + 0.03958 + 0.00052 = 0.0411417\text{ h}\cdot\text{°F/Btu}$$

The thermal resistances through the wall with windows are



$$A_{windows} = 4(3\times 5) = 60\text{ ft}^2$$

$$A_{wall} = A_{total} - A_{windows} = 480 - 60 = 420\text{ ft}^2$$

$$R_2 = R_{glass} = \frac{L}{kA} = \frac{0.25/12\text{ ft}}{(0.45\text{ Btu/h.ft.°F})(60\text{ ft}^2)} = 0.0007716\text{ h}\cdot\text{°F/Btu}$$

$$R_4 = R_{wall} = \frac{L}{kA} = \frac{19\text{ h.ft}^2\text{.°F/Btu}}{(420\text{ ft}^2)} = 0.04524\text{ h}\cdot\text{°F/Btu}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{glass}} + \frac{1}{R_{wall}} = \frac{1}{0.0007716} + \frac{1}{0.04524} \longrightarrow R_{eqv} = 0.00076\cdot\text{h°F/Btu}$$

$$R_{total,2} = R_i + R_{eqv} + R_o = 0.001047 + 0.00076 + 0.00052 = 0.002327\text{ h}\cdot\text{°F/Btu}$$

Then the ratio of the heat transfer through the walls with and without windows becomes

$$\frac{\dot{Q}_{total,2}}{\dot{Q}_{total,1}} = \frac{\Delta T / R_{total,2}}{\Delta T / R_{total,1}} = \frac{R_{total,1}}{R_{total,2}} = \frac{0.0411417}{0.002327} = 17.7$$

15-33 Two of the walls of a house have no windows while the other two walls have single- or double-pane windows. The average rate of heat transfer through each wall, and the amount of money this household will save per heating season by converting the single pane windows to double pane windows are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be $k = 0.26$ W/m·°C for air, and 0.76 W/m·°C for glass.

Analysis The rate of heat transfer through each wall can be determined by applying thermal resistance network. The convection resistances at the inner and outer surfaces are common in all cases.

Walls without windows :

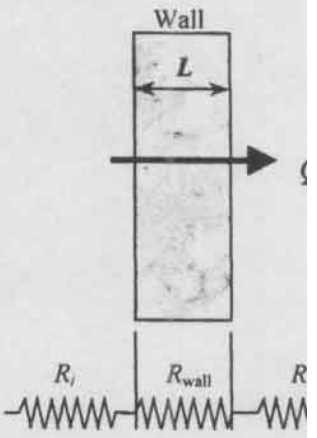
$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \text{W/m}^2.^\circ\text{C})(10\times 4\ \text{m}^2)} = 0.003571\ ^\circ\text{C/W}$$

$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\ \text{m}^2\,^\circ\text{C/W}}{(10\times 4\ \text{m}^2)} = 0.05775\ ^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(15\ \text{W/m}^2.^\circ\text{C})(10\times 4\ \text{m}^2)} = 0.001667\ ^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{wall}} + R_o = 0.00357 + 0.05775 + 0.001667 = 0.062988\ ^\circ\text{C/W}$$

Then $$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(22-5)^\circ\text{C}}{0.062988^\circ\text{C/W}} = \mathbf{270\ W}$$



Wall with single pane windows:

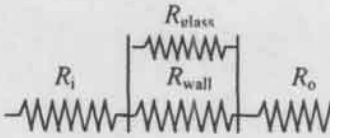
$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\ \text{m}^2\,^\circ\text{C/W}}{(20\times 4)-5(1.2\times 1.8)\ \text{m}^2} = 0.033382\ ^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\ \text{m}}{(0.78\ \text{W/m}^2.^\circ\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968\ ^\circ\text{C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{glass}}} = \frac{1}{0.033382} + 5\frac{1}{0.002968} \rightarrow R_{eqv} = 0.00058\ ^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.000583 + 0.000833 = 0.003202\ ^\circ\text{C/W}$$

Then $$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(22-5)^\circ\text{C}}{0.003202^\circ\text{C/W}} = \mathbf{5309\ W}$$



4th wall with double pane windows:

$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\ \text{m}^2\,^\circ\text{C/W}}{(20\times 4)-5(1.2\times 1.8)\text{m}^2} = 0.033382\ ^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\ \text{m}}{(0.78\ \text{W/m}^2.^\circ\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968\ ^\circ\text{C/W}$$

$$R_{\text{air}} = \frac{L_{\text{air}}}{kA} = \frac{0.015\ \text{m}}{(0.026\ \text{W/m}^2.^\circ\text{C})(1.2\times 1.8)\text{m}^2} = 0.267094\ ^\circ\text{C/W}$$

$$R_{\text{window}} = 2R_{\text{glass}} + R_{\text{air}} = 2\times 0.002968 + 0.267094 = 0.27303\ ^\circ\text{C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{window}}} = \frac{1}{0.033382} + 5\frac{1}{0.27303} \longrightarrow R_{\text{eqv}} = 0.020717\ ^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.020717 + 0.000833 = 0.023336\ ^\circ\text{C/W}$$

Then $$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(22-5)^\circ\text{C}}{0.023336^\circ\text{C/W}} = \mathbf{729\ W}$$

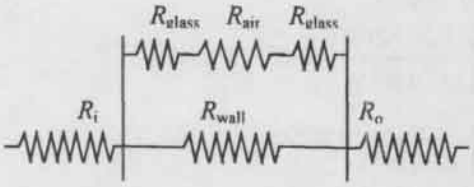
The rate of heat transfer which will be saved if the single pane windows are converted to double pane windows is

$$\dot{Q}_{\text{save}} = \dot{Q}_{\substack{\text{single}\\\text{pane}}} - \dot{Q}_{\substack{\text{double}\\\text{pane}}} = 5309 - 729 = 4580\ \text{W}$$

The amount of energy and money saved during a 7-month long heating season by switching from single pane to double pane windows become

$$Q_{save} = \dot{Q}_{save}\Delta t = (4.580\ \text{kW})(7 \times 30 \times 24\ \text{h}) = 23{,}083\ \text{kWh}$$

Money savings = (Energy saved)(Unit cost of energy) = (23,085 kWh)($0.09/kWh) = **$2078**



15-34 The wall of a refrigerator is constructed of fiberglass insulation sandwiched between two layers of sheet metal. The minimum thickness of insulation that needs to be used in the wall in order to avoid condensation on the outer surfaces is to be determined. √

Assumptions **1** Heat transfer through the refrigerator walls is steady since the temperatures of the food compartment and the kitchen air remain constant at the specified values. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation effects.

Properties The thermal conductivities are given to be k = 1.51 W/m·°C for sheet metal and 0.035 W/m·°C for fiberglass insulation.

Analysis The minimum thickness of insulation can be determined by assuming the outer surface temperature of the refrigerator to be 10°C. In steady operation, the rate of heat transfer through the refrigerator wall is constant, and thus heat transfer between the room and the refrigerated space is equal to the heat transfer between the room and the outer surface of the refrigerator. Considering a unit surface area,

$$\dot{Q} = h_o A(T_{room} - T_{s,out}) = (9\ \text{W/m}^2.^\circ\text{C})(1\ \text{m}^2)(25-20)^\circ\text{C} = 45\ \text{W}$$

Using the thermal resistance network, heat transfer between the room and the refrigerated space can be expressed as

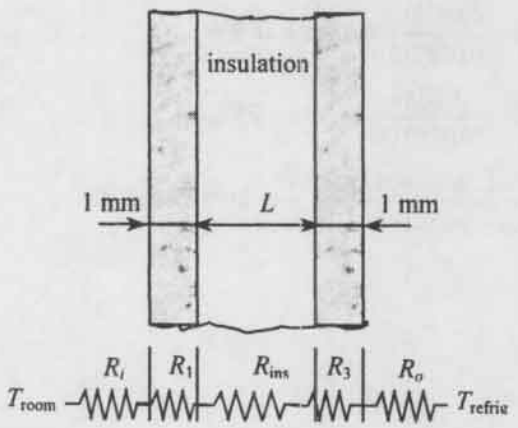
$$\dot{Q} = \frac{T_{room} - T_{refrig}}{R_{total}} \rightarrow \dot{Q}/A = \frac{T_{room} - T_{refrig}}{\frac{1}{h_o} + 2\left(\frac{L}{k}\right)_{metal} + \left(\frac{L}{k}\right)_{insulation} + \frac{1}{h_i}}$$

Substituting,

$$45\ \text{W/m}^2 = \frac{(25-3)^\circ\text{C}}{\frac{1}{9\ \text{W/m}^2.^\circ\text{C}} + \frac{2\times 0.001\ \text{m}}{15.1\ \text{W/m}^2.^\circ\text{C}} + \frac{L}{0.035\ \text{W/m}^2.^\circ\text{C}} + \frac{1}{4\ \text{W/m}^2.^\circ\text{C}}}$$

Solv ing for L, the minimum thickness of insulation is determined to be

L = 0.0045 m = **0.45 cm**



15-35 Heat is to be conducted along a circuit board with a copper layer on one side. The percentages of heat

conduction along the copper and epoxy layers as well as the effective thermal conductivity of the board are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces is disregarded **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 386 W/m·°C for copper and 0.26 W/m·°C for epoxy layers.

Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer board can be expressed as

$$\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} = \left(kA\frac{\Delta T}{L}\right)_{\text{copper}} + \left(kA\frac{\Delta T}{L}\right)_{\text{epoxy}} = \left[(kt)_{\text{copper}} + (kt)_{\text{epoxy}}\right]w\frac{\Delta T}{L}$$

Heat conduction along an "equivalent" board of thick ness $t = t_{\text{copper}} + t_{\text{epoxy}}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA\frac{\Delta T}{L}\right)_{\text{board}} = k_{\text{eff}}(t_{\text{copper}} + t_{\text{epoxy}})w\frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{eff}(t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to kt. Substituting, the fractions of heat conducted along the copper and epoxy layers as well as the effective thermal conductivity of the board are determined to be

$$(kt)_{copper} = (386\ \text{W/m.°C})(0.0001\ \text{m}) = 0.0386\ \text{W/°C}$$

$$(kt)_{epoxy} = (0.26\ \text{W/m.°C})(0.0012\ \text{m}) = 0.000312\ \text{W/°C}$$

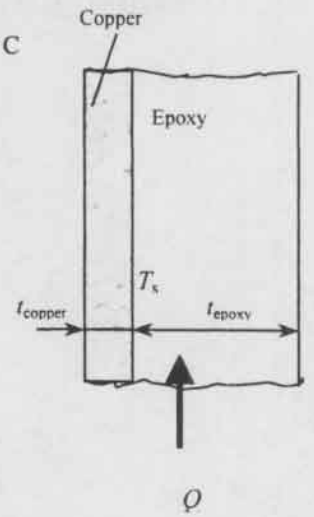
$$(kt)_{total} = (kt)_{copper} + (kt)_{epoxy} = 0.0386 + 0.000312 = 0.038912\ \text{W/°C}$$

$$f_{epoxy} = \frac{(kt)_{epoxy}}{(kt)_{total}} = \frac{0.000312}{0.038912} = 0.008 = \mathbf{0.8\%}$$

$$f_{copper} = \frac{(kt)_{copper}}{(kt)_{total}} = \frac{0.0386}{0.038912} = 0.992 = \mathbf{99.2\%}$$

and

$$k_{eff} = \frac{(386 \times 0.0001 + 0.26 \times 0.0012)\ \text{W/°C}}{(0.0001 + 0.0012)\ \text{m}} = \mathbf{29.9\ W/m.°C}$$



15-36E A thin copper plate is sandwiched between two layers of epoxy boards. The effective thermal conductivity of the board along its 9 in long side and the fraction of the heat conducted through copper along that side are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces are disregarded **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper and 0.15 Btu/h·ft·°F for epoxy layers.

Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer plate can be expressed as (we treat the two layers of epoxy as a single layer that is twice as thick)

$$\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} = \left(kA\frac{\Delta T}{L}\right)_{\text{copper}} + \left(kA\frac{\Delta T}{L}\right)_{\text{epoxy}} = \left[(kt)_{\text{copper}} + (kt)_{\text{epoxy}}\right]w\frac{\Delta T}{L}$$

Heat conduction along an "equivalent" plate of thick ness $t = t_{\text{copper}} + t_{\text{epoxy}}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA\frac{\Delta T}{L}\right)_{\text{board}} = k_{\text{eff}}(t_{\text{copper}} + t_{\text{epoxy}})w\frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{\text{eff}}(t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{\text{eff}} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

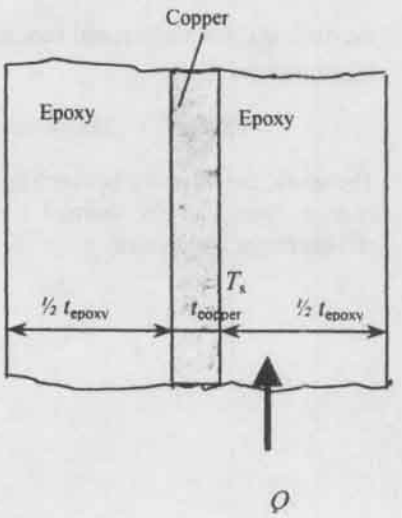
Note that heat conduction is proportional to kt. Substituting, the fraction of heat conducted along the copper layer and the effective thermal conductivity of the plate are determined to be

$$(kt)_{\text{copper}} = (223\ \text{Btu/h.ft.°F})(0.03/12\ \text{ft}) = 0.5575\ \text{Btu/h.°F}$$
$$(kt)_{\text{epoxy}} = 2(0.15\ \text{Btu/h.ft.°F})(0.1/12\ \text{ft}) = 0.0025\ \text{Btu/h.°F}$$
$$(kt)_{\text{total}} = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} = (0.5575 + 0.0025) = 0.56\ \text{Btu/h.°F}$$

and

$$k_{\text{eff}} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}} = \frac{0.56\ \text{Btu/h.°F}}{[(0.03/12) + 2(0.1/12)]\ \text{ft}} = \mathbf{29.2\ Btu/h.ft^2.°F}$$

$$f_{\text{copper}} = \frac{(kt)_{\text{copper}}}{(kt)_{\text{total}}} = \frac{0.5575}{0.56} = 0.996 = \mathbf{99.6\%}$$



Thermal Contact Resistance

15-37C The resistance that an interface offers to heat transfer per unit interface area is called thermal contact resistance, R_c. The inverse of thermal contact resistance is called the thermal contact conductance.

15-38C The thermal contact resistance will be greater for rough surfaces because an interface with rough surfaces will contain more air gaps whose thermal conductivity is low.

15-39C An interface acts like a very thin layer of insulation, and thus the thermal contact resistance has significance only for highly conducting materials like metals. Therefore, the thermal contact resistance can be ignored for two layers of insulation pressed against each other.

15-40C An interface acts like a very thin layer of insulation, and thus the thermal contact resistance is significant for highly conducting materials like metals. Therefore, the thermal contact resistance must be considered for two layers of metals pressed against each other.

15-41C Heat transfer through the voids at an interface is by conduction and radiation. Evacuating the interface eliminates heat transfer by conduction, and thus increases the thermal contact resistance.

15-42C Thermal contact resistance can be minimized by (1) applying a thermally conducting liquid on the surfaces before they are pressed against each other, (2) by replacing the air at the interface by a better conducting gas such as helium or hydrogen, (3) by increasing the interface pressure, and (4) by inserting a soft metallic foil such as tin, silver, copper, nickel, or aluminum between the two surfaces.

15-43 The thickness of copper plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is determined to be

$$R_c = \frac{1}{h_c} = \frac{1}{18{,}000\ \text{W/m}^2.^\circ\text{C}} = 5.556 \times 10^{-5}\ \text{m}^2.^\circ\text{C/W}$$

For a unit surface area, the thermal resistance of a flat plate is defined as $R = \frac{L}{k}$ where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR = kR_c = (386\ \text{W/m.}^\circ\text{C})(5.556 \times 10^{-5}\ \text{m}^2.^\circ\text{C/W}) = 0.0214\ \text{m} = \mathbf{2.14\ cm}$$

Therefore, the interface between the two plates offers as much resistance to heat transfer as a 2.14 cm thick copper. Note that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

15-44 Six identical power transistors are attached on a copper plate. For a maximum case temperature of 85°C, the maximum power dissipation and the temperature jump at the interface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer can be approximated as being one-dimensional, although it is recognized that heat conduction in some parts of the plate will be two-dimensional since the plate area is much larger than the base area of the transistor. But the large thermal conductivity of copper will minimize this effect. **3** All the heat generated at the junction is dissipated through the back surface of the plate since the transistors are covered by a thick plexiglas layer. **4** Thermal conductivities are constant.

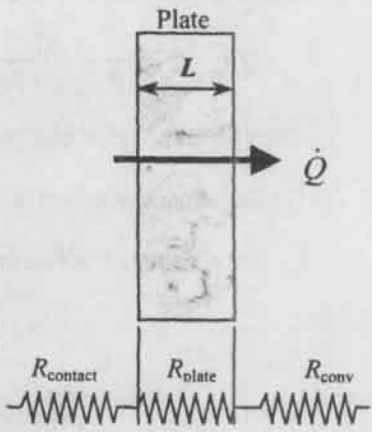
Properties The thermal conductivity of copper is given to be k = 386 W/m·°C. The contact conductance at the interface of copper-aluminum plates for the case of 1.15-1.4 μm roughness and 10 MPa pressure is h_c = 49,000 W/m^2·°C (Table 15-2).

Analysis The contact area between the case and the plate is given to be 9 cm^2, and the plate area for each transistor is 100 cm^2. The thermal resistance network of this problem consists of three resistances in series (contact, plate, and convection) which are determined to be

$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(49{,}000\ \text{W/m}^2\cdot^\circ\text{C})(9\times10^{-4}\ \text{m}^2)} = 0.0227\ ^\circ\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.012\ \text{m}}{(386\ \text{W/m.}^\circ\text{C})(0.01\ \text{m}^2)} = 0.0031\ ^\circ\text{C/W}$$

$$R_{\text{convection}} = \frac{1}{h_o A} = \frac{1}{(30\ \text{W/m}^2\cdot^\circ\text{C})(0.01\ \text{m}^2)} = 3.333\ ^\circ\text{C/W}$$



The total thermal resistance is then

$$R_{\text{total}} = R_{\text{contact}} + R_{\text{plate}} + R_{\text{convection}} = 0.0227 + 0.0031 + 3.333 = 3.359\ ^\circ\text{C/W}$$

Note that the thermal resistance of copper plate is very small and can be ignored all together. Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{(85-15)^\circ\text{C}}{3.359\ ^\circ\text{C/W}} = \mathbf{20.8\ W}$$

Therefore, the power transistor should not be operated at power levels greater than 20.8 W if the case temperature is not to exceed 85°C.

The temperature jump at the interface is determined from

$$\Delta T_{\text{interface}} = \dot{Q} R_{\text{contact}} = (20.8\ \text{W})(0.0227\ ^\circ\text{C/W}) = \mathbf{0.47^\circ C}$$

which is not very large. Therefore, even if we eliminate the thermal contact resistance at the interface completely, we will lower the operating temperature of the transistor in this case by less than 1°C.

15-45 Two cylindrical aluminum bars with ground surfaces are pressed against each other in an insulation sleeve. For specified top and bottom surface temperatures, the rate of heat transfer along the cylinders and the temperature drop at the interface are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional in the axial direction since the lateral surfaces of both cylinders are well-insulated. **3** Thermal conductivities are constant.

Properties The thermal conductivity of aluminum bars is given to be k = 176 W/m·°C. The contact conductance at the interface of aluminum-aluminum plates for the case of ground surfaces and of 20 atm ≈ 2 MPa pressure is h_c = 11,400 W/m²·°C (Table 15-2).

Analysis (a) The thermal resistance network in this case consists of two conduction resistance and the contact resistance, and are determined to be

$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(11{,}400\ \text{W/m}^2.^\circ\text{C})[\pi(0.05\ \text{m})^2/4]} = 0.0447\ ^\circ\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.15\ \text{m}}{(176\ \text{W/m.}^\circ\text{C})[\pi(0.05\ \text{m})^2/4]} = 0.4341\ ^\circ\text{C/W}$$

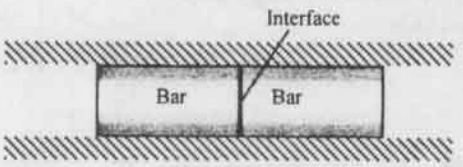
Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{\text{contact}} + 2R_{\text{bar}}} = \frac{(150-20)^\circ\text{C}}{(0.0447+2\times 0.4341)^\circ\text{C/W}} = \mathbf{142.4\ W}$$

Therefore, the rate of heat transfer through the bars is 142.4 W.

(b) The temperature drop at the interface is determined to be

$$\Delta T_{\text{interface}} = \dot{Q} R_{\text{contact}} = (142.4\ \text{W})(0.0447\ ^\circ\text{C/W}) = \mathbf{6.4^\circ C}$$





15-46 A thin copper plate is sandwiched between two epoxy boards. The error involved in the total thermal resistance of the plate if the thermal contact conductances are ignored is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since the plate is large. **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 386 W/m·°C for copper plates and k = 0.26 W/m·°C for epoxy boards. The contact conductance at the interface of copper-epoxy layers is given to be $h_c = 4000$ W/m^2·°C.

Analysis (a) The thermal resistances of different layers for unit surface area of 1 m^2 are

$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(4{,}000\ \text{W/m}^2.^\circ\text{C})(1\ \text{m}^2)} = 0.00025\ ^\circ\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.001\ \text{m}}{(386\ \text{W/m.}^\circ\text{C})(1\ \text{m}^2)} = 2.6\times10^{-6}\ ^\circ\text{C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.005\ \text{m}}{(0.26\ \text{W/m.}^\circ\text{C})(1\ \text{m}^2)} = 0.01923\ ^\circ\text{C/W}$$

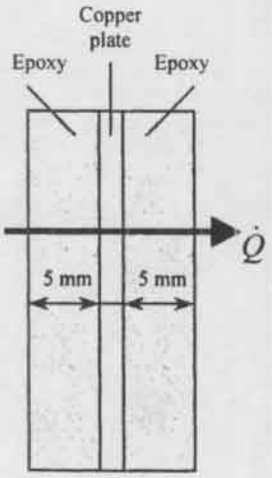
The total thermal resistance is

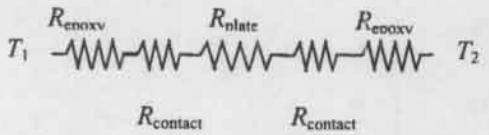
$$R_{\text{total}} = 2R_{\text{contact}} + R_{\text{plate}} + 2R_{\text{epoxy}} = 2\times0.00025 + 2.6\times10^{-6} + 2\times0.01923 = 0.03896\ ^\circ\text{C/W}$$

Then the percent error involved in the total thermal resistance of the plate if the thermal contact resistances are ignored is determined to be

$$\%\text{Error} = \frac{2R_{\text{contact}}}{R_{\text{total}}}\times100 = \frac{2\times0.00025}{0.03896}\times100 = \mathbf{1.3\%}$$

which is negligible.





Generalized Thermal Resistance Networks

15-47C Parallel resistances indicate simultaneous heat transfer (such as convection and radiation on a surface). Series resistances indicate sequential heat transfer (such as two homogeneous layers of a wall).

15-48C The thermal resistance network approach will give adequate results for multi-dimensional heat transfer problems if heat transfer occurs predominantly in one direction.

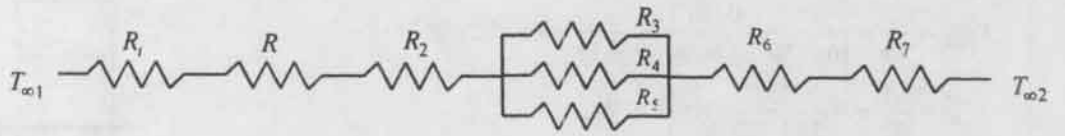
15-49C Two approaches used in development of the thermal resistance network in the x-direction for multi-dimensional problems are (1) to assume any plane wall normal to the x-axis to be isothermal and (2) to assume any plane parallel to the x-axis to be adiabatic.

15-50 A wall consists of horizontal bricks separated by plaster layers. There are also plaster layers on each side of the wall, and a rigid foam on the inner side of the wall. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be $k = 0.72$ W/m·°C for bricks, $k = 0.22$ W/m·°C for plaster layers, and $k = 0.026$ W/m·°C for the rigid foam.

Analysis We consider 1 m deep and 0.33 m high portion of wall which is representative of the entire wall. The thermal resistance network and individual resistances are



$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2.^\circ\text{C})(0.33 \times 1\ \text{m}^2)} = 0.303\ ^\circ\text{C/W}$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.026\ \text{W/m.}^\circ\text{C})(0.33 \times 1\ \text{m}^2)} = 2.33\ ^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\substack{plaster \\ side}} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m.}^\circ\text{C})(0.30 \times 1\ \text{m}^2)} = 0.303\ ^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\substack{plaster \\ center}} = \frac{L}{h_o A} = \frac{0.18\ \text{m}}{(0.22\ \text{W/m.}^\circ\text{C})(0.015 \times 1\ \text{m}^2)} = 54.55\ ^\circ\text{C/W}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.18\ \text{m}}{(0.72\ \text{W/m.}^\circ\text{C})(0.30 \times 1\ \text{m}^2)} = 0.833\ ^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(20\ \text{W/m.}^\circ\text{C})(0.33 \times 1\ \text{m}^2)} = 0.152\ ^\circ\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{54.55} + \frac{1}{0.833} + \frac{1}{54.55} \longrightarrow R_{mid} = 0.81\ ^\circ\text{C/W}$$

$$R_{total} = R_i + R_1 + 2R_2 + R_{mid} + R_o = 0.303 + 2.33 + 2(0.303) + 0.81 + 0.152$$
$$= 4.201\ ^\circ\text{C/W}$$

The steady rate of heat transfer through the wall per $0.33\ \text{m}^2$ is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[(22 - (-4)]^\circ\text{C}}{4.201^\circ\text{C/W}} = 6.19\ \text{W}$$

Then steady rate of heat transfer through the entire wall becomes

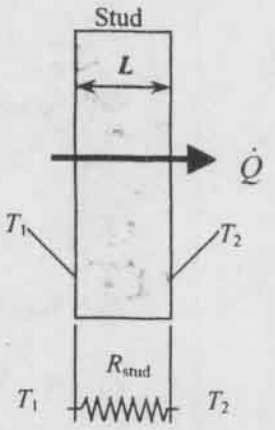
$$\dot{Q}_{total} = (6.19\ \text{W})\frac{(4 \times 6)\text{m}^2}{0.33\ \text{m}^2} = \mathbf{450\ W}$$

15-51 A wall is to be constructed of 10-cm thick wood studs or with pairs of 5-cm thick wood studs nailed to each other. The rate of heat transfer through the solid stud and through a stud pair nailed to each other, as well as the effective conductivity of the nailed stud pair are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the x direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance between the two layers is negligible. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be $k = 0.11$ W/m·°C for wood studs and $k = 50$ W/m·°C for manganese steel nails.

Analysis (a) The heat transfer area of the stud is $A = (0.1\text{ m})(2.5\text{ m}) = 0.25\text{ m}^2$. The thermal resistance and heat transfer rate through the solid stud are



$$R_{stud} = \frac{L}{kA} = \frac{0.1\text{ m}}{(0.11\text{ W/m.°C})(0.25\text{ m}^2)} = 3.64\text{ °C/W}$$

$$\dot{Q} = \frac{\Delta T}{R_{stud}} = \frac{15\text{°C}}{3.64\text{ °C/W}} = \mathbf{4.1\text{ W}}$$

(b) The thermal resistances of stud pair and nails are in parallel

$$A_{nails} = 50\frac{\pi D^2}{4} = 50\left[\frac{\pi(0.004\text{ m})^2}{4}\right] = 0.000628\text{ m}^2$$

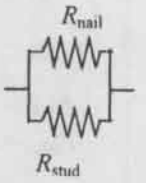
$$R_{nails} = \frac{L}{kA} = \frac{0.1\text{ m}}{(50\text{ W/m.°C})(0.000628\text{ m}^2)} = 3.18\text{ °C/W}$$

$$R_{stud} = \frac{L}{kA} = \frac{0.1\text{ m}}{(0.11\text{ W/m.°C})(0.25 - 0.000628\text{ m}^2)} = 3.65\text{ °C/W}$$

$$\frac{1}{R_{total}} = \frac{1}{R_{stud}} + \frac{1}{R_{nails}} = \frac{1}{3.65} + \frac{1}{3.18} \longrightarrow R_{total} = 1.70\text{ °C/W}$$

$$\dot{Q} = \frac{\Delta T}{R_{stud}} = \frac{15\text{°C}}{1.70\text{ °C/W}} = \mathbf{8.8\text{ W}}$$

(c) The effective conductivity of the nailed stud pair can be determined from



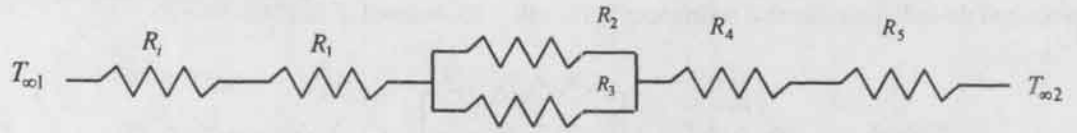
$$\dot{Q} = k_{eff} A\frac{\Delta T}{L} \longrightarrow k_{eff} = \frac{\dot{Q}L}{\Delta TA} = \frac{(8.83\text{ W})(0.1\text{ m})}{(15\text{ W})(0.25\text{ m}^2)} = \mathbf{0.235\text{ W/m.°C}}$$

15-52 A wall is constructed of two layers of sheetrock spaced by 5 cm × 12 cm wood studs. The space between the studs is filled with fiberglass insulation. The thermal resistance of the wall and the rate of heat transfer through the wall are to be determined. √

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be $k = 0.17$ W/m·°C for sheetrock, $k = 0.11$ W/m·°C for wood studs, and $k = 0.035$ W/m·°C for fiberglass insulation.

Analysis (a) The representative surface area is $A = 1 \times 0.65 = 0.65\ \text{m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_i = \frac{1}{h_i A} = \frac{1}{(8.3\ \text{W/m}^2.^\circ\text{C})(0.65\ \text{m}^2)} = 0.185\ ^\circ\text{C/W}$$

$$R_1 = R_4 = R_{sheetrock} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(0.17\ \text{W/m.°C})(0.65\ \text{m}^2)} = 0.09\ ^\circ\text{C/W}$$

$$R_2 = R_{stud} = \frac{L}{kA} = \frac{0.12\ \text{m}}{(0.11\ \text{W/m.°C})(0.05\ \text{m}^2)} = 21.8\ ^\circ\text{C/W}$$

$$R_3 = R_{fiberglass} = \frac{L}{kA} = \frac{0.12\ \text{m}}{(0.035\ \text{W/m.°C})(0.60\ \text{m}^2)} = 5.7\ ^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(34\ \text{W/m}^2.^\circ\text{C})(0.65\ \text{m}^2)} = 0.045\ ^\circ\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{21.8} + \frac{1}{5.7} \longrightarrow R_{mid} = 4.52\ ^\circ\text{C/W}$$

$$R_{total} = R_i + R_1 + R_{mid} + R_4 + R_o = 0.185 + 0.09 + 4.52 + 0.09 + 0.045 = \mathbf{4.93\ °C/W}\ \text{(for a 1 m} \times \text{0.65 m section)}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20 - (-5)]^\circ\text{C}}{4.93\ ^\circ\text{C/W}} = 5.07\ \text{W}$$

Then steady rate of heat transfer through entire wall becomes

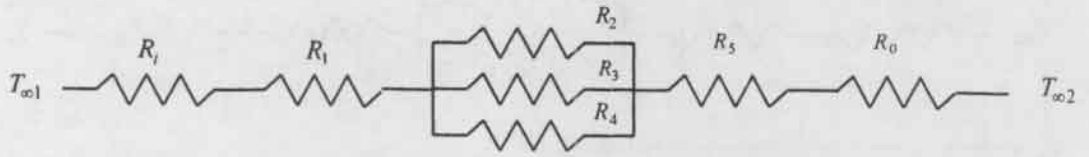
$$\dot{Q}_{total} = (5.07\ \text{W})\frac{(12\ \text{m})(5\ \text{m})}{0.65\ \text{m}^2} = \mathbf{468\ W}$$

15-53E A wall is to be constructed using solid bricks or identical size bricks with 9 square air holes. There is a 0.5 in thick sheetrock layer between two adjacent bricks on all four sides, and on both sides of the wall. The rates of heat transfer through the wall constructed of solid bricks and of bricks with air holes are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be $k = 0.40$ Btu/h·ft·°F for bricks, $k = 0.015$ Btu/h·ft·°F for air, and $k = 0.10$ Btu/h·ft·°F for sheetrock.

Analysis (*a*) The representative surface area is $A = (7.5/12)(7.5/12) = 0.3906\ \text{ft}^2$. The thermal resistance network and the individual thermal resistances if the wall is constructed of solid bricks are



$$R_i = \frac{1}{h_i A} = \frac{1}{(15\ \text{Btu/h.ft}^2\text{.°F})(0.3906\ \text{ft}^2)} = 0.1707\ \text{h°F/Btu}$$

$$R_1 = R_5 = R_{plaster} = \frac{L}{kA} = \frac{0.5/12\ \text{ft}}{(0.10\ \text{Btu/h.ft.°F})(0.3906\ \text{ft}^2)} = 1.0667\ \text{h°F/Btu}$$

$$R_2 = R_{plaster} = \frac{L}{kA} = \frac{9/12\ \text{ft}}{(0.10\ \text{Btu/h.ft.°F})[(7.5/12)\times(0.5/12)]\text{ft}^2} = 288\ \text{h°F/Btu}$$

$$R_3 = R_{plaster} = \frac{L}{kA} = \frac{9/12\ \text{ft}}{(0.10\ \text{Btu/h.ft.°F})[(7/12)\times(0.5/12)]\text{ft}^2} = 308.57\ \text{h°F/Btu}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{9/12\ \text{ft}}{(0.40\ \text{Btu/h.ft.°F})[(7/12)\times(7/12)]\text{ft}^2} = 5.51\ \text{h°F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(4\ \text{Btu/h.ft}^2\text{.°F})(0.3906\ \text{ft}^2)} = 0.64\ \text{h°F/Btu}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{288} + \frac{1}{308.57} + \frac{1}{5.51} \longrightarrow R_{mid} = 5.3135\ \text{h°F/Btu}$$

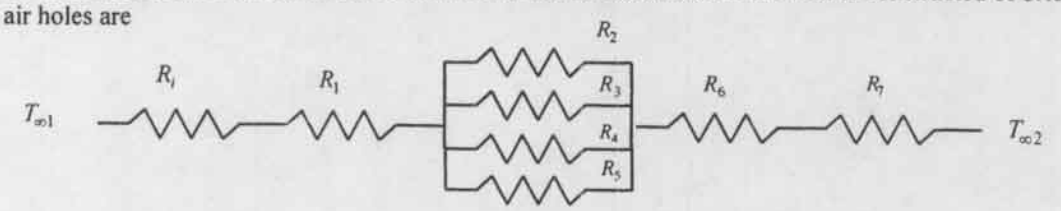
$$R_{total} = R_i + R_1 + R_{mid} + R_5 + R_o = 0.1707 + 1.0667 + 5.3135 + 1.0667 + 0.64 = 8.2576\ \text{h°F/Btu}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(75-35)°\text{F}}{8.2576\ \text{h°F/Btu}} = 4.844\ \text{Btu/h}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (4.844\ \text{W})\frac{(30\ \text{ft})(10\ \text{ft})}{0.3906\ \text{m}^2} = \mathbf{3720\ Btu/h}$$

(b) The thermal resistance network and the individual thermal resistances if the wall is constructed of bricks with air holes are



$$A_{airholes} = 9(1.25/12) \times (1.25/12) = 0.0977 \text{ ft}^2$$

$$A_{bricks} = (7/12 \text{ ft})^2 - 0.0977 = 0.2426 \text{ ft}^2$$

$$R_4 = R_{airholes} = \frac{L}{kA} = \frac{9/12 \text{ ft}}{(0.015 \text{ Btu/h.ft.°F})(0.0977 \text{ ft}^2)} = 511.77 \text{ h°F/Btu}$$

$$R_5 = R_{brick} = \frac{L}{kA} = \frac{9/12 \text{ ft}}{(0.40 \text{ Btu/h.ft.°F})(0.2426 \text{ ft}^2)} = 7.729 \text{ h°F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(4 \text{ Btu/h.ft}^2\text{.°F})(0.3906 \text{ ft}^2)} = 0.64 \text{ h°F/Btu}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{288} + \frac{1}{308.57} + \frac{1}{511.77} + \frac{1}{7.729} \longrightarrow R_{mid} = 7.244 \text{ h°F/Btu}$$

$$R_{total} = R_i + R_1 + R_{mid} + R_6 + R_o = 0.1707 + 1.0667 + 7.244 + 1.0677 + 0.64 = 10.1881 \text{ h°F/Btu}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(75-35)°\text{F}}{10.1881 \text{ h°F/Btu}} = 3.926 \text{ Btu/h}$$

Then steady rate of heat transfer through entire wall becomes

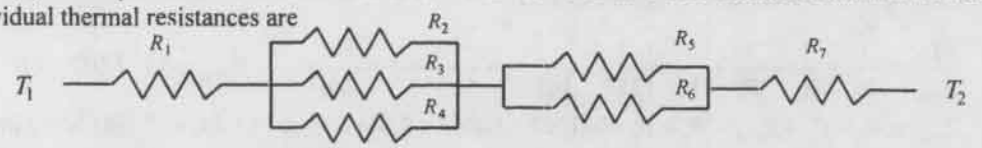
$$\dot{Q}_{total} = (3.926 \text{ Btu/h}) \frac{(30 \text{ ft})(10 \text{ ft})}{0.3906 \text{ ft}^2} = \mathbf{3015 \text{ Btu/h}}$$

15-54 A composite wall consists of several horizontal and vertical layers. The left and right surfaces of the wall are maintained at uniform temperatures. The rate of heat transfer through the wall, the interface temperatures, and the temperature drop across the section F are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Thermal contact resistances at the interfaces are disregarded.

Properties The thermal conductivities of various materials used are given to be $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$ W/m·°C.

Analysis (*a*) The representative surface area is $A = 0.12 \times 1 = 0.12\ \text{m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_1 = R_A = \frac{L}{kA} = \frac{0.01\ \text{m}}{(2\ \text{W/m.°C})(0.12\ \text{m}^2)} = 0.04\ \text{°C/W}$$

$$R_2 = R_C = \frac{L}{kA} = \frac{0.05\ \text{m}}{(20\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.06\ \text{°C/W}$$

$$R_3 = R_B = \frac{L}{kA} = \frac{0.05\ \text{m}}{(8\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.16\ \text{°C/W}$$

$$R_4 = R_D = \frac{L}{kA} = \frac{0.05\ \text{m}}{(15\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.08\ \text{°C/W}$$

$$R_5 = R_D = \frac{L}{kA} = \frac{0.1\ \text{m}}{(15\ \text{W/m.°C})(0.06\ \text{m}^2)} = 0.11\ \text{°C/W}$$

$$R_6 = R_E = \frac{L}{kA} = \frac{0.1\ \text{m}}{(35\ \text{W/m.°C})(0.06\ \text{m}^2)} = 0.05\ \text{°C/W}$$

$$R_7 = R_F = \frac{L}{kA} = \frac{0.06\ \text{m}}{(2\ \text{W/m.°C})(0.12\ \text{m}^2)} = 0.25\ \text{°C/W}$$

$$\frac{1}{R_{mid,1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.06} + \frac{1}{0.16} + \frac{1}{0.08} \longrightarrow R_{mid,1} = 0.028\ \text{°C/W}$$

$$\frac{1}{R_{mid,2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.11} + \frac{1}{0.05} \longrightarrow R_{mid,2} = 0.034\ \text{°C/W}$$

$$R_{total} = R_1 + R_{mid,1} + R_{mid,2} + R_7 = 0.04 + 0.028 + 0.034 + 0.25 = 0.352\ \text{°C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(300 - 100)\text{°C}}{0.352\ \text{°C/W}} = 568\ \text{W}\ \text{(for a 0.12 m} \times \text{1 m section)}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (568\ \text{W})\frac{(5\ \text{m})(8\ \text{m})}{0.12\ \text{m}^2} = \mathbf{1.89 \times 10^5\ W}$$

(*b*) The total thermal resistance between left surface and the point where the sections B, D, and E meet is

$$R_{total} = R_1 + R_{mid,1} = 0.04 + 0.028 = 0.068\ \text{°C/W}$$

Then the temperature at the point where the sections B, D, and E meet becomes

$$\dot{Q} = \frac{T_1 - T}{R_{total}} \longrightarrow T = T_1 - \dot{Q}R_{total} = 300\text{°C} - (568\ \text{W})(0.068\ \text{°C/W}) = \mathbf{261°C}$$

(*c*) The temperature drop across the section F can be determined from

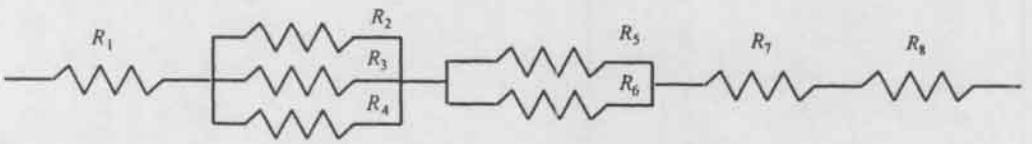
$$\dot{Q} = \frac{\Delta T}{R_F} \rightarrow \Delta T = \dot{Q}R_F = (568\ \text{W})(0.25\ \text{°C/W}) = \mathbf{142°C}$$

15-55 A composite wall consists of several horizontal and vertical layers. The left and right surfaces of the wall are maintained at uniform temperatures. The rate of heat transfer through the wall, the interface temperatures, and the temperature drop across the section F are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Thermal contact resistances at the interfaces are to be considered.

Properties The thermal conductivities of various materials used are given to be $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$ W/m·°C.

Analysis The representative surface area is $A = 0.12 \times 1 = 0.12\ \text{m}^2$



(*a*) The thermal resistance network and the individual thermal resistances are

$$R_1 = R_A = \frac{L}{kA} = \frac{0.01\ \text{m}}{(2\ \text{W/m.°C})(0.12\ \text{m}^2)} = 0.04\ °\text{C/W}$$

$$R_2 = R_C = \frac{L}{kA} = \frac{0.05\ \text{m}}{(20\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.06\ °\text{C/W}$$

$$R_3 = R_B = \frac{L}{kA} = \frac{0.05\ \text{m}}{(8\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.16\ °\text{C/W}$$

$$R_4 = R_D = \frac{L}{kA} = \frac{0.05\ \text{m}}{(15\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.08\ °\text{C/W}$$

$$R_5 = R_D = \frac{L}{kA} = \frac{0.1\ \text{m}}{(15\ \text{W/m.°C})(0.06\ \text{m}^2)} = 0.11\ °\text{C/W}$$

$$R_6 = R_E = \frac{L}{kA} = \frac{0.1\ \text{m}}{(35\ \text{W/m.°C})(0.06\ \text{m}^2)} = 0.05\ °\text{C/W}$$

$$R_7 = R_F = \frac{L}{kA} = \frac{0.06\ \text{m}}{(2\ \text{W/m.°C})(0.12\ \text{m}^2)} = 0.25\ °\text{C/W}$$

$$R_8 = \frac{0.00012\ \text{m}^2\text{°C/W}}{0.12\ \text{m}^2} = 0.001\ °\text{C/W}$$

$$\frac{1}{R_{mid,1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.06} + \frac{1}{0.16} + \frac{1}{0.08} \longrightarrow R_{mid,1} = 0.028\ °\text{C/W}$$

$$\frac{1}{R_{mid,2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.11} + \frac{1}{0.05} \longrightarrow R_{mid,2} = 0.034\ °\text{C/W}$$

$$R_{total} = R_1 + R_{mid,1} + R_{mid,2} + R_7 + R_8 = 0.04 + 0.028 + 0.034 + 0.25 + 0.001$$
$$= 0.353\ °\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(300 - 100)°\text{C}}{0.353\ °\text{C/W}} = 567\ \text{W}\ \text{(for a 0.12 m} \times \text{1 m section)}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (567\ \text{W})\frac{(5\ \text{m})(8\ \text{m})}{0.12\ \text{m}^2} = \mathbf{1.89 \times 10^5\ W}$$

(*b*) The total thermal resistance between left surface and the point where the sections B, D, and E meet is

$$R_{total} = R_1 + R_{mid,1} = 0.04 + 0.028 = 0.068\ ^\circ\text{C/W}$$

Then the temperature at the point where The sections B, D, and E meet becomes

$$\dot{Q} = \frac{T_1 - T}{R_{total}} \longrightarrow T = T_1 - \dot{Q}R_{total} = 300^\circ\text{C} - (567\ \text{W})(0.068\ ^\circ\text{C/W}) = \mathbf{261^\circ C}$$

(*c*) The temperature drop across the section F can be determined from

$$\dot{Q} = \frac{\Delta T}{R_F} \longrightarrow \Delta T = \dot{Q}R_F = (567\ \text{W})(0.25\ ^\circ\text{C/W}) = \mathbf{142^\circ C}$$

15-56 A coat is made of 5 layers of 0.1 mm thick synthetic fabric separated by 1.5 mm thick air space. The rate of heat loss through the jacket is to be determined, and the result is to be compared to the heat loss through a jackets without the air space. Also, the equivalent thickness of a wool coat is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the jacket is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.13 W/m·°C for synthetic fabric, k = 0.026 W/m·°C for air, and k = 0.035 W/m·°C for wool fabric.

Analysis The thermal resistance network and the individual thermal resistances are



$$R_{fabric} = R_1 = R_3 = R_5 = R_7 = R_9 = \frac{L}{kA} = \frac{0.0001\ \text{m}}{(0.13\ \text{W/m.°C})(1.1\ \text{m}^2)} = 0.0007\ °\text{C/W}$$

$$R_{air} = R_2 = R_4 = R_6 = R_8 = \frac{L}{kA} = \frac{0.0015\ \text{m}}{(0.026\ \text{W/m.°C})(1.1\ \text{m}^2)} = 0.0524\ °\text{C/W}$$

$$R_o = \frac{1}{hA} = \frac{1}{(25\ \text{W/m}^2\text{.°C})(1.1\ \text{m}^2)} = 0.0364\ °\text{C/W}$$

$$R_{total} = 5R_{fabric} + 4R_{air} + R_o = 5 \times 0.0007 + 4 \times 0.0524 + 0.0364 = 0.2495\ °\text{C/W}$$

and

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[(28-(-5)]°\text{C}}{0.2495\ °\text{C/W}} = \mathbf{132.3\ W}$$

If the jacket is made of a single layer of 0.5 mm thick synthetic fabric, the rate of heat transfer would be

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{T_{s1} - T_{\infty 2}}{5 \times R_{fabric} + R_o} = \frac{[(28-(-5)]°\text{C}}{(5 \times 0.0007 + 0.0364)\ °\text{C/W}} = 827\ \text{W}$$

The thickness of a wool fabric that has the same thermal resistance is determined from

$$R_{total} = R_{\substack{wool \\ fabric}} + R_o = \frac{L}{kA} + \frac{1}{hA}$$

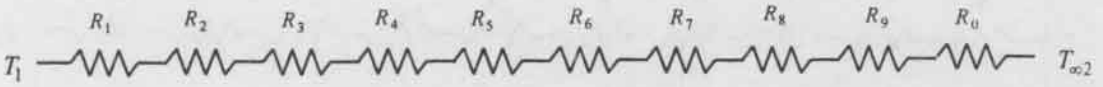
$$0.2495\ °\text{C/W} = \frac{L}{(0.035\ \text{W/m.°C})(1.1\ \text{m}^2)} + 0.0364 \longrightarrow L = 0.00820\ \text{m} = \mathbf{8.2\ mm}$$

15-57 A coat is made of 5 layers of 0.1 mm thick cotton fabric separated by 1.5 mm thick air space. The rate of heat loss through the jacket is to be determined, and the result is to be compared to the heat loss through a jackets without the air space. Also, the equivalent thickness of a wool coat is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the jacket is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be $k = 0.06$ W/m·°C for cotton fabric, $k = 0.026$ W/m·°C for air, and $k = 0.035$ W/m·°C for wool fabric.

Analysis The thermal resistance network and the individual thermal resistances are



$$R_{cotton} = R_1 = R_3 = R_5 = R_7 = R_9 = \frac{L}{kA} = \frac{0.0001\ \text{m}}{(0.06\ \text{W/m.°C})(1.1\ \text{m}^2)} = 0.00152\ °\text{C/W}$$

$$R_{air} = R_2 = R_4 = R_6 = R_8 = \frac{L}{kA} = \frac{0.0015\ \text{m}}{(0.026\ \text{W/m.°C})(1.1\ \text{m}^2)} = 0.0524\ °\text{C/W}$$

$$R_o = \frac{1}{hA} = \frac{1}{(25\ \text{W/m}^2\text{.°C})(1.1\ \text{m}^2)} = 0.0364\ °\text{C/W}$$

$$R_{total} = 5R_{fabric} + 4R_{air} + R_o = 5 \times 0.00152 + 4 \times 0.0524 + 0.0364 = 0.2536\ °\text{C/W}$$

and

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[(28-(-5)]°\text{C}}{0.2536\,°\text{C/W}} = \mathbf{130\ W}$$

If the jacket is made of a single layer of 0.5 mm thick cotton fabric, the rate of heat transfer will be

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{T_{s1} - T_{\infty 2}}{5 \times R_{fabric} + R_o} = \frac{[(28-(-5)]°\text{C}}{(5 \times 0.00152 + 0.0364)\ °\text{C/W}} = \mathbf{750\ W}$$

The thickness of a wool fabric for that case can be determined from

$$R_{\text{total}} = R_{\substack{\text{wool}\\\text{fabric}}} + R_o = \frac{L}{kA} + \frac{1}{hA}$$

$$0.2536\ °\text{C/W} = \frac{L}{(0.035\ \text{W/m.°C})(1.1\ \text{m}^2)} + 0.0364 \longrightarrow L = 0.0084\ \text{m} = \mathbf{8.4\ mm}$$

15-58 A kiln is made of 20 cm thick concrete walls and ceiling. The two ends of the kiln are made of thin sheet metal covered with 2-cm thick styrofoam. For specified indoor and outdoor temperatures, the rate of heat transfer from the kiln is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the walls and ceiling is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer. **5** Heat loss through the floor is negligible. **6** Thermal resistance of sheet metal is negligible.

Properties The thermal conductivities are given to be $k = 0.9$ W/m·°C for concrete and $k = 0.033$ W/m·°C for styrofoam insulation.

Analysis In this problem there is a question of which surface area to use. We will use the outer surface area for outer convection resistance, the inner surface area for inner convection resistance, and the average area for the conduction resistance. Or we could use the inner or the outer surface areas in the calculation of all thermal resistances with little loss in accuracy. For top and the two side surfaces:



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3000\ \text{W/m}^2.^\circ\text{C})[(40\ \text{m})(13-0.6)\ \text{m}]} = 0.0067\times10^{-4}\ ^\circ\text{C/W}$$

$$R_{concrete} = \frac{L}{kA_{ave}} = \frac{0.2\ \text{m}}{(0.9\ \text{W/m.}^\circ\text{C})[(40\ \text{m})(13-0.3)\ \text{m}]} = 4.37\times10^{-4}\ ^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\ \text{W/m}^2.^\circ\text{C})[(40\ \text{m})(13\ \text{m})]} = 0.769\times10^{-4}\ ^\circ\text{C/W}$$

$$R_{total} = R_i + R_{concrete} + R_o = (0.0067 + 4.37 + 0.769)\times10^{-4} = 5.146\times10^{-4}\ ^\circ\text{C/W}$$

and $$\dot{Q}_{top+sides} = \frac{T_{in} - T_{out}}{R_{total}} = \frac{[40-(-4)]^\circ\text{C}}{5.146\times10^{-4}\ ^\circ\text{C/W}} = 85{,}500\ \text{W}$$

Heat loss through the end surface of the kiln with styrofoarn:



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3000\ \text{W/m}^2.^\circ\text{C})[(4-0.4)(5-0.4)\ \text{m}^2]} = 0.201\times10^{-4}\ ^\circ\text{C/W}$$

$$R_{styrofoam} = \frac{L}{kA_{ave}} = \frac{0.02\ \text{m}}{(0.033\ \text{W/m.}^\circ\text{C})[(4-0.2)(5-0.2)\ \text{m}^2]} = 0.0332\ ^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\ \text{W/m}^2.^\circ\text{C})[4\times5\ \text{m}^2]} = 0.0020\ ^\circ\text{C/W}$$

$$R_{total} = R_i + R_{styrpfoam} + R_o = 0.201\times10^{-4} + 0.0332 + 0.0020 = 0.0352\ ^\circ\text{C/W}$$

and $$\dot{Q}_{end\ surface} = \frac{T_{in} - T_{out}}{R_{total}} = \frac{[40-(-4)]^\circ\text{C}}{0.0352\ ^\circ\text{C/W}} = 1250\ \text{W}$$

Then the total rate of heat transfer from the kiln becomes

$$\dot{Q}_{total} = \dot{Q}_{top+sides} + 2\dot{Q}_{side} = 85{,}500 + 2\times1250 = \mathbf{88{,}000\ W}$$

15-59E The thermal resistance of an epoxy glass laminate across its thickness is to be reduced by planting cylindrical copper fillings throughout. The thermal resistance of the epoxy board for heat conduction across its thickness as a result of this modification is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the plate is one-dimensional. **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be $k = 0.10$ Btu/h·ft·°F for epoxy glass laminate and $k =$ 223 Btu/h·ft·°F for copper fillings.

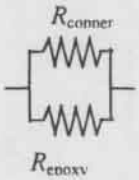
Analysis The thermal resistances of copper fillings and the epoxy board are in parallel. The number of copper fillings in the board and the area they comprise are

$$A_{total} = (6/12 \text{ ft})(8/12 \text{ ft}) = 0.33 \text{ m}^2$$

$$n_{copper} = \frac{0.33 \text{ ft}^2}{(0.06/12 \text{ ft})(0.06/12 \text{ ft})} = 13{,}200 \text{ (number of copper fillings)}$$

$$A_{copper} = n\frac{\pi D^2}{4} = 13{,}200\frac{\pi(0.02/12 \text{ ft})^2}{4} = 0.0288 \text{ ft}^2$$

$$A_{epoxy} = A_{total} - A_{copper} = 0.33 - 0.0288 = 0.301 \text{ ft}^2$$



The thermal resistances are evaluated to be

$$R_{copper} = \frac{L}{kA} = \frac{0.05/12 \text{ ft}}{(223 \text{ Btu/h.ft.°F})(0.0288 \text{ ft}^2)} = 0.00065 \text{ h.°F/Btu}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.05/12 \text{ ft}}{(0.10 \text{ Btu/h.ft.°F})(0.301 \text{ ft}^2)} = 0.138 \text{ h.°F/Btu}$$

Then the thermal resistance of the entire epoxy board becomes

$$\frac{1}{R_{board}} = \frac{1}{R_{copper}} + \frac{1}{R_{epoxy}} = \frac{1}{0.00065} + \frac{1}{0.138} \longrightarrow R_{board} = \mathbf{0.00065 \text{ h.°F/Btu}}$$

Heat Conduction in Cylinders and Spheres

15-60C During one dimensional heat conduction, heat transfer surface area is constant for a plane wall, and variable for a sphere and cylinder (in radial direction).

15-61C When the diameter of cylinder is very small compared to its length, it can be treated as an indefinitely long cylinder. Cylindrical rods can also be treated as being infinitely long when dealing with heat transfer at locations far from the top or bottom surfaces. However, it is not proper to use this model when finding temperatures near the bottom and the top of the cylinder.

15-62C Heat transfer in this short cylinder is one-dimensional since there will be no heat transfer in the axial and tangential directions.

15-63C No. In steady-operation the temperature of a solid cylinder or sphere does not change in radial direction (unless there is heat generation).

15-64 A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be $k = 15$ W/m·°C. The heat of fusion of water at 1 atm is $h_{if} = 333.7$ kJ / kg. The outer surface of the tank is black and thus its emissivity is $\varepsilon = 1$.

Analysis (a) The inner and the outer surface areas of sphere are

$$A_i = \pi D_i^2 = \pi(5\text{ m})^2 = 78.5\text{ m}^2$$

$$A_o = \pi D_o^2 = \pi(5.03\text{ m})^2 = 79.5\text{ m}^2$$

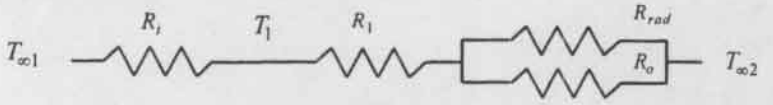
We assume the outer surface temperature T_2 to be 4°C after comparing convection heat transfer coefficients at the inner and the outer surfaces of the tank. With this assumption, the radiation heat transfer coefficient can be determined from

$$h_{rad} = \varepsilon\sigma(T_2^2 + T_{surr}^2)(T_2 + T_{surr})$$

$$= 1(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(273+4\text{ K})^2 + (273+20\text{ K})^2](273+20\text{ K})(273+4\text{ K})]$$

$$= 5.25\text{ W/m}^2.\text{K}$$

The individual thermal resistances are



$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(80\text{ W/m}^2.\text{°C})(78.5\text{ m}^2)} = 0.000159\text{ °C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(2.515-2.5)\text{ m}}{4\pi(15\text{ W/m.°C})(2.515\text{ m})(2.5\text{ m})} = 0.000013\text{ °C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(10\text{ W/m}^2.\text{°C})(79.5\text{ m}^2)} = 0.00126\text{ °C/W}$$

$$R_{rad} = \frac{1}{h_{rad} A} = \frac{1}{(5.25\text{ W/m}^2.\text{°C})(79.5\text{ m}^2)} = 0.00240\text{ °C/W}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{conv,o}} + \frac{1}{R_{rad}} = \frac{1}{0.00126} + \frac{1}{0.00240} \longrightarrow R_{eqv} = 0.000826\text{ °C/W}$$

$$R_{total} = R_{conv,i} + R_1 + R_{eqv} = 0.000159 + 0.000013 + 0.000826 = 0.000998\text{ °C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(20-0)\text{°C}}{0.000998\text{ °C/W}} = \mathbf{2.004\times10^4\text{ W}}$$

(b) The total amount of heat transfer during a 24-hour period and the amount of ice that will melt during this period are

$$Q = \dot{Q}\Delta t = (20.04\text{ kJ/s})(24\times3600\text{ s}) = 1.732\times10^6\text{ kJ}$$

$$m_{ice} = \frac{Q}{h_{if}} = \frac{1.732\times10^6\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{5189\text{ kg}}$$

Check: The outer surface temperature of the tank is

$$\dot{Q} = h_{conv+rad} A_o (T_{\infty1} - T_s) \rightarrow T_s = T_{\infty1} - \frac{\dot{Q}}{h_{conv+rad} A_o} = 20\text{°C} - \frac{20{,}040\text{ W}}{(10+5.25\text{ W/m}^2.\text{°C})(79.5\text{ m}^2)} = 3.5\text{°C}$$

which is very close to the assumed temperature of 4°C for the outer surface temperature used in the evaluation of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations.

15-65 A steam pipe covered with 15-cm thick glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

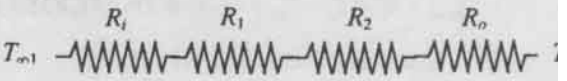
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 15$ W/m·°C for steel and $k = 0.038$ W/m·°C for glass wool insulation

Analysis (a) The inner and the outer surface areas of the insulated pipe per unit length are

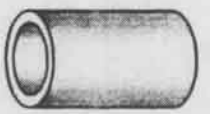
$$A_i = \pi D_i L = \pi(0.05\text{ m})(1\text{ m}) = 0.157\text{ m}^2$$

$$A_o = \pi D_o L = \pi(0.055+0.06\text{ m})(1\text{ m}) = 0.361\text{ m}^2$$



The individual thermal resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(80\text{ W/m}^2\cdot\text{°C})(0.157\text{ m}^2)} = 0.08\text{ °C/W}$$



$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(15\text{ W/m.°C})(1\text{ m})} = 0.00101\text{ °C/W}$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.038\text{ W/m.°C})(1\text{ m})} = 3.089\text{ °C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(15\text{ W/m}^2\cdot\text{°C})(0.361\text{ m}^2)} = 0.1847\text{ °C/W}$$

$$R_{total} = R_i + R_1 + R_2 + R_o = 0.08+0.00101+3.089+0.1847 = 3.355\text{ °C/W}$$

Then the steady rate of heat loss from the steam per m. pipe length becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(320-5)\text{°C}}{3.355\text{ °C/W}} = \mathbf{93.9\text{ W}}$$

The temperature drops across the pipe and the insulation are

$$\Delta T_{pipe} = \dot{Q} R_{pipe} = (93.9\text{ W})(0.00101\text{ °C/W}) = \mathbf{0.095°C}$$

$$\Delta T_{insulation} = \dot{Q} R_{insulation} = (93.9\text{ W})(3.089\text{ °C/W}) = \mathbf{290°C}$$

15-66 A 50-m long section of a steam pipe passes through an open space at 15°C. The rate of heat loss from the steam pipe, the annual cost of this heat loss, and the thickness of fiberglass insulation needed to save 90 percent of the heat lost are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivity is constant. **4** The thermal contact resistance at the interface is negligible. **5** The pipe temperature remains constant at about 150°C with or without insulation. **6** The combined heat transfer coefficient on the outer surface remains constant even after the pipe is insulated.

Properties The thermal conductivity of fiberglass insulation is given to be $k = 0.035$ W/m·°C.

Analysis (a) The rate of heat loss from the steam pipe is

$$A_o = \pi DL = \pi(0.1\text{ m})(50\text{ m}) = 15.7\text{ m}^2$$



$$\dot{Q}_{bare} = h_o A(T_s - T_{air}) = (20\text{ W/m}^2.^\circ\text{C})(15.7\text{ m}^2)(150-15)^\circ\text{C} = \mathbf{42{,}390\text{ W}}$$

(b) The amount of heat loss per year is

$$Q = \dot{Q}\Delta t = (42.390\text{ kJ/s})(365\times 24\times 3600\text{ s/yr}) = 1.336\times 10^9\text{ kJ/yr}$$

The amount of gas consumption from the natural gas furnace that has an efficiency of 75% is

$$Q_{gas} = \frac{1.336\times 10^9\text{ kJ/yr}}{0.75}\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right) = 16{,}885\text{ therms/yr}$$

The annual cost of this energy lost is

$$\begin{aligned}\text{Energy cost} &= (\text{Energy used})(\text{Unit cost of energy})\\ &= (16{,}885\text{ therms/yr})(\$0.52/\text{therm}) = \mathbf{\$8780/yr}\end{aligned}$$

(c) In order to save 90% of the heat loss and thus to reduce it to 0.1×42,390 = 4239 W, the thickness of insulation needed is determined from



$$\dot{Q}_{insulated} = \frac{T_s - T_{air}}{R_o + R_{insulation}} = \frac{T_s - T_{air}}{\dfrac{1}{h_o A_o} + \dfrac{\ln(r_2/r_1)}{2\pi kL}}$$

Substituting and solving for r_2, we get

$$4239\text{ W} = \frac{(150-15)^\circ\text{C}}{\dfrac{1}{(20\text{ W/m}^2.^\circ\text{C})[(2\pi r_2(50\text{ m})]} + \dfrac{\ln(r_2/0.05)}{2\pi(0.035\text{ W/m.}^\circ\text{C})(50\text{ m})}} \rightarrow r_2 = 0.069\text{ m}$$

Then the thickness of insulation becomes

$$t_{insulation} = r_2 - r_1 = 6.9 - 5 = \mathbf{1.9\text{ cm}}$$

15-67 An electric hot water tank is made of two concentric cylindrical metal sheets with foam insulation in between. The fraction of the hot water cost that is due to the heat loss from the tank and the payback period of the do-it-yourself insulation kit are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal resistances of the water tank and the outer thin sheet metal shell are negligible. **5** Heat loss from the top and bottom surfaces is negligible.

Properties The thermal conductivities are given to be $k = 0.03$ W/m·°C for foam insulation and $k = 0.035$ W/m·°C for fiber glass insulation

Analysis We consider only the side surfaces of the water heater for simplicity, and disregard the top and bottom surfaces (it will make difference of about 10 percent). The individual thermal resistances are

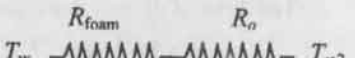
$$A_o = \pi D_o L = \pi(0.46\text{ m})(2\text{ m}) = 2.89\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.^\circ\text{C})(2.89\text{ m}^2)} = 0.029\,^\circ\text{C/W}$$

$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2.^\circ\text{C})(2\text{ m})} = 0.37\,^\circ\text{C/W}$$

$$R_{total} = R_o + R_{foam} = 0.029 + 0.37 = 0.40\,^\circ\text{C/W}$$

The rate of heat loss from the hot water tank is



$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(55-27)^\circ\text{C}}{0.40\,^\circ\text{C/W}} = 70\text{ W}$$

The amount and cost of heat loss per year are

$$Q = \dot{Q}\Delta t = (0.07\text{ kW})(365\times 24\text{ h/yr}) = 613.2\text{ kWh/yr}$$

$$\text{Cost of Energy} = (\text{Amount of energy})(\text{Unit cost}) = (613.2\text{ kWh})(\$0.08/\text{kWh}) = \$49.56$$

$$f = \frac{\$49.056}{\$280} = 0.1752 = \mathbf{17.5\%}$$

If 3 cm thick fiber glass insulation is used to wrap the entire tank, the individual resistances becomes



$$A_o = \pi D_o L = \pi(0.52\text{ m})(2\text{ m}) = 3.27\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.^\circ\text{C})(3.27\text{ m}^2)} = 0.026\,^\circ\text{C/W}$$

$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2.^\circ\text{C})(2\text{ m})} = 0.37\,^\circ\text{C/W}$$

$$R_{fiberglass} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(26/23)}{2\pi(0.035\text{ W/m}^2.^\circ\text{C})(2\text{ m})} = 0.28\,^\circ\text{C/W}$$

$$R_{total} = R_o + R_{foam} + R_{fiberglass} = 0.026 + 0.37 + 0.28 = 0.676\,^\circ\text{C/W}$$

The rate of heat loss from the hot water heater in this case is

$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(55-27)^\circ\text{C}}{0.676\,^\circ\text{C/W}} = 41.4\text{ W}$$

The energy saving is

$$\text{saving} = 70 - 41.4 = 28.6\text{ W}$$

The time necessary for this additional insulation to pay for its cost of \$30 is then determined to be

$$\text{Cost} = (0.0286\text{ kW})(\text{Time period})(\$0.08/\text{kWh}) = \$30$$

Then, Time period = 13,021 hours = **543 days ≈ 1.5 years**

15-68 A cold aluminum canned drink that is initially at a uniform temperature of 3°C is brought into a room air at 25°C. The time it will take for the average temperature of the drink to rise to 10°C with and without rubber insulation is to be determined.

Assumptions **1** The drink is at a uniform temperature at all times. **2** The thermal resistance of the can and the internal convection resistance are negligible so that the can is at the same temperature as the drink inside. **3** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **4** Thermal properties are constant. **5** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of rubber insulation is given to be $k = 0.13$ W/m·°C. For the drink, we use the properties of water at room temperature, $\rho = 1000$ kg/m^3 and $C_p = 4180$ J/kg.°C.

Analysis This is a transient heat conduction, and the rate of heat transfer will decrease as the drink warms up and the temperature difference between the drink and the surroundings decreases. However, we can solve this problem approximately by assuming a constant average temperature of (3+10)/2 = 6.5°C during the process. Then the average rate of heat transfer into the drink is

$$A_o = \pi D_o L + 2\frac{\pi D^2}{4} = \pi(0.06\text{ m})(0.125\text{ m}) + 2\frac{\pi(0.06\text{ m})^2}{4} = 0.0292\text{ m}^2$$

$$\dot{Q}_{bare,ave} = h_o A(T_{air} - T_{can,ave}) = (10\text{ W/m}^2.^\circ\text{C})(0.0292\text{ m}^2)(25-6.5)^\circ\text{C} = 5.40\text{ W}$$

The amount of heat that must be supplied to the drink to raise its temperature to 10 °C is

$$m = \rho V = \rho\pi r^2 L = (1000\text{ kg/m}^3)\pi(0.03\text{ m})^2(0.125\text{ m}) = 0.353\text{ kg}$$

$$Q = mC_p\Delta T = (0.353\text{ kg})(4180\text{ J/kg})(10-3)^\circ\text{C} = 10{,}329\text{ J}$$

Then the time required for this much heat transfer to take place is

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{10{,}329\text{ J}}{5.4\text{ J/s}} = 1912\text{ s} = \mathbf{31.9\text{ min}}$$

We now repeat calculations after wrapping the can with 1-cm thick rubber insulation, except the top surface. The rate of heat transfer from the top surface is

$$\dot{Q}_{top,ave} = h_o A_{top}(T_{air} - T_{can,ave}) = (10\text{ W/m}^2.^\circ\text{C})[\pi(0.03\text{ m})^2](25-6.5)^\circ\text{C} = 0.52\text{ W}$$

Heat transfer through the insulated side surface is

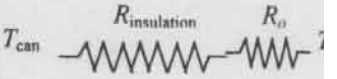
$$A_o = \pi D_o L = \pi(0.08\text{ m})(0.125\text{ m}) = 0.03142\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(10\text{ W/m}^2.^\circ\text{C})(0.03142\text{ m}^2)} = 3.183^\circ\text{C/W}$$

$$R_{insulation,side} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(4/3)}{2\pi(0.13\text{ W/m}^2.^\circ\text{C})(0.125\text{ m})} = 2.818^\circ\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 3.183 + 2.818 = 6.001^\circ\text{C/W}$$

$$\dot{Q}_{side} = \frac{T_{air} - T_{can,ave}}{R_{conv,o}} = \frac{(25-6.5)^\circ\text{C}}{6.001^\circ\text{C/W}} = 3.08\text{ W}$$



The ratio of bottom to the side surface areas is $(\pi r^2)/(2\pi rL) = r/(2L) = 3/(2\times 12.5) = 0.12$. Therefore, the effect of heat transfer through the bottom surface can be accounted for approximately by increasing the heat transfer from the side surface by 12%. Then,

$$\dot{Q}_{insulated} = \dot{Q}_{side+bottom} + \dot{Q}_{top} = 1.12\times 3.08 + 0.52 = 3.97\text{ W}$$

Then the time of heating becomes

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{10{,}329\text{ J}}{3.97\text{ J/s}} = 2602\text{ s} = \mathbf{43.4\text{ min}}$$

15-69 A cold aluminum canned drink that is initially at a uniform temperature of 3°C is brought into a room air at 25°C. The time it will take for the average temperature of the drink to rise to 10°C with and without rubber insulation is to be determined.

Assumptions **1** The drink is at a uniform temperature at all times. **2** The thermal resistance of the can and the internal convection resistance are negligible so that the can is at the same temperature as the drink inside. **3** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **4** Thermal properties are constant. **5** The thermal contact resistance at the interface is to be considered.

Properties The thermal conductivity of rubber insulation is given to be $k = 0.13$ W/m·°C. For the drink, we use the properties of water at room temperature, $\rho = 1000$ kg/m^3 and $C_p = 4180$ J/kg.°C.

Analysis This is a transient heat conduction, and the rate of heat transfer will decrease as the drink warms up and the temperature difference between the drink and the surroundings decreases. However, we can solve this problem approximately by assuming a constant average temperature of (3+10)/2 = 6.5°C during the process. Then the average rate of heat transfer into the drink is

$$A_o = \pi D_o L + 2\frac{\pi D^2}{4} = \pi(0.06\text{ m})(0.125\text{ m}) + 2\frac{\pi(0.06\text{ m})^2}{4} = 0.0292\text{ m}^2$$

$$\dot{Q}_{bare,ave} = h_o A(T_{air} - T_{can,ave}) = (10\text{ W/m}^2.^\circ\text{C})(0.0292\text{ m}^2)(25-6.5)^\circ\text{C} = 5.40\text{ W}$$

The amount of heat that must be supplied to the drink to raise its temperature to 10 °C is

$$m = \rho V = \rho\pi r^2 L = (1000\text{ kg/m}^3)\pi(0.03\text{ m})^2(0.125\text{ m}) = 0.353\text{ kg}$$

$$Q = mC_p\Delta T = (0.353\text{ kg})(4180\text{ J/kg})(10-3)^\circ\text{C} = 10{,}329\text{ J}$$

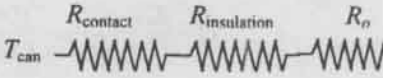
Then the time required for this much heat transfer to take place is

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{10{,}329\text{ J}}{5.4\text{ J/s}} = 1912\text{ s} = \mathbf{31.9\text{ min}}$$

We now repeat calculations after wrapping the can with 1-cm thick rubber insulation, except the top surface. The rate of heat transfer from the top surface is

$$\dot{Q}_{top,ave} = h_o A_{top}(T_{air} - T_{can,ave}) = (10\text{ W/m}^2.^\circ\text{C})[\pi(0.03\text{ m})^2](25-6.5)^\circ\text{C} = 0.52\text{ W}$$

Heat transfer through the insulated side surface is



$$A_o = \pi D_o L = \pi(0.08\text{ m})(0.125\text{ m}) = 0.03142\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(10\text{ W/m}^2.^\circ\text{C})(0.03142\text{ m}^2)} = 3.183\,^\circ\text{C/W}$$

$$R_{insulation,side} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(4/3)}{2\pi(0.13\text{ W/m}^2.^\circ\text{C})(0.125\text{ m})} = 2.818\,^\circ\text{C/W}$$

$$R_{contact} = \frac{0.00008\text{ m}^2.^\circ\text{C/W}}{0.0236\text{ m}^2} = 0.0034\,^\circ\text{C/W}$$

$$R_{total} = R_o + R_{insulation} + R_{contact} = 3.183 + 2.818 + 0.0034 = 6.004\,^\circ\text{C/W}$$

$$\dot{Q}_{side} = \frac{T_{air} - T_{can,ave}}{R_{conv,o}} = \frac{(25-6.5)^\circ\text{C}}{6.004\,^\circ\text{C/W}} = 3.08\text{ W}$$

The ratio of bottom to the side surface areas is $(\pi r^2)/(2\pi rL) = r/(2L) = 3/(2\times 12.5) = 0.12$. Therefore, the effect of heat transfer through the bottom surface can be accounted for approximately by increasing the heat transfer from the side surface by 12%. Then,

$$\dot{Q}_{insulated} = \dot{Q}_{side+bottom} + \dot{Q}_{top} = 1.12 \times 3.08 + 0.52 = 3.97\text{ W}$$

Then the time of heating becomes

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{10{,}329\text{ J}}{3.97\text{ J/s}} = 2602\text{ s} = \mathbf{43.4\text{ min}}$$

Discussion The thermal contact resistance did not have any effect on heat transfer.

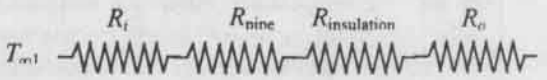
15-70E A steam pipe covered with 2-in thick fiberglass insulation is subjected to convection on its surfaces. The rate of heat loss from the steam per unit length and the error involved in neglecting the thermal resistance of the steel pipe in calculations are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 8.7$ Btu/h·ft·°F for steel and $k = 0.020$ Btu/h·ft·°F for fiberglass insulation.

Analysis The inner and outer surface areas of the insulated pipe are

$$A_i = \pi D_i L = \pi(3.5/12 \text{ ft})(1 \text{ ft}) = 0.916 \text{ ft}^2$$
$$A_o = \pi D_o L = \pi(8/12 \text{ ft})(1 \text{ ft}) = 2.094 \text{ ft}^2$$



The individual resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(30 \text{ Btu/h.ft}^2.^\circ\text{F})(0.916 \text{ ft}^2)} = 0.036 \text{ h} \cdot ^\circ\text{F/Btu}$$

$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2/1.75)}{2\pi(8.7 \text{ Btu/h.ft.}^\circ\text{F})(1 \text{ ft})} = 0.002 \text{ h} \cdot ^\circ\text{F/Btu}$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(4/2)}{2\pi(0.020 \text{ Btu/h.ft.}^\circ\text{F})(1 \text{ ft})} = 5.516 \text{ h} \cdot ^\circ\text{F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(5 \text{ Btu/h.ft}^2.^\circ\text{F})(2.094 \text{ ft}^2)} = 0.096 \text{ h} \cdot ^\circ\text{F/Btu}$$

$$R_{total} = R_i + R_1 + R_2 + R_o = 0.036 + 0.002 + 5.516 + 0.096 = 5.65 \text{ h} \cdot ^\circ\text{F/Btu}$$

Then the steady rate of heat loss from the steam per ft. pipe length becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(600 - 60)^\circ\text{F}}{5.65 \text{ h}^\circ\text{F/Btu}} = \mathbf{95.58 \text{ Btu/h}}$$

If the thermal resistance of the steel pipe is neglected, the new value of total thermal resistance will be

$$R_{total} = R_i + R_2 + R_o = 0.036 + 5.516 + 0.096 = 5.648 \text{ h}^\circ\text{F/Btu}$$

Then the percentage error involved in calculations becomes

$$error\% = \frac{(5.65 - 5.648)\text{h}^\circ\text{F/Btu}}{5.65 \text{ h}^\circ\text{F/Btu}} \times 100 = \mathbf{0.035\%}$$

which is insignificant.

15-71 Hot water is flowing through a 15-m section of a cast iron pipe. The pipe is exposed to cold air and surfaces in the basement. The rate of heat loss from the hot water and the average velocity of the water in the pipe as it passes through the basement are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal properties are constant.

Properties The thermal conductivity and emissivity of cast iron are given to be $k = 52$ W/m·°C and $\varepsilon = 0.7$.

Analysis The individual resistances are

$$A_i = \pi D_i L = \pi(0.04\text{ m})(15\text{ m}) = 1.885\text{ m}^2$$

$$A_o = \pi D_o L = \pi(0.046\text{ m})(15\text{ m}) = 2.168\text{ m}^2$$



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(120\text{ W/m}^2.^\circ\text{C})(1.885\text{ m}^2)} = 0.0044\,^\circ\text{C/W}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.3/2)}{2\pi(52\text{ W/m.}^\circ\text{C})(15\text{ m})} = 0.00003\,^\circ\text{C/W}$$

The outer surface temperature of the pipe will be somewhat below the water temperature. Assuming the outer surface temperature of the pipe to be 80 °C (we will check this assumption later), the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_2^2 + T_{surr}^2)(T_2 + T_{surr}) \\ &= (0.7)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(353\text{ K})^2 + (283\text{ K})^2](353+283) \\ &= 5.167\text{ W/m}^2.\text{K} \end{aligned}$$

Since the surrounding medium and surfaces are at the same temperature, the radiation and convection heat transfer coefficients can be added and the result can be taken as the combined heat transfer coefficient. Then,

$$h_{combined} = h_{rad} + h_{conv,2} = 5.167 + 15 = 20.167\text{ W/m}^2.^\circ\text{C}$$

$$R_o = \frac{1}{h_{combined} A_o} = \frac{1}{(20.167\text{ W/m}^2.^\circ\text{C})(2.168\text{ m}^2)} = 0.023\,^\circ\text{C/W}$$

$$R_{total} = R_i + R_{pipe} + R_o = 0.0044 + 0.00003 + 0.023 = 0.027\,^\circ\text{C/W}$$

The rate of heat loss from the hot water pipe then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(90-10)^\circ\text{C}}{0.0271\,^\circ\text{C/W}} = \mathbf{2963\text{ W}}$$

For a temperature drop of 3°C, the mass flow rate of water and the average velocity of water must be

$$\dot{Q} = \dot{m}C_p\Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p\Delta T} = \frac{2963\text{ J/s}}{(4180\text{ J/kg.}^\circ\text{C})(3\,^\circ\text{C})} = 0.236\text{ kg/s}$$

$$\dot{m} = \rho V A_c \longrightarrow V = \frac{\dot{m}}{\rho A_c} = \frac{0.236\text{ kg/s}}{(1000\text{ kg/m}^3)\dfrac{\pi(0.04\text{ m})^2}{4}} = \mathbf{0.188\text{ m/s}}$$

Discussion The outer surface temperature of the pipe is

$$\dot{Q} = \frac{T_{\infty 1} - T_s}{R_i + R_{pipe}} \rightarrow 2963\text{ W} = \frac{(90-T_s)^\circ\text{C}}{(0.0044+0.00003)^\circ\text{C/W}} \rightarrow T_s = 77^\circ\text{C}$$

which is very close to the value assumed for the surface temperature in the evaluation of the radiation resistance. Therefore, there is no need to repeat the calculations.

15-72 Hot water is flowing through a 15 m section of a copper pipe. The pipe is exposed to cold air and surfaces in the basement. The rate of heat loss from the hot water and the average velocity of the water in the pipe as it passes through the basement are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant.

Properties The thermal conductivity and emissivity of copper are given to be $k = 52$ W/m·°C and $\varepsilon = 0.7$.

Analysis The individual resistances are

$$A_i = \pi D_i L = \pi(0.04\text{ m})(15\text{ m}) = 1.885\text{ m}^2$$

$$A_o = \pi D_o L = \pi(0.046\text{ m})(15\text{ m}) = 2.168\text{ m}^2$$



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(120\text{ W/m}^2.^\circ\text{C})(1.885\text{ m}^2)} = 0.004\,^\circ\text{C/W}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(2.3/2)}{2\pi(386\text{ W/m.°C})(15\text{ m})} = 0.0000038\,^\circ\text{C/W}$$

The outer surface temperature of the pipe will be somewhat below the water temperature. Assuming the outer surface temperature of the pipe to be 80 °C (we will check this assumption later), the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_2^2 + T_{surr}^2)(T_2 + T_{surr}) \\ &= (0.7)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(353\text{ K})^2 + (283\text{ K})^2](353+283) \\ &= 5.167\text{ W/m}^2.\text{K} \end{aligned}$$

Since the surrounding medium and surfaces are at the same temperature, the radiation and convection heat transfer coefficients can be added and the result can be taken as the combined heat transfer coefficient. Then,

$$h_{combined} = h_{rad} + h_{conv,2} = 5.167 + 15 = 20.167\text{ W/m}^2.^\circ\text{C}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(20.167\text{ W/m}^2.^\circ\text{C})(2.168\text{ m}^2)} = 0.023\,^\circ\text{C/W}$$

$$R_{total} = R_i + R_{pipe} + R_o = 0.004 + 0.0000038 + 0.023 = 0.027\,^\circ\text{C/W}$$

The rate of heat loss from the hot tank water then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(90-10)^\circ\text{C}}{0.0271\,^\circ\text{C/W}} = \mathbf{2963\text{ W}}$$

For a temperature drop of 3°C, the mass flow rate of water and the average velocity of water must be

$$\dot{Q} = \dot{m}C_p\Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p\Delta T} = \frac{2963\text{ J/s}}{(4180\text{ J/kg.}^\circ\text{C})(3\,^\circ\text{C})} = 0.236\text{ kg/s}$$

$$\dot{m} = \rho V A_c \longrightarrow V = \frac{\dot{m}}{\rho A_c} = \frac{0.236\text{ kg/s}}{(1000\text{ kg/m}^3)\left[\dfrac{\pi(0.04\text{ m})^2}{4}\right]} = \mathbf{0.188\text{ m/s}}$$

Discussion The outer surface temperature of the pipe is

$$\dot{Q} = \frac{T_{\infty 1} - T_s}{R_i + R_{pipe}} \rightarrow 2963\text{ W} = \frac{(90-T_s)^\circ\text{C}}{(0.0044+0.0000)^\circ\text{C/W}} \rightarrow T_s = 77^\circ\text{C}$$

which is very close to the value assumed for the surface temperature in the evaluation of the radiation resistance. Therefore, there is no need to repeat the calculations.

15-73E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper tubes. For specified heat transfer coefficients, the length of the tube required to condense steam at a rate of 400 lbm/h is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivity of copper tube is given to be k = 223 Btu/h·ft·°F. The heat of vaporization of water at 100°F is given to be 1037 Btu/lbm.

Analysis The individual resistances are



$$A_i = \pi D_i L = \pi(0.4/12 \text{ ft})(1 \text{ ft}) = 0.105 \text{ ft}^2$$

$$A_o = \pi D_o L = \pi(0.6/12 \text{ ft})(1 \text{ ft}) = 0.157 \text{ ft}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(35 \text{ Btu/h.ft}^2\text{.°F})(0.105 \text{ ft}^2)} = 0.272 \text{ h°F/Btu}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3/2)}{2\pi(223 \text{ Btu/h.ft.°F})(1 \text{ ft})} = 0.00029 \text{ h°F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(1500 \text{ Btu/h.ft}^2\text{.°F})(0.157 \text{ ft}^2)} = 0.0042 \text{ h°F/Btu}$$

$$R_{total} = R_i + R_{pipe} + R_o = 0.272 + 0.00029 + 0.0042 = 0.277 \text{ h°F/Btu}$$

The heat transfer rate per ft length of the tube is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(100 - 70)° \text{F}}{0.277 \text{ °F/Btu}} = 108.3 \text{ Btu/h}$$

The total rate of heat transfer required to condense steam at a rate of 400 lbm/h and the length of the tube required is determined to be

$$\dot{Q}_{total} = \dot{m} h_{fg} = (400 \text{ lbm/h})(1037 \text{ Btu/lbm}) = 414{,}800 \text{ Btu/h}$$

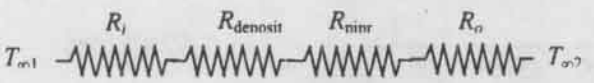
$$\text{Tube length} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{414{,}800}{108.3} = \mathbf{3830 \text{ ft}}$$

15-74E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper tubes. For specified heat transfer coefficients and 0.01-in thick scale build up on the inner surface, the length of the tube required to condense steam at a rate of 400 lbm/h is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper tube and be k = 0.5 Btu/h·ft·°F for the mineral deposit. The heat of vaporization of water at 100°F is given to be 1037 Btu/lbm.

Analysis When a 0.01-in thick layer of deposit forms on the inner surface of the pipe, the inner diameter of the pipe will reduce from 0.4 in to 0.38 in. The individual thermal resistances are



$$A_i = \pi D_i L = \pi(0.4/12 \text{ ft})(1 \text{ ft}) = 0.105 \text{ ft}^2$$

$$A_o = \pi D_o L = \pi(0.6/12 \text{ ft})(1 \text{ ft}) = 0.157 \text{ ft}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(35 \text{ Btu/h.ft}^2.°\text{F})(0.105 \text{ ft}^2)} = 0.272 \text{ h°F/Btu}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k L} = \frac{\ln(3/2)}{2\pi(223 \text{ Btu/h.ft.°F})(1 \text{ ft})} = 0.00029 \text{ h°F/Btu}$$

$$R_{deposit} = \frac{\ln(r_1/r_{dep})}{2\pi k_2 L} = \frac{\ln(0.2/0.19)}{2\pi(0.5 \text{ Btu/h.ft.°F})(1 \text{ ft})} = 0.0163 \text{ h.°F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(1500 \text{ Btu/h.ft}^2.°\text{F})(0.157 \text{ ft}^2)} = 0.0042 \text{ h°F/Btu}$$

$$R_{total} = R_i + R_{pipe} + R_{deposit} + R_o = 0.272 + 0.00029 + 0.0163 + 0.0042 = 0.293 \text{ h°F/Btu}$$

The heat transfer rate per ft length of the tube is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(100 - 70)° \text{F}}{0.293 \text{ °F/Btu}} = 102.4 \text{ Btu/h}$$

The total rate of heat transfer required to condense steam at a rate of 400 lbm/h and the length of the tube required can be determined to be

$$\dot{Q}_{total} = \dot{m} h_{fg} = (400 \text{ lbm/h})(1037 \text{ Btu/lbm}) = 414{,}800 \text{ Btu/h}$$

$$\text{Tube length} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{414{,}800}{102.4} = \mathbf{4051 \text{ ft}}$$

15-75 A 15-m diameter spherical tank filled with liquid nitrogen at 1 atm and -196°C is exposed to convection and radiation with the surrounding air and surfaces. The rate of evaporation of liquid nitrogen in the tank as a result of the heat gain from the surroundings for the cases of no insulation, 5-cm thick fiberglass insulation, and 2-cm thick superinsulation are to be determined.

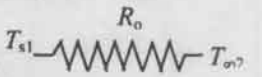
Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the nitrogen inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid nitrogen at 1 atm are given to be 198 kJ/kg and 810 kg/m^3, respectively. The thermal conductivities are given to be k = 0.035 W/m·°C for fiberglass insulation and k = 0.00005 W/m·°C for super insulation.

Analysis (a) The heat transfer rate and the rate of evaporation of the liquid without insulation are

$$A = \pi D^2 = \pi(3\text{ m})^2 = 28.27\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.^\circ\text{C})(28.27\text{ m}^2)} = 0.00101^\circ\text{C/W}$$

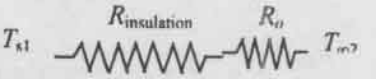


$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_o} = \frac{[15-(-196)]^\circ\text{C}}{0.00101^\circ\text{C/W}} = 208{,}910\text{ W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{208.910\text{ kJ/s}}{198\text{ kJ/kg}} = \mathbf{1.055\text{ kg/s}}$$

(b) The heat transfer rate and the rate of evaporation of the liquid with a 5-cm thick layer of fiberglass insulation are

$$A = \pi D^2 = \pi(3.1\text{ m})^2 = 30.19\text{ m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.^\circ\text{C})(30.19\text{ m}^2)} = 0.000946^\circ\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.55-1.5)\text{ m}}{4\pi(0.035\text{ W/m.°C})(1.55\text{ m})(1.5\text{ m})} = 0.0489^\circ\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000946 + 0.0489 = 0.0498^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]^\circ\text{C}}{0.0498^\circ\text{C/W}} = 4237\text{ W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{4.237\text{ kJ/s}}{198\text{ kJ/kg}} = \mathbf{0.0214\text{ kg/s}}$$

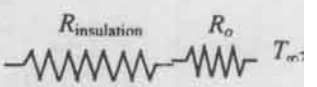
(c) The heat transfer rate and the rate of evaporation of the liquid with 2-cm thick layer of superinsulation is

$$A = \pi D^2 = \pi(3.04\text{ m})^2 = 29.03\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.^\circ\text{C})(29.03\text{ m}^2)} = 0.000984^\circ\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52-1.5)\text{ m}}{4\pi(0.00005\text{ W/m.°C})(1.52\text{ m})(1.5\text{ m})} = 13.96^\circ\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000984 + 13.96 = 13.96^\circ\text{C/W}$$



$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]^\circ\text{C}}{13.96^\circ\text{C/W}} = 15.11\text{ W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.01511\text{ kJ/s}}{198\text{ kJ/kg}} = \mathbf{0.000076\text{ kg/s}}$$

15-76 A 15-m diameter spherical tank filled with liquid oxygen at 1 atm and -183°C is exposed to convection and radiation with the surrounding air and surfaces. The rate of evaporation of liquid oxygen in the tank as a result of the heat gain from the surroundings for the cases of no insulation, 5-cm thick fiberglass insulation, and 2-cm thick superinsulation are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the oxygen inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

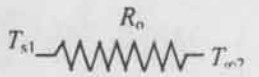
Properties The heat of vaporization and density of liquid oxygen at 1 atm are given to be 213 kJ/kg and 1140 kg/m^3, respectively. The thermal conductivities are given to be k = 0.035 W/m·°C for fiberglass insulation and k = 0.00005 W/m·°C for super insulation.

Analysis (*a*) The heat transfer rate and the rate of evaporation of the liquid without insulation are

$$A = \pi D^2 = \pi(3\text{ m})^2 = 28.27\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.^\circ\text{C})(28.27\text{ m}^2)} = 0.00101^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_o} = \frac{[15-(-183)]^\circ\text{C}}{0.00101^\circ\text{C/W}} = 196{,}040\text{ W}$$



$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{196.040\text{ kJ/s}}{213\text{ kJ/kg}} = \mathbf{0.920\text{ kg/s}}$$

(*b*) The heat transfer rate and the rate of evaporation of the liquid with a 5-cm thick layer of fiberglass insulation are

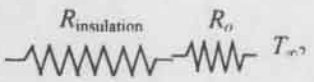
$$A = \pi D^2 = \pi(3.1\text{ m})^2 = 30.19\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.^\circ\text{C})(30.19\text{ m}^2)} = 0.000946^\circ\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.55-1.5)\text{ m}}{4\pi(0.035\text{ W/m.°C})(1.55\text{ m})(1.5\text{ m})} = 0.0489^\circ\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000946 + 0.0489 = 0.0498^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-183)]^\circ\text{C}}{0.0498^\circ\text{C/W}} = 3976\text{ W}$$



$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{3.976\text{ kJ/s}}{213\text{ kJ/kg}} = \mathbf{0.0187\text{ kg/s}}$$

(*c*) The heat transfer rate and the rate of evaporation of the liquid with a 2-cm superinsulation is

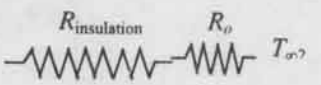
$$A = \pi D^2 = \pi(3.04\text{ m})^2 = 29.03\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.^\circ\text{C})(29.03\text{ m}^2)} = 0.000984^\circ\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52-1.5)\text{ m}}{4\pi(0.00005\text{ W/m.°C})(1.52\text{ m})(1.5\text{ m})} = 13.96^\circ\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000984 + 13.96 = 13.96^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-183)]^\circ\text{C}}{13.96^\circ\text{C/W}} = 14.18\text{ W}$$



$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.01418\text{ kJ/s}}{213\text{ kJ/kg}} = \mathbf{0.000067\text{ kg/s}}$$

Critical Radius Of Insulation

15-77C In a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of insulation, but decreases the convection resistance of the surface because of the increase in the outer surface area. Due to these opposite effects, a critical radius of insulation is defined as the outer radius that provides maximum rate of heat transfer. For a cylindrical layer, it is defined as $r_{cr} = k / h$ where k is the thermal conductivity of insulation and h is the external convection heat transfer coefficient.

15-78C It will decrease.

15-79C Yes, the measurements can be right. If the radius of insulation is less than critical radius of insulation of the pipe, the rate of heat loss will increase.

15-80C No.

15-81C For a cylindrical pipe, the critical radius of insulation is defined as $r_{cr} = k / h$. On windy days, the external convection heat transfer coefficient is greater compared to calm days. Therefore critical radius of insulation will be greater on calm days.

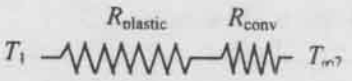
15-82 An electric wire is tightly wrapped with a 1-mm thick plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined. √

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient accounts for the radiation effects, if any.

Properties The thermal conductivity of plastic cover is given to be $k = 0.15$ W/m·°C.

Analysis In steady operation, the rate of heat transfer from the wire is equal to the heat generated within the wire,

$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ V}) = 80\text{ W}$$



The total thermal resistance is

$$R_{conv} = \frac{1}{h_o A_o} = \frac{1}{(18\text{ W/m}^2.^\circ\text{C})[\pi(0.004\text{ m})(10\text{ m})]} = 0.442\,^\circ\text{C/W}$$

$$R_{plastic} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(2/1)}{2\pi(0.15\text{ W/m.°C})(10\text{ m})} = 0.0735\,^\circ\text{C/W}$$

$$R_{total} = R_{conv} + R_{plastic} = 0.442 + 0.0735 = 0.5156\,^\circ\text{C/W}$$

Then the interface temperature becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total} = 30^\circ\text{C} + (80\text{ W})(0.5156\,^\circ\text{C/W}) = \mathbf{71.2^\circ C}$$

The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.15\text{ W/m.°C}}{18\text{ W/m}^2.^\circ\text{C}} = 0.0083\text{ m} = 8.3\text{ mm}$$

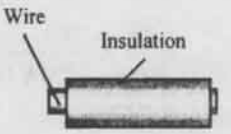
Doubling the thickness of the plastic cover will increase the outer radius of the wire to 3 mm, which is less than the critical radius of insulation. Therefore, doubling the thickness of plastic cover will increase the rate of heat loss and decrease the interface temperature (it can be shown that the interface temperature in this case will be 62.9°C).

15-83E An electrical wire is covered with 0.02-in thick plastic insulation. It is to be determined if the plastic insulation on the wire will increase or decrease heat transfer from the wire.

Assumptions **1** Heat transfer from the wire is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of plastic cover is given to be $k = 0.075$ Btu/h·ft·°F.

Analysis The critical radius of plastic insulation is



$$r_{cr} = \frac{k}{h} = \frac{0.075\ \text{Btu/h.ft.°F}}{2.5\ \text{Btu/h.ft}^2\text{.°F}} = 0.03\ \text{ft} = 0.36\ \text{in} > r_2\ (= 0.0615\ \text{in})$$

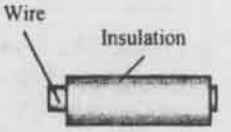
Since the outer radius of the wire with insulation is smaller than critical radius of insulation, plastic insulation will **increase** heat transfer from the wire.

15-84E An electrical wire is covered with 0.02-in thick plastic insulation. By considering the effect of thermal contact resistance, it is to be determined if the plastic insulation on the wire will increase or decrease heat transfer from the wire.

Assumptions **1** Heat transfer from the wire is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant

Properties The thermal conductivity of plastic cover is given to be $k = 0.075$ Btu/h·ft·°F.

Analysis Without insulation, the total thermal resistance is (per ft length of the wire)



$$R_{\text{tot}} = R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(2.5\ \text{Btu/h.ft}^2\text{.°F})[\pi(0.083/12\ \text{ft})(1\ \text{ft})]} = 18.4\ \text{h.°F/Btu}$$

With insulation, the total thermal resistance is

$$R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(2.5\ \text{Btu/h.ft}^2\text{.°F})[\pi(0.123/12\ \text{ft})(1\ \text{ft})]} = 12.42\ \text{h.°F/Btu}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(0.123/0.083)}{2\pi(0.075\ \text{Btu/h.ft.°F})(1\ \text{ft})} = 0.835\ \text{h.°F/Btu}$$



$$R_{\text{interface}} = \frac{h_c}{A_c} = \frac{0.0001\ \text{h.ft}^2\text{.°F/Btu}}{[\pi(0.083/12\ \text{ft})(1\ \text{ft})]} = 0.0046\ \text{h.°F/Btu}$$

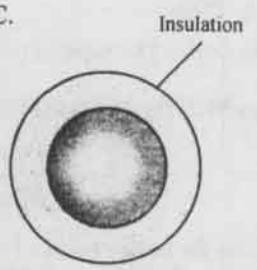
$$R_{\text{total}} = R_{\text{conv}} + R_{\text{plastic}} + R_{\text{interface}} = 12.42 + 0.835 + 0.0046 = 13.26\ \text{h.°F/Btu}$$

Since the total thermal resistance decreases after insulation, plastic insulation **will increase** heat transfer from the wire. The thermal contact resistance appears to have negligible effect in this case.

15-85 A spherical ball is covered with 1-mm thick plastic insulation. It is to be determined if the plastic insulation on the ball will increase or decrease heat transfer from it.

Assumptions **1** Heat transfer from the ball is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of plastic cover is given to be $k = 0.13$ W/m·°C.



Analysis The critical radius of plastic insulation for the spherical ball is

$$r_{cr} = \frac{2k}{h} = \frac{2(0.13\ \text{W/m.°C})}{20\ \text{W/m}^2\text{.°C}} = 0.013\ \text{m} = 13\ \text{mm} > r_2 (= 7\ \text{mm})$$

Since the outer temperature of the ball with insulation is smaller than critical radius of insulation, plastic insulation will **increase** heat transfer from the wire.

Thermal Insulation

15-86C Thermal insulation is a material that is used primarily to provide resistance to heat flow. It differs from other kinds of insulators in that the purpose of an electrical insulator is to halt the flow of electric current, and the purpose of a sound insulator is to slow down the propagation of sound waves.

15-87C In *cold surfaces* such as chilled water lines, refrigerated trucks, and air conditioning ducts, insulation saves energy since the source of "coldness" is *refrigeration* that requires energy input. In this case heat is transferred from the surroundings to the cold surfaces, and the refrigeration unit must now work harder and longer to make up for this heat gain and thus it must consume more electrical energy.

15-88C The *R*-value of insulation is the *thermal resistance* of the insulating material *per unit surface area.* For *flat insulation* the *R*-value is obtained by simply dividing the thickness of the insulation by its thermal conductivity. That is, *R*-value = L/k. Doubling the thickness L doubles the *R*-value of flat insulation.

15-89C The R-value of an insulation represents the thermal resistance of insulation *per unit surface area* (or per unit length in the case of pipe insulation).

15-90C Superinsulations are obtained by using layers of highly reflective sheets separated by glass fibers in an evacuated space. Radiation between two surfaces is inversely proportional to the number of sheets used and thus heat loss by radiation will be very low by using this highly reflective sheets. Evacuating the space between the layers forms a vacuum which minimize conduction or convection through the air space.

15-91C Yes, hair or any other cover reduces heat loss from the head, and thus serves as insulation for the head. The insulating ability of hair or feathers is most visible in birds and hairy animals.

15-92C The primary reasons for insulating are energy conservation, personnel protection and comfort, maintaining process temperature, reducing temperature variation and fluctuations, condensation and corrosion prevention, fire protection, freezing protection, and reducing noise and vibration.

15-93C The *optimum* thickness of insulation is the thickness that corresponds to a minimum combined cost of insulation and heat lost. The cost of insulation increases roughly linearly with thickness while the cost of heat lost decreases exponentially. The total cost, which is the sum of the two, decreases first, reaches a minimum, and then increases. The thickness that corresponds to the minimum total cost is the optimum thickness of insulation, and this is the recommended thickness of insulation to be installed.

15-94 The thickness of flat *R*-8 insulation in SI units is to be determined when the thermal conductivity of the material is known.

Assumptions Thermal properties are constant.

Properties The thermal conductivity of the insulating material is given to be $k = 0.04$ W/m·°C.



Analysis The thickness of flat *R*-8 insulation (in m^2.°C/W) is determined from the definition of *R*-value to be

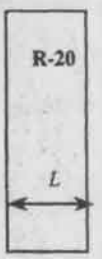
$$R_{value} = \frac{L}{k} \rightarrow L = R_{value}k = (8\ m^2.°C/W)(0.04\ W/m.°C) = \mathbf{0.32\ m}$$

15-95E The thickness of flat *R*-20 insulation in English units is to be determined when the thermal conductivity of the material is known.

Assumptions Thermal properties are constant.

Properties The thermal conductivity of the insulating material is given to be $k = 0.02$ Btu/h·ft·°F.

Analysis The thickness of flat *R*-20 insulation (in h·ft^2·°F/Btu) is determined from the definition of *R*-value to be



$$R_{value} = \frac{L}{k} \rightarrow L = R_{value}k = (20\ h.ft^2.°F/Btu)(0.02\ Btu/h.ft.°F) = \mathbf{0.4\ ft}$$

15-96 A steam pipe is to be covered with enough insulation to reduce the exposed surface temperature to 30°C . The thickness of insulation that needs to be installed is to be determined.

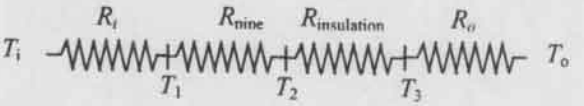
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be k = 52 W/m·°C for cast iron pipe and k = 0.038 W/m·°C for fiberglass insulation.

Analysis The thermal resistance network for this problem involves 4 resistances in series. The inner radius of the pipe is r_1 = 2.0 cm and the outer radius of the pipe and thus the inner radius of insulation is r_2 = 2.3 cm. Letting r_3 represent the outer radius of insulation, the areas of the surfaces exposed to convection for a L = 1 m long section of the pipe become

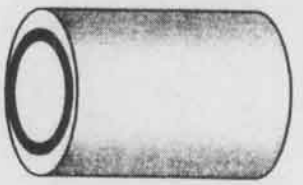
$$A_1 = 2\pi r_1 L = 2\pi(0.02\ \text{m})(1\ \text{m}) = 0.1257\ \text{m}^2$$

$$A_3 = 2\pi r_3 L = 2\pi r_3(1\ \text{m}) = 2\pi r_3\ \text{m}^2 \quad (r_3 \text{ in m})$$



Then the individual thermal resistances are determined to be

$$R_i = R_{conv,1} = \frac{1}{h_i A_1} = \frac{1}{(80\ \text{W/m}^2\cdot^\circ\text{C})(0.1257\ \text{m}^2)} = 0.995\ ^\circ\text{C/W}$$



$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(0.023/0.02)}{2\pi(15\ \text{W/m}\cdot^\circ\text{C})(1\ \text{m})} = 0.00148\ ^\circ\text{C/W}$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(r_3/0.023)}{2\pi(0.038\ \text{W/m}\cdot^\circ\text{C})(1\ \text{m})} = 4.188\ln(r_3/0.023)\ ^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_o A_3} = \frac{1}{(22\ \text{W/m}^2\cdot^\circ\text{C})(2\pi r_3\ \text{m}^2)} = \frac{1}{138.2 r_3}\ ^\circ\text{C/W}$$

Noting that all resistances are in series, the total resistance is

$$R_{total} = R_i + R_1 + R_2 + R_o = 0.995 + 0.00148 + 4.188\ln(r_3/0.023) + 1/(138.2r_3)\ ^\circ\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_i - T_o}{R_{total}} = \frac{(110-22)^\circ\text{C}}{[0.995 + 0.00148 + 4.188\ln(r_3/0.023) + 1/(138.2r_3)]^\circ\text{C/W}}$$

Noting that the outer surface temperature of insulation is specified to be 30°C, the rate of heat loss can also be expressed as

$$\dot{Q} = \frac{T_3 - T_o}{R_o} = \frac{(30-22)^\circ\text{C}}{1/(138.2r_3)^\circ\text{C/W}} = 1106 r_3$$

Setting the two relations above equal to each other and solving for r_3 gives r_3 = 0.0314 m. Then the minimum thickness of fiberglass insulation required is

$$t = r_3 - r_2 = 0.0314 - 0.0230 = 0.0084\ \text{m} = \mathbf{0.84\ cm}$$

Therefore, insulating the pipe with at least 0.84 cm thick fiberglass insulation will ensure that the outer surface temperature of the pipe will be at 30°C or below.

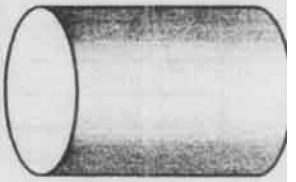
15-97 A cylindrical oven is to be insulated to reduce heat losses. The optimum thickness of insulation and the amount of money saved per year are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulation is one-dimensional. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** The surface temperature of the furnace and the heat transfer coefficient remain constant. **6** The surfaces of the cylindrical oven can be treated as plain surfaces since its diameter is greater than 1 m.

Properties The thermal conductivity of insulation is given to be $k = 0.038$ W/m·°C.

Analysis We treat the surfaces of this cylindrical furnace as plain surfaces since its diameter is greater than 1 m, and disregard the curvature effects. The exposed surface area of the furnace is

$$A = 2A_{\text{base}} + A_{\text{side}} = 2\pi r^2 + 2\pi rL = 2\pi(1.5\text{ m})^2 + 2\pi(1.5\text{ m})(6\text{ m}) = 70.69\text{ m}^2$$

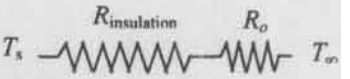


The rate of heat loss from the furnace before the insulation is installed is

$$\dot{Q} = h_o A(T_s - T_\infty) = (30\text{ W/m}^2.^\circ\text{C})(70.69\text{ m}^2)(90-27)^\circ\text{C} = 133{,}600\text{ W}$$

Noting that the plant operates 52×24 = 4160 h/yr, the annual heat lost from the furnace is

$$Q = \dot{Q}\Delta t = (133.6\text{ kJ/s})(4160\times 3600\text{ s/yr}) = 2.001\times 10^9\text{ kJ/yr}$$



The efficiency of the furnace is given to be 78 percent. Therefore, to generate this much heat, the furnace must consume energy (in the form of natural gas) at a rate of

$$Q_{\text{in}} = Q/\eta_{\text{oven}} = (2.001\times 10^9\text{ kJ/yr})/0.78 = 2.565\times 10^9\text{ kJ/yr} = 24{,}314\text{ therms/yr}$$

since 1 therm = 105,500 kJ. Then the annual fuel cost of this furnace before insulation becomes

$$\text{Annual Cost} = \text{Q}_{\text{in}} \times \text{Unit cost} = (24{,}314\text{ therm/yr})(\$0.50/\text{therm}) = \$12{,}157/\text{yr}$$

We expect the surface temperature of the furnace to increase, and the heat transfer coefficient to decrease somewhat when insulation is installed. We assume these two effects to counteract each other. Then the rate of heat loss for 1-cm thick insulation becomes

$$\dot{Q}_{\text{ins}} = \frac{T_s - T_\infty}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{A(T_s - T_\infty)}{\frac{t_{\text{ins}}}{k_{\text{ins}}} + \frac{1}{h_o}} = \frac{(70.69\text{ m}^2)(90-27)^\circ\text{C}}{\frac{0.01\text{ m}}{0.038\text{ W/m.}^\circ\text{C}} + \frac{1}{30\text{ W/m}^2.^\circ\text{C}}} = 15{,}021\text{ W}$$

Also, the total amount of heat loss from the furnace per year and the amount and cost of energy consumption of the furnace become

$$Q_{\text{ins}} = \dot{Q}_{\text{ins}}\Delta t = (15.021\text{ kJ/s})(4160\times 3600\text{ s/yr}) = 2.249\times 10^8\text{ kJ/yr}$$

$$Q_{\text{in,ins}} = Q_{\text{ins}}/\eta_{\text{oven}} = (2.249\times 10^8\text{ kJ/yr})/0.78 = 2.884\times 10^8\text{ kJ/yr} = 2734\text{ therms}$$

$$\text{Annual Cost} = \text{Q}_{\text{in,ins}} \times \text{Unit cost} = (2734\text{ therm/yr})(\$0.50/\text{therm}) = \$1367/\text{yr}$$

Cost savings = Energy cost w/o insulation - Energy cost w/insulation = 12,157 - 1367 = $10,790/yr

The unit cost of insulation is given to be $10/m^2 per cm thickness, plus $30/m^2 for labor. Then the total cost of insulation becomes

$$\text{Insulation Cost} = (\text{Unit cost})(\text{Surface area}) = [(\$10/\text{cm})(1\text{ cm}) + \$30/\text{m}^2](70.69\text{ m}^2) = \$2828$$

To determine the thickness of insulation whose cost is equal to annual energy savings, we repeat the calculations above for 2, 3, ... 15 cm thick insulations, and list the results in the table below.

Insulation thickness	Rate of heat loss W	Cost of heat lost $/yr	Cost savings $/yr	Insulation cost $
0 cm	133,600	12,157	0	0
1 cm	15,021	1367	10,790	2828
5 cm	3301	300	11,850	3535
10 cm	1671	152	12,005	9189
11 cm	1521	138	12,019	9897
12 cm	1396	127	12,030	10,604
13 cm	1289	117	12,040	11,310
14 cm	1198	109	12,048	12,017
15 cm	1119	102	12,055	12,724

Therefore, the thickest insulation that will pay for itself in one year is the one whose thickness is **14 cm**.

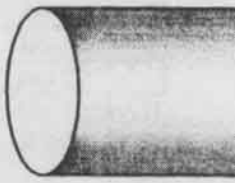
15-98 A cylindrical oven is to be insulated to reduce heat losses. The optimum thickness of insulation and the amount of money saved per year are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulation is one-dimensional. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** The surface temperature of the furnace and the heat transfer coefficient remain constant. **6** The surfaces of the cylindrical oven can be treated as plain surfaces since its diameter is greater than 1 m.

Properties The thermal conductivity of insulation is given to be $k = 0.038$ W/m·°C.

Analysis We treat the surfaces of this cylindrical furnace as plain surfaces since its diameter is greater than 1 m, and disregard the curvature effects. The exposed surface area of the furnace is

$$A = 2A_{\text{base}} + A_{\text{side}} = 2\pi r^2 + 2\pi rL = 2\pi(1.5\text{ m})^2 + 2\pi(1.5\text{ m})(6\text{ m}) = 70.69\text{ m}^2$$

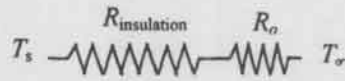


The rate of heat loss from the furnace before the insulation is installed is

$$\dot{Q} = h_o A(T_s - T_\infty) = (30\text{ W/m}^2\cdot\text{°C})(70.69\text{ m}^2)(70-27)\text{°C} = 101{,}794\text{ W}$$

Noting that the plant operates 52×24 = 4160 h/yr, the annual heat lost from the furnace is

$$Q = \dot{Q}\Delta t = (101.794\text{ kJ/s})(4160\times 3600\text{ s/yr}) = 1.524\times 10^9\text{ kJ/yr}$$



The efficiency of the furnace is given to be 78 percent. Therefore, to generate this much heat, the furnace must consume energy (in the form of natural gas) at a rate of

$$Q_{\text{in}} = Q/\eta_{\text{oven}} = (1.524\times 10^9\text{ kJ/yr})/0.78 = 1.954\times 10^9\text{ kJ/yr} = 18{,}526\text{ therms/yr}$$

since 1 therm = 105,500 kJ. Then the annual fuel cost of this furnace before insulation becomes

$$\text{Annual Cost} = \text{Q}_{\text{in}} \times \text{Unit cost} = (18{,}526\text{ therm/yr})(\$0.50/\text{therm}) = \$9{,}263/\text{yr}$$

We expect the surface temperature of the furnace to increase, and the heat transfer coefficient to decrease somewhat when insulation is installed. We assume these two effects to counteract each other. Then the rate of heat loss for 1-cm thick insulation becomes

$$\dot{Q}_{\text{ins}} = \frac{T_s - T_\infty}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{A(T_s - T_\infty)}{\frac{t_{\text{ins}}}{k_{\text{ins}}} + \frac{1}{h_o}} = \frac{(70.69\text{ m}^2)(75-27)\text{°C}}{\frac{0.01\text{ m}}{0.038\text{ W/m.°C}} + \frac{1}{30\text{ W/m}^2\text{.°C}}} = 11{,}445\text{ W}$$

Also, the total amount of heat loss from the furnace per year and the amount and cost of energy consumption of the furnace become

$$Q_{\text{ins}} = \dot{Q}_{\text{ins}}\Delta t = (11.445\text{ kJ/s})(4160\times 3600\text{ s/yr}) = 1.714\times 10^8\text{ kJ/yr}$$

$$Q_{\text{in,ins}} = Q_{\text{ins}}/\eta_{\text{oven}} = (1.714\times 10^8\text{ kJ/yr})/0.78 = 2.197\times 10^8\text{ kJ/yr} = 2082\text{ therms}$$

$$\text{Annual Cost} = Q_{\text{in,ins}} \times \text{Unit cost} = (2082\text{ therm/yr})(\$0.50/\text{therm}) = \$1041/\text{yr}$$

Cost savings = Energy cost w/o insulation - Energy cost w/insulation = 9263 - 1041 = $8227/yr

The unit cost of insulation is given to be $10/m^2 per cm thickness, plus $30/m^2 for labor. Then the total cost of insulation becomes

$$\text{Insulation Cost} = (\text{Unit cost})(\text{Surface area}) = [(\$10/\text{cm})(1\text{ cm}) + \$30/\text{m}^2](70.69\text{ m}^2) = \$2828$$

To determine the thickness of insulation whose cost is equal to annual energy savings, we repeat the calculations above for 2, 3, ... 15 cm thick insulations, and list the results in the table below.

Insulation Thickness	Rate of heat loss W	Cost of heat lost $/yr	Cost savings $/yr	Insulation cost $
0 cm	101,794	9263	0	0
1 cm	11,445	1041	8222	2828
5 cm	2515	228	9035	3535
9 cm	**1413**	**129**	**9134**	**8483**
10 cm	1273	116	9147	9189
11 cm	1159	105	9158	9897
12 cm	1064	97	9166	10,604

Therefore, the thickest insulation that will pay for itself in one year is the one whose thickness is **9 cm**. The 10-cm thick insulation will come very close to paying for itself in one year.

15-99E Steam is flowing through an insulated steel pipe, and it is proposed to add another 1-in thick layer of fiberglass insulation on top of the existing one to reduce the heat losses further and to save energy and money. It is to be determined if the new insulation will pay for itself within 2 years.

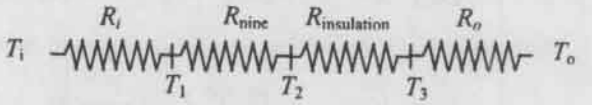
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The heat transfer coefficients remain constant. **5** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be k = 8.7 Btu/h·ft·°F for steel pipe and k = 0.020 Btu/h·ft·°F for fiberglass insulation.

Analysis The inner radius of the pipe is r_1 = 1.75 in, the outer radius of the pipe is r_2 = 2 in, and the outer radii of the existing and proposed insulation layers are r_3 = 3 in and 4 in, respectively. Considering a unit pipe length of L = 1 ft, the individual thermal resistances are determined to be

$$R_i = R_{\text{conv},1} = \frac{1}{h_i A_1} = \frac{1}{h_i(2\pi r_1 L)} = \frac{1}{(30\ \text{Btu/h.ft}^2\text{.°F})[2\pi(1.75/12\ \text{ft})(1\ \text{ft})]} = 0.0364\ \text{h.°F/Btu}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2/1.75)}{2\pi(8.7\ \text{Btu/h.ft.°F})(1\ \text{ft})} = 0.00244\ \text{h.°F/Btu}$$



Current Case:

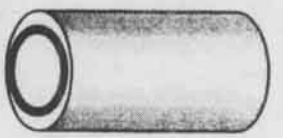
$$R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_{\text{ins}} L} = \frac{\ln(3/2)}{2\pi(0.020\ \text{Btu/h.ft.°F})(1\ \text{ft})} = 3.227\ \text{h.°F/Btu}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_o A_3} = \frac{1}{h_o(2\pi r_3)} = \frac{1}{(5\ \text{Btu/h.ft}^2\text{.°F})[2\pi(3/12\ \text{ft})(1\ \text{ft})]} = 0.1273\ \text{h.°F/Btu}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q}_{\text{current}} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_i - T_o}{R_i + R_{\text{pipe}} + R_{\text{ins}} + R_o} = \frac{(400-60)°\text{F}}{(0.0364+0.00244+3.227+0.1273)\ \text{h.°F/Btu}} = 100.2\ \text{Btu/h}$$

Proposed Case:



$$R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_{ins} L} = \frac{\ln(4/2)}{2\pi(0.020\ \text{Btu/h.ft.°F})(1\ \text{ft})} = 5.516\ \text{h.°F/Btu}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_o A_3} = \frac{1}{h_o(2\pi r_3)} = \frac{1}{(5\ \text{Btu/h.ft}^2\text{.°F})[2\pi(4/12\ \text{ft})(1\ \text{ft})]} = 0.0955\ \text{h.°F/Btu}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q}_{\text{prop}} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_i - T_o}{R_i + R_{\text{pipe}} + R_{\text{ins}} + R_o} = \frac{(400-60)°\text{F}}{(0.0364+0.00244+5.516+0.0955)\ \text{h.°F/Btu}} = 60.2\ \text{Btu/h}$$

Therefore, the amount of energy and money saved by the additional insulation per year are

$$\dot{Q}_{\text{saved}} = \dot{Q}_{\text{prop}} - \dot{Q}_{\text{current}} = 100.2 - 60.2 = 40.0\ \text{Btu/h}$$

$$Q_{\text{saved}} = \dot{Q}_{\text{saved}}\Delta t = (40.0\ \text{Btu/h})(8760\ \text{h/yr}) = 350{,}400\ \text{Btu/yr}$$

$$\text{Money saved} = Q_{\text{saved}} \times (\text{Unit cost}) = (350{,}400\ \text{Btu/yr})(\$0.01/1000\ \text{Btu}) = \$3.504/\text{yr}$$

or $7.01 per 2 years, which is barely more than the $7.0 minimum required. But the criteria is satisfied, and the proposed additional insulation is **justified**.

15-100 The plumbing system of a plant involves some section of a plastic pipe exposed to the ambient air. The pipe is to be insulated with adequate fiber glass insulation to prevent freezing of water in the pipe. The thickness of insulation that will protect the water from freezing under worst conditions is to be determined.

Assumptions **1** Heat transfer is transient, but can be treated as steady at average conditions. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The water in the pipe is stationary, and its initial temperature is 15°C. **5** The thermal contact resistance at the interface is negligible. **6** The convection resistance inside the pipe is negligible.

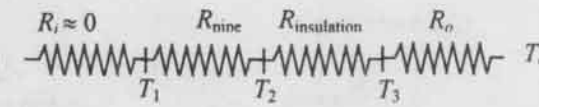
Properties The thermal conductivities are given to be k = 0.16 W/m·°C for plastic pipe and k = 0.035 W/m·°C for fiberglass insulation. The density and specific heat of water are to be ρ = 1000 kg/m^3 and C_p = 4.18 kJ/kg.°C (Table A-15).

Analysis The inner radius of the pipe is r_1 = 3.0 cm and the outer radius of the pipe and thus the inner radius of insulation is r_2 = 3.3 cm. We let r_3 represent the outer radius of insulation. Considering a 1-m section of the pipe, the amount of heat that must be transferred from the water as it cools from 15 to 0°C is determined to be

$$m = \rho V = \rho(2\pi r_1 L) = (1000 \text{ kg/m}^3)[2\pi(0.03 \text{ m})(1 \text{ m})] = 2.827 \text{ kg}$$
$$Q_{\text{total}} = mC_p\Delta T = (2.827 \text{ kg})(4.18 \text{ kJ/kg.°C})(15-0)°\text{C} = 177.3 \text{ kJ}$$

Then the average rate of heat transfer during 60 h becomes

$$\dot{Q}_{\text{ave}} = \frac{Q_{\text{total}}}{\Delta t} = \frac{177{,}300 \text{ J}}{(60 \times 3600 \text{ s})} = 0.821 \text{ W}$$

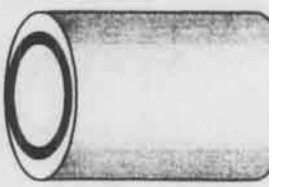


The individual thermal resistances are

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_{\text{pipe}} L} = \frac{\ln(0.033/0.03)}{2\pi(0.16 \text{ W/m.°C})(1 \text{ m})} = 0.0948 \text{ °C/W}$$

$$R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(r_3/0.033)}{2\pi(0.035 \text{ W/m.°C})(1 \text{ m})} = 4.55\ln(r_3/0.033) \text{ °C/W}$$

$$R_o = R_{\text{conv}} = \frac{1}{h_o A_3} = \frac{1}{(30 \text{ W/m}^2\text{.°C})(2\pi r_3 \text{ m}^2)} = \frac{1}{188.5 r_3} \text{ °C/W}$$



Then the rate of average heat transfer from the water can be expressed as

$$\dot{Q} = \frac{T_{i,ave} - T_o}{R_{\text{total}}} \rightarrow 0.821 \text{ W} = \frac{[7.5-(-10)]°\text{C}}{[0.0948 + 4.55\ln(r_3/0.033) + 1/(188.5r_3)]°\text{C/W}} \rightarrow r_3 = 3.50 \text{ m}$$

Therefore, the minimum thickness of fiberglass needed to protect the pipe from freezing is

$$t = r_3 - r_2 = 3.50 - 0.033 = \mathbf{3.467 \text{ m}}$$

which is too large. Installing such a thick insulation is not practical, however, and thus other freeze protection methods should be considered.

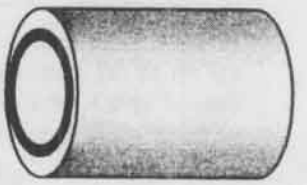
15-101 The plumbing system of a plant involves some section of a plastic pipe exposed to the ambient air. The pipe is to be insulated with adequate fiber glass insulation to prevent freezing of water in the pipe. The thickness of insulation that will protect the water from freezing more than 20% under worst conditions is to be determined.

Assumptions **1** Heat transfer is transient, but can be treated as steady at average conditions. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The water in the pipe is stationary, and its initial temperature is 15°C. **5** The thermal contact resistance at the interface is negligible. **6** The convection resistance inside the pipe is negligible.

Properties The thermal conductivities are given to be k = 0.16 W/m·°C for plastic pipe and k = 0.035 W/m·°C for fiberglass insulation. The density and specific heat of water are to be ρ = 1000 kg/m^3 and C_p = 4.18 kJ/kg.°C (Table A-15).

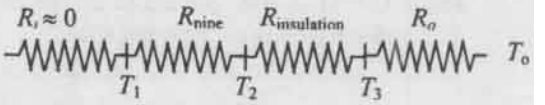
Analysis The inner radius of the pipe is r_1 = 3.0 cm and the outer radius of the pipe and thus the inner radius of insulation is r_2 = 3.3 cm. We let r_3 represent the outer radius of insulation. The latent heat of freezing of water is 33.7 kJ/kg. Considering a 1-m section of the pipe, the amount of heat that must be transferred from the water as it cools from 15 to 0°C is determined to be

$$m = \rho V = \rho(2\pi r_1 L) = (1000 \text{ kg/m}^3)[2\pi(0.03 \text{ m})(1 \text{ m})] = 2.827 \text{ kg}$$
$$Q_{\text{cooling}} = mC_p\Delta T = (2.827 \text{ kg})(4.18 \text{ kJ/kg.°C})(15-0)°\text{C} = 177.3 \text{ kJ}$$
$$Q_{\text{freezing}} = 0.2 \times mh_{if} = 0.2 \times (2.827 \text{ kg})(333.7 \text{ kJ/kg}) = 188.7 \text{ kJ}$$
$$Q_{\text{total}} = Q_{\text{cooling}} + Q_{\text{freezing}} = 177.3 + 188.7 = 366.0 \text{ kJ}$$



Then the average rate of heat transfer during 60 h becomes

$$\dot{Q}_{\text{ave}} = \frac{Q_{\text{total}}}{\Delta t} = \frac{366{,}000 \text{ J}}{(60 \times 3600 \text{ s})} = 1.694 \text{ W}$$



The individual thermal resistances are

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_{\text{pipe}} L} = \frac{\ln(0.033/0.03)}{2\pi(0.16 \text{ W/m.°C})(1 \text{ m})} = 0.0948 \text{ °C/W}$$

$$R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(r_3/0.033)}{2\pi(0.035 \text{ W/m.°C})(1 \text{ m})} = 4.55\ln(r_3/0.033) \text{ °C/W}$$

$$R_o = R_{\text{conv}} = \frac{1}{h_o A_3} = \frac{1}{(30 \text{ W/m}^2\text{.°C})(2\pi r_3 \text{ m}^2)} = \frac{1}{188.5 r_3} \text{ °C/W}$$

Then the rate of average heat transfer from the water can be expressed as

$$\dot{Q} = \frac{T_{i,ave} - T_o}{R_{\text{total}}} \rightarrow 1.694 \text{ W} = \frac{[7.5-(-10)]°\text{C}}{[0.0948 + 4.55\ln(r_3/0.033) + 1/(188.5r_3)]°\text{C/W}} \rightarrow r_3 = 0.312 \text{ m}$$

Therefore, the minimum thickness of fiberglass needed to protect the pipe from freezing is

$$t = r_3 - r_2 = 0.312 - 0.033 = \mathbf{0.279 \text{ m}}$$

which is too large. Installing such a thick insulation is not practical, however, and thus other freeze protection methods should be considered.

Heat Transfer From Finned Surfaces

15-102C Increasing the rate of heat transfer from a surface by increasing the heat transfer surface area.

15-103C The fin efficiency is defined as the ratio of actual heat transfer rate from the fin to the ideal heat transfer rate from the fin if the entire fin were at base temperature, and its value is between 0 and 1. Fin effectiveness is defined as the ratio of heat transfer rate from a finned surface to the heat transfer rate from the same surface if there were no fins, and its value is expected to be greater than 1.

15-104C Heat transfer rate will decrease since a fin effectiveness smaller than 1 indicates that the fin acts as insulation.

15-105C Fins enhance heat transfer from a surface by increasing heat transfer surface area for convection heat transfer. However, adding too many fins on a surface can suffocate the fluid and retard convection, and thus it may cause the overall heat transfer coefficient and heat transfer to decrease.

15-106C Effectiveness of a single fin is the ratio of the heat transfer rate from the entire exposed surface of the fin to the heat transfer rate from the fin base area. The overall effectiveness of a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins.

15-107C Fins should be attached on the air side since the convection heat transfer coefficient is lower on the air side than it is on the water side.

15-108C Fins should be attached to the outside since the heat transfer coefficient inside the tube will be higher due to forced convection. Fins should be added to both sides of the tubes when the convection coefficients at the inner and outer surfaces are comparable in magnitude.

15-109C Welding or tight fitting introduces thermal contact resistance at the interface, and thus retards heat transfer. Therefore, the fins formed by casting or extrusion will provide greater enhancement in heat transfer.

15-110C If the fin is too long, the temperature of the fin tip will approach the surrounding temperature and we can neglect heat transfer from the fin tip. Also, if the surface area of the fin tip is very small compared to the total surface area of the fin, heat transfer from the tip can again be neglected.

15-111C Increasing the length of a fin decreases its efficiency but increases its effectiveness.

15-112C Increasing the diameter of a fin will increase its efficiency but decrease its effectiveness.

15-113C The thicker fin will have higher efficiency; the thinner one will have higher effectiveness.

15-114C The fin with the lower heat transfer coefficient will have the higher efficiency and the higher effectiveness.

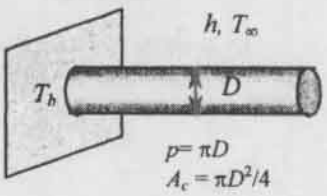
15-115 A relation is to be obtained for the fin efficiency for a fin of constant cross-sectional area A_s, perimeter p, length L, and thermal conductivity k exposed to convection to a medium at T_∞ with a heat transfer coefficient h. The relation is to be simplified for circular fin of diameter D and for a rectangular fin of thickness t.

Assumptions **1** The fins are sufficiently long so that the temperature of the fin at the tip is nearly T_∞. **2** Heat transfer from the fin tips is negligible.

Analysis Taking the temperature of the fin at the base to be T_b and using the heat transfer relation for a long fin, fin efficiency for long fins can be expressed as

$$\eta_{\text{fin}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin}\\ \text{if the entire fin were at base temperature}}$$

$$= \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} = \frac{\sqrt{hpkA_c}}{hpL} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}}$$



This relation can be simplified for a circular fin of diameter D and rectangular fin of thickness t and width w to be

$$\eta_{\text{fin,circular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(\pi D^2/4)}{(\pi D)h}} = \frac{1}{2L}\sqrt{\frac{kD}{h}}$$

$$\eta_{\text{fin,rectangular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(wt)}{2(w+t)h}} \cong \frac{1}{L}\sqrt{\frac{k(wt)}{2wh}} = \frac{1}{L}\sqrt{\frac{kt}{2h}}$$

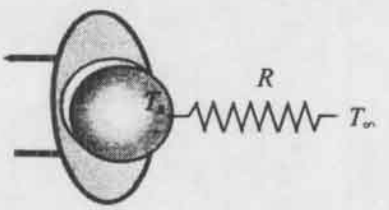
15-116 The maximum power rating of a transistor whose case temperature is not to exceed 80°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 80°C.

Properties The case-to-ambient thermal resistance is given to be 20°C / W.

Analysis The maximum power at which this transistor can be operated safely is

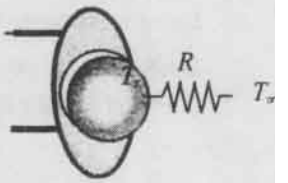
$$\dot{Q} = \frac{\Delta T}{R_{\text{case-ambient}}} = \frac{T_{\text{case}} - T_\infty}{R_{\text{case-ambient}}} = \frac{(80-30)\,^\circ\text{C}}{25\,^\circ\text{C/W}} = \mathbf{2\ W}$$



15-117 A commercially available heat sink from Table 15-7 is to be selected to keep the case temperature of a transistor below 90°C in an environment at 20°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 90°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The thermal resistance between the transistor attached to the sink and the ambient air is determined to be



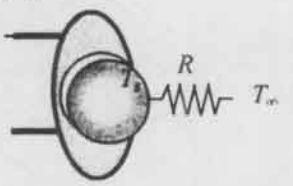
$$\dot{Q} = \frac{\Delta T}{R_{\text{case-ambient}}} \longrightarrow R_{\text{case-ambient}} = \frac{T_{\text{transistor}} - T_\infty}{\dot{Q}} = \frac{(90-20)^\circ\text{C}}{40\ \text{W}} = \mathbf{1.75\ ^\circ C/W}$$

The thermal resistance of the heat sink must be below 1.75 °C/W. Table 15-7 reveals that HS6071 in vertical position, HS5030 and HS6115 in both horizontal and vertical position can be selected.

15-118 A commercially available heat sink from Table 15-7 is to be selected to keep the case temperature of a transistor below 80°C in an environment at 35°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 80°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The thermal resistance between the transistor attached to the sink and the ambient air is determined to be



$$\dot{Q} = \frac{\Delta T}{R_{\text{case-ambient}}} \longrightarrow R_{\text{case-ambient}} = \frac{T_{\text{transistor}} - T_\infty}{\dot{Q}} = \frac{(80-35)^\circ\text{C}}{30\ \text{W}} = \mathbf{1.5\ ^\circ C/W}$$

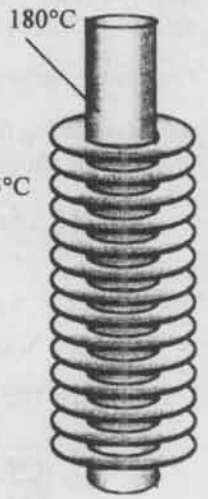
The thermal resistance of the heat sink must be below 1.5 °C/W. Table 15-7 reveals that HS5030 in both horizontal and vertical positions, HS6071 in vertical position, and HS6115 in both horizontal and vertical positions can be selected.

15-119 Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be $k = 186$ W/m·°C.

Analysis In case of no fins, heat transfer from the tube per meter of its length is



$$A_{\text{no fin}} = \pi D_1 L = \pi(0.05\text{ m})(1\text{ m}) = 0.1571\text{ m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (40\text{ W/m}^2.^\circ\text{C})(0.1571\text{ m}^2)(180-25)^\circ\text{C} = 974\text{ W}$$

The efficiency of these circular fins can be determined from Fig. 15-70 to be

$$\left.\begin{aligned} L &= (D_2 - D_1)/2 = (0.06-0.05)/2 = 0.005\text{ m} \\ \frac{r_2 + (t/2)}{r_1} &= \frac{0.03+(0.001/2)}{0.025} = 1.22 \\ \left(L+\frac{t}{2}\right)\sqrt{\frac{h}{kt}} &= \left(0.005+\frac{0.001}{2}\right)\sqrt{\frac{40\text{ W/m}^2\,^\circ\text{C}}{(186\text{ W/m}^\circ\text{C})(0.001\text{ m})}} = 0.08 \end{aligned}\right\} \eta_{\text{fin}} = 0.97$$

Heat transfer from a single fin is

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi(0.03^2 - 0.025^2) + 2\pi(0.03)(0.001) = 0.00192\text{ m}^2$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= 0.97(40\text{ W/m}^2.^\circ\text{C})(0.00192\text{ m}^2)(180-25)^\circ\text{C}$$
$$= 11.53\text{ W}$$

Heat transfer from a single unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 s = \pi(0.025\text{ m})(0.003\text{ m}) = 0.000236\text{ m}^2$$

$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty) = (40\text{ W/m}^2.^\circ\text{C})(0.000236\text{ m}^2)(180-25)^\circ\text{C} = 1.46\text{ W}$$

There are 250 fins and thus 250 interfin spacings per meter length of the tube. The total heat transfer from the finned tube is then determined from

$$\dot{Q}_{\text{total,fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 250(11.53 + 1.46) = 3248\text{ W}$$

Therefore the increase in heat transfer from the tube per meter of its length as a result of the addition of the fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total,fin}} - \dot{Q}_{\text{no fin}} = 3248 - 974 = \mathbf{2274\text{ W}}$$

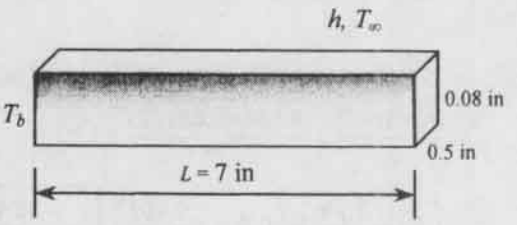
15-120E The handle of a stainless steel spoon partially immersed in boiling water extends 7 in. in the air from the free surface of the water. The temperature difference across the exposed surface of the spoon handle is to be determined.

Assumptions **1** The temperature of the submerged portion of the spoon is equal to the water temperature. **2** The temperature in the spoon varies in the axial direction only (along the spoon), $T(x)$. **3** The heat transfer from the tip of the spoon is negligible. **4** The heat transfer coefficient is constant and uniform over the entire spoon surface. **5** The thermal properties of the spoon are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the spoon..

Properties The thermal conductivity of the spoon is given to be $k = 8.7$ Btu/h·ft·°F.

Analysis Noting that the cross-sectional area of the spoon is constant and measuring x from the free surface of water, the variation of temperature along the spoon can be expressed as

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh a(L-x)}{\cosh aL}$$



where

$$p = 2(0.5/12 \text{ ft} + 0.08/12 \text{ ft}) = 0.0967 \text{ ft}$$

$$A_c = (0.5/12 \text{ ft})(0.08/12 \text{ ft}) = 0.000278 \text{ ft}^2$$

$$a=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{(3 \text{ Btu/h.ft}^2\text{.°F})(0.0967 \text{ ft})}{(8.7 \text{ Btu/h.ft.°F})(0.000278 \text{ ft}^2)}}=10.95 \text{ ft}^{-1}$$

Noting that $x = L = 7/12 = 0.583$ ft at the tip and substituting, the tip temperature of the spoon is determined to be

$$T(L)=T_\infty+(T_b-T_\infty)\frac{\cosh a(L-L)}{\cosh aL}$$

$$=75°\text{F}+(200-75)\frac{\cosh 0}{\cosh(10.95\times 0.583)}=75°\text{F}+(200-75)\frac{1}{296}=75.4°\text{F}$$

Therefore, the temperature difference across the exposed section of the spoon handle is

$$\Delta T = T_b - T_{\text{tip}} = (200-75.4)°\text{F} = \mathbf{124.6°F}$$

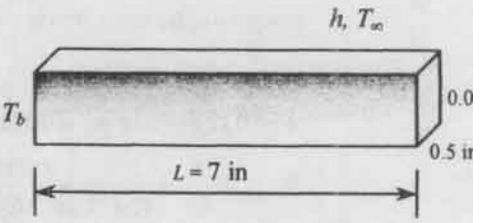
15-121E The handle of a silver spoon partially immersed in boiling water extends 7 in. in the air from the free surface of the water. The temperature difference across the exposed surface of the spoon handle is to be determined.

Assumptions **1** The temperature of the submerged portion of the spoon is equal to the water temperature. **2** The temperature in the spoon varies in the axial direction only (along the spoon), $T(x)$. **3** The heat transfer from the tip of the spoon is negligible. **4** The heat transfer coefficient is constant and uniform over the entire spoon surface. **5** The thermal properties of the spoon are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the spoon..

Properties The thermal conductivity of the spoon is given to be $k = 247$ Btu/h·ft·°F.

Analysis Noting that the cross-sectional area of the spoon is constant and measuring x from the free surface of water, the variation of temperature along the spoon can be expressed as

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh a(L-x)}{\cosh aL}$$



where

$$p = 2(0.5/12 \text{ ft} + 0.08/12 \text{ ft}) = 0.0967 \text{ ft}$$

$$A_c = (0.5/12 \text{ ft})(0.08/12 \text{ ft}) = 0.000278 \text{ ft}^2$$

$$a=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{(3 \text{ Btu/h.ft}^2\text{.°F})(0.0967 \text{ ft})}{(247 \text{ Btu/h.ft.°F})(0.000278 \text{ ft}^2)}}=2.055 \text{ ft}^{-1}$$

Noting that $x = L = 0.7/12=0.583$ ft at the tip and substituting, the tip temperature of the spoon is determined to be

$$T(L)=T_\infty+(T_b-T_\infty)\frac{\cosh a(L-L)}{\cosh aL}$$

$$=75°\text{F}+(200-75)\frac{\cosh 0}{\cosh(2.055\times 0.583)}=75°\text{F}+(200-75)\frac{1}{1.81}=\mathbf{144.1°F}$$

Therefore, the temperature difference across the exposed section of the spoon handle is

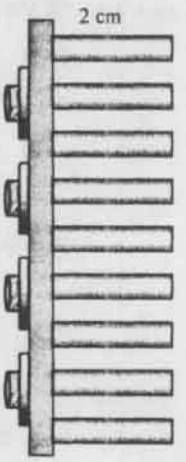
$$\Delta T = T_b - T_{\text{tip}} = (200-144.1)°\text{C} = \mathbf{55.9°F}$$

15-122 A circuit board houses 80 logic chips on one side, dissipating 0.04 W each through the back side of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 864 aluminum pin fins on the back surface.

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the back side of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivities are given to be $k = 20$ W/m·°C for the circuit board, $k = 237$ W/m·°C for the aluminum plate and fins, and $k = 1.8$ W/m·°C for the epoxy adhesive.

Analysis (a) The total rate of heat transfer dissipated by the chips is



$$\dot{Q} = 80 \times (0.04 \text{ W}) = 3.2 \text{ W}$$

The individual resistances are

$$A = (0.12 \text{ m})(0.18 \text{ m}) = 0.0216 \text{ m}^2$$

$$R_{\text{board}} = \frac{L}{kA} = \frac{0.003 \text{ m}}{(20 \text{ W/m.°C})(0.0216 \text{ m}^2)} = 0.00694 \text{ °C/W}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(50 \text{ W/m}^2\text{.°C})(0.0216 \text{ m}^2)} = 0.9259 \text{ °C/W}$$

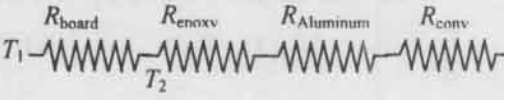
$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.00694 + 0.9259 = 0.93284 \text{ °C/W}$$

The temperatures on the two sides of the circuit board are

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40°\text{C} + (3.2 \text{ W})(0.93284 \text{ °C/W}) = \mathbf{43.0°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 43.0°\text{C} - (3.2 \text{ W})(0.00694 \text{ °C/W}) = 40.5 - 0.02 \cong \mathbf{43.0°C}$$

Therefore, the board is nearly isothermal.



(b) Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(50 \text{ W/m}^2\text{.°C})}{(237 \text{ W/m.°C})(0.0025 \text{ m})}} = 18.37 \text{ m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(18.37 \text{ m}^{-1} \times 0.02 \text{ m})}{18.37 \text{ m}^{-1} \times 0.02 \text{ m}} = 0.957$$

The fins can be assumed to be at base temperature provided that the fin area is modified by multiplying it by 0.957. Then the various thermal resistances are

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0002 \text{ m}}{(1.8 \text{ W/m.°C})(0.0216 \text{ m}^2)} = 0.0051 \text{ °C/W}$$

$$R_{\text{Al}} = \frac{L}{kA} = \frac{0.002 \text{ m}}{(237 \text{ W/m.°C})(0.0216 \text{ m}^2)} = 0.00039 \text{ °C/W}$$

$$A_{\text{finned}} = \eta_{\text{fin}} n \pi D L = 0.957 \times 864 \pi (0.0025 \text{ m})(0.02 \text{ m}) = 0.130 \text{ m}^2$$

$$A_{\text{unfinned}} = 0.0216 - 864 \frac{\pi D^2}{4} = 0.0216 - 864 \times \frac{\pi (0.0025)^2}{4} = 0.0174 \text{ m}^2$$

$$A_{\text{total,with fins}} = A_{\text{finned}} + A_{\text{unfinned}} = 0.130 + 0.017 = 0.147 \text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{h A_{\text{total,with fins}}} = \frac{1}{(50 \text{ W/m}^2.^\circ\text{C})(0.147 \text{ m}^2)} = 0.1361 \,^\circ\text{C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{epoxy}} + R_{\text{aluminum}} + R_{\text{conv}}$$

$$= 0.00694 + 0.0051 + 0.00039 + 0.1361 = 0.1484 \,^\circ\text{C/W}$$

Then the temperatures on the two sides of the circuit board becomes

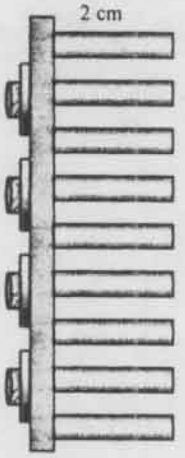
$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q} R_{\text{total}} = 40^\circ\text{C} + (3.2 \text{ W})(0.1484 \,^\circ\text{C/W}) = \mathbf{40.5^\circ C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q} R_{\text{board}} = 40.5^\circ\text{C} - (3.2 \text{ W})(0.00694 \,^\circ\text{C/W}) = 40.5 - 0.02 \cong \mathbf{40.5^\circ C}$$

15-123 A circuit board houses 80 logic chips on one side, dissipating 0.04 W each through the back side of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 864 copper pin fins on the back surface.

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the back side of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivities are given to be $k = 20$ W/m·°C for the circuit board, $k = 386$ W/m·°C for the copper plate and fins, and $k = 1.8$ W/m·°C for the epoxy adhesive.



Analysis (a) The total rate of heat transfer dissipated by the chips is

$$\dot{Q} = 80 \times (0.04 \text{ W}) = 3.2 \text{ W}$$

The individual resistances are

$$A = (0.12 \text{ m})(0.18 \text{ m}) = 0.0216 \text{ m}^2$$

$$R_{\text{board}} = \frac{L}{kA} = \frac{0.003 \text{ m}}{(20 \text{ W/m.°C})(0.0216 \text{ m}^2)} = 0.00694 \text{ °C/W}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(50 \text{ W/m}^2\text{.°C})(0.0216 \text{ m}^2)} = 0.9259 \text{ °C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.00694 + 0.9259 = 0.93284 \text{ °C/W}$$

The temperatures on the two sides of the circuit board are

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40°\text{C} + (3.2 \text{ W})(0.93284 \text{ °C/W}) = \mathbf{43.0°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 43.0°\text{C} - (3.2 \text{ W})(0.00694 \text{ °C/W}) = 40.5 - 0.02 \cong \mathbf{43.0°C}$$

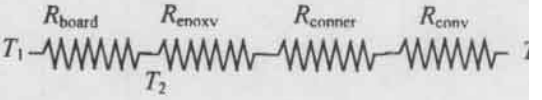
Therefore, the board is nearly isothermal.

(b) We assume (1) heat transfer from the fin tips is negligible and (2) the thermal properties of the fins are constant. Then noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(50 \text{ W/m}^2\text{.°C})}{(386 \text{ W/m.°C})(0.0025 \text{ m})}} = 14.40 \text{ m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(14.40 \text{ m}^{-1} \times 0.02 \text{ m})}{14.40 \text{ m}^{-1} \times 0.02 \text{ m}} = 0.973$$

The fins can be assumed to be at base temperature provided that the fin area is modified by multiplying it by 0.973. Then the various thermal resistances are



$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0002 \text{ m}}{(1.8 \text{ W/m.°C})(0.0216 \text{ m}^2)} = 0.0051 \text{ °C/W}$$

$$R_{\text{copper}} = \frac{L}{kA} = \frac{0.002 \text{ m}}{(386 \text{ W/m.°C})(0.0216 \text{ m}^2)} = 0.00024 \text{ °C/W}$$

$$A_{\text{finned}} = \eta_{\text{fin}} n \pi DL = 0.973 \times 864 \pi (0.0025 \text{ m})(0.02 \text{ m}) = 0.132 \text{ m}^2$$

$$A_{\text{unfinned}} = 0.0216 - 864 \frac{\pi D^2}{4} = 0.0216 - 864 \times \frac{\pi (0.0025)^2}{4} = 0.0174 \text{ m}^2$$

$$A_{\text{total,with fins}} = A_{\text{finned}} + A_{\text{unfinned}} = 0.132 + 0.017 = 0.149 \text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_{\text{total,with fins}}} = \frac{1}{(50 \text{ W/m}^2.\text{°C})(0.149 \text{ m}^2)} = 0.1342 \text{ °C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{epoxy}} + R_{\text{copper}} + R_{\text{conv}}$$

$$= 0.00694 + 0.0051 + 0.00024 + 0.1342 = 0.1465 \text{ °C/W}$$

Then the temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q} R_{\text{total}} = 40°\text{C} + (3.2 \text{ W})(0.1465 \text{ °C/W}) = \mathbf{40.5°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q} R_{\text{board}} = 40.5°\text{C} - (3.2 \text{ W})(0.00694 \text{ °C/W}) = 40.5 - 0.02 \cong \mathbf{40.5°C}$$

15-124 A hot plate is to be cooled by attaching aluminum pin fins on one side. The rate of heat transfer from the 1 m by 1 m section of the plate and the effectiveness of the fins are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** Heat transfer from the fin tips is negligible. **4** The heat transfer coefficient is constant and uniform over the entire fin surface. **5** The thermal properties of the fins are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum plate and fins is given to be $k = 237$ W/m·°C.

Analysis Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(35\ \text{W/m}^2.^\circ\text{C})}{(237\ \text{W/m.}^\circ\text{C})(0.0025\ \text{m})}} = 15.37\ \text{m}^{-1}$$

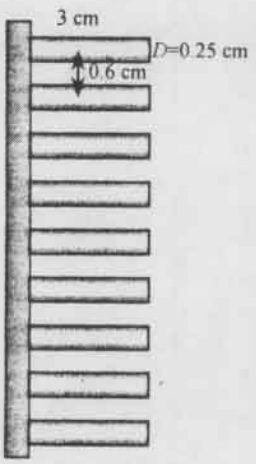
$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(15.37\ \text{m}^{-1} \times 0.03\ \text{m})}{15.37\ \text{m}^{-1} \times 0.03\ \text{m}} = 0.935$$

The number of fins, finned and unfinned surface areas, and heat transfer rates from those areas are

$$n = \frac{1\ \text{m}^2}{(0.006\ \text{m})(0.006\ \text{m})} = 27{,}777$$

$$A_{\text{fin}} = 27777\left[\pi DL + \frac{\pi D^2}{4}\right] = 27777\left[\pi(0.0025)(0.03) + \frac{\pi(0.0025)^2}{4}\right] = 6.68\ \text{m}^2$$

$$A_{\text{unfinned}} = 1 - 27777\left(\frac{\pi D^2}{4}\right) = 1 - 27777\left[\frac{\pi(0.0025)^2}{4}\right] = 0.86\ \text{m}^2$$



$$\dot{Q}_{\text{finned}} = \eta_{\text{fin}} \dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= 0.935(35\ \text{W/m}^2.^\circ\text{C})(6.68\ \text{m}^2)(100-30)^\circ\text{C}$$
$$= 15{,}300\ \text{W}$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_b - T_\infty) = (35\ \text{W/m}^2.^\circ\text{C})(0.86\ \text{m}^2)(100-30)^\circ\text{C}$$
$$= 2107\ \text{W}$$

Then the total heat transfer from the finned plate becomes

$$\dot{Q}_{\text{total,fin}} = \dot{Q}_{\text{finned}} + \dot{Q}_{\text{unfinned}} = 15{,}300 + 2107 = 1.74 \times 10^4\ \text{W} = \mathbf{17.4\ kW}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$A_{\text{no fin}} = (1\ \text{m})(1\ \text{m}) = 1\ \text{m}^2$$
$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (35\ \text{W/m}^2.^\circ\text{C})(1\ \text{m}^2)(100-30)^\circ\text{C} = 2450\ \text{W}$$

Then the fin effectiveness becomes

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{17{,}400}{2450} = \mathbf{7.10}$$

15-125 A hot plate is to be cooled by attaching aluminum pin fins on one side. The rate of heat transfer from the 1 m by 1 m section of the plate and the effectiveness of the fins are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** Heat transfer from the fin tips is negligible. **4** The heat transfer coefficient is constant and uniform over the entire fin surface. **5** The thermal properties of the fins are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum plate and fins is given to be $k = 237$ W/m·°C.

Analysis Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(35\ \text{W/m}^2.^\circ\text{C})}{(386\ \text{W/m.}^\circ\text{C})(0.0025\ \text{m})}} = 12.04\ \text{m}^{-1}$$

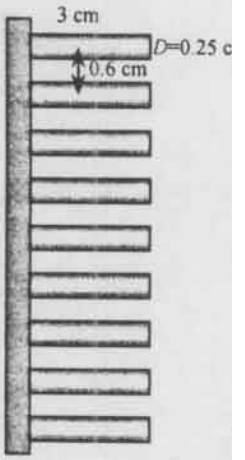
$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(12.04\ \text{m}^{-1} \times 0.03\ \text{m})}{12.04\ \text{m}^{-1} \times 0.03\ \text{m}} = 0.959$$

The number of fins, finned and unfinned surface areas, and heat transfer rates from those areas are

$$n = \frac{1\ \text{m}^2}{(0.006\ \text{m})(0.006\ \text{m})} = 27777$$

$$A_{\text{fin}} = 27777\left[\pi DL + \frac{\pi D^2}{4}\right] = 27777\left[\pi(0.0025)(0.03) + \frac{\pi(0.0025)^2}{4}\right] = 6.68\ \text{m}^2$$

$$A_{\text{unfinned}} = 1 - 27777\left(\frac{\pi D^2}{4}\right) = 1 - 27777\left[\frac{\pi(0.0025)^2}{4}\right] = 0.86\ \text{m}^2$$



$$\dot{Q}_{\text{finned}} = \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= 0.959(35\ \text{W/m}^2.^\circ\text{C})(6.68\ \text{m}^2)(100-30)^\circ\text{C}$$
$$= 15{,}467\ \text{W}$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_b - T_\infty) = (35\ \text{W/m}^{2\circ}\text{C})(0.86\ \text{m}^2)(100-30)^\circ\text{C} = 2107\ \text{W}$$

Then the total heat transfer from the finned plate becomes

$$\dot{Q}_{\text{total,fin}} = \dot{Q}_{\text{finned}} + \dot{Q}_{\text{unfinned}} = 15{,}467 + 2107 = 1.76\times10^4\ \text{W} = \mathbf{17.6\ W}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$A_{\text{no fin}} = (1\ \text{m})(1\ \text{m}) = 1\ \text{m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (35\ \text{W/m}^2.^\circ\text{C})(1\ \text{m}^2)(100-30)^\circ\text{C} = 2450\ \text{W}$$

Then the fin effectiveness becomes

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{17600}{2450} = \mathbf{7.17}$$

15-126 Two cast iron steam pipes are connected to each other through two 1-cm thick flanges exposed to cold ambient air. The average outer surface temperature of the pipe, the fin efficiency, the rate of heat transfer from the flanges, and the equivalent pipe length of the flange for heat transfer are to be determined.

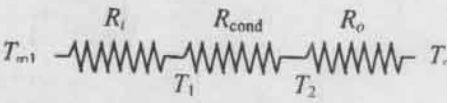
Assumptions **1** Steady operating conditions exist. **2** The temperature along the flanges (fins) varies in one direction only (normal to the pipe). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the cast iron is given to be $k = 52$ W/m·°C.

Analysis (a) We treat the flanges as fins. The individual thermal resistances are

$$A_i = \pi D_i L = \pi(0.092\ \text{m})(6\ \text{m}) = 1.73\ \text{m}^2$$

$$A_o = \pi D_o L = \pi(0.1\ \text{m})(6\ \text{m}) = 1.88\ \text{m}^2$$



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(180\ \text{W/m}^2.^\circ\text{C})(1.73\ \text{m}^2)} = 0.0032\ ^\circ\text{C/W}$$

$$R_{cond} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(5/4.6)}{2\pi(52\ \text{W/m.}^\circ\text{C})(6\ \text{m})} = 0.00004\ ^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\ \text{W/m}^2.^\circ\text{C})(1.88\ \text{m}^2)} = 0.0213\ ^\circ\text{C/W}$$

$$R_{total} = R_i + R_{cond} + R_o = 0.0032 + 0.00004 + 0.0213 = 0.0245\ ^\circ\text{C/W}$$

The rate of heat transfer and average outer surface temperature of the pipe are

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(200-8)^\circ\text{C}}{0.0245\ ^\circ\text{C}} = 7824\ \text{W}$$

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_o} \longrightarrow T_2 = T_{\infty 2} + \dot{Q}R_o = 8\ ^\circ\text{C} + (7824\ \text{W})(0.0213\ ^\circ\text{C/W}) = \mathbf{174.7^\circ C}$$

(b) The fin efficiency can be determined from Fig. 15-70 to be

$$\left.\begin{aligned} \frac{r_2 + \frac{t}{2}}{r_1} &= \frac{0.1 + \frac{0.02}{2}}{0.05} = 2.23 \\ \xi &= \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} = \left(0.05\ \text{m} + \frac{0.02}{2}\ \text{m}\right)\sqrt{\frac{25\ \text{W/m}^2\,^\circ\text{C}}{(52\ \text{W/m}^\circ\text{C})(0.02\ \text{m})}} = 0.29 \end{aligned}\right\} \eta_{fin} = 0.88$$

$$A_{fin} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi[(0.1\ \text{m})^2 - (0.05\ \text{m})^2] + 2\pi(0.1\ \text{m})(0.02\ \text{m}) = 0.0597\ \text{m}^2$$

The heat transfer rate from the flanges is

$$\dot{Q}_{finned} = \eta_{fin}\dot{Q}_{fin,max} = \eta_{fin} h A_{fin}(T_b - T_\infty)$$
$$= 0.88(25\ \text{W/m}^2.^\circ\text{C})(0.0597\ \text{m}^2)(174.7-8)^\circ\text{C} = \mathbf{219\ W}$$

(c) A 6-m long section of the steam pipe is losing heat at a rate of 7842 W or 7842/6 = 1307 W per m length. Then for heat transfer purposes the flange section is equivalent to

$$\text{Equivalent legth} = \frac{218\ \text{W}}{1307\ \text{W/m}} = 0.167\ \text{m} = \mathbf{16.7\ cm}$$

Therefore, the flange acts like a fin and increases the heat transfer by 16.7/2 = 8.35 times.

Review Problems

15-127E Steam is produced in copper tubes by heat transferred from another fluid condensing outside the tubes at a high temperature. The rate of heat transfer per foot length of the tube when a 0.01 in thick layer of limestone is formed on the inner surface of the tube is to be determined.

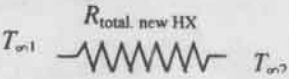
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper tubes and k = 1.7 Btu/h·ft·°F for limestone.

Analysis The total thermal resistance of the new heat exchanger is

$$\dot{Q}_{\text{new}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,new}}} \longrightarrow R_{\text{total,new}} = \frac{T_{\infty 1} - T_{\infty 2}}{\dot{Q}_{\text{new}}} = \frac{(350-250)^\circ\text{F}}{2\times 10^4\ \text{Btu/h}} = 0.005\ \text{h.}^\circ\text{F/Btu}$$

After 0.01 in thick layer of limestone forms, the new value of thermal resistance and heat transfer rate are determined to be



$$R_{\text{limestone,i}} = \frac{\ln(r_1/r_i)}{2\pi kL} = \frac{\ln(0.5/0.49)}{2\pi(1.7\ \text{Btu/h.ft.}^\circ\text{F})(1\ \text{ft})} = 0.00189\ \text{h}^\circ\text{F/Btu}$$

$$R_{\text{total,w/lime}} = R_{\text{total,new}} + R_{\text{limestone,i}} = 0.005 + 0.00189 = 0.00689\ \text{h}^\circ\text{F/Btu}$$

$$\dot{Q}_{\text{w/lime}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,w/lime}}} = \frac{(350-250)^\circ\text{F}}{0.00689\ \text{h}^\circ\text{F/Btu}} = \mathbf{1.45\times 10^4\ Btu/h}\ \text{(a decline of 27\%)}$$

Discussion Note that the limestone layer will change the inner surface area of the pipe and thus the internal convection resistance slightly, but this effect should be negligible.



15-128E Steam is produced in copper tubes by heat transferred from another fluid condensing outside the tubes at a high temperature. The rate of heat transfer per foot length of the tube when a 0.01 in thick layer of limestone is formed on the inner and outer surfaces of the tube is to be determined.

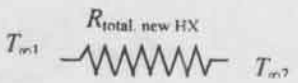
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper tubes and k = 1.7 Btu/h·ft·°F for limestone.

Analysis The total thermal resistance of the new heat exchanger is

$$\dot{Q}_{\text{new}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,new}}} \longrightarrow R_{\text{total,new}} = \frac{T_{\infty 1} - T_{\infty 2}}{\dot{Q}_{\text{new}}} = \frac{(350-250)^\circ\text{F}}{2\times 10^4 \text{ Btu/h}} = 0.005 \text{ h.}^\circ\text{F/Btu}$$

After 0.01 in thick layer of limestone forms, the new value of thermal resistance and heat transfer rate are determined to be



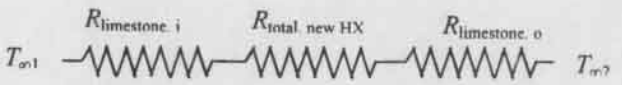
$$R_{\text{limestone,i}} = \frac{\ln(r_1/r_i)}{2\pi kL} = \frac{\ln(0.5/0.49)}{2\pi(1.7 \text{ Btu/h.ft.}^\circ\text{F})(1 \text{ ft})} = 0.00189 \text{ h.}^\circ\text{F/Btu}$$

$$R_{\text{limestone,i}} = \frac{\ln(r_o/r_2)}{2\pi kL} = \frac{\ln(0.66/0.65)}{2\pi(1.7 \text{ Btu/h.ft.}^\circ\text{F})(1 \text{ ft})} = 0.00143 \text{ h.}^\circ\text{F/Btu}$$

$$R_{\text{total,w/lime}} = R_{\text{total,new}} + R_{\text{limestone,i}} + R_{\text{limestone,o}} = 0.005 + 0.00189 + 0.00143 = 0.00832 \text{ h.}^\circ\text{F/Btu}$$

$$\dot{Q}_{\text{w/lime}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,w/lime}}} = \frac{(350-250)^\circ\text{F}}{0.00832 \text{ h}^\circ\text{F/Btu}} = \mathbf{1.20\times 10^4 \text{ Btu/h}} \text{ (a decline of 40\%)}$$

Discussion Note that the limestone layer will change the inner surface area of the pipe and thus the internal convection resistance slightly, but this effect should be negligible.



15-129 A cylindrical tank filled with liquid propane at 1 atm and -42°C is exposed to convection and radiation with the surrounding air and surfaces. The time it will take for the propane to evaporate completely as a result of the heat gain from the surroundings for the cases of no insulation and 7.5-cm thick glass wool insulation are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the propane inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid propane at 1 atm are given to be 425 kJ/kg and 581 kg/m^3, respectively. The thermal conductivity of glass wool insulation is given to be $k = 0.038$ W/m·°C.

Analysis (a) If the tank is not insulated, the heat transfer rate is determined to be

$$A_{\text{tank}} = \pi DL + 2\pi(\pi D^2/4) = \pi(1.2\text{ m})(6\text{ m}) + 2\pi(1.2\text{ m})^2/4 = 24.88\text{ m}^2$$

$$\dot{Q} = hA_{\text{tank}}(T_{\infty 1} - T_{\infty 2}) = (20\text{ W/m}^2.^\circ\text{C})(24.88\text{ m}^2)[15-(-42)]^\circ\text{C} = 28{,}360\text{ W}$$



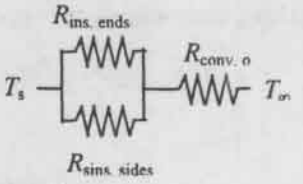
The volume of the tank and the mass of the propane are

$$V = \pi r^2 L = \pi(0.6\text{ m})^2(6\text{ m}) = 6.79\text{ m}^3$$

$$m = \rho V = (581\text{ kg/m}^3)(6.79\text{ m}^3) = 3942.6\text{ kg}$$

The rate of vaporization of propane is

$$\dot{Q} = \dot{m}h_{fg} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{28.360\text{ kJ/s}}{425\text{ kJ/kg}} = 0.06674\text{ kg/s}$$



Then the time period for the propane tank to empty becomes

$$\Delta t = \frac{m}{\dot{m}} = \frac{3942.6\text{ kg}}{0.06674\text{ kg/s}} = 59{,}074\text{ s} = \mathbf{16.4\ hours}$$

(*b*) We now repeat calculations for the case of insulated tank with 7.5-cm thick insulation.

$$A_o = \pi DL + 2\pi(\pi D^2/4) = \pi(1.35\text{ m})(6\text{ m}) + 2\pi(1.35\text{ m})^2/4 = 28.31\text{ m}^2$$

$$R_{\text{conv,o}} = \frac{1}{h_o A_o} = \frac{1}{(20\text{ W/m}^2.^\circ\text{C})(28.31\text{ m}^2)} = 0.001766\ ^\circ\text{C/W}$$

$$R_{\text{insulation,side}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(67.5/60)}{2\pi(0.038\text{ W/m.}^\circ\text{C})(6\text{ m})} = 0.0822\ ^\circ\text{C/W}$$

$$R_{\text{insulation,ends}} = 2\frac{L}{kA_{ave}} = \frac{2\times 0.075\text{ m}}{(0.038\text{ W/m.}^\circ\text{C})[\pi(1.275\text{ m})^2/4]} = 3.0917\ ^\circ\text{C/W}$$

Noting that the insulation on the side surface and the end surfaces are in parallel, the equivalent resistance for the insulation is determined to be

$$R_{\text{insulation}} = \left(\frac{1}{R_{\text{insulation,side}}} + \frac{1}{R_{\text{insulation,ends}}}\right)^{-1} = \left(\frac{1}{0.0822\,^\circ\text{C/W}} + \frac{1}{3.0917\,^\circ\text{C/W}}\right)^{-1} = 0.08007\,^\circ\text{C/W}$$

Then the total thermal resistance and the heat transfer rate become

$$R_{\text{total}} = R_{\text{conv,o}} + R_{\text{insulation}} = 0.001766 + 0.08007 = 0.08184\,^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_\infty - T_s}{R_{\text{total}}} = \frac{[15-(-42)]^\circ\text{C}}{0.08184\,^\circ\text{C/W}} = 696.5\text{ W}$$

Then the time period for the propane tank to empty becomes

$$\dot{Q} = \dot{m}h_{fg} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.6965\text{ kJ/s}}{425\text{ kJ/kg}} = 0.001639\text{ kg/s}$$

$$\Delta t = \frac{m}{\dot{m}} = \frac{3942.6\text{ kg}}{0.001639\text{ kg/s}} = 2{,}405{,}500\text{ s} = 668.2\text{ hours} = \mathbf{27.8\ days}$$

15-130 Hot water is flowing through a 15-m section of a cast iron pipe. The pipe is exposed to cold air and surfaces in the basement, and it experiences a 3°C-temperature drop. The combined convection and radiation heat transfer coefficient at the outer surface of the pipe is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any significant change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no significant variation in the axial direction. **3** Thermal properties are constant.

Properties The thermal conductivity of cast iron is given to be $k = 52$ W/m·°C.

Analysis Using water properties at room temperature, the mass flow rate of water and rate of heat transfer from the water are determined to be

$$\dot{m} = \rho \dot{V}_c = \rho V A_c = (1000 \text{ kg/m}^3)(1.5 \text{ m/s})[\pi(0.03)^2/4]\text{m}^2 = 1.06 \text{ kg/s}$$

$$\dot{Q} = \dot{m} C_p \Delta T = (1.06 \text{ kg/s})(4180 \text{ J/kg.°C})(70-67)°\text{C} = 13{,}296 \text{ W}$$



The thermal resistances for convection in the pipe and the pipe itself are

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(1.75/1.5)}{2\pi(52 \text{ W/m.°C})(15 \text{ m})} = 0.000031 \text{ °C/W}$$

$$R_{conv,i} = \frac{1}{h_i A_i} = \frac{1}{(400 \text{ W/m}^2\text{.°C})[\pi(0.03)(15)]\text{m}^2} = 0.001768 \text{ °C/W}$$

Using arithmetic mean temperature (70+67)/2 = 68.5°C for water, the heat transfer can be expressed as

$$\dot{Q} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{total}} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{conv,i} + R_{pipe} + R_{combined,o}} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{conv,i} + R_{pipe} + \dfrac{1}{h_{combined} A_o}}$$

Substituting,

$$13{,}296 \text{ W} = \frac{(68.5-15)°\text{C}}{(0.000031 \text{ °C/W}) + (0.001768 \text{ °C/W}) + \dfrac{1}{h_{combined}[\pi(0.035)(15)]\text{m}^2}}$$

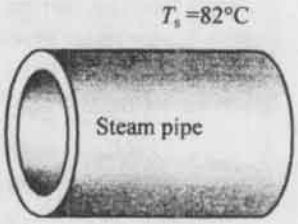
Solving for the combined heat transfer coefficient gives

$$h_{combined} = \mathbf{273 \text{ W/m}^2\text{.°C}}$$

15-131 An 10-m long section of a steam pipe exposed to the ambient is to be insulated to reduce the heat loss through that section of the pipe by 90 percent. The amount of heat loss from the steam in 10 h and the amount of saved per year by insulating the steam pipe.

Assumptions **1** Heat transfer through the pipe is steady and one-dimensional. **2** Thermal conductivities are constant. **3** The furnace operates continuously. **4** The given heat transfer coefficients accounts for the radiation effects. **5** The temperatures of the pipe surface and the surroundings are representative of annual average during operating hours. **6** The plant operates 110 days a year.

Analysis The rate of heat transfer for the uninsulated case is



$$A_o = \pi D_o L = \pi(0.12\text{ m})(10\text{ m}) = 3.77\text{ m}^2$$

$$\dot{Q} = hA_o(T_s - T_{air}) = (25\text{ W/m}^2.^\circ\text{C})(3.77\text{ m}^2)(82-5)^\circ\text{C} = 7257\text{ W}$$

The amount of heat loss during a 10-hour period is

$$Q = \dot{Q}\Delta t = (7.257\text{ kJ/s})(10\times 3600\text{ s}) = \mathbf{2.613\times 10^5\ kJ}\ \text{(per day)}$$

The steam generator has an efficiency of 80%, and steam heating is used for 110 days a year. Then the amount of natural gas consumed per year and its cost are

$$\text{Fuel used} = \frac{2.613\times 10^5\text{ kJ}}{0.80}\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right)(110\text{ days/yr}) = 3405\text{ therms/yr}$$

$$\text{Cost of fuel} = (\text{Amount of fuel})(\text{Unit cost of fuel})$$
$$= (340.5\text{ therms/yr})(\$0.60/\text{therm}) = \$204.3/\text{yr}$$

Then the money saved by reducing the heat loss by 90% by insulation becomes

$$\text{Money saved} = 0.9\times(\text{Cost of fuel}) = 0.9\times\$204.3/\text{yr} = \mathbf{\$184}$$

15-132 A multilayer circuit board dissipating 27 W of heat consists of 4 layers of copper and 3 layers of epoxy glass sandwiched together. The circuit board is attached to a heat sink from both ends maintained at 35°C. The magnitude and location of the maximum temperature that occurs in the board is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer can be approximated as being one-dimensional. **3** Thermal conductivities are constant. **4** Heat is generated uniformly in the epoxy layers of the board. **5** Heat transfer from the top and bottom surfaces of the board is negligible. **6** The thermal contact resistances at the copper-epoxy interfaces are negligible.

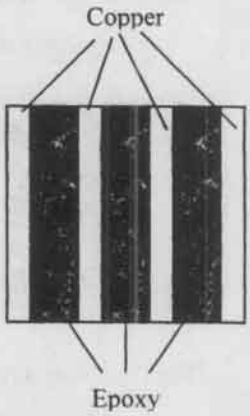
Properties The thermal conductivities are given to be k = 386 W/m·°C for copper layers and k = 0.26 W/m·°C for epoxy glass boards.

Analysis The effective conductivity of the multilayer circuit board is first determined to be

$$(kt)_{\text{copper}} = 4[(386\ \text{W/m.°C})(0.0002\ \text{m})] = 0.3088\ \text{W/°C}$$

$$(kt)_{\text{epoxy}} = 3[(0.26\ \text{W/m.°C})(0.0015\ \text{m})] = 0.00117\ \text{W/°C}$$

$$k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}} = \frac{(0.3088 + 0.00117)\text{W/°C}}{[4(0.0002) + 3(0.0015)\text{m}} = 58.48\ \text{W/m.°C}$$



The maximum temperature will occur at the midplane of the board that is the farthest to the heat sink. Its value is

$$A = 0.18[4(0.0002) + 3(0.0015)] = 0.000954\ \text{m}^2$$

$$\dot{Q} = \frac{k_{\text{eff}} A}{L}(T_1 - T_2)$$

$$T_{\max} = T_1 = T_2 + \frac{\dot{Q}L}{k_{\text{eff}} A} = 35°\text{C} + \frac{(27/2\ \text{W})(0.18/2\ \text{m})}{(58.48\ \text{W/m.°C})(0.000954\ \text{m}^2)} = \mathbf{56.8°C}$$

15-133 The plumbing system of a house involves some section of a plastic pipe exposed to the ambient air. The pipe is initially filled with stationary water at 0°C. It is to be determined if the water in the pipe will completely freeze during a cold night.

Assumptions **1** Heat transfer is transient, but can be treated as steady since the water temperature remains constant during freezing. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties of water are constant. **4** The water in the pipe is stationary, and its initial temperature is 0°C. **5** The convection resistance inside the pipe is negligible so that the inner surface temperature of the pipe is 0°C.

Properties The thermal conductivity of the pipe is given to be $k = 0.16$ W/m·°C. The density and latent heat of fusion of water at 0°C are $\rho = 1000$ kg/m^3 and $h_{if} = 333.7$ kJ/kg (Table A-15).

Analysis We assume the inner surface of the pipe to be at 0°C at all times. The thermal resistances involved and the rate of heat transfer are

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(1.2/1)}{2\pi(0.16\ \text{W/m.°C})(0.5\ \text{m})} = 0.3627\ °\text{C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(40\ \text{W/m}^2.°\text{C})[\pi(0.024\ \text{m})(0.5\ \text{m})]} = 0.6631\ °\text{C/W}$$

$$R_{total} = R_{pipe} + R_{conv,o} = 0.3627 + 0.6631 = 1.0258\ °\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[0-(-5)]°\text{C}}{1.0258\ °\text{C/W}} = 4.87\ \text{W}$$

The total amount of heat lost by the water during a 14-h period that night is

$$Q = \dot{Q}\Delta t = (4.87\ \text{J/s})(14 \times 3600\ \text{s}) = 245.65\ \text{kJ}$$

The amount of heat required to freeze the water in the pipe completely is

$$m = \rho V = \rho\pi r^2 L = (1000\ \text{kg/m}^3)\pi(0.01\ \text{m})^2(0.5\ \text{m}) = 0.157\ \text{kg}$$
$$Q = mh_{fg} = (0.157\ \text{kg})(333.7\ \text{kJ/kg}) = 52.4\ \text{kJ}$$

The water in the pipe will **freeze completely** that night since the amount heat loss is greater than the amount it takes to freeze the water completely (245.65 > 52.4) .

15-134 The plumbing system of a house involves some section of a plastic pipe exposed to the ambient air. The pipe is initially filled with stationary water at 0°C. It is to be determined if the water in the pipe will completely freeze during a cold night.

Assumptions **1** Heat transfer is transient, but can be treated as steady since the water temperature remains constant during freezing. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties of water are constant. **4** The water in the pipe is stationary, and its initial temperature is 0°C. **5** The convection resistance inside the pipe is negligible so that the inner surface temperature of the pipe is 0°C.

Properties The thermal conductivity of the pipe is given to be $k = 0.16$ W/m·°C. The density and latent heat of fusion of water at 0°C are $\rho = 1000$ kg/m^3 and $h_{if} = 333.7$ kJ/kg (Table A-15).

Analysis We assume the inner surface of the pipe to be at 0°C at all times. The thermal resistances involved and the rate of heat transfer are

$$R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(1.2/1)}{2\pi(0.16\ \text{W/m.°C})(0.5\ \text{m}^2)} = 0.3627\ \text{°C/W}$$

$$R_{\text{conv,o}} = \frac{1}{h_o A} = \frac{1}{(10\ \text{W/m}^2\text{.°C})[\pi(0.024\ \text{m})(0.5\ \text{m})]} = 2.6526\ \text{°C/W}$$

$$R_{\text{total}} = R_{\text{pipe}} + R_{\text{conv,o}} = 0.3627 + 2.6526 = 3.0153\ \text{°C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[0-(-5)]\text{°C}}{3.0153\ \text{°C/W}} = 1.658\ \text{W}$$

$$Q = \dot{Q}\Delta t = (4.87\ \text{J/s})(14 \times 3600\ \text{s}) = 245.65\ \text{kJ}$$

$$Q = \dot{Q}\Delta t = (1.658\ \text{J/s})(14 \times 3600\ \text{s}) = 83.57\ \text{kJ}$$

The amount of heat required to freeze the water in the pipe completely is

$$m = \rho V = \rho\pi r^2 L = (1000\ \text{kg/m}^3)\pi(0.01\ \text{m})^2(0.5\ \text{m}) = 0.157\ \text{kg}$$
$$Q = mh_{fg} = (0.157\ \text{kg})(333.7\ \text{kJ/kg}) = 52.4\ \text{kJ}$$

The water in the pipe will **freeze completely** that night since the amount heat loss is greater than the amount it takes to freeze the water completely $(83.57 > 52.4)$.

15-135E The surface temperature of a baked potato drops from 300°F to 200°F in 5 minutes in an environment at 70°F. The average heat transfer coefficient and the cooling time of the potato if it is wrapped completely in a towel are to be determined.

Assumptions 1Thermal properties of potato are constant, and can be taken to be the properties of water. 2 The thermal contact resistance at the interface is negligible. 3 The heat transfer coefficients for wrapped and unwrapped potatoes are the same.

Properties The thermal conductivity of a thick towel is given to be k = 0.035 Btu/h·ft·°F. We take the properties of potato to be those of water at room temperature, ρ = 62.2 lbm/ft^3 and C_p = 0.998 Btu/lbm·°F.

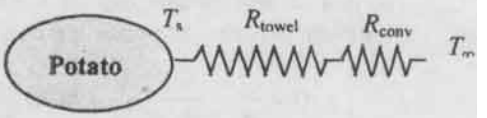
Analysis This is a transient heat conduction problem, and the rate of heat transfer will decrease as the potato cools down and the temperature difference between the potato and the surroundings decreases. However, we can solve this problem approximately by assuming a constant average temperature of (300+200)/2 = 250°F for the potato during the process. The mass of the potato is

$$m = \rho V = \rho \frac{4}{3}\pi r^3 = (62.2 \text{ lbm/ft}^3)\frac{4}{3}\pi(1.5/12 \text{ ft})^3 = 0.5089 \text{ lbm}$$

The amount of heat lost as the potato is cooled from 300 to 200°F is

$$Q = mC_p\Delta T = (0.5089 \text{ lbm})(0.998 \text{ Btu/lbm.°F})(300-200)°\text{F} = 50.8 \text{ Btu}$$

The rate of heat transfer and the average heat transfer coefficient between the potato and its surroundings are



$$\dot{Q} = \frac{Q}{\Delta t} = \frac{50.8 \text{ Btu}}{(5/60 \text{ h})} = 609.4 \text{ Btu/h}$$

$$\dot{Q} = hA_o(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_o(T_s - T_\infty)} = \frac{609.4 \text{ Btu/h}}{\pi(3/12 \text{ ft})^2(250-70)°\text{F}} = \mathbf{17.2 \text{ Btu/h.ft}^2\text{.°F}}$$

When the potato is wrapped in a towel, the thermal resistance and heat transfer rate are determined to be

$$R_{towel} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{[(1.5+0.12)/12]\text{ft} - (1.5/12)\text{ft}}{4\pi(0.035 \text{ Btu/h.ft.°F})[(1.5+0.12)/12]\text{ft}(1.5/12)\text{ft}} = 1.3473 \text{ h°F/Btu}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{(17.2 \text{ Btu/h.ft}^2\text{.°F})\pi(3.24/12)^2 \text{ ft}^2} = 0.2539 \text{ h.°F/Btu}$$

$$R_{total} = R_{towel} + R_{conv} = 1.3473 + 0.2539 = 1.6012 \text{ h°F/Btu}$$

$$\dot{Q} = \frac{T_s - T_\infty}{R_{total}} = \frac{(250-75)°\text{F}}{1.6012 \text{ h°F/Btu}} = 109.3 \text{ Btu/h}$$

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{50.8 \text{ Btu}}{109.3 \text{ Btu/h}} = 0.465 \text{ h} = \mathbf{27.9 \text{ min}}$$

This result is conservative since the heat transfer coefficient will be lower in this case because of the smaller exposed surface temperature.

15-136E The surface temperature of a baked potato drops from 300°F to 200°F in 5 minutes in an environment at 70°F. The average heat transfer coefficient and the cooling time of the potato if it is loosely wrapped completely in a towel are to be determined.

Assumptions **1** Thermal properties of potato are constant, and can be taken to be the properties of water. **2** The heat transfer coefficients for wrapped and unwrapped potatoes are the same.

Properties The thermal conductivity of a thick towel is given to be k = 0.035 Btu/h·ft·°F. The thermal conductivity of air is given to be k = 0.015 Btu/h·ft·°F. We take the properties of potato to be those of water at room temperature, ρ = 62.2 lbm/ft^3 and C_p = 0.998 Btu/lbm·°F.

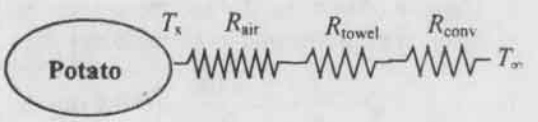
Analysis This is a transient heat conduction problem, and the rate of heat transfer will decrease as the potato cools down and the temperature difference between the potato and the surroundings decreases. However, we can solve this problem approximately by assuming a constant average temperature of (300+200)/2 = 250°F for the potato during the process. The mass of the potato is

$$m = \rho V = \rho \frac{4}{3}\pi r^3 = (62.2 \text{ lbm/ft}^3)\frac{4}{3}\pi(1.5/12 \text{ ft})^3 = 0.5089 \text{ lbm}$$

The amount of heat lost as the potato is cooled from 300 to 200°F is

$$Q = mC_p\Delta T = (0.5089 \text{ lbm})(0.998 \text{ Btu/lbm.°F})(300-200)°\text{F} = 50.8 \text{ Btu}$$

The rate of heat transfer and the average heat transfer coefficient between the potato and its surroundings are



$$\dot{Q} = \frac{Q}{\Delta t} = \frac{50.8 \text{ Btu}}{(5/60 \text{ h})} = 609.4 \text{ Btu/h}$$

$$\dot{Q} = hA_o(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_o(T_s - T_\infty)} = \frac{609.4 \text{ Btu/h}}{\pi(3/12 \text{ ft})^2(250-70)°\text{F}} = \mathbf{17.2 \text{ Btu/h.ft}^2\text{.°F}}$$

When the potato is wrapped in a towel, the thermal resistance and heat transfer rate are determined to be

$$R_{air} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{[(1.50+0.02)/12]\text{ft} - (1.50/12)\text{ft}}{4\pi(0.015 \text{ Btu/h.ft.°F})[(1.50+0.02)/12]\text{ft}(1.50/12)\text{ft}} = 1.5584 \text{ h.°F/Btu}$$

$$R_{towel} = \frac{r_3 - r_2}{4\pi k r_2 r_3} = \frac{[(1.52+0.12)/12]\text{ft} - (1.52/12)\text{ft}}{4\pi(0.035 \text{ Btu/h.ft.°F})[(1.52+0.12)/12]\text{ft}(1.52/12)\text{ft}} = 1.3134 \text{ h°F/Btu}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{(17.2 \text{ Btu/h.ft}^2\text{.°F})\pi(3.28/12)^2 \text{ ft}^2} = 0.2477 \text{ h.°F/Btu}$$

$$R_{total} = R_{air} + R_{towel} + R_{conv} = 0.5584 + 1.3134 + 0.2477 = 1.561 \text{ h°F/Btu}$$

$$\dot{Q} = \frac{T_s - T_\infty}{R_{total}} = \frac{(250-75)°\text{F}}{1.561 \text{ h.°F/Btu}} = 112.1 \text{ Btu/h}$$

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{50.8 \text{ Btu}}{112.1 \text{ Btu/h}} = 0.443 \text{ h} = \mathbf{27.2 \text{ min}}$$

This result is conservative since the heat transfer coefficient will be lower because of the smaller exposed surface temperature.

15-137 An ice chest made of 15-cm thick styrofoam is initially filled with 45 kg of ice at 0°C. The length of time it will take for the ice in the chest to melt completely is to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional. **3** Thermal conductivity is constant. **4** The inner surface temperature of the ice chest can be taken to be 0°C at all times. **5** Heat transfer from the base of the ice chest is negligible.

Properties The thermal conductivity of styrofoam is given to be $k = 0.033$ W/m·°C. The heat of fusion of water at 1 atm is $h_{if} = 333.7$ kJ / kg.

Analysis Disregarding any heat loss through the bottom of the ice chest, the total thermal resistance and the heat transfer rate are determined to be

$$A_i = 2(0.3-0.03)(0.4-0.06)+2(0.3-0.03)(0.5-0.06)+(0.4-0.06)(0.5-0.06) = 0.5708\ \text{m}^2$$

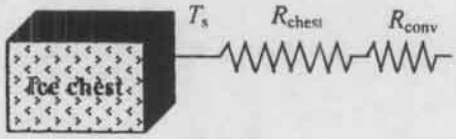
$$A_o = 2(0.3)(0.4)+2(0.3)(0.5)+(0.4)(0.5) = 0.74\ \text{m}^2$$

$$R_{\text{chest}} = \frac{L}{kA_i} = \frac{0.03\ \text{m}}{(0.033\ \text{W/m.°C})(0.5708\ \text{m}^2)} = 1.5927\ \text{°C/W}$$

$$R_{\text{conv}} = \frac{1}{hA_o} = \frac{1}{(20\ \text{W/m}^2\text{.°C})(0.74\ \text{m}^2)} = 0.06757\ \text{°C/W}$$

$$R_{\text{total}} = R_{\text{chest}} + R_{\text{conv}} = 1.5927 + 0.06757 = 1.6603\ \text{°C/W}$$

$$\dot{Q} = \frac{T_s - T_\infty}{R_{\text{total}}} = \frac{(30-0)\text{°C}}{1.6603\ \text{°C/W}} = 18.07\ \text{W}$$



The total amount of heat necessary to melt the ice completely is

$$Q = mh_{if} = (45\ \text{kg})(333.7\ \text{kJ/kg}) = 15{,}016.5\ \text{kJ}$$

Then the time period to transfer this much heat to the cooler to melt the ice completely becomes

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{15{,}016{,}500\ \text{J}}{18.07\ \text{kJ/s}} = 831{,}050\ \text{s} = 230.8\ \text{h} = \mathbf{9.6\ \text{days}}$$

15-138 A wall is constructed of two large steel plates separated by 1-cm thick steel bars placed 99 cm apart. The remaining space between the steel plates is filled with fiberglass insulation. The rate of heat transfer through the wall is to be determined, and it is to be assessed if the steel bars between the plates can be ignored in heat transfer analysis since they occupy only 1 percent of the heat transfer surface area.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall can be approximated to be one-dimensional. **3** Thermal conductivities are constant. **4** The surfaces of the wall are maintained at constant temperatures.

Properties The thermal conductivities are given to be $k = 15$ W/m·°C for steel plates and $k = 0.035$ W/m·°C for fiberglass insulation.

Analysis We consider 1 m high and 1 m wide portion of the wall which is representative of entire wall. Thermal resistance network and individual resistances are



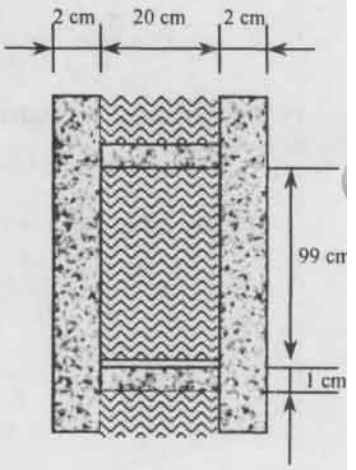
$$R_1 = R_4 = R_{steel} = \frac{L}{kA} = \frac{0.02\,\text{m}}{(15\,\text{W/m.°C})(1\,\text{m}^2)} = 0.00133\,\text{°C/W}$$

$$R_2 = R_{steel} = \frac{L}{kA} = \frac{0.2\,\text{m}}{(15\,\text{W/m.°C})(0.01\,\text{m}^2)} = 1.333\,\text{°C/W}$$

$$R_3 = R_{insulation} = \frac{L}{kA} = \frac{0.2\,\text{m}}{(0.035\,\text{W/m.°C})(0.99\,\text{m}^2)} = 5.772\,\text{°C/W}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.333} + \frac{1}{5.772} \longrightarrow R_{eq} = 1.083\,\text{°C/W}$$

$$R_{total} = R_1 + R_{eqv} + R_4 = 0.00133 + 1.083 + 0.00133 = 1.0856\,\text{°C/W}$$



The rate of heat transfer per m^2 surface area of the wall is

$$\dot{Q} = \frac{\Delta T}{R_{total}} = \frac{15\,\text{°C}}{1.0856\,\text{°C/W}} = 13.82\ \text{W}$$

The total rate of heat transfer through the entire wall is then determined to be

$$\dot{Q}_{total} = (4 \times 6)\dot{Q} = 24(13.82\ \text{W}) = \mathbf{331.7\ W}$$

If the steel bars were ignored since they constitute only 1% of the wall section, the R_{equiv} would simply be equal to the thermal resistance of the insulation, and the heat transfer rate in this case would be

$$\dot{Q} = \frac{\Delta T}{R_{total}} = \frac{\Delta T}{R_1 + R_{insulation} + R_4} = \frac{15\,\text{°C}}{(0.00133 + 5.772 + 0.00133)\,\text{°C/W}} = 2.60\ \text{W}$$

which is mush less than 13.82 W obtained earlier. Therefore, (13.82-2.60)/13.82 = 81.2% of the heat transfer occurs through the steel bars across the wall despite the negligible space that they occupy, and obviously their effect cannot be neglected. The connecting bars are serving as "thermal bridges."

15-139 A circuit board houses electronic components on one side, dissipating a total of 15 W through the backside of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 20 aluminum fins of rectangular profile on the backside. √

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the backside of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

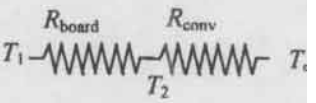
Properties The thermal conductivities are given to be $k = 12$ W/m·°C for the circuit board, $k = 237$ W/m·°C for the aluminum plate and fins, and $k = 1.8$ W/m·°C for the epoxy adhesive.

Analysis (*a*) The thermal resistance of the board and the convection resistance on the backside of the board are

$$R_{\text{board}} = \frac{L}{kA} = \frac{0.002\ \text{m}}{(12\ \text{W/m.°C})(0.1\ \text{m})(0.15\ \text{m})} = 0.011\ °\text{C/W}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(45\ \text{W/m.°C})(0.1\ \text{m})(0.15\ \text{m})} = 1.481\ °\text{C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.011 + 1.481 = 1.492\ °\text{C/W}$$

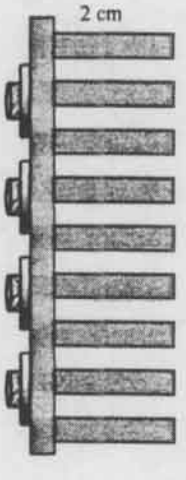


Then surface temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}} = 37°\text{C} + (15\ \text{W})(1.492\ °\text{C/W}) = \mathbf{59.4°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 59.4°\text{C} - (15\ \text{W})(0.011\ °\text{C/W}) = \mathbf{59.2°C}$$

(*b*) Noting that the cross-sectional areas of the fins are constant, the efficiency of these rectangular fins is determined to be



$$a = \sqrt{\frac{hp}{kA_c}} \cong \sqrt{\frac{h(2w)}{k(tw)}} = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2(45\ \text{W/m}^2\text{.°C})}{(237\ \text{W/m.°C})(0.002\ \text{m})}} = 13.78\ \text{m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(13.78\ \text{m}^{-1} \times 0.02\ \text{m})}{13.78\ \text{m}^{-1} \times 0.02\ \text{m}} = 0.975$$

The finned and unfinned surface areas are

$$A_{\text{finned}} = (20)2w\left(L + \frac{t}{2}\right) = (20)2(0.15)\left(0.02 + \frac{0.002}{2}\right) = 0.126\ \text{m}^2$$

$$A_{\text{unfinned}} = (0.1)(0.15) - 20(0.002)(0.15) = 0.0090\ \text{m}^2$$

Then,

$$\dot{Q}_{\text{finned}} = \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}}hA_{\text{fin}}(T_{\text{base}} - T_\infty)$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_{\text{base}} - T_\infty)$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{unfinned}} + \dot{Q}_{\text{finned}} = h(T_{\text{base}} - T_\infty)(\eta_{\text{fin}}A_{\text{fin}} + A_{\text{unfinned}})$$



Substituting, the base temperature of the finned surfaces is determined to be

$$T_{\text{base}} = T_\infty + \frac{\dot{Q}_{\text{total}}}{h(\eta_{\text{fin}}A_{\text{fin}} + A_{\text{unfinned}})}$$

$$= 37°\text{C} + \frac{15\ \text{W}}{(45\ \text{W/m}^2\text{.°C})[(0.975)(0.126\ \text{m}^2) + (0.0090\ \text{m}^2)]} = \mathbf{39.5°C}$$

Then the temperatures on both sides of the board are determined using the thermal resistance network to be

$$R_{\text{aluminum}} = \frac{L}{kA} = \frac{0.001\text{ m}}{(237\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.00028\text{ °C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.00015\text{ m}}{(1.8\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.00555\text{ °C/W}$$

$$\dot{Q} = \frac{T_1 - T_{\text{base}}}{R_{\text{aluminum}} + R_{\text{epoxy}} + R_{\text{board}}} = \frac{(T_1 - 39.5)°\text{C}}{(0.00028 + 0.00555 + 0.011)\,°\text{C/W}}$$

$$\longrightarrow T_1 = 39.5°\text{C} + (15\text{ W})(0.0168\,°\text{C/W}) = \mathbf{39.8°C}$$

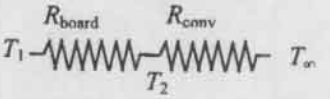
$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 39.8°\text{C} - (15\text{ W})(0.011°\text{C/W}) = \mathbf{39.6°C}$$

15-140 A circuit board houses electronic components on one side, dissipating a total of 15 W through the backside of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 20 copper fins of rectangular profile on the backside. √

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the backside of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivities are given to be $k = 12$ W/m·°C for the circuit board, $k = 386$ W/m·°C for the copper plate and fins, and $k = 1.8$ W/m·°C for the epoxy adhesive.

Analysis (*a*) The thermal resistance of the board and the convection resistance on the backside of the board are



$$R_{board} = \frac{L}{kA} = \frac{0.002\ \text{m}}{(12\ \text{W/m.°C})(0.1\ \text{m})(0.15\ \text{m})} = 0.011\ °\text{C/W}$$

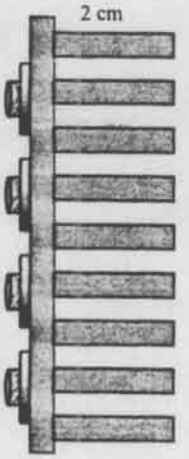
$$R_{conv} = \frac{1}{hA} = \frac{1}{(45\ \text{W/m.°C})(0.1\ \text{m})(0.15\ \text{m})} = 1.481\ °\text{C/W}$$

$$R_{total} = R_{board} + R_{conv} = 0.011 + 1.481 = 1.492\ °\text{C/W}$$

Then surface temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total} = 37°\text{C} + (15\ \text{W})(1.492\ °\text{C/W}) = \mathbf{59.4°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{board}} \longrightarrow T_2 = T_1 - \dot{Q}R_{board} = 59.4°\text{C} - (15\ \text{W})(0.011\ °\text{C/W}) = \mathbf{59.2°C}$$



(*b*) Noting that the cross-sectional areas of the fins are constant, the efficiency of these rectangular fins is determined to be

$$a = \sqrt{\frac{hp}{kA_c}} \cong \sqrt{\frac{h(2w)}{k(tw)}} = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2(45\ \text{W/m}^2.°\text{C})}{(386\ \text{W/m.°C})(0.002\ \text{m})}} = 10.80\ \text{m}^{-1}$$

$$\eta_{fin} = \frac{\tanh aL}{aL} = \frac{\tanh(10.80\ \text{m}^{-1} \times 0.02\ \text{m})}{10.80\ \text{m}^{-1} \times 0.02\ \text{m}} = 0.985$$

The finned and unfinned surface areas are

$$A_{finned} = (20)2w\left(L + \frac{t}{2}\right) = (20)2(0.15)\left(0.02 + \frac{0.002}{2}\right) = 0.126\ \text{m}^2$$

$$A_{unfinned} = (0.1)(0.15) - 20(0.002)(0.15) = 0.0090\ \text{m}^2$$

Then,

$$\dot{Q}_{finned} = \eta_{fin}\dot{Q}_{fin,max} = \eta_{fin}hA_{fin}(T_{base} - T_\infty)$$

$$\dot{Q}_{unfinned} = hA_{unfinned}(T_{base} - T_\infty)$$

$$\dot{Q}_{total} = \dot{Q}_{unfinned} + \dot{Q}_{finned} = h(T_{base} - T_\infty)(\eta_{fin}A_{fin} + A_{unfinned})$$



Substituting, the base temperature of the finned surfaces determine to be

$$T_{base} = T_\infty + \frac{\dot{Q}_{total}}{h(\eta_{fin}A_{fin} + A_{unfinned})}$$

$$= 37°\text{C} + \frac{15\ \text{W}}{(45\ \text{W/m}^2.°\text{C})[(0.985)(0.126\ \text{m}^2) + (0.0090\ \text{m}^2)]} = \mathbf{39.5°C}$$

Then the temperatures on both sides of the board are determined using the thermal resistance network to be

$$R_{\text{copper}} = \frac{L}{kA} = \frac{0.001\text{ m}}{(386\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.00017\text{ °C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.00015\text{ m}}{(1.8\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.00555\text{ °C/W}$$

$$\dot{Q} = \frac{T_1 - T_{base}}{R_{\text{copper}} + R_{\text{epoxy}} + R_{\text{board}}} = \frac{(T_1 - 39.5)°\text{C}}{(0.00017 + 0.00555 + 0.011)\text{°C/W}}$$

$$\longrightarrow T_1 = 39.5°\text{C} + (15\text{ W})(0.0167\text{°C/W}) = \mathbf{39.8°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 39.8°\text{C} - (15\text{ W})(0.011\text{°C/W}) = \mathbf{39.6°C}$$



Chapter 16
TRANSIENT HEAT CONDUCTION

Lumped System Analysis

16-1C In transient conduction, the temperature of a body varies with time as well as position. In steady conduction, the temperature of a body varies with position only. In one-dimensional heat conduction problems, the temperature varies in one direction only whereas in two dimensional problems the temperature varies in two dimensions.

16-2C In heat transfer analysis, some bodies are observed to behave like a "lump" whose entire body temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only. Heat transfer analysis which utilizes this idealization is known as the lumped system analysis. It is applicable when the Biot number (the ratio of conduction resistance within the body to convection resistance at the surface of the body) is less than or equal to 0.1.

16-3C The lumped system analysis is more likely to be applicable for the body cooled naturally since the Biot number is proportional to the convection heat transfer coefficient, which is proportional to the air velocity. Therefore, the Biot number is more likely to be less than 0.1 for the case of natural convection.

16-4C The lumped system analysis is more likely to be applicable for the body allowed to cool in the air since the Biot number is proportional to the convection heat transfer coefficient, which is larger in water than it is in air because of the larger thermal conductivity of water. Therefore, the Biot number is more likely to be less than 0.1 for the case of the solid cooled in the air

16-5C The temperature drop of the potato during the second minute will be less than $4\,^{\circ}C$ since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

16-6C The temperature rise of the potato during the second minute will be less than $5\,^{\circ}C$ since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

16-7C Biot number represents the ratio of conduction resistance within the body to convection resistance at the surface of the body. The Biot number is more likely to be larger for poorly conducting solids since such bodies have larger resistances against heat conduction.

16-8C The heat transfer is proportional to the surface area. Two half pieces of the roast have a much larger surface area than the single piece and thus a higher rate of heat transfer. As a result, the two half pieces will cook much faster than the single large piece.

16-9C The cylinder will cool faster than the sphere since heat transfer rate is proportional to the surface area, and the sphere has the smallest area for a given volume.

16-10C The lumped system analysis is more likely to be applicable in air than in water since the convection heat transfer coefficient and thus the Biot number is much smaller in air.

16-11C The lumped system analysis is more likely to be applicable for a golden apple than for an actual apple since the thermal conductivity is much larger and thus the Biot number is much smaller for gold.

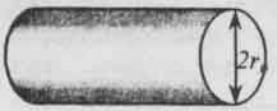
16-12C The lumped system analysis is more likely to be applicable to slender bodies than the well-rounded bodies since the characteristic length (ratio of volume to surface area) and thus the Biot number is much smaller for slender bodies.

16-13 Relations are to be obtained for the characteristic lengths of a large plane wall of thickness 2L, a very long cylinder of radius r_o and a sphere of radius r_o

Analysis Relations for the characteristic lengths of a large plane wall of thickness 2L, a very long cylinder of radius r_o and a sphere of radius r_o are

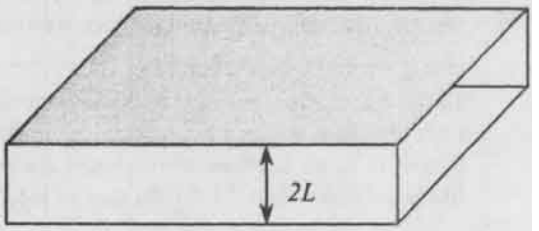


$$L_{c,wall} = \frac{V}{A_{surface}} = \frac{2LA}{2A} = L$$



$$L_{c,cylinder} = \frac{V}{A_{surface}} = \frac{\pi r_o^2 h}{2\pi r_o h} = \frac{r_o}{2}$$

$$L_{c,sphere} = \frac{V}{A_{surface}} = \frac{4\pi r_o^3/3}{4\pi r_o^2} = \frac{r_o}{3}$$

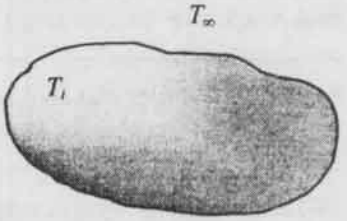


16-14 A relation for the time period for a lumped system to reach the average temperature $(T_i + T_\infty)/2$ is to be obtained.

Analysis The relation for time period for a lumped system to reach the average temperature $(T_i + T_\infty)/2$ can be determined as

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{\frac{T_i + T_\infty}{2} - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T_i - T_\infty}{2(T_i - T_\infty)} = e^{-bt} \longrightarrow \frac{1}{2} = e^{-bt}$$

$$-bt = -\ln 2 \longrightarrow t = \frac{\ln 2}{\mathbf{b}} = \frac{0.693}{\mathbf{b}}$$



16-15 The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT is to be determined.

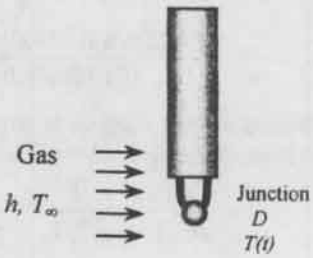
Assumptions **1** The junction is spherical in shape with a diameter of D = 0.0012 m. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be $k = 35\ \text{W/m.°C}$, $\rho = 8500\ \text{kg/m}^3$, and $C_p = 320\ \text{J/kg.°C}$.

Analysis The characteristic length of the junction and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.0012\ \text{m}}{6} = 0.0002\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(65\ \text{W/m}^2.\text{°C})(0.0002\ \text{m})}{(35\ \text{W/m.°C})} = 0.00037 < 0.1$$



Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{65\ \text{W/m}^2.\text{°C}}{(8500\ \text{kg/m}^3)(320\ \text{J/kg.°C})(0.0002\ \text{m})} = 0.1195\ \text{s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow 0.01 = e^{-(0.1195\ \text{s}^{-1})t} \longrightarrow t = \mathbf{38.5\ s}$$

16-16E A number of brass balls are to be quenched in a water bath at a specified rate. The temperature of the balls after quenching and the rate at which heat needs to be removed from the water in order to keep its temperature constant are to be determined.

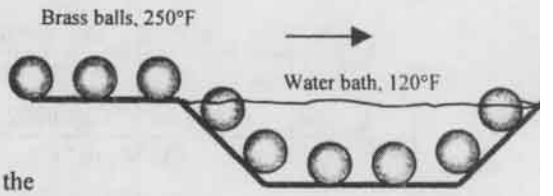
Assumptions **1** The balls are spherical in shape with a radius of r_0 = 1 in. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the brass balls are given to be k = 64.1 Btu/h.ft.°F, ρ = 532 lbm/ft^3, and C_p = 0.092 Btu/lbm.°F.

Analysis (*a*) The characteristic length and the Biot number for the brass balls are

$$L_c = \frac{V}{A} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{2/12\text{ ft}}{6} = 0.02778\text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(42\text{ Btu/h.ft}^2\text{.°F})(0.02778\text{ ft})}{(64.1\text{ Btu/h.ft.°F})} = 0.01820 < 0.1$$



The lumped system analysis is applicable since Bi < 0.1. Then the temperature of the balls after quenching becomes

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{42\text{ Btu/h.ft}^2\text{.°F}}{(532\text{ lbm/ft}^3)(0.092\text{ Btu/lbm.°F})(0.02778\text{ ft})} = 30.9\text{ h}^{-1} = 0.00858\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{T(t)-120}{250-120} = e^{-(0.00858\text{ s}^{-1})(120\text{ s})} \longrightarrow T(t) = \mathbf{166\,°F}$$

(*b*) The total amount of heat transfer from a ball during a 2-minute period is

$$m = \rho V = \rho\frac{\pi D^3}{6} = (532\text{ lbm/ft}^3)\frac{\pi(2/12\text{ ft})^3}{6} = 1.290\text{ lbm}$$

$$Q = mC_p[T_i - T(t)] = (1.29\text{ lbm})(0.092\text{ Btu/lbm.°F})(250-166)\text{°F} = 9.97\text{ Btu}$$

Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{ball} Q_{ball} = (100\text{ balls/min}) \times (9.97\text{ Btu}) = \mathbf{997\ Btu/min}$$

Therefore, heat must be removed from the water at a rate of 997 Btu/min in order to keep its temperature constant at 120 °F.

16-17E A number of aluminum balls are to be quenched in a water bath at a specified rate. The temperature of balls after quenching and the rate at which heat needs to be removed from the water in order to keep its temperature constant are to be determined.

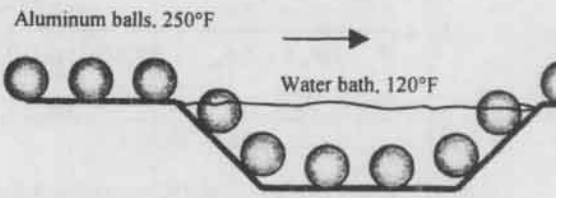
Assumptions **1** The balls are spherical in shape with a radius of r_0 = 1 in. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the aluminum balls are k = 137 Btu/h.ft.°F, ρ = 168 lbm/ft^3, and C_p = 0.216 Btu/lbm.°F (Table A-19E).

Analysis (a) The characteristic length and the Biot number for the aluminum balls are

$$L_c = \frac{V}{A} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{2/12\text{ ft}}{6} = 0.02778\text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(42\text{ Btu/h.ft}^2\text{.°F})(0.02778\text{ ft})}{(137\text{ Btu/h.ft.°F})} = 0.00852 < 0.1$$



The lumped system analysis is applicable since Bi < 0.1. Then the temperature of the balls after quenching becomes

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{42\text{ Btu/h.ft}^2\text{.°F}}{(168\text{ lbm/ft}^3)(0.216\text{ Btu/lbm.°F})(0.02778\text{ ft})} = 41.66\text{ h}^{-1} = 0.01157\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{T(t)-120}{250-120} = e^{-(0.01157\text{ s}^{-1})(120\text{ s})} \longrightarrow T(t) = \mathbf{152°F}$$

(*b*) The total amount of heat transfer from a ball during a 2-minute period is

$$m = \rho V = \rho\frac{\pi D^3}{6} = (168\text{ lbm/ft}^3)\frac{\pi(2/12\text{ ft})^3}{6} = 0.4072\text{ lbm}$$

$$Q = mC_p[T_i - T(t)] = (0.4072\text{ lbm})(0.216\text{ Btu/lbm.°F})(250-152)°\text{F} = 8.62\text{ Btu}$$

Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{\text{ball}} Q_{\text{ball}} = (100\text{ balls/min}) \times (8.62\text{ Btu}) = \mathbf{862\text{ Btu/min}}$$

Therefore, heat must be removed from the water at a rate of 862 Btu/min in order to keep its temperature constant at 120 °F.

16-18 Milk in a thin-walled glass container is to be warmed up by placing it into a large pan filled with hot water. The warming time of the milk is to be determined.

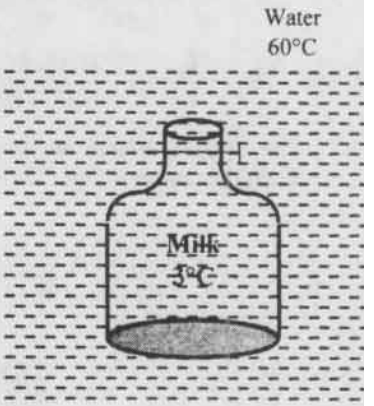
Assumptions **1** The glass container is cylindrical in shape with a radius of r_0 = 3 cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.

Properties The thermal conductivity, density, and specific heat of the milk are k = 0.607 W/m.°C, ρ = 997 kg/m^3, and C_p = 4.180 kJ/kg.°C (Table A-15).

Analysis The characteristic length and Biot number for the glass of milk are

$$L_c = \frac{V}{A} = \frac{\pi r_o^{\,2} L}{2\pi r_o L + 2\pi r_o^{\,2}} = \frac{\pi(0.03\,\text{m})^2(0.07\,\text{m})}{2\pi(0.03\,\text{m})(0.07\,\text{m}) + 2\pi(0.03\,\text{m})^2} = 0.01050\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(120\ \text{W/m}^2.^\circ\text{C})(0.0105\,\text{m})}{(0.607\ \text{W/m.}^\circ\text{C})} = 2.07 > 0.1$$



For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C:

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{120\ \text{W/m}^2.^\circ\text{C}}{(997\ \text{kg/m}^3)(4180\ \text{J/kg.}^\circ\text{C})(0.0105\,\text{m})} = 0.002742\ \text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{38-60}{3-60} = e^{-(0.002742\,\text{s}^{-1})t} \longrightarrow t = \mathbf{347\ s = 5.79\ min}$$

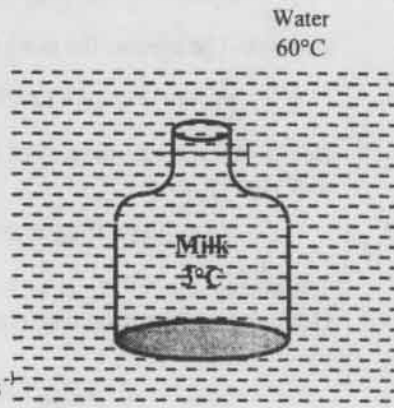
Therefore, it will take about 6 minutes to warm the milk from 3 to 38°C.

16-19 A thin-walled glass containing milk is placed into a large pan filled with hot water to warm up the milk. The warming time of the milk is to be determined.

Assumptions **1** The glass container is cylindrical in shape with a radius of r_0 = 3 cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.

Properties The thermal conductivity, density, and specific heat of the milk are k = 0.607 W/m.°C, ρ = 997 kg/m^3, and C_p = 4.180 kJ/kg.°C (Table A-15).

Analysis The characteristic length and the Biot number are



$$L_c = \frac{V}{A} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.03\text{ m})^2(0.07\text{ m})}{2\pi(0.03\text{ m})(0.07\text{ m}) + 2\pi(0.03\text{ m})^2} = 0.01050\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(240\text{ W/m}^2.^\circ\text{C})(0.0105\text{ m})}{(0.607\text{ W/m.}^\circ\text{C})} = 4.14 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C:

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{240\text{ W/m}^2.^\circ\text{C}}{(997\text{ kg/m}^3)(4180\text{ J/kg.}^\circ\text{C})(0.0105\text{ m})} = 0.005485\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{38-60}{3-60} = e^{-(0.005485\text{ s}^{-1})t} \longrightarrow t = \mathbf{174\text{ s} = 2.89\text{ min}}$$

Therefore, it will take about 3 minutes to warm the milk from 3 to 38°C.

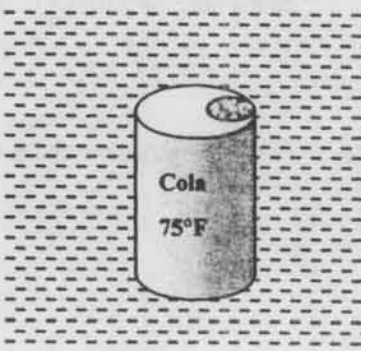
16-20E A person shakes a can of drink in a iced water to cool it. The cooling time of the drink is to be determined.

Assumptions **1** The can containing the drink is cylindrical in shape with a radius of r_0 = 1.25 in. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.

Properties The density and specific heat of water at room temperature are ρ = 62.22 lbm/ft^3, and C_p = 0.999 Btu/lbm.°F (Table A-15E).

Analysis Application of lumped system analysis in this case gives

$$L_c = \frac{V}{A} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(1.25/12\text{ ft})^2(5/12\text{ ft})}{2\pi(1.25/12\text{ ft})(5/12\text{ ft}) + 2\pi(1.25/12\text{ ft})^2} = 0.04167\text{ ft}$$



$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{30\text{ Btu/h.ft}^2.^\circ\text{F}}{(62.22\text{ lbm/ft}^3)(0.999\text{ Btu/lbm.}^\circ\text{F})(0.04167\text{ ft})}$$

$$= 11.599\text{ h}^{-1} = 0.00322\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{45-32}{75-32} = e^{-(0.00322\text{ s}^{-1})t} \longrightarrow t = \mathbf{372\text{ s}}$$

Therefore, it will take 6 minutes and 12 seconds to cool the canned drink to 45°F.

16-21 An iron whose base plate is made of an aluminum alloy is turned on. The time for the plate temperature to reach 140°C and whether it is realistic to assume the plate temperature to be uniform at all times are to be determined.

Assumptions **1** 85 percent of the heat generated in the resistance wires is transferred to the plate. **2** The thermal properties of the plate are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The density, specific heat, and thermal diffusivity of the aluminum alloy plate are given to be ρ = 2770 kg/m^3, C_p = 875 kJ/kg.°C, and α = 7.3×10^{-5} m^2/s. The thermal conductivity of the plate can be determined from $\alpha = k/\rho.C_p$ = 177 W/m.°C (or it can be read from Table A-19).

Analysis The mass of the iron's base plate is

$$m = \rho V = \rho LA = (2770\ \text{kg/m}^3)(0.005\ \text{m})(0.03\ \text{m}^2) = 0.4155\ \text{kg}$$

Noting that only 85 percent of the heat generated is transferred to the plate, the rate of heat transfer to the iron's base plate is

$$\dot{Q}_{\text{in}} = 0.85 \times 1000\ \text{W} = 850\ \text{W}$$

The temperature of the plate, and thus the rate of heat transfer from the plate, changes during the process. Using the average plate temperature, the average rate of heat loss from the plate is determined from

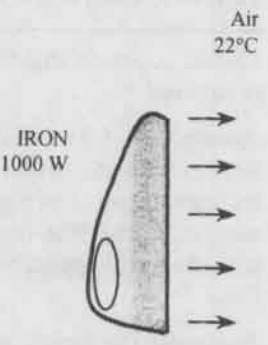
$$\dot{Q}_{\text{loss}} = hA(T_{\text{plate, ave}} - T_\infty) = (12\ \text{W/m}^2.^\circ\text{C})(0.03\ \text{m}^2)\left(\frac{140+22}{2} - 22\right)^\circ\text{C} = 21.2\ \text{W}$$

Energy balance on the plate can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{plate}} \quad \rightarrow \quad \dot{Q}_{\text{in}}\Delta t - \dot{Q}_{\text{out}}\Delta t = \Delta E_{\text{plate}} = mC_p\Delta T_{\text{plate}}$$

Solving for Δt and substituting,

$$\Delta t = \frac{mC_p\Delta T_{\text{plate}}}{\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}} = \frac{(0.4155\ \text{kg})(875\ \text{J/kg.°C})(140-22)^\circ\text{C}}{(850-21.2)\ \text{J/s}} = \mathbf{51.8\ s}$$



which is the time required for the plate temperature to reach 140 °C.

To determine whether it is realistic to assume the plate temperature to be uniform at all times, we need to calculate the Biot number,

$$L_c = \frac{V}{A} = \frac{LA}{A} = L = 0.005\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\ \text{W/m}^2.^\circ\text{C})(0.005\ \text{m})}{(177.0\ \text{W/m.°C})} = 0.00034 < 0.1$$

It is realistic to assume uniform temperature for the plate since Bi < 0.1.

Discussion This problem can also be solved by obtaining the differential equation from an energy balance on the plate for a differential time interval, and solving the differential equation. It gives

$$T(t) = T_\infty + \frac{\dot{Q}_{\text{in}}}{hA}\left(1 - \exp(-\frac{hA}{mC_p}t)\right)$$

Substituting the known quantities and solving for t again gives 51.8 s.

16-22 Ball bearings leaving the oven at a uniform temperature of 900°C are exposed to air for a while before they are dropped into the water for quenching. The time they can stand in the air before their temperature falls below 850°C is to be determined.

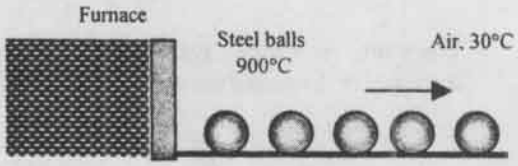
Assumptions **1** The bearings are spherical in shape with a radius of $r_0 = 0.6$ cm. **2** The thermal properties of the bearings are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the bearings are given to be $k = 15.1$ W/m.°C, $\rho = 8085$ kg/m^3, and $C_p = 0.480$ kJ/kg.°F.

Analysis The characteristic length of the steel ball bearings and Biot number are

$$L_c = \frac{V}{A} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.012\ \text{m}}{6} = 0.002\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(125\ \text{W/m}^2.^\circ\text{C})(0.002\ \text{m})}{(15.1\ \text{W/m.}^\circ\text{C})} = 0.0167 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the allowable time is determined to be

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{125\ \text{W/m}^2.^\circ\text{C}}{(8085\ \text{kg/m}^3)(480\ \text{J/kg.}^\circ\text{C})(0.002\ \text{m})} = 0.01610\ \text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{850-30}{900-30} = e^{-(0.0161\ \text{s}^{-1})t} \longrightarrow t = \mathbf{3.68\ s}$$

The result indicates that the ball bearing can stay in the air about 4 s before being dropped into the water.

16-23 A number of carbon steel balls are to be annealed by heating them first and then allowing them to cool slowly in ambient air at a specified rate. The time of annealing and the total rate of heat transfer from the balls to the ambient air are to be determined.

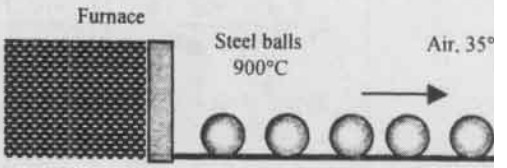
Assumptions **1** The balls are spherical in shape with a radius of $r_0 = 4$ mm. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be $k = 54$ W/m.°C, $\rho = 7833$ kg/m^3, and $C_p = 0.465$ kJ/kg.°C.

Analysis (*a*) The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.008 \text{ m}}{6} = 0.0013 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(75 \text{ W/m}^2.^\circ\text{C})(0.0013 \text{ m})}{(54 \text{ W/m.}^\circ\text{C})} = 0.0018 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the time for the annealing process is determined to be

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{75 \text{ W/m}^2.^\circ\text{C}}{(7833 \text{ kg/m}^3)(465 \text{ J/kg.}^\circ\text{C})(0.0013 \text{ m})} = 0.01584 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100 - 35}{900 - 35} = e^{-(0.01584 \text{ s}^{-1})t} \longrightarrow t = \mathbf{163 \text{ s} = 2.7 \text{ min}}$$

(*b*) The amount of heat transfer from a single ball is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (7833 \text{ kg/m}^3)\frac{\pi(0.008 \text{ m})^3}{6} = 0.0021 \text{ kg}$$

$$Q = mC_p[T_f - T_i] = (0.0021 \text{ kg})(465 \text{ J/kg.}^\circ\text{C})(900 - 100)^\circ\text{C} = 781 \text{ J} = 0.781 \text{ kJ (per ball)}$$

Then the total rate of heat transfer from the balls to the ambient air becomes

$$\dot{Q} = \dot{n}_{\text{ball}} Q = (2500 \text{ balls/h}) \times (0.781 \text{ kJ/ball}) = 1{,}953 \text{ kJ/h} = \mathbf{542 \text{ W}}$$

16-24 An electronic device is on for 5 minutes, and off for several hours. The temperature of the device at the end of the 5-min operating period is to be determined for the cases of operation with and without a heat sink.

Assumptions **1** The device and the heat sink are isothermal. **2** The thermal properties of the device and of the sink are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The specific heat of the device is given to be C_p = 850 J/kg.°C. The specific heat of the aluminum sink is 903 J/kg.°C (Table A-19), but can be taken to be 850 J/kg.°C for simplicity in analysis.

Analysis **(*a*) Approximate solution**

This problem can be solved approximately by using an average temperature for the device when evaluating the heat loss. An energy balance on the device can be expressed as

$$E_{\text{in}} - E_{\text{out}} + E_{\text{generation}} = \Delta E_{\text{device}} \quad \rightarrow \quad -\dot{Q}_{\text{out}}\Delta t + \dot{E}_{\text{generation}}\Delta t = mC_p\Delta T_{\text{device}}$$

or,

$$\dot{E}_{\text{generation}}\Delta t - hA\left(\frac{T+T_\infty}{2} - T_\infty\right)\Delta t = mC_p(T - T_\infty)$$

Substituting the given values,

$$(30\text{ J/s})(5\times 60\text{ s}) - (12\text{ W/m}^2.^\circ\text{C})(0.00005\text{ m}^2)\left(\frac{T-25}{2}\right)^\circ\text{C}(5\times 60\text{ s}) = (0.02\text{ kg})(850\text{ J/kg.}^\circ\text{C})(T-25)^\circ\text{C}$$

which gives $T = \mathbf{552^\circ C}$

If the device were attached to an aluminum heat sink, the temperature of the device would be

$$(30\text{ J/s})(5\times 60\text{ s}) - (12\text{ W/m}^2.^\circ\text{C})(0.0055\text{m}^2)\left(\frac{T-25}{2}\right)^\circ\text{C}(5\times 60\text{ s}) = (0.20+0.02)\text{kg}\times(850\text{ J/kg.}^\circ\text{C})(T-25)^\circ\text{C}$$

which gives $T = \mathbf{70.7^\circ C}$

Note that the temperature of the electronic device drops considerably as a result of attaching it to a heat sink.

(*b*) Exact solution

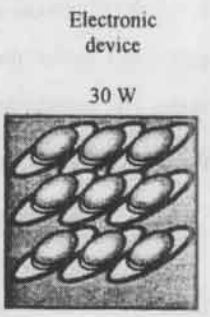
This problem can be solved exactly by obtaining the differential equation from an energy balance on the device for a differential time interval *dt*. We will get

$$\frac{d(T-T_\infty)}{dt} + \frac{Ah}{mC_p}(T-T_\infty) = \frac{\dot{E}_{\text{generation}}}{mC_p}$$

It can be solved to give

$$T(t) = T_\infty + \frac{\dot{E}_{\text{generation}}}{hA}\left(1 - \exp(-\frac{hA}{mC_p}t)\right)$$

Substituting the known quantities and solving for *t* gives **527°C** for the first case and **75.2°C** for the second case, which are fairly close to the results obtained from the approximate analysis.



Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres

16-25C A cylinder whose diameter is small relative to its length can be treated as an infinitely long cylinder. When the diameter and length of the cylinder are comparable, it is not proper to treat the cylinder as being infinitely long. It is also not proper to use this model when finding the temperatures near the bottom or top surfaces of a cylinder since heat transfer at those locations can be two-dimensional.

16-26C Yes. A plane wall whose one side is insulated is equivalent to a plane wall that is twice as thick and is exposed to convection from both sides. The midplane in the latter case will behave like an insulated surface because of thermal symmetry.

16-27C The solution for determination of the one-dimensional transient temperature distribution involves many variables that make the graphical representation of the results impractical. In order to reduce the number of parameters, some variables are grouped into dimensionless quantities.

16-28C The Fourier number is a measure of heat conducted through a body relative to the heat stored. Thus a large value of Fourier number indicates faster propagation of heat through body. Since Fourier number is proportional to time, doubling the time will also double the Fourier number.

16-29C This case can be handled by setting the heat transfer coefficient h to infinity ∞ since the temperature of the surrounding medium in this case becomes equivalent to the surface temperature.

16-30C The maximum possible amount of heat transfer will occur when the temperature of the body reaches the temperature of the medium, and can be determined from $Q_{\max} = mC_p(T_\infty - T_i)$.

16-31C When the Biot number is less than 0.1, the temperature of the sphere will be nearly uniform at all times. Therefore, it is more convenient to use the lumped system analysis in this case.

16-32 A student calculates the total heat transfer from a spherical copper ball. It is to be determined whether his/her result is reasonable.

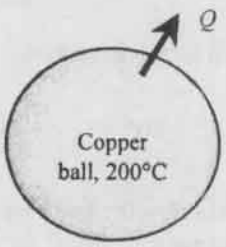
Assumptions The thermal properties of the copper ball are constant at room temperature.

Properties The density and specific heat of the copper ball are $\rho = 8933\ \text{kg/m}^3$, and $C_p = 0.385\ \text{kJ/kg.°C}$.

Analysis The mass of the copper ball and the maximum amount of heat transfer from the copper ball are

$$m = \rho V = \rho\left(\frac{\pi D^3}{6}\right) = (8933\ \text{kg/m}^3)\left[\frac{\pi(0.15\ \text{m})^3}{6}\right] = 15.79\ \text{kg}$$

$$Q_{\max} = mC_p[T_i - T_\infty] = (15.79\ \text{kg})(0.385\ \text{kJ/kg.°C})(200 - 25)\text{°C} = 1064\ \text{kJ}$$



Discussion The student's result of 4200 kJ is **not reasonable** since it is greater than the maximum possible amount of heat transfer.

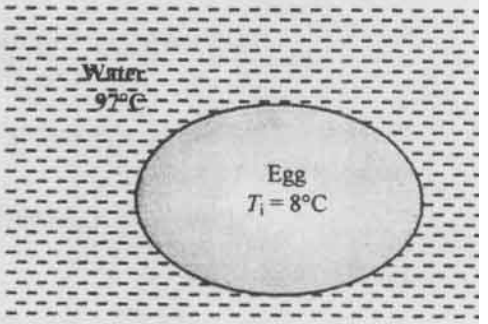
16-33 An egg is dropped into boiling water. The cooking time of the egg is to be determined. √

Assumptions **1** The egg is spherical in shape with a radius of $r_0 = 2.75$ cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs are given to be $k = 0.6$ W/m.°C and $\alpha = 0.14\times10^{-6}$ m^2/s.

Analysis The Biot number for this process is

$$Bi = \frac{hr_o}{k} = \frac{(1400\ \text{W/m}^2.^\circ\text{C})(0.0275\ \text{m})}{(0.6\ \text{W/m.}^\circ\text{C})} = 64.2$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 3.0877 \quad \text{and} \quad A_1 = 1.9969$$

Then the Fourier number becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70-97}{8-97} = (1.9969)e^{-(3.0877)^2\tau} \longrightarrow \tau = 0.198 \approx 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the time required for the temperature of the center of the egg to reach 70°C is determined to be

$$t = \frac{\tau r_o^{\,2}}{\alpha} = \frac{(0.198)(0.0275\ \text{m})^2}{(0.14\times10^{-6}\ \text{m}^2/\text{s})} = 1070\ \text{s} = \mathbf{17.8\ min}$$

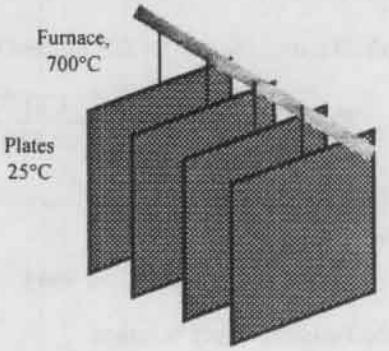
16-34 Large brass plates are heated in an oven. The surface temperature of the plates leaving the oven is to be determined.

Assumptions **1** Heat conduction in the plate is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the plate are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of brass at room temperature are given to be $k = 110$ W/m.°C, $\alpha = 33.9\times10^{-6}$ m^2/s

Analysis The Biot number for this process is

$$Bi = \frac{hL}{k} = \frac{(80\ \text{W/m}^2.^\circ\text{C})(0.015\ \text{m})}{(110\ \text{W/m.}^\circ\text{C})} = 0.0109$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 0.1039 \quad \text{and} \quad A_1 = 1.0018$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(33.9\times10^{-6}\ \text{m}^2/\text{s})(10\ \text{min}\times60\ \text{s/min})}{(0.015\ \text{m})^2} = 90.4 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the surface of the plates becomes

$$\theta(L,t)_{wall} = \frac{T(x,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0018)e^{-(0.1039)^2(90.4)}\cos(0.1039) = 0.376$$

$$\frac{T(L,t)-700}{25-700} = 0.376 \longrightarrow T(L,t) = \mathbf{446\,^\circ C}$$

Discussion This problem can be solved easily using the lumped system analysis since Bi < 0.1, and thus the lumped system analysis is applicable. It gives

$$\alpha = \frac{k}{\rho C_p} \rightarrow \rho C_p = \frac{k}{\alpha} = \frac{110\ \text{W/m}\cdot^\circ\text{C}}{33.9\times10^{-6}\ \text{m}^2/s} = 3.245\times10^6\ \text{W}\cdot\text{s/m}^3\cdot^\circ\text{C}$$

$$b = \frac{hA}{\rho V C_p} = \frac{hA}{\rho(LA)C_p} = \frac{h}{\rho L C_p} = \frac{h}{L(k/\alpha)} = \frac{80\ \text{W/m}^2\cdot^\circ\text{C}}{(0.015\ \text{m})(3.245\times10^6\ \text{W}\cdot\text{s/m}^3\cdot^\circ\text{C})} = 0.001644\ \text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \rightarrow T(t) = T_\infty + (T_i-T_\infty)e^{-bt} = 700^\circ\text{C} + (25-700^\circ\text{C})e^{-(0.001644\ \text{s}^{-1})(600\ \text{s})} = \mathbf{448\,^\circ C}$$

which is almost identical to the result obtained above.

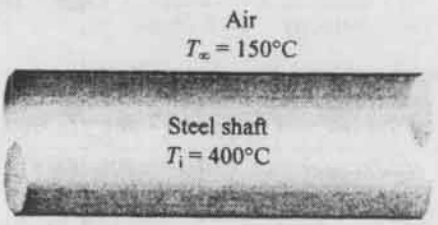
16-35 A long cylindrical shaft at 400°C is allowed to cool slowly. The center temperature and the heat transfer per unit length of the cylinder are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the shaft are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of stainless steel 304 at room temperature are given to be $k = 14.9$ W/m.°C, $\rho = 7900$ kg/m^3, $C_p = 477$ J/kg.°C, $\alpha = 3.95\times10^{-6}$ m^2/s

Analysis First the Biot number is calculated to be

$$Bi = \frac{hr_o}{k} = \frac{(60\ \text{W/m}^2.^\circ\text{C})(0.175\ \text{m})}{(14.9\ \text{W/m.}^\circ\text{C})} = 0.71$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 1.0935 \quad \text{and} \quad A_1 = 1.1558$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.95\times10^{-6}\ \text{m}^2/\text{s})(1200\ \text{s})}{(0.175\ \text{m})^2} = 0.1548$$

which is very close to the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the temperature at the center of the shaft becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.1558)e^{-(1.0935)^2(0.1548)} = 0.9605$$

$$\frac{T_0 - 150}{400 - 150} = 0.9605 \longrightarrow T_0 = \mathbf{390\ °C}$$

The maximum heat can be transferred from the cylinder per meter of its length is

$$m = \rho V = \rho \pi r_o^2 L = (7900\ \text{kg/m}^3)[\pi(0.175\ \text{m})^2(1\ \text{m})] = 760.1\ \text{kg}$$

$$Q_{\max} = mC_p[T_\infty - T_i] = (760.1\ \text{kg})(0.477\ \text{kJ/kg.°C})(400-150)°\text{C} = 9.064\times10^4\ \text{kJ}$$

Once the constant $J_1 = 0.4709$ is determined from Table 16-2 corresponding to the constant λ_1, the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\left(\frac{390-150}{400-150}\right)\frac{0.4709}{1.0935} = 0.173$$

$$Q = 0.173(9.064\times10^4\ \text{kJ}) = \mathbf{1.57\times10^4\ kJ}$$

16-36E Long cylindrical steel rods are heat-treated in an oven. Their centerline temperature when they leave the oven is to be determined.

Assumptions **1** Heat conduction in the rods is one-dimensional since the rods are long and they have thermal symmetry about the center line. **2** The thermal properties of the rod are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

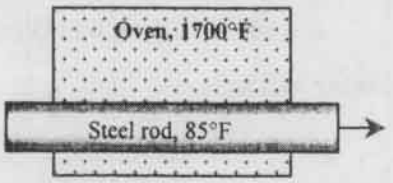
Properties The properties of AISI stainless steel rods are given to be $k = 7.74$ Btu/h.ft.°F, $\alpha = 0.135$ ft^2/h.

Analysis The time the steel rods stays in the oven can be determined from

$$t = \frac{\text{length}}{\text{velocity}} = \frac{30 \text{ ft}}{10 \text{ ft/min}} = 3 \text{ min} = 180 \text{ s}$$

The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(20 \text{ Btu/h.ft}^2\text{.°F})(2/12 \text{ ft})}{(7.74 \text{ Btu/h.ft.°F})} = 0.4307$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 0.8784 \quad \text{and} \quad A_1 = 1.0995$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.135 \text{ ft}^2/\text{h})(3/60 \text{ h})}{(2/12 \text{ ft})^2} = 0.243$$

Then the temperature at the center of the rods becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0995) e^{-(0.8784)^2 (0.243)} = 0.912$$

$$\frac{T_0 - 1700}{85 - 1700} = 0.912 \longrightarrow T_0 = \mathbf{227°F}$$

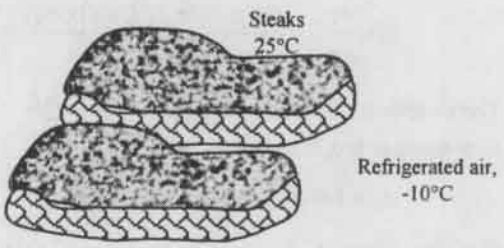
16-37 Steaks are cooled by passing them through a refrigeration room. The time of cooling is to be determined.

Assumptions **1** Heat conduction in the steaks is one-dimensional since the steaks are large relative to their thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the steaks are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of steaks are given to be $k = 0.45$ W/m.°C and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis The Biot number is

$$Bi = \frac{hL}{k} = \frac{(9\ \text{W/m}^2.^\circ\text{C})(0.01\,\text{m})}{(0.45\ \text{W/m.}^\circ\text{C})} = 0.200$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 0.4328 \quad \text{and} \quad A_1 = 1.0311$$

The Fourier number is

$$\frac{T(L,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) \longrightarrow \frac{3-(-10)}{25-(-10)} = (1.0311)e^{-(0.4328)^2\tau}\cos(0.4328) \longrightarrow \tau = 4.934 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the length of time for the steaks to be kept in the refrigerator is determined to be

$$t = \frac{\tau L^2}{\alpha} = \frac{(4.934)(0.01\ \text{m})^2}{(0.91\times10^{-7}\ \text{m}^2/\text{s})} = 5422\ \text{s} = \mathbf{90.4\ min}$$

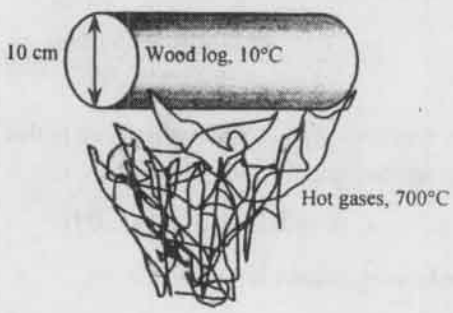
16-38 A long cylindrical wood log is exposed to hot gases in a fireplace. The time for the ignition of the wood is to be determined. √

Assumptions **1** Heat conduction in the wood is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the wood are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of wood are given to be $k = 0.17$ W/m.°C, $\alpha = 1.28\times10^{-7}$ m^2/s

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(13.6\ \text{W/m}^2.^\circ\text{C})(0.05\ \text{m})}{(0.17\ \text{W/m.}^\circ\text{C})} = 4.00$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 1.9081 \quad \text{and} \quad A_1 = 1.4698$$

Once the constant J_0 is determined from Table 16-2 corresponding to the constant λ_1, the Fourier number is determined to be

$$\frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r_o / r_o) \longrightarrow \frac{420 - 500}{10 - 500} = (1.4698) e^{-(1.9081)^2 \tau} (0.2771) \longrightarrow \tau = 0.251$$

which is above the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can be used. Then the length of time before the log ignites is

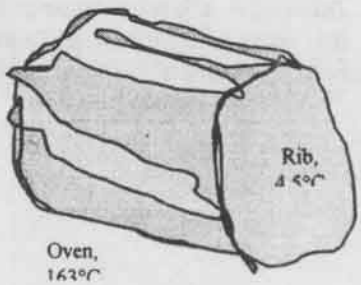
$$t = \frac{\tau r_o^{\,2}}{\alpha} = \frac{(0.251)(0.05\ \text{m})^2}{(1.28\times10^{-7}\ \text{m}^2/\text{s})} = 4902\ \text{s} = \mathbf{81.7\ min}$$

16-39 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is rare done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the rib are given to be $k = 0.45$ W/m.°C, $\rho = 1200$ kg/m^3, $C_p = 4.1$ kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s.

Analysis (a) The radius of the roast is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.002667\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.002667\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2/\text{s})(2\times3600+45\times60)\text{s}}{(0.08603\text{ m})^2} = 0.1217$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{60-163}{4.5-163} = 0.65 = A_1 e^{-\lambda_1^2(0.1217)}$$

It is determined from Table 16-1 by trial and error that this equation is satisfied when $Bi = 30$, which corresponds to $\lambda_1 = 3.0372$ and $A_1 = 1.9898$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(30)}{(0.08603\text{ m})} = \mathbf{157\ W/m^2.°C}$$

This value seems to be larger than expected for problems of this kind. This is probably due to the Fourier number being less than 0.2.

(b) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9898)e^{-(3.0372)^2(0.1217)}\frac{\sin(3.0372\text{ rad})}{3.0372}$$

$$\frac{T(r_o,t)-163}{4.5-163} = 0.0222 \longrightarrow T(r_o,t) = \mathbf{159.5\,°C}$$

(c) The maximum possible heat transfer is

$$Q_{\max} = mC_p(T_i - T_\infty) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(4.5-163)\text{°C} = -2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1)-\lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.65)\frac{\sin(3.0372)-(3.0372)\cos(3.0372)}{(3.0372)^3} = 0.783$$

$$Q = 0.783Q_{\max} = (0.783)(2080\text{ kJ}) = \mathbf{1629\ kJ}$$

Prob. 16-39 (*continued*)

(d) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{71-163}{4.5-163} = (1.9898) e^{-(3.0372)^2 \tau} \longrightarrow \tau = 0.1336$$

$$t = \frac{r_o^{\,2}}{\alpha} = \frac{(0.1336)(0.08603\,\text{m})^2}{(0.91\times 10^{-7}\ \text{m}^2/\text{s})} = 10{,}866\,\text{s} = 181\,\text{min} \cong \mathbf{3\,hr}$$

This result is close to the listed value of 3 hours and 20 minutes. The difference between the two results is due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

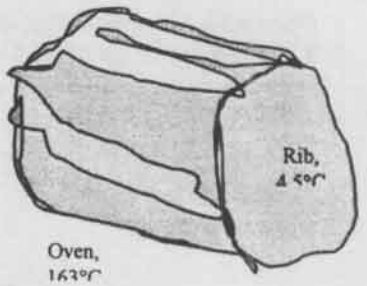
Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

16-40 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is well-done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the rib are given to be $k = 0.45$ W/m.°C, $\rho = 1200$ kg/m^3, $C_p = 4.1$ kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis (a) The radius of the rib is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.00267\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^{\,3} \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.00267\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^{\,2}} = \frac{(0.91\times10^{-7}\text{ m}^2/\text{s})(4\times3600+15\times60)\text{s}}{(0.08603\text{ m})^2} = 0.1881$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution formulation can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{77-163}{4.5-163} = 0.543 = A_1 e^{-\lambda_1^2 (0.1881)}$$

It is determined from Table 16-1 by trial and error that this equation is satisfied when $Bi = 4.5$, which corresponds to $\lambda_1 = 2.5130$ and $A_1 = 1.7536$. Then the heat transfer coefficient can be determined from.

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(4.5)}{(0.08603\text{ m})} = \mathbf{23.5\ W/m^2.°C}$$

(b) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.7536) e^{-(2.513)^2(0.1883)} \frac{\sin(2.513)}{2.513}$$

$$\frac{T(r_o,t) - 163}{4.5-163} = 0.125 \longrightarrow T(r_o,t) = \mathbf{143.5\ °C}$$

(c) The maximum possible heat transfer is

$$Q_{\max} = mC_p(T_i - T_\infty) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(4.5-163)\text{°C} = 2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph} \frac{\sin(\lambda_1) - \lambda_1 \cos(\lambda_1)}{\lambda_1^{\,3}} = 1 - 3(0.543) \frac{\sin(2.513) - (2.513)\cos(2.513)}{(2.513)^3} = 0.731$$

$$Q = 0.731 Q_{\max} = (0.731)(2080\text{ kJ}) = \mathbf{1521\ kJ}$$

Prob. 16-40 (*continued*)

(d) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{71-163}{4.5-163} = (1.7536)e^{-(2.5130)^2 \tau} \longrightarrow \tau = 0.173$$

$$t = \frac{r_o^{\,2}}{\alpha} = \frac{(0.173)(0.08603\,\text{m})^2}{(0.91\times10^{-7}\ \text{m}^2/\text{s})} = 14{,}070\,\text{s} = 234.5\,\text{min} = \mathbf{3\ hr\ 55\ min}$$

This result is close to the listed value of 3 hours and 20 minutes. The difference between the two results is probably due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

16-41 An egg is dropped into boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical in shape with a radius of $r_0 = 2.75$ cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs can be approximated by those of water at room temperature to be $k = 0.607$ W/m.°C, $\alpha = k/\rho C_p = 0.146\times10^{-6}$ m²/s (Table A-15).

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(500\ \text{W/m}^2.^\circ\text{C})(0.0275\ \text{m})}{(0.607\ \text{W/m.}^\circ\text{C})} = 22.7$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 2.9945 \quad \text{and} \quad A_1 = 1.9815$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{60-100}{8-100} = (1.9815)e^{-(2.9945)^2 \tau} \longrightarrow \tau = 0.1691$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the length of time for the egg to be kept in boiling water is determined to be

$$t = \frac{r_o^2}{\alpha} = \frac{(0.1691)(0.0275\ \text{m})^2}{(0.146\times10^{-6}\ \text{m}^2/\text{s})} = 876\ \text{s} = \mathbf{14.6\ min}$$

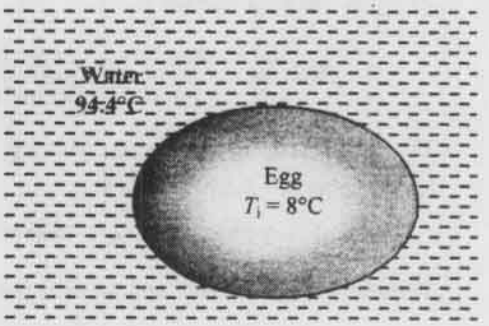
16-42 An egg is cooked in boiling water. The cooking time of the egg is to be determined for a location at 1610-m elevation.

Assumptions **1** The egg is spherical in shape with a radius of $r_0 = 2.75$ cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg and heat transfer coefficient are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs can be approximated by those of water at room temperature to be $k = 0.607$ W/m.°C, $\alpha = k/\rho C_p = 0.146\times10^{-6}$ m^2/s (Table A-15).

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(500\ \text{W/m}^2.^\circ\text{C})(0.0275\ \text{m})}{(0.607\ \text{W/m.}^\circ\text{C})} = 22.7$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 2.9945 \quad \text{and} \quad A_1 = 1.9815$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{60 - 94.4}{8 - 94.4} = (1.9815)e^{-(2.9945)^2 \tau} \longrightarrow \tau = 0.1790$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the length of time for the egg to be kept in boiling water is determined to be

$$t = \frac{r_o^2}{\alpha} = \frac{(0.1790)(0.0275\ \text{m})^2}{(0.146\times10^{-6}\ \text{m}^2/\text{s})} = 927\ \text{s} = \mathbf{15.5\ min}$$

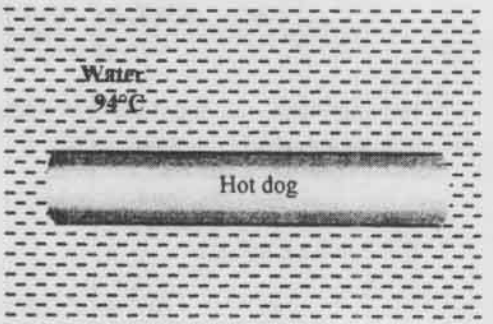
16-43 A hot dog is dropped into boiling water, and temperature measurements are taken at certain time intervals. The thermal diffusivity and thermal conductivity of the hot dog and the convection heat transfer coefficient are to be determined. √

Assumptions **1** Heat conduction in the hot dog is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of hot dog available are given to be $\rho = 980$ kg/m^3 and $C_p = 3900$ J/kg.°C.

Analysis (a) From Fig. 16-14b we have

$$\left.\begin{aligned} \frac{T-T_\infty}{T_o-T_\infty} &= \frac{88-94}{59-94} = 0.17 \\ \frac{r}{r_o} &= \frac{r_o}{r_o} = 1 \end{aligned}\right\} \frac{1}{Bi} = \frac{k}{hr_o} = 0.15$$



The Fourier number is determined from Fig. 16-14a to be

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.15 \\ \frac{T_o-T_\infty}{T_i-T_\infty} &= \frac{59-94}{20-94} = 0.47 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.20$$

The thermal diffusivity of the hot dog is determined to be

$$\frac{\alpha t}{r_o^2} = 0.20 \longrightarrow \alpha = \frac{0.2 r_o^2}{t} = \frac{(0.2)(0.011\,\text{m})^2}{120\,\text{s}} = \mathbf{2.0\times10^{-7}\ m^2/s}$$

(*b*) The thermal conductivity of the hot dog is determined from

$$k = \alpha\rho C_p = (2\times10^{-7}\ \text{m}^2/\text{s})(980\,\text{kg/m}^3)(3900\,\text{J/kg.°C}) = \mathbf{0.76\ W/m.°C}$$

(*c*) From part (*a*) we have $\frac{1}{Bi} = \frac{k}{hr_o} = 0.15$. Then,

$$\frac{k}{h} = 0.15 r_0 = (0.15)(0.011\,\text{m}) = 0.00165\,\text{m}$$

Therefore, the heat transfer coefficient is

$$\frac{k}{h} = 0.00165 \longrightarrow h = \frac{0.76\ \text{W/m.°C}}{0.00165\,\text{m}} = \mathbf{461\ W/m^2.°C}$$

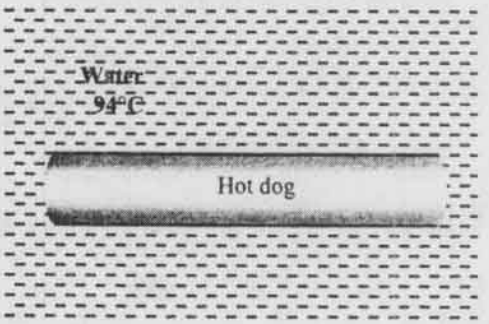
16-44 Using the data and the answers given in Prob. 16-43, the center and the surface temperatures of the hot dog 4 min after the start of the cooking and the amount of heat transferred to the hot dog are to be determined.

Assumptions **1** Heat conduction in the hot dog is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of hot dog and the convection heat transfer coefficient are given to be $k = 0.76$ W/m.°C, $\rho = 980$ kg/m^3, $C_p = 3900$ J/kg.°C, $\alpha = 2\times10^{-7}$ m^2/s, and $h = 658$ W/m^2.°C.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(658\ \text{W/m}^2.^\circ\text{C})(0.011\ \text{m})}{(0.76\ \text{W/m.}^\circ\text{C})} = 9.524$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 2.1681 \quad \text{and} \quad A_1 = 1.5644$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(2\times10^{-7}\ \text{m}^2/\text{s})(4\ \text{min}\times 60\ \text{s/min})}{(0.011\ \text{m})^2} = 0.3967 > 0.2$$

Then the temperature at the center of the hot dog is determined to be

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5644)e^{-(2.1681)^2(0.3967)} = 0.242$$

$$\frac{T_0 - 94}{20 - 94} = 0.242 \longrightarrow T_0 = \mathbf{76.1\ °C}$$

From Table 16-2 we read J_0 =0.1273 corresponding to the constant λ_1. Then the temperature at the surface of the hot dog becomes

$$\frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r_o / r_o) = (1.5644)e^{-(2.1681)^2(0.3967)}(0.1273) = 0.03085$$

$$\frac{T(r_o,t) - 94}{20 - 94} = 0.03085 \longrightarrow T(r_o,t) = \mathbf{91.7\ °C}$$

The maximum possible amount of heat transfer is

$$m = \rho V = \rho \pi r_o^2 L = (980\ \text{kg/m}^3)\left[\pi(0.011\ \text{m})^2 (0.125\ \text{m})\right] = 0.04657\ \text{kg}$$

$$Q_{\max} = mC_p(T_i - T_\infty) = (0.04657\ \text{kg})(3900\ \text{J/kg.°C})(94 - 20)^\circ\text{C} = 13{,}440\ \text{J}$$

From Table 16-2 we read J_1= 0.5597 corresponding to the constant λ_1. Then the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.242)\frac{0.5597}{2.1681} = 0.875 \longrightarrow Q = 0.875(13{,}440\ \text{kJ}) = \mathbf{11{,}760\ kJ}$$

16-45E Whole chickens are to be cooled in the racks of a large refrigerator. Heat transfer coefficient that will enable to meet temperature constraints of the chickens while keeping the refrigeration time to a minimum is to be determined.

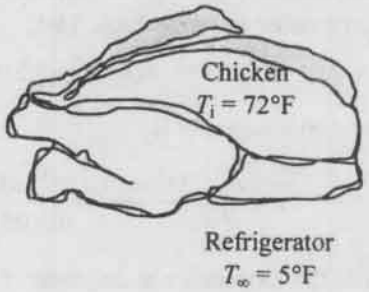
Assumptions **1** The chicken is a homogeneous spherical object. **2** Heat conduction in the chicken is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the chicken are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the chicken are given to be $k = 0.26$ Btu/h.ft.°F, $\rho = 74.9$ lbm/ft^3, $C_p = 0.98$ Btu/lbm.°F, and $\alpha = 0.0035$ ft^2/h.

Analysis The radius of the chicken is determined to be

$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{5\ \text{lbm}}{74.9\ \text{lbm/ft}^3} = 0.06676\ \text{ft}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.06676\ \text{ft}^3)}{4\pi}} = 0.2517\ \text{ft}$$



From Fig. 16-15b we have

$$\left.\begin{aligned} \frac{T-T_\infty}{T_o-T_\infty} &= \frac{35-5}{45-5} = 0.75 \\ \frac{x}{r_o} &= \frac{r_o}{r_o} = 1 \end{aligned}\right\} \frac{1}{Bi} = \frac{k}{hr_o} = 1.75$$

Then the heat transfer coefficients becomes

$$h = \frac{k}{1.75 r_o} = \frac{(0.26\ \text{Btu/h.ft.°F})}{1.75(0.2517\ \text{ft})} = \mathbf{0.590\ Btu/h.ft^2.°F}$$

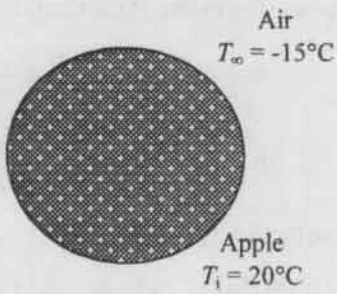
16-46 A person puts apples into the freezer to cool them quickly. The center and surface temperatures of the apples, and the amount of heat transfer from each apple in 1 h are to be determined. √

Assumptions **1** The apples are spherical in shape with a diameter of 9 cm. **2** Heat conduction in the apples is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the apples are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the apples are given to be k = 0.418 W/m.°C, ρ = 840 kg/m^3, C_p = 3.81 kJ/kg.°C, and $\alpha = 1.3\times10^{-7}$ m^2/s.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(8\ \text{W/m}^2.^\circ\text{C})(0.045\ \text{m})}{(0.418\ \text{W/m.}^\circ\text{C})} = 0.861$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 1.476 \quad \text{and} \quad A_1 = 1.2390$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_0^2} = \frac{(1.3\times10^{-7}\ \text{m}^2/\text{s})(1\ \text{h}\times3600\ \text{s/h})}{(0.045\ \text{m})^2} = 0.231 > 0.2$$

Then the temperature at the center of the apples becomes

$$\theta_{o,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - (-15)}{20-(-15)} = (1.239)e^{-(1.476)^2(0.231)} = 0.749 \longrightarrow T_0 = \mathbf{11.2^\circ C}$$

The temperature at the surface of the apples is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.239)e^{-(1.476)^2(0.231)} \frac{\sin(1.476\ \text{rad})}{1.476} = 0.505$$

$$\frac{T(r_o,t)-(-15)}{20-(-15)} = 0.505 \longrightarrow T(r_o,t) = \mathbf{2.7^\circ C}$$

The maximum possible heat transfer is

$$m = \rho V = \rho \frac{4}{3}\pi r_o^{\,3} = (840\ \text{kg/m}^3)\left[\frac{4}{3}\pi(0.045\ \text{m})^3\right] = 0.3206\ \text{kg}$$

$$Q_{\max} = mC_p(T_i - T_\infty) = (0.3206\ \text{kg})(3.6\ \text{kJ/kg.}^\circ\text{C})[20-(-15)]^\circ\text{C} = 40.4\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^{\,3}} = 1 - 3(0.749)\frac{\sin(1.476\ \text{rad}) - (1.476)\cos(1.476\ \text{rad})}{(1.476)^3} = 0.402$$

$$Q = 0.402 Q_{\max} = (0.402)(40.4\ \text{kJ}) = \mathbf{16.2\ kJ}$$

Note: Values for k and C_p are corrected to match α.

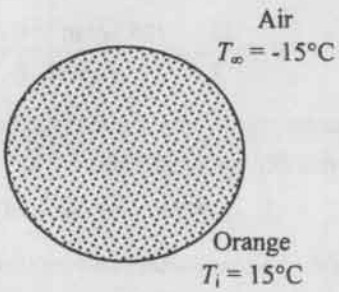
16-47 An orange is exposed to very cold ambient air. It is to be determined whether the orange will freeze in 4 h in subfreezing temperatures.

Assumptions **1** The orange is spherical in shape with a diameter of 8 cm. **2** Heat conduction in the orange is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the orange are constant, and are those of water. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the orange are approximated by those of water at the average temperature of about 5°C, $k = 0.571$ W/m.°C and $\alpha = k/\rho C_p = 0.571/(1000 \times 4205) = 0.136 \times 10^{-6}$ m^2/s (Table A-15).

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(15 \text{ W/m}^2.^\circ\text{C})(0.04 \text{ m})}{(0.571 \text{ W/m.}^\circ\text{C})} = 1.051 \approx 1.0$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 1.5708 \quad \text{and} \quad A_1 = 1.2732$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.136 \times 10^{-6} \text{ m}^2/\text{s})(4 \text{ h} \times 3600 \text{ s/h})}{(0.04 \text{ m})^2} = 1.22 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the surface of the oranges becomes

$$\theta(r_o, t)_{sph} = \frac{T(r_o, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.2732) e^{-(1.5708)^2 (1.22)} \frac{\sin(1.5708 \text{ rad})}{1.5708} = 0.0399$$

$$\frac{T(r_o, t) - (-6)}{15 - (-6)} = 0.0399 \longrightarrow T(r_o, t) = \mathbf{-5.2\ ^\circ C}$$

which is less than 0°C. Therefore, the oranges will freeze.

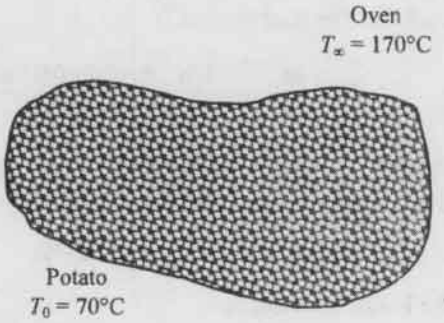
16-48 A hot baked potato is taken out of the oven and wrapped so that no heat is lost from it. The time the potato is baked in the oven and the final equilibrium temperature of the potato after it is wrapped are to be determined.

Assumptions **1** The potato is spherical in shape with a diameter of 8 cm. **2** Heat conduction in the potato is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the potato are given to be $k = 0.6$ W/m.°C, $\rho = 1100$ kg/m^3, $C_p = 3.9$ kJ/kg.°C, and $\alpha = 1.4\times10^{-7}$ m^2/s.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(25\ \text{W/m}^2.^\circ\text{C})(0.04\ \text{m})}{(0.6\ \text{W/m.}^\circ\text{C})} = 1.67$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 1.8777 \quad \text{and} \quad A_1 = 1.4113$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70-170}{25-170} = (1.4113)e^{-(1.8777)^2 \tau} \longrightarrow \tau = 0.203 > 0.2$$

The baking time of the potatoes is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.203)(0.04\ \text{m})^2}{(1.4\times10^{-7}\ \text{m}^2/\text{s})} = 2320\ \text{s} = \mathbf{38.7\ min}$$

The maximum amount of heat transfer is

$$m = \rho V = \rho \frac{4}{3}\pi r_o^3 = (1100\ \text{kg/m}^3)\left[\frac{4}{3}\pi(0.04\ \text{m})^3\right] = 0.295\ \text{kg}$$

$$Q_{\max} = mC_p(T_i - T_\infty) = (0.295\ \text{kg})(3.900\ \text{kJ/kg.°C})(25-170)^\circ\text{C} = -166.76\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.69)\frac{\sin(1.8777) - (1.8777)\cos(1.8777)}{(1.8777)^3} = 0.525$$

$$Q = 0.525 Q_{\max} = (0.525)(166.76\ \text{kJ}) = \mathbf{87.6\ kJ}$$

The final equilibrium temperature of the potato after it is wrapped is

$$Q = mC_p(T_{eqv} - T_i) \longrightarrow T_{eqv} = T_i + \frac{Q}{mC_p} = 25^\circ\text{C} + \frac{87.6\ \text{kJ}}{(0.29\ \text{kg})(3.9\ \text{kJ/kg.°C})} = \mathbf{102^\circ C}$$

Transient Heat Conduction in Semi-Infinite Solids

16-49C A semi-infinite medium is an idealized body which has a single exposed plane surface and extends to infinity in all directions. The earth and thick walls can be considered to be semi-infinite media.

16-50C A thick plane wall can be treated as a semi-infinite medium if all we are interested in is the variation of temperature in a region near one of the surfaces for a time period during which the temperature in the mid section of the wall does not experience any change.

16-51C The total amount of heat transfer from a semi-infinite solid up to a specified time t_0 can be determined by integration from

$$Q = \int_0^{t_0} Ah[T(0,t) - T_\infty]dt$$

where the surface temperature $T(0,t)$ is obtained from Eq. 16-22 by substituting $x = 0$.

16-52 The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

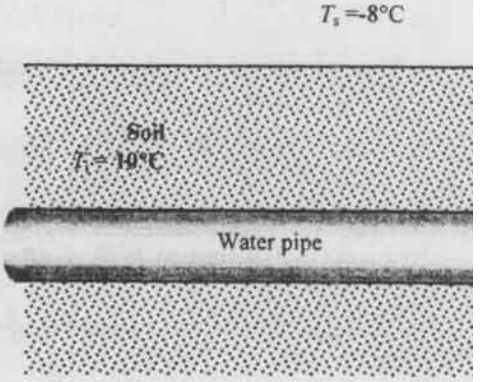
Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be $k = 0.4$ W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s.

Analysis The length of time the snow pack stays on the ground is

$$t = (60 \text{ days})(24 \text{ hr / days})(3600 \text{ s / hr}) = 5.184\times10^6 \text{ s}$$

The surface is kept at -18°C at all times. The depth at which freezing at 0°C occurs can be determined from the analytical solution,



$$\frac{T(x,t) - T_i}{T_s - T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right)$$

$$\frac{0-10}{-8-10} = erfc\left(\frac{x}{2\sqrt{(0.15\times10^{-6} \text{ m}^2/\text{s})(5.184\times10^6 \text{ s})}}\right)$$

$$\rightarrow 0.556 = erfc\left(\frac{x}{1.7636}\right)$$

Then from Table 16-3 we get $\frac{x}{1.7636} = 0.416 \rightarrow x = \mathbf{0.734 \text{ m}}$

Discussion The solution could also be determined using the chart, but it would be subject to reading error.

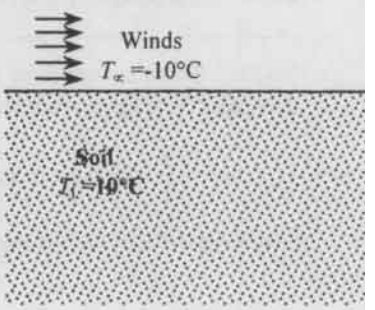
16-53 An area is subjected to cold air for a 10-h period. The soil temperatures at distances 0, 10, 20, and 50 cm from the earth's surface are to be determined. √

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be $k = 0.9$ W/m.°C and $\alpha = 1.6\times10^{-5}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the ground can be determined from

$$\frac{T(x,t)-T_i}{T_\infty-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)-\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$



where

$$\frac{h\sqrt{\alpha t}}{k}=\frac{(40\text{ W/m}^2.^\circ\text{C})\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\times3600\text{ s})}}{0.9\text{ W/m.°C}}=33.7$$

$$\frac{h^2\alpha t}{k^2}=\left(\frac{h\sqrt{\alpha t}}{k}\right)^2=33.7^2=1138$$

Then we conclude that the last term in the temperature distribution relation above must be zero regardless of x despite the exponential term tending to infinity since (1) $erfc(\xi)\to0$ for $\xi>4$ (see Table 16-3) and (2) the term has to remain less than 1 to have physically meaningful solutions. That is,

$$\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]=\exp\left(\frac{hx}{k}+1138\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+33.3\right)\right]\cong0$$

Therefore, the temperature distribution relation simplifies to

$$\frac{T(x,t)-T_i}{T_\infty-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)\to T(x,t)=T_i+(T_\infty-T_i)erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Then the temperatures at 0, 10, 20, and 50 cm depth from the ground surface become

$x=0$: $$T(0,10\text{ h})=T_i+(T_\infty-T_i)erfc\left(\frac{0}{2\sqrt{\alpha t}}\right)=T_i+(T_\infty-T_i)erfc(0)=T_i+(T_\infty-T_i)\times1=T_\infty=\mathbf{-10^\circ C}$$

$x=0.1$m: $$T(0.1\text{ m},10\text{ h})=10+(-10-10)erfc\left(\frac{0.1\text{ m}}{2\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\text{ h}\times3600\text{ s/h})}}\right)$$
$$=10-20erfc(0.066)=10-20\times0.9257=\mathbf{-8.5^\circ C}$$

$x=0.2$ m: $$T(0.2\text{ m},10\text{ h})=10+(-10-10)erfc\left(\frac{0.2\text{ m}}{2\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\text{ h}\times3600\text{ s/h})}}\right)$$
$$=10-20erfc(0.132)=10-20\times0.8519=\mathbf{-7.0^\circ C}$$

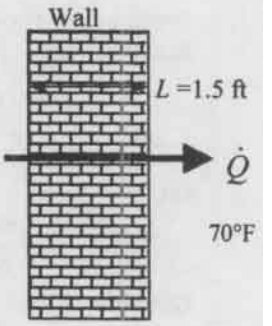
$x=0.5$ m: $$T(0.5\text{ m},10\text{ h})=10+(-10-10)erfc\left(\frac{0.5\text{ m}}{2\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\text{ h}\times3600\text{ s/h})}}\right)$$
$$=10-20erfc(0.329)=10-20\times0.6418=\mathbf{-2.8^\circ C}$$

16-54E The walls of a furnace made of concrete are exposed to hot gases at the inner surfaces. The time it will take for the temperature of the outer surface of the furnace to change is to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at inner surfaces only and the convection heat transfer coefficient inside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature of 1800°F. **2** The thermal properties of the concrete wall are constant.

Properties The thermal properties of the concrete are given to be $k = 0.64$ Btu/h.ft.°F and $\alpha = 0.023$ ft²/h.

Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from



$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

But,

$$\frac{T(x,t)-T_i}{T_s-T_i} = \frac{70.1-70}{1800-70} = 0.00006 \rightarrow 0.00006 = erfc(2.85) \quad \text{(Table 3)}$$

Therefore,

$$\frac{x}{2\sqrt{\alpha t}} = 2.85 \longrightarrow t = \frac{x^2}{4\times(2.85)^2\alpha} = \frac{(1.5\text{ ft})^2}{4\times(2.85)^2(0.023\text{ ft}^2/\text{h})} = 3.01\text{ h} = \mathbf{181\text{ min}}$$

16-55 A thick wood slab is exposed to hot gases for a period of 5 minutes. It is to be determined whether the wood will ignite.

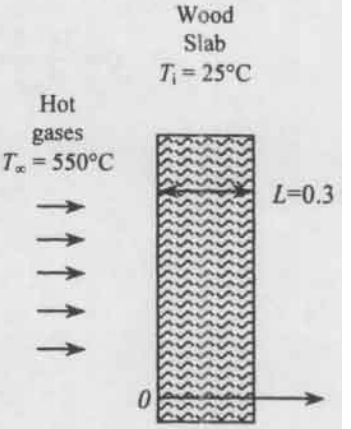
Assumptions **1** The wood slab is treated as a semi-infinite medium subjected to convection at the exposed surface. **2** The thermal properties of the wood slab are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The thermal properties of the wood are $k = 0.17$ W/m.°C and $\alpha = 1.28\times10^{-7}$ m²/s.

Analysis The one-dimensional transient temperature distribution in the wood can be determined from

$$\frac{T(x,t)-T_i}{T_\infty-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

where



$$\frac{h\sqrt{\alpha t}}{k} = \frac{(35\text{ W/m}^2.^\circ\text{C})\sqrt{(1.28\times10^{-7}\text{ m}^2/\text{s})(5\times60\text{ s})}}{0.17\text{ W/m.°C}} = 1.276$$

$$\frac{h^2\alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = 1.276^2 = 1.628$$

Noting that $x = 0$ at the surface and using Table 16-3 for *erfc* values,

$$\frac{T(x,t)-25}{550-25} = erfc(0) - \exp(0+1.628)erfc(0+1.276)$$
$$= 1-(5.0921)(0.0727)$$
$$= 0.630$$

Solving for $T(x, t)$ gives

$$T(x,t) = \mathbf{356^\circ C}$$

which is less than the ignition temperature of 420°C. Therefore, the wood will not ignite.

16-56 The outer surfaces of a large cast iron container filled with ice are exposed to hot water. The time before the ice starts melting and the rate of heat transfer to the ice are to be determined.

Assumptions **1** The temperature in the container walls is affected by the thermal conditions at outer surfaces only and the convection heat transfer coefficient outside inside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the wall are constant.

Properties The thermal properties of the cast iron are given to be $k = 52$ W/m.°C and $\alpha = 1.28\times10^{-7}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

But,

$$\frac{T(x,t)-T_i}{T_s-T_i} = \frac{0.1-0}{60-0} = 0.00167 \rightarrow \ 0.00167 = erfc(2.225) \quad \text{(Table 3)}$$

Therefore,

$$\frac{x}{2\sqrt{\alpha t}} = 2.225 \longrightarrow t = \frac{x^2}{4\times(2.225)^2\alpha} = \frac{(0.05\text{ m})^2}{(2.225)^2(1.7\times10^{-5}\text{ m}^2/\text{s})} = \mathbf{7.4\ s}$$

Ice chest



The rate of heat transfer to the ice when steady operation conditions are reached can be determined by applying the thermal resistance network concept as

$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(250\text{ W/m}^2.\text{°C})(1.2\times2\text{ m}^2)} = 0.00167\text{°C/W}$$

$$R_{wall} = \frac{L}{kA} = \frac{0.05\text{ m}}{(52\text{ W/m.°C})(1.2\times2\text{ m}^2)} = 0.00040\text{°C/W}$$

$$R_{conv,0} = \frac{1}{h_o A} = \frac{1}{(\infty)(1.2\times2\text{ m}^2)} \cong 0\text{°C/W}$$

$$R_{total} = R_{conv,1} + R_{wall} + R_{conv,2} = 0.00167 + 0.00040 + 0 = 0.00207\text{°C/W}$$

$$\dot{Q} = \frac{T_2 - T_1}{R_{total}} = \frac{(60-0)\text{°C}}{0.00207\text{°C/W}} = \mathbf{28{,}990\ W}$$



Transient Heat Conduction in Multidimensional Systems

16-57C The product solution enables us to determine the dimensionless temperature of two- or three-dimensional heat transfer problems as the product of dimensionless temperatures of one-dimensional heat transfer problems. The dimensionless temperature for a two-dimensional problem is determined by determining the dimensionless temperatures in both directions, and taking their product.

16-58C The dimensionless temperature for a three-dimensional heat transfer is determined by determining the dimensionless temperatures of one-dimensional geometries whose intersection is the three dimensional geometry, and taking their product.

16-59C This short cylinder is physically formed by the intersection of a long cylinder and a plane wall. The dimensionless temperatures at the center of plane wall and at the center of the cylinder are determined first. Their product yields the dimensionless temperature at the center of the short cylinder.

16-60C The heat transfer in this short cylinder is one-dimensional since there is no heat transfer in the axial direction. The temperature will vary in the radial direction only.

16-61 A short cylinder is allowed to cool in atmospheric air. The temperatures at the centers of the cylinder and the top surface as well as the total heat transfer from the cylinder for 15 min of cooling are to be determined.

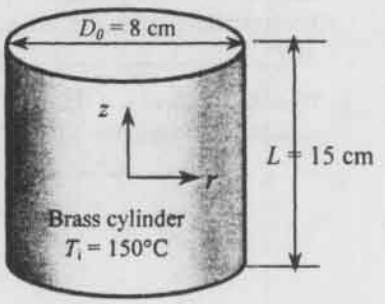
Assumptions **1** Heat conduction in the short cylinder is two-dimensional, and thus the temperature varies in both the axial x- and the radial r- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of brass are given to be $\rho = 8530\ \text{kg/m}^3, C_p = 0.389\ \text{kJ/kg}\cdot°\text{C}$, $k = 110\ \text{W/m}\cdot°\text{C}$, and $\alpha = 3.39\times10^{-5}\ \text{m}^2/\text{s}$.

Analysis This short cylinder can physically be formed by the intersection of a long cylinder of radius $D/2$ = 4 cm and a plane wall of thickness $2L$ = 15 cm. We measure x from the midplane.

(*a*) The Biot number is calculated for the plane wall to be

$$Bi = \frac{hL}{k} = \frac{(40\ \text{W/m}^2\cdot°\text{C})(0.075\ \text{m})}{(110\ \text{W/m}\cdot°\text{C})} = 0.02727$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 0.164 \quad \text{and} \quad A_1 = 1.0050$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.39\times10^{-5}\ \text{m}^2/\text{s})(15\ \text{min}\times60\ \text{s/min})}{(0.075\ \text{m})^2} = 5.424 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0050)e^{-(0.164)^2(5.424)} = 0.868$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_0}{k} = \frac{(40\ \text{W/m}^2\cdot°\text{C})(0.04\ \text{m})}{(110\ \text{W/m}\cdot°\text{C})} = 0.01455$$

$$\lambda_1 = 0.1704 \quad \text{and} \quad A_1 = 1.0038$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.39\times10^{-5}\ \text{m}^2/\text{s})(15\times60\ \text{s})}{(0.04\ \text{m})^2} = 19.069 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0038)e^{-(0.1704)^2(19.069)} = 0.577$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta_{o,wall}\times\theta_{o,cyl} = 0.868\times0.577 = 0.501$$

$$\frac{T(0,0,t) - 20}{150 - 20} = 0.501 \longrightarrow T(0,0,t) = \mathbf{85.1°C}$$

Prob. 16-61 (*continued*)

(*b*) The center of the top surface of the cylinder is still at the center of the long cylinder ($r = 0$), but at the outer surface of the plane wall ($x = L$). Therefore, we first need to determine the dimensionless temperature at the surface of the wall.

$$\theta(L,t)_{wall} = \frac{T(x,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 L/L) = (1.0050)e^{-(0.164)^2(5.424)}\cos(0.164) = 0.857$$

Then the center temperature of the top surface of the cylinder becomes

$$\left[\frac{T(L,0,t)-T_\infty}{T_i-T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{wall}\times\theta_{o,cyl} = 0.857\times 0.577 = 0.494$$

$$\frac{T(L,0,t)-20}{150-20} = 0.494 \longrightarrow T(L,0,t) = \mathbf{84.2°C}$$

(*c*) We first need to determine the maximum heat can be transferred from the cylinder

$$m = \rho V = \rho\pi r_o L = (8530\ \text{kg/m}^3)\left[\pi(0.04\ \text{m})^2(0.15\ \text{m})\right] = 6.43\ \text{kg}$$

$$Q_{\max} = mC_p(T_i - T_\infty) = (6.43\ \text{kg})(0.389\ \text{kJ/kg.°C})(150-20)°\text{C} = 325\ \text{kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1-\theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1-(0.848)\frac{\sin(0.2282)}{0.2282} = 0.136$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1-2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1-2(0.577)\frac{0.0846}{0.1704} = 0.427$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cylinder}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{long\\cylinder}}\left[1-\left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}}\right] = 0.136+(0.427)(1-0.136) = 0.503$$

Then the total heat transfer from the short cylinder during the first 15 minutes of cooling becomes

$$Q = 0.503Q_{\max} = (0.503)(325\ \text{kJ}) = \mathbf{164\ kJ}$$

16-62 A semi-infinite aluminum cylinder is cooled by water. The temperature at the center of the cylinder 10 cm from the end surface is to be determined.

Assumptions **1** Heat conduction in the semi-infinite cylinder is two-dimensional, and thus the temperature varies in both the axial x- and the radial r- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of aluminum are given to be $k = 237$ W/m.°C and $\alpha = 9.71\times10^{-6}$ m^2/s.

Analysis This semi-infinite cylinder can physically be formed by the intersection of a long cylinder of radius $r_O = D/2 = 7.5$ cm and a semi-infinite medium. The dimensionless temperature 10 cm from the surface of a semi-infinite medium is first determined from

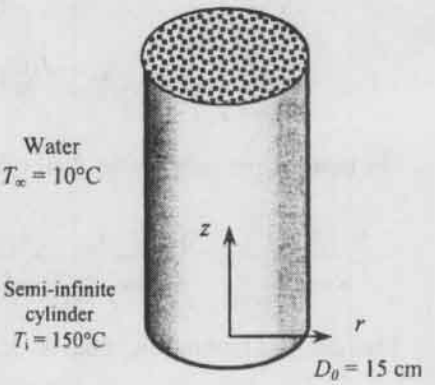
$$\frac{T(x,t)-T_i}{T_\infty-T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$$= erfc\left(\frac{0.1}{2\sqrt{(9.71\times10^{-5})(8\times60)}}\right) - \exp\left(\frac{(140)(0.1)}{237}+\frac{(140)^2(9.71\times10^{-5})(8\times60)}{(237)^2}\right)$$

$$\times\left[erfc\left(\frac{0.1}{2\sqrt{(9.71\times10^{-5})(8\times60)}}+\frac{(140)\sqrt{(9.71\times10^{-5})(8\times60)}}{237}\right)\right]$$

$$= erfc(0.23) - \exp(0.0753)erfc(0.36) = 0.745 - (1.078)(0.6107) = 0.0867$$

$$\theta_{semi-inf} = \frac{T(x,t)-T_\infty}{T_i-T_\infty} = 1 - 0.0867 = 0.913$$



The Biot number is calculated for the long cylinder to be

$$Bi = \frac{hr_o}{k} = \frac{(140\ \text{W/m}^2.\text{°C})(0.075\ \text{m})}{(237\ \text{W/m.°C})} = 0.0443$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 0.2948 \quad \text{and} \quad A_1 = 1.0110$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(9.71\times10^{-5}\ \text{m}^2/\text{s})(8\times60\ \text{s})}{(0.075\ \text{m})^2} = 8.286 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,cyl} = \frac{T_0-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0110)e^{-(0.2948)^2(8.286)} = 0.492$$

The center temperature of the semi-infinite cylinder then becomes

$$\left[\frac{T(x,0,t)-T_\infty}{T_i-T_\infty}\right]_{\substack{\text{semi-infinite}\\\text{cylinder}}} = \theta_{\text{semi-inf}}(x,t)\times\theta_{o,cyl} = 0.913\times0.492 = 0.449$$

$$\left[\frac{T(x,0,t)-10}{150-10}\right]_{\substack{\text{semi-infinite}\\\text{cylinder}}} = 0.449 \longrightarrow T(x,0,t) = \mathbf{72.9°C}$$

16-63E A hot dog is dropped into boiling water. The center temperature of the hot dog is do be determined by treating hot dog as a finite cylinder and also as an infinitely long cylinder.

Assumptions **1** When treating hot dog as a finite cylinder, heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial x- and the radial r- directions. When treating hot dog as an infinitely long cylinder, heat conduction is one-dimensional in the radial r- direction. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

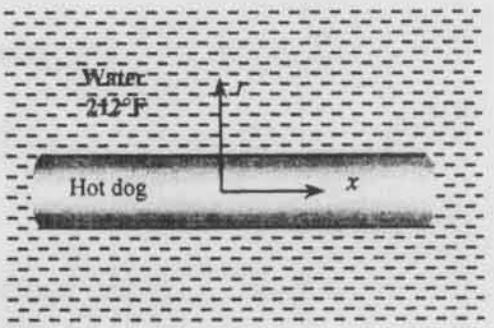
Properties The thermal properties of the hot dog are given to be $k = 0.44$ Btu/h.ft.°F, $\rho = 61.2$ lbm/ft^3 $C_p = 0.93$ Btu/lbm.°F, and $\alpha = 0.0077$ ft^2/h.

Analysis (a) This hot dog can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = (0.4/12)$ ft and a plane wall of thickness $2L = (5/12)$ ft. The distance x is measured from the midplane.

After 5 minutes

First the Biot number is calculated for the plane wall to be

$$Bi = \frac{hL}{k} = \frac{(120 \text{ Btu/h.ft}^2\text{.°F})(2.5/12 \text{ ft})}{(0.44 \text{ Btu/h.ft.°F})} = 56.8$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 9-1,

$$\lambda_1 = 1.5503 \quad \text{and} \quad A_1 = 1.2730$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(5/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.015 < 0.2 \quad \text{(Be cautious!)}$$

Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2730) e^{-(1.5503)^2 (0.015)} \cong 1$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(120 \text{ Btu/h.ft}^2\text{.°F})(0.4/12 \text{ ft})}{(0.44 \text{ Btu/h.ft.°F})} = 9.1$$

$$\lambda_1 = 2.1589 \quad \text{and} \quad A_1 = 1.5618$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(5/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 0.578 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618) e^{-(2.1589)^2 (0.578)} = 0.106$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short} \\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.106 = 0.106$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.106 \longrightarrow T(0,0,t) = \mathbf{194°F}$$

Prob. 16-63E (*continued*)

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(10/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.03 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2730) e^{-(1.5503)^2 (0.03)} \cong 1$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(10/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.156 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618) e^{-(2.1589)^2 (1.156)} = 0.007$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short} \\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.007 = 0.007$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.007 \longrightarrow T(0,0,t) = \mathbf{211°F}$$

After 15 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.045 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2730) e^{-(1.5503)^2 (0.045)} \cong 1$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.734 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618) e^{-(2.1589)^2 (1.734)} = 0.0005$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short} \\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.0005 = 0.0005$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.0005 \longrightarrow T(0,0,t) = \mathbf{212°F}$$

(*b*) Treating the hot dog as an infinitely long cylinder will not change the results obtained in the part (a) since dimensionless temperatures for the plane wall is 1 for all cases.

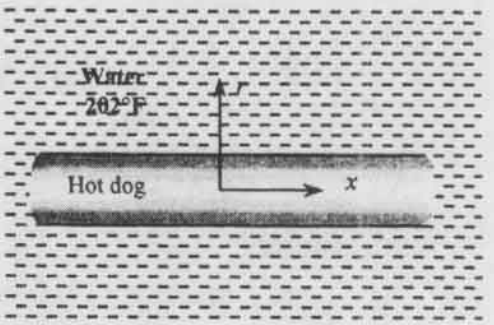
16-64E A hot dog is dropped into boiling water. The center temperature of the hot dog is do be determined by treating hot dog as a finite cylinder and an infinitely long cylinder.

Assumptions **1** When treating hot dog as a finite cylinder, heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial x- and the radial r- directions. When treating hot dog as an infinitely long cylinder, heat conduction is one-dimensional in the radial r- direction. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be $k = 0.44$ Btu/h.ft.°F, $\rho = 61.2$ lbm/ft^3 $C_p = 0.93$ Btu/lbm.°F, and $\alpha = 0.0077$ ft^2/h.

Analysis (a) This hot dog can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = (0.4/12)$ ft and a plane wall of thickness $2L = (5/12)$ ft. The distance x is measured from the midplane.

After 5 minutes



First the Biot number is calculated for the plane wall to be

$$Bi = \frac{hL}{k} = \frac{(120 \text{ Btu/h.ft}^2\text{.°F})(2.5/12 \text{ ft})}{(0.44 \text{ Btu/h.ft.°F})} = 56.8$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 1.5503 \quad \text{and} \quad A_1 = 1.2730$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(5/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.015 < 0.2 \quad \text{(Be cautious!)}$$

Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2730) e^{-(1.5503)^2 (0.015)} \cong 1$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(120 \text{ Btu/h.ft}^2\text{.°F})(0.4/12 \text{ ft})}{(0.44 \text{ Btu/h.ft.°F})} = 9.1$$

$$\lambda_1 = 2.1589 \quad \text{and} \quad A_1 = 1.5618$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(5/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 0.578 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618) e^{-(2.1589)^2 (0.578)} = 0.106$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty} \right]_{\substack{\text{short} \\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.106 = 0.106$$

$$\frac{T(0,0,t) - 202}{40 - 202} = 0.106 \longrightarrow T(0,0,t) = \mathbf{185°F}$$

Prob. 16-64E (*continued*)

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(10/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.03 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2730) e^{-(1.5503)^2 (0.03)} \cong 1$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(10/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.156 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618) e^{-(2.1589)^2 (1.156)} = 0.007$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short} \\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.007 = 0.007$$

$$\frac{T(0,0,t) - 202}{40 - 202} = 0.007 \longrightarrow T(0,0,t) = \mathbf{201°F}$$

After 15 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.045 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2730) e^{-(1.5503)^2 (0.045)} \cong 1$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.734 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618) e^{-(2.1589)^2 (1.734)} = 0.0005$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short} \\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.0005 = 0.0005$$

$$\frac{T(0,0,t) - 202}{40 - 202} = 0.0005 \longrightarrow T(0,0,t) = \mathbf{202\,°F}$$

(b) Treating the hot dog as an infinitely long cylinder will not change the results obtained in the part (a) since dimensionless temperatures for the plane wall is 1 for all cases.

16-65 A rectangular ice block is placed on a table. The time the ice block starts melting is to be determined.√

Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both x- and y- directions. **2** The thermal properties of the ice block are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the ice are given to be $k = 2.22$ W/m.°C and $\alpha = 0.124\times10^{-7}$ m^2/s.

Analysis This rectangular ice block can be treated as a short rectangular block that can physically be formed by the intersection of two infinite plane wall of thickness $2L = 4$ cm and an infinite plane wall of thickness $2L = 10$ cm. We measure x from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness $2L = 10$ cm. Since the melting starts at the corner of the top surface, we need to determine the time required to melt ice block which will happen when the temperature drops below 0°C at this location. The Biot numbers and the corresponding constants are first determined to be

$$Bi_{\text{wall},1} = \frac{hL_1}{k} = \frac{(12\ \text{W/m}^2.^\circ\text{C})(0.02\ \text{m})}{(2.22\ \text{W/m.}^\circ\text{C})} = 0.1081 \longrightarrow \lambda_1 = 0.3208 \quad \text{and} \quad A_1 = 1.0173$$

$$Bi_{\text{wall},3} = \frac{hL_3}{k} = \frac{(12\ \text{W/m}^2.^\circ\text{C})(0.05\ \text{m})}{(2.22\ \text{W/m.}^\circ\text{C})} = 0.2703 \longrightarrow \lambda_1 = 0.4951 \quad \text{and} \quad A_1 = 1.0408$$

The ice will start melting at the corners because of the maximum exposed surface area there. Noting that $\tau = \alpha t / L^2$ and assuming that $\tau > 0.2$ in all dimensions so that the one-term approximate solution for transient heat conduction is applicable, the product solution method can be written for this problem as

$$\theta(L_1, L_2, L_3, t)_{block} = \theta(L_1, t)_{\text{wall},1}{}^2\, \theta(L_3, t)_{\text{wall},2}$$

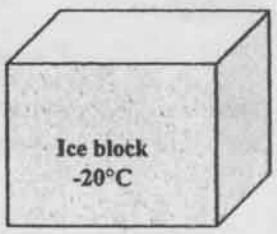
$$\frac{0-18}{-20-18} = \left[A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 L_1 / L_1)\right]^2 \left[A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 L_3 / L_3)\right]$$

$$0.4737 = \left\{(1.0173)\exp\left[-(0.3208)^2 \frac{(0.124\times10^{-7})t}{(0.02)^2}\right]\cos(0.3208)\right\}^2$$

$$\times \left\{(1.0408)\exp\left[-(0.4951)^2 \frac{(0.124\times10^{-7})t}{(0.05)^2}\right]\cos(0.4951)\right\}$$

$$\longrightarrow t \approx \mathbf{21.2\ hours}$$

Therefore, the ice will start melting in about 21 hours.



Discussion Note that

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7}\ \text{m}^2/\text{s})(23\ \text{h}\times3600\ \text{s/h})}{(0.05\ \text{m})^2} = 0.4 > 0.2$$

and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified.

16-66 A cylindrical ice block is placed on a table. The initial temperature of the ice block to avoid melting for 2 h is to be determined.

Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both x- and r- directions. **2** Heat transfer from the base of the ice block to the table is negligible. **3** The thermal properties of the ice block are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the ice are given to be $k = 2.22$ W/m.°C and $\alpha = 0.124\times10^{-7}$ m^2/s.

Analysis This cylindrical ice block can be treated as a short cylinder that can physically be formed by the intersection of a long cylinder of diameter $D = 2$ cm and an infinite plane wall of thickness $2L = 4$ cm. We measure x from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness $2L = 4$ cm. The melting starts at the outer surfaces of the top surface when the temperature drops below 0°C at this location. The Biot numbers, the corresponding constants, and the Fourier numbers are

$$Bi_{\text{wall}} = \frac{hL}{k} = \frac{(13\ \text{W/m}^2.^\circ\text{C})(0.02\ \text{m})}{(2.22\ \text{W/m.}^\circ\text{C})} = 0.1171 \longrightarrow \lambda_1 = 0.3318 \quad \text{and} \quad A_1 = 1.0187$$

$$Bi_{\text{cyl}} = \frac{hr_o}{k} = \frac{(13\ \text{W/m}^2.^\circ\text{C})(0.01\ \text{m})}{(2.22\ \text{W/m.}^\circ\text{C})} = 0.05856 \longrightarrow \lambda_1 = 0.3407 \quad \text{and} \quad A_1 = 1.0146$$

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7}\ \text{m}^2/\text{s})(2\ \text{h}\times3600\ \text{s/h})}{(0.02\ \text{m})^2} = 0.2232 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(0.124\times10^{-7}\ \text{m}^2/\text{s})(2\ \text{h}\times3600\ \text{s/h})}{(0.01\ \text{m})^2} = 0.8928 > 0.2$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable. The product solution for this problem can be written as

$$\theta(L, r_o, t)_{\text{block}} = \theta(L, t)_{\text{wall}}\,\theta(r_o, t)_{\text{cyl}}$$

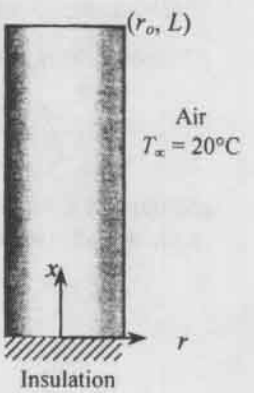
$$\frac{0-20}{T_i - 20} = \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L)\right]\left[A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o/r_o)\right]$$

$$\frac{0-20}{T_i - 20} = \left[(1.0187)e^{-(0.3318)^2(0.2232)}\cos(0.3318)\right]\left[(1.0146)e^{-(0.3407)^2(0.8928)}(0.9707)\right]$$

which gives

$$T_i = \mathbf{-4^\circ C}$$

Therefore, the ice will not start melting for at least 2 hours if its initial temperature is -4°C or below.



16-67 A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

Assumptions **1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all x-, y, and z- directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial x- and radial r- directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the granite are given to be $k = 2.5$ W/m.°C and $\alpha = 1.15\times10^{-6}$ m^2/s.

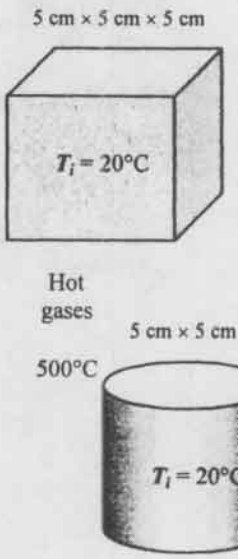
***Analysis*:**

Cubic block: This cubic block can physically be formed by the intersection of three infinite plane walls of thickness $2L = 5$ cm.

After 10 minutes: The Biot number, the corresponding constants, and the Fourier number are

$$Bi = \frac{hL}{k} = \frac{(40\ \text{W/m}^2.^\circ\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.}^\circ\text{C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \quad \text{and} \quad A_1 = 1.0580$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(10\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 1.104 > 0.2$$



To determine the center temperature, the product solution can be written as

$$\theta(0,0,0,t)_{\text{block}} = [\theta(0,t)_{\text{wall}}]^3$$

$$\frac{T(0,0,0,t)-T_\infty}{T_i-T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)^3$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}^3 = 0.369 \longrightarrow T(0,0,0,t) = \mathbf{323°C}$$

After 20 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(20\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}^3 = 0.115 \longrightarrow T(0,0,0,t) = \mathbf{445°C}$$

After 60 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(60\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}^3 = 0.00109 \longrightarrow T(0,0,0,t) = \mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

Prob. 16-67 (*continued*)

Cylinder: This cylindrical block can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 2.5$ cm and a plane wall of thickness $2L = 5$ cm.

After 10 minutes: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.^\circ\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.}^\circ\text{C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \quad \text{and} \quad A_1 = 1.0931$$

To determine the center temperature, the product solution can be written as

$$\theta(0,0,t)_{block} = [\theta(0,t)_{wall}][\theta(0,t)_{cyl}]$$

$$\frac{T(0,0,t)-T_\infty}{T_i-T_\infty} = \left(A_1 e^{-\lambda_1^2 \tau}\right)_{wall}\left(A_1 e^{-\lambda_1^2 \tau}\right)_{cyl}$$

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}\left\{(1.0931)e^{-(0.8516)^2(1.104)}\right\} = 0.352 \longrightarrow T(0,0,t) = \mathbf{331^\circ C}$$

After 20 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}\left\{(1.0931)e^{-(0.8516)^2(2.208)}\right\} = 0.107 \longrightarrow T(0,0,t) = \mathbf{449^\circ C}$$

After 60 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}\left\{(1.0931)e^{-(0.8516)^2(6.624)}\right\} = 0.00092 \longrightarrow T(0,0,t) = \mathbf{500^\circ C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

16-68 A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

Assumptions **1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all x-, y, and z- directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial x- and radial r- directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the granite are $k = 2.5$ W/m.°C and $\alpha = 1.15\times10^{-6}$ m^2/s.

***Analysis*:**

Cubic block: This cubic block can physically be formed by the intersection of three infinite plane wall of thickness $2L = 5$ cm. Two infinite plane walls are exposed to the hot gases with a heat transfer coefficient of $h = 40$ W/m^2.°C and one with $h = 80$ W/m^2.°C.

After 10 minutes: The Biot number and the corresponding constants for $h = 40$ W/m^2.°C are

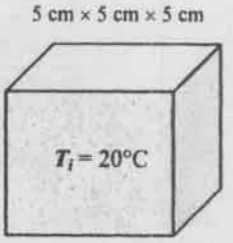
$$Bi = \frac{hL}{k} = \frac{(40\ \text{W/m}^2.^\circ\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \quad \text{and} \quad A_1 = 1.0580$$

The Biot number and the corresponding constants for $h = 80$ W/m^2.°C are

$$Bi = \frac{hL}{k} = \frac{(80\ \text{W/m}^2.^\circ\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.°C})} = 0.800 \longrightarrow \lambda_1 = 0.7910 \quad \text{and} \quad A_1 = 1.1016$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(10\ \text{min}\times60\ \text{s/min})}{(0.025\ \text{m})^2} = 1.104 > 0.2$$



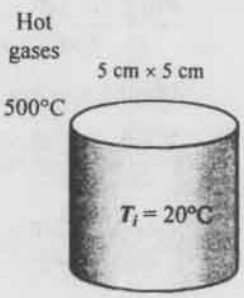
To determine the center temperature, the product solution method can be written as

$$\theta(0,0,0,t)_{\text{block}} = [\theta(0,t)_{\text{wall}}]^2[\theta(0,t)_{\text{wall}}]$$

$$\frac{T(0,0,0,t)-T_\infty}{T_i-T_\infty} = \left(A_1e^{-\lambda_1^2\tau}\right)^2\left(A_1e^{-\lambda_1^2\tau}\right)$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}^2\left\{(1.1016)e^{-(0.7910)^2(1.104)}\right\} = 0.284$$

$$\longrightarrow T(0,0,0,t) = \mathbf{364°C}$$



After 20 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(20\ \text{min}\times60\ \text{s/min})}{(0.025\ \text{m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}^2\left\{(1.1016)e^{-(0.7910)^2(2.208)}\right\} = 0.0654$$

$$\longrightarrow T(0,0,0,t) = \mathbf{469°C}$$

Prob. 16-68 (*continued*)

After 60 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(60\ \text{min}\times60\ \text{s/min})}{(0.025\ \text{m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}^2\left\{(1.1016)e^{-(0.7910)^2(6.624)}\right\} = 0.000186$$

$$\longrightarrow T(0,0,0,t) = \mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

Cylinder: This cylindrical block can physically be formed by the intersection of a long cylinder of radius $r_O = D/2 = 2.5$ cm exposed to the hot gases with a heat transfer coefficient of $h = 40\ \text{W/m}^2.°\text{C}$ and a plane wall of thickness $2L = 5$ cm exposed to the hot gases with $h = 80\ \text{W/m}^2.°\text{C}$.

After 10 minutes: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \quad \text{and} \quad A_1 = 1.0931$$

To determine the center temperature, the product solution method can be written as

$$\theta(0,0,t)_{\text{block}} = [\theta(0,t)_{\text{wall}}][\theta(0,t)_{cyl}]$$

$$\frac{T(0,0,t)-T_\infty}{T_i-T_\infty} = \left(A_1e^{-\lambda_1^2\tau}\right)_{\text{wall}}\left(A_1e^{-\lambda_1^2\tau}\right)_{cyl}$$

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(1.104)}\right\}\left\{(1.0931)e^{-(0.8516)^2(1.104)}\right\} = 0.271$$

$$T(0,0,t) = \mathbf{370°C}$$

After 20 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(2.208)}\right\}\left\{(1.0931)e^{-(0.8516)^2(2.208)}\right\} = 0.06094 \longrightarrow T(0,0,t) = \mathbf{471°C}$$

After 60 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(6.624)}\right\}\left\{(1.0931)e^{-(0.8516)^2(6.624)}\right\} = 0.0001568 \longrightarrow T(0,0,t) = \mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

16-69 A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transfer to the block are to be determined.

Assumptions **1** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial x- and radial r- directions. **2** The thermal properties of the aluminum are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (it will be verified).

Properties The thermal properties of the aluminum block are given to be $k = 236$ W/m.°C, $\rho = 2702$ kg/m^3, $C_p = 0.896$ kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness $2L = 20$ cm, and a long cylinder of radius $r_o = D/2 = 7.5$ cm. The Biot numbers and the corresponding constants are first determined to be

$$Bi = \frac{hL}{k} = \frac{(50\ \text{W/m}^2.^\circ\text{C})(0.1\ \text{m})}{(2.36\ \text{W/m.}^\circ\text{C})} = 0.02119 \quad \longrightarrow \lambda_1 = 0.1439 \ \text{ and } \ A_1 = 1.0035$$

$$Bi = \frac{hr_0}{k} = \frac{(50\ \text{W/m}^2.^\circ\text{C})(0.075\ \text{m})}{236\ \text{W/m.}^\circ\text{C}} = 0.01589 \longrightarrow \lambda_1 = 0.1762 \ \text{ and } \ A_1 = 1.0040$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}}\,\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2 \tau}\right)_{\text{wall}} \left(A_1 e^{-\lambda_1^2 \tau}\right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{(1.0035)\exp\left[-(0.1439)^2\frac{(9.75\times10^{-5})t}{(0.1)^2}\right]\right\}\left\{(1.0040)\exp\left[-(0.1762)^2\frac{(9.75\times10^{-5})t}{(0.075)^2}\right]\right\} = 0.7627$$

Solving for the time t gives $t = 372$ s = **6.2 min.** We note that

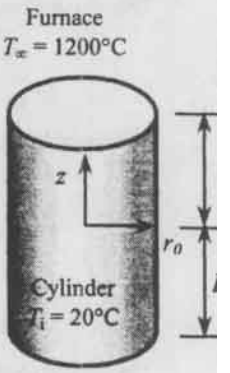
$$\tau = \frac{\alpha t}{L^2} = \frac{(9.75\times10^{-5}\ \text{m}^2/\text{s})(6.2\ \text{min}\times 60\ \text{s/min})}{(0.1\ \text{m})^2} = 3.6 > 0.2$$

and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified.

The maximum amount of heat transfer is

$$m = \rho V = \rho\pi r_0^2 L = (2702\ \text{kg/m}^3)\left[\pi(0.075\ \text{m})^2(0.2\ \text{m})\right] = 9.550\ \text{kg}$$

$$Q_{\max} = mC_p(T_i - T_\infty) = (9.550\ \text{kg})(0.896\ \text{kJ/kg.°C})(20-1200)^\circ\text{C} = 10{,}100\ \text{kJ}$$



Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1-\theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1-(0.7627)\frac{\sin(0.1439)}{0.1439} = 0.2400$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1-2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1-2(0.7627)\frac{0.0876}{0.1762} = 0.2416$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cylinder}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{long\\cylinder}}\left[1-\left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}}\right] = 0.24 + (0.2416)(1-0.24) = 0.4236$$

Then the total heat transfer from the short cylinder as it is cooled from 300°C at the center to 20°C becomes

$$Q = 0.4236 Q_{\max} = (0.4236)(10{,}100\ \text{kJ}) = \mathbf{4278\ kJ}$$

which is identical to the heat transfer to the cylinder as the cylinder at 20°C is heated to 300°C at the center.

16-70 A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transferred to the block are to be determined.

Assumptions **1** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial x- and radial r- directions. **2** Heat transfer from the bottom surface of the block is negligible. **3** The thermal properties of the aluminum are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the aluminum block are given to be $k = 236$ W/m.°C, $\rho = 2702$ kg/m^3, $C_p = 0.896$ kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness $2L = 40$ cm and a long cylinder of radius $r_0 = D/2 = 7.5$ cm. Note that the height of the short cylinder represents the half thickness of the infinite plane wall where the bottom surface of the short cylinder is adiabatic. The Biot numbers and corresponding constants are first determined to be

$$Bi = \frac{hL}{k} = \frac{(50\ \text{W/m}^2.^\circ\text{C})(0.2\ \text{m})}{(236\ \text{W/m.}^\circ\text{C})} = 0.04237 \longrightarrow \lambda_1 = 0.2031 \ \text{ and } \ A_1 = 1.0069$$

$$Bi = \frac{hr_0}{k} = \frac{(50\ \text{W/m}^2.^\circ\text{C})(0.075\ \text{m})}{(236\ \text{W/m.}^\circ\text{C})} = 0.01589 \longrightarrow \lambda_1 = 0.1762 \ \text{ and } \ A_1 = 1.0040$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}}\,\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2 \tau}\right)_{\text{wall}} \left(A_1 e^{-\lambda_1^2 \tau}\right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{(1.0069)\exp\left[-(0.2031)^2\frac{(9.75\times10^{-5})t}{(0.2)^2}\right]\right\}\left\{(1.0040)\exp\left[-(0.1762)^2\frac{(9.75\times10^{-5})t}{(0.075)^2}\right]\right\} = 0.7627$$

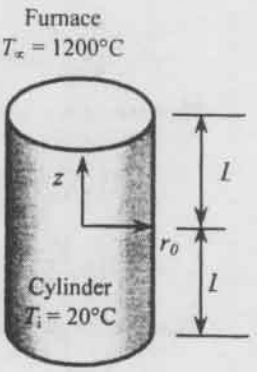
Solving for the time t gives $t = 441$ s = **7.35 min.** We note that

$$\tau = \frac{\alpha t}{L^2} = \frac{(9.75\times10^{-5}\ \text{m}^2/\text{s})(7.35\ \text{min}\times 60\ \text{s/min})}{(0.2\ \text{m})^2} = 1.1 > 0.2$$

and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified. The maximum amount of heat transfer is

$$m = \rho V = \rho\pi r_o L = (2702\ \text{kg/m}^3)\left[\pi(0.075\ \text{m})^2(0.2\ \text{m})\right] = 9.55\ \text{kg}$$

$$Q_{\max} = mC_p(T_i - T_\infty) = (9.55\ \text{kg})(0.896\ \text{kJ/kg.}^\circ\text{C})(20-1200)^\circ\text{C} = 10{,}100\ \text{kJ}$$



Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1 - \theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1 - (0.7627)\frac{\sin(0.2031)}{0.2031} = 0.2425$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.7627)\frac{0.0876}{0.1762} = 0.2416$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{short}\\\text{cylinder}}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\\text{wall}}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{long}\\\text{cylinder}}}\left[1 - \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\\text{wall}}}\right] = 0.2425 + (0.2416)(1-0.2425) = 0.4255$$

Then the total heat transfer from the short cylinder as it is cooled from 300°C at the center to 20°C becomes

$$Q = 0.4255 Q_{\max} = (0.4255)(10{,}100\ \text{kJ}) = \mathbf{4298\ kJ}$$

which is identical to the heat transfer to the cylinder as the cylinder at 20°C is heated to 300°C at the center.

Review Problems

16-71 Two large steel plates are stuck together because of the freezing of the water between the two plates. Hot air is blown over the exposed surface of the plate on the top to melt the ice. The length of time the hot air should be blown is to be determined.

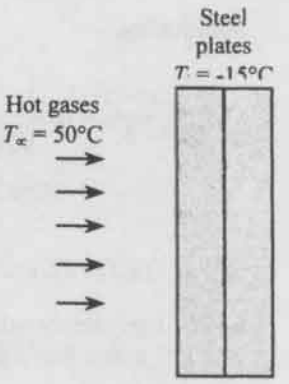
Assumptions **1** Heat conduction in the plates is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the steel plates are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of steel plates are given to be $k = 43$ W/m.°C and $\alpha = 1.17\times10^{-5}$ m^2/s

Analysis The characteristic length of the plates and the Biot number are

$$L_c = \frac{V}{A} = L = 0.02\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(40\ \text{W/m}^2.^\circ\text{C})(0.02\,\text{m})}{(43\ \text{W/m.}^\circ\text{C})} = 0.019 < 0.1$$



Since $\text{Bi} < 0.1$, the lumped system analysis is applicable. Therefore,

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{40\ \text{W/m}^2.^\circ\text{C}}{(8055\,\text{kg/m}^3)(480\,\text{J/kg.}^\circ\text{C})(0.02\,\text{m})} = 0.000517\,\text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{0-50}{-15-50} = e^{-(0.000517\,\text{s}^{-1})t} \longrightarrow t = 507\,\text{s} = \mathbf{8.5\,min}$$

Alternative solution: This problem can also be solved using the transient chart Fig. 16-13a,

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{1}{0.019} = 52.6 \\ \frac{T_o-T_\infty}{T_i-T_\infty} &= \frac{0-50}{-15-50} = 0.769 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 15 > 0.2$$

Then,

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(15)(0.02\,\text{m})^2}{(1.17\times10^{-5}\ \text{m}^2/\text{s})} = \mathbf{513\ s}$$

The small difference is due to the reading error of the chart.

16-72 A curing kiln is heated by injecting steam into it and raising its inner surface temperature to a specified value. It is to be determined whether the temperature at the outer surfaces of the kiln changes during the curing period.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at inner surfaces only and the convection heat transfer coefficient inside is very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature of 45°C. **2** The thermal properties of the concrete wall are constant.

Properties The thermal properties of the concrete wall are given to be $k = 0.9$ W/m.°C and $\alpha = 0.23\times10^{-5}$ m^2/s.

Analysis We determine the temperature at a depth of $x = 0.3$ m in 3 h using the analytical solution,

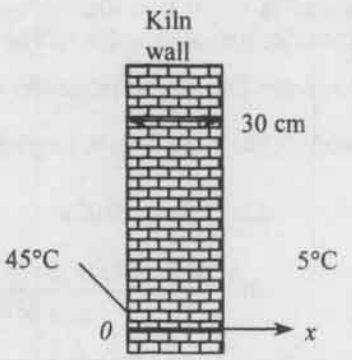
$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Substituting,

$$\frac{T(x,t)-5}{44-5} = erfc\left(\frac{0.3\text{ m}}{2\sqrt{(0.23\times10^{-5}\text{ m}^2/\text{s})(3\text{ h}\times3600\text{ s/h})}}\right)$$

$$= erfc(0.952) = 0.1782$$

$$T(x,t) = \mathbf{12.1\,°C}$$



which is greater than the initial temperature of 5°C. Therefore, heat will propagate through the 0.3 m thick wall in 3 h, and thus it may be desirable to insulate the outer surface of the wall to save energy.

16-73 The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature of -10°C. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be $k = 0.7$ W/m.°C and $\alpha = 1.4\times10^{-5}$ m^2/s.

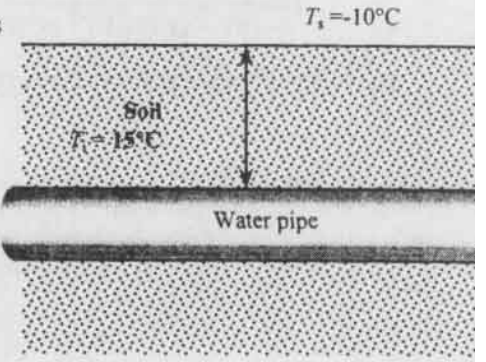
Analysis The depth at which the temperature drops to 0°C in 75 days is determined using the analytical solution,

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Substituting,

$$\frac{0-15}{-10-15} = erfc\left(\frac{x}{2\sqrt{(1.4\times10^{-5}\text{ m}^2/\text{s})(75\text{ day}\times24\text{ h/day}\times3600\text{ s/h})}}\right)$$

$$\longrightarrow x = \mathbf{7.05\text{ m}}$$



Therefore, the pipes must be buried at a depth of at least 7.05 m.

16-74 A hot dog is to be cooked by dropping it into boiling water. The time of cooking is to be determined.

Assumptions **1** Heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial x- and the radial r- directions. **2** The thermal properties of the hot dog are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be $k = 0.76$ W/m.°C, $\rho = 980$ kg/m^3, $C_p = 3.9$ kJ/kg.°C, and $\alpha = 2\times10^{-7}$ m^2/s.

Analysis This hot dog can physically be formed by the intersection of an infinite plane wall of thickness $2L = 12$ cm, and a long cylinder of radius $r_o = D/2 = 1$ cm. The Biot numbers and corresponding constants are first determined to be

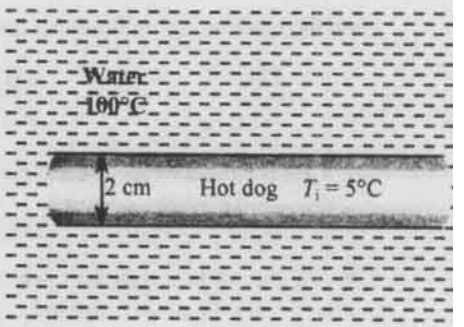
$$Bi = \frac{hL}{k} = \frac{(600\ \text{W/m}^2.^\circ\text{C})(0.06\ \text{m})}{(0.76\ \text{W/m.}^\circ\text{C})} = 47.37 \longrightarrow \lambda_1 = 1.5381 \ \text{ and } \ A_1 = 1.2726$$

$$Bi = \frac{hr_o}{k} = \frac{(600\ \text{W/m}^2.^\circ\text{C})(0.01\ \text{m})}{(0.76\ \text{W/m.}^\circ\text{C})} = 7.895 \longrightarrow \lambda_1 = 2.1251 \ \text{ and } \ A_1 = 1.5515$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{block} = \theta(0,t)_{wall}\,\theta(0,t)_{cyl} = \left(A_1 e^{-\lambda_1^2 \tau}\right)\left(A_1 e^{-\lambda_1^2 \tau}\right)$$

$$\frac{80-100}{5-100} = \left\{(1.2726)\exp\left[-(1.5381)^2\,\frac{(2\times10^{-7})t}{(0.06)^2}\right]\right\} \times \left\{(1.5515)\exp\left[-(2.1251)^2\,\frac{(2\times10^{-7})t}{(0.01)^2}\right]\right\} = 0.2105$$



which gives

$$t = \mathbf{244\ s = 4.1\ min}$$

Therefore, it will take about 4.1 min for the hot dog to cook. Note that

$$\tau_{cyl} = \frac{\alpha t}{r_o^2} = \frac{(2\times10^{-7}\ \text{m}^2/\text{s})(244\ \text{s})}{(0.01\ \text{m})^2} = 0.49 > 0.2$$

and thus the assumption $\tau > 0.2$ for the applicability of the one-term approximate solution is verified.

Discussion This problem could also be solved by treating the hot dog as an infinite cylinder since heat transfer through the end surfaces will have little effect on the mid section temperature because of the large distance.

16-75 A long roll of large 1-Mn manganese steel plate is to be quenched in an oil bath at a specified rate. The temperature of the sheet metal after quenching and the rate at which heat needs to be removed from the oil in order to keep its temperature constant are to be determined.

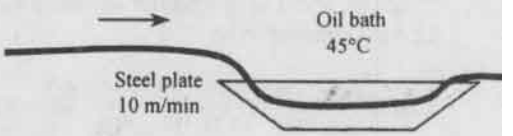
Assumptions **1** The thermal properties of the balls are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface. **3** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be checked).

Properties The properties of the steel plate are given to be $k = 60.5$ W/m.°C, $\rho = 7854$ kg/m^3, and $C_p = 434$ J/kg.°C.

Analysis The characteristic length of the steel plate and the Biot number are

$$L_c = \frac{V}{A} = L = 0.0025\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(860\ \text{W/m}^2.^\circ\text{C})(0.0025\,\text{m})}{60.5\ \text{W/m.}^\circ\text{C}} = 0.036 < 0.1$$



Since Bi < 0.1, the lumped system analysis is applicable. Therefore,

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{860\ \text{W/m}^2.^\circ\text{C}}{(7854\,\text{kg/m}^3)(434\,\text{J/kg.}^\circ\text{C})(0.0025\,\text{m})} = 0.10092\,\text{s}^{-1}$$

$$\text{time} = \frac{\text{length}}{\text{velocity}} = \frac{8\,\text{m}}{10\,\text{m/min}} = 0.8\,\text{min} = 48\,\text{s}$$

Then the temperature of the sheet metal when it leaves the oil bath is determined to be

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t) - 45}{820 - 45} = e^{-(0.10092\,\text{s}^{-1})(48\,\text{s})} \longrightarrow T(t) = \mathbf{51.1^\circ C}$$

The mass flow rate of the sheet metal through the oil bath is

$$\dot{m} = \rho \dot{V} = \rho w t V = (785\,\text{kg/m}^3)(2\,\text{m})(0.005\,\text{m})(10\,\text{m/min}) = 78.5\,\text{kg/min}$$

Then the rate of heat transfer from the sheet metal to the oil bath and thus the rate at which heat needs to be removed from the oil in order to keep its temperature constant at 45 °C becomes

$$\dot{Q} = \dot{m} C_p [T(t) - T_\infty] = (78.5\,\text{kg/min})(434\,\text{J/kg.}^\circ\text{C})(51.1 - 45)^\circ\text{C} = 207{,}800\,\text{J/min} = \mathbf{3.46\,kW}$$

16-76E A stuffed turkey is cooked in an oven. The average heat transfer coefficient at the surface of the turkey, the temperature of the skin of the turkey in the oven and the total amount of heat transferred to the turkey in the oven are to be determined.

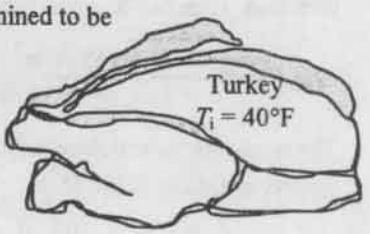
Assumptions **1** The turkey is a homogeneous spherical object. **2** Heat conduction in the turkey is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the turkey are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the turkey are given to be $k = 0.26$ Btu/h.ft.°F, $\rho = 75$ lbm/ft^3, $C_p = 0.98$ Btu/lbm.°F, and $\alpha = 0.0035$ ft^2/h.

Analysis (*a*) Assuming the turkey to be spherical in shape, its radius is determined to be

$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{14 \text{ lbm}}{75 \text{ lbm/ft}^3} = 0.1867 \text{ ft}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.1867 \text{ ft}^3)}{4\pi}} = 0.3545 \text{ ft}$$



The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.5\times10^{-3} \text{ ft}^2/\text{h})(5 \text{ h})}{(0.3545 \text{ ft})^2} = 0.1392$$

which is close to 0.2 but a little below it. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the one-term solution formulation at one-third the radius from the center of the turkey can be expressed as

$$\theta(x,t)_{sph} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r / r_o)}{\lambda_1 r / r_o}$$

$$\frac{185 - 325}{40 - 325} = 0.491 = A_1 e^{-\lambda_1^2 (0.14)} \frac{\sin(0.333\lambda_1)}{0.333\lambda_1}$$

By trial and error, it is determined from Table 16-1 that the equation above is satisfied when $Bi = 20$ corresponding to $\lambda_1 = 2.9857$ and $A_1 = 1.9781$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.26 \text{ Btu/h.ft.°F})(20)}{(0.3545 \text{ ft})} = \mathbf{14.7 \text{ Btu/h.ft}^2\text{.°F}}$$

(*b*) The temperature at the surface of the turkey is

$$\frac{T(r_o,t) - 325}{40 - 325} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.9781) e^{-(2.9857)^2 (0.14)} \frac{\sin(2.9857)}{2.9857} = 0.02953$$

$$\longrightarrow T(r_o,t) = \mathbf{317 \text{ °F}}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mC_p(T_i - T_\infty) = (14 \text{ lbm})(0.98 \text{ Btu/lbm.°F})(40 - 325)°F = 3910 \text{ Btu}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph} \frac{\sin(\lambda_1) - \lambda_1 \cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.491) \frac{\sin(2.9857) - (2.9857)\cos(2.9857)}{(2.9857)^3} = 0.828$$

$$Q = 0.828 Q_{\max} = (0.828)(3910 \text{ Btu}) = \mathbf{3237 \text{ Btu}}$$

Discussion The temperature of the outer parts of the turkey will be greater than that of the inner parts when the turkey is taken out of the oven. Then heat will continue to be transferred from the outer parts of the turkey to the inner as a result of temperature difference. Therefore, after 5 minutes, the thermometer reading will probably be more than 185 °F.

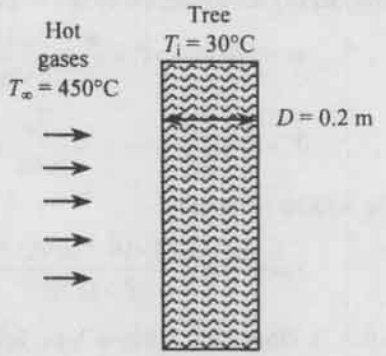
16-77 The trunks of some dry oak trees are exposed to hot gases. The time for the ignition of the trunks is to be determined.

Assumptions **1** Heat conduction in the trunks is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the trunks are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the trunks are given to be $k = 0.17$ W/m.°C and $\alpha = 1.28\times10^{-7}$ m^2/s.

Analysis We treat the trunks of the trees as an infinite cylinder since heat transfer is primarily in the radial direction. Then the Biot number becomes

$$Bi = \frac{hr_o}{k} = \frac{(65\ \text{W/m}^2.^\circ\text{C})(0.1\ \text{m})}{(0.17\ \text{W/m.}^\circ\text{C})} = 38.24$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 2.3420 \quad \text{and} \quad A_1 = 1.5989$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_0^2} = \frac{(1.28\times10^{-7}\ \text{m}^2/\text{s})(4\ \text{h}\times3600\ \text{s/h})}{(0.1\ \text{m})^2} = 0.184$$

which is slightly below 0.2 but close to it. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the temperature at the surface of the trees in 4 h becomes

$$\theta(r_o,t)_{cyl} = \frac{T(r_o,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r/r_o)$$

$$\frac{T(r_o,t)-450}{30-450} = (1.5989)e^{-(2.3420)^2(0.184)}(0.0343) = 0.02 \longrightarrow T(r_o,t) = \mathbf{442\ °C} > 410°\text{C}$$

Therefore, the trees will ignite. (Note: J_0 is read from Table 16-2).

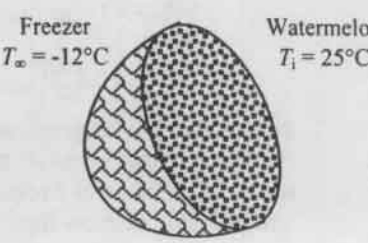
16-78 A spherical watermelon that is cut into two equal parts is put into a freezer. The time it will take for the center of the exposed cut surface to cool from 25 to 3°C is to be determined.

Assumptions **1** The temperature of the exposed surfaces of the watermelon is affected by the convection heat transfer at those surfaces only. Therefore, the watermelon can be considered to be a semi-infinite medium **2** The thermal properties of the watermelon are constant.

Properties The thermal properties of the water is closely approximated by those of water at room temperature, $k = 0.607$ W/m.°C and $\alpha = k/\rho C_p = 0.146\times10^{-6}$ m^2/s.

Analysis We use the transient chart in Fig. 16-23 in this case for convenience (instead of the analytic solution),

$$\left.\begin{aligned} 1-\frac{T(x,t)-T_\infty}{T_i-T_\infty} &= 1-\frac{3-(-12)}{25-(-12)} = 0.595 \\ \xi &= \frac{x}{2\sqrt{\alpha t}} = 0 \end{aligned}\right\} \frac{h\sqrt{\alpha t}}{k} = 1$$



Therefore,

$$t = \frac{(1)^2 k^2}{h^2 \alpha} = \frac{(0.607\ \text{W/m.}^\circ\text{C})^2}{(30\ \text{W/m}^2.^\circ\text{C})^2(0.146\times10^{-6}\ \text{m}^2/\text{s})} = 2804\ \text{s} = \mathbf{46.7\ min}$$

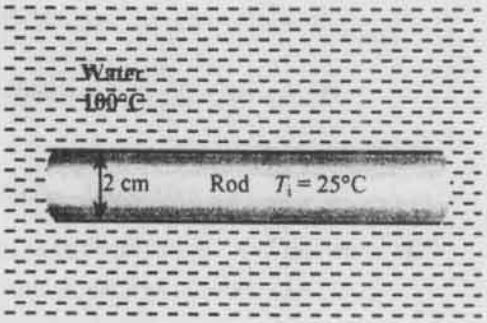
16-79 A cylindrical rod is dropped into boiling water. The thermal diffusivity and the thermal conductivity of the rod are to be determined.

Assumptions **1** Heat conduction in the rod is one-dimensional since the rod is sufficiently long, and thus temperature varies in the radial direction only. **2** The thermal properties of the rod are constant.

Properties The thermal properties of the rod available are given to be $\rho = 3700$ kg/m^3 and $C_p = 920$ J/kg.°C.

Analysis From Fig. 16-14b we have

$$\left.\begin{aligned} \frac{T-T_\infty}{T_o-T_\infty} &= \frac{93-100}{75-100} = 0.28 \\ \frac{x}{r_o} &= \frac{r_o}{r_o} = 1 \end{aligned}\right\} \frac{1}{Bi} = \frac{k}{hr_o} = 0.25$$



From Fig. 16-14a we have

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.25 \\ \frac{T_o-T_\infty}{T_i-T_\infty} &= \frac{75-100}{25-100} = 0.33 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.40$$

Then the thermal diffusivity and the thermal conductivity of the material become

$$\alpha = \frac{0.40 r_o^2}{t} = \frac{(0.40)(0.01\,\text{m})^2}{3\,\text{min} \times 60\,\text{s/min}} = \mathbf{2.22 \times 10^{-7}\ m^2/s}$$

$$\alpha = \frac{k}{\rho C_p} \longrightarrow k = \alpha \rho C_p = (2.22 \times 10^{-7}\ \text{m}^2/\text{s})(3700\,\text{kg/m}^3)(920\,\text{J/kg.°C}) = \mathbf{0.756\ W/m.°C}$$

16-80 The time it will take for the diameter of a raindrop to reduce to a certain value as it falls through ambient air is to be determined.

Assumptions **1** The water temperature remains constant. **2** The thermal properties of the water are constant.

Properties The density and heat of vaporization of the water are $\rho = 1000\ \text{kg/m}^3$ and $h_{fg} = 2490\ \text{kJ/kg}$.

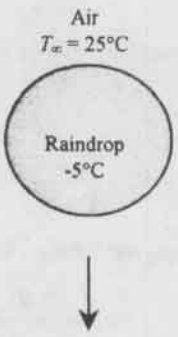
Analysis The initial and final masses of the raindrop are

$$m_i = \rho V_i = \rho \frac{4}{3}\pi r_i^3 = (1000\ \text{kg/m}^3)\frac{4}{3}\pi(0.0025\ \text{m})^3 = 0.0000654\ \text{kg}$$

$$m_f = \rho V_f = \rho \frac{4}{3}\pi r_f^3 = (1000\ \text{kg/m}^3)\frac{4}{3}\pi(0.0015\ \text{m})^3 = 0.0000141\ \text{kg}$$

whose difference is

$$m = m_i - m_f = 0.0000654 - 0.0000141 = 0.0000513\ \text{kg}$$



The amount of heat transfer required to cause this much evaporation is

$$Q = (0.0000513\ \text{kg})(2490\ \text{kJ/kg}) = 0.1277\ \text{kJ}$$

The average heat transfer surface area and the rate of heat transfer are

$$A = \frac{4\pi(r_i^2 + r_f^2)}{2} = \frac{4\pi[(0.0025\ \text{m})^2 + (0.0015\ \text{m})^2}{2} = 5.34\times10^{-5}\ \text{m}^2$$

$$\dot{Q} = hA(T_i - T_\infty) = (400\ \text{W/m}^2.°\text{C})(5.34\times10^{-5}\ \text{m}^2)(25-5)°\text{C} = 0.4272\ \text{J/s}$$

Then the time required for the raindrop to experience this reduction in size becomes

$$\dot{Q} = \frac{Q}{\Delta t} \longrightarrow \Delta t = \frac{Q}{\dot{Q}} = \frac{127.7\ \text{J}}{0.4272\ \text{J/s}} = 299\ \text{s} = \mathbf{5\ min}$$

16-81E A plate, a long cylinder, and a sphere are exposed to cool air. The center temperature of each geometry is to be determined.

Assumptions **1** Heat conduction in each geometry is one-dimensional. **2** The thermal properties of the bodies are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

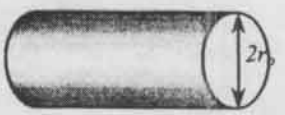
Properties The properties of bronze are given to be $k = 15$ Btu/h.ft.°F and $\alpha = 0.333$ ft²/h.



Analysis **After 5 minutes**

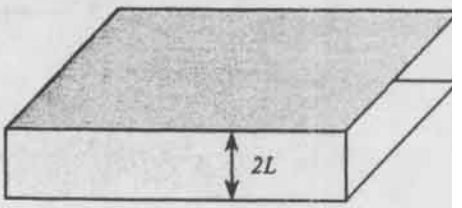
Plate: First the Biot number is calculated to be

$$Bi = \frac{hL}{k} = \frac{(7\ \text{Btu/h.ft}^2\text{.°F})(0.5/12\ \text{ft})}{(15\ \text{Btu/h.ft.°F})} = 0.01944$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 0.1410 \quad \text{and} \quad A_1 = 1.0033$$



The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.333\ \text{ft}^2/\text{h})(5\ \text{min}/60\ \text{min/h})}{(0.5/12\ \text{ft})^2} = 15.98 > 0.2$$

Then the center temperature of the plate becomes

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0033)e^{-(0.1410)^2(15.98)} = 0.730 \longrightarrow T_0 = \mathbf{312°F}$$

Cylinder:

$$Bi = 0.02 \xrightarrow{\text{Table 9-1}} \lambda_1 = 0.1995 \quad \text{and} \quad A_1 = 1.0050$$

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0050)e^{-(0.1995)^2(15.98)} = 0.532 \longrightarrow T_0 = \mathbf{248°F}$$

Sphere:

$$Bi = 0.02 \xrightarrow{\text{Table 9-1}} \lambda_1 = 0.2445 \quad \text{and} \quad A_1 = 1.0060$$

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0060)e^{-(0.2445)^2(15.98)} = 0.387 \longrightarrow T_0 = \mathbf{201°F}$$

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.333\ \text{ft}^2/\text{h})(10\ \text{min}/60\ \text{min/h})}{(0.5/12\ \text{ft})^2} = 31.97 > 0.2$$

Plate:

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0033)e^{-(0.1410)^2(31.97)} = 0.531 \longrightarrow T_0 = \mathbf{248°F}$$

Cylinder:

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0050)e^{-(0.1995)^2(31.97)} = 0.282 \longrightarrow T_0 = \mathbf{167°F}$$

Prob. 16-81E (*continued*)

Sphere:

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0060) e^{-(0.2445)^2 (31.97)} = 0.149 \longrightarrow T_0 = \mathbf{123°F}$$

After 30 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.333 \text{ ft}^2/\text{h})(30 \text{ min}/60 \text{ min}/\text{h})}{(0.5/12 \text{ ft})^2} = 95.9 > 0.2$$

Plate:

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0033) e^{-(0.1410)^2 (95.9)} = 0.149 \longrightarrow T_0 = \mathbf{123°F}$$

Cylinder:

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0050) e^{-(0.1995)^2 (95.9)} = 0.0221 \longrightarrow T_0 = \mathbf{82°F}$$

Sphere:

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0060) e^{-(0.2445)^2 (95.9)} = 0.00326 \longrightarrow T_0 = \mathbf{76°F}$$

The sphere has the largest surface area through which heat is transferred per unit volume, and thus the highest rate of heat transfer. Consequently, the center temperature of the sphere is always the lowest.

16-82E A plate, a long cylinder, and a sphere are exposed to cool air. The center temperature of each geometry is to be determined. √

Assumptions **1** Heat conduction in each geometry is one-dimensional. **2** The thermal properties of the geometries are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

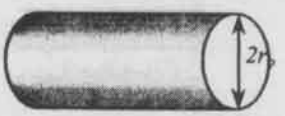
Properties The properties of cast iron are given to be $k = 29$ Btu/h.ft.°F and $\alpha = 0.61$ ft²/h.



Analysis **After 5 minutes**

Plate: First the Biot number is calculated to be

$$Bi = \frac{hL}{k} = \frac{(7\ \text{Btu/h.ft}^2\text{.°F})(0.5/12\ \text{ft})}{(29\ \text{Btu/h.ft.°F})} = 0.01006$$

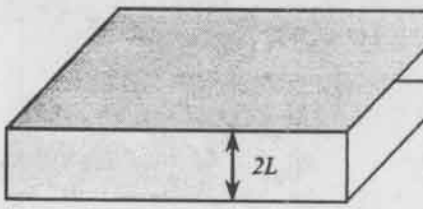


The constants λ_1 and A_1 corresponding to this Biot number are, from Table 16-1,

$$\lambda_1 = 0.0998 \quad \text{and} \quad A_1 = 1.0017$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.61\ \text{ft}^2/\text{h})(5\ \text{min}/60\ \text{min/h})}{(0.5/12\ \text{ft})^2} = 29.28 > 0.2$$



Then the center temperature of the plate becomes

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0017)e^{-(0.0998)^2(29.28)} = 0.748 \longrightarrow T_0 = \mathbf{318°F}$$

Cylinder:

$$Bi = 0.01 \xrightarrow{\text{Table 4-1}} \lambda_1 = 0.1412 \quad \text{and} \quad A_1 = 1.0025$$

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0025)e^{-(0.1412)^2(29.28)} = 0.559 \longrightarrow T_0 = \mathbf{257°F}$$

Sphere:

$$Bi = 0.01 \xrightarrow{\text{Table 4-1}} \lambda_1 = 0.1730 \quad \text{and} \quad A_1 = 1.0030$$

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0030)e^{-(0.1730)^2(29.28)} = 0.418 \longrightarrow T_0 = \mathbf{211°F}$$

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.61\ \text{ft}^2/\text{h})(10\ \text{min}/60\ \text{min/h})}{(0.5/12\ \text{ft})^2} = 58.56 > 0.2$$

Plate:

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0017)e^{-(0.0998)^2(58.56)} = 0.559 \longrightarrow T_0 = \mathbf{257°F}$$

Cylinder:

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0025)e^{-(0.1412)^2(58.56)} = 0.312 \longrightarrow T_0 = \mathbf{176°F}$$

Prob. 16-82E (*continued*)

Sphere:

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0030) e^{-(0.1730)^2 (58.56)} = 0.174 \longrightarrow T_0 = \mathbf{132°F}$$

After 30 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.61 \text{ ft}^2/\text{h})(30 \text{ min}/60 \text{ min/h})}{(0.5/12 \text{ ft})^2} = 175.68 > 0.2$$

Plate:

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0017) e^{-(0.0998)^2 (175.68)} = 0.174 \longrightarrow T_0 = \mathbf{132°F}$$

Cylinder:

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0025) e^{-(0.1412)^2 (175.68)} = 0.030 \longrightarrow T_0 = \mathbf{84.8°F}$$

Sphere:

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0030) e^{-(0.1730)^2 (175.68)} = 0.0052 \longrightarrow T_0 = \mathbf{76.7°F}$$

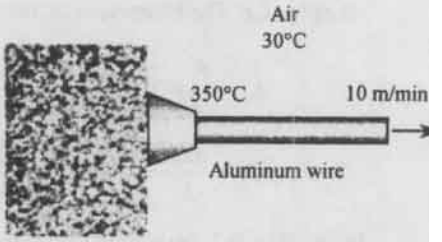
The sphere has the largest surface area through which heat is transferred per unit volume, and thus the highest rate of heat transfer. Consequently, the center temperature of the sphere is always the lowest.

16-83 Long aluminum wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the aluminum are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of aluminum are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, C_p = 0.896 kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis (*a*) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A} = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015\,\text{m}}{2} = 0.00075\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35\,\text{W/m}^2.^\circ\text{C})(0.00075\,\text{m})}{236\,\text{W/m.}^\circ\text{C}} = 0.00011 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then,

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{35\,\text{W/m}^2.^\circ\text{C}}{(2702\,\text{kg/m}^3)(896\,\text{J/kg.}^\circ\text{C})(0.00075\,\text{m})} = 0.0193\,\text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0193\,\text{s}^{-1})t} \longrightarrow t = \mathbf{144\,s}$$

(*b*) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \rightarrow \text{length} = (10/60\,\text{m/s})(144\,\text{s}) = \mathbf{24\,m}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(*c*) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho\dot{V} = \rho(\pi r_0^2/4)V = (2702\,\text{kg/m}^3)\pi(0.0015\,\text{m})^2(10\,\text{m/min}) = 0.191\,\text{kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

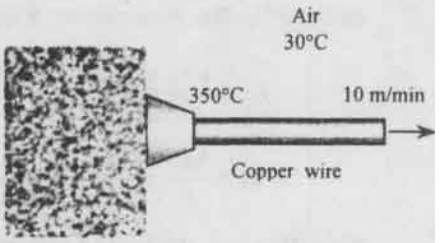
$$\dot{Q} = \dot{m}C_p[T(t)-T_\infty] = (0.191\,\text{kg/min})(0.896\,\text{kJ/kg.}^\circ\text{C})(350-50)^\circ\text{C} = 51.3\,\text{kJ/min} = \mathbf{0.856\,kW}$$

16-84 Long copper wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the copper are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of copper are given to be $k = 386$ W/m.°C, $\rho = 8950$ kg/m^3, $C_p = 0.383$ kJ/kg.°C, and $\alpha = 1.13\times10^{-4}$ m^2/s.

Analysis (*a*) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A} = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015\,\text{m}}{2} = 0.00075\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35\,\text{W/m}^2.^\circ\text{C})(0.00075\,\text{m})}{386\,\text{W/m.}^\circ\text{C}} = 0.000068 < 0.1$$

Since Bi < 0.1 the lumped system analysis is applicable. Then,

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{35\,\text{W/m}^2.^\circ\text{C}}{(8950\,\text{kg/m}^3)(383\,\text{J/kg.}^\circ\text{C})(0.00075\,\text{m})} = 0.0136\,\text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0136\,\text{s}^{-1})t} \longrightarrow t = \mathbf{204\,s}$$

(*b*) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \longrightarrow \text{length} = \left(\frac{10\,\text{m/min}}{60\,\text{s/min}}\right)(204\,\text{s}) = \mathbf{34\,m}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(*c*) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho\dot{V} = \rho(\pi r_0^2/4)V = (8950\,\text{kg/m}^3)\pi(0.0015\,\text{m})^2(10\,\text{m/min}) = 0.633\,\text{kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

$$\dot{Q} = \dot{m}C_p[T(t)-T_\infty] = (0.633\,\text{kg/min})(0.383\,\text{kJ/kg.}^\circ\text{C})(350-50)^\circ\text{C} = 72.7\,\text{kJ/min} = \mathbf{1.21\,kW}$$

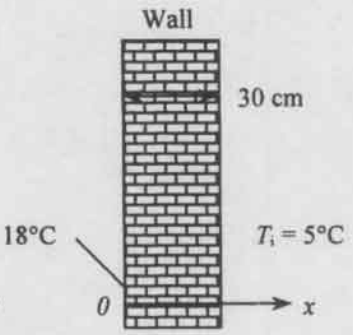
16-85 A brick house made of brick that was initially cold is exposed to warm atmospheric air at the outer surfaces. The time it will take for the temperature of the inner surfaces of the house to start changing is to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at outer surfaces only, and thus the wall can be considered to be a semi-infinite medium with a specified outer surface temperature of 18°C. **2** The thermal properties of the brick wall are constant.

Properties The thermal properties of the brick are given to be $k = 0.72$ W/m.°C and $\alpha = 0.45\times10^{-4}$ m^2/s.

Analysis The exact analytical solution to this problem is

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right)$$



Substituting,

$$\frac{5.1-5}{18-5} = 0.0077 = erfc\left(\frac{0.3\text{ m}}{2\sqrt{(0.45\times10^{-6}\text{ m}^2/\text{s})t}}\right)$$

Noting from Table 16-3 that 0.0077 = erfc(1.88), the time is determined to be

$$\left(\frac{0.3\text{ m}}{2\sqrt{(0.45\times10^{-6}\text{ m}^2/\text{s})t}}\right) = 1.88 \rightarrow t = 14{,}147\text{ s} = \mathbf{236\text{ min}}$$

Chapter 17
FORCED CONVECTION

Physical Mechanisms of Forced Convection

17-1C In forced convection, the fluid is forced to flow over a surface or in a tube by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect that manifests itself as the rise of the warmer fluid and the fall of the cooler fluid. The convection caused by winds is natural convection for the earth, but it is forced convection for bodies subjected to the winds since for the body it makes no difference whether the air motion is caused by a fan or by the winds.

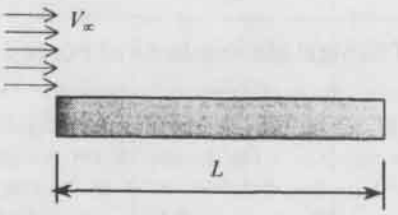
17-2C If the fluid is forced to flow over a surface, it is called external forced convection. If it is forced to flow in a tube, it is called internal forced convection. A heat transfer system can involve both internal and external convection simultaneously. Example: A pipe transporting a fluid in a windy area.

17-3C The convection heat transfer coefficient will usually be higher in forced convection since heat transfer coefficient depends on the fluid velocity, and forced convection involves higher fluid velocities.

17-4C The potato will normally cool faster by blowing warm air to it despite the smaller temperature difference in this case since the fluid motion caused by blowing enhances the heat transfer coefficient considerably.

17-5C The Prandtl number $\Pr = \nu/\alpha$ is a measure of the relative magnitudes of the diffusivity of momentum (and thus the development of the velocity boundary layer) and the diffusivity of heat (and thus the development of the thermal boundary layer). The Pr is a fluid property, and thus its value is independent of the type of flow and flow geometry. The Pr changes with temperature, but not pressure.

17-6C Nusselt number is the dimensionless convection heat transfer coefficient, and it represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. It is defined as follows:



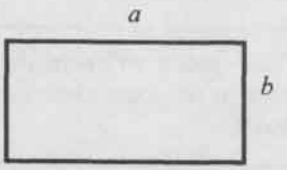
(*a*) For flow over a flat plate of length L: $Nu = \dfrac{hL}{k}$

(*b*, *c*) For flow over a cylinder of outer diameter D or for flow in a circular tube of inner diameter D: $Nu = \dfrac{hD}{k}$



(*d*) For flow in a rectangular tube of cross-section (a×b): $Nu = \dfrac{hD_h}{k}$ where the hydraulic diameter D_h is

$$D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$$



17-7C Heat transfer through a fluid is conduction in the absence of bulk fluid motion, and convection in the presence of it. The rate of heat transfer is higher in convection because of fluid motion. The value of the convection heat transfer coefficient depends on the fluid motion as well as the fluid properties. Thermal conductivity is a fluid property, and its value does not depend on the flow.

17-8C A fluid motion is laminar when it involves smooth streamlines and highly ordered motion of molecules, and turbulent when it involves velocity fluctuations and highly disordered motion. The heat transfer coefficient is higher in turbulent flow.

17-9C A thermal boundary layer will not develop in flow over a surface if both the fluid and the surface are at the same temperature since there will be no heat transfer in that case.

Flow Over Flat Plates

17-10C In the laminar forced convection from a horizontal flat plate, the heat flux will be higher at the leading edge because of the higher convection heat transfer coefficient.

17-11C During flow over a flat plate, the fluid temperature changes in the thermal boundary layer, but it remains constant outside the boundary layer.

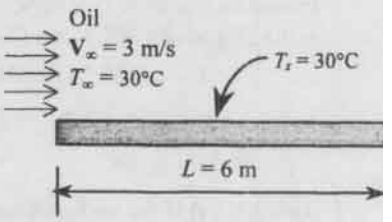
17-12 Hot engine oil flows over a flat plate. The rate of heat transfer per unit width of the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible.

Properties The properties of engine oil at the film temperature of $(T_s + T_\infty)/2 = (80+30)/2 = 55°C = 328$ K are (Table A-16)

$$\rho = 867 \text{ kg/m}^3 \qquad \upsilon = 123\times10^{-6} \text{ m}^2/\text{s}$$
$$k = 0.141 \text{ W/m.°C} \quad \text{Pr} = 1505$$

Analysis Noting that $L = 6$ m, the Reynolds number at the end of the plate is



$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(3 \text{ m/s})(6 \text{ m})}{123\times10^{-6} \text{ m}^2/\text{s}} = 1.46\times10^5$$

which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate. The average Nusselt number and the heat transfer coefficient are determined using the laminar flow relations for a flat plate,

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664(1.46\times10^5)^{0.5}(1505)^{1/3} = 2908$$

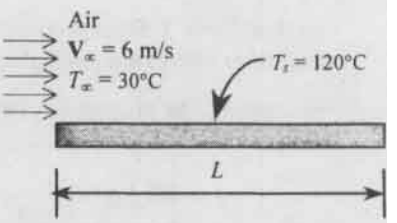
$$h = \frac{k}{L}Nu = \frac{0.141 \text{ W/m.°C}}{6 \text{ m}}(2908) = 68.3 \text{ W/m}^2.°\text{C}$$

The rate of heat transfer is then determined from Newton's law of cooling to be

$$\dot{Q} = hA(T_\infty - T_s) = (68.3 \text{ W/m}^2.°\text{C})(6\times1 \text{ m}^2)(80-30)°\text{C} = 2.05\times10^4 \text{ W} = \mathbf{20.5\,kW}$$

17-13 The top surface of a hot block is to be cooled by forced air. The rate of heat transfer is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties.



Properties The atmospheric pressure in atm is

$$P = (83.4 \text{ kPa})\frac{1 \text{ atm}}{101.325 \text{ kPa}} = 0.823 \text{ atm}$$

For an ideal gas, the thermal conductivity and the Prandtl number are independent of pressure, but the kinematic viscosity is inversely proportional to the pressure. With these considerations, the properties of air at 0.823 atm and 350 K are (Table A-18)

$$k = 0.0297 \text{ W/m.°C}$$

$$\upsilon = \upsilon_{@1atm} / P_{atm} = (2.06\times10^{-5} \text{ m}^2/\text{s}) / 0.823 = 2.5\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.706$$

Analysis (*a*) If the air flows parallel to the 8 m side, the Reynolds number in this case becomes

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(6 \text{ m/s})(8 \text{ m})}{2.5\times10^{-5} \text{ m}^2/\text{s}} = 1.92\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\text{Pr}^{1/3} = [0.037(1.92\times10^6)^{0.8} - 871](0.706)^{1/3} = 2727$$

$$h = \frac{k}{L}Nu = \frac{0.0297 \text{ W/m.°C}}{8 \text{ m}}(2727) = 10.1 \text{ W/m}^2\text{.°C}$$

$$A = wL = (2.5 \text{ m})(8 \text{ m}) = 20 \text{ m}^2$$

$$\dot{Q} = hA(T_\infty - T_s) = (10.1 \text{ W/m}^2\text{.°C})(20 \text{ m}^2)(120-30)°C = 1.82\times10^4 = \mathbf{1.82\,kW}$$

(*b*) If the air flows parallel to the 2.5 m side, the Reynolds number is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(6 \text{ m/s})(2.5 \text{ m})}{2.5\times10^{-5} \text{ m}^2/\text{s}} = 6\times10^5$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\text{Pr}^{1/3} = [0.037(6\times10^5)^{0.8} - 871](0.706)^{1/3} = 606$$

$$h = \frac{k}{L}Nu = \frac{0.0297 \text{ W/m.°C}}{2.5 \text{ m}}(606) = 7.20 \text{ W/m}^2\text{.°C}$$

$$A = wL = (8 \text{ m})(2.5 \text{ m}) = 20 \text{ m}^2$$

$$\dot{Q} = hA(T_\infty - T_s) = (7.20 \text{ W/m}^2\text{.°C})(20 \text{ m}^2)(120-30)°C = 1.30\times10^4 \text{ W} = \mathbf{13.0\,kW}$$

17-14 Wind is blowing parallel to the wall of a house. The rate of heat loss from that wall is to be determined for two cases.

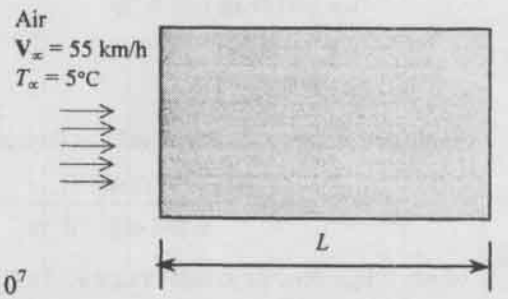
Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (12+5)/2 = 8.5°C = 281.5$ K are (Table A-18)

$k = 0.0246$ W/m.°C

$\upsilon = 1.4\times10^{-5}$ m^2/s

Pr = 0.717



Analysis *Air flows parallel to the 10 m side*:

The Reynolds number in this case is

$$Re_L = \frac{V_\infty L}{\upsilon} = \frac{[(55\times1000/3600)\text{m/s}](10\text{ m})}{1.4\times10^{-5}\text{ m}^2/\text{s}} = 1.09\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, heat transfer coefficient and then heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,Re_L^{0.8} - 871)\,Pr^{1/3} = [0.037(1.09\times10^7)^{0.8} - 871](0.717)^{1/3} = 1.335\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.0246\text{ W/m.°C}}{10\text{ m}}(1.335\times10^4) = 32.8\text{ W/m}^2\text{.°C}$$

$$A = wL = (4\text{ m})(10\text{ m}) = 40\text{ m}^2$$

$$\dot{Q} = hA(T_\infty - T_s) = (32.8\text{ W/m}^2\text{.°C})(40\text{ m}^2)(12-5)°C = 9.18\times10^3 = \mathbf{9.18\,kW}$$

If the wind velocity is doubled:

$$Re_L = \frac{V_\infty L}{\upsilon} = \frac{[(110\times1000/3600)\text{m/s}](10\text{ m})}{1.4\times10^{-5}\text{ m}^2/\text{s}} = 2.18\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,Re_L^{0.8} - 871)\,Pr^{1/3} = [0.037(2.18\times10^7)^{0.8} - 871](0.719)^{1/3} = 2.38\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.0246\text{ W/m.°C}}{10\text{ m}}(2.38\times10^4) = 58.6\text{ W/m}^2\text{.°C}$$

$$A = wL = (10\text{ m})(4\text{ m}) = 40\text{ m}^2$$

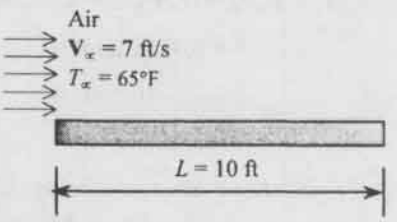
$$\dot{Q} = hA(T_\infty - T_s) = (58.6\text{ W/m}^2\text{.°C})(40\text{ m}^2)(12-5)°C = 1.64\times10^4 = \mathbf{16.4\,kW}$$

17-15E Air flows over a flat plate. The local heat transfer coefficients at intervals of 1 ft are to be determined and plotted against the distance from the leading edge.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and 65°F are (Table A-18E)

$$k = 0.0147 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.163\times10^{-3} \text{ ft}^2/\text{s}$$
$$\text{Pr} = 0.72$$



Analysis For the first 1 ft interval, the Reynolds number is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(7 \text{ ft/s})(1 \text{ ft})}{0.163\times10^{-3} \text{ ft}^2/\text{s}} = 4.29\times10^4$$

which is less than the critical value of 5×10^5. Therefore, the flow is laminar. The local Nusselt number is then determined from

$$Nu_x = \frac{hx}{k} = 0.332 \text{ Re}_x^{0.5} \text{ Pr}^{1/3} = 0.332(4.29\times10^4)^{0.5}(0.72)^{1/3} = 61.7$$

The local heat transfer coefficient is

$$h_x = \frac{k}{x} Nu = \frac{0.0147 \text{ Btu/h.ft.°F}}{1 \text{ ft}}(61.7) = 0.91 \text{ Btu/h.ft}^2\text{.°F}$$

We repeat calculations for all 1-ft intervals. The results are

$x = 2.0$ ft, $h_x = 0.64$ Btu/h.ft^2.°F

$x = 3.0$ ft, $h_x = 0.52$ Btu/h.ft^2.°F

$x = 4.0$ ft, $h_x = 0.45$ Btu/h.ft^2.°F

$x = 5.0$ ft, $h_x = 0.40$ Btu/h.ft^2.°F

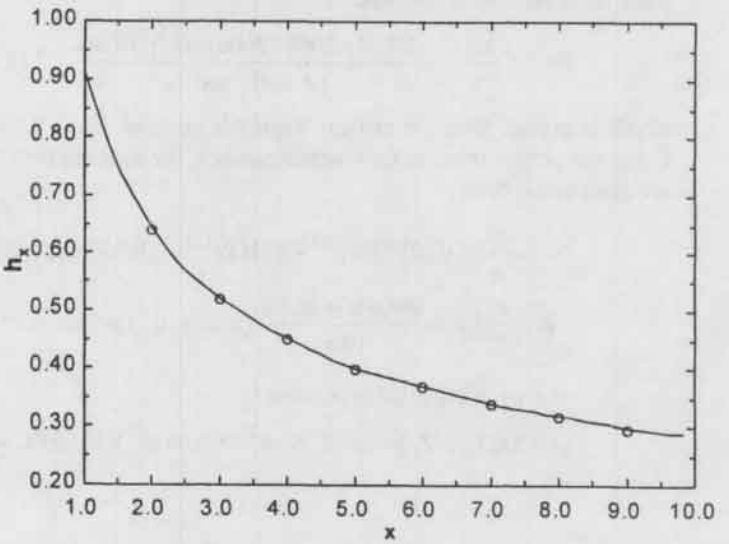
$x = 6.0$ ft, $h_x = 0.37$ Btu/h.ft^2.°F

$x = 7.0$ ft, $h_x = 0.34$ Btu/h.ft^2.°F

$x = 8.0$ ft, $h_x = 0.32$ Btu/h.ft^2.°F

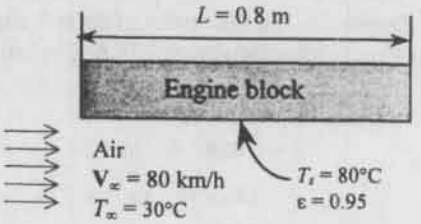
$x = 9.0$ ft, $h_x = 0.30$ Btu/h.ft^2.°F

$x = 10$ ft, $h_x = 0.29$ Btu/h.ft^2.°F



17-16 A car travels at a velocity of 80 km/h. The rate of heat transfer from the bottom surface of the hot automotive engine block is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas with constant properties. **4** The flow is turbulent over the entire surface because of the constant agitation of the engine block.



Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (80+30)/2 = 55°C = 328.5$ K are (Table A-18)

$$k = 0.0282 \text{ W/m.°C}$$
$$\upsilon = 1.86\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.708$$

Analysis Air flows parallel to the 0.4 m side. The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(80\times1000/3600)\text{ m/s}](0.8\text{ m})}{1.86\times10^{-5}\text{ m}^2/\text{s}} = 9.6\times10^5$$

which is less than the critical Reynolds number. But the flow is assumed to be turbulent over the entire surface because of the constant agitation of the engine block. Using the proper relations, the Nusselt number, the heat transfer coefficient, and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = 0.037\,\text{Re}_L^{0.8}\,\text{Pr}^{1/3} = 0.037(9.6\times10^5)^{0.8}(0.708)^{1/3} = 2014$$

$$h = \frac{k}{L}Nu = \frac{0.0282\text{ W/m.°C}}{0.4\text{ m}}(2014) = 71.0\text{ W/m}^2\text{.°C}$$

$$A = wL = (0.8\text{ m})(0.4\text{ m}) = 0.32\text{ m}^2$$

$$\dot{Q}_{conv} = hA(T_\infty - T_s) = (71.0\text{ W/m}^2\text{.°C})(0.32\text{ m}^2)(80-30)°\text{C} = \mathbf{1136\ W}$$

The radiation heat transfer from the same surface is

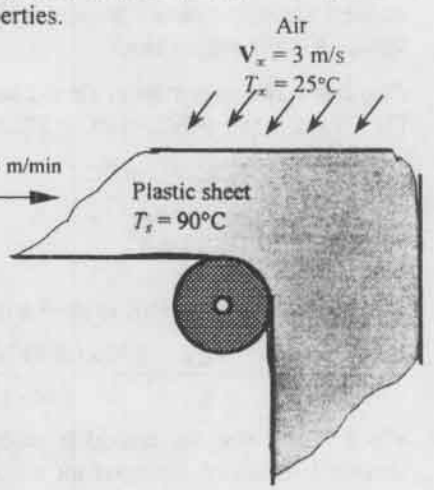
$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4) = (0.95)(0.32\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(80+273\text{ K})^4 - (25+273\text{ K})^4]$$
$$= \mathbf{132\ W}$$

Then the total rate of heat transfer from that surface becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = (1136+132)\text{W} = \mathbf{1268\ W}$$

17-17 Air flows on both sides of a continuous sheet of plastic. The rate of heat transfer from the plastic sheet is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties.



Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (90+25)/2 = 57.5°C = 330.5$ K are (Table A-18)

$$\rho = 1.076 \text{ kg/m}^3$$
$$k = 0.0282 \text{ W/m.°C}$$
$$\upsilon = 1.86\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.708$$

Analysis The length of the cooling section is first determined from

$$L = \mathbf{V}\Delta t = [(15/60) \text{ m/s}](2 \text{ s}) = 0.5 \text{ m}$$

The Reynolds number is

$$\text{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(3 \text{ m/s})(1.2 \text{ m})}{1.86\times10^{-5} \text{ m}^2/\text{s}} = 1.935\times10^5$$

which is less than the critical Reynolds number. Thus the flow is laminar. Using the proper relation in laminar flow for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664(1.935\times10^5)^{0.5}(0.708)^{1/3} = 260.4$$

$$h = \frac{k}{L}Nu = \frac{0.0282 \text{ W/m.°C}}{0.5 \text{ m}}(260.4) = 14.7 \text{ W/m}^2\text{.°C}$$

$$A = 2wL = 2(1.2 \text{ m})(0.5 \text{ m}) = 1.2 \text{ m}^2$$

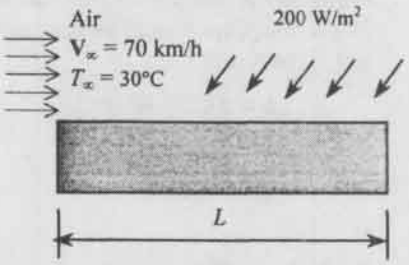
$$\dot{Q}_{conv} = hA(T_\infty - T_s) = (14.7 \text{ W/m}^2\text{.°C})(1.2 \text{ m}^2)(90-25)°\text{C} = \mathbf{1145 \text{ W}}$$

17-18 The top surface of the passenger car of a train in motion is absorbing solar radiation. The equilibrium temperature of the top surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation heat exchange with the surroundings is negligible. **4** Air is an ideal gas with constant properties.

Properties The properties of air at $30°C \cong 300$ K are (Table A-18)

$$k = 0.0261 \text{ W/m.°C}$$
$$\upsilon = 1.57\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.712$$



Analysis The rate of convection heat transfer from the top surface of the car to the air must be equal to the solar radiation absorbed by the same surface in order to reach steady operation conditions. The Reynolds number is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[70\times1000/3600)\text{ m/s}](8\text{ m})}{1.57\times10^{-5}\text{ m}^2/\text{s}} = 9.9\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\Pr^{1/3} = [0.037(9.9\times10^6)^{0.8} - 871](0.712)^{1/3} = 1.227\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.0261\text{ W/m.°C}}{8\text{ m}}(1.227\times10^4) = 40.0\text{ W/m}^2\text{.°C}$$

The equilibrium temperature of the top surface is then determined by taking convection and radiation heat fluxes to be equal to each other

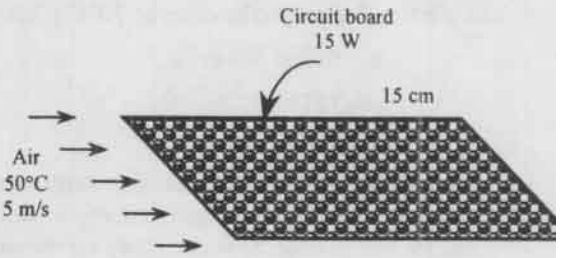
$$\dot{q}_{rad} = \dot{q}_{conv} = h(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{q}_{conv}}{h} = 30°C + \frac{200\text{ W/m}^2}{40\text{ W/m}^2\text{.°C}} = \mathbf{35°C}$$

17-19 A circuit board is cooled by air. The surface temperatures of the electronic components at the leading edge and the end of the board are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Any heat transfer from the back surface of the board is disregarded. **5** Air is an ideal gas with constant properties.

Properties Assuming the film temperature to be approximately 350 K, the properties of air are evaluated at this temperature to be (Table A-18)

$$k = 0.0297 \text{ W/m.°C}$$
$$\upsilon = 2.06\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.706$$



Analysis (*a*) The convection heat transfer coefficient at the leading edge approaches infinity, and thus the surface temperature there must approach the air temperature which is 50°C.

(*b*) The Reynolds number is

$$\text{Re}_x = \frac{V_\infty x}{\upsilon} = \frac{(5 \text{ m/s})(0.15 \text{ m})}{2.06\times10^{-5} \text{ m}^2/\text{s}} = 3.641\times10^4$$

which is less than the critical Reynolds number but we assume the flow to be turbulent since the electronic components are expected to act as turbulators. Using the Nusselt number uniform heat flux, the local heat transfer coefficient at the end of the board is determined to be

$$Nu_x = \frac{h_x x}{k} = 0.0308\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3} = 0.0308(3.641\times10^4)^{0.8}(0.706)^{1/3} = 122$$

$$h_x = \frac{k_x}{x} Nu_x = \frac{0.0297 \text{ W/m.°C}}{0.15 \text{ m}}(122) = 24.2 \text{ W/m}^2\text{.°C}$$

Then the surface temperature at the end of the board becomes

$$\dot{q} = h_x(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{q}}{h_x} = 50°\text{C} + \frac{(15 \text{ W})/(0.15 \text{ m})^2}{24.2 \text{ W/m}^2\text{.°C}} = \mathbf{77.5°C}$$

Discussion The heat flux can also be determined approximately using the relation for isothermal surfaces,

$$Nu_x = \frac{h_x x}{k} = 0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3} = 0.0296(36{,}408)^{0.8}(0.706)^{1/3} = 117.4$$

$$h_x = \frac{k_x}{x} Nu_x = \frac{0.0297 \text{ W/m.°C}}{0.15 \text{ m}}(117.4) = 23.2 \text{ W/m}^2\text{.°C}$$

Then the surface temperature at the end of the board becomes

$$\dot{q} = h_x(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{q}}{h_x} = 50°\text{C} + \frac{(15 \text{ W})/(0.15 \text{ m})^2}{23.2 \text{ W/m}^2\text{.°C}} = \mathbf{78.7°C}$$

Note that the two results are close to each other.

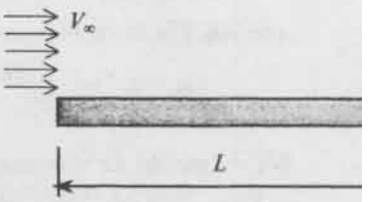
17-20 Laminar flow of a fluid over a flat plate is considered. The change in the rate of heat transfer is to be determined when the free-stream velocity of the fluid is doubled.

Analysis For the laminar flow of a fluid over a flat plate maintained at a constant temperature, the rate of heat transfer is given by

$$\dot{Q}_1 = hA(T_s - T_\infty) = \left(\frac{k}{L}Nu\right)A(T_s - T_\infty) = \left(\frac{k}{L}\right)\left(0.664\,\mathrm{Re}^{0.5}\,\mathrm{Pr}^{1/3}\right)A(T_s - T_\infty)$$

$$= \frac{k}{L}0.664\left(\frac{\mathbf{V}_\infty L}{\upsilon}\right)^{0.5}\mathrm{Pr}^{1/3}\,A(T_s - T_\infty)$$

$$= 0.664\mathbf{V}_\infty{}^{0.5}\frac{k}{L^{0.5}\upsilon^{0.5}}\mathrm{Pr}^{1/3}\,A(T_s - T_\infty)$$



When the free-stream velocity of the fluid is doubled, the new value of the heat transfer rate between the fluid and the plate becomes

$$\dot{Q}_2 = 0.664(2\mathbf{V}_\infty)^{0.5}\frac{k}{L^{0.5}\upsilon^{0.5}}\mathrm{Pr}^{1/3}\,A(T_s - T_\infty)$$

Then the ratio is

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2\mathbf{V}_\infty)^{0.5}}{\mathbf{V}_\infty{}^{0.5}} = 2^{0.5} = \sqrt{2}$$

17-21E A refrigeration truck is traveling at 55 mph. The average temperature of the outer surface of the refrigeration compartment of the truck is to be determined.

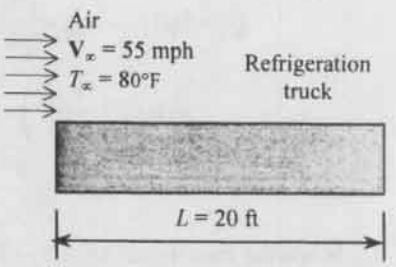
Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties. **5** The local atmospheric pressure is 1 atm.

Properties Assuming the film temperature to be approximately 80°F, the properties of air at this temperature and 1 atm are (Table A-18E)

$$k = 0.015 \text{ Btu/h.ft.°F}$$

$$\upsilon = 0.17\times10^{-3} \text{ ft}^2/\text{s}$$

$$\text{Pr} = 0.72$$



Analysis The Reynolds number is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[55\times5280/3600) \text{ ft/s}](20 \text{ ft})}{0.17\times10^{-3} \text{ ft}^2/\text{s}} = 9.49\times10^6$$

We assume the air flow over the entire outer surface to be turbulent. Therefore using the proper relation in turbulent flow for Nusselt number, the average heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = 0.037\,\text{Re}_L^{0.8}\,\text{Pr}^{1/3} = 0.037(9.49\times10^6)^{0.8}(0.72)^{1/3} = 1.27\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.015 \text{ Btu/h.ft.°F}}{20 \text{ ft}}(1.27\times10^4) = 9.53 \text{ Btu/h.ft}^2\text{.°F}$$

Since the refrigeration system is operated at half the capacity, we will take half of the heat removal rate

$$\dot{Q} = \frac{(600\times60) \text{ Btu/h}}{2} = 18{,}000 \text{ Btu/h}$$

The total heat transfer surface area and the average surface temperature of the refrigeration compartment of the truck are determined from

$$A = 2[(20 \text{ ft})(9 \text{ ft}) + (20 \text{ ft})(8 \text{ ft}) + (9 \text{ ft})(8 \text{ ft})] = 824 \text{ ft}^2$$

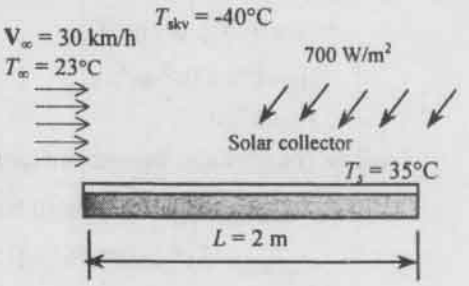
$$\dot{Q} = hA(T_\infty - T_s) \longrightarrow T_s = T_\infty - \frac{\dot{q}_{conv}}{hA} = 80°\text{F} - \frac{18{,}000 \text{ Btu/h}}{(9.53 \text{ Btu/h.ft}^2\text{.°F})(824 \text{ ft}^2)} = 77.7°\text{F}$$

17-22 Solar radiation is incident on the glass cover of a solar collector. The total rate of heat loss from the collector, the collector efficiency, and the temperature rise of water as it flows through the collector are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Heat exchange on the back surface of the absorber plate is negligible. **4** Air is an ideal gas with constant properties. **5** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $(35+23)/2 = 29\,^\circ\text{C}$ are (Table A-18)

$$k = 0.0261\ \text{W/m.°C}$$
$$\upsilon = 1.57\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.712$$



Analysis (*a*) Assuming wind flows across 2 m surface, the Reynolds number is determined from

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(30\times1000/3600)\text{m/s}(2\text{ m})}{1.57\times10^{-5}\ \text{m}^2/\text{s}} = 1.062\times10^6$$

which is greater than the critical Reynolds number (5×10^5). Using the Nusselt number relation for combined laminar and turbulent flow, the average heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}^{0.8} - 871)\,\text{Pr}^{1/3} = [0.037(1.062\times10^6)^{0.8} - 871](0.712)^{1/3} = 1410$$
$$h = \frac{k}{L}Nu = \frac{0.0261\ \text{W/m.°C}}{2\ \text{m}}(1410) = 18.4\ \text{W/m}^2.^\circ\text{C}$$

Then the rate of heat loss from the collector by convection is

$$\dot{Q}_{conv} = hA(T_\infty - T_s) = (18.4\ \text{W/m}^2.^\circ\text{C})(2\times1.2\ \text{m}^2)(35\text{-}23)^\circ\text{C} = 530\ \text{W}$$

The rate of heat loss from the collector by radiation is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.90)(2\times1.2\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.^\circ\text{C})\left[(35+273\ \text{K})^4 - (-40+273\ \text{K})^4\right]$$
$$= 732\ \text{W}$$

and

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 530 + 732 = \mathbf{1262\ W}$$

(*b*) The net rate of heat transferred to the water is

$$\dot{Q}_{net} = \dot{Q}_{in} - \dot{Q}_{out} = \alpha A I - \dot{Q}_{out}$$
$$= (0.88)(2\times1.2\ \text{m}^2)(700\ \text{W/m}^2) - 1262\ \text{W}$$
$$= 1478 - 1262 = 216\ \text{W}$$
$$\eta_{collector} = \frac{\dot{Q}_{net}}{\dot{Q}_{in}} = \frac{216\ \text{W}}{1478\ \text{W}} = \mathbf{0.15}$$

(*c*) The temperature rise of water as it flows through the collector is

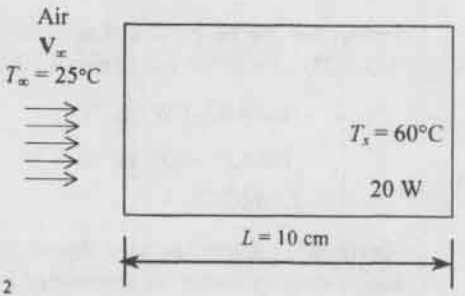
$$\dot{Q}_{net} = \dot{m}C_p\Delta T \longrightarrow \dot{m} = \frac{\dot{Q}_{net}}{C_p\Delta T} = \frac{216.4\ \text{W}}{(1/60\ \text{kg/s})(4180\ \text{J/kg.°C})} = \mathbf{3.1\ °C}$$

17-23 A fan blows air parallel to the passages between the fins of a heat sink attached to a transformer. The minimum free-stream velocity that the fan should provide to avoid overheating is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** The fins and the base plate are nearly isothermal (fin efficiency is equal to 1) **5** Air is an ideal gas with constant properties. **6** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (60+25)/2 = 42.5°C = 315.5$ K are (Table A-18)

$$k = 0.0275 \text{ W/m.°C}$$
$$\upsilon = 1.77\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$



Analysis The total heat transfer surface area for this finned surface is

$$A_{s,\text{finned}} = (2\times7)(0.1\text{ m})(0.005\text{ m}) = 0.007\text{ m}^2$$
$$A_{s,\text{unfinned}} = (0.1\text{ m})(0.062\text{ m}) - 7\times(0.002\text{ m})(0.1\text{ m}) = 0.0048\text{ m}^2$$
$$A_{s,\text{total}} = A_{s,\text{finned}} + A_{s,\text{unfinned}} = 0.007\text{ m}^2 + 0.0048\text{ m}^2 = 0.0118\text{ m}^2$$

The convection heat transfer coefficient can be determined from Newton's law of cooling relation for a finned surface.

$$\dot{Q} = \eta hA(T_\infty - T_s) \longrightarrow h = \frac{\dot{Q}}{\eta A(T_\infty - T_s)} = \frac{20\text{ W}}{(1)(0.0118\text{ m}^2)(60-25)°\text{C}} = 48.4\text{ W/m}^2.°\text{C}$$

Starting from heat transfer coefficient, Nusselt number, Reynolds number and finally free-stream velocity will be determined. We assume the flow is laminar over the entire finned surface of the transformer.

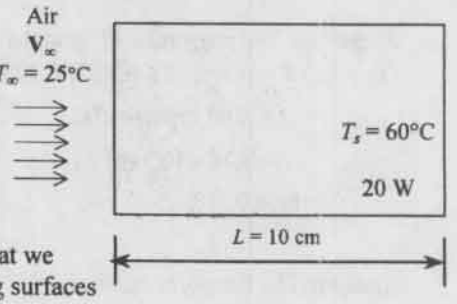
$$Nu = \frac{hL}{k} = \frac{(48.4\text{ W/m}^2.°\text{C})(0.1\text{ m})}{0.0275\text{ W/m.°C}} = 176$$

$$Nu = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} \longrightarrow \text{Re}_L = \frac{Nu^2}{0.664^2\,\text{Pr}^{2/3}} = \frac{176^2}{(0.664)^2(0.710)^{2/3}} = 8.828\times10^4$$

$$\text{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} \longrightarrow \mathbf{V}_\infty = \frac{\text{Re}_L\,\upsilon}{L} = \frac{(8.828\times10^4)(1.77\times10^{-5}\text{ m}^2/\text{s})}{0.1\text{ m}} = \mathbf{15.6\text{ m/s}}$$

17-24 A fan blows air parallel to the passages between the fins of a heat sink attached to a transformer. The minimum free-stream velocity that the fan should provide to avoid overheating is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** The fins and the base plate are nearly isothermal (fin efficiency is equal to 1) **4** Air is an ideal gas with constant properties. **5** The local atmospheric pressure is 1 atm.



Properties The properties of air at the film temperature of $(T_s + T_\infty)/2 = (60+25)/2 = 42.5°C = 315.5$ K are

$$k = 0.0275 \text{ W/m.°C}$$
$$\upsilon = 1.77\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$

Analysis We first need to determine radiation heat transfer rate. Note that we will use the base area and we assume the temperature of the surrounding surfaces are at the same temperature with the air ($T_{surr} = 25\,°C$)

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.90)[(0.1\,\text{m})(0.062\,\text{m})](5.67\times10^{-8}\ \text{W/m}^2.°\text{C})[(60+273\,\text{K})^4 - (25+273\,\text{K})^4]$$
$$= 1.4\ \text{W}$$

The heat transfer rate by convection will be 1.4 W less than total rate of heat transfer from the transformer. Therefore

$$\dot{Q}_{conv} = \dot{Q}_{total} - \dot{Q}_{rad} = 20 - 1.4 = 18.6\ \text{W}$$

The total heat transfer surface area for this finned surface is

$$A_{s,finned} = (2\times7)(0.1\,\text{m})(0.005\,\text{m}) = 0.007\,\text{m}^2$$
$$A_{s,unfinned} = (0.1\,\text{m})(0.062\,\text{m}) - 7\times(0.002\,\text{m})(0.1\,\text{m}) = 0.0048\,\text{m}^2$$
$$A_{s,total} = A_{s,finned} + A_{s,unfinned} = 0.007\,\text{m}^2 + 0.0048\,\text{m}^2 = 0.0118\,\text{m}^2$$

The convection heat transfer coefficient can be determined from Newton's law of cooling relation for a finned surface.

$$\dot{Q}_{conv} = \eta hA(T_\infty - T_s) \longrightarrow h = \frac{\dot{Q}_{conv}}{\eta A(T_\infty - T_s)} = \frac{18.6\ \text{W}}{(1)(0.0118\,\text{m}^2)(60-25)°\text{C}} = 45\ \text{W/m}^2.°\text{C}$$

Starting from heat transfer coefficient, Nusselt number, Reynolds number and finally free-stream velocity will be determined. We assume the flow is laminar over the entire finned surface of the transformer.

$$Nu = \frac{hL}{k} = \frac{(45\ \text{W/m}^2.°\text{C})(0.1\,\text{m})}{0.0275\ \text{W/m.°C}} = 163.6$$

$$Nu = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} \longrightarrow \text{Re}_L = \frac{Nu^2}{0.664^2\,\text{Pr}^{2/3}} = \frac{(163.6)^2}{(0.664)^2(0.710)^{2/3}} = 7.628\times10^4$$

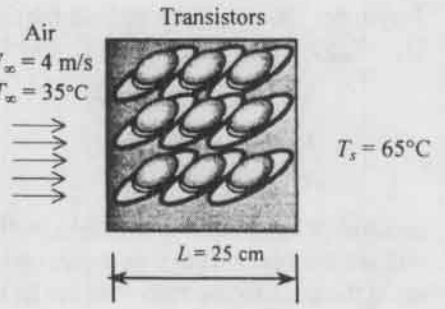
$$\text{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} \rightarrow \mathbf{V}_\infty = \frac{\text{Re}_L\,\upsilon}{L} = \frac{(7.628\times10^4)(1.77\times10^{-5}\ \text{m}^2/\text{s})}{0.1\,\text{m}} = \mathbf{13.5\ m/s}$$

17-25 Air is blown over an aluminum plate mounted on an array of power transistors. The number of transistors that can be placed on this plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible **4** Heat transfer from the back side of the plate is negligible. **5** Air is an ideal gas with constant properties. **6** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $(T_s + T_\infty)/2 = (65+35)/2 = 50°C = 323$ K are (Table A-18)

$$k = 0.0275 \text{ W/m.°C}$$
$$\upsilon = 1.77\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$



Analysis The Reynolds number is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(4 \text{ m/s})(0.25 \text{ m})}{1.77\times10^{-5} \text{ m}^2/\text{s}} = 56{,}497$$

which is less than the critical Reynolds number (5×10^5). Thus the flow is laminar. Using the proper relation in laminar flow for Nusselt number, heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664(56{,}497)^{0.5}(0.710)^{1/3} = 140.8$$

$$h = \frac{k}{L}Nu = \frac{0.0275 \text{ W/m.°C}}{0.25 \text{ m}}(140.8) = 15.5 \text{ W/m}^2\text{.°C}$$

$$A = wL = (0.25 \text{ m})(0.25 \text{ m}) = 0.0625 \text{ m}^2$$

$$\dot{Q}_{conv} = hA(T_\infty - T_s) = (15.5 \text{ W/m}^2\text{.°C})(0.0625 \text{ m}^2)(65-35)°\text{C} = 29.1 \text{ W}$$

Considering that each transistor dissipates 3 W of power, the number of transistors that can be placed on this plate becomes

$$n = \frac{29.1 \text{ W}}{3 \text{ W}} = 9.7 \longrightarrow \mathbf{9}$$

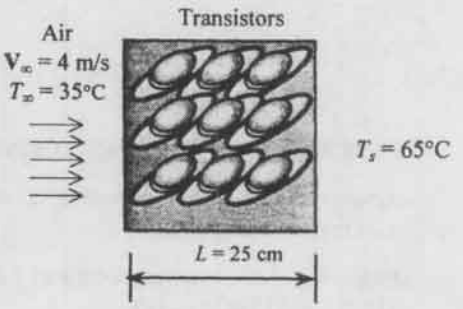
This result is conservative since the transistors will cause the flow to be turbulent, and the rate of heat transfer to be higher.

17-26 Air is blown over an aluminum plate mounted on an array of power transistors. The number of transistors that can be placed on this plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible **4** Heat transfer from the backside of the plate is negligible. **5** Air is an ideal gas with constant properties. **6** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (65+35)/2 = 50°C = 323$ K are (Table A-18)

$$k = 0.0275 \text{ W/m.°C}$$
$$\upsilon = 1.77\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$



Note that the atmospheric pressure will only affect the kinematic viscosity. The atmospheric pressure in atm is

$$P = (83.4 \text{ kPa})\frac{1 \text{ atm}}{101.325 \text{ kPa}} = 0.823 \text{ atm}$$

The kinematic viscosity at this atmospheric pressure will be

$$\upsilon = (1.77\times10^{-5} \text{ m}^2/\text{s})/0.823 = 2.15\times10^{-5} \text{ m}^2/\text{s}$$

Analysis The Reynolds number is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(4 \text{ m/s})(0.25 \text{ m})}{2.15\times10^{-5} \text{ m}^2/\text{s}} = 4.65\times10^4$$

which is less than the critical Reynolds number (5×10^5). Thus the flow is laminar. Using the proper relation in laminar flow for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664(4.65\times10^4)^{0.5}(0.710)^{1/3} = 128$$

$$h = \frac{k}{L}Nu = \frac{0.0275 \text{ W/m.°C}}{0.25 \text{ m}}(128) = 14.1 \text{ W/m}^2.°C$$

$$A = wL = (0.25 \text{ m})(0.25 \text{ m}) = 0.0625 \text{ m}^2$$

$$\dot{Q}_{conv} = hA(T_\infty - T_s) = (14.1 \text{ W/m}^2.°C)(0.0625 \text{ m}^2)(65-35)°C = 26.3 \text{ W}$$

Considering that each transistor dissipates 3 W of power, the number of transistors that can be placed on this plate becomes

$$n = \frac{26.3 \text{ W}}{3 \text{ W}} = 8.8 \longrightarrow \mathbf{8}$$

This result is conservative since the transistors will cause the flow to be turbulent, and the rate of heat transfer to be higher.

Flow Across Cylinders And Spheres

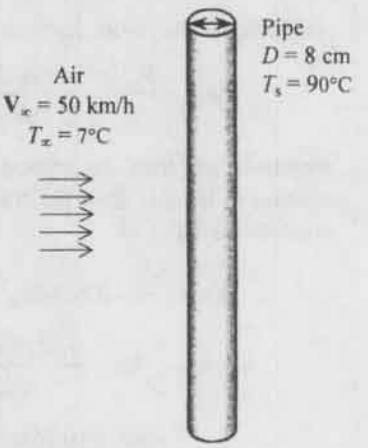
17-27C For the laminar flow, the heat transfer coefficient will be the highest at the stagnation point which corresponds to $\theta \approx 0°$. In turbulent flow, on the other hand, it will be highest when θ is between 90° and 120°, as can be seen from Fig. 17-14.

17-28 A steam pipe is exposed to windy air. The rate of heat loss from the steam is to be determined.√

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (90+7)/2 = 48.5°C =$ 321.5 K are (Table A-18)

$$k = 0.0275 \text{ W/m.°C}$$
$$\upsilon = 1.77 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(50 \text{ km/h})(1000 \text{ m/km})/(3600 \text{ s/h})](0.08 \text{ m})}{1.77 \times 10^{-5} \text{ m}^2/\text{s}} = 6.277 \times 10^4$$

The Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62 \text{ Re}^{0.5} \text{ Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(6.277 \times 10^4)^{0.5}(0.710)^{1/3}}{\left[1+(0.4/0.710)^{2/3}\right]^{1/4}} \left[1+\left(\frac{6.277 \times 10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 159$$

The heat transfer coefficient and the heat transfer rate become

$$h = \frac{k}{D} Nu = \frac{0.0275 \text{ W/m.°C}}{0.08 \text{ m}} (159) = 54.6 \text{ W/m}^2\text{.°C}$$

$$A = \pi DL = \pi(0.08 \text{ m})(1 \text{ m}) = 0.251 \text{ m}^2$$

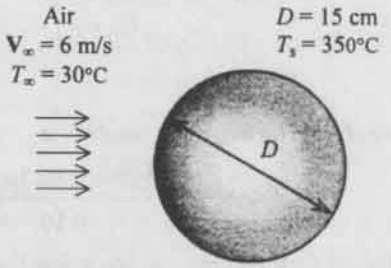
$$\dot{Q}_{conv} = hA(T_s - T_\infty) = (54.6 \text{ W/m}^2\text{.°C})(0.251 \text{ m}^2)(90-7)°C = \mathbf{1138 \text{ W}} \text{ (per m length)}$$

17-29 A hot stainless steel ball is cooled by forced air. The average convection heat transfer coefficient and the cooling time are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The outer surface temperature of the ball is uniform at all times.

Properties The average surface temperature is (350+250)/2 = 300°C, and the properties of air at 1 atm pressure and the free stream temperature of 30°C are (Table A-18)

$$k = 0.0261 \text{ W/m.°C}$$
$$\upsilon = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu_\infty = 1.85 \times 10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@300\,°C} = 2.94 \times 10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.712$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{(6 \text{ m/s})(0.15 \text{ m})}{1.57 \times 10^{-5} \text{ m}^2/\text{s}} = 5.733 \times 10^4$$

The Nusselt number corresponding this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 2 + \left[0.4 \text{Re}^{0.5} + 0.06 \text{Re}^{2/3}\right] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(5.733 \times 10^4)^{0.5} + 0.06(5.733 \times 10^4)^{2/3}\right](0.712)^{0.4} \left(\frac{1.85 \times 10^{-5}}{2.94 \times 10^{-5}}\right)^{1/4} = 146$$

Heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.0261 \text{ W/m.°C}}{0.15 \text{ m}} (146) = \mathbf{25.4 \text{ W/m}^2\text{.°C}}$$

The average rate of heat transfer can be determined from Newton's law of cooling by using average surface temperature of the ball

$$A = \pi D^2 = \pi (0.15 \text{ m})^2 = 0.0707 \text{ m}^2$$
$$\dot{Q}_{ave} = hA(T_s - T_\infty) = (25.4 \text{ W/m}^2\text{.°C})(0.0707 \text{ m}^2)(300 - 30)°\text{C} = 485 \text{ W}$$

Assuming the ball temperature to be nearly uniform , the total heat transferred from the ball during the cooling from 350 °C to 250 °C can be determined from

$$Q_{\text{total}} = mC_p(T_1 - T_2)$$

where $$m = \rho V = \rho \frac{\pi D^3}{6} = (8055 \text{ kg/m}^3) \frac{\pi (0.15 \text{ m})^3}{6} = 14.2 \text{ kg}$$

Therefore, $$Q_{\text{total}} = mC_p(T_1 - T_2) = (14.2 \text{ kg})(0.480 \text{ kJ/kg.°C})(350 - 250)°\text{C} = 681.6 \text{ kJ}$$

Then the time of cooling becomes

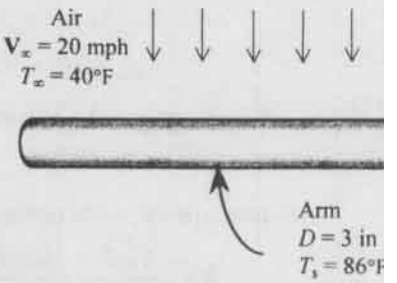
$$\Delta t = \frac{Q}{\dot{Q}} = \frac{681{,}600 \text{ J}}{485 \text{ J/s}} = 1406 \text{ s} = \mathbf{23.4 \text{ min}}$$

17-30E A person extends his uncovered arms into the windy air outside. The rate of heat loss from the arm is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. 4 The arm is treated as a 2-ft-long and 3-in.-diameter cylinder with insulated ends. **5** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (86+40)/2 = 63°F$ are (Table A-18E)

$$k = 0.0146 \text{ Btu / h.ft.°F}$$
$$\upsilon = 0.161\times10^{-3} \text{ ft}^2/\text{s}$$
$$\text{Pr} = 0.72$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(20\times5280/3600) \text{ ft/s}](3/12) \text{ ft}}{0.161\times10^{-3} \text{ ft}^2/\text{s}} = 4.555\times10^4$$

The Nusselt number corresponding this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+\left(\frac{0.4}{\text{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(4.555\times10^4)^{0.5}(0.72)^{1/3}}{\left[1+\left(\frac{0.4}{0.72}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{4.555\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 130.5$$

Then the heat transfer coefficient and the heat transfer rate from the arm becomes

$$h = \frac{k}{D}Nu = \frac{0.0146 \text{ Btu/h.ft.°F}}{(3/12) \text{ ft}}(130.5) = 7.62 \text{ Btu/h.ft}^2\text{.°F}$$

$$A = \pi DL = \pi(3/12 \text{ ft})(2 \text{ ft}) = 1.57 \text{ ft}^2$$

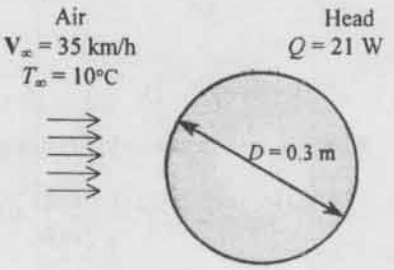
$$\dot{Q}_{conv} = hA(T_s - T_\infty) = (7.62 \text{ Btu/h.ft}^2\text{.°F})(1.57 \text{ ft}^2)(86-40)°F = \mathbf{551 \text{ Btu/h}}$$

17-31 The average surface temperature of the head of a person when it is not covered and is subjected to winds is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** One-quarter of the heat the person generates is lost from the head. **5** The head can be approximated as a 30-cm-diameter sphere. **6** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 10°C are (Table A-18)

$$k = 0.0248 \text{ W/m.°C}$$
$$\upsilon = 1.42 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu_\infty = 1.77 \times 10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@300\text{ K}} = 1.89 \times 10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.716$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(35\times 1000/3600)\text{ m/s}](0.3\text{ m})}{1.42\times 10^{-5}\text{ m}^2/\text{s}} = 2.054\times 10^5$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\text{Re}^{0.5} + 0.06\,\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(2.054\times 10^5)^{0.5} + 0.06(2.054\times 10^4)^{2/3}\right](0.716)^{0.4}\left(\frac{1.77\times 10^{-5}}{1.85\times 10^{-5}}\right)^{1/4} = 339.6$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.0248\text{ W/m.°C}}{0.3\text{ m}}(339.6) = 28.1\text{ W/m}^2\text{.°C}$$

Then the surface temperature of the head is determined to be

$$A = \pi D^2 = \pi(0.3\text{ m})^2 = 0.282\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 10\text{ °C} + \frac{(84/4)\text{ W}}{(28.1\text{ W/m}^2\text{.°C})(0.282\text{ m}^2)} = \mathbf{12.7\text{ °C}}$$

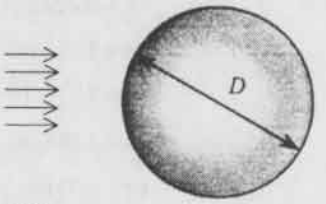
17-32 The flow of a fluid across an isothermal cylinder is considered. The change in the rate of heat transfer when the free-stream velocity of the fluid is doubled is to be determined. √

Analysis The rate of heat transfer between the fluid and the cylinder is given by Newton's law of cooling. We assume the Nusselt number is proportional to the *n*th power of the Reynolds number with $0.33 < n < 0.805$ (Table 17-3). Then,

$$\dot{Q}_1 = hA(T_s - T_\infty) = \left(\frac{k}{D}Nu\right)A(T_s - T_\infty) = \frac{k}{D}(\mathrm{Re})^n A(T_s - T_\infty)$$

$$= \frac{k}{D}\left(\frac{\mathbf{V}_\infty D}{\upsilon}\right)^n A(T_s - T_\infty)$$

$$= \mathbf{V}_\infty{}^n \frac{k}{D}\left(\frac{D}{\upsilon}\right)^n A(T_s - T_\infty)$$



When the free-stream velocity of the fluid is doubled, the heat transfer rate becomes

$$\dot{Q}_2 = (2\mathbf{V}_\infty)^n \frac{k}{D}\left(\frac{D}{\upsilon}\right)^n A(T_s - T_\infty)$$

Taking the ratio of them yields

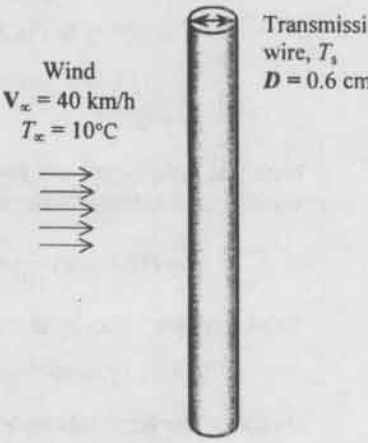
$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2\mathbf{V}_\infty)^n}{\mathbf{V}_\infty{}^n} = \mathbf{2^n}$$

17-33 The wind is blowing across the wire of a transmission line. The surface temperature of the wire is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties We assume the film temperature to be 300 K. The properties of air at this temperature are (Table A-18)

$$\rho = 1.177 \text{ kg/m}^3$$
$$k = 0.0261 \text{ W/m.°C}$$
$$\upsilon = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.712$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(40 \times 1000/3600) \text{ m/s}](0.006 \text{ m})}{1.57 \times 10^{-5} \text{ m}^2/\text{s}} = 4246$$

The Nusselt number corresponding this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62 \text{Re}^{0.5} \text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(4246)^{0.5}(0.712)^{1/3}}{\left[1+(0.4/0.712)^{2/3}\right]^{1/4}} \left[1+\left(\frac{4246}{282{,}000}\right)^{5/8}\right]^{4/5} = 33.8$$

The heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.0261 \text{ W/m.°C}}{0.006 \text{ m}} (33.8) = 147 \text{ W/m}^2\text{.°C}$$

The rate of heat generated in the electrical transmission lines per meter length is

$$\dot{W} = \dot{Q} = I^2 R = (50 \text{ A})^2 (0.002 \text{ Ohm}) = 5.0 \text{ W}$$

The entire heat generated in electrical transmission line has to be transferred to the ambient air. The surface temperature of the wire then becomes

$$A = \pi DL = \pi(0.006 \text{ m})(1 \text{ m}) = 0.01885 \text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 10°\text{C} + \frac{5 \text{ W}}{(147 \text{ W/m}^2\text{.°C})(0.01885 \text{ m}^2)} = \mathbf{11.8°C}$$

17-34 An aircraft is cruising at 900 km/h. A heating system keeps the wings above freezing temperatures. The average convection heat transfer coefficient on the wing surface and the average rate of heat transfer per unit surface area are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The wing is approximated as a cylinder of elliptical cross section whose minor axis is 30 cm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (0\text{-}55.4)/2 = -27.7°C = 245.3$ K are (Table A-18)

$$k = 0.0223 \text{ W/m.°C}$$
$$\upsilon = 1.14 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.724$$



Note that the atmospheric pressure will only affect the kinematic viscosity. The atmospheric pressure in atm unit is

$$P = (18.8 \text{ kPa})\frac{1 \text{ atm}}{101.325 \text{ kPa}} = 0.1855 \text{ atm}$$

The kinematic viscosity at this atmospheric pressure is

$$\upsilon = (0.177 \times 10^{-5} \text{ m}^2/\text{s})/0.1855 = 6.14 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(900 \times 1000/3600) \text{ m/s}](0.3 \text{ m})}{6.14 \times 10^{-5} \text{ m}^2/\text{s}} = 1.277 \times 10^6$$

The Nusselt number relation for a cylinder of elliptical cross-section is limited to Re < 15,000, and the relation below is not really applicable in this case. However, this relation is all we have for elliptical shapes, and we will use it with the understanding that the results may not be accurate.

$$Nu = \frac{hD}{k} = 0.248 \text{Re}^{0.612} \text{Pr}^{1/3} = 0.248(1.277 \times 10^6)^{0.612}(0.724)^{1/3} = 1215$$

The average heat transfer coefficient on the wing surface is

$$h = \frac{k}{D} Nu = \frac{0.0223 \text{ W/m.°C}}{0.3 \text{ m}}(1215) = \mathbf{88.7 \text{ W/m}^2\text{.°C}}$$

Then the average rate of heat transfer per unit surface area becomes

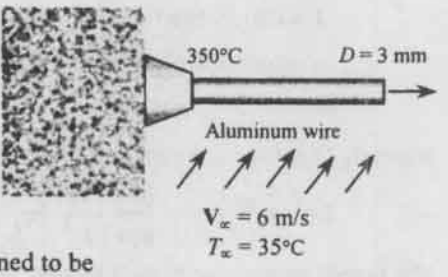
$$\dot{q} = h(T_s - T_\infty) = (88.7 \text{ W/m}^2\text{.°C})[0\text{-}(\text{-}55.4)]°C = \mathbf{4914 \text{ W/m}^2}$$

17-35 A long aluminum wire is cooled by cross air flowing over it. The rate of heat transfer from the wire per meter length when it is first exposed to the air is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (350+35)/2 = 192.5°C = 465.5$ K are (Table A-18)

$$k = 0.0373 \text{ W/m.°C}$$
$$\upsilon = 3.37 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.700$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{(6 \text{ m/s})(0.003 \text{ m})}{3.37 \times 10^{-5} \text{ m}^2/\text{s}} = 534$$

The Nusselt number corresponding this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62 \text{ Re}^{0.5} \text{ Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(534)^{0.5}(0.700)^{1/3}}{\left[1+(0.4/0.700)^{2/3}\right]^{1/4}} \left[1+\left(\frac{534}{282{,}000}\right)^{5/8}\right]^{4/5} = 11.6$$

Then the heat transfer coefficient and the heat transfer rate from the wire per meter length become

$$h = \frac{k}{D} Nu = \frac{0.0373 \text{ W/m.°C}}{0.003 \text{ m}} (11.6) = 145 \text{ W/m}^2\text{.°C}$$

$$A = \pi DL = \pi(0.003 \text{ m})(1 \text{ m}) = 0.00942 \text{ m}^2$$

$$\dot{Q}_{conv} = hA(T_s - T_\infty) = (145 \text{ W/m}^2\text{.°C})(0.00942 \text{ m}^2)(350-35)°C = \mathbf{430 \text{ W}}$$

17-36E A fan is blowing air over the entire body of a person. The average temperature of the outer surface of the person is to be determined for two cases. √

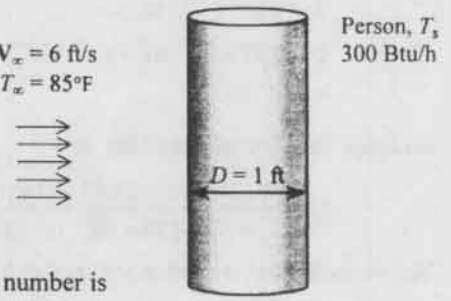
Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The average human body can be treated as a 1-ft-diamter cylinder with an exposed surface area of 18 ft^2. **5** The local atmospheric pressure is 1 atm.

Properties We assume the film temperature to be 100 °F. The properties of air at this temperature are (Table A-18)

$k = 0.0154$ Btu / h.ft.°F

$\upsilon = 0.180 \times 10^{-3}$ ft^2 / s

$\Pr = 0.72$



Analysis The Reynolds number is

$$\mathrm{Re} = \frac{V_\infty D}{\upsilon} = \frac{(6\ \text{ft/s})(1\ \text{ft})}{0.180\times10^{-3}\ \text{ft}^2/\text{s}} = 3.333\times10^4$$

The proper relation for Nusselt number corresponding this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{0.5}\,\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(3.333\times10^4)^{0.5}(0.72)^{1/3}}{\left[1+(0.4/0.72)^{2/3}\right]^{1/4}}\left[1+\left(\frac{3.333\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 108$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.0154\ \text{Btu/h.ft.°F}}{(1\ \text{ft})}(108) = 1.66\ \text{Btu/h.ft}^2\text{.°F}$$

Then the average temperature of the outer surface of the person becomes

$$\dot{Q} = hA(T_s - T_\infty) \rightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 85°\text{F} + \frac{300\ \text{Btu/h}}{(1.66\ \text{Btu/h.ft}^2\text{.°F})(18\ \text{ft}^2)} = \mathbf{95.0°F}$$

If the air velocity were doubled, the Reynolds number would be

$$\mathrm{Re} = \frac{V_\infty D}{\upsilon} = \frac{(12\ \text{ft/s})(1\ \text{ft})}{0.180\times10^{-3}\ \text{ft}^2/\text{s}} = 6.667\times10^4$$

The proper relation for Nusselt number corresponding this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{0.5}\,\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(6.667\times10^4)^{0.5}(0.72)^{1/3}}{\left[1+(0.4/0.72)^{2/3}\right]^{1/4}}\left[1+\left(\frac{6.667\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 166$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.0154\ \text{Btu/h.ft.°F}}{(1\ \text{ft})}(166) = 2.56\ \text{Btu/h.ft}^2\text{.°F}$$

Then the average temperature of the outer surface of the person becomes

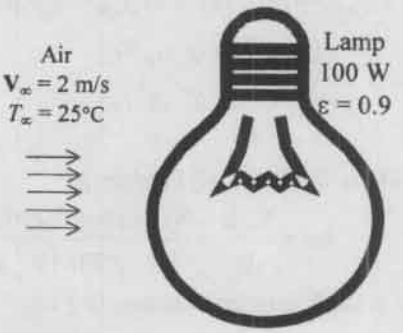
$$\dot{Q} = hA(T_s - T_\infty) \rightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 85°\text{F} + \frac{300\ \text{Btu/h}}{(2.56\ \text{Btu/h.ft}^2\text{.°F})(18\ \text{ft}^2)} = \mathbf{91.5°F}$$

17-37 A light bulb is cooled by a fan. The equilibrium temperature of the glass bulb is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The light bulb is in spherical shape. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 25°C are (Table A-18)

$$k = 0.0261 \text{ W/m.°C}$$
$$\upsilon = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu_\infty = 1.85 \times 10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@350\text{ K}} = 2.08 \times 10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.712$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{(2 \text{ m/s})(0.1 \text{ m})}{1.57 \times 10^{-5} \text{ m}^2/\text{s}} = 1.274 \times 10^4$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 2 + \left[0.4 \text{Re}^{0.5} + 0.06 \text{Re}^{2/3}\right] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(1.274 \times 10^4)^{0.5} + 0.06(1.274 \times 10^4)^{2/3}\right](0.712)^{0.4} \left(\frac{1.85 \times 10^{-5}}{2.08 \times 10^{-5}}\right)^{1/4} = 68$$

The heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.0261 \text{ W/m.°C}}{0.1 \text{ m}} (68) = 17.8 \text{ W/m}^2\text{.°C}$$

Noting that 90 % of electrical energy is converted to heat,

$$\dot{Q} = (0.90)(100 \text{ W}) = 90 \text{ W}$$

The bulb loses heat by both convection and radiation. The equilibrium temperature of the glass bulb can be determined by iteration,

$$A = \pi D^2 = \pi (0.1 \text{ m})^2 = 0.0314 \text{ m}^2$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA(T_s - T_\infty) + \varepsilon A \sigma (T_s^4 - T_{surr}^4)$$

$$90 \text{ W} = (17.8 \text{ W/m}^2\text{.°C})(0.0314 \text{ m}^2)\left[T_s - (25+273)\text{K}\right]$$
$$+(0.9)(0.0314 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2\text{.K}^4)\left[T_s^4 - (25+273 \text{ K})^4\right]$$

$$T_s = 405 \text{ K} = \mathbf{132°C}$$

17-38 A steam pipe is exposed to a light winds in the atmosphere. The amount of heat loss from the steam during a certain period and the money the facility will save a year as a result of insulating the steam pipe are to be determined.

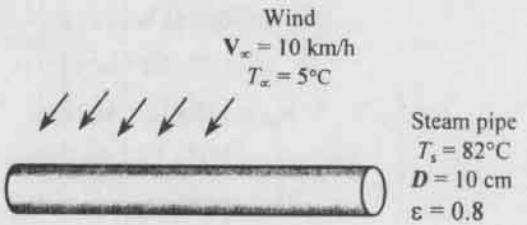
Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The plant operates every day of the year for 10 h a day. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (82+5)/2 = 43.5°C = 316.5$ K are (Table A-18)

$$k = 0.0273 \text{ W/m.°C}$$

$$\upsilon = 1.73 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.710$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(10 \times 1000/3600) \text{ m/s}](0.1 \text{ m})}{1.73 \times 10^{-5} \text{ m}^2/\text{s}} = 1.606 \times 10^4$$

The Nusselt number corresponding this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62 \text{ Re}^{0.5} \text{ Pr}^{1/3}}{\left[1 + (0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(1.606 \times 10^4)^{0.5}(0.710)^{1/3}}{\left[1 + (0.4/0.71)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{1.606 \times 10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 69.9$$

The heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.0273 \text{ W/m.°C}}{0.1 \text{ m}} (69.9) = 19.1 \text{ W/m}^2\text{.°C}$$

The rate of heat loss by convection is

$$A = \pi DL = \pi(0.1 \text{ m})(12 \text{ m}) = 3.77 \text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) = (19.1 \text{ W/m}^2\text{.°C})(3.77 \text{ m}^2)(82 - 5)°C = 5541 \text{ W}$$

The rate of heat loss by radiation is

$$\dot{Q}_{rad} = \varepsilon A \sigma (T_s^4 - T_{surr}^4)$$

$$= (0.8)(3.77 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2\text{.K}^4)\left[(82 + 273 \text{ K})^4 - (0 + 273 \text{ K})^4\right] = 1766 \text{ W}$$

The total rate of heat loss then becomes

$$\dot{Q}_{\text{total}} = \dot{Q}_{conv} + \dot{Q}_{rad} = 5541 + 1766 = 7307 \text{ W}$$

The amount of heat loss from the steam during a 10 hour work day is

$$Q = \dot{Q}_{total} \Delta t = (7.307 \text{ kJ/s})(10 \text{ h/day} \times 3600 \text{ s/h}) = \mathbf{2.63 \times 10^5 \text{ kJ/day}}$$

The total amount of heat loss from the steam per year is

$$Q_{total} = \dot{Q}_{day} (\text{no. of days}) = (2.63 \times 10^5 \text{ kJ/day})(365 \text{ days/yr}) = 9.60 \times 10^7 \text{ kJ/yr}$$

Noting that the steam generator has an efficiency of 80 %, the amount of gas used is

$$Q_{gas} = \frac{Q_{total}}{0.80} = \frac{9.60 \times 10^7 \text{ kJ/yr}}{0.80} \left(\frac{1 \text{ therm}}{105{,}500 \text{ kJ}}\right) = 1138 \text{ therms/yr}$$

Insulation reduces this amount by 90 %. The amount of energy and money saved becomes

Energy saved = $(0.90)Q_{gas}$ = (0.90)(1138 therms/yr) = 1024 therms/yr

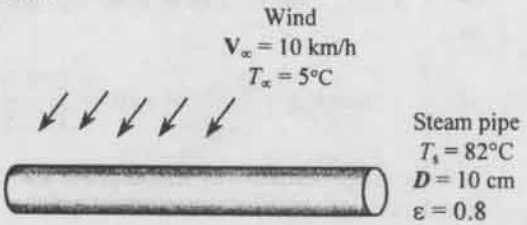
Money saved = (Energy saved)(Unit cost of energy) = (1024 therms/yr)($0.54/therm) = **$553**

17-39 A steam pipe is exposed to light winds in the atmosphere. The amount of heat loss from the steam during a certain period and the money the facility will save a year as a result of insulating the steam pipes are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The plant operates every day of the year for 10 h. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (82+5)/2 = 43.5°C = 316.5$ K are (Table A-18)

$$k = 0.0273 \text{ W/m.°C}$$
$$\upsilon = 1.73\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(10\times1000/3600)\text{ m/s}](0.1\text{ m})}{1.73\times10^{-5}\text{ m}^2/\text{s}} = 1.606\times10^4$$

The Nusselt number corresponding this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(1.606\times10^4)^{0.5}(0.710)^{1/3}}{\left[1+(0.4/0.71)^{2/3}\right]^{1/4}}\left[1+\left(\frac{1.606\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 69.9$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.0273\text{ W/m.°C}}{0.1\text{ m}}(69.9) = 19.1\text{ W/m}^2\text{.°C}$$

The rate of heat loss by convection is

$$A = \pi DL = \pi(0.1\text{ m})(12\text{ m}) = 3.77\text{ m}^2$$
$$\dot{Q} = hA(T_s - T_\infty) = (19.1\text{ W/m}^2\text{.°C})(3.77\text{ m}^2)(82-5)°C = 5541\text{ W}$$

The rate of heat loss by radiation is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.8)(3.77\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)\left[(82+273\text{ K})^4 - (0+273\text{ K})^4\right] = 1766\text{ W}$$

The total rate of heat loss then becomes

$$\dot{Q}_{\text{total}} = \dot{Q}_{conv} + \dot{Q}_{rad} = 5541 + 1766 = 7307\text{ W}$$

For an average surrounding temperature of 0 °C, the rate of heat loss by radiation and the total rate of heat loss are

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.8)(3.77\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)\left[(82+273\text{ K})^4 - (0+273\text{ K})^4\right]$$
$$= 1766\text{ W}$$
$$\dot{Q}_{\text{total}} = \dot{Q}_{conv} + \dot{Q}_{rad} = 7307 + 1766 = 9073\text{ W}$$

If the average surrounding temperature is −20 °C, the rate of heat loss by radiation and the total rate of heat loss become

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.8)(3.77\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)\left[(82+273\ \text{K})^4 - (-20+273\ \text{K})^4\right]$$
$$= 2015\ \text{W}$$
$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 7307 + 2015 = 9322\ \text{W}$$

which is 9322 - 9073 = 249 W more than the value for a surrounding temperature of 0°C. This corresponds to

$$\%\text{change} = \frac{\dot{Q}_{\text{difference}}}{\dot{Q}_{\text{total},0°\text{C}}}\times100 = \frac{249\ \text{W}}{9073\ \text{W}}\times100 = \mathbf{2.7}\ \ \text{(increase)}$$

If the average surrounding temperature is 25°C, the rate of heat loss by radiation and the total rate of heat loss become

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.8)(3.77\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)\left[(82+273\ \text{K})^{4^4} - (25+273\ \text{K})^4\right]$$
$$= 1367\ \text{W}$$
$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 7307 + 1367 = 8674\ \text{W}$$

which is 9073 - 8674 = 399 W less than the value for a surrounding temperature of 0°C. This corresponds to

$$\%\text{change} = \frac{\dot{Q}_{\text{difference}}}{\dot{Q}_{\text{total},0°\text{C}}}\times100 = \frac{399\ \text{W}}{9073\ \text{W}}\times100 = \mathbf{4.4}\ \ \text{(decrease)}$$

Therefore, the effect of the temperature variations of the surrounding surfaces on the total heat transfer is less than 5%.

17-40E An electrical resistance wire is cooled by a fan. The surface temperature of the wire is to be determined. √

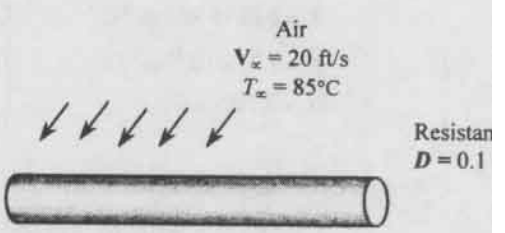
Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties We assume the film temperature to be 100 °F. The properties of air at this temperature are (Table A-18)

$$k = 0.0154 \text{ Btu/h.ft.°F}$$

$$\upsilon = 0.180 \times 10^{-3} \text{ ft}^2/\text{s}$$

$$\text{Pr} = 0.72$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{(20 \text{ ft/s})(0.1/12 \text{ ft})}{0.180 \times 10^{-3} \text{ ft}^2/\text{s}} = 926$$

The proper relation for Nusselt number corresponding this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62 \text{Re}^{0.5} \text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(926)^{0.5}(0.72)^{1/3}}{\left[1+(0.4/0.72)^{2/3}\right]^{1/4}} \left[1+\left(\frac{926}{282{,}000}\right)^{5/8}\right]^{4/5} = 15.5$$

The heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.0154 \text{ Btu/h.ft.°F}}{(0.1/12 \text{ ft})}(15.5) = 28.6 \text{ Btu/h.ft}^2\text{.°F}$$

Then the average temperature of the outer surface of the person becomes

$$A = \pi DL = \pi(0.1/12 \text{ ft})(12 \text{ ft}) = 0.314 \text{ ft}^2$$

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 85°\text{F} + \frac{(1500 \times 3.41214) \text{ Btu/h}}{(28.6 \text{ Btu/h.ft}^2\text{.°F})(0.314 \text{ ft}^2)} = \mathbf{654°F}$$

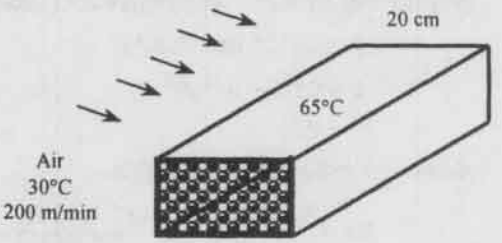
The result can be improved by repeating the calculations using air properties evaluated at the new film temperature of (85 + 654)/2 = 370°F.

17-41 The components of an electronic system located in a horizontal duct is cooled by air flowing over the duct. The total power rating of the electronic device is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (65+30)/2 = 47.5°C = 320.5$ K are (Table A-18)

$$k = 0.0275 \text{ W/m.°C}$$
$$\upsilon = 1.77 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(200/60) \text{ m/s}](0.2 \text{ m})}{1.77 \times 10^{-5} \text{ m}^2/\text{s}} = 3.767 \times 10^4$$

Using the relation for a square duct from Table 17-3, the Nusselt number is determined to be

$$Nu = \frac{hD}{k} = 0.102 \, \text{Re}^{0.675} \, \text{Pr}^{1/3} = 0.102(3.767 \times 10^4)^{0.675}(0.710)^{1/3} = 111.6$$

The heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.0275 \text{ W/m.°C}}{0.2 \text{ m}}(111.6) = 15.3 \text{ W/m}^2\text{.°C}$$

Then the rate of heat transfer from the duct becomes

$$A = (4 \times 0.2 \text{ m})(1.5 \text{ m}) = 1.2 \text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) = (15.3 \text{ W/m}^2\text{.°C})(1.2 \text{ m}^2)(65 - 30)°C = \mathbf{643 \text{ W}}$$

17-42 The components of an electronic system located in a horizontal duct is cooled by air flowing over the duct. The total power rating of the electronic device is to be determined. √

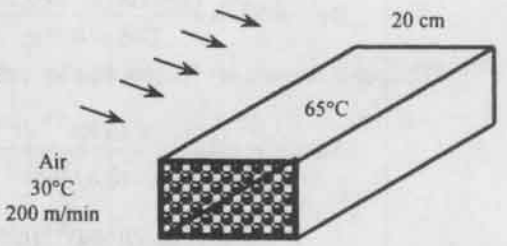
Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (65+30)/2 = 47.5°C = 320.5$ K are (Table A-18)

$$k = 0.0275 \text{ W/m.°C}$$
$$\upsilon = 1.77 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$

For a location at 4000 m altitude where the atmospheric pressure is 61.66 kPa, only kinematic viscosity of air will be affected. Thus,

$$\upsilon_{@\,61.66\,\text{kPa}} = \left(\frac{101.325}{61.66}\right)(1.77\times10^{-5}) = 2.916\times10^{-5} \text{ m}^2/\text{s}$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(200/60)\text{ m/s}](0.2\text{ m})}{2.916\times10^{-5} \text{ m}^2/\text{s}} = 2.286\times10^4$$

Using the relation for a square duct from Table 17-3, the Nusselt number is determined to be

$$Nu = \frac{hD}{k} = 0.102\,\text{Re}^{0.675}\,\text{Pr}^{1/3} = 0.102(2.286)^{0.675}(0.710)^{1/3} = 79.7$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.0275 \text{ W/m.°C}}{0.2 \text{ m}}(79.7) = 11.0 \text{ W/m}^2\text{.°C}$$

Then the rate of heat transfer from the duct becomes

$$A = (4 \times 0.2 \text{ m})(1.5 \text{ m}) = 1.2 \text{ m}^2$$
$$\dot{Q} = hA(T_s - T_\infty) = (11.0 \text{ W/m}^2\text{.°C})(1.2 \text{ m}^2)(65-30)°C = \mathbf{461\ W}$$

17-43 A cylindrical electronic component mounted on a circuit board is cooled by air flowing across it. The surface temperature of the component is to be determined. √

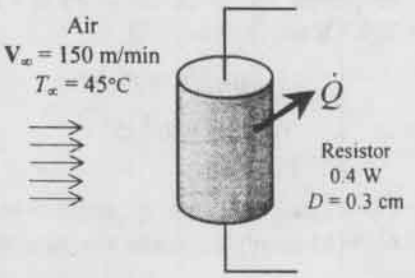
Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties We assume the film temperature to be 350 K. The properties of air at 1 atm and at this temperature are (Table A-18)

$$k = 0.0297 \text{ W/m.°C}$$

$$\upsilon = 2.06 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.706$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{(150/60 \text{ m/s})(0.003 \text{ m})}{2.06 \times 10^{-5} \text{ m}^2/\text{s}} = 364$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62 \text{Re}^{0.5} \text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(364)^{0.5}(0.706)^{1/3}}{\left[1+(0.4/0.706)^{2/3}\right]^{1/4}} \left[1+\left(\frac{364}{282{,}000}\right)^{5/8}\right]^{4/5} = 9.66$$

The heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.0297 \text{ W/m.°C}}{0.003 \text{ m}} (9.66) = 95.6 \text{ W/m}^2\text{.°C}$$

Then the surface temperature of the component becomes

$$A = \pi DL = \pi(0.003 \text{ m})(0.018 \text{ m}) = 0.0001696 \text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 45\,°\text{C} + \frac{0.4 \text{ W}}{(95.6 \text{ W/m}^2\text{.°C})(0.0001696 \text{ m}^2)} = \mathbf{69.7°C}$$

Flow In Tubes

17-44C In the fully developed region of flow in a circular tube, the velocity profile will not change in the flow direction but the temperature profile may.

17-45C The hydrodynamic and thermal entry lengths are given as $L_h = 0.05\,\mathrm{Re}\,D$ and $L_t = 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D$ for laminar flow, and $L_h \approx L_t \approx 4.4D\,\mathrm{Re}^{1/6}$ in turbulent flow. Noting that $\mathrm{Pr} >> 1$ for oils, the thermal entry length is larger than the hydrodynamic entry length in laminar flow. In turbulent, the hydrodynamic and thermal entry lengths are independent of Re or Pr numbers, and are comparable in magnitude.

17-46C The hydrodynamic and thermal entry lengths are given as $L_h = 0.05\,\mathrm{Re}\,D$ and $L_t = 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D$ for laminar flow, and $L_h \approx L_t \approx 4.4D\,\mathrm{Re}^{1/6}$ in turbulent flow. Noting that $\mathrm{Pr} << 1$ for liquid metals, the thermal entry length is smaller than the hydrodynamic entry length in laminar flow. In turbulent, the hydrodynamic and thermal entry lengths are independent of Re or Pr numbers, and are comparable in magnitude.

17-47C In fluid flow, it is convenient to work with an average or mean velocity V_m and an average or mean temperature T_m which remain constant in incompressible flow when the cross-sectional area of the tube is constant. The V_m and T_m represent the velocity and temperature, respectively, at a cross section if all the particles were at the same velocity and temperature.

17-48C When the surface temperature of tube is constant, the appropriate temperature difference for use in the Newton's law of cooling is logarithmic mean temperature difference that can be expressed as

$$\Delta T_{\ln} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

17-49C The number of transfer units NTU is a measure of the heat transfer area and effectiveness of a heat transfer system. A small value of NTU (NTU < 5) indicates more opportunities for heat transfer whereas a large NTU value (NTU >5) indicates that heat transfer will not increase no matter how much we extend the length of the tube.

17-50C The logarithmic mean temperature difference $\Delta T_{\ln}$ is an exact representation of the average temperature difference between the fluid and the surface for the entire tube. It truly reflects the exponential decay of the local temperature difference. The error in using the arithmetic mean temperature increases to undesirable levels when ΔT_e differs from ΔT_i by great amounts. Therefore we should always use the logarithmic mean temperature.

17-51C The region of flow over which the thermal boundary layer develops and reaches the tube center is called the thermal entry region, and the length of this region is called the thermal entry length. The region in which the flow is both hydrodynamically (the velocity profile is fully developed and remains unchanged) and thermally (the dimensionless temperature profile remains unchanged) developed is called the fully developed region.

17-52C The heat flux will be higher near the inlet because the heat transfer coefficient is highest at the tube inlet where the thickness of thermal boundary layer is zero, and decreases gradually to the fully developed value.

17-53C The heat flux will be higher near the inlet because the heat transfer coefficient is highest at the tube inlet where the thickness of thermal boundary layer is zero, and decreases gradually to the fully developed value.

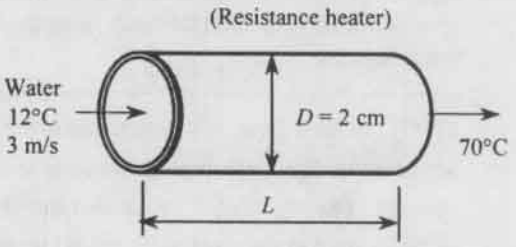
17-54C The tubes with rough surfaces have much higher heat transfer coefficients than the tubes with smooth surfaces. In the case of laminar flow, the effect of surface roughness on the heat transfer coefficient is negligible.

17-55 Water is to be heated in a tube equipped with an electric resistance heater on its surface. The power rating of the heater and the inner surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the average temperature of $(70+12)/2 = 41°C \approx 40°C$ are (Table A-15)

$$\rho = 992.1\ \text{kg/m}^3$$
$$k = 0.631\ \text{W/m.°C}$$
$$\upsilon = \mu/\rho = 0.658\times10^{-6}\ \text{m}^2/\text{s}$$
$$C_p = 4179\ \text{J/kg.°C}$$
$$\text{Pr} = 4.32$$



Analysis The power rating of the resistance heater is

$$\dot{m} = \rho\dot{V} = (992.1\ \text{kg/m}^3)(0.008\ \text{m}^3/\text{min}) = 7.937\ \text{kg/min} = 0.132\ \text{kg/s}$$

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (0.132\ \text{kg/s})(4179\ \text{J/kg.°C})(70-12)°C$$
$$= \mathbf{32{,}000\ W}$$

The velocity of water and the Reynolds number are

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{(8\times10^{-3}/60)\ \text{m}^3/\text{s}}{\pi(0.02\ \text{m})^2/4} = 0.4244\ \text{m/s}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(0.4244\ \text{m/s})(0.02\ \text{m})}{0.658\times10^{-6}\ \text{m}^2/\text{s}} = 12{,}890$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4D\,\text{Re}^{1/6} = 4.4(0.02\ \text{m})(12{,}890)^{1/6} = 0.43\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(12{,}890)^{0.8}(4.32)^{0.4} = 80.2$$

Heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.631\ \text{W/m.°C}}{0.02\ \text{m}}(80.2) = 2530\ \text{W/m}^2\text{.°C}$$

Then the inner surface temperature of the pipe at the exit becomes

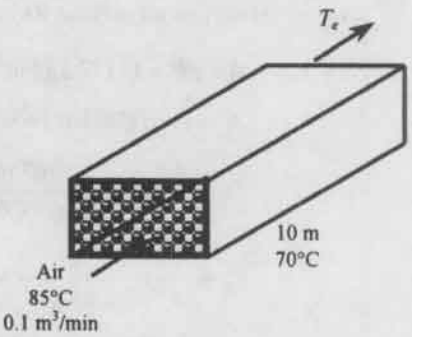
$$\dot{Q} = hA(T_{s,e} - T_e)$$
$$32{,}000\ \text{W} = (2530\ \text{W/m}^2\text{.°C})[\pi(0.02\ \text{m})(7\ \text{m})](T_s - 70)°C$$
$$T_{s,e} = \mathbf{98.8°C}$$

17-56 Flow of hot air through uninsulated square ducts of a heating system in the attic is considered. The exit temperature and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 350 K since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at 1 atm and this temperature are (Table A-18)

$$\rho = 1.009 \text{ kg/m}^3$$
$$k = 0.0297 \text{ W/m.°C}$$
$$\upsilon = 2.06 \times 10^{-5} \text{ m}^2/\text{s}$$
$$C_p = 1008 \text{ J/kg.°C}$$
$$\text{Pr} = 0.706$$



Analysis The characteristic length that is the hydraulic diameter, the mean velocity of air, and the Reynolds number are

$$D_h = \frac{4A_c}{P} = \frac{4a^2}{4a} = a = 0.15 \text{ m}$$

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{0.10 \text{ m}^3/\text{s}}{(0.15 \text{ m})^2} = 4.44 \text{ m/s}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(4.44 \text{ m/s})(0.15 \text{ m})}{2.06 \times 10^{-5} \text{ m}^2/\text{s}} = 32{,}362$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4 D \text{Re}^{1/6} = 4.4(0.15 \text{ m})(32{,}363)^{1/6} = 3.7 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.3} = 0.023(32{,}362)^{0.8}(0.706)^{0.3} = 84.0$$

Heat transfer coefficient is

$$h = \frac{k}{D_h} Nu = \frac{0.0297 \text{ W/m.°C}}{0.15 \text{ m}}(84.0) = 16.6 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of air,

$$A = 4aL = 4(0.15 \text{ m})(10 \text{ m}) = 6 \text{ m}^2$$
$$\dot{m} = \rho \dot{V} = (1.009 \text{ kg/m}^3)(0.10 \text{ m}^3/\text{s}) = 0.1009 \text{ kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}C_p)} = 70 - (70 - 85)e^{-\frac{(16.6)(6)}{(0.1009)(1008)}} = \mathbf{75.6°C}$$

Then the logarithmic mean temperature difference and the rate of heat loss from the air becomes

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{75.6 - 85}{\ln\left(\frac{70 - 75.6}{70 - 85}\right)} = 9.5°\text{C}$$

$$\dot{Q} = hA\Delta T_{\ln} = (16.6 \text{ W/m}^2\text{.°C})(6 \text{ m}^2)(9.5°\text{C}) = \mathbf{946 \text{ W}}$$

Note that the temperature of air drops by almost 10°C as it flows in the duct as a result of heat loss.

17-57 Air enters the constant spacing between the glass cover and the plate of a solar collector. The net rate of heat transfer and the temperature rise of air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the spacing are smooth. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

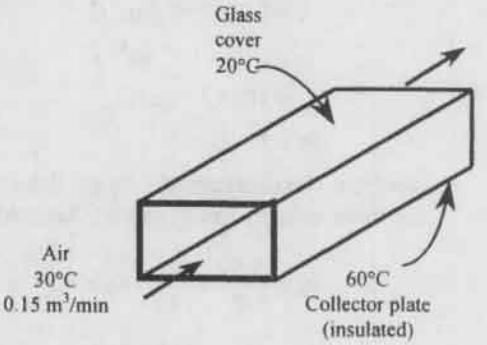
Properties The properties of air at 1 atm and 300 K are (Table A-18)

$\rho = 1.177\ \text{kg/m}^3$ $\quad C_p = 1005\ \text{J/kg.°C}$

$k = 0.0261\ \text{W/m.°C}$ $\quad \text{Pr} = 0.712$

$\upsilon = 1.57\times10^{-5}\ \text{m}^2/\text{s}$

Analysis Mass flow rate, cross sectional area, hydraulic diameter, mean velocity of air and the Reynolds number are



$$\dot{m} = \rho\dot{V} = (1.177\ \text{kg/m}^3)(0.15\ \text{m}^3/\text{s}) = 0.177\ \text{kg/s}$$

$$A_c = (1\ \text{m})(0.03\ \text{m}) = 0.03\ \text{m}^2$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.03\ \text{m}^2)}{2(1\ \text{m}+0.03\ \text{m})} = 0.05825\ \text{m}$$

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.15\ \text{m}^3/\text{s}}{0.03\ \text{m}^2} = 5\ \text{m/s}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(5\ \text{m/s})(0.05825\ \text{m})}{1.57\times10^{-5}\ \text{m}^2/\text{s}} = 18{,}550\ >\ 4000$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4D\,\text{Re}^{1/6} = 4.4(0.05825\,\text{m})(18{,}850)^{1/6} = 1.3\,\text{m}$$

which are much shorter than the total length of the collector. Therefore, we can assume fully developed turbulent flow in the entire collector, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(18{,}550)^{0.8}(0.712)^{0.3} = 54.0$$

and $$h = \frac{k}{D_h}Nu = \frac{0.0261\ \text{W/m.°C}}{0.05825\ \text{m}}(54.0) = 24.2\ \text{W/m}^2.\text{°C}$$

The exit temperature of air can be calculated using the "average" surface temperature as

$$A = 2(5\ \text{m})(1\ \text{m}) = 10\ \text{m}^2$$

$$T_{s,ave} = \frac{60+20}{2} = 40°\text{C}$$

$$T_e = T_{s,ave} - (T_{s,ave} - T_i)\exp\left(-\frac{hA}{\dot{m}C_p}\right) = 40 - (40-30)\exp\left(-\frac{24.2\times10}{0.177\times1005}\right) = 37.4°\text{C}$$

The temperature rise of air is

$$\Delta T = 37.4°\text{C} - 30°\text{C} = \mathbf{7.4°C}$$

The logarithmic mean temperature difference and the heat loss from the glass are

$$\Delta T_{\ln,glass} = \frac{T_e - T_i}{\ln\dfrac{T_s - T_e}{T_s - T_i}} = \frac{37.4-30}{\ln\dfrac{20-37.4}{20-30}} = 13.36°\text{C}$$

$$\dot{Q}_{glass} = hA\Delta T_{\ln} = (24.2\ \text{W/m}^2.\text{°C})(5\ \text{m}^2)(13.36°\text{C}) = 1617\ \text{W}$$

The logarithmic mean temperature difference and the heat gain of the absorber are

Prob. 17-57 (*continued*)

$$\Delta T_{\ln,absorber} = \frac{T_e - T_i}{\ln \dfrac{T_s - T_e}{T_s - T_i}} = \frac{37.4 - 30}{\ln \dfrac{60 - 37.4}{60 - 30}} = 26.1°\text{C}$$

$$\dot{Q}_{absorber} = hA\Delta T_{\ln} = (24.2\ \text{W/m}^2.°\text{C})(5\ \text{m}^2)(26.1°\text{C}) = 3158\ \text{W}$$

Then the net rate of heat transfer becomes

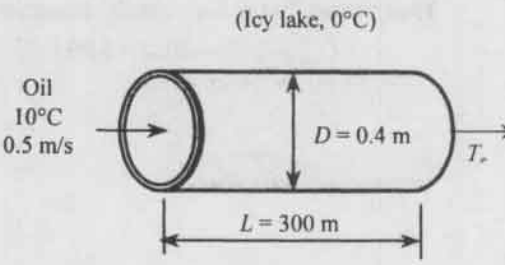
$$\dot{Q}_{net} = 3158 - 1617 = \mathbf{1541\ W}$$

17-58 Oil flows through a pipeline that passes through icy waters of a lake. The exit temperature of the oil and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is very nearly 0°C. **3** The thermal resistance of the pipe is negligible. **4** The inner surfaces of the pipeline are smooth. **5** The flow is hydrodynamically developed when the pipeline reaches the lake.

Properties The properties of oil at 10°C are (Table A-16)

$$\rho = 894\ \text{kg/m}^3, \qquad k = 0.146\ \text{W/m.°C}$$
$$\mu = 2.325\ \text{kg/m.s}, \qquad \upsilon = 2591\times10^{-6}\ \text{m}^2/\text{s}$$
$$C_p = 1838\ \text{J/kg.°C}, \qquad \text{Pr} = 28750$$



Analysis (a) The Reynolds number in this case is

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(0.5\ \text{m/s})(0.4\ \text{m})}{2591\times10^{-6}\ \text{m}^2/\text{s}} = 77.2$$

which is less than 2300. Therefore, the flow is laminar, and the thermal entry length is roughly

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D = 0.05(77.2)(28750)(0.4\ \text{m}) = 44{,}390\ \text{m}$$

which is much longer than the total length of the pipe. Therefore, we assume thermally developing flow, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 1.86\left(\frac{\text{Re}\,\text{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left[\frac{(77.2)(28{,}750)(0.4)}{300}\right]^{1/3}\left(\frac{2.325}{3.85}\right)^{0.14} = 24.9$$

and $$h = \frac{k}{D}Nu = \frac{0.146\ \text{W/m.°C}}{0.4\ \text{m}}(24.9) = 9.09\ \text{W/m}^2\text{.°C}$$

Next we determine the exit temperature of oil

$$A = \pi DL = \pi(0.4\ \text{m})(300\ \text{m}) = 377\ \text{m}^2$$

$$\dot{m} = \rho\dot{V} = \rho A_c \mathbf{V}_m = \rho\left(\frac{\pi D^2}{4}\right)\mathbf{V}_m = (894\ \text{kg/m}^3)\frac{\pi(0.4\ \text{m})^2}{4}(0.5\ \text{m/s}) = 56.2\ \text{kg/s}$$

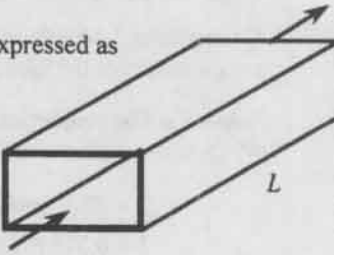
$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}C_p)} = 0-(0-10)e^{-\frac{(9.09)(377)}{(56.2)(1838)}} = \mathbf{9.67\ °C}$$

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the oil are

$$\Delta T_{\text{ln}} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{9.67-10}{\ln\left(\frac{0-9.67}{0-10}\right)} = 9.95°\text{C}$$

$$\dot{Q} = hA\Delta T_{\text{ln}} = (9.09\ \text{W/m}^2\text{.°C})(377\ \text{m}^2)(9.95°\text{C}) = 3.41\times10^4\ \text{W} = \mathbf{34.1\ kW}$$

17-59 Laminar flow of a fluid through an isothermal square channel is considered. The change in the rate of heat transfer is to be determined when the mean velocity is doubled.



Laminar flow
$\mathbf{V}_m$

Analysis The rate of heat transfer between the fluid and the walls of the channel is expressed as

$$\dot{Q}_1 = hA\Delta T_{\ln} = \frac{k}{D} NuA\Delta T_{\ln} = \frac{k}{D} 1.86 \left(\frac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0.4} A\Delta T_{\ln}$$

$$= \frac{\mathbf{V}_m^{\,1/3} D^{1/3}}{\upsilon^{1/3}} \frac{k}{D} 1.86 \left(\frac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0.4} A\Delta T_{\ln}$$

When the free-stream velocity of the fluid is doubled, the heat transfer rate becomes

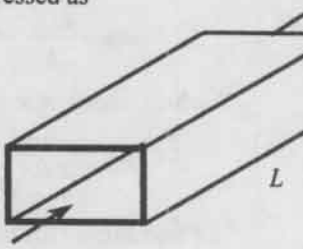
$$\dot{Q}_2 = \frac{(2\mathbf{V}_m)^{1/3} D^{1/3}}{\upsilon^{1/3}} \frac{k}{D} 1.86 \left(\frac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0.4} A\Delta T_{\ln}$$

Their ratio is

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2\mathbf{V}_m)^{1/3}}{\mathbf{V}_m^{\,1/3}} = 2^{1/3} = \mathbf{1.26}$$

Therefore, doubling the velocity will double the pressure drop but it will increase the heat transfer rate by only 26%.

17-60 Turbulent flow of a fluid through an isothermal square channel is considered. The change in the rate of heat transfer is to be determined when the mean velocity is doubled.



Turbulent flow
$\mathbf{V}_m$

Analysis The rate of heat transfer between the fluid and the walls of the channel is expressed as

$$\dot{Q}_1 = hA\Delta T_{\ln} = \frac{k}{D} NuA\Delta T_{\ln} = \frac{k}{D} 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3} A\Delta T_{\ln}$$

$$= 0.023\mathbf{V}_m^{\,0.8} \left(\frac{D}{\upsilon}\right)^{0.8} \frac{k}{D} \mathrm{Pr}^{1/3} A\Delta T_{\ln}$$

When the free-stream velocity of the fluid is doubled, the heat transfer rate becomes

$$\dot{Q}_2 = 0.023(2\mathbf{V}_m)^{0.8} \left(\frac{D}{\upsilon}\right)^{0.8} \frac{k}{D} \mathrm{Pr}^{1/3} A\Delta T_{\ln}$$

Their ratio is

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2\mathbf{V}_m)^{0.8}}{\mathbf{V}_m^{\,0.8}} = 2^{0.8} = \mathbf{1.74}$$

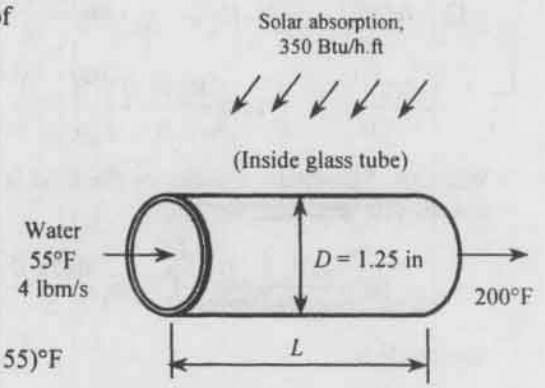
Therefore, doubling the velocity will increase the pressure drop 3.8 times but it will increase the heat transfer rate by only 74%.

17-61E Water is heated in a parabolic solar collector. The required length of parabolic collector and the surface temperature of the collector tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal resistance of the tube is negligible. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the average temperature of (55+200)/2 = 127.5°F ≈ 130°F are (Table A-15E)

$$\rho = 61.6 \text{ lbm/ft}^3$$
$$k = 0.375 \text{ Btu/ft.°F}$$
$$\upsilon = \mu/\rho = 0.555\times10^{-5} \text{ ft}^2/\text{s}$$
$$C_p = 1.00 \text{ Btu/lbm.°F}$$
$$\text{Pr} = 3.28$$



Analysis The total rate of heat transfer is

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (4 \text{ lbm/s})(1.00 \text{ Btu/lbm.°F})(200-55)\text{°F}$$
$$= 580 \text{ Btu/s} = 2.088\times10^6 \text{ Btu/h}$$

The length of the tube required is

$$L = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{2.088\times10^4 \text{ Btu/h}}{350 \text{ Btu/h.ft}} = \mathbf{5966 \text{ ft}}$$

The velocity of water and the Reynolds number are

$$\mathbf{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{4 \text{ lbm/s}}{(61.6 \text{ lbm/m}^3)\pi\dfrac{(1.25/12 \text{ ft})^2}{4}} = 7.62 \text{ ft/s}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(7.62 \text{ m/s})(1.25/12 \text{ ft})}{0.555\times10^{-5} \text{ ft}^2/\text{s}} = 1.43\times10^5$$

which is greater than 4000. Therefore, we can assume fully developed turbulent flow in the entire tube, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4} = 0.023(1.43\times10^4)^{0.8}(3.28)^{0.4} = 492$$

The heat transfer coefficient is

$$h = \frac{k}{D_h} Nu = \frac{0.375 \text{ Btu/h.ft.°F}}{1.25/12 \text{ ft}}(492) = 1770 \text{ Btu/h.ft}^2\text{.°F}$$

The heat flux on the tube is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{2.088\times10^4 \text{ Btu/h}}{\pi(1.25/12 \text{ ft})(5966 \text{ ft})} = 1069 \text{ Btu/h.ft}^2$$

Then the surface temperature of the tube at the exit becomes

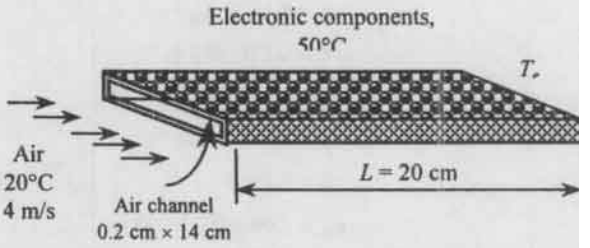
$$\dot{q} = h(T_s - T_e) \longrightarrow T_s = T_e + \frac{\dot{q}}{h} = 200\text{°F} + \frac{1069 \text{ Btu/h.ft}^2}{1770 \text{ Btu/h.ft}^2\text{.°F}} = \mathbf{200.6°F}$$

17-62 A circuit board is cooled by passing cool air through a channel drilled into the board. The maximum total power of the electronic components is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat flux at the top surface of the channel is uniform, and heat transfer through other surfaces is negligible. **3** The inner surfaces of the channel are smooth. **4** Air is an ideal gas with constant properties. **5** The pressure of air in the channel is 1 atm.

Properties The properties of air at 1 atm and 300 K are (Table A-18)

$$\rho = 1.177 \text{ kg/m}^3$$
$$k = 0.0261 \text{ W/m.°C}$$
$$\upsilon = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$$
$$C_p = 1005 \text{ J/kg.°C}$$
$$\text{Pr} = 0.712$$
$$\mu_b = 1.85 \times 10^{-5} \text{ kg/s}$$
$$\mu_{s,@50\,°C} = 1.96 \times 10^{-5} \text{ kg/s}$$



Analysis The cross-sectional and heat transfer surface areas are

$$A_c = (0.002 \text{ m})(0.14 \text{ m}) = 0.00028 \text{ m}^2$$
$$A = (0.14 \text{ m})(0.2 \text{ m}) = 0.028 \text{ m}^2$$

To determine heat transfer coefficient, we first need to find the Reynolds number,

$$D_h = \frac{4A_c}{P} = \frac{4(0.00028 \text{ m}^2)}{2(0.002 \text{ m} + 0.14 \text{ m})} = 0.00394 \text{ m}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(4 \text{ m/s})(0.00394 \text{ m})}{1.57 \times 10^{-5} \text{ m}^2/\text{s}} = 1004$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length is

$$L_t = 0.05 \,\text{Re}\,\text{Pr}\, D_h = 0.05(1004)(0.712)(0.00394 \text{ m}) = 0.14 \text{ m} \; < 0.20 \text{ m}$$

Therefore, we have developing flow through most of the channel. However, we take the conservative approach and assume fully developed flow, and from Table 17-3 we read Nu = 8.24. Then the heat transfer coefficient becomes

$$h = \frac{k}{D_h} Nu = \frac{0.0261 \text{ W/m.°C}}{0.00394 \text{ m}}(8.24) = 54.6 \text{ W/m}^2\text{.°C}$$

Also,

$$\dot{m} = \rho \mathbf{V} A_c = (1.177 \text{ kg/m}^3)(4 \text{ m/s})(0.00028 \text{ m}^2) = 0.001318 \text{ kg/s}$$

Heat flux at the exit can be written as $\dot{q} = h(T_s - T_e)$ where $T_s = 50°\text{C}$ at the exit. Then the heat transfer rate can be expressed as $\dot{Q} = \dot{q}A = hA_s(T_s - T_e)$, and the exit temperature of the air can be determined from

$$hA(T_s - T_e) = \dot{m}C_p(T_e - T_i)$$
$$(54.6 \text{ W/m}^2\text{.°C})(0.028 \text{ m}^2)(50°\text{C} - T_e) = (0.001318 \text{ kg/s})(1005 \text{ J/kg.°C})(T_e - 20 \text{ °C})$$
$$T_e = 36.1°\text{C}$$

Then the maximum total power of the electronic components that can safely be mounted on this circuit board becomes

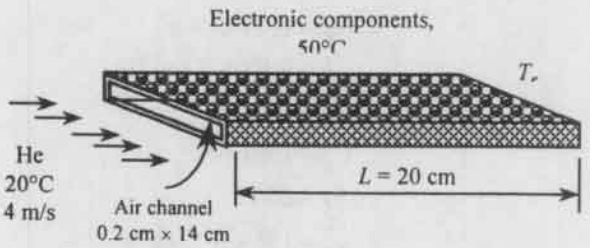
$$\dot{Q}_{\max} = \dot{m}C_p(T_e - T_i) = (0.001318 \text{ kg/s})(1005 \text{ J/kg.°C})(36.1 - 20°\text{C}) = \mathbf{21.3 \text{ W}}$$

17-63 A circuit board is cooled by passing cool helium gas through a channel drilled into the board. The maximum total power of the electronic components is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat flux at the top surface of the channel is uniform, and heat transfer through other surfaces is negligible. **3** The inner surfaces of the channel are smooth. **4** Helium is an ideal gas. **5** The pressure of helium in the channel is 1 atm.

Properties The properties of helium at 300 K are (Table A-18)

$$\rho = 0.1627 \text{ kg/m}^3$$
$$k = 0.150 \text{ W/m.°C}$$
$$\upsilon = 1.22 \times 10^{-4} \text{ m}^2/\text{s}$$
$$C_p = 5197 \text{ J/kg.°C}$$
$$\text{Pr} = 0.690$$
$$\mu_b = 1.99 \times 10^{-5} \text{ kg/s}$$
$$\mu_{s,@50\,°C} = 2.09 \times 10^{-5} \text{ kg/s}$$



Analysis The cross-sectional and heat transfer surface areas are

$$A_c = (0.002 \text{ m})(0.14 \text{ m}) = 0.00028 \text{ m}^2$$
$$A = (0.14 \text{ m})(0.2 \text{ m}) = 0.028 \text{ m}^2$$

To determine heat transfer coefficient, we need to first find the Reynolds number

$$D_h = \frac{4A_c}{P} = \frac{4(0.00028 \text{ m}^2)}{2(0.002 \text{ m} + 0.14 \text{ m})} = 0.00394 \text{ m}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(4 \text{ m/s})(0.00394 \text{ m})}{1.22 \times 10^{-4} \text{ m}^2/\text{s}} = 129.2$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length is

$$L_t = 0.05 \text{Re Pr } D_h = 0.05(129.2)(0.690)(0.00394 \text{ m}) = 0.0175 \text{ m} << 0.20 \text{ m}$$

Therefore, the flow is fully developed flow, and from Table 17-3 we read Nu = 8.24. Then the heat transfer coefficient becomes

$$h = \frac{k}{D_h} Nu = \frac{0.150 \text{ W/m.°C}}{0.00394 \text{ m}}(8.24) = 314 \text{ W/m}^2\text{.°C}$$

Also,

$$\dot{m} = \rho \mathbf{V} A_c = (0.1627 \text{ kg/m}^3)(4 \text{ m/s})(0.00028 \text{ m}^2) = 0.000182 \text{ kg/s}$$

Heat flux at the exit can be written as $\dot{q} = h(T_s - T_e)$ where $T_s = 50°\text{C}$ at the exit. Then the heat transfer rate can be expressed as $\dot{Q} = \dot{q}A = hA_s(T_s - T_e)$, and the exit temperature of the air can be determined from

$$\dot{m}C_p(T_e - T_i) = hA_s(T_s - T_e)$$
$$(0.000182 \text{ kg/s})(5197 \text{ J/kg.°C})(T_e - 20°\text{C}) = (314 \text{ W/m}^2\text{.°C})(0.0568 \text{ m}^2)(50°\text{C} - T_e)$$
$$T_e = 48.5°\text{C}$$

Then the maximum total power of the electronic components that can safely be mounted on this circuit board becomes

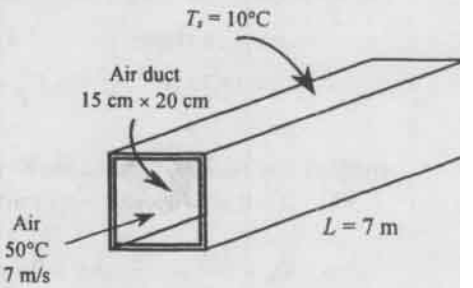
$$\dot{Q}_{\max} = \dot{m}C_p(T_e - T_i) = (0.000182 \text{ kg/s})(5197 \text{ J/kg.°C})(48.5 - 20°\text{C}) = \mathbf{27.0 \text{ W}}$$

17-64 Air enters a rectangular duct. The exit temperature of the air, the rate of heat transfer is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas with constant properties. **4** The pressure of air in the duct is 1 atm.

Properties We assume the bulk mean temperature for air to be 320 K since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at this temperature and 1 atm are

$$\rho = 1.010\ \text{kg/m}^3 \qquad C_p = 1006\ \text{J/kg.°C}$$
$$k = 0.0276\ \text{W/m.°C} \qquad \text{Pr} = 0.710$$
$$\upsilon = 1.77\times10^{-5}\ \text{m}^2/\text{s}$$



Analysis (*a*) The hydraulic diameter, the mean velocity of air, and the Reynolds number are

$$D_h = \frac{4A_c}{P} = \frac{4(0.15\ \text{m})(0.20\ \text{m})}{2[(0.15\ \text{m})+(0.20\ \text{m})]} = 0.1714\ \text{m}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(7\ \text{m/s})(0.1714\ \text{m})}{1.77\times10^{-5}\ \text{m}^2/\text{s}} = 6.78\times10^4$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4D\,\text{Re}^{1/6} = 4.4(0.1714\ \text{m})(6.78\times10^4)^{1/6} = 4.8\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(6.78\times10^4)^{0.8}(0.710)^{0.3} = 152$$

Heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.0276\ \text{W/m.°C}}{0.1714\ \text{m}}(152) = 24.5\ \text{W/m}^2\text{.°C}$$

Next we determine the exit temperature of air

$$A = 2\times7[(0.15\ \text{m})+(0.20\ \text{m})] = 4.9\ \text{m}^2$$
$$A_c = (0.15\ \text{m})(0.20\ \text{m}) = 0.03\ \text{m}^2$$
$$\dot{m} = \rho V A_c = (1.010\ \text{kg/m}^3)(7\ \text{m/s})(0.03\ \text{m}^2) = 0.2121\ \text{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}C_p)} = 10-(10-50)e^{-\frac{(24.5)(4.9)}{(0.2121)(1006)}} = \mathbf{32.8°C}$$

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the air are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{32.8-50}{\ln\left(\frac{10-32.8}{10-50}\right)} = 30.6°\text{C}$$

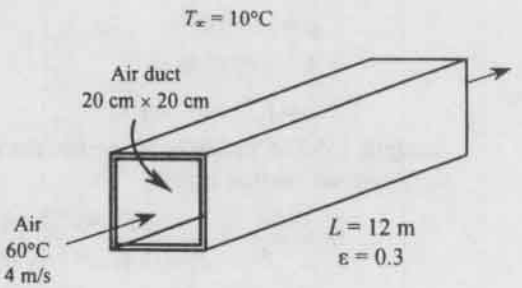
$$\dot{Q} = hA\Delta T_{\ln} = (24.5\ \text{W/m}^2\text{.°C})(4.9\ \text{m}^2)(30.6°\text{C}) = \mathbf{3674\ W}$$

17-65 Hot air enters a sheet metal duct located in a basement. The exit temperature of hot air and the rate of heat loss are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We expect the air temperature to drop somewhat, and evaluate the air properties at 1 atm and the estimated bulk mean temperature of 37°C = 310 K (Table A-18),

$$\rho = 1.143\ \text{kg/m}^3; \qquad k = 0.0268\ \text{W/m.°C}$$
$$\upsilon = 1.67\times10^{-5}\ \text{m}^2/\text{s}; \quad C_p = 1006\ \text{J/kg.°C}$$
$$\text{Pr} = 0.711$$



Analysis The surface area and the Reynolds number are

$$A = 4aL = 4\times(0.2\ \text{m})(12\ \text{m}) = 9.6\ \text{m}$$

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 0.2\ \text{m}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(4\ \text{m/s})(0.20\ \text{m})}{1.67\times10^{-5}\ \text{m}^2/\text{s}} = 47{,}900\ >>\ 4{,}000$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4D\,\text{Re}^{1/6} = 4.4(0.2\ \text{m})(47{,}900)^{1/6} = 5.3\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow for the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(47{,}900)^{0.8}(0.711)^{0.3} = 111.4$$

and

$$h = \frac{k}{D_h}Nu = \frac{0.0268\ \text{W/m.°C}}{0.2\ \text{m}}(111.4) = 14.9\ \text{W/m}^2\text{.°C}$$

The mass flow rate of air is

$$\dot{m} = \rho A V = (1.143\ \text{kg/m}^3)(0.2\times0.2)\text{m}^2(4\ \text{m/s}) = 0.183\ \text{kg/s}$$

In steady operation, heat transfer from hot air to the duct must be equal to the heat transfer from the duct to the surrounding (by convection and radiation), which must be equal to the energy loss of the hot air in the duct. That is,

$$\dot{Q} = \dot{Q}_{\text{conv,in}} = \dot{Q}_{\text{conv+rad,out}} = \Delta\dot{E}_{\text{hot air}}$$

Assuming the duct to be at an average temperature of T_s, the quantities above can be expressed as

$\dot{Q}_{\text{conv,in}}$: $$\dot{Q} = h_i A\Delta T_{\ln} = h_i A\frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} \rightarrow \dot{Q} = (14.9\ \text{W/m}^2\text{.°C})(9.6\ \text{m}^2)\frac{T_e - 60}{\ln\left(\dfrac{T_s - T_e}{T_s - 60}\right)}$$

$\dot{Q}_{\text{conv+rad,out}}$: $$\dot{Q} = h_o A(T_s - T_o) + \varepsilon A\sigma\left(T_s^4 - T_o^4\right) \rightarrow \dot{Q} = (10\ \text{W/m}^2\text{.°C})(9.6\ \text{m}^2)(T_s - 10)\text{°C}$$
$$+0.3(9.6\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\text{.K}^4)\left[(T_s + 273)^4 - (10+273)^4\right]\text{K}^4$$

$\Delta\dot{E}_{\text{hot air}}$: $$\dot{Q} = \dot{m}C_p(T_e - T_i) \rightarrow \dot{Q} = (1.183\ \text{kg/s})(1006\ \text{J/kg.°C})(60 - T_e)\text{°C}$$

This is a system of three equations with three unknowns whose solution is

$$\dot{Q} = \mathbf{3072\ W},\ T_e = \mathbf{57.4°C},\ \text{and}\ T_s = 37.2\text{°C}$$

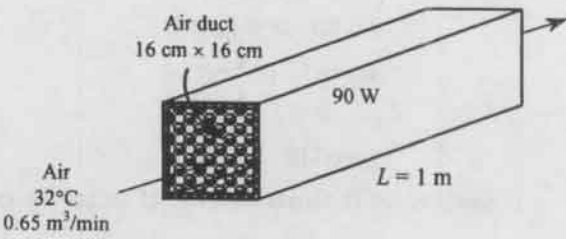
Therefore, the hot air will lose heat at a rate of 3072 W and exit the duct at 57.4°C.

17-66 The components of an electronic system located in a rectangular horizontal duct are cooled by forced air. The exit temperature of the air and the highest component surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 310 K since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the duct whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-18)

$$\rho = 1.143 \text{ kg/m}^3$$
$$k = 0.0268 \text{ W/m.°C}$$
$$\upsilon = 1.67 \times 10^{-5} \text{ m}^2/\text{s}$$
$$C_p = 1006 \text{ J/kg.°C}$$
$$\text{Pr} = 0.710$$



Analysis (*a*) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho \dot{V} = (1.143 \text{ kg/m}^3)(0.65 \text{ m}^3/\text{min}) = 0.74295 \text{ kg/min} = 0.0124 \text{ kg/s}$$

$$\dot{Q} = \dot{m} C_p (T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m} C_p} = 32°\text{C} + \frac{(0.85)(90 \text{ W})}{(0.0124 \text{ kg/s})(1006 \text{ J/kg.°C})} = \mathbf{38.1°C}$$

(*b*) The mean fluid velocity and hydraulic diameter are

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.65 \text{ m/min}}{(0.16 \text{ m})(0.16 \text{ m})} = 25.4 \text{ m/min} = 0.423 \text{ m/s}$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.16 \text{ m})(0.16 \text{ m})}{4(0.16 \text{ m})} = 0.16 \text{ m}$$

Then

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(0.423 \text{ m/s})(0.16 \text{ m})}{1.67 \times 10^{-5} \text{ m}^2/\text{s}} = 4053$$

which is greater than 4000. Also, the components will cause turbulence and thus we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = 0.023(4053)^{0.8}(0.710)^{0.4} = 15.4$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.0268 \text{ W/m.°C}}{0.16 \text{ m}}(15.4) = 2.58 \text{ W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Assuming uniform surface heat flux, its value is determined from

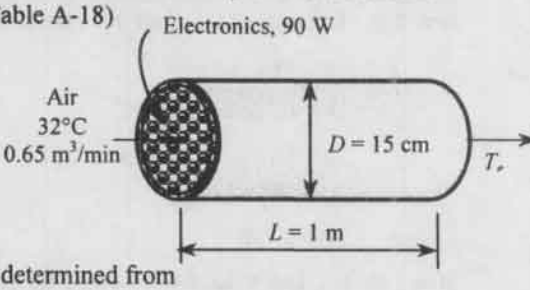
$$\dot{q} = h(T_{s,highest} - T_e) \rightarrow T_{s,highest} = T_e + \frac{\dot{q}}{h} = 38.1°\text{C} + \frac{(0.85)(90 \text{ W})/[4(0.16 \text{ m})(1 \text{ m})]}{(2.58 \text{ W/m}^2\text{.°C})} = \mathbf{84.4°C}$$

17-67 The components of an electronic system located in a circular horizontal duct are cooled by forced air. The exit temperature of the air and the highest component surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 310 K since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the duct whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-18)

$$\rho = 1.143 \text{ kg/m}^3$$
$$k = 0.0268 \text{ W/m.°C}$$
$$\upsilon = 1.67 \times 10^{-5} \text{ m}^2/\text{s}$$
$$C_p = 1006 \text{ J/kg.°C}$$
$$\text{Pr} = 0.710$$



Analysis (a) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho \dot{V} = (1.143 \text{ kg/m}^3)(0.65 \text{ m}^3/\text{min}) = 0.74295 \text{ kg/min} = 0.0124 \text{ kg/s}$$

$$\dot{Q} = \dot{m} C_p (T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m} C_p} = 32\,^\circ\text{C} + \frac{(0.85)(90 \text{ W})}{(0.0124 \text{ kg/s})(1006 \text{ J/kg.°C})} = \mathbf{38.1\,^\circ C}$$

(b) The mean fluid velocity is

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.65 \text{ m/min}}{\pi (0.15 \text{ m})^2 / 4} = 36.7 \text{ m/min} = 0.612 \text{ m/s}$$

Then,

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(0.612 \text{ m/s})(0.15 \text{ m})}{1.67 \times 10^{-5} \text{ m}^2/\text{s}} = 5497$$

which is greater than 4000. Also, the components will cause turbulence and thus we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = 0.023(5497)^{0.8}(0.710)^{0.4} = 19.7$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.0268 \text{ W/m.°C}}{0.15 \text{ m}} (19.7) = 3.52 \text{ W/m}^2.^\circ\text{C}$$

The highest component surface temperature will occur at the exit of the duct. Assuming uniform heat flux, its value is determined from

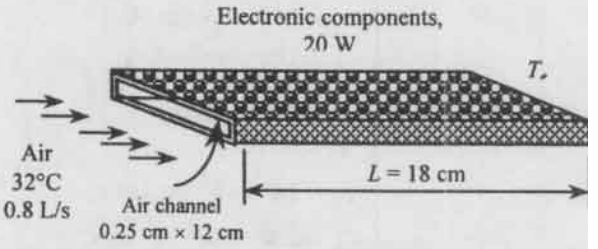
$$\dot{q} = h(T_{s,highest} - T_e) \rightarrow T_{s,highest} = T_e + \frac{\dot{q}}{h} = 38.1^\circ\text{C} + \frac{(0.85)(90 \text{ W}) / [\pi (0.15 \text{ m})(1 \text{ m})]}{(3.52 \text{ W/m}^2.^\circ\text{C})} = \mathbf{84.2^\circ C}$$

17-68 Air enters a hollow-core printed circuit board. The exit temperature of the air and the highest temperature on the inner surface are to be determined. √

Assumptions **1** Steady flow conditions exist. **2** Heat generated is uniformly distributed over the two surfaces of the PCB. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 310 K since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the hollow core whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-18)

$$\rho = 1.143\ \text{kg/m}^3$$
$$k = 0.0268\ \text{W/m.°C}$$
$$\upsilon = 1.67\times10^{-5}\ \text{m}^2/\text{s}$$
$$C_p = 1006\ \text{J/kg.°C}$$
$$\text{Pr} = 0.710$$
$$\mu_b = 1.89\times10^{-5}\ \text{kg/m.s}$$
$$\mu_{s,@350\,\text{K}} = 2.08\times10^{-5}\ \text{kg/m.s}$$



Analysis (a) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho\dot{V} = (1.143\ \text{kg/m}^3)(0.8\times10^{-3}\ \text{m}^3/\text{s}) = 9.14\times10^{-4}\ \text{kg/s}$$

$$\dot{Q} = \dot{m}C_p(T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m}C_p} = 32\ °\text{C} + \frac{20\ \text{W}}{(9.14\times10^{-4}\ \text{kg/s})(1006\ \text{J/kg.°C})} = \mathbf{53.7°C}$$

(b) The mean fluid velocity and hydraulic diameter are

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.8\times10^{-3}\ \text{m}^3/\text{s}}{(0.12\ \text{m})(0.0025\ \text{m})} = 2.67\ \text{m/s}$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.12\ \text{m})(0.0025\ \text{m})}{2[(0.12\ \text{m}) + (0.0025\ \text{m})]} = 0.0049\ \text{m}$$

Then,

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(2.67\ \text{m/s})(0.0049\ \text{m})}{1.67\times10^{-5}\ \text{m}^2/\text{s}} = 783$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length in this case is

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D_h = 0.05(783)(0.71)(0.0049\ \text{m}) = 0.14\ \text{m}$$

which is shorter than the total length of the duct. Therefore, we assume thermally developing flow , and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 1.86\left(\frac{\text{Re}\,\text{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left[\frac{(783)(0.71)(0.0049)}{0.18}\right]^{1/3}\left(\frac{1.89\times10^{-5}}{2.08\times10^{-5}}\right)^{0.14} = 8.24$$

and,

$$h = \frac{k}{D_h}Nu = \frac{0.0268\ \text{W/m.°C}}{0.0049\ \text{m}}(8.24) = 46.2\ \text{W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Its value is determined from

$$\dot{Q} = hA(T_{s,highest} - T_e) \rightarrow T_{s,highest} = T_e + \frac{\dot{Q}}{hA}$$

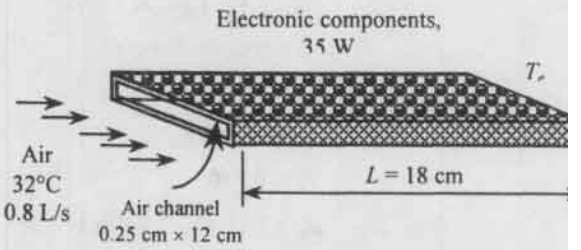
$$= 53.7°\text{C} + \frac{20\ \text{W}}{(46.2\ \text{W/m}^2\text{.°C})\left[2(0.12\times0.18 + 0.0025\times0.18)\text{m}^2\right]} = \mathbf{64.0°C}$$

17-69 Air enters a hollow-core printed circuit board. The exit temperature of the air and the highest temperature on the inner surface are to be determined.

Assumptions **1** Steady flow conditions exist. **2** Heat generated is uniformly distributed over the two surfaces of the PCB. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 310 K since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the hollow core whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-18)

$$\rho = 1.143 \text{ kg/m}^3$$
$$k = 0.0268 \text{ W/m.°C}$$
$$\upsilon = 1.67 \times 10^{-5} \text{ m}^2/\text{s}$$
$$C_p = 1006 \text{ J/kg.°C}$$
$$\text{Pr} = 0.710$$
$$\mu_b = 1.89 \times 10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@350\text{ K}} = 2.08 \times 10^{-5} \text{ kg/m.s}$$



Analysis (a) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho \dot{V} = (1.143 \text{ kg/m}^3)(0.8 \times 10^{-3} \text{ m}^3/\text{s}) = 9.14 \times 10^{-4} \text{ kg/s}$$

$$\dot{Q} = \dot{m} C_p (T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m} C_p} = 32°\text{C} + \frac{35 \text{ W}}{(9.14 \times 10^{-4} \text{ kg/s})(1006 \text{ J/kg.°C})} = \mathbf{70.1°C}$$

(b) The mean fluid velocity and hydraulic diameter are

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.8 \times 10^{-3} \text{ m}^3/\text{s}}{(0.12 \text{ m})(0.0025 \text{ m})} = 2.67 \text{ m/s}$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.12 \text{ m})(0.0025 \text{ m})}{2[(0.12 \text{ m}) + (0.0025 \text{ m})]} = 0.0049 \text{ m}$$

Then,

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(2.67 \text{ m/s})(0.0049 \text{ m})}{1.67 \times 10^{-5} \text{ m}^2/\text{s}} = 783$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length in this case is

$$L_t = 0.05 \text{Re Pr} D_h = 0.05(783)(0.71)(0.0049 \text{ m}) = 0.14 \text{ m}$$

which is shorter than the total length of the duct. Therefore, we assume thermally developing flow , and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 1.86\left(\frac{\text{Re Pr } D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left[\frac{(783)(0.71)(0.0049)}{0.18}\right]^{1/3}\left(\frac{1.89 \times 10^{-5}}{2.08 \times 10^{-5}}\right)^{0.14} = 4.54$$

and,

$$h = \frac{k}{D_h} Nu = \frac{0.0268 \text{ W/m.°C}}{0.0049 \text{ m}}(4.54) = 24.8 \text{ W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Its value is determined from

$$\dot{Q} = hA(T_{s,highest} - T_e) \longrightarrow T_{s,highest} = T_e + \frac{\dot{Q}}{hA}$$

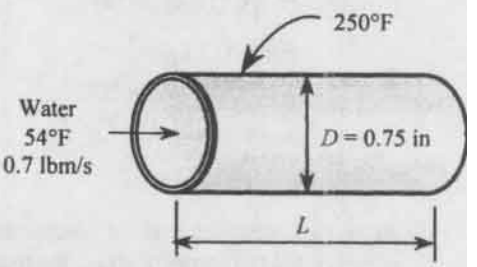
$$= 70.1°\text{C} + \frac{35 \text{ W}}{(24.8 \text{ W/m}^2\text{.°C})[2(0.12 \times 0.18 + 0.0025 \times 0.18)\text{m}^2]} = \mathbf{102.1°C}$$

17-70E Water is heated by passing it through thin-walled copper tubes. The length of the copper tube that needs to be used is to be determined. √

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the tube are smooth. **3** The thermal resistance of the tube is negligible. **4** The temperature at the tube surface is constant.

Properties The properties of water at the bulk mean fluid temperature of $T_{b,ave} = (54+140)/2 = 97°F \approx 100°F$ are (Table A-9E)

$$\rho = 62.0 \text{ lbm/ft}^3$$
$$k = 0.363 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.738\times10^{-5} \text{ ft}^2/\text{s}$$
$$C_p = 0.999 \text{ Btu/lbm.°F}$$
$$\text{Pr} = 4.54$$



Analysis (*a*) The mass flow rate and the Reynolds number are

$$\dot{m} = \rho A_c V_m \rightarrow V_m = \frac{\dot{m}}{\rho A_c} = \frac{0.7 \text{ lbm/s}}{(62 \text{ lbm/ft}^3)[\pi(0.75/12 \text{ ft})^2/4]} = 3.68 \text{ ft/s}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(3.68 \text{ ft/s})(0.75/12 \text{ ft})}{0.738\times10^{-5} \text{ ft}^2/\text{s}} = 31{,}165$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4 D \text{Re}^{1/6} = 4.4(0.75 \text{ in})(31{,}165)^{1/6} = 18.5 \text{ in}$$

which is probably shorter than the total length of the pipe we will determine. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = 0.023(31{,}165)^{0.8}(4.54)^{0.4} = 165.8$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.363 \text{ Btu/h.ft.°F}}{(0.75/12) \text{ ft}} (165.8) = 963 \text{ Btu/h.ft}^2\text{.°F}$$

The logarithmic mean temperature difference and then the rate of heat transfer per ft length of the tube are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{140-54}{\ln\left(\frac{250-140}{250-54}\right)} = 148.9°F$$

$$\dot{Q} = hA\Delta T_{\ln} = (963 \text{ Btu/h.ft}^2\text{.°F})[\pi(0.75/12 \text{ ft})(1 \text{ ft})](148.9°F) = 28{,}150 \text{ Btu/h}$$

The rate of heat transfer needed to raise the temperature of water from 54 °F to 140 °F is

$$\dot{Q} = \dot{m} C_p (T_e - T_i) = (0.7\times3600 \text{ lbm/h})(0.999 \text{ Btu/lbm.°F})(140-54)°F = 216{,}500 \text{ Btu/h}$$

Then the length of the copper tube that needs to be used becomes

$$Length = \frac{216{,}500 \text{ Btu/h}}{28{,}150 \text{ Btu/h}} = \mathbf{7.69 \text{ ft}}$$

17-71 A computer is cooled by a fan blowing air through the case of the computer. The flow rate of the air, the fraction of the temperature rise of air that is due to heat generated by the fan, and the highest allowable inlet air temperature are to be determined. √

Assumptions **1** Steady flow conditions exist. **2** Heat flux is uniformly distributed. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 300 K. The properties of air at 1 atm and this temperature are (Table A-18)

$$\rho = 1.177\ \text{kg/m}^3 \qquad \text{Pr} = 0.712$$
$$k = 0.0261\ \text{W/m.°C} \qquad \mu_b = 1.85\times10^{-5}\ \text{kg/m.s}$$
$$\upsilon = 1.57\times10^{-5}\ \text{m}^2/\text{s} \qquad \mu_{s,@350\,\text{K}} = 2.08\times10^{-5}\ \text{kg/m.s}$$
$$C_p = 1005\ \text{J/kg.°C}$$

Analysis (*a*) Noting that the electric energy consumed by the fan is converted to thermal energy, the mass flow rate of air is

$$\dot{Q} = \dot{m}C_p(T_e - T_i) \rightarrow \dot{m} = \frac{\dot{Q} + \dot{W}_{\text{elect, fan}}}{C_p(T_e - T_i)} = \frac{(8\times10 + 25)\ \text{W}}{(1005\ \text{J/kg.°C})(10\text{°C})} = \mathbf{0.01045\ kg/s}$$

(*b*) The fraction of temperature rise of air that is due to the heat generated by the fan and its motor can be determined from

$$\dot{Q} = \dot{m}C_p\Delta T \rightarrow \Delta T = \frac{\dot{Q}}{\dot{m}C_p} = \frac{25\ \text{W}}{(0.01045\ \text{kg/s})(1005\ \text{J/kg.°C})} = \mathbf{2.38°C}$$

$$f = \frac{2.38\text{°C}}{10\text{°C}} = 0.238 = \mathbf{23.8\%}$$

(*c*) The mean velocity of air is

$$\dot{m} = \rho A_c V_m \rightarrow V_m = \frac{\dot{m}}{\rho A_c} = \frac{(0.01045/8)\ \text{kg/s}}{(1.177\ \text{kg/m}^3)[(0.003\ \text{m})(0.12\ \text{m})]} = 3.08\ \text{m/s}$$

and,

$$D_h = \frac{4A_c}{P} = \frac{4(0.003\ \text{m})(0.12\ \text{m})}{2(0.003\ \text{m} + 0.12\ \text{m})} = 0.00585\ \text{m}$$



Therefore,

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(3.08\ \text{m/s})(0.00585\ \text{m})}{1.57\times10^{-5}\ \text{m}^2/\text{s}} = 1148$$

which is less than 4000. Therefore, the flow is laminar. Assuming fully developed flow, the Nusselt number from is determined from Table 17-4 corresponding to a/b = 12/0.3 = 40 to be Nu = 8.24. Then,

$$h = \frac{k}{D_h}Nu = \frac{0.0261\ \text{W/m.°C}}{0.00585\ \text{m}}(8.24) = 36.8\ \text{W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Assuming uniform heat flux, the air temperature at the exit is determined from

$$\dot{q} = h(T_{s,\max} - T_e) \rightarrow T_e = T_{s,\max} - \frac{\dot{q}}{h} = 70\text{°C} - \frac{[(80+25)\ \text{W}]/[8\times2(0.12\times0.18 + 0.003\times0.18)\ \text{m}^2]}{36.8\ \text{W/m}^2\text{.°C}} = 61.9\text{°C}$$

The highest allowable inlet temperature then becomes

$$T_e - T_i = 10\text{°C} \rightarrow T_i = T_e - 10\text{°C} = 61.9\text{°C} - 10\text{°C} = \mathbf{51.9°C}$$

Discussion Although the Reynolds number is less than 4000, the flow in this case will most likely be turbulent because of the electronic components that that protrude into flow. Therefore, the heat transfer coefficient determined above is probably conservative.

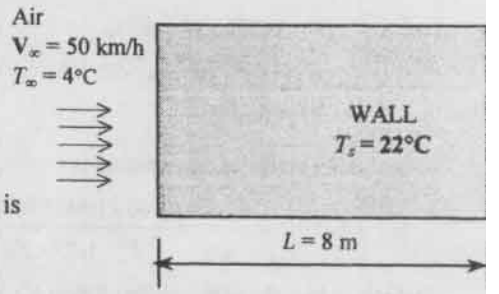
Review Problems

17-72 Wind is blowing parallel to the walls of a house. The rate of heat loss from the wall is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties Assuming a film temperature of $T_f = 275$ K for the outdoors, the properties of air are evaluated to be (Table A-18)

$$k = 0.0242 \text{ W/m.°C}$$
$$\upsilon = 1.36\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.718$$



Analysis Air flows along 8-m side. The Reynolds number in this case is

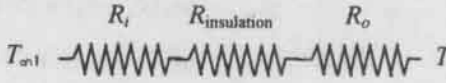
$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(50\times1000/3600) \text{ m/s}](8 \text{ m})}{1.36\times10^{-5} \text{ m}^2/\text{s}} = 8.17\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be

$$Nu = \frac{h_0 L}{k} = (0.037\,\text{Re}_L^{\,0.8} - 871)\,\Pr^{1/3} = [0.037(8.17\times10^6)^{0.8} - 871](0.718)^{1/3} = 1.044\times10^4$$

$$h_0 = \frac{k}{L}Nu = \frac{0.0242 \text{ W/m.°C}}{8 \text{ m}}(1.044\times10^4) = 31.6 \text{ W/m}^2\text{.°C}$$

The thermal resistances are



$$A = wL = (3 \text{ m})(8 \text{ m}) = 24 \text{ m}^2$$

$$R_i = \frac{1}{h_i A} = \frac{1}{(8 \text{ W/m}^2\text{.°C})(24 \text{ m}^2)} = 0.0052 \text{ °C/W}$$

$$R_{insulation} = \frac{(R-3.38)_{value}}{A} = \frac{3.38 \text{ m}^2\text{.°C/W}}{24 \text{ m}^2} = 0.1408 \text{ °C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(31.6 \text{ W/m}^2\text{.°C})(24 \text{ m}^2)} = 0.0013 \text{ °C/W}$$

Then the total thermal resistance and the heat transfer rate through the wall are determined from

$$R_{total} = R_i + R_{insulation} + R_o = 0.0052 + 0.1408 + 0.0013 = 0.1473 \text{ °C/W}$$

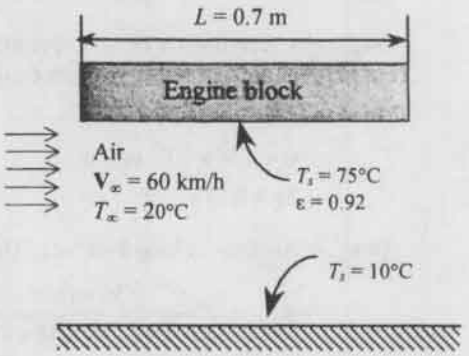
$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(22-4)°\text{C}}{0.1473 \text{ °C/W}} = \mathbf{122 \text{ W}}$$

17-73 A car travels at a velocity of 60 km/h. The rate of heat transfer from the bottom surface of the hot automotive engine block is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm. **5** The flow is turbulent over the entire surface because of the constant agitation of the engine block. **6** The bottom surface of the engine is a flat surface.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (75+20)/2 = 47.5°C = 320.5$ K are (Table A-18)

$$k = 0.0275 \text{ W/m.°C}$$
$$\upsilon = 1.77\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$



Analysis The Reynolds number is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(60\times1000/3600) \text{ m/s}](0.7 \text{ m})}{1.77\times10^{-5} \text{ m}^2/\text{s}} = 6.591\times10^5$$

which is less than the critical Reynolds number. But we will assume turbulent flow because of the constant agitation of the engine block.

$$Nu = \frac{hL}{k} = 0.037 \text{ Re}_L^{0.8} \text{ Pr}^{1/3} = 0.037(6.591\times10^5)^{0.8}(0.710)^{1/3} = 1492$$

$$h = \frac{k}{L} Nu = \frac{0.0275 \text{ W/m.°C}}{0.7 \text{ m}}(1492) = 58.6 \text{ W/m}^2\text{.°C}$$

$$\dot{Q}_{conv} = hA_s(T_\infty - T_s) = (58.6 \text{ W/m}^2\text{.°C})[(0.6 \text{ m})(0.7 \text{ m})](75-20)°C = 1354 \text{ W}$$

The heat loss by radiation is then determined from Stefan-Boltzman law to be

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.92)(0.42 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(75+273 \text{ K})^4 - (10+273 \text{ K})^4] = 181 \text{ W}$$

Then the total rate of heat loss from the bottom surface of the engine block becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 1303 + 181 = \mathbf{1484 \text{ W}}$$

The gunk will introduce an additional resistance to heat dissipation from the engine. The total heat transfer rate in this case can be calculated from

$$\dot{Q} = \frac{T_\infty - T_s}{\frac{1}{hA} + \frac{L}{kA}} = \frac{(75-20)°C}{\frac{1}{(58.6 \text{ W/m}^2\text{.°C})[(0.6 \text{ m})(0.7 \text{ m})]} + \frac{(0.002 \text{ m})}{(3 \text{ W/m.°C})(0.6 \text{ m}\times0.7 \text{ m})}} = 1303 \text{ W}$$

The decrease in the heat transfer rate is

$$1354-1303 = \mathbf{51 \text{ W}}$$

17-74E A minivan is traveling at 60 mph. The rate of heat transfer to the van is to be determined.

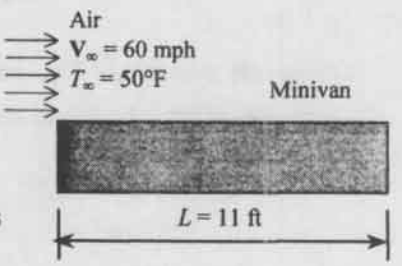
Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air flow is turbulent because of the intense vibrations involved. **5** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties Assuming a film temperature of $T_f = 80°F$, the properties of air are evaluated to be (Table A-18)

$$k = 0.015 \text{ Btu / h.ft.°F}$$

$$\upsilon = 0.17\times10^{-3} \text{ ft}^2/\text{s}$$

$$\text{Pr} = 0.72$$



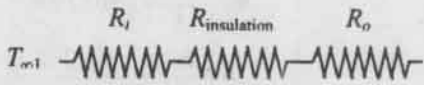
Analysis Air flows along 11 ft long side. The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(60\times5280/3600)\text{ ft/s}](11\text{ ft})}{0.17\times10^{-3}\text{ ft}^2/\text{s}} = 5.694\times10^6$$

which is greater than the critical Reynolds number. The air flow is assumed to be entirely turbulent because of the intense vibrations involved. Then the Nusselt number and the heat transfer coefficient are determined to be

$$Nu = \frac{h_o L}{k} = 0.037\,\text{Re}_L^{0.8}\,\text{Pr}^{1/3} = 0.037(5.694\times10^5)^{0.8}(0.72)^{1/3} = 8414$$

$$h_o = \frac{k}{L}Nu = \frac{0.015\text{ Btu/h.ft.°F}}{11\text{ ft}}(8414) = 11.5\text{ Btu/h.ft}^2\text{.°F}$$



The thermal resistances are

$$A_{total} = 2[(3.2\text{ ft})(6\text{ ft}) + (3.2\text{ ft})(11\text{ ft}) + (6\text{ ft})(11\text{ ft})] = 240.8\text{ ft}^2$$

$$R_i = \frac{1}{h_i A} = \frac{1}{(1.2\text{ Btu/h.ft}^2\text{.°F})(240.8\text{ ft}^2)} = 0.0035\text{ h.°F/Btu}$$

$$R_{insulation} = \frac{(R-3)_{value}}{A} = \frac{3\text{ h.ft}^2\text{.°F/Btu}}{(240.8\text{ ft}^2)} = 0.0125\text{ h.°F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(11.5\text{ Btu/h.ft}^2\text{.°F})(240.8\text{ ft}^2)} = 0.0004\text{ h.°F/Btu}$$

Then the total thermal resistance and the heat transfer rate into the minivan are determined to be

$$R_{total} = R_i + R_{insulation} + R_o = 0.0035 + 0.0125 + 0.0004 = 0.0164\text{ h.°F/Btu}$$

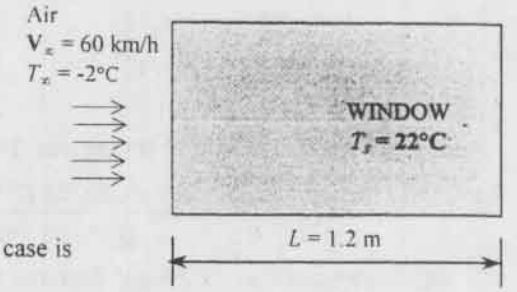
$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(90-70)°\text{F}}{0.0164\text{ h.°F/Btu}} = \mathbf{1220\text{ Btu/h}}$$

17-75 Wind is blowing parallel to the walls of a house with windows. The rate of heat loss through the window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air flow is turbulent because of the intense vibrations involved. **5** The minivan is modeled as a rectangular box. **6** Air is an ideal gas with constant properties. **6** The pressure of air is 1 atm.

Properties Assuming a film temperature of $T_f = 275$ K, the properties of air at 1 atm and this temperature are evaluated to be (Table A-18)

$$k = 0.0242 \text{ W/m.°C}$$
$$\upsilon = 1.36\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.718$$



Analysis Air flows along 1.2 m side. The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(60\times1000/3600) \text{ m/s}](1.2 \text{ m})}{1.36\times10^{-5} \text{ m}^2/\text{s}} = 1.47\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\Pr^{1/3} = \left[0.037(1.47\times10^6)^{0.8} - 871\right](0.718)^{1/3} = 2065$$

$$h = \frac{k}{L}Nu = \frac{0.0242 \text{ W/m.°C}}{1.2 \text{ m}}(2065) = 41.6 \text{ W/m}^2\text{.°C}$$

The thermal resistances are

$$A_{total} = 3(1.2 \text{ m})(1.5 \text{ m}) = 5.4 \text{ m}^2$$

$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(8 \text{ W/m}^2\text{.°C})(5.4 \text{ m}^2)} = 0.0231 \text{ °C/W}$$

$$R_{cond} = \frac{L}{kA} = \frac{0.005 \text{ m}}{(0.78 \text{ W/m.°C})(5.4 \text{ m}^2)} = 0.0012 \text{ °C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(41.6 \text{ W/m}^2\text{.°C})(5.4 \text{ m}^2)} = 0.0044 \text{ °C/W}$$

Then the total thermal resistance and the heat transfer rate through the 3 windows become

$$R_{total} = R_{conv,i} + R_{cond} + R_{conv,o} = 0.0231 + 0.0012 + 0.0044 = 0.0287 \text{ °C/W}$$

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{[22-(-2)]\text{°C}}{0.0287 \text{ °C/W}} = \mathbf{836 \text{ W}}$$

17-98 A compressor is connected to the outside through a circular duct. The power used by compressor to overcome the pressure drop, the rate of heat transfer, and the temperature rise of air are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties.

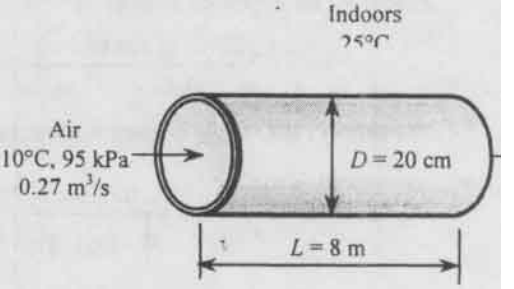
Properties We take the bulk mean temperature for air to be 290 K since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the duct whose surface is exposed to a higher temperature. The properties of air at this temperature and 1 atm pressure are

$$\rho = 1.224 \text{ kg/m}^3 \qquad C_p = 1005 \text{ J/kg.°C}$$
$$k = 0.0253 \text{ W/m.°C} \qquad \text{Pr} = 0.714$$
$$\upsilon = 1.48 \times 10^{-5} \text{ m}^2/\text{s}$$

Note that the atmospheric pressure other than 1 atm will change the density and kinematic viscosity values. First the pressure in atm is

$$P = \frac{95 \text{ kPa}}{101.325 \text{ kPa}} = 0.938 \text{ atm}$$

Then,
$$\rho = (1.224 \text{ kg/m}^3)(0.938) = 1.148 \text{ kg/m}^3$$
$$\upsilon = (1.48 \times 10^{-5} \text{ m}^2/\text{s})/(0.938) = 1.58 \times 10^{-5} \text{ m}^2/\text{s}$$



Analysis The mean velocity of air is

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.27 \text{ m}^3/\text{s}}{\pi(0.2 \text{ m})^2/4} = 8.6 \text{ m/s}$$

Then
$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(8.6 \text{ m/s})(0.2 \text{ m})}{1.58 \times 10^{-5} \text{ m}^2/\text{s}} = 1.089 \times 10^5$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4 D \,\text{Re}^{1/6} = 4.4(0.2 \text{ m})(1.089 \times 10^6)^{1/6} = 6.1 \text{ m}$$

which is shorter than the total length of the duct. Therefore, we assume fully developed flow in a smooth pipe, and determine friction factor from

$$f = 0.184 \,\text{Re}^{-2} = 0.184(1.089 \times 10^5)^{-0.2} = 0.018$$

The pressure drop and the compressor power required to overcome this pressure drop are

$$\dot{m} = \rho \dot{V} = (1.148 \text{ kg/m}^3)(0.27 \text{ m}^3/\text{s}) = 0.31 \text{ kg/s}$$

$$\Delta P = f \frac{L}{D} \frac{\rho \mathbf{V}_m^2}{2} = (0.018) \frac{(8 \text{ m})}{(0.2 \text{ m})} \frac{(1.148 \text{ kg/m}^3)(8.6 \text{ m/s})^2}{2} = 30.72 \text{ N/m}^2$$

$$\dot{W}_{pump} = \frac{\dot{m} \Delta P}{\rho} = \frac{(0.31 \text{ kg/s})(30.72 \text{ N/m}^2)}{1.148 \text{ kg/m}^3} = \mathbf{8.3 \text{ W}}$$

(*b*) For the fully developed turbulent flow, the Nusselt number is

$$Nu = \frac{hD}{k} = 0.023 \,\text{Re}^{0.8} \,\text{Pr}^{0.4} = 0.023(1.089 \times 10^5)^{0.8}(0.714)^{0.4} = 215$$

and
$$h = \frac{k}{D_h} Nu = \frac{0.0253 \text{ W/m.°C}}{0.2 \text{ m}} (215) = 27.2 \text{ W/m}^2\text{.°C}$$

Disregarding the thermal resistance of the duct, the rate of heat transfer to the air in the duct becomes

$$A = \pi DL = \pi(0.2 \text{ m})(8 \text{ m}) = 5.0265 \text{ m}^2$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 A} + \frac{1}{h_2 A}} = \frac{25 - 10}{\frac{1}{(27.2)(5.0265)} + \frac{1}{(10)(5.0265)}} = \mathbf{551 \text{ W}}$$

(*c*) The temperature rise of air in the duct is

$$\dot{Q} = \dot{m} C_p \Delta T \rightarrow 551 \text{ W} = (0.31 \text{ kg/s})(1005 \text{ J/kg.°C}) \Delta T \rightarrow \Delta T = \mathbf{1.8°C}$$

17-76 A fan is blowing air over the entire body of a person. The average temperature of the outer surface of the person is to be determined. √

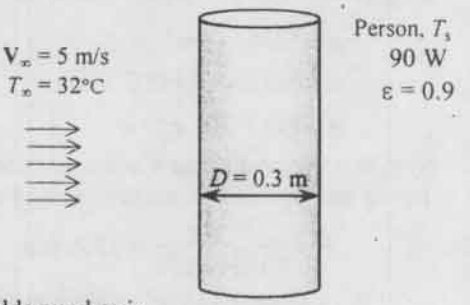
Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm. **4** The average human body can be treated as a 30-cm-diameter cylinder with an exposed surface area of 1.7 m^2.

Properties We assume the film temperature to be 310 K. The properties of air at 1 atm and this temperature are (Table A-18)

$k = 0.0268\ \text{W/m.°C}$

$\upsilon = 1.67 \times 10^{-5}\ \text{m}^2/\text{s}$

$\text{Pr} = 0.710$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{(5\ \text{m/s})(0.3\ \text{m})}{1.67 \times 10^{-5}\ \text{m}^2/\text{s}} = 8.982 \times 10^4$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{28200}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(8.982\times10^4)^{0.5}(0.710)^{1/3}}{\left[1+(0.4/0.710)^{2/3}\right]^{1/4}}\left[1+\left(\frac{8.982\times10^4}{28200}\right)^{5/8}\right]^{4/5} = 200.5$$

Then

$$h = \frac{k}{D}Nu = \frac{0.0268\ \text{W/m.°C}}{0.3\ \text{m}}(200.5) = 17.9\ \text{W/m}^2\text{.°C}$$

Considering that there is heat generation in that person's body at a rate of 90 W and body gains heat by radiation from the surrounding surfaces, an energy balance can be written as

$$\dot{Q}_{generated} + \dot{Q}_{radiation} = \dot{Q}_{convection}$$

Substituting values with proper units and then application of trial & error method yields the average temperature of the outer surface of the person.

$$90\ \text{W} + \varepsilon A\sigma(T_{surr}^{\ 4} - T_s^{\ 4}) = hA(T_s - T_\infty)$$

$$90 + (0.9)(1.7)(5.67\times10^{-8})[(40+273)^4 - T_s^{\ 4}] = (17.9)(1.7)[T_s - (32+273)]$$

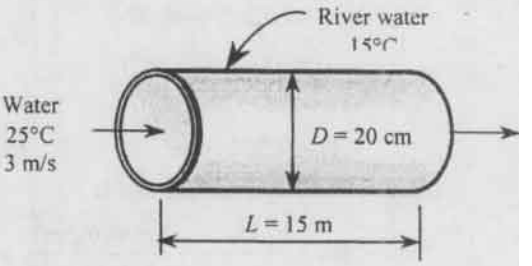
$$T_s = \mathbf{309.3\ K = 36.3°C}$$

17-78 Air enters the underwater section of a duct. The outlet temperature of the air and the fan power needed to overcome the flow resistance are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** The surface of the duct is at the temperature of the water. **5** Air is an ideal gas with constant properties. **6** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 290 K since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at 1 atm and this temperature are (Table A-18)

$$\rho = 1.224 \text{ kg/m}^3$$
$$k = 0.0253 \text{ W/m.°C}$$
$$\upsilon = 1.48\times10^{-5} \text{ m}^2/\text{s}$$
$$C_p = 1005 \text{ J/kg.°C}$$
$$\text{Pr} = 0.714$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(3 \text{ m/s})(0.2 \text{ m})}{1.48\times10^{-5} \text{ m}^2/\text{s}} = 4.054\times10^4$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4 D \text{Re}^{1/6} = 4.4(0.2 \text{ m})(4.054\times10^4)^{1/6} = 5.2 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.3} = 0.023(4.054\times10^4)^{0.8}(0.714)^{0.3} = 101$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.0253 \text{ W/m.°C}}{0.2 \text{ m}}(101) = 12.8 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of air,

$$A = \pi DL = \pi(0.2 \text{ m})(15 \text{ m}) = 9.42 \text{ m}^2$$

$$\dot{m} = \rho V_m A_c = (1.224 \text{ kg/m}^3)(3 \text{ m/s})\left(\frac{\pi(0.2 \text{ m})^2}{4}\right) = 0.115 \text{ kg/s}$$

and

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}C_p)} = 15 - (15-25)e^{-\frac{(12.8)(9.42)}{(0.115)(1005)}} = \mathbf{18.5°C}$$

The friction factor, pressure drop, and the fan power required to overcome this pressure drop can be determined for the case of fully developed turbulent flow in smooth pipes to be

$$f = 0.184 \text{Re}^{-0.2} = 0.184(4.054\times10^4)^{-0.2} = 0.0220$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_m^2}{2} = 0.022\frac{(15 \text{ m})}{(0.2 \text{ m})}\frac{(1.224 \text{ kg/m}^3)(3 \text{ m/s})^2}{2} = 9.09 \text{ N/m}^2$$

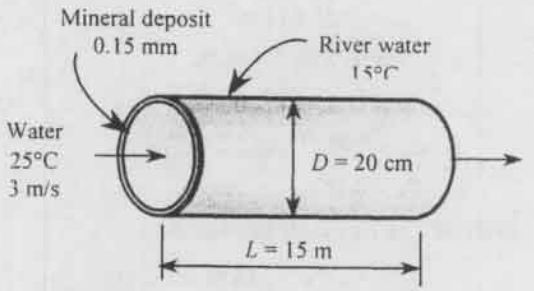
$$\dot{W}_{pump} = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.115 \text{ kg/s})(9.095 \text{ N/m}^2)}{1.224 \text{ kg/m}^3} = \mathbf{0.854 \text{ W}}$$

17-79 Air enters the underwater section of a duct. The outlet temperature of the air and the fan power needed to overcome the flow resistance are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 290 K since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at 1 atm and this temperature are (Table A-18)

$$\rho = 1.224\ \text{kg/m}^3$$
$$k = 0.0253\ \text{W/m.°C}$$
$$\upsilon = 1.48\times10^{-5}\ \text{m}^2/\text{s}$$
$$C_p = 1005\ \text{J/kg.°C}$$
$$\text{Pr} = 0.714$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(3\ \text{m/s})(0.2\ \text{m})}{1.48\times10^{-5}\ \text{m}^2/\text{s}} = 4.054\times10^4$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4D\,\text{Re}^{1/6} = 4.4(0.2\ \text{m})(4.054\times10^4)^{1/6} = 5.2\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number and h from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(4.054\times10^4)^{0.8}(0.714)^{0.3} = 101$$

$$h = \frac{k}{D_h}Nu = \frac{0.0253\ \text{W/m.°C}}{0.2\ \text{m}}(101) = 12.8\ \text{W/m}^2\text{.°C}$$

$$A = \pi DL = \pi(0.2\ \text{m})(15\ \text{m}) = 9.42\ \text{m}^2$$

Also,

$$\dot{m} = \rho V_m A_c = (1.224\ \text{kg/m}^3)(3\ \text{m/s})\left(\frac{\pi(0.2\ \text{m})^2}{4}\right) = 0.115\ \text{kg/s}$$

The unit thermal resistance of the mineral deposit is

$$R_{\text{mineral}} = \frac{L}{k} = \frac{0.0015\ \text{m}}{3\ \text{W/m.°C}} = 0.0005\ \text{m}^2\text{.°C/W}$$

which is much less than (under 1%) the unit convection resistance,

$$R_{\text{conv}} = \frac{1}{h} = \frac{1}{12.8\ \text{W/m}^2\text{.°C}} = 0.0780\ \text{m}^2\text{.°C/W}$$

Therefore, the effect of 0.15 mm thick mineral deposit on heat transfer is negligible.

Next we determine the exit temperature of air,

$$A = \pi DL = \pi(0.2\ \text{m})(15\ \text{m}) = 9.42\ \text{m}^2$$

$$\dot{m} = \rho V_m A_c = (1.224\ \text{kg/m}^3)(3\ \text{m/s})\left(\frac{\pi(0.2\ \text{m})^2}{4}\right) = 0.115\ \text{kg/s}$$

and $$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}C_p)} = 15 - (15-25)e^{-\frac{(12.8)(9.42)}{(0.115)(1005)}} = \mathbf{18.5^\circ C}$$

The friction factor, pressure drop, and the fan power required to overcome this pressure drop can be determined for the case of fully developed turbulent flow in smooth pipes to be

$$f = 0.184\,\mathrm{Re}^{-0.2} = 0.184(40{,}541)^{-0.2} = 0.022$$

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^{\,2}}{2} = 0.022\frac{(15\text{ m})}{(0.2\text{ m})}\frac{(1.224\text{ kg/m}^3)(3\text{ m/s})^2}{2} = 9.09\text{ N/m}^2$$

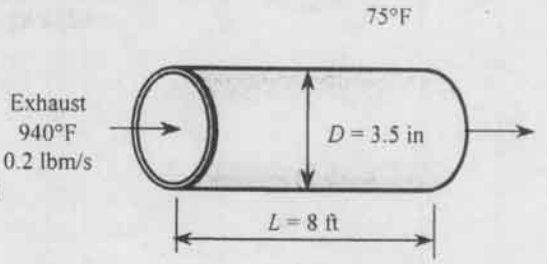
$$\dot{W}_{pump} = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.115\text{ kg/s})(9.095\text{ N/m}^2)}{1.224\text{ kg/m}^3} = \mathbf{0.854\text{ W}}$$

17-80E The exhaust gases of an automotive engine enter a steel exhaust pipe. The velocity of exhaust gases at the inlet and the temperature of exhaust gases at the exit are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the pipe are smooth. **3** The thermal resistance of the pipe is negligible. **4** Exhaust gases have the properties of air, which is an ideal gas with constant properties.

Properties We take the bulk mean temperature for exhaust gases to be 800 °F since the mean temperature of gases at the inlet will drop somewhat as a result of heat loss through the exhaust pipe whose surface is at a lower temperature. The properties of air at this temperature and 1 atm pressure are (Table A-18)

$$\rho = 0.0314 \text{ lbm/ft}^3$$
$$k = 0.0286 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.717 \times 10^{-3} \text{ ft}^2/\text{s}$$
$$C_p = 0.256 \text{ Btu/lbm.°F}$$
$$\text{Pr} = 0.697$$



Noting that 1 atm = 14.7 psia, the pressure in atm is P = (16.1 psia)/(14.7 psia) = 1.096 atm. Then,

$$\rho = (0.0314 \text{ lbm/ft}^3)(1.096) = 0.0344 \text{ lbm/ft}^3$$
$$\upsilon = (0.717 \times 10^{-3} \text{ ft}^2/\text{s})/(1.096) = 0.654 \times 10^{-3} \text{ ft}^2/\text{s}$$

Analysis (a) The velocity of exhaust gases at the inlet of the exhaust pipe is

$$\dot{m} = \rho V_m A_c \longrightarrow V_m = \frac{\dot{m}}{\rho A_c} = \frac{0.2 \text{ lbm/s}}{(0.0344 \text{ lbm/ft}^3)(\pi(3.5/12 \text{ ft})^2/4)} = \mathbf{87.0 \text{ ft/s}}$$

(b) The Reynolds number is

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(87.0 \text{ ft/s})(3.5/12 \text{ ft})}{0.654 \times 10^{-3} \text{ ft}^2/\text{s}} = 38{,}800$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4 D \,\text{Re}^{1/6} = 4.4(3.5/12 \text{ ft})(38{,}800)^{1/6} = 7.5 \text{ ft}$$

which are shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \,\text{Re}^{0.8} \,\text{Pr}^{0.3} = 0.023(38{,}800)^{0.8}(0.697)^{0.3} = 96.8$$

and

$$h_i = h = \frac{k}{D_h} Nu = \frac{0.0286 \text{ Btu/h.ft.°F}}{(3.5/12) \text{ ft}}(96.8) = 9.50 \text{ Btu/h.ft}^2\text{.°F}$$

$$A = \pi DL = \pi(3.5/12 \text{ ft})(8 \text{ ft}) = 7.33 \text{ ft}^2$$

In steady operation, heat transfer from exhaust gases to the duct must be equal to the heat transfer from the duct to the surroundings, which must be equal to the energy loss of the exhaust gases in the pipe. That is,

$$\dot{Q} = \dot{Q}_{\text{internal}} = \dot{Q}_{\text{external}} = \Delta \dot{E}_{\text{exhaust gases}}$$

Assuming the duct to be at an average temperature of T_s , the quantities above can be expressed as

$\dot{Q}_{\text{internal}}$: $$\dot{Q} = h_i A \Delta T_{\ln} = h_i A \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} \rightarrow \dot{Q} = (9.50 \text{ Btu/h.ft}^2\text{.°F})(7.33 \text{ ft}^2) \frac{T_e - 940°\text{F}}{\ln\left(\frac{T_s - T_e}{T_s - 940}\right)}$$

$\dot{Q}_{\text{external}}$: $$\dot{Q} = h_o A(T_s - T_o) \rightarrow \dot{Q} = (3 \text{ Btu/h.ft}^2\text{.°F})(7.33 \text{ ft}^2)(T_s - 75)°\text{F}$$

$\Delta \dot{E}_{\text{exhaust gases}}$: $$\dot{Q} = \dot{m} C_p (T_e - T_i) \rightarrow \dot{Q} = (0.2 \times 3600 \text{ lbm/h})(0.256 \text{ Btu/lbm.°F})(940 - T_e)°\text{F}$$

This is a system of three equations with three unknowns whose solution is

$$\dot{Q} = \mathbf{13{,}790 \text{ Btu/h}}, \; T_e = \mathbf{865°F}, \text{ and } T_s = 702°\text{F}$$

Therefore, the exhaust gases will leave the pipe at 865°F.

17-81 Hot water enters a cast iron pipe whose outer surface is exposed to cold air with a specified heat transfer coefficient. The rate of heat loss from the water and the exit temperature of the water are to be determined.

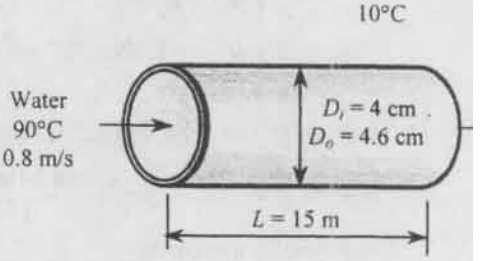
Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the pipe are smooth.

Properties We assume the water temperature not to drop significantly since the pipe is not very long. We will check this assumption later. The properties of water at 90°C are (Table A-15)

$$\rho = 965.3 \text{ kg/m}^3; \qquad k = 0.675 \text{ W/m.°C}$$

$$\upsilon = \mu / \rho = 0.326\times10^{-6} \text{ m}^2/\text{s}; \quad C_p = 4206 \text{ J/kg.°C}$$

$$\text{Pr} = 1.96$$



Analysis (a) The mass flow rate of water is

$$\dot{m} = \rho A_c V = (965.3 \text{ kg/m}^3)\frac{\pi(0.04 \text{ m})^2}{4}(0.8 \text{ m/s}) = 0.970 \text{ kg/s}$$

The Reynolds number is

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(0.8 \text{ m/s})(0.04 \text{ m})}{0.326\times10^{-6} \text{ m}^2/\text{s}} = 98{,}160$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4 D \text{Re}^{1/6} = 4.4(0.04 \text{ m})(98{,}160)^{1/6} = 1.2 \text{ m}$$

which are much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe. The friction factor corresponding to Re = 98,160 and ε/D = (0.026 cm)/(4 cm) = 0.0065 is determined from the Moody chart to be f = 0.034. Then the Nusselt number becomes

$$Nu = \frac{hD_h}{k} = 0.125 f \text{ Re Pr}^{1/3} = 0.125\times0.034\times98{,}160\times1.96^{1/3} = 522$$

and

$$h_i = h = \frac{k}{D_h} Nu = \frac{0.675 \text{ W/m.°C}}{0.04 \text{ m}}(522) = 8809 \text{ W/m}^2\text{.°C}$$

which is much greater than the convection heat transfer coefficient of 15 W/m².°C. Therefore, the convection thermal resistance inside the pipe is negligible, and thus the inner surface temperature of the pipe can be taken to be equal to the water temperature. Also, we expect the pipe to be nearly isothermal since it is made of thin metal (we check this later). Then the rate of heat loss from the pipe will be the sum of the convection and radiation from the outer surface at a temperature of 90°C, and is determined to be

$$A_o = \pi D_0 L = \pi(0.046 \text{ m})(15 \text{ m}) = 2.168 \text{ m}^2$$

$$\dot{Q}_{conv} = h_o A_o (T_s - T_{surr}) = (15 \text{ W/m}^2\text{.°C})(2.168 \text{ m}^2)(90-10)\text{°C} = 2602 \text{ W}$$

$$\dot{Q}_{rad} = \varepsilon A_0 \sigma (T_s^4 - T_{surr}^4)$$

$$= (0.7)(2.168 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(90+273 \text{ K})^4 - (10+273 \text{ K})^4\right] = 942 \text{ W}$$

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 2602 + 942 = \mathbf{3544 \text{ W}}$$

(b) The temperature at which water leaves the basement is

$$\dot{m} = \rho A_c V = (965.3 \text{ kg/m}^3)[\pi(0.04 \text{ m})^2/4](0.8 \text{ m/s}) = 0.970 \text{ kg/s}$$

$$\dot{Q} = \dot{m} C_p (T_i - T_e) \longrightarrow T_e = T_i - \frac{\dot{Q}}{\dot{m} C_p} = 90\text{°C} - \frac{3544 \text{ W}}{(0.970 \text{ kg/s})(4206 \text{ J/kg.°C})} = \mathbf{89.1°C}$$

The result justifies our assumption that the temperature drop of water is negligible. Also, the thermal resistance of the pipe and temperature drop across it are

$$R_{pipe} = \frac{\ln(D_2/D_1)}{4\pi k L} = \frac{\ln(4.6/4)}{4\pi(52 \text{ W/m.°C})(15 \text{ m})} = 1.65\times10^{-5} \text{ °C/W}$$

$$\Delta T_{pipe} = \dot{Q}_{total} R_{pipe} = (3544 \text{ W})(1.65\times10^{-5} \text{ °C/W}) = 0.06\text{°C}$$

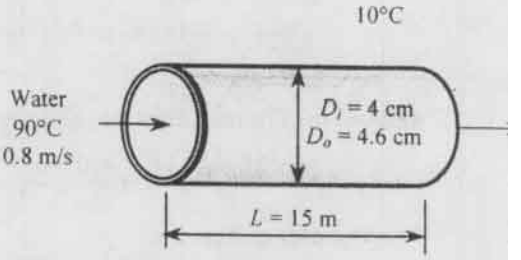
which justifies our assumption that the temperature drop across the pipe is negligible.

17-82 Hot water enters a copper pipe whose outer surface is exposed to cold air with a specified heat transfer coefficient. The rate of heat loss from the water and the exit temperature of the water are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the pipe are smooth.

Properties We assume the water temperature not to drop significantly since the pipe is not very long. We will check this assumption later. The properties of water at 90°C are (Table A-15)

$$\rho = 965.3\ \text{kg/m}^3; \qquad k = 0.675\ \text{W/m.°C}$$
$$\upsilon = \mu/\rho = 0.326\times10^{-6}\ \text{m}^2/\text{s}; \quad C_p = 4206\ \text{J/kg.°C}$$
$$\text{Pr} = 1.96$$



Analysis (a) The mass flow rate of water is

$$\dot{m} = \rho A_c V = (965.3\ \text{kg/m}^3)\frac{\pi(0.04\ \text{m})^2}{4}(0.8\ \text{m/s}) = 0.970\ \text{kg/s}$$

The Reynolds number is

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(0.8\ \text{m/s})(0.04\ \text{m})}{0.326\times10^{-6}\ \text{m}^2/\text{s}} = 98{,}160$$

which is greater than 4000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 4.4D\,\text{Re}^{1/6} = 4.4(0.04\ \text{m})(98{,}160)^{1/6} = 1.2\ \text{m}$$

which are much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe. Assuming the copper pipe to be smooth, the Nusselt number is determined to be

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023\times98{,}160^{0.8}\times1.96^{0.3} = 277$$

and
$$h_i = h = \frac{k}{D_h}Nu = \frac{0.675\ \text{W/m.°C}}{0.04\ \text{m}}(277) = 4674\ \text{W/m}^2\text{.°C}$$

which is much greater than the convection heat transfer coefficient of 15 W/m².°C. Therefore, the convection thermal resistance inside the pipe is negligible, and thus the inner surface temperature of the pipe can be taken to be equal to the water temperature. Also, we expect the pipe to be nearly isothermal since it is made of thin metal (we check this later). Then the rate of heat loss from the pipe will be the sum of the convection and radiation from the outer surface at a temperature of 90°C, and is determined to be

$$A_o = \pi D_0 L = \pi(0.046\ \text{m})(15\ \text{m}) = 2.168\ \text{m}^2$$

$$\dot{Q}_{conv} = h_o A_o (T_s - T_{surr}) = (15\ \text{W/m}^2\text{.°C})(2.168\ \text{m}^2)(90-10)\text{°C} = 2602\ \text{W}$$

$$\dot{Q}_{rad} = \varepsilon A_0 \sigma (T_s^{\,4} - T_{surr}^{\,4})$$
$$= (0.7)(2.168\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\text{.K}^4)[(90+273\ \text{K})^4 - (10+273\ \text{K})^4] = 942\ \text{W}$$

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 2602 + 942 = \mathbf{3544\ W}$$

(b) The temperature at which water leaves the basement is

$$\dot{m} = \rho A_c V = (965.3\ \text{kg/m}^3)[\pi(0.04\ \text{m})^2/4](0.8\ \text{m/s}) = 0.970\ \text{kg/s}$$

$$\dot{Q} = \dot{m}C_p(T_i - T_e) \longrightarrow T_e = T_i - \frac{\dot{Q}}{\dot{m}C_p} = 90\text{°C} - \frac{3544\ \text{W}}{(0.970\ \text{kg/s})(4206\ \text{J/kg.°C})} = \mathbf{89.1°C}$$

The result justifies our assumption that the temperature drop of water is negligible. Also, the thermal resistance of the pipe and temperature drop across it are

$$R_{pipe} = \frac{\ln(D_2/D_1)}{4\pi kL} = \frac{\ln(4.6/4)}{4\pi(386\ \text{W/m.°C})(15\ \text{m})} = 1.92\times10^{-6}\ \text{°C/W}$$

$$\Delta T_{pipe} = \dot{Q}_{total} R_{pipe} = (3544\ \text{W})(1.92\times10^{-6}\ \text{°C/W}) = 0.007\text{°C}$$

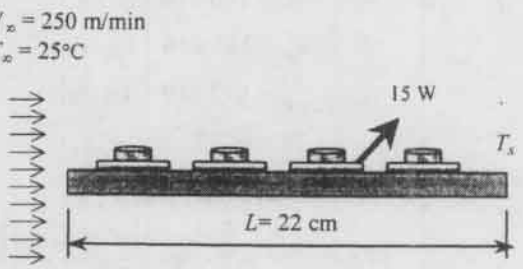
which justifies our assumption that the temperature drop across the pipe is negligible.

17-83 The heat generated by four transistors mounted on a thin vertical plate is dissipated by air blown over the plate on both surfaces. The temperature of the aluminum plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** The entire plate is nearly isothermal. **5** The exposed surface area of the transistor is taken to be equal to its base area. **6** Air is an ideal gas with constant properties. **7** The pressure of air is 1 atm.

Properties Assuming a film temperature of $T_f = 320\,\text{K}$, the properties of air are evaluated to be

$$k = 0.0275\ \text{W/m.°C}$$
$$\upsilon = 1.77\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.710$$



Analysis The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(250/60)\,\text{m/s}](0.22\,\text{m})}{1.77\times10^{-5}\ \text{m}^2/\text{s}} = 5.179\times10^4$$

which is smaller than the critical Reynolds number. Thus we have laminar flow. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664(5.179\times10^4)^{0.5}(0.710)^{1/3} = 134.8$$

$$h = \frac{k}{L}Nu = \frac{0.0275\ \text{W/m.°C}}{0.22\,\text{m}}(134.8) = 16.9\ \text{W/m}^2\text{.°C}$$

The temperature of aluminum plate then becomes

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 25°\text{C} + \frac{(4\times15)\,\text{W}}{(16.9\ \text{W/m}^2\text{.°C})[2(0.22\,\text{m})^2]} = \mathbf{61.7°C}$$

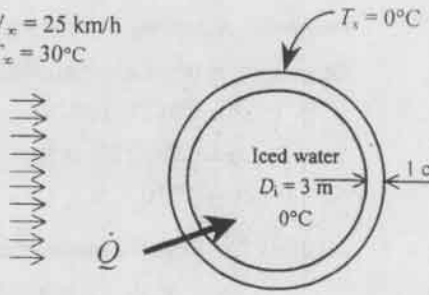
Discussion In reality, the heat transfer coefficient will be higher since the transistors will cause turbulence in the air.

17-84 A spherical tank used to store iced water is subjected to winds. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Thermal resistance of the tank is negligible. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 30°C are (Table A-18)

$$k = 0.0262 \text{ W/m.°C}$$
$$\upsilon = 1.60 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu_\infty = 1.86 \times 10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@273\text{ K}} = 1.72 \times 10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.712$$



Analysis (a) The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(25 \times 1000/3600) \text{ m/s}](3.02 \text{ m})}{1.60 \times 10^{-5} \text{ m}^2/\text{s}} = 1.31 \times 10^6$$

The Nusselt number corresponding to this Reynolds number is determined from

$$Nu = \frac{hD}{k} = 2 + \left[0.4 \text{ Re}^{0.5} + 0.06 \text{ Re}^{2/3}\right] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(1.31 \times 10^6)^{0.5} + 0.06(1.31 \times 10^6)^{2/3}\right](0.712)^{0.4} \left(\frac{1.86 \times 10^{-5}}{1.72 \times 10^{-5}}\right)^{1/4} = 1049$$

and $$h = \frac{k}{D} Nu = \frac{0.0262 \text{ W/m.°C}}{3.02 \text{ m}} (1049) = 9.10 \text{ W/m}^2\text{.°C}$$

The rate of heat transfer to the iced water is

$$\dot{Q} = hA(T_s - T_\infty) = h(\pi D^2)(T_s - T_\infty) = (9.10 \text{ W/m}^2\text{.°C})[\pi(3.02 \text{ m})^2](30 - 0)\text{°C} = \mathbf{7822 \text{ W}}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (7.822 \text{ kJ/s})(24 \times 3600 \text{ s}) = 675{,}800 \text{ kJ}$$

Then the amount of ice that melts during this period becomes

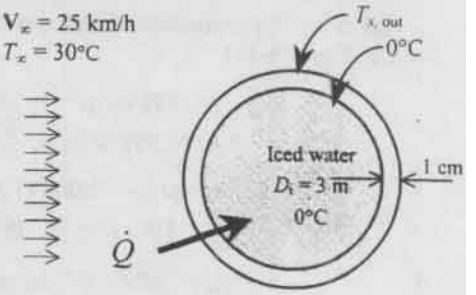
$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{675{,}800 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{2025 \text{ kg}}$$

17-85 A spherical tank used to store iced water is subjected to winds. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **7** The pressure of air is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 30°C are (Table A-18)

$$k = 0.0262 \text{ W/m.°C}$$
$$\upsilon = 1.60 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu_\infty = 1.86 \times 10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@273\text{ K}} = 1.72 \times 10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.712$$



Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(50 \times 1000/3600) \text{ m/s}](3.02 \text{ m})}{1.60 \times 10^{-5} \text{ m}^2/\text{s}} = 2.62 \times 10^6$$

The Nusselt number corresponding to this Reynolds number is determined from

$$Nu = \frac{hD}{k} = 2 + \left[0.4 \text{ Re}^{0.5} + 0.06 \text{ Re}^{2/3}\right] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(2.62 \times 10^6)^{0.5} + 0.06(2.62 \times 10^6)^{2/3}\right](0.712)^{0.4} \left(\frac{1.86 \times 10^{-5}}{1.72 \times 10^{-5}}\right)^{1/4} = 1594$$

and $$h = \frac{k}{D} Nu = \frac{0.0262 \text{ W/m.°C}}{3.02 \text{ m}}(1594) = 13.8 \text{ W/m}^2\text{.°C}$$

In steady operation, heat transfer through the tank by conduction is equal to the heat transfer from the outer surface of the tank by convection and radiation. Therefore,

$$\dot{Q} = \dot{Q}_{\text{through tank}} = \dot{Q}_{\text{from tank, conv+rad}}$$

$$\dot{Q} = \frac{T_{s,out} - T_{s,in}}{R_{sphere}} = h_o A_o (T_{surr} - T_{s,out}) + \varepsilon A_o \sigma (T_{surr}^4 - T_{s,out}^4)$$

where $$R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.51 - 1.50) \text{ m}}{4\pi(15 \text{ W/m.°C})(1.51 \text{ m})(1.50 \text{ m})} = 2.34 \times 10^{-5} \text{ °C/W}$$

Substituting,

$$\dot{Q} = \frac{T_{s,out} - 0°\text{C}}{2.34 \times 10^{-5} \text{ °C/W}} = (13.8 \text{ W/m}^2\text{.°C})(28.65 \text{ m}^2)(30 - T_{s,out})°\text{C}$$
$$+ (0.9)(28.65 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2\text{.K}^4)[(15 + 273 \text{ K})^4 - (T_{s,out} + 273 \text{ K})^4]$$

whose solution is

$$T_s = 0.3°\text{C} \text{ and } \dot{Q} = 1.36 \times 10^4 \text{ W} = \mathbf{13.6 \text{ kW}}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (13.6 \text{ kJ/s})(24 \times 3600 \text{ s}) = 1.175 \times 10^6 \text{ kJ}$$

Then the amount of ice that melts during this period becomes

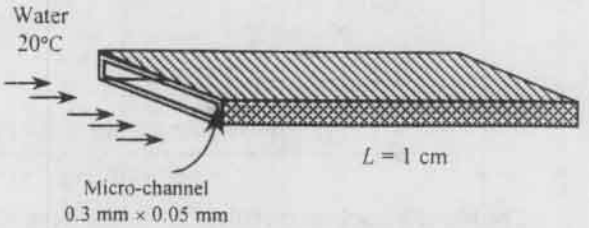
$$Q = m h_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{1.175 \times 10^6 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{3520 \text{ kg}}$$

17-86 Integrated circuits are cooled by water flowing through a series of microscopic channels. The temperature rise of water across the microchannels and the average surface temperature of the microchannels are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the microchannels are smooth. **3** Entrance effects are disregarded. **4** Any heat transfer from the side and cover surfaces are neglected.

Properties We assume the bulk mean temperature of water to be the inlet temperature of 20°C since the mean temperature of water at the inlet will rise somewhat as a result of heat gain through the microscopic channels. The properties of water at 20°C and the viscosity at the anticipated surface temperature of 25°C are (Table A-15)

$$\rho = 998\ \text{kg/m}^3$$
$$k = 0.598\ \text{W/m.°C}$$
$$\upsilon = \mu/\rho = 1.004\times10^{-6}\ \text{m}^2/\text{s}$$
$$C_p = 4182\ \text{J/kg.°C};\ \text{Pr} = 7.01$$
$$\mu_b = 1.002\times10^{-3}\ \text{kg/m.s}$$
$$\mu_{s,@25°C} = 0.891\times10^{-3}\ \text{kg/m.s}$$



Analysis (a) The mass flow rate of water is

$$\dot{m} = \rho\dot{V} = (998\ \text{kg/m}^3)(0.01\times10^{-3}\ \text{m}^3/\text{s}) = 0.00998\ \text{kg/s}$$

The temperature rise of water as it flows through the micro channels is

$$\dot{Q} = \dot{m}C_p\Delta T \longrightarrow \Delta T = \frac{\dot{Q}}{\dot{m}C_p} = \frac{50\ \text{J/s}}{(0.00998\ \text{kg/s})(4182\ \text{J/kg°C})} = \mathbf{1.2°C}$$

(b) The Reynolds number is

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.01\times10^{-3}\ \text{m}^3/\text{s}}{(0.05\times10^{-3}\ \text{m})(0.3\times10^{-3}\ \text{m})\times100} = 6.67\ \text{m/s}$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.05\times10^{-3}\ \text{m})(0.3\times10^{-3}\ \text{m})}{2(0.05\times10^{-3}\ \text{m}+0.3\times10^{-3}\ \text{m})} = 8.57\times10^{-5}\ \text{m}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(6.67\ \text{m/s})(8.57\times10^{-5}\ \text{m})}{1.004\times10^{-6}\ \text{m}^2/\text{s}} = 569$$

which is less than 4000. Therefore, the flow is laminar, and the thermal entry length in this case is

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D_h = 0.05(569)(7.01)(8.57\times10^{-5}\ \text{m}) = 0.017\ \text{m}$$

which is longer than the total length of the channels. Therefore, we can assume thermally developing flow, and determine the Nusselt number from (actually, the relation below is for circular tubes)

$$Nu = 1.86\left(\frac{\text{Re}\,\text{Pr}\,D_h}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left[\frac{(569)(7.01)(8.57\times10^{-5})}{0.01}\right]^{1/3}\left(\frac{1.002\times10^{-3}}{0.891\times10^{-3}}\right)^{0.14} = 6.14$$

and $$h = \frac{k}{D_h}Nu = \frac{0.598\ \text{W/m.°C}}{8.57\times10^{-5}\ \text{m}}(6.14) = 42{,}800\ \text{W/m}^2\text{.°C}$$

Then the average surface temperature of the base of the micro channels is determined to be

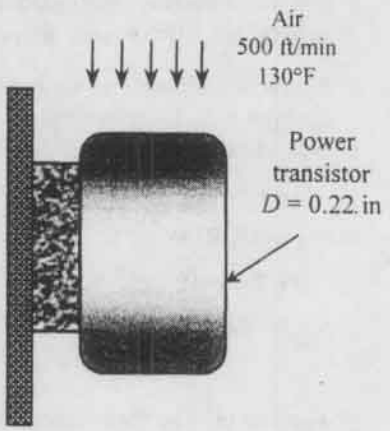
$$A = pL = 2(0.3+0.05)\times10^{-3}\times0.01 = 7\times10^{-6}\ \text{m}^2$$

$$\dot{Q} = hA(T_{s,ave} - T_{m,ave})$$

$$T_{s,ave} = T_{m,ave} + \frac{\dot{Q}}{hA} = \left(\frac{20+21.2}{2}\right)°\text{C} + \frac{(50/100)\ \text{W}}{(42{,}800\ \text{W/m}^2\text{.°C})(7\times10^{-6}\ \text{m}^2)} = \mathbf{22.2\ °C}$$

17-87E A cylindrical transistor mounted on a circuit board is cooled by air flowing over it. The maximum power rating of the transistor is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $T_f = (165+130)/2 = 147.5\,^\circ\text{F}$ are (Table A-18)

$$k = 0.0164 \text{ Btu/h.ft.}^\circ\text{F}$$
$$\upsilon = 0.208\times10^{-3} \text{ ft}^2/\text{s}$$
$$\text{Pr} = 0.72$$

Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{(500/60 \text{ ft/s})(0.22/12 \text{ ft})}{0.208\times10^{-3} \text{ ft}^2/\text{s}} = 735$$

The Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(735)^{0.5}(0.72)^{1/3}}{\left[1+(0.4/0.72)^{2/3}\right]^{1/4}}\left[1+\left(\frac{735}{282{,}000}\right)^{5/8}\right]^{4/5} = 13.8$$

and $$h = \frac{k}{D}Nu = \frac{0.0164 \text{ Btu/h.ft.}^\circ\text{F}}{(0.22/12 \text{ ft})}(13.8) = 12.3 \text{ Btu/h.ft}^2\text{.}^\circ\text{F}$$

Then the amount of power this transistor can dissipate safely becomes

$$\dot{Q} = hA(T_s - T_\infty) = h(\pi DL)(T_s - T_\infty)$$
$$= (12.3 \text{ Btu/h.ft}^2\text{.}^\circ\text{F})[\pi(0.22/12 \text{ ft})(0.25/12 \text{ ft})](165-130)^\circ\text{C}$$
$$= \mathbf{0.52\ Btu/h = 0.15\ W} \quad (1\text{ W} = 3.412 \text{ Btu/h})$$

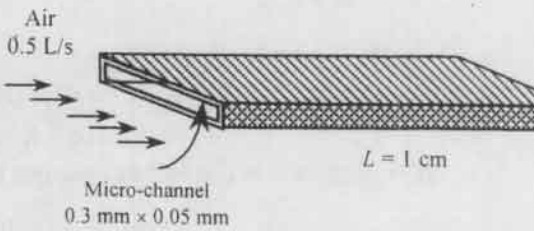
17-88 Integrated circuits are cooled by air flowing through a series of microscopic channels. The temperature rise of air across the microchannels and the average surface temperature of the microchannels are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the microchannels are smooth. **3** Entrance effects are disregarded. **4** Any heat transfer from the side and cover surfaces are neglected. **5** Air is an ideal gas with constant properties. **6** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 350 K since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the microscopic channels whose base areas are exposed to uniform heat flux. The properties of air at 1 atm and 350 K are (Table A-18)

$\rho = 1.009\ \text{kg/m}^3$
$k = 0.0297\ \text{W/m.°C}$
$\upsilon = 2.06\times10^{-5}\ \text{m}^2/\text{s}$
$C_p = 1008\ \text{J/kg.°C}$

$\text{Pr} = 0.706$
$\mu_b = 2.08\times10^{-5}\ \text{kg/m.s}$
$\mu_{s,@400\text{K}} = 2.29\times10^{-5}\ \text{kg/m.s}$



Analysis (a) The mass flow rate of air is

$$\dot{m} = \rho\dot{V} = (1.009\ \text{kg/m}^3)(0.5\times10^{-3}\ \text{m}^3/\text{s}) = 5.045\times10^{-4}\ \text{kg/s}$$

The temperature rise of air as it flows through the micro channels is

$$\dot{Q} = \dot{m}C_p\Delta T \ \rightarrow\ \Delta T = \frac{\dot{Q}}{\dot{m}C_p} = \frac{50\ \text{J/s}}{(5.045\times10^{-4}\ \text{kg/s})(1008\ \text{J/kg.°C})} = \mathbf{98.3°C}$$

(b) The Reynolds number is

$$V_m = \frac{\dot{V}}{A_c} = \frac{(0.5\times10^{-3}/100)\ \text{m}^3/\text{s}}{(0.05\times10^{-3}\ \text{m})(0.3\times10^{-3}\ \text{m})} = 333.3\ \text{m/s}$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.05\times10^{-3}\ \text{m})(0.3\times10^{-3}\ \text{m})}{2(0.05\times10^{-3}\ \text{m}+0.3\times10^{-3}\ \text{m})} = 8.57\times10^{-5}\ \text{m}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(333.3\ \text{m/s})(8.57\times10^{-5}\ \text{m})}{2.06\times10^{-5}\ \text{m}^2/\text{s}} = 1387$$

which is smaller than 4000. Therefore, the flow is laminar and the thermal entry length in this case is

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D_h = 0.05(1387)(0.706)(8.57\times10^{-5}\ \text{m}) = 0.0042\ \text{m}$$

which is 42% of the total length of the channels. Therefore, we can assume thermally developing flow, and determine the Nusselt number from (actually, the relation below is for circular tubes)

$$Nu = 1.86\left(\frac{\text{Re}\,\text{Pr}\,D_h}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left[\frac{(1387)(0.706)(8.57\times10^{-5})}{0.01}\right]^{1/3}\left(\frac{2.08\times10^{-5}}{2.29\times10^{-5}}\right)^{0.14} = 3.73$$

and $$h = \frac{k}{D_h}Nu = \frac{0.0297\ \text{W/m.°C}}{8.57\times10^{-5}\ \text{m}}(3.73) = 1293\ \text{W/m}^2\text{.°C}$$

Then the average surface temperature of the base of the micro channels becomes

$$A = pL = 2(0.3+0.05)\times10^{-3}\times0.01 = 7\times10^{-6}\ \text{m}^2$$

$$\dot{Q} = hA(T_{s,ave} - T_{m,ave})$$

$$T_{s,ave} = T_{m,ave} + \frac{\dot{Q}}{hA} = \left(\frac{20+98.3}{2}\right)°\text{C} + \frac{(50/100)\ \text{W}}{(1293\ \text{W/m}^2\text{.°C})(7\times10^{-6}\ \text{m}^2)} = \mathbf{114°C}$$

17-89 A computer is cooled by a fan blowing air through the case of the computer. The flow rate of the fan and the diameter of the casing of the fan are to be determined.

Assumptions **1** Steady flow conditions exist. **2** Heat flux is uniformly distributed. **3** Air is an ideal gas with constant properties.

Properties The relevant properties of air are (Tables A-1 and A-18)

$$C_p = 1006 \text{ J/kg.°C}$$

$$R = 0.287 \text{ kPa.m}^3/\text{kg.K}$$

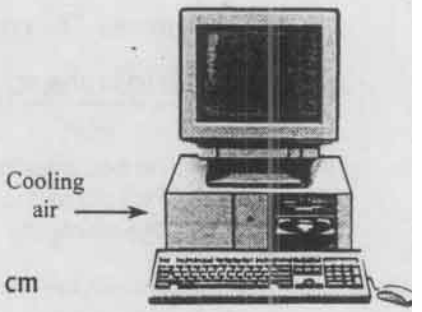
Analysis We need to determine the flow rate of air for the worst case scenario. Therefore, we assume the inlet temperature of air to be 50°C, the atmospheric pressure to be 70.12 kPa, and disregard any heat transfer from the outer surfaces of the computer case. The mass flow rate of air required to absorb heat at a rate of 45 W can be determined from

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in}) \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})} = \frac{45 \text{ J/s}}{(1006 \text{ J/kg.°C})(60-50)\text{°C}} = 0.00447 \text{ kg/s}$$

In the worst case the exhaust fan will handle air at 60°C. Then the density of air entering the fan and the volume flow rate becomes

$$\rho = \frac{P}{RT} = \frac{70.12 \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg.K})(60+273)\text{K}} = 0.7337 \text{ kg/m}^3$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.00447 \text{ kg/s}}{0.7337 \text{ kg/m}^3} = 0.00609 \text{ m}^3/\text{s} = 0.366 \text{ m}^3/\text{min}$$



For an average velocity of 120 m/min, the diameter of the duct in which the fan is installed can be determined from

$$\dot{V} = A_c V = \frac{\pi D^2}{4} V \longrightarrow D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{4(0.366 \text{ m}^3/\text{min})}{\pi(120 \text{ m/min})}} = 0.062 \text{ m} = \mathbf{6.2 \text{ cm}}$$

17-90 Wind is blowing over the roof of a house. The rate of heat transfer through the roof and the cost of this heat loss for 14-h period are to be determined.

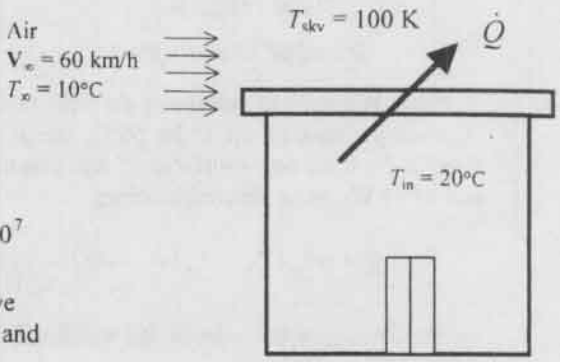
Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties Assuming a film temperature of $T_f = 285$ K, the properties of air are

$k = 0.0250$ W/m.°C

$\upsilon = 1.44\times10^{-5}$ m^2/s

Pr = 0.716



Analysis The Reynolds number is

$$Re_L = \frac{V_\infty L}{\upsilon} = \frac{[(60\times1000/3600)\text{ m/s}](20\text{ m})}{1.44\times10^{-5}\text{ m}^2/\text{s}} = 2.31\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Then the Nusselt number and the heat transfer coefficient are determined to be

$$Nu = \frac{hL}{k} = (0.037\,Re_L^{0.8} - 871)\,Pr^{1/3} = [0.037(2.31\times10^7)^{0.8} - 871](0.716)^{1/3} = 2.50\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.0250\text{ W/m.°C}}{20\text{ m}}(2.50\times10^4) = 31.3\text{ W/m}^2.\text{°C}$$

In steady operation, heat transfer from the room to the roof (by convection and radiation) must be equal to the heat transfer from the roof to the surroundings (by convection and radiation), which must be equal to the heat transfer through the roof by conduction. That is,

$$\dot{Q} = \dot{Q}_{\text{room to roof, conv+rad}} = \dot{Q}_{\text{roof, cond}} = \dot{Q}_{\text{roof to surroundings, conv+rad}}$$

Taking the inner and outer surface temperatures of the roof to be $T_{s,in}$ and $T_{s,out}$, respectively, the quantities above can be expressed as

$$\dot{Q}_{\text{room to roof, conv+rad}} = h_i A(T_{room} - T_{s,in}) + \varepsilon A\sigma(T_{room}^4 - T_{s,in}^4) = (5\text{ W/m}^2.\text{°C})(300\text{ m}^2)(20 - T_{s,in})\text{°C}$$
$$+ (0.9)(300\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\left[(20+273\text{ K})^4 - (T_{s,in}+273\text{ K})^4\right]$$

$$\dot{Q}_{\text{roof, cond}} = kA\frac{T_{s,in} - T_{s,out}}{L} = (2\text{ W/m.°C})(300\text{ m}^2)\frac{T_{s,in} - T_{s,out}}{0.15\text{ m}}$$

$$\dot{Q}_{\text{roof to surr, conv+rad}} = h_o A(T_{s,out} - T_{surr}) + \varepsilon A\sigma(T_{s,out}^4 - T_{surr}^4) = (31.3\text{ W/m}^2.\text{°C})(300\text{ m}^2)(T_{s,out} - 10)\text{°C}$$
$$+ (0.9)(300\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\left[(T_{s,out}+273\text{ K})^4 - (100\text{ K})^4\right]$$

Solving the equations above simultaneously gives

$$\dot{Q} = 2.80\times10^4\text{ W} = \mathbf{28.0\,kW}, \qquad T_{s,in} = 10.5\text{°C}, \text{ and } T_{s,out} = 3.5\text{°C}$$

The total amount of natural gas consumption during a 14-hour period is

$$Q_{gas} = \frac{Q_{total}}{0.85} = \frac{\dot{Q}\Delta t}{0.85} = \frac{(28.0\text{ kJ/s})(14\times3600\text{ s})}{0.85}\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right) = 15.7\text{ therms}$$

Finally, the money lost through the roof during that period is

Money lost = (17.1 therms)($0.60 / therm) = **$9.44**

17-91 Steam is flowing in a stainless steel pipe while air is flowing across the pipe. The rate of heat loss from the steam per unit length of the pipe is to be determined. √

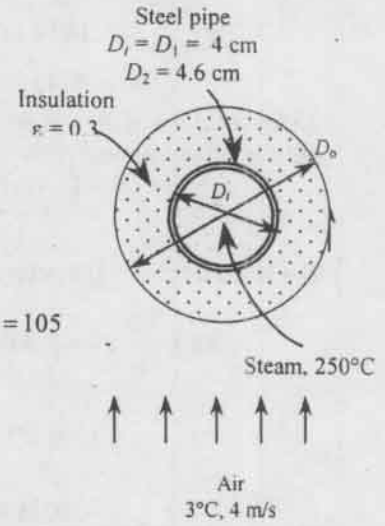
Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm.

Properties Assuming a film temperature of $T_f = 280$ K, the properties of air are (Table A-18)

$$k = 0.0248 \text{ W/m.°C}, \qquad \upsilon = 1.44\times10^{-5} \text{ m}^2/\text{s}, \quad \text{and} \quad \text{Pr} = 0.716$$

Analysis The outer diameter of insulated pipe is $D_o = 4.6+2\times3.5=11.6$ cm = 0.116 m. The Reynolds number is

$$\text{Re} = \frac{V_\infty D_o}{\upsilon} = \frac{(4 \text{ m/s})(0.116 \text{ m})}{1.44\times10^{-5} \text{ m}^2/\text{s}} = 3.222\times10^4$$



The Nusselt number for flow across a cylinder is determined from

$$Nu = \frac{hD_o}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(3.222\times10^4)^{0.5}(0.716)^{1/3}}{\left[1+(0.4/0.716)^{2/3}\right]^{1/4}}\left[1+\left(\frac{3.222\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 105$$

and $$h_o = \frac{k}{D_o}Nu = \frac{0.0248 \text{ W/m}\cdot\text{°C}}{0.116 \text{ m}}(105) = 22.5 \text{ W/m}^2\cdot\text{°C}$$

Area of the outer surface of the pipe per m length of the pipe is

$$A_o = \pi D_o L = \pi(0.116 \text{ m})(1 \text{ m}) = 0.3644 \text{ m}^2$$

In steady operation, heat transfer from the steam through the pipe and the insulation to the outer surface (by first convection and then conduction) must be equal to the heat transfer from the outer surface to the surroundings (by simultaneous convection and radiation). That is,

$$\dot{Q} = \dot{Q}_{\text{pipe and insulation}} = \dot{Q}_{\text{surface to surroundings}}$$

Using the thermal resistance network, heat transfer from the steam to the outer surface is expressed as

$$R_{conv,i} = \frac{1}{h_i A_i} = \frac{1}{(80 \text{ W/m}^2.\text{°C})[\pi(0.04 \text{ m})(1 \text{ m})]} = 0.0995 \text{ °C/W}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(2.3/2)}{2\pi(15 \text{ W/m.°C})(1 \text{ m})} = 0.0015 \text{ °C/W}$$

$$R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi kL} = \frac{\ln(5.8/2.3)}{2\pi(0.038 \text{ W/m.°C})(1 \text{ m})} = 3.874 \text{ °C/W}$$

and $$\dot{Q}_{\text{pipe and ins}} = \frac{T_{\infty1} - T_s}{R_{conv,i} + R_{pipe} + R_{insulation}} = \frac{(250 - T_s)\text{°C}}{(0.0995+0.0015+3.874)\text{°C/W}}$$

Heat transfer from the outer surface can be expressed as

$$\dot{Q}_{\text{surface to surr, conv+rad}} = h_o A_o(T_s - T_{surr}) + \varepsilon A_o\sigma(T_s^4 - T_{surr}^4) = (43.6 \text{ W/m}^2.\text{°C})(0.3644 \text{ m}^2)(T_s - 3)\text{°C}$$
$$+(0.9)(0.3644 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(3+273 \text{ K})^4 - (T_{s,in}+273 \text{ K})^4\right]$$

Solving the two equations above simultaneously, the surface temperature and the heat transfer rate per m length of the pipe are determined to be

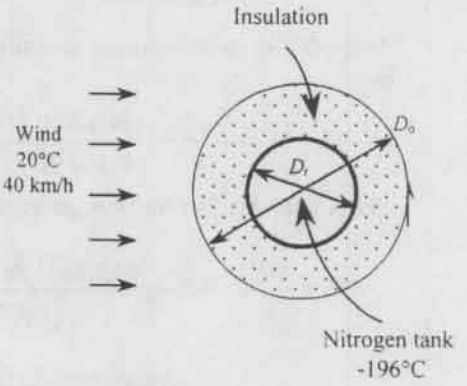
$$T_s = 9.9\text{°C} \text{ and } \dot{Q} = \mathbf{60.4 \text{ W}} \text{ (per m length)}$$

17-92 A spherical tank filled with liquid nitrogen is exposed to winds. The rate of evaporation of the liquid nitrogen due to heat transfer from the air is to be determined for three cases. √

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 20°C are

$$k = 0.0255 \text{ W/m.°C}$$
$$\upsilon = 1.50 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu_\infty = 1.82 \times 10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@196\,°C} = 1.05 \times 10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.713$$



Analysis (*a*) When there is no insulation, $D = D_i = 4$ m, and the Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(40 \times 1000/3600) \text{ m/s}](4 \text{ m})}{1.50 \times 10^{-5} \text{ m}^2/\text{s}} = 2.963 \times 10^6$$

The Nusselt number is determined from

$$Nu = \frac{hD}{k} = 2 + \left[0.4 \text{ Re}^{0.5} + 0.06 \text{ Re}^{2/3}\right] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(2.963 \times 10^6)^{0.5} + 0.06(2.963 \times 10^6)^{2/3}\right](0.713)^{0.4} \left(\frac{1.82 \times 10^{-5}}{1.05 \times 10^{-5}}\right)^{1/4} = 1933$$

and $$h = \frac{k}{D} Nu = \frac{0.0255 \text{ W/m.°C}}{4 \text{ m}} (1933) = 12.3 \text{ W/m}^2\text{.°C}$$

The rate of heat transfer to the liquid nitrogen is

$$\dot{Q} = hA(T_s - T_\infty) = h(\pi D^2)(T_s - T_\infty)$$
$$= (12.3 \text{ W/m}^2\text{.°C})[\pi(4 \text{ m})^2][(20 - (-196)]\text{°C} = 134 \text{ kW}$$

The rate of evaporation of liquid nitrogen then becomes

$$\dot{Q} = \dot{m} h_{if} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{134 \text{ kJ/s}}{198 \text{ kJ/kg}} = \mathbf{0.676 \text{ kg/s}}$$

(*b*) Note that after insulation the outer surface temperature and diameter will change. Therefore we need to evaluate dynamic viscosity at a new surface temperature which we will assume to be 200 K. At 200 K, $\mu = 1.34 \times 10^{-5}$ kg/m.s. Noting that $D = D_0 = 4.1$ m, the Nusselt number becomes

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(40 \times 1000/3600) \text{ m/s}](4.1 \text{ m})}{1.50 \times 10^{-5} \text{ m}^2/\text{s}} = 3.037 \times 10^6$$

$$Nu = \frac{hD}{k} = 2 + \left[0.4 \text{ Re}^{0.5} + 0.06 \text{ Re}^{2/3}\right] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(3.037 \times 10^6)^{0.5} + 0.06(3.037 \times 10^6)^{2/3}\right](0.713)^{0.4} \left(\frac{1.82 \times 10^{-5}}{1.34 \times 10^{-5}}\right)^{1/4} = 1846$$

and $$h = \frac{k}{D} Nu = \frac{0.0255 \text{ W/m.°C}}{4.1 \text{ m}} (1846) = 11.5 \text{ W/m}^2\text{.°C}$$

The rate of heat transfer to the liquid nitrogen is

$$A = \pi D^2 = \pi(4.1\,\text{m})^2 = 52.81\,\text{m}^2$$

$$\dot{Q} = \frac{T_\infty - T_{s,\tan k}}{R_{insulation} + R_{conv}} = \frac{T_\infty - T_{s,\tan k}}{\dfrac{r_2 - r_1}{4\pi k r_1 r_2} + \dfrac{1}{hA}}$$

$$= \frac{[20-(-196)]^\circ\text{C}}{\dfrac{(2.05-2)\,\text{m}}{4\pi(0.035\,\text{W/m.°C})(2.05\,\text{m})(2\,\text{m})} + \dfrac{1}{(11.5\,\text{W/m}^2\text{.°C})(52.81\,\text{m}^2)}} = 7353\,\text{W}$$

The rate of evaporation of liquid nitrogen then becomes

$$\dot{Q} = \dot{m}h_{if} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{7.353\,\text{kJ/s}}{198\,\text{kJ/kg}} = \mathbf{0.0371\,kg/s}$$

(*c*) Again we use the dynamic viscosity value at the estimated surface temperature of 200 K. Noting that $D = D_0 = 4.04$ m in this case, the Nusselt number becomes

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(40\times1000/3600)\,\text{m/s}](4.04\,\text{m})}{1.50\times10^{-5}\,\text{m}^2/\text{s}} = 2.993\times10^6$$

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\text{Re}^{0.5} + 0.06\,\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(2.993\times10^6)^{0.5} + 0.06(2.993\times10^6)^{2/3}\right](0.713)^{0.4}\left(\frac{1.82\times10^{-5}}{1.34\times10^{-5}}\right)^{1/4} = 1829$$

and $$h = \frac{k}{D}Nu = \frac{0.0255\,\text{W/m.°C}}{4.04\,\text{m}}(1829) = 11.6\,\text{W/m}^2\text{.°C}$$

The rate of heat transfer to the liquid nitrogen is

$$A = \pi D^2 = \pi(4.04\,\text{m})^2 = 51.28\,\text{m}^2$$

$$\dot{Q} = \frac{T_\infty - T_{s,\tan k}}{R_{insulation} + R_{conv}} = \frac{T_\infty - T_{s,\tan k}}{\dfrac{r_2 - r_1}{4\pi k r_1 r_2} + \dfrac{1}{hA}}$$

$$= \frac{[20-(-196)]^\circ\text{C}}{\dfrac{(2.02-2)\,\text{m}}{4\pi(0.00005\,\text{W/m.°C})(2.02\,\text{m})(2\,\text{m})} + \dfrac{1}{(11.6\,\text{W/m}^2\text{.°C})(51.28\,\text{m}^2)}} = 27.4\,\text{W}$$

The rate of evaporation of liquid nitrogen then becomes

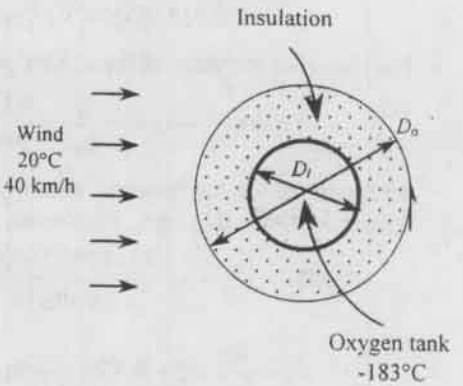
$$\dot{Q} = \dot{m}h_{if} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{0.0274\,\text{kJ/s}}{198\,\text{kJ/kg}} = \mathbf{1.38\times10^{-4}\,kg/s}$$

17-93 A spherical tank filled with liquid oxygen is exposed to ambient winds. The rate of evaporation of the liquid oxygen due to heat transfer from the air is to be determined for three cases. √

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **7** The pressure of air is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 20 °C are

$$k = 0.0255 \text{ W/m.°C}$$
$$\upsilon = 1.50 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu_\infty = 1.82 \times 10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@196\,°C} = 1.05 \times 10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.713$$



Analysis (*a*) When there is no insulation, $D = D_i = 4$ m, and the Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(40 \times 1000/3600) \text{ m/s}](4 \text{ m})}{1.50 \times 10^{-5} \text{ m}^2/\text{s}} = 2.963 \times 10^6$$

The Nusselt number is determined from

$$Nu = \frac{hD}{k} = 2 + \left[0.4 \,\text{Re}^{0.5} + 0.06 \,\text{Re}^{2/3}\right] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(2.963 \times 10^6)^{0.5} + 0.06(2.963 \times 10^6)^{2/3}\right](0.713)^{0.4} \left(\frac{1.82 \times 10^{-5}}{1.05 \times 10^{-5}}\right)^{1/4} = 1933$$

and $$h = \frac{k}{D} Nu = \frac{0.0255 \text{ W/m.°C}}{4 \text{ m}} (1933) = 12.3 \text{ W/m}^2\text{.°C}$$

The rate of heat transfer to the liquid oxygen is

$$\dot{Q} = hA(T_s - T_\infty) = h(\pi D^2)(T_s - T_\infty) = (12.3 \text{ W/m}^2\text{.°C})[\pi(4 \text{ m})^2][(20 - (-183)]\text{°C} = 126 \text{ kW}$$

The rate of evaporation of liquid oxygen then becomes

$$\dot{Q} = \dot{m} h_{if} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{126 \text{ kJ/s}}{213 \text{ kJ/kg}} = \mathbf{0.59 \text{ kg/s}}$$

(*b*) Note that after insulation the outer surface temperature and diameter will change. Therefore we need to evaluate dynamic viscosity at a new surface temperature which we will assume to be 200 K. At 200 K, $\mu = 1.34 \times 10^{-5}$ kg/m.s. Noting that $D = D_0 = 4.1$ m, the Nusselt number becomes

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(40 \times 1000/3600) \text{ m/s}](4.1 \text{ m})}{1.50 \times 10^{-5} \text{ m}^2/\text{s}} = 3.037 \times 10^6$$

$$Nu = \frac{hD}{k} = 2 + \left[0.4 \,\text{Re}^{0.5} + 0.06 \,\text{Re}^{2/3}\right] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(3.037 \times 10^6)^{0.5} + 0.06(3.037 \times 10^6)^{2/3}\right](0.713)^{0.4} \left(\frac{1.82 \times 10^{-5}}{1.34 \times 10^{-5}}\right)^{1/4} = 1846$$

and $$h = \frac{k}{D} Nu = \frac{0.0255 \text{ W/m.°C}}{4.1 \text{ m}} (1846) = 11.5 \text{ W/m}^2\text{.°C}$$

The rate of heat transfer to the liquid nitrogen is

$$A = \pi D^2 = \pi(4.1\,\text{m})^2 = 52.81\,\text{m}^2$$

$$\dot{Q} = \frac{T_\infty - T_{s,\text{tan}\,k}}{R_{insulation} + R_{conv}} = \frac{T_\infty - T_{s,\text{tan}\,k}}{\dfrac{r_2 - r_1}{4\pi k r_1 r_2} + \dfrac{1}{hA}}$$

$$= \frac{[20-(-183)]^\circ\text{C}}{\dfrac{(2.05-2)\,\text{m}}{4\pi(0.035\,\text{W/m.°C})(2.05\,\text{m})(2\,\text{m})} + \dfrac{1}{(11.5\,\text{W/m}^2\text{.°C})(52.81\,\text{m}^2)}} = 6910\,\text{W}$$

The rate of evaporation of liquid nitrogen then becomes

$$\dot{Q} = \dot{m} h_{if} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{6.910\,\text{kJ/s}}{213\,\text{kJ/kg}} = \mathbf{0.0324\ kg/s}$$

(*c*) Again we use the dynamic viscosity value at the estimated surface temperature of 200 K. Noting that $D = D_0 = 4.04$ m in this case, the Nusselt number becomes

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(40\times1000/3600)\,\text{m/s}](4.04\,\text{m})}{1.50\times10^{-5}\,\text{m}^2/\text{s}} = 2.993\times10^6$$

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\text{Re}^{0.5} + 0.06\,\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(2.993\times10^6)^{0.5} + 0.06(2.993\times10^6)^{2/3}\right](0.713)^{0.4}\left(\frac{1.82\times10^{-5}}{1.34\times10^{-5}}\right)^{1/4} = 1829$$

and $$h = \frac{k}{D} Nu = \frac{0.0255\,\text{W/m.°C}}{4.04\,\text{m}}(1829) = 11.6\,\text{W/m}^2\text{.°C}$$

The rate of heat transfer to the liquid nitrogen is

$$A = \pi D^2 = \pi(4.04\,\text{m})^2 = 51.28\,\text{m}^2$$

$$\dot{Q} = \frac{T_\infty - T_{s,\text{tan}\,k}}{R_{insulation} + R_{conv}} = \frac{T_\infty - T_{s,\text{tan}\,k}}{\dfrac{r_2 - r_1}{4\pi k r_1 r_2} + \dfrac{1}{hA}}$$

$$= \frac{[20-(-183)]^\circ\text{C}}{\dfrac{(2.02-2)\,\text{m}}{4\pi(0.00005\,\text{W/m.°C})(2.02\,\text{m})(2\,\text{m})} + \dfrac{1}{(11.6\,\text{W/m}^2\text{.°C})(51.28\,\text{m}^2)}} = 25.8\,\text{W}$$

The rate of evaporation of liquid oxygen then becomes

$$\dot{Q} = \dot{m} h_{if} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{0.0258\,\text{kJ/s}}{213\,\text{kJ/kg}} = \mathbf{1.21\times10^{-4}\ kg/s}$$

17-94 A circuit board houses 80 closely spaced logic chips on one side. All the heat generated is conducted across the circuit board and is dissipated from the back side of the board to the ambient air, which is forced to flow over the surface by a fan. The temperatures on the two sides of the circuit board are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties. **7** The pressure of air is 1 atm.

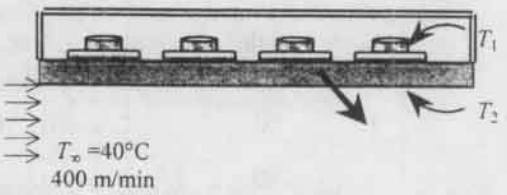
Properties Assuming a film temperature of $T_f = 315$ K, the properties of air are (Table A-18)

$$k = 0.0272 \text{ W/m.°C}$$
$$\upsilon = 1.72\times10^{-5} \text{ m}^2/\text{s}$$
$$\mathrm{Pr} = 0.710$$

Analysis The Reynolds number is

$$\mathrm{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(400/60)\text{ m/s}](0.18\text{ m})}{1.72\times10^{-5}\text{ m}^2/\text{s}} = 6.977\times10^4$$

which is less than the critical Reynolds number. Therefore, the flow is laminar. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be



$$Nu = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} = 0.664(6.977\times10^4)^{0.5}(0.710)^{1/3} = 156$$
$$h = \frac{k}{L}Nu = \frac{0.0272\text{ W/m.°C}}{0.18\text{ m}}(156) = 23.6\text{ W/m}^2\text{.°C}$$

The temperatures on the two sides of the circuit board are

$$\dot{Q} = hA(T_2 - T_\infty) \rightarrow T_2 = T_\infty + \frac{\dot{Q}}{hA}$$
$$= 40°\text{C} + \frac{(80\times0.04)\text{ W}}{(23.6\text{ W/m}^2\text{.°C})(0.12\text{ m})(0.18\text{ m})} = \mathbf{46.28°C}$$
$$\dot{Q} = \frac{kA}{L}(T_1 - T_2) \rightarrow T_1 = T_2 + \frac{\dot{Q}L}{kA}$$
$$= 46.28°\text{C} + \frac{(80\times0.04\text{ W})(0.003\text{ m})}{(16\text{ W/m.°C})(0.12\text{ m})(0.18\text{ m})} = \mathbf{46.31°C}$$

17-95E The equivalent wind chill temperature of an environment at 10°F at various winds speeds are

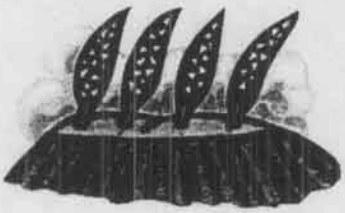
V = 10 mph: $$T_{equiv} = 91.4 - (91.4 - T_{ambient})(0.475 - 0.0203V + 0.304\sqrt{V})$$
$$= 91.4 - [91.4 - (10°\text{F})][0.475 - 0.0203(10\text{ mph}) + 0.304\sqrt{10\text{ mph}}] = \mathbf{-9°\,F}$$

V = 20 mph: $$T_{equiv} = 91.4 - [91.4 - (10°\text{F})][0.475 - 0.0203(20\text{ mph}) + 0.304\sqrt{20\text{ mph}}] = \mathbf{-24.9°\,F}$$

V = 30 mph: $$T_{equiv} = 91.4 - [91.4 - (10°\text{F})][0.475 - 0.0203(30\text{ mph}) + 0.304\sqrt{30\text{ mph}}] = \mathbf{-33.2°\,F}$$

V = 40 mph: $$T_{equiv} = 91.4 - [91.4 - (10°\text{F})][0.475 - 0.0203(40\text{ mph}) + 0.304\sqrt{40\text{ mph}}] = \mathbf{-37.7°\,F}$$

In the last 3 cases, the person needs to be concerned about the possibility of freezing.



Chapter 18
NATURAL CONVECTION

Physical Mechanisms of Natural Convection

18-1C Natural convection is the mode of heat transfer that occurs between a solid and a fluid which moves under the influence of natural means. Natural convection differs from forced convection in that fluid motion in natural convection is caused by natural effects such as buoyancy.

18-2C The convection heat transfer coefficient is usually higher in forced convection because of the higher fluid velocities involved.

18-3C The hot boiled egg in a spacecraft will cool faster when the spacecraft is on the ground since there is no gravity in space, and thus there will be no natural convection currents which is due to the buoyancy force.

18-4C. The upward force exerted by a fluid on a body completely or partially immersed in it is called the buoyancy or "lifting" force. The buoyancy force is proportional to the density of the medium. Therefore, the buoyancy force is the largest in mercury, followed by in water, air, and the evacuated chamber. Note that in an evacuated chamber there will be no buoyancy force because of absence of any fluid in the medium.

18-5C The buoyancy force is proportional to the density of the medium, and thus is larger in sea water than it is in fresh water. Therefore, the hull of a ship will sink deeper in fresh water because of the smaller buoyancy force acting upwards.

18-6C A spring scale measures the "weight" force acting on it, and the person will weigh less in water because of the upward buoyancy force acting on the person's body.

18-7C The greater the volume expansion coefficient, the greater the change in density with temperature, the greater the buoyancy force, and thus the greater the natural convection currents.

18-8C There cannot be any natural convection heat transfer in a medium that experiences no change in volume with temperature.

18-9C The lines on an interferometer photograph represent isotherms (constant temperature lines) for a gas, which correspond to the lines of constant density. Closely packed lines on a photograph represent a large temperature gradient.

18-10C The Grashof number represents the ratio of the buoyancy force to the viscous force acting on a fluid. The inertial forces in Reynolds number is replaced by the buoyancy forces in Grashof number.

18-11 The volume expansion coefficient is defined as $\beta = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$. For an ideal gas, $P = \rho RT$ or $\rho = \frac{P}{RT}$, and thus $\beta = -\frac{1}{\rho}\left(\frac{\partial (P/RT)}{\partial T}\right)_P = \frac{-1}{\rho}\left(\frac{-P}{RT^2}\right) = \frac{1}{\rho T}\left(\frac{P}{RT}\right) = \frac{1}{\rho T}(\rho) = \frac{1}{T}$

Natural Convection Over Surfaces

18-12C Rayleigh number is the product of the Grashof and Prandtl numbers.

18-13C A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr^{1/4}}$.

18-14C No, a hot surface will cool slower when facing down since the warmer air in this position cannot rise and escape easily.

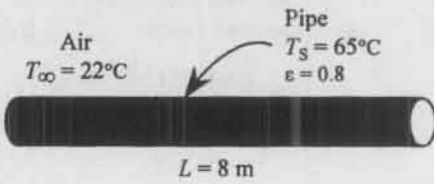
18-15C The heat flux will be higher at the bottom of the plate since the thickness of the boundary layer which is a measure of thermal resistance is the lowest there.

18-16 A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the pipe is constant.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (65+22)/2 = 43.5°C = 316.5$ K are (Table A-18)

$$k = 0.0272 \text{ W/m.°C}$$
$$\upsilon = 1.72 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.710$$
$$\beta = \frac{1}{T_f} = \frac{1}{316.5 \text{ K}} = 0.00316 \text{ K}^{-1}$$



Analysis (*a*) The characteristic length in this case is the outer diameter of the pipe, $\delta = D = 0.06$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.00316 \text{ K}^{-1})(65-22 \text{ K})(0.06 \text{ m})^3}{(1.72\times10^{-5} \text{ m}^2/\text{s})^2}(0.710) = 690{,}298$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(690{,}298)^{1/6}}{\left[1+(0.559/0.710)^{9/16}\right]^{8/27}}\right\}^2 = 13.1$$

$$h = \frac{k}{\delta}Nu = \frac{0.0272 \text{ W/m.°C}}{0.06 \text{ m}}(13.1) = 5.94 \text{ W/m}^2.°C$$

$$A = \pi DL = \pi(0.06 \text{ m})(8 \text{ m}) = 1.51 \text{ m}^2$$

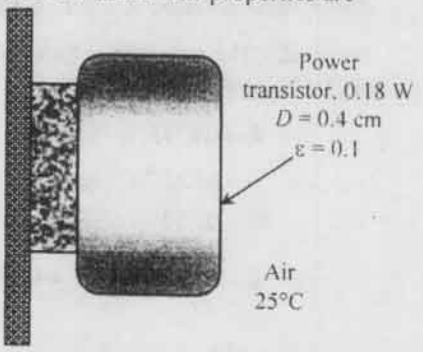
$$\dot{Q} = hA(T_s - T_\infty) = (5.94 \text{ W/m}^2.°C)(1.51 \text{ m}^2)(65-22)°C = \mathbf{386\ W}$$

(*b*) The radiation heat loss from the pipe is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4) = (0.8)(1.51 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(65+273 \text{ K})^4 - (22+273 \text{ K})^4\right] = \mathbf{375\ W}$$

18-17 A power transistor mounted on the wall dissipates 0.18 W. The surface temperature of the transistor is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Any heat transfer from the base surface is disregarded. **4** The local atmospheric pressure is 1 atm. **5** Air properties are evaluated at 100°C.



Properties The properties of air at 1 atm and the anticipated film temperature of 100°C = 373 K are (Table A-18)

$$k = 0.0313\ \text{W/m.°C}$$

$$\upsilon = 2.31\times10^{-5}\ \text{m}^2/\text{s}$$

$$\text{Pr} = 0.705$$

$$\beta = \frac{1}{T_f} = \frac{1}{373\ \text{K}} = 0.00268\ \text{K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 150°C for the evaluation of h. We will check the accuracy of this guess later and repeat the calculations if necessary.

The transistor loses heat through its cylindrical surface as well as its top surface. For convenience, we take the heat transfer coefficient at the top surface of the transistor to be the same as that of its side surface. (The alternative is to treat the top surface as a vertical plate, but this will double the amount of calculations without providing much improvement in accuracy since the area of the top surface is much smaller and it is circular in shape instead of being rectangular). The characteristic length in this case is the outer diameter of the transistor, $\delta = D = 0.004$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.00268\ \text{K}^{-1})(150-35\ \text{K})(0.004\ \text{m})^3}{(2.31\times10^{-5}\ \text{m}^2/\text{s})^2}(0.705) = 278$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(278)^{1/6}}{\left[1+(0.559/0.705)^{9/16}\right]^{8/27}}\right\}^2 = 2.02$$

$$h = \frac{k}{\delta}Nu = \frac{0.0313\ \text{W/m.°C}}{0.004\ \text{m}}(2.02) = 15.8\ \text{W/m}^2.\text{°C}$$

$$A = \pi DL + \pi D^2/4 = \pi(0.004\ \text{m})(0.0045\ \text{m}) + \pi(0.004\ \text{m})^2/4 = 0.0000691\ \text{m}^2$$

and

$$\begin{aligned}\dot{Q} &= hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)\\ &= (15.8\ \text{W/m}^2.\text{°C})(0.0000691\ \text{m}^2)(150-35)\ \text{°C}\\ &\quad + (0.1)(0.0000691\ \text{m}^2)(5.67\times10^{-8})\left[(150+273)^4 - (25+273\ \text{K})^4\right]\\ &= 0.15\ \text{W}\end{aligned}$$

which is lower that 0.18 W. Therefore, the surface temperature must be higher. By trying other temperatures, the surface temperature that results in a total heat transfer rate of 0.18 W is determined to be

$$T_s = \mathbf{174°C}$$

Discussion The film temperature in this case is $(T_s+T_\infty)/2 = (174+25)/2 = 99.5$°C, which is very close to the value of 100°C used in the evaluation of the properties. Therefore, there is no need to repeat calculations in this case.

18-18E A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

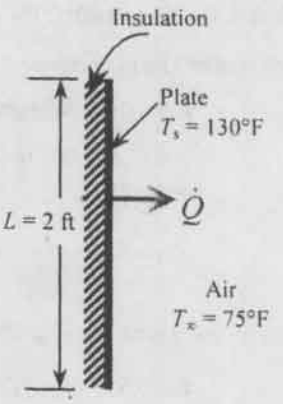
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (130+75)/2 = 102.5°F$ are (Table A-18)

$$k = 0.0154 \text{ Btu / h.ft.°F}$$

$$\upsilon = 0.180 \times 10^{-3} \text{ ft}^2/\text{s}$$

$$\text{Pr} = 0.72$$

$$\beta = \frac{1}{T_f} = \frac{1}{102.5+460} = 0.00178 \text{ R}^{-1}$$



Analysis (a) When the plate is vertical, the characteristic length is the height of the plate. $\delta = L = 2$ ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.00178 \text{ R}^{-1})(130-75 \text{ R})(2 \text{ ft})^3}{(0.180\times10^{-3} \text{ ft}^2/\text{s})^2}(0.72) = 5.6\times10^8$$

$$Nu = 0.59Ra^{1/4^2} = 0.59(5.6\times10^8)^{1/4} = 90.8$$

$$h = \frac{k}{\delta}Nu = \frac{0.0154 \text{ Btu / h.ft.°F}}{2 \text{ ft}}(90.8) = 0.70 \text{ Btu / h.ft}^2\text{.°F}$$

$$A = L^2 = (2 \text{ ft})^2 = 4 \text{ ft}^2$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (0.70 \text{ Btu / h.ft}^2\text{.°F})(4 \text{ ft}^2)(130-75)°\text{C} = \mathbf{154 \text{ Btu /h}}$$

(b) When the plate is horizontal with hot surface facing up, the characteristic length is determined from

$$\delta = \frac{A}{P} = \frac{L^2}{4L} = \frac{L}{4} = \frac{2 \text{ ft}}{4} = 0.5 \text{ ft}.$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.00178 \text{ R}^{-1})(130-75 \text{ R})(0.5 \text{ ft})^3}{(0.180\times10^{-3} \text{ ft}^2/\text{s})^2}(0.72) = 8.76\times10^6$$

$$Nu = 0.54Ra^{1/4} = 0.54(8.76\times10^6)^{1/4} = 29.3$$

$$h = \frac{k}{\delta}Nu = \frac{0.0154 \text{ Btu / h.ft.°F}}{0.5 \text{ ft}}(29.3) = 0.90 \text{ Btu / h.ft}^2\text{.°F}$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (0.90 \text{ Btu / h.ft}^2\text{.°F})(4 \text{ ft}^2)(130-75)°\text{C} = \mathbf{198 \text{ Btu /h}}$$

(c) When the plate is horizontal with hot surface facing down, the characteristic length is again $\delta = 0.5$ ft and the Rayleigh number is $Ra = 8.71\times10^6$. Then,

$$Nu = 0.27Ra^{1/4^2} = 0.27(8.71\times10^6)^{1/4} = 14.7$$

$$h = \frac{k}{\delta}Nu = \frac{0.0154 \text{ Btu / h.ft.°F}}{0.5 \text{ ft}}(14.7) = 0.45 \text{ Btu / h.ft}^2\text{.°F}$$

and

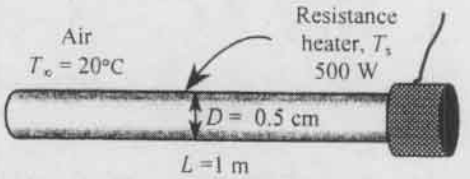
$$\dot{Q} = hA(T_s - T_\infty) = (0.45 \text{ Btu / h.ft}^2\text{.°F})(4 \text{ ft}^2)(130-75)°\text{C} = \mathbf{99.4 \text{ Btu /h}}$$

18-19 A cylindrical resistance heater is placed horizontally in a fluid. The outer surface temperature of the resistance wire is to be determined for two different fluids. √

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer by radiation is ignored. **5** Properties are evaluated at 1000 K for air and 40°C for water.

Properties The properties of air at 1 atm and 1000 K are (Table A-18)

$$k = 0.0672 \text{ W/m.°C}$$
$$\upsilon = 11.8\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.709$$
$$\beta = \frac{1}{T_f} = \frac{1}{1000 \text{ K}} = 0.001 \text{ K}^{-1}$$



The properties of water at 40°C are

$$k = 0.631 \text{ W/m.°C}$$
$$\upsilon = \mu/\rho = 0.658\times10^{-6} \text{ m}^2/\text{s}$$
$$\text{Pr} = 4.32$$
$$\beta = 0.000377 \text{ K}^{-1}$$

Analysis (a) The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 1500 K for the calculation of h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the wire, $\delta = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.001 \text{ K}^{-1})(1500 - 293 \text{ K})(0.005 \text{ m})^3}{(11.8\times10^{-5} \text{ m}^2/\text{s})^2}(0.709) = 92.4$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(92.4)^{1/6}}{\left[1+(0.559/0.709)^{9/16}\right]^{8/27}}\right\}^2 = 1.65$$

$$h = \frac{k}{\delta}Nu = \frac{0.0672 \text{ W/m.°C}}{0.005 \text{ m}}(1.65) = 22.1 \text{ W/m}^2\text{.°C}$$
$$A = \pi DL = \pi(0.005 \text{ m})(1 \text{ m}) = 0.0157 \text{ m}^2$$

and

$$\dot{Q} = hA(T_s - T_\infty)$$
$$500 \text{ W} = (22.1 \text{ W/m}^2\text{.°C})(0.0157 \text{ m}^2)(T_s - 293)\text{K}$$
$$T_s = \mathbf{1734 \text{ K} = 1461°C}$$

which is sufficiently close to the assumed value of 1500 K used in the evaluation of h, and thus it is not necessary to repeat calculations. If we iterate (or use and equation solver), we will get $Ra = 90.2$, $h = 22.0$, and $T_s = 1465$°C. The film temperature in this case is $(T_s+T_\infty)/2 = (1465+20)/2 = 743$°C = 1016 K, which is very close to the value of 1000 K used in the evaluation of the properties.

Prob. 18-19 (*continued*)

(*b*) For the case of water, we "guess" the surface temperature to be 50°C. The characteristic length in this case is the outer diameter of the wire, $\delta = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\,\mathrm{Pr} = \frac{(9.81\,\mathrm{m/s^2})(0.000377\,\mathrm{K^{-1}})(50-20\,\mathrm{K})(0.005\,\mathrm{m})^3}{(0.658\times10^{-6}\,\mathrm{m^2/s})^2}(4.32) = 138{,}380$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(138{,}380)^{1/6}}{\left[1+(0.559/4.32)^{9/16}\right]^{8/27}}\right\}^2 = 10.02$$

$$h = \frac{k}{\delta}Nu = \frac{0.631\,\mathrm{W/m.°C}}{0.005\,\mathrm{m}}(10.02) = 1265\,\mathrm{W/m^2.°C}$$

and

$$\dot{Q} = hA(T_s - T_\infty)$$

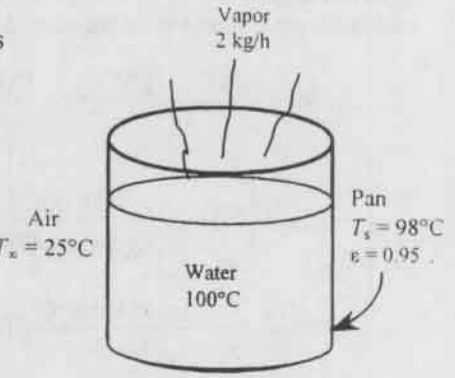
$$500\,\mathrm{W} = (1265\,\mathrm{W/m^2.°C})(0.0157\,\mathrm{m^2})(T_s - 20)°\mathrm{C}$$

$$T_s = \mathbf{45.2°C}$$

which is sufficiently close to the assumed value of 50°C in the evaluation of the properties and h. If we iterate (or use and equation solver), we will get Ra = 120,521, h = 1218, and T_s = 46.1°C. The film temperature in this case is $(T_s+T_\infty)/2$ = (46+20)/2 =33°C, which is close to the value of 40°C used in the evaluation of the properties.

18-20 Water is boiling in a pan that is placed on top of a stove. The rate of heat loss from the cylindrical side surface of the pan by natural convection and radiation and the ratio of heat lost from the side surfaces of the pan to that by the evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (98+25)/2 = 61.5°C = 334.5$ K are (Table A-18)

$$k = 0.0286 \text{ W/m.°C}$$
$$\upsilon = 1.91\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.708$$
$$\beta = \frac{1}{T_f} = \frac{1}{334.5 \text{ K}} = 0.00299 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the pan, $\delta = L = 0.12$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.00299 \text{ K}^{-1})(98-25 \text{ K})(0.12 \text{ m})^3}{(1.91\times10^{-5} \text{ m}^2/\text{s})^2}(0.708) = 7.17\times10^6$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(0.12)}{(7.17\times10^6/0.708)^{1/4}} = 0.074 < 0.25 \quad \text{and thus } D \geq \frac{35L}{Gr^{1/4}} =$$

Therefore,

$$Nu = 0.59Ra^{1/4} = 0.59(7.17\times10^6)^{1/4} = 30.5$$
$$h = \frac{k}{\delta}Nu = \frac{0.0286 \text{ W/m.°C}}{0.12 \text{ m}}(30.5) = 7.3 \text{ W/m}^2\text{.°C}$$
$$A = \pi DL = \pi(0.25 \text{ m})(0.12 \text{ m}) = 0.09425 \text{ m}^2$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (7.3 \text{ W/m}^2\text{.°C})(0.09425 \text{ m}^2)(98-25)°\text{C} = \mathbf{50 \text{ W}}$$

(*b*) The radiation heat loss from the pan is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.95)(0.09425 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(98+273 \text{ K})^4 - (25+273 \text{ K})^4\right] = \mathbf{56.1 \text{ W}}$$

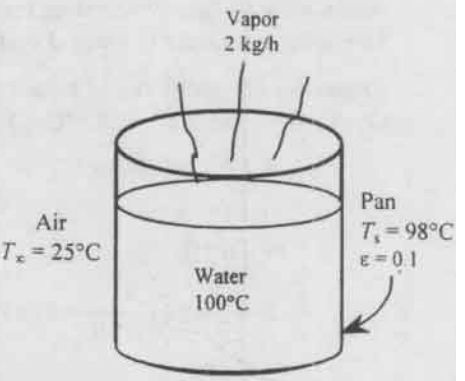
(*c*) The heat loss by the evaporation of water is

$$\dot{Q} = \dot{m}h_{if} = (2/3600 \text{ kg/s})(2257 \text{ kJ/kg}) = 1.254 \text{ kW} = 1254 \text{ W}$$

Then the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water then becomes

$$f = \frac{50+56.1}{1254} = 0.085 = \mathbf{8.5\ \%}$$

18-21 Water is boiling in a pan that is placed on top of a stove. The rate of heat loss from the cylindrical side surface of the pan by natural convection and radiation and the ratio of heat lost from the side surfaces of the pan to that by the evaporation of water are to be determined.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (98+25)/2 = 61.5°C = 334.5$ K are (Table A-18)

$$k = 0.0286 \text{ W/m.°C}$$
$$\upsilon = 1.91\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.708$$
$$\beta = \frac{1}{T_f} = \frac{1}{334.5 \text{ K}} = 0.00299 \text{ K}^{-1}$$

Analysis (a) The characteristic length in this case is the height of the pan, $\delta = L = 0.12$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00299 \text{ K}^{-1})(98-25 \text{ K})(0.12 \text{ m})^3}{(1.91\times10^{-5} \text{ m}^2/\text{s})^2}(0.708) = 7.17\times10^6$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(0.12)}{(7.17\times10^6/0.708)^{1/4}} = 0.074 < 0.25 \quad \text{and thus } D \ge \frac{35L}{Gr^{1/4}} =$$

Therefore,

$$Nu = 0.59Ra^{1/4} = 0.59(7.17\times10^6)^{1/4} = 30.5$$
$$h = \frac{k}{\delta}Nu = \frac{0.0286 \text{ W/m.°C}}{0.12 \text{ m}}(30.5) = 7.3 \text{ W/m}^2\text{.°C}$$
$$A = \pi DL = \pi(0.25 \text{ m})(0.12 \text{ m}) = 0.09425 \text{ m}^2$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (7.3 \text{ W/m}^2\text{.°C})(0.09425 \text{ m}^2)(98-25)°\text{C} = \mathbf{50 \text{ W}}$$

(b) The radiation heat loss from the pan is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.10)(0.09425 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(98+273 \text{ K})^4 - (25+273 \text{ K})^4\right] = \mathbf{5.9 \text{ W}}$$

(c) The heat loss by the evaporation of water is

$$\dot{Q} = \dot{m}h_{if} = (2/3600 \text{ kg/s})(2257 \text{ kJ/kg}) = 1.254 \text{ kW} = 1254 \text{ W}$$

Then the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water then becomes

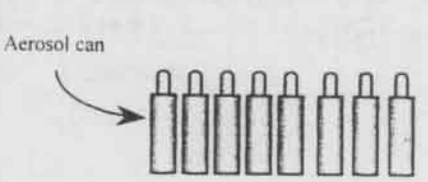
$$f = \frac{50+5.9}{1254} = 0.045 = \mathbf{4.5\ \%}$$

18-22 Some cans move slowly in a hot water container made of sheet metal. The rate of heat loss from the four side surfaces of the container and the annual cost of those heat losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** Heat loss from the top surface is disregarded.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (55+20)/2 = 37.5°C = 310.5$ K are

$$k = 0.0267 \text{ W/m.°C}$$
$$\upsilon = 1.67 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.710$$
$$\beta = \frac{1}{T_f} = \frac{1}{310.5 \text{ K}} = 0.00322 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of the bath, $\delta = L = 0.5$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00322 \text{ K}^{-1})(55-20 \text{ K})(0.5 \text{ m})^3}{(1.67\times10^{-5} \text{ m}^2/\text{s})^2}(0.710) = 3.5\times10^8$$

$$Nu = 0.59Ra^{1/4} = 0.59(3.5\times10^8)^{1/4} = 80.8$$

$$h = \frac{k}{\delta}Nu = \frac{0.0267 \text{ W/m.°C}}{0.5 \text{ m}}(80.8) = 4.3 \text{ W/m}^2\text{.°C}$$

$$A = 2[(0.5 \text{ m})(1 \text{ m}) + (0.5 \text{ m})(3.5 \text{ m})] = 4.5 \text{ m}^2$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (4.3 \text{ W/m}^2\text{.°C})(4.5 \text{ m}^2)(55-20)°C = 677 \text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.7)(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(55+273 \text{ K})^4 - (20+273 \text{ K})^4\right] = 751 \text{ W}$$

Then the total rate of heat loss becomes

$$\dot{Q}_{total} = \dot{Q}_{\substack{natural \\ convection}} + \dot{Q}_{rad} = 677 + 751 = \mathbf{1428 \text{ W}}$$

The amount and cost of the heat loss during one year is

$$Q_{total} = \dot{Q}_{total}\Delta t = (1.428 \text{ kW})(8760 \text{ h}) = 12{,}511 \text{ kWh}$$

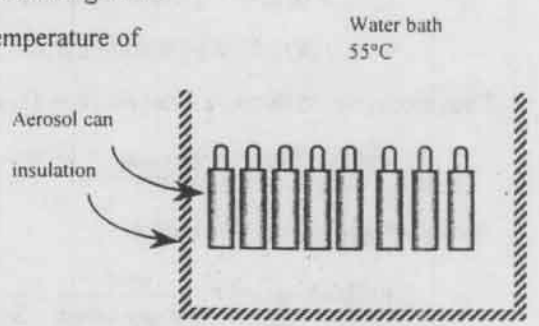
$$\text{Cost} = (12{,}511 \text{ kWh})(\$0.085/\text{kWh}) = \mathbf{\$1063}$$

18-23 Some cans move slowly in a hot water container made of sheet metal. It is proposed to insulate the side and bottom surfaces of the container for $350. The simple payback period of the insulation to pay for itself from the energy it saves is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** Heat loss from the top surface is disregarded.

Properties The properties of air at 1 atm the anticipated film temperature of $(T_s+T_\infty)/2 = (55+20)/2 = 37.5°C = 310.5$ K are (Table A-18)

$$k = 0.0267 \text{ W/m.°C}$$
$$\upsilon = 1.67 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.710$$
$$\beta = \frac{1}{T_f} = \frac{1}{310.5 \text{ K}} = 0.00322 \text{ K}^{-1}$$



Analysis Insulation will drop the outer surface temperature to a value close to the ambient temperature. The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. For convenience, we use the properties of air at 300 K (the anticipated film temperature),

$$k = 0.0261 \text{ W/m.°C}$$
$$\upsilon = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.712$$

We start the solution process by "guessing" the outer surface temperature to be 26 °C. We will check the accuracy of this guess later and repeat the calculations if necessary with a better guess based on the results obtained. Note that $\beta = 1/T_f = 1/(23+273 \text{ K}) = 0.00338 \text{ K}^{-1}$. The characteristic length in this case is the height of the tank, $\delta = L = 0.5$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.00338 \text{ K}^{-1})(26-20 \text{ K})(0.5 \text{ m})^3}{(1.57 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.712) = 7.12 \times 10^7$$

$$Nu = 0.59 Ra^{1/4} = 0.59(7.12 \times 10^7)^{1/4} = 54.2$$

$$h = \frac{k}{\delta} Nu = \frac{0.0261 \text{ W/m.°C}}{0.5 \text{ m}}(54.2) = 2.83 \text{ W/m}^2.°C$$

$$A = 2[(0.5 \text{ m})(1.10 \text{ m}) + (0.5 \text{ m})(3.60 \text{ m})] = 4.7 \text{ m}^2$$

Then the total rate of heat loss from the outer surface of the insulated tank by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (2.83 \text{ W/m}^2.°C)(4.7 \text{ m}^2)(26-20)°C$$
$$+ (0.1)(4.7 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)[(26+273 \text{ K})^4 - (20+273 \text{ K})^4]$$
$$= 95.6 \text{ W}$$

In steady operation, the heat lost by the side surfaces of the tank must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. The second conditions requires the surface temperature to be

$$\dot{Q} = \dot{Q}_{insulation} = kA\frac{T_{tank} - T_s}{L} \quad \rightarrow \quad 95.6 \text{ W} = (0.035 \text{ W/m.°C})(4.7 \text{ m}^2)\frac{(55 - T_s)°C}{0.05 \text{ m}}$$

Prob. 18-23 (*cotinued*)

It gives $T_s = 25.94°C \approx 26°C$, which is the value we assumed. Therefore, the surface temperature is 26°C.

The total amount of heat loss and its cost during one year are

$$Q_{total} = \dot{Q}_{total}\Delta t = (95.6\ \text{W})(8760\ \text{h}) = 837.5\ \text{kWh}$$

$$\text{Cost} = (837.6\ \text{kWh})(\$0.085/\text{kWh}) = \$71.2$$

Then money saved during a one-year period due to insulation becomes

$$\text{Money saved} = \text{Cost}_{\substack{without\\insulation}} - \text{Cost}_{\substack{with\\insulation}} = \$1063 - \$71.2 = \$991.8$$

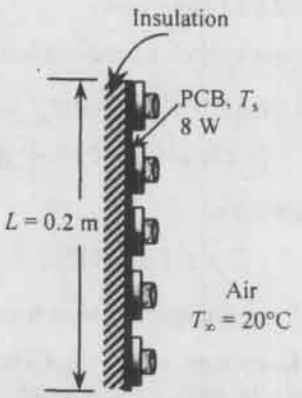
The insulation will pay for itself in

$$\text{Payback period} = \frac{\text{Cost}}{\text{Money saved}} = \frac{\$350}{\$991.8/yr} = \mathbf{0.35\ yr\ =\ 129\ days}$$

Discussion We would definitely recommend the installation of insulation in this case.

18-24 A printed circuit board (PCB) is placed in a room. The average temperature of the hot surface of the board is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** The heat loss from the back surface of the board is negligible.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (45+20)/2 = 32.5°C = 305.5$ K are

$$k = 0.0265 \text{ W/m.°C}$$

$$\upsilon = 1.62 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.711$$

$$\beta = \frac{1}{T_f} = \frac{1}{305.5 \text{ K}} = 0.00327 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown

(a) **Vertical PCB** . We start the solution process by "guessing" the surface temperature to be 45°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the PCB, $\delta = L = 0.2$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00327 \text{ K}^{-1})(45-20 \text{ K})(0.2 \text{ m})^3}{(1.62\times10^{-5} \text{ m}^2/\text{s})^2}(0.711) = 1.74\times10^7$$

$$Nu = 0.599Ra^{1/4} = 0.59(1.74\times10^7)^{1/4} = 38.1$$

$$h = \frac{k}{\delta}Nu = \frac{0.0265 \text{ W/m.°C}}{0.2 \text{ m}}(38.1) = 5.1 \text{ W/m}^2.°\text{C}$$

$$A = (0.15 \text{ m})(0.2 \text{ m}) = 0.03 \text{ m}^2$$

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$8 \text{ W} = (5.1 \text{ W/m}^2.°\text{C})(0.03 \text{ m}^2)\left[T_s - (20+273 \text{ K})\right] + (0.8)(0.03 \text{ m}^2)(5.67\times10^{-8})\left[T_s^4 - (20+273 \text{ K})^4\right]$$

Its solution is

$$T_s = 319 \text{ K} = \mathbf{46°C}$$

which is sufficiently close to the assumed value of 45°C for the evaluation of the properties and h.

(b) **Horizontal, hot surface facing up** Again we assume the surface temperature to be 45 °C and use the properties evaluated above. The characteristic length in this case is

$$\delta = \frac{A}{P} = \frac{(0.20 \text{ m})(0.15 \text{ m})}{2(0.2 \text{ m}+0.15 \text{ m})} = 0.0429 \text{ m}.$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00327 \text{ K}^{-1})(45-20 \text{ K})(0.0429 \text{ m})^3}{(1.62\times10^{-5} \text{ m}^2/\text{s})^2}(0.711) = 1.71\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.71\times10^5)^{1/4} = 11.0$$

$$h = \frac{k}{\delta} Nu = \frac{0.0265 \text{ W/m.°C}}{0.0429 \text{ m}}(11.0) = 6.8 \text{ W/m}^2.\text{°C}$$

Prob. 18-24 (*continued*)

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$8 \text{ W} = (6.8 \text{ W/m}^2.\text{°C})(0.03 \text{ m}^2)[T_s - (20+273 \text{ K})] + (0.8)(0.03 \text{ m}^2)(5.67\times10^{-8})[T_s^4 - (20+273 \text{ K})^4]$$

Its solution is

$$T_s = 315 \text{ K} = \mathbf{42°C}$$

which is sufficiently close to the assumed value of 45°C in the evaluation of the properties and h.

(c) **Horizontal, hot surface facing down** This time we expect the surface temperature to be higher, and assume the surface temperature to be 50 °C. We will check this assumption after obtaining result and repeat calculations with a better assumption, if necessary. The properties of air at the film temperature of

$$T_f = \frac{T_s + T_\infty}{2} = \frac{50+20}{2} = 35°\text{C} = 308 \text{ K are}$$

$$k = 0.0267 \text{ W/m.°C}$$

$$\upsilon = 1.67\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.710$$

$$\beta = \frac{1}{T_f} = \frac{1}{308 \text{ K}} = 0.003247 \text{ K}^{-1}$$

The characteristic length in this case is, from part (b), $\delta = 0.0429$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.003247 \text{ K}^{-1})(50-20 \text{ K})(0.0429 \text{ m})^3}{(1.67\times10^{-5} \text{ m}^2/\text{s})^2}(0.710) = 191{,}879$$

$$Nu = 0.27Ra^{1/4} = 0.27(191{,}879)^{1/4} = 5.7$$

$$h = \frac{k}{\delta} Nu = \frac{0.0267 \text{ W/m.°C}}{0.0429 \text{ m}}(5.7) = 3.5 \text{ W/m}^2.\text{°C}$$

Considering both natural convection and radiation heat loses

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$= (3.5 \text{ W/m}^2.\text{°C})(0.03 \text{ m}^2)[T_s - (20+273 \text{ K})] + (0.8)(0.03 \text{ m}^2)(5.67\times10^{-8})[T_s^4 - (20+273 \text{ K})^4] = 8 \text{ W}$$

Its solution is

$$T_s = 323 \text{ K} = \mathbf{50°C}$$

which is identical to the assumed value. Therefore, there is no need to repeat calculations.

18-25 Absorber plates whose back side is heavily insulated is placed horizontally outdoors. Solar radiation is incident on the plate. The equilibrium temperature of the plate is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

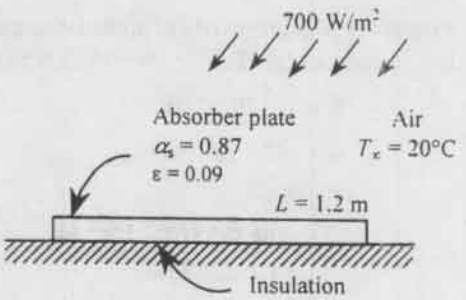
Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (115+25)/2 = 70°C = 343$ K are

$$k = 0.0291 \text{ W/m.°C}$$

$$\upsilon = 1.98\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.707$$

$$\beta = \frac{1}{T_f} = \frac{1}{343 \text{ K}} = 0.002915 \text{ K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 115°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is $\delta = \frac{A}{P} = \frac{(1.2 \text{ m})(0.8 \text{ m})}{2(1.2 \text{ m}+0.8 \text{ m})} = 0.24$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.002915 \text{ K}^{-1})(115-25 \text{ K})(0.24 \text{ m})^3}{(1.98\times10^{-5} \text{ m}^2/\text{s})^2}(0.707) = 6.4\times10^7$$

$$Nu = 0.54\,Ra^{1/4} = 0.54(6.4\times10^7)^{1/4} = 48.3$$

$$h = \frac{k}{\delta}Nu = \frac{0.0291 \text{ W/m.°C}}{0.24 \text{ m}}(48.3) = 5.9 \text{ W/m}^2\text{.°C}$$

$$A = (0.8 \text{ m})(1.2 \text{ m}) = 0.96 \text{ m}^2$$

In steady operation, the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation. Therefore,

$$\dot{Q} = \alpha\dot{q}A = (0.87)(700 \text{ W/m}^2)(0.96 \text{ m}^2) = 585 \text{ W}$$

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$585 \text{ W} = (5.9 \text{ W/m}^2\text{.°C})(0.96 \text{ m}^2)[T_s - (25+273 \text{ K})] + (0.09)(0.96 \text{ m}^2)(5.67\times10^{-8})[T_s^4 - (25+273 \text{ K})^4]$$

Its solution is $T_s = 388 \text{ K} = 115°C$

which is identical to the assumed value. Therefore there is no need to repeat calculations.

If the absorber plate is made of ordinary aluminum which has a solar absorptivity of 0.28 and an emissivity of 0.07, the rate of solar gain becomes

$$\dot{Q} = \alpha\dot{q}A = (0.28)(700 \text{ W/m}^2)(0.96 \text{ m}^2) = 188.2 \text{ W}$$

Again noting that in steady operation the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation, and using the convection coefficient determined above for convenience (actually, we should calculate the new h using data at a lower temperature, and iterating if necessary for better accuracy),

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$188.2 \text{ W} = (5.9 \text{ W/m}^2\text{.°C})(0.96 \text{ m}^2)[T_s - (25+273 \text{ K})] + (0.07)(0.96 \text{ m}^2)(5.67\times10^{-8})[T_s^4 - (25+273 \text{ K})^4]$$

Its solution is $T_s = 329 \text{ K} = 56°C$

Discussion If we recalculated the h using properties at 45°C, we would obtain h = 5.9 W/m².°C and T_s = 61°C.

18-26 An absorber plate whose back side is heavily insulated is placed horizontally outdoors. Solar radiation is incident on the plate. The equilibrium temperature of the plate is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

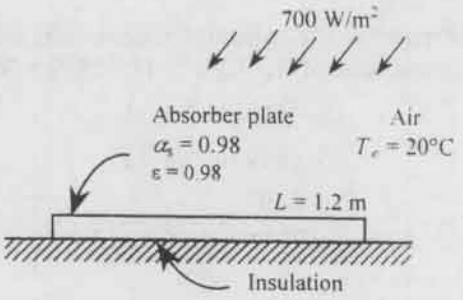
Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (80+25)/2 = 52.5°C = 325.5$ K are

$$k = 0.0279 \text{ W/m.°C}$$

$$\upsilon = 1.82\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.709$$

$$\beta = \frac{1}{T_f} = \frac{1}{325.5 \text{ K}} = 0.003072 \text{ K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 80°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is $\delta = \frac{A}{P} = \frac{(1.2 \text{ m})(0.8 \text{ m})}{2(1.2 \text{ m}+0.8 \text{ m})} = 0.24$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.003072 \text{ K}^{-1})(80-25 \text{ K})(0.24 \text{ m})^3}{(1.82\times10^{-5} \text{ m}^2/\text{s})^2}(0.709) = 4.9\times10^7$$

$$Nu = 0.54Ra^{1/4} = 0.54(4.9\times10^7)^{1/4} = 45.2$$

$$h = \frac{k}{\delta}Nu = \frac{0.0279 \text{ W/m.°C}}{0.24 \text{ m}}(45.2) = 5.3 \text{ W/m}^2\text{.°C}$$

$$A = (0.8 \text{ m})(1.2 \text{ m}) = 0.96 \text{ m}^2$$

In steady operation, the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation. Therefore,

$$\dot{Q} = \alpha\dot{q}A = (0.98)(700 \text{ W/m}^2)(0.96 \text{ m}^2) = 659 \text{ W}$$

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$659 \text{ W} = (5.3 \text{ W/m}^2\text{.°C})(0.96 \text{ m}^2)[T_s - (25+273 \text{ K})] + (0.98)(0.96 \text{ m}^2)(5.67\times10^{-8})[T_s^4 - (25+273 \text{ K})^4]$$

Its solution is $T_s = 351 \text{ K} = \mathbf{78°C}$

which is identical to the assumed value. Therefore there is no need to repeat calculations.

For a white painted absorber plate, the solar absorptivity is 0.26 and the emissivity is 0.90. Then the rate of solar gain becomes

$$\dot{Q} = \alpha\dot{q}A = (0.26)(700 \text{ W/m}^2)(0.96 \text{ m}^2) = 175 \text{ W}$$

Again noting that in steady operation the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation, and using the convection coefficient determined above for convenience (actually, we should calculate the new h using data at a lower temperature, and iterating if necessary for better accuracy),

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

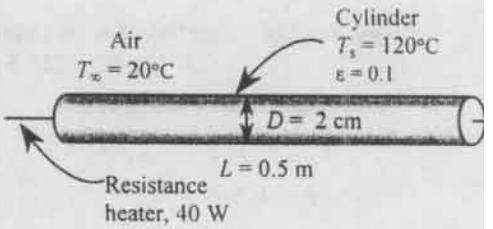
$$175 \text{ W} = (5.3 \text{ W/m}^2\text{.°C})(0.96 \text{ m}^2)[T_s - (25+273 \text{ K})] + (0.90)(0.96 \text{ m}^2)(5.67\times10^{-8})[T_s^4 - (25+273 \text{ K})^4]$$

Its solution is $T_s = 314 \text{ K} = \mathbf{41°C}$

Discussion If we recalculated the h using air properties at 32.5°C, we would obtain $h = 3.9$ W/m².°C and $T_s = 44$°C.

18-27 A resistance heater is placed along the centerline of a horizontal cylinder whose two circular side surfaces are well insulated. The natural convection heat transfer coefficient and whether the radiation effect is negligible are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Analysis The heat transfer surface area of the cylinder is

$$A = \pi DL = \pi(0.02 \text{ m})(0.5 \text{ m}) = 0.0314 \text{ m}^2$$

Noting that in steady operation the heat dissipated from the outer surface must equal to the electric power consumed, and radiation is negligible, the convection heat transfer is determined to be

$$\dot{Q} = hA(T_s - T_\infty) \rightarrow \quad h = \frac{\dot{Q}}{A(T_s - T_\infty)} = \frac{40 \text{ W}}{(0.0314 \text{ m}^2)(120 - 20)°\text{C}} = \mathbf{12.7 \text{ W/m}^2.°\text{C}}$$

The radiation heat loss from the cylinder is

$$\dot{Q}_{rad} = \varepsilon A \sigma (T_s^4 - T_{surr}^4)$$
$$= (0.1)(0.0314 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)[(120 + 273 \text{ K})^4 - (20 + 273 \text{ K})^4] = 2.9 \text{ W}$$

Therefore, the fraction of heat loss by radiation is

$$\text{Radiation fraction} = \frac{\dot{Q}_{radiation}}{\dot{Q}_{total}} = \frac{2.9 \text{ W}}{40 \text{ W}} = 0.0725 = 7.25\%$$

which is greater than 5%. Therefore, the radiation effect is still more than acceptable, and corrections must be made for the radiation effect.

18-28 A thick fluid flows through a pipe in calm ambient air. The pipe is heated electrically. The power rating of the electric resistance heater and the cost of electricity during a 10-h period are to be determined.√

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

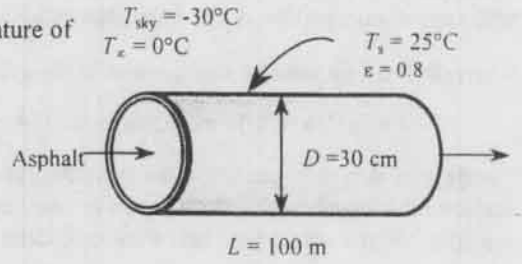
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (25+0)/2 = 12.5°C = 285.5$ K are (Table A-18)

$$k = 0.0250 \text{ W/m.°C}$$

$$\upsilon = 1.45\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.715$$

$$\beta = \frac{1}{T_f} = \frac{1}{285.5 \text{ K}} = 0.003503 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the outer diameter of the pipe, $\delta = D = 0.3$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.003503 \text{ K}^{-1})(25-0 \text{ K})(0.3 \text{ m})^3}{(1.45\times10^{-5} \text{ m}^2/\text{s})^2}(0.715) = 7.9\times10^7$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(7.9\times10^7)^{1/6}}{\left[1+(0.559/0.715)^{9/16}\right]^{8/27}}\right\}^2 = 52.6$$

$$h = \frac{k}{\delta}Nu = \frac{0.0250 \text{ W/m.°C}}{0.3 \text{ m}}(52.6) = 4.39 \text{ W/m}^2.°\text{C}$$

$$A = \pi DL = \pi(0.3 \text{ m})(100 \text{ m}) = 94.24 \text{ m}^2$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (4.39 \text{ W/m}^2.°\text{C})(94.24 \text{ m}^2)(25-0)°\text{C} = 10{,}344 \text{ W}$$

The radiation heat loss from the cylinder is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.8)(94.24 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(25+273 \text{ K})^4 - (-30+273 \text{ K})^4] = 18{,}806 \text{ W}$$

Then,

$$\dot{Q}_{total} = \dot{Q}_{\substack{natural\\convection}} + \dot{Q}_{radiation} = 10{,}344 + 18{,}806 = 29150 \text{ W} = \mathbf{29.2 \text{ kW}}$$

The total amount and cost of heat loss during a 10 hour period is

$$Q = \dot{Q}\Delta t = (29.2 \text{ kW})(10 \text{ h}) = 292 \text{ kWh}$$

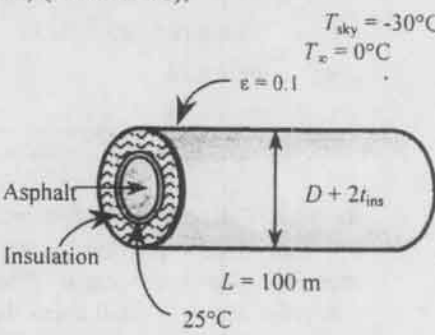
$$\text{Cost} = (292 \text{ kWh})(\$0.09/\text{kWh}) = \mathbf{\$26.3}$$

18-29 A thick fluid flows through a pipe in calm ambient air. The pipe is heated electrically. It is proposed to insulate the pipe. The thickness of the insulation that needs to be used to reduce the heat losses by 85 percent and the money the insulation will save during a 10-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties Insulation will drop the outer surface temperature to a value close to the ambient temperature, and possible below it because of the very low sky temperature for radiation heat loss. For convenience, we use the properties of air at 1 atm and 280 K (the anticipated film temperature) (Table A-18),

$$k = 0.0246 \text{ W/m.°C}$$
$$\upsilon = 1.40 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.717$$
$$\beta = \frac{1}{T_f} = \frac{1}{280 \text{ K}} = 0.00357 \text{ K}^{-1}$$



Analysis The rate of heat loss in the previous problem was obtained to be 29,150 W. Noting that insulation will cut down the heat losses by 85%, the rate of heat loss after insulation will be

$$\dot{Q} = (1 - 0.85)\dot{Q}_{\text{no insulation}} = 0.15 \times 29{,}150 \text{ W} = 4373 \text{ W}$$

The amount of energy and money insulation will save during a 10-h period is simply determined from

$$Q_{saved,total} = \dot{Q}_{saved}\Delta t = (0.85 \times 29.150 \text{ kW})(10 \text{ h}) = 248 \text{ kWh}$$

Money saved = (Energy saved)(Unit cost of energy) = (248 kWh)($0.09 / kWh) = **$22.3**

The characteristic length in this case is the outer diameter of the insulated pipe, $\delta = D + 2t_{insul} = 0.3 + 2t_{insul}$ where t_{insul} is the thickness of insulation in m. Then the problem can be formulated for T_s and t_{insul} as follows:

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00357 \text{ K}^{-1})[(T_s - 273)\text{K}](0.3 + 2t_{insul})^3}{(1.40 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.717)$$

$$Nu = 0.59Ra^{1/4}$$

$$h = \frac{k}{\delta}Nu = \frac{0.0244 \text{ W/m.°C}}{\delta}Nu$$

$$A = \pi D_0 L = \pi(0.3 + 2t_{insul})(100 \text{ m})$$

The total rate of heat loss from the outer surface of the insulated pipe by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA(T_s - T_\infty) + \hat{\varepsilon}A\sigma(T_s^4 - T_{surr}^4)$$

$$4373 = hA(T_s - 273) + (0.1)A(5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)[T_s^4 - (-30 + 273 \text{ K})^4]$$

In steady operation, the heat lost by the side surfaces of the pipe must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. Therefore,

$$\dot{Q} = \dot{Q}_{insulation} = \frac{2\pi kL(T_{\text{tank}} - T_s)}{\ln(D_o / D)} \quad \rightarrow \quad 4373 \text{ W} = \frac{2\pi(0.035 \text{ W/m.°C})(100 \text{ m})(298 - T_s)\text{K}}{\ln[(0.3 + 2t_{insul})/0.3]}$$

The solution of all of the equations above simultaneously using an equation solver gives T_s = 281.5 K = 8.5°C and t_{insul} = **0.013 m = 1.3 cm**.

Note that the film temperature then becomes (8.5+0)/2 = 4.25°C = 277.25 K which is very closed to the assumed value of 280 K. Therefore, there is no need to repeat the calculations using properties at this new film temperature.

18-30E An industrial furnace that resembles a horizontal cylindrical enclosure whose end surfaces are well insulated. The highest allowable surface temperature of the furnace and the annual cost of this loss to the plant are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

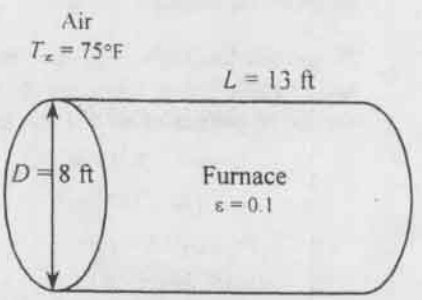
Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (140+75)/2 = 107.5°F$ are (Table A-18)

$$k = 0.0156 \text{ Btu/h.ft.°F}$$

$$\upsilon = 0.184 \times 10^{-3} \text{ ft}^2/\text{s}$$

$$\Pr = 0.72$$

$$\beta = \frac{1}{T_f} = \frac{1}{(107.5+460)\text{R}} = 0.001762 \text{ R}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 140°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the furnace, $\delta = D = 8$ ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(32.2 \text{ ft/s}^2)(0.001762 \text{ R}^{-1})(140-75 \text{ R})(8 \text{ ft})^3}{(0.184\times10^{-3} \text{ ft}^2/\text{s})^2}(0.72) = 4.02\times10^{10}$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(4.02\times10^{10})^{1/6}}{\left[1+(0.559/0.72)^{9/16}\right]^{8/27}}\right\}^2 = 377.2$$

$$h = \frac{k}{\delta}Nu = \frac{0.0156 \text{ Btu/h.ft.°F}}{8 \text{ ft}}(377.2) = 0.74 \text{ Btu/h.ft}^2\text{.°F}$$

$$A = \pi DL = \pi(8 \text{ ft})(13 \text{ ft}) = 326.7 \text{ ft}^2$$

The total rate of heat generated in the furnace is

$$\dot{Q}_{gen} = (0.82)(48 \text{ therms/h})(100{,}000 \text{ Btu/therm}) = 3{,}936{,}000 \text{ Btu/h}$$

Noting that 1% of the heat generated can be dissipated by natural convection and radiation ,

$$\dot{Q} = (0.01)(3{,}936{,}000 \text{ W}) = 39{,}360 \text{ Btu/h}$$

The total rate of heat loss from the furnace by natural convection and radiation can be expressed as

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$39{,}360 \text{ Btu/h} = (0.74 \text{ Btu/h.ft}^2\text{.°F})(326.7 \text{ ft}^2)[T_s - (75+460 \text{ R})]$$
$$+ (0.85)(326.7 \text{ m}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2\text{.R}^4)[T_s^4 - (75+460 \text{ R})^4]$$

Its solution is

$$T_s = 600 \text{ R} = \mathbf{140°F}$$

which is identical to the assumed value. Therefore, there is no need to repeat calculations.

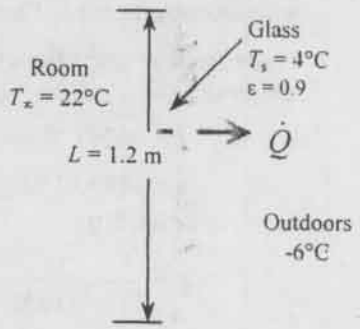
The total amount of heat loss and its cost during a-3000 hour period is

$$Q_{total} = \dot{Q}_{total}\Delta t = (39{,}360 \text{ Btu/h})(3000 \text{ h}) = 1.18\times10^8 \text{ Btu}$$

$$\text{Cost} = (1.18\times10^8/100{,}000 \text{ therm})(\$0.48/\text{therm}) = \mathbf{\$567}$$

18-31 A glass window is considered. The convection heat transfer coefficient on the inner side of the window, the rate of total heat transfer through the window, and the combined natural convection and radiation heat transfer coefficient on the outer surface of the window are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (4+22)/2 = 13°C = 286$ K are (Table A-18)

$$k = 0.0250 \text{ W/m.°C}$$

$$\upsilon = 1.45\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.715$$

$$\beta = \frac{1}{T_f} = \frac{1}{286 \text{ K}} = 0.00350 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the window, $\delta = L = 1.2$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.00350 \text{ K}^{-1})(22-4 \text{ K})(1.2 \text{ m})^3}{(1.45\times10^{-5} \text{ m}^2/\text{s})^2}(0.715) = 3.6\times10^9$$

$$Nu = 0.1Ra^{1/3} = 0.1(3.6\times10^9)^{1/3} = 154$$

$$h = \frac{k}{\delta}Nu = \frac{0.0250 \text{ W/m.°C}}{1.2 \text{ m}}(154) = \mathbf{3.21 \text{ W/m}^2.°C}$$

$$A = (1.2 \text{ m})(2 \text{ m}) = 2.4 \text{ m}^2$$

(*b*) The sum of the natural convection and radiation heat transfer from the room to the window is

$$\dot{Q}_{\text{convection}} = hA(T_\infty - T_s) = (3.21 \text{ W/m}^2.°C)(2.4 \text{ m}^2)(22-4)°C = 139 \text{ W}$$

$$\dot{Q}_{\text{radiation}} = \varepsilon A\sigma(T_{surr}^4 - T_s^4)$$
$$= (0.9)(2.4 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(22+273 \text{ K})^4 - (4+273 \text{ K})^4] = 207 \text{ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{convection}} + \dot{Q}_{\text{radiation}} = 139+207 = \mathbf{346 \text{ W}}$$

(*c*) The outer surface temperature of the window can be determined from

$$\dot{Q}_{\text{total}} = \frac{kA}{L}(T_{s,i} - T_{s,o}) \longrightarrow T_{s,o} = T_{s,i} - \frac{\dot{Q}_{\text{total}}L}{kA} = 4°C - \frac{(346 \text{ W})(0.006 \text{ m})}{(0.78 \text{ W/m.°C})(2.4 \text{ m}^2)} = 2.9°C$$

Then the combined natural convection and radiation heat transfer coefficient on the outer surface of the window can be determined from

$$\dot{Q}_{\text{total}} = h_{\text{combined}}A(T_{s,o} - T_{\infty,o})$$

or
$$h_{\text{combined}} = \frac{\dot{Q}_{\text{total}}}{A(T_{s,o} - T_{\infty,o})} = \frac{346 \text{ W}}{(2.4 \text{ m}^2)[2.9-(-6)]°C} = \mathbf{16.1 \text{ W/m}^2.°C}$$

Note that $\Delta T = \dot{Q}R$ and thus the thermal resistance R of a layer is proportional to the temperature drop across that layer. Therefore, the fraction of thermal resistance of the glass is equal to the ratio of the temperature drop across the glass to the overall temperature difference,

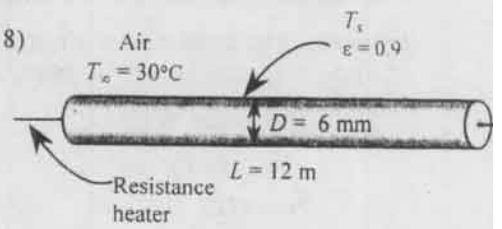
$$\frac{R_{\text{glass}}}{R_{\text{total}}} = \frac{\Delta T_{\text{glass}}}{\Delta T R_{\text{total}}} = \frac{4-2.9}{22-(-6)} = 0.039 \quad (\text{or } 3.9\%)$$

which is low. Thus it is reasonable to neglect the thermal resistance of the glass.

18-32 An insulated electric wire is exposed to calm air. The temperature at the interface of the wire and the plastic insulation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (44+30)/2 = 37°C = 310$ K are (Table A-18)



$$k = 0.0267 \text{ W/m.°C}$$

$$\upsilon = 1.67 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.710$$

$$\beta = \frac{1}{T_f} = \frac{1}{310 \text{ K}} = 0.003226 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 44°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the insulated wire $\delta = D\ (3 + 3)$ mm = 0.006 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.003226 \text{ K}^{-1})(44 - 30 \text{ K})(0.006 \text{ m})^3}{(1.67 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.710) = 242$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(242)^{1/6}}{\left[1+(0.559/0.710)^{9/16}\right]^{8/27}}\right\}^2 = 1.97$$

$$h = \frac{k}{\delta}Nu = \frac{0.0267 \text{ W/m.°C}}{0.006 \text{ m}}(1.97) = 8.77 \text{ W/m}^2\text{.°C}$$

$$A = \pi DL = \pi(0.006 \text{ m})(12 \text{ m}) = 0.226 \text{ m}^2$$

The rate of heat generation, and thus the rate of heat transfer is

$$\dot{Q} = VI = (8 \text{ V})(10 \text{ A}) = 80 \text{ W}$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$80 \text{ W} = (8.77 \text{ W/m}^2\text{.°C})(0.226 \text{ m}^2)[2.9 - (-6)] + (0.9)(0.226 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2\text{.K}^4)[T_s^4 - (30 + 273 \text{ K})^4]$$

Its solution is

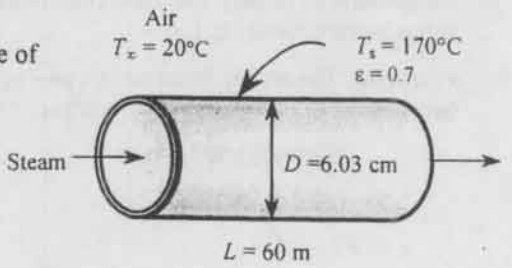
$$T_s = 326 \text{ K} = 53°C$$

which is close to the assumed value of 44°C. To improve the result, we repeat the calculations for a surface temperature of 50 °C. It gives $T_s = 52°C$. Therefore, we can take surface temperature to be $T_s = (50+52)/2 = 51°C$. Then the temperature at the interface of the wire and the plastic cover in steady operation becomes

$$\dot{Q} = \frac{2\pi kL}{\ln(D_2/D_1)}(T_i - T_s) \longrightarrow T_i = T_s + \frac{\dot{Q}\ln(D_2/D_1)}{2\pi kL} = 51°C + \frac{(80 \text{ W})\ln(6/3)}{2\pi(0.15 \text{ W/m.°C})(12 \text{ m})} = \mathbf{55.9°C}$$

18-33 A steam pipe extended from one end of a plant to the other with no insulation on it. The rate of heat loss from the steam pipe and the annual cost of those heat losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (170+20)/2 = 95°C = 368$ K are (Table A-18)

$$k = 0.0309 \text{ W/m.°C}$$
$$\upsilon = 2.25\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.705$$
$$\beta = \frac{1}{T_f} = \frac{1}{368 \text{ K}} = 0.002717 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $\delta = D = 0.0603$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.002717 \text{ K}^{-1})(170-20 \text{ K})(0.0603 \text{ m})^3}{(2.25\times10^{-5} \text{ m}^2/\text{s})^2}(0.705) = 1.22\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.22\times10^6)^{1/6}}{\left[1+(0.559/0.705)^{9/16}\right]^{8/27}}\right\}^2 \doteq 15.4$$

$$h = \frac{k}{\delta}Nu = \frac{0.0309 \text{ W/m.°C}}{0.0603 \text{ m}}(15.4) = 7.89 \text{ W/m}^2\text{.°C}$$
$$A = \pi DL = \pi(0.0603 \text{ m})(60 \text{ m}) = 11.37 \text{ m}^2$$

Then the total rate of heat transfer by natural convection and radiation becomes

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (7.89 \text{ W/m}^2\text{.°C})(11.37 \text{ m}^2)(170-20)°C$$
$$+ (0.7)(11.37 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(170+273 \text{ K})^4 - (20+273 \text{ K})^4]$$
$$= 2.75\times10^4 W = \mathbf{27.5 \text{ kW}}$$

The total amount of gas consumption and its cost during a one-year period is

$$Q_{gas} = \frac{\dot{Q}\Delta t}{\eta} = \frac{27.5 \text{ kJ/s}}{0.78}\left(\frac{1 \text{ therm}}{105{,}500 \text{ kJ}}\right)(8760 \text{ h/yr}\times3600 \text{ s/h}) = 1.054\times10^4 \text{ therms/yr}$$

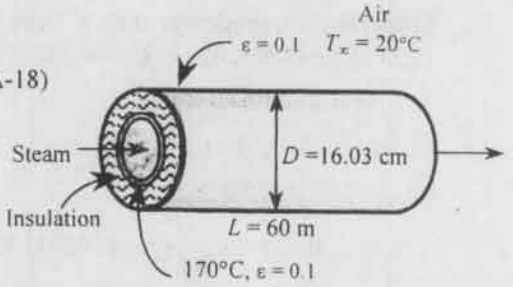
$$\text{Cost} = (1.054\times10^4 \text{ therms/yr})(\$0.538/\text{therm}) = \mathbf{\$5670/yr}$$

18-34 A steam pipe extended from one end of a plant to the other. It is proposed to insulate the steam pipe for $750. The simple payback period of the insulation to pay for itself from the energy it saves are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (34+20)/2 = 17°C = 290$ K are (Table A-18)

$$k = 0.0253 \text{ W/m.°C}$$
$$\upsilon = 1.47\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.714$$
$$\beta = \frac{1}{T_f} = \frac{1}{290 \text{ K}} = 0.00345 \text{ K}^{-1}$$



Analysis Insulation will drop the outer surface temperature to a value close to the ambient temperature. The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the outer surface temperature to be 34°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the insulated pipe, $\delta = D = 0.1603$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.00345 \text{ K}^{-1})(34-20 \text{ K})(0.1603 \text{ m})^3}{(1.47\times10^{-5} \text{ m}^2/\text{s})^2}(0.714) = 6.44\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(6.44\times10^6)^{1/6}}{\left[1+(0.559/0.714)^{9/16}\right]^{8/27}}\right\}^2 = 24.5$$

$$h = \frac{k}{\delta}Nu = \frac{0.0253 \text{ W/m.°C}}{0.1603 \text{ m}}(24.5) = 3.87 \text{ W/m}^2\text{.°C}$$

$$A = \pi DL = \pi(0.1603 \text{ m})(60 \text{ m}) = 30.22 \text{ m}^2$$

Then the total rate of heat loss from the outer surface of the insulated pipe by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (3.87 \text{ W/m}^2\text{.°C})(30.22 \text{ m}^2)(34-20)°C$$
$$+ (0.1)(30.22 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(34+273 \text{ K})^4 - (20+273 \text{ K})^4]$$
$$= 1895 \text{ W}$$

In steady operation, the heat lost from the exposed surface of the insulation by convection and radiation must be equal to the heat conducted through the insulation. This requirement gives the surface temperature to be

$$\dot{Q} = \dot{Q}_{\text{insulation}} = \frac{T_{s,i} - T_s}{R_{ins}} = \frac{T_{s,i} - T_s}{\dfrac{\ln(D_2/D_1)}{2\pi kL}} \quad \rightarrow \quad 1895 \text{ W} = \frac{(170 - T_s)°C}{\dfrac{\ln(16.03/6.03)}{2\pi(0.038 \text{ W/m.°C})(60 \text{ m})}}$$

It gives 40.7°C for the surface temperature, which is somewhat different than the assumed value of 34°C. Repeating the calculations with other surface temperatures gives

$$T_s = 34.6°C \text{ and } \dot{Q} = 1985 \text{ W}$$

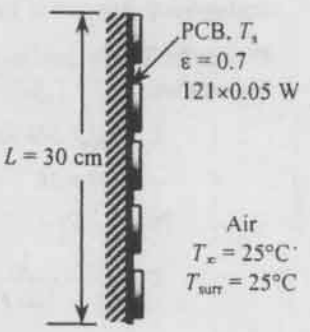
Heat loss and its cost without insulation was determined in the Prob. 18-33 to be 27.5 kW and $5670. Then the reduction in the heat losses becomes

$$\dot{Q}_{\text{saved}} = 27.5 - 1.985 \cong 25.5 \text{ kW} \quad \text{or} \quad 25.5/27.5 = 0.921 \quad (92.1\%)$$

Therefore, the money saved by insulation will be 0.921×($5670/yr) = **$5222/yr** which will pay for the cost of $750 in $750/($5222/yr)=0.144 year = **52.4 days**.

18-35 A circuit board containing square chips is mounted on a vertical wall in a room. The surface temperature of the chips is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer from the back side of the circuit board is negligible.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+25)/2 = 30°C = 303$ K are (Table A-18)

$$k = 0.0262 \text{ W/m.°C}$$

$$\upsilon = 1.60 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.712$$

$$\beta = \frac{1}{T_f} = \frac{1}{303 \text{ K}} = 0.0033 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 35°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the board, $\delta = L = 0.3$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.0033 \text{ K}^{-1})(35-25 \text{ K})(0.3 \text{ m})^3}{(1.60\times10^{-5} \text{ m}^2/\text{s})^2}(0.710) = 2.4\times10^7$$

$$Nu = 0.59 Ra^{1/4} = 0.59(2.4\times10^7)^{1/4} = 41.3$$

$$h = \frac{k}{\delta}Nu = \frac{0.0262 \text{ W/m.°C}}{0.3 \text{ m}}(41.3) = 3.61 \text{ W/m}^2\text{.°C}$$

$$A = (0.3 \text{ m})^2 = 0.09 \text{ m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$(121\times0.05) \text{ W} = (3.61 \text{ W/m}^2\text{.°C})(0.09 \text{ m}^2)(T_s - 25) + (0.7)(0.09 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[T_s^4 - (25+273 \text{ K})^4]$$

Its solution is

$$T_s = 306.4 \text{ K} = \mathbf{33.4°C}$$

which is sufficiently close to the assumed value in the evaluation of properties and h. Therefore, there is no need to repeat calculations by reevaluating the properties and h at the new film temperature.

18-36 A circuit board containing square chips is positioned horizontally in a room. The surface temperature of the chips is to be determined for two orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer from the back side of the circuit board is negligible.

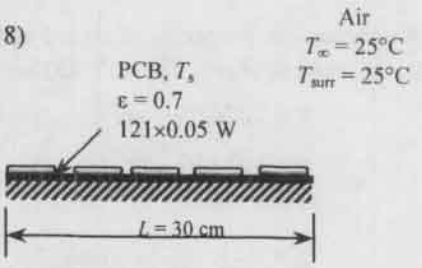
Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+25)/2 = 30°C = 303$ K are (Table A-18)

$$k = 0.0262 \text{ W/m.°C}$$

$$\upsilon = 1.60 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.712$$

$$\beta = \frac{1}{T_f} = \frac{1}{303 \text{ K}} = 0.0033 \text{ K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 35°C for the evaluation of the properties and h. The characteristic length for both cases is determined from

$$\delta = \frac{A}{P} = \frac{(0.3 \text{ m})^2}{2[(0.3 \text{ m}) + (0.3 \text{ m})]} = 0.075 \text{ m}.$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.00333 \text{ K}^{-1})(35-25 \text{ K})(0.075 \text{ m})^3}{(1.60\times10^{-5} \text{ m}^2/\text{s})^2}(0.712) = 3.83\times10^5$$

(*a*) Chips (hot surface) facing up:

$$Nu = 0.54 Ra^{1/4} = 0.54(3.83\times10^5)^{1/4} = 13.4$$

$$h = \frac{k}{\delta}Nu = \frac{0.0262 \text{ W/m.°C}}{0.075 \text{ m}}(13.4) = 4.68 \text{ W/m}^2\text{.°C}$$

$$A = (0.3 \text{ m})^2 = 0.09 \text{ m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$(121\times0.05) \text{ W} = (4.68 \text{ W/m}^2\text{.°C})(0.09 \text{ m}^2)(T_s - 25) + (0.7)(0.09 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[T_s^4 - (25+273 \text{ K})^4]$$

Its solution is

$$T_s = 305.4 \text{ K} = \mathbf{32.4°C}$$

which is sufficiently close to the assumed value. Therefore, there is no need to repeat calculations.

(*b*) Chips (hot surface) facing up:

$$Nu = 0.27 Ra^{1/4} = 0.27(3.83\times10^5)^{1/4} = 6.72$$

$$h = \frac{k}{\delta}Nu = \frac{0.0262 \text{ W/m.°C}}{0.075 \text{ m}}(6.72) = 2.35 \text{ W/m}^2\text{.°C}$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$(121\times0.05) \text{ W} = (2.35 \text{ W/m}^2\text{.°C})(0.09 \text{ m}^2)(T_s - 25) + (0.7)(0.09 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[T_s^4 - (25+273 \text{ K})^4]$$

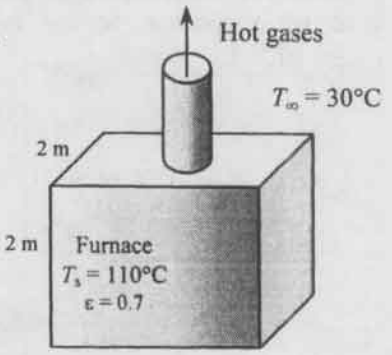
Its solution is

$$T_s = 307.9 \text{ K} = \mathbf{34.9°C}$$

which is sufficiently close to the assumed value in the evaluation of properties and h. Therefore, there is no need to repeat calculations.

18-37 It is proposed that the side surfaces of a cubic industrial furnace be insulated for $550 in order to reduce the heat loss by 90 percent. The thickness of the insulation and the payback period of the insulation to pay for itself from the energy it saves are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (110+30)/2 = 70°C = 343$ K are (Table A-18)

$$k = 0.0292 \text{ W/m.°C}$$
$$\upsilon = 1.99\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.707$$
$$\beta = \frac{1}{T_f} = \frac{1}{343 \text{ K}} = 0.002915 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the height of the furnace, $\delta = L = 2$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.002915 \text{ K}^{-1})(110-30 \text{ K})(2 \text{ m})^3}{(1.99\times10^{-5} \text{ m}^2/\text{s})^2}(0.707) = 3.26\times10^{10}$$

$$Nu = 0.1Ra^{1/3} = 0.1(3.26\times10^{10})^{1/3} = 319$$

$$h = \frac{k}{\delta}Nu = \frac{0.0292 \text{ W/m.°C}}{2 \text{ m}}(319) = 4.66 \text{ W/m}^2\text{.°C}$$
$$A = 4(2 \text{ m})^2 = 16 \text{ m}^2$$

Then the heat loss by combined natural convection and radiation becomes

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (4.66 \text{ W/m}^2\text{.°C})(16 \text{ m}^2)(110-25)°C$$
$$+ (0.7)(16 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(110+273 \text{ K})^4 - (30+273 \text{ K})^4]$$
$$= 1.43\times10^4 \text{ W} = 14.3 \text{ kW}$$

Noting that insulation will reduce the heat losses by 90%, the rate of heat loss after insulation will be

$$\dot{Q}_{\text{saved}} = 0.9\dot{Q}_{\text{no insulation}} = 0.9\times14.3 \text{ kW} = 12.9 \text{ kW}$$
$$\dot{Q}_{\text{loss}} = (1-0.9)\dot{Q}_{\text{no insulation}} = 0.1\times14.3 \text{ kW} = 1.43 \text{ kW}$$

The furnace operates continuously and thus 8760 h. Then the amount of energy and money the insulation will save becomes

$$\text{Energy saved} = \dot{Q}_{saved}\Delta t = \frac{12.9 \text{ kJ/s}}{0.78}\left(\frac{1 \text{ therm}}{105{,}500 \text{ kJ}}\right)(8760\times3600 \text{ s/yr}) = 4944 \text{ therms/yr}$$

Money saved = (Energy saved)(Unit cost of energy) = (4944 therms)($0.55 / therm) = **$2719**

Therefore, the money saved by insulation will pay for the cost of $550 in $550/($2719/yr)=0.202 yr = **74 days**.

Prob. 18-37 (*continued*)

Insulation will lower the outer surface temperature, the Rayleigh and Nusselt numbers, and thus the convection heat transfer coefficient. For the evaluation of the heat transfer coefficient, we assume the surface temperature in this case to be 50 °C. The properties of air at the film temperature of $T_f = \frac{T_s + T_\infty}{2} = \frac{50+0}{2} = 25°\text{C} = 298\ \text{K}$ are

$$k = 0.0260\ \text{W/m.°C}$$
$$\upsilon = 1.56 \times 10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.712$$
$$\beta = \frac{1}{T_f} = \frac{1}{298\ \text{K}} = 0.003355\ \text{K}^{-1}$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8\ \text{m/s}^2)(0.003355\ \text{K}^{-1})(50-30\ \text{K})(2\ \text{m})^3}{(1.56\times10^{-5}\ \text{m}^2/\text{s})^2}(0.712) = 1.54\times10^{10}$$

$$Nu = 0.1Ra^{1/3} = 0.1(1.54\times10^{10})^{1/3} = 249$$

$$h = \frac{k}{\delta}Nu = \frac{0.0260\ \text{W/m.°C}}{2\ \text{m}}(249) = 3.23\ \text{W/m}^2\text{.°C}$$

$$A = 4\times(2\ \text{m})(2+2t_{insul})\ \text{m}$$

The total rate of heat loss from the outer surface of the insulated furnace by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$1428\ \text{W} = (3.23\ \text{W/m}^2\text{.°C})A(T_s - 298) + (0.7)A(5.67\times10^{-8}\ \text{W/m}^2\text{.K}^4)[T_s^4 - (298\ \text{K})^4]$$

In steady operation, the heat lost by the side surfaces of the pipe must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. Therefore,

$$\dot{Q} = \dot{Q}_{insulation} = kA\frac{(T_{\text{furnace}} - T_s)}{t_{ins}} \quad \rightarrow \quad 1428\ \text{W} = (0.038\ \text{W/m.°C})A\frac{[(110+273) - T_s]\text{K}}{t_{insul}}$$

Solving the two equations above by trial-and error (or better yet, an equation solver) gives T_s = 309.3 K = 36.3°C and t_{insul} = 0.032 m or 3.2 cm. The accuracy can be improved somewhat by reevaluating the properties and h at the new film temperature of (36.3+25)/2 = 30.7°C.

18-38 A cylindrical propane tank is exposed to calm ambient air. The propane is slowly vaporized due to a crack developed at the top of the tank. The time it will take for the tank to empty is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation heat transfer is negligible.

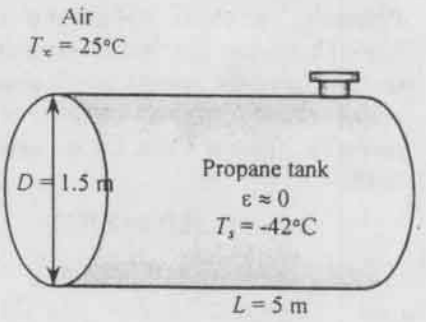
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (-42+25)/2 = -8.5°C = 264.5$ K are (Table A-18)

$$k = 0.0234 \text{ W/m.°C}$$

$$\upsilon = 1.26\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.721$$

$$\beta = \frac{1}{T_f} = \frac{1}{264.5 \text{ K}} = 0.003781 \text{ K}^{-1}$$



Analysis The tank gains heat through its cylindrical surface as well as its circular end surfaces. For convenience, we take the heat transfer coefficient at the end surfaces of the tank to be the same as that of its side surface. (The alternative is to treat the end surfaces as a vertical plate, but this will double the amount of calculations without providing much improvement in accuracy since the area of the end surfaces is much smaller and it is circular in shape rather than being rectangular). The characteristic length in this case is the outer diameter of the tank, $\delta = D = 1.5$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.003781 \text{ K}^{-1})[(25-(-42) \text{ K}](1.5 \text{ m})^3}{(1.26\times10^{-5} \text{ m}^2/\text{s})^2}(0.721) = 3.8\times10^{10}$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(3.8\times10^{10})^{1/6}}{\left[1+(0.559/0.721)^{9/16}\right]^{8/27}}\right\}^2 = 371$$

$$h = \frac{k}{\delta}Nu = \frac{0.0234 \text{ W/m.°C}}{1.5 \text{ m}}(371) = 5.78 \text{ W/m}^2.°\text{C}$$

$$A = \pi DL + 2\pi D^2/4 = \pi(1.5 \text{ m})(5 \text{ m}) + 2\pi(1.5 \text{ m})^2/4 = 27.10 \text{ m}^2$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (5.78 \text{ W/m}^2.°\text{C})(27.1 \text{ m}^2)[(25-(-42)]°\text{C} = 1.05\times10^4 \text{ W} = 10.5 \text{ kW}$$

The total mass and the rate of evaporation of propane are

$$m = \rho V = \rho\frac{\pi D^2}{4}L = (581 \text{ kg/m}^3)\frac{\pi(1.5 \text{ m})^2}{4}(5 \text{ m}) = 5134 \text{ kg}$$

$$\dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{10.5 \text{ kJ/s}}{425 \text{ kJ/kg}} = 0.0247 \text{ kg/s}$$

and it will take

$$\Delta t = \frac{m}{\dot{m}} = \frac{5134 \text{ kg}}{0.0247 \text{ kg/s}} = 207{,}850 \text{ s} = \mathbf{57.7 \text{ hours}}$$

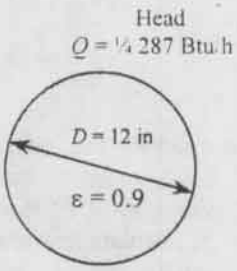
for the propane tank to empty.

18-39E The average surface temperature of a human head is to be determined when it is not covered.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The head can be approximated as a 12-in.-diameter sphere.

Properties The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 95°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (95+77)/2 = 86°F$ are (Table A-18)

$$k = 0.0151 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.173\times10^{-3} \text{ ft}^2/\text{s}$$
$$\Pr = 0.72$$
$$\beta = \frac{1}{T_f} = \frac{1}{(86+460)\text{R}} = 0.001832 \text{ R}^{-1}$$



Analysis The characteristic length for a spherical object is $\delta = D = 1$ ft. Then,

$$Ra = \frac{g\beta(T_s-T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(32.2 \text{ ft/s}^2)(0.001832 \text{ R}^{-1})(95-77 \text{ R})(1.57 \text{ ft})^3}{(0.173\times10^{-3} \text{ ft}^2/\text{s})^2}(0.72)$$
$$= 0.255\times10^8$$

$$Nu = 2+\frac{0.589Ra^{1/4}}{\left[1+\left(\frac{0.469}{\Pr}\right)^{9/16}\right]^{4/9}} = 2+\frac{0.589(0.255\times10^8)^{1/4}}{\left[1+\left(\frac{0.469}{0.712}\right)^{9/16}\right]^{4/9}} = 59.5$$

$$h = \frac{k}{\delta}Nu = \frac{0.0151 \text{ Btu/h.ft.°F}}{1 \text{ ft}}(59.5) = 0.898 \text{ Btu/h.ft}^2\text{.°F}$$
$$A = \pi D^2 = \pi(1 \text{ ft})^2 = 3.142 \text{ ft}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be written as

$$\dot{Q} = hA(T_s-T_\infty)+\varepsilon A\sigma(T_s^4-T_{surr}^4)$$
$$(287/4 \text{ Btu/h}) = (0.898 \text{ Btu/h.ft}^2\text{.°F})(3.142 \text{ ft}^2)(T_s-77)°\text{F}$$
$$+(0.9)(3.142 \text{ m}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2\text{.R}^4)[(T_s+460 \text{ R})^4-(77+460 \text{ R})^4]$$

Its solution is

$$T_s = \mathbf{89.1°F}$$

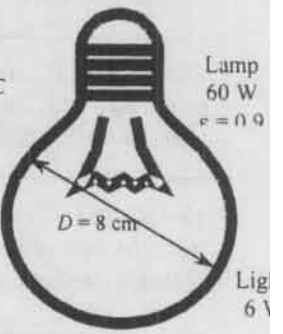
which is sufficiently close to the assumed value in the evaluation of the properties and h. Therefore, there is no need to repeat calculations.

18-40 The equilibrium temperature of a light glass bulb in a room is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The light bulb is approximated as an 8-cm-diameter sphere.

Properties The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 170°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (170+25)/2 = 97.5°C = 370.5$ K are (Table A-18)

$$k = 0.0304 \text{ W/m.°C}$$
$$\upsilon = 2.12\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.705$$
$$\beta = \frac{1}{T_f} = \frac{1}{370.5 \text{ K}} = 0.002699 \text{ K}^{-1}$$



Analysis The characteristic length in this case is $\delta = D = 0.08$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.002699 \text{ K}^{-1})(170-25 \text{ K})(0.08 \text{ m})^3}{(2.16\times10^{-5} \text{ m}^2/\text{s})^2}(0.705)$$
$$= 2.97\times10^6$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+(0.469/\Pr)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(2.97\times10^6)^{1/4}}{\left[1+(0.469/0.705)^{9/16}\right]^{4/9}} = 20.9$$

Then

$$h = \frac{k}{\delta}Nu = \frac{0.0304 \text{ W/m.°C}}{0.08 \text{ m}}(20.9) = 7.94 \text{ W/m}^2\text{.°C}$$
$$A = \pi D^2 = \pi(0.08 \text{ m})^2 = 0.0201 \text{ m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be written as

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$(0.90\times60) \text{ W} = (7.94 \text{ W/m}^2\text{.°C})(0.0201 \text{ m}^2)(T_s - 25)$$
$$+ (0.9)(0.0201 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s+273)^4 - (25+273 \text{ K})^4]$$

Its solution is

$$T_s = \mathbf{169°C}$$

which is sufficiently close to the value assumed in the evaluation of properties and h. Therefore, there is no need to repeat calculations.

Natural Convection Inside Enclosures

18-41C We would recommend putting the hot fluid into the upper compartment of the container. In this case no convection currents will develop in the enclosure since the lighter (hot) fluid will always be on top of the heavier (cold) fluid.

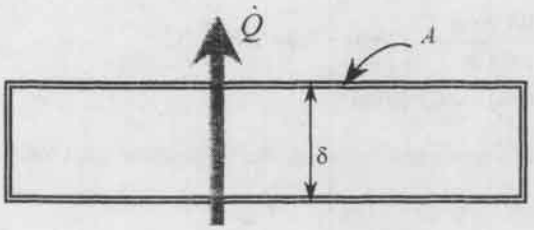
18-42C We would disagree with this recommendation since the air space introduces some thermal resistance to heat transfer. The thermal resistance of air space will be zero only when the convection coefficient approaches infinity, which is never the case. However, when the air space is eliminated, so is its thermal resistance.

18-43C Yes, dividing the air space into two compartments will retard air motion in the air space, and thus slow down heat transfer by natural convection. The vinyl sheet will also act as a radiation shield and reduce heat transfer by radiation.

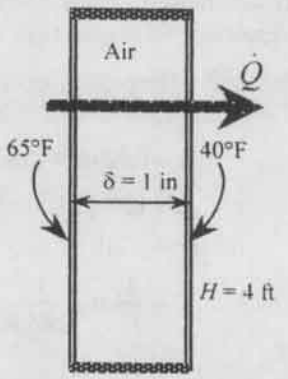
18-44C The effective thermal conductivity of an enclosure represents the enhancement on heat transfer as result of convection currents relative to conduction. The ratio of the effective thermal conductivity to the ordinary thermal conductivity yields Nusselt number $Nu = k_{eff} / k$.

18-45 Conduction thermal resistance of a medium is expressed as $R = L/(kA)$. Thermal resistance of a rectangular enclosure can be expressed by replacing L with characteristic length of enclosure δ, and thermal conductivity k with effective thermal conductivity k_{eff} to give

$$R = \delta/(k_{eff} A) = \delta/(kNuA)$$



18-46E Two glasses of a double pane window are maintained at specified temperatures. The rate of heat transfer through the window by natural convection and radiation, and the R-value of insulation are to be determined. √



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm.

Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (65+40)/2 = 52.5°F$ are (Table A-18)

$$k = 0.0144 \text{ Btu/h.ft.}^\circ\text{F}$$

$$\upsilon = 0.156 \times 10^{-3} \text{ ft}^2/\text{s}$$

$$\text{Pr} = 0.72$$

$$\beta = \frac{1}{T_f} = \frac{1}{(52.5+460)\text{ R}} = 0.001951 \text{ R}^{-1}$$

Analysis (*a*) The characteristic length in this case is the distance between the two glasses, $\delta = 1$ in. Then,

$$Ra = \frac{g\beta(T_1 - T_2)\delta^3}{\upsilon^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001951 \text{ R}^{-1})(65-40 \text{ R})(1/12 \text{ ft})^3}{(0.156\times10^{-3} \text{ ft}^2/\text{s})^2}(0.72) = 2.689\times10^4$$

$$Nu = 0.197 Ra^{1/4}\left(\frac{H}{\delta}\right)^{-1/9} = 0.197(2.689\times10^4)^{1/4}\left(\frac{4 \text{ ft}}{1/12 \text{ ft}}\right)^{-1/9} = 1.64$$

Then,

$$A = HL = (4 \text{ ft})(6 \text{ ft}) = 24 \text{ ft}^2$$

$$\dot{Q} = kNuA\frac{T_1 - T_2}{\delta} = (0.0144 \text{ Btu/h.ft.°F})(1.64)(24 \text{ ft}^2)\frac{(65-40)°\text{F}}{(1/12)\text{ft}} = \mathbf{170 \text{ Btu/h}}$$

(*b*) The rate of heat transfer by radiation is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_1^4 - T_2^4)$$

$$= (0.82)(24 \text{ ft}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2.\text{R}^4)[(65+460 \text{ R})^4 - (40+460 \text{ R})^4] = \mathbf{454 \text{ Btu/h}}$$

Then the total rate of heat transfer is

$$\dot{Q}_{total} = \dot{Q}_{convection} + \dot{Q}_{rad} = 170 + 454 = 624 \text{ Btu/h}$$

Then the effective thermal conductivity of the air which also accounts for the radiation effect and the *R*-value become

$$\dot{Q}_{total} = k_{eff} A\frac{T_1 - T_2}{\delta} \longrightarrow k_{eff} = \frac{\dot{Q}\delta}{A(T_1 - T_2)} = \frac{(624 \text{ Btu/h})(1/12 \text{ ft})}{(24 \text{ ft}^2)(65-40)°\text{F}} = \mathbf{0.0867 \text{ Btu/h.ft.°F}}$$

$$R_{value} = \frac{\delta}{k_{eff}} = \frac{(1/12 \text{ ft})}{0.0867 \text{ Btu/h.ft.°F}} = 0.96 \text{ h.ft}^2.°\text{F/Btu} = \mathbf{R-0.96}$$

18-47 Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclusure is 1 atm.

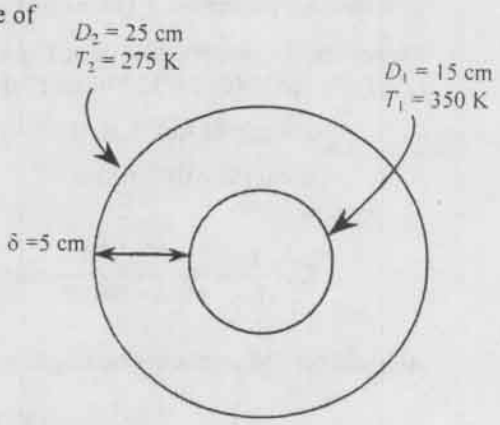
Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (350+275)/2 = 312.5$ K are (Table A-18)

$$k = 0.0270 \text{ W/m.°C}$$

$$\upsilon = 1.69\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.710$$

$$\beta = \frac{1}{T_f} = \frac{1}{312.5 \text{ K}} = 0.00320 \text{ K}^{-1}$$



Analysis The characteristic length in this case is determined from

$$\delta = \frac{D_2 - D_1}{2} = \frac{25-15}{2} = 5 \text{ cm}.$$

Then,

$$Ra = \frac{g\beta(T_1 - T_2)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.00320 \text{ K}^{-1})(350-275 \text{ K})(0.05 \text{ m})^3}{(1.69\times10^{-5} \text{ m}^2/\text{s})^2}(0.710) = 7.309\times10^5$$

$$Nu = 0.228Ra^{0.226} = 0.228(7.309\times10^4)^{0.226} = 4.82$$

$$A = \pi D_1 D_2 = \pi(0.15 \text{ m})(0.25 \text{ m}) = 0.1178 \text{ m}^2$$

and

$$\dot{Q} = kNuA\frac{T_1 - T_2}{\delta} = (0.0270 \text{ W/m.°C})(4.82)(0.1178 \text{ m}^2)\frac{(350-275)°\text{C}}{0.05 \text{ m}} = \mathbf{23 \text{ W}}$$

18-48 The absorber plate and the glass cover of a flat-plate solar collector are maintained at specified temperatures. The rate of heat loss from the absorber plate by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat loss by radiation is negligible. **4** The air pressure in the enclusure is 1 atm.

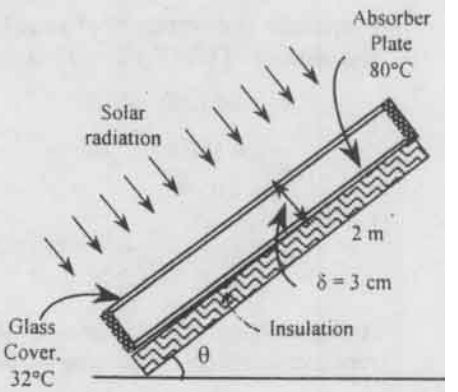
Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (80+32)/2 = 56°C = 329$ K are (Table A-18)

$$k = 0.0282 \text{ W/m.°C}$$

$$\upsilon = 1.85\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.709$$

$$\beta = \frac{1}{T_f} = \frac{1}{329 \text{ K}} = 0.00304 \text{ K}^{-1}$$



Analysis For $\theta = 0°$, we have horizontal rectangular enclosure. The characteristic length in this case is the distance between the two glasses $\delta = 0.025$ m. Then,

$$Ra = \frac{g\beta(T_1 - T_2)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00304 \text{ K}^{-1})(80-32 \text{ K})(0.025 \text{ m})^3}{(1.85\times10^{-5} \text{ m}^2/\text{s})^2}(0.709) = 4.628\times10^4$$

$$Nu = 0.212Ra^{1/4} = 0.212(4.628\times10^4)^{1/4} = 3.11$$

Then

$$A = HL = (2 \text{ m})(3 \text{ m}) = 6 \text{ m}^2$$

$$\dot{Q} = kNuA\frac{T_1 - T_2}{\delta} = (0.0282 \text{ W/m.°C})(3.11)(6 \text{ m}^2)\frac{(80-32)°\text{C}}{0.025 \text{ m}} = \mathbf{1010 \text{ W}}$$

For $\theta = 20°$, we need to replace g by $g\cos\theta$ in Rayleigh number relation. Then,

$$Ra = \frac{g\cos\theta\beta(T_1 - T_2)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(\cos 20°)(0.00304 \text{ K}^{-1})(80-32 \text{ K})(0.025 \text{ m})^3}{(1.85\times10^{-5} \text{ m}^2/\text{s})^2}(0.709) = 4.349\times10^4$$

$$Nu = 0.212Ra^{1/4} = 0.212(4.349\times10^4)^{1/4} = 3.06$$

$$\dot{Q} = kNuA\frac{T_1 - T_2}{\delta} = (0.0282 \text{ W/m.°C})(3.06)(6 \text{ m}^2)\frac{(80-32)°\text{C}}{0.025 \text{ m}} = \mathbf{994 \text{ W}}$$

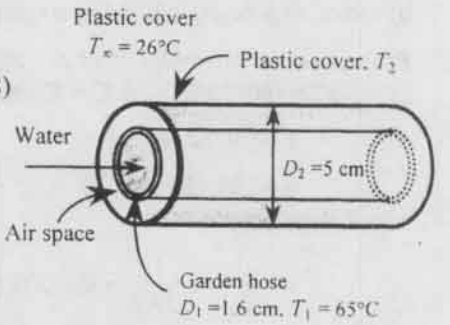
For $\theta = 90°$, we have vertical rectangular enclosure. The Nusselt number for this geometry and orientation can be determined from ($Ra = 46{,}280$, same a that for horizontal case)

$$Nu = 0.197Ra^{1/4}\left(\frac{H}{\delta}\right)^{-1/9} = 0.197(4.628\times10^4)^{1/4}\left(\frac{2 \text{ m}}{0.025 \text{ m}}\right)^{-1/9} = 1.78$$

$$\dot{Q} = kNuA\frac{T_1 - T_2}{\delta} = (0.0282 \text{ W/m.°C})(1.78)(6 \text{ m}^2)\frac{(80-32)°\text{C}}{0.025 \text{ m}} = \mathbf{578 \text{ W}}$$

18-49 A simple solar collector is built by placing a clear plastic tube around a garden hose. The rate of heat loss from the water in the hose per meter of its length by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat loss by radiation is negligible. **3** The air pressure in the enclosure is 1 atm.



Properties The properties of air at 1 atm and the anticipated average temperature of $(T_1+T_2)/2 = (65+35)/2 = 50°C = 323$ K are (Table A-18)

$$k = 0.0278 \text{ W/m.°C}$$

$$\upsilon = 1.80 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.709$$

$$\beta = \frac{1}{T_f} = \frac{1}{323 \text{ K}} = 0.00309 \text{ K}^{-1}$$

Analysis We assume the plastic tube temperature to be 35°C. We will check this assumption later, and repeat calculations, if necessary. The characteristic length in this case is determined from

$$\delta = \frac{D_2 - D_1}{2} = \frac{5 - 1.6}{2} = 1.7 \text{ cm}.$$

Then,

$$Ra = \frac{g\beta(T_1 - T_2)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00309 \text{ K}^{-1})(65-35 \text{ K})(0.017 \text{ m})^3}{(1.80\times10^{-5} \text{ m}^2/\text{s})^2}(0.709) = 9768$$

$$Nu = 0.11Ra^{0.29} = 0.11(9768)^{0.29} = 1.58$$

$$A = \frac{\pi L(D_2 - D_1)}{\ln(D_2/D_1)} = \frac{\pi(1 \text{ m})(0.05 \text{ m} - 0.016 \text{ m})}{\ln(0.05/0.016)} = 0.0937 \text{ m}^2$$

$$\dot{Q} = kNuA\frac{T_1 - T_2}{\delta} = (0.0278 \text{ W/m.°C})(1.58)(0.0937 \text{ m}^2)\frac{(65-35)°\text{C}}{0.017 \text{ m}} = 7.26 \text{ W}$$

Now we will calculate heat transfer from plastic tube to the ambient air by natural convection. Note that we should find a result close to the value we have already calculated since in steady operation they must be equal to each other. Also note that we neglect radiation heat transfer. We will use the same assumption for the plastic tube temperature (i.e., 35 °C). The properties of air at 1 atm and the film temperature of $T_{ave} = (T_s + T_\infty)/2 = (35+26)/2 = 30.5°\text{C} = 303.5$ K are

$$k = 0.0264 \text{ W/m.°C}, \ \upsilon = 1.61\times10^{-5} \text{ m}^2/\text{s}, \ \text{Pr} = 0.712, \text{ and } \beta = 1/T_f = 1/303.5 \text{ K} = 0.0033 \text{ K}^{-1}$$

The characteristic length in this case is the outer diameter of the solar collector $\delta = 0.05$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.0033 \text{ K}^{-1})(35-26 \text{ K})(0.05 \text{ m})^3}{(1.61\times10^{-5} \text{ m}^2/\text{s})^2}(0.712) = 9.962\times10^4$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(9.962\times10^4)^{1/6}}{\left[1+(0.559/0.712)^{9/16}\right]^{8/27}}\right\}^2 = 7.77$$

$$A = \pi D_2 L = \pi(0.05 \text{ m})(1 \text{ m}) = 0.1571 \text{ m}^2$$

$$h = \frac{k}{\delta}Nu = \frac{0.0264 \text{ W/m.°C}}{0.05 \text{ m}}(7.77) = 4.10 \text{ W/m}^2\text{.°C}$$

$$\dot{Q} = hA(T_s - T_\infty) = (4.10 \text{ W/m}^2\text{.°C})(0.1571 \text{ m}^2)(35-26)°\text{C} = 5.80 \text{ W}$$

The two results, 7.26 W and 5.80 W, are somewhat different, and the calculations should be repeated using a different cover temperature. But as a rough approximation, we can take the solution to be the arithmetic average of them, $\dot{Q} = (7.26+5.80)/2 = \mathbf{6.53 \text{ W}}$.

Combined Natural and Forced Convection

18-50C In combined natural and forced convection, the natural convection is negligible when $Gr / \mathrm{Re}^2 < 0.1$. Otherwise it is not.

18-51C In assisting or transverse flows, natural convection enhances forced convection heat transfer while in opposing flow it hurts forced convection.

18-52C When neither natural nor forced convection is negligible, it is not correct to calculate each separately and to add them to determine the total convection heat transfer. Instead, the correlation

$$\mathrm{Nu}_{\text{combined}} = \left(\mathrm{Nu}^n_{\text{forced}} + \mathrm{Nu}^n_{\text{natural}}\right)^{1/n}$$

based on the experimental studies should be used.

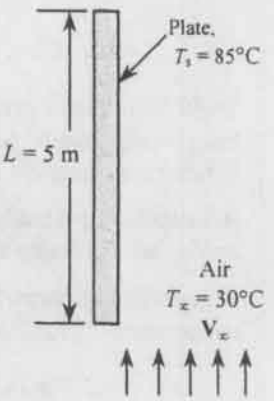
18-53 A vertical plate in air is considered. The forced motion velocity above which natural convection heat transfer from the plate is negligible is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.

Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2 = (85+30)/2 = 57.5°\mathrm{C} = 330.5$ K are (Table A-18)

$$\upsilon = 1.87 \times 10^{-5}\ \mathrm{m^2/s}$$

$$\beta = \frac{1}{T_f} = \frac{1}{330.5\ \mathrm{K}} = 0.003026\ \mathrm{K^{-1}}$$



Analysis The characteristic length is the height of the plate, $\delta = 5$ m. The Grashof and Reynolds numbers are

$$Gr = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2} = \frac{(9.8\ \mathrm{m/s^2})(0.003026\ \mathrm{K^{-1}})(85-30\ \mathrm{K})(5\ \mathrm{m})^3}{(1.87\times10^{-5}\ \mathrm{m^2/s})^2} = 5.8\times10^{11}$$

$$\mathrm{Re} = \frac{V_\infty \delta}{\upsilon} = \frac{V_\infty (5\ \mathrm{m})}{1.87\times10^{-5}\ \mathrm{m^2/s}} = 2.67\times10^5 V_\infty$$

and the forced motion velocity above which natural convection heat transfer from this plate is negligible is

$$\frac{Gr}{\mathrm{Re}^2} = 0.1 \longrightarrow \frac{5.8\times10^{11}}{(2.67\times10^5 V_\infty)^2} = 0.1 \longrightarrow V_\infty = \mathbf{9.02\ m/s}$$

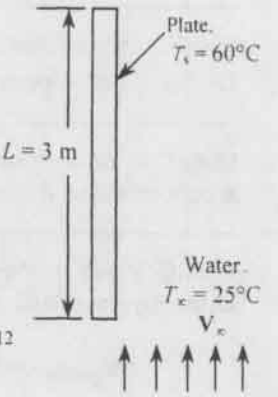
18-54 A vertical plate in water is considered. The forced motion velocity above which natural convection heat transfer from the plate is negligible is to be determined.

Assumptions **1** Steady operating conditions exist.

Properties The properties of water at the film temperature of $(T_s+T_\infty)/2$ $= (60+25)/2 = 42.5°C = 315.5$ K are (Table A-18)

$$\upsilon = 0.65\times10^{-6}\ \text{m}^2/\text{s}$$

$$\beta = 0.00040\ \text{K}^{-1}$$



Analysis The characteristic length is the height of the plate $\delta = 3$ m.
The Grashof and Reynolds numbers are

$$Gr = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2} = \frac{(9.8\ \text{m/s}^2)(0.0004\ \text{K}^{-1})(60-25\ \text{K})(3\ \text{m})^3}{(0.65\times10^{-6}\ \text{m}^2/\text{s})^2} = 8.77\times10^{12}$$

$$\text{Re} = \frac{V_\infty\delta}{\upsilon} = \frac{V_\infty(3\ \text{m})}{0.65\times10^{-6}\ \text{m}^2/\text{s}} = 4.6\times10^6 V_\infty$$

and the forced motion velocity above which natural convection heat transfer from this plate is negligible is

$$\frac{Gr}{\text{Re}^2} = 0.1 \longrightarrow \frac{8.77\times10^{12}}{(4.6\times10^6 V_\infty)^2} = 0.1 \longrightarrow V_\infty = \mathbf{2.03\ m/s}$$

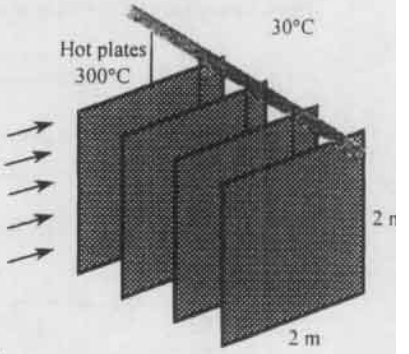
18-55 Thin square plates coming out of the oven in a production facility are cooled by blowing ambient air horizontally parallel to their surfaces. The air velocity above which the natural convection effects on heat transfer are negligible is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.

Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2 = (300+30)/2 = 165°C = 438$ K are (Table A-18)

$$\upsilon = 3.04\times10^{-5}\ \text{m}^2/\text{s}$$

$$\beta = \frac{1}{T_f} = \frac{1}{438\ \text{K}} = 0.002283\ \text{K}^{-1}$$



Analysis The characteristic length is the height of the plate $\delta = 2$ m.
The Grashof and Reynolds numbers are

$$Gr = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2} = \frac{(9.81\ \text{m/s}^2)(0.002283\ \text{K}^{-1})(300-30\ \text{K})(2\ \text{m})^3}{(3.04\times10^{-5}\ \text{m}^2/\text{s})^2} = 5.23\times10^{10}$$

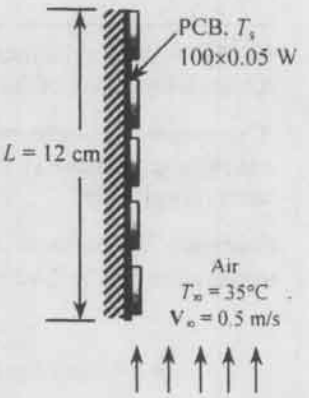
$$\text{Re} = \frac{V_\infty\delta}{\upsilon} = \frac{V_\infty(2\ \text{m})}{3.04\times10^{-5}\ \text{m}^2/\text{s}} = 6.58\times10^5 V_\infty$$

and the forced motion velocity above which natural convection heat transfer from this plate is negligible is

$$\frac{Gr}{\text{Re}^2} = 0.1 \longrightarrow \frac{5.23\times10^{10}}{(6.58\times10^5 V_\infty)^2} = 0.1 \longrightarrow V_\infty = \mathbf{1.1\ m/s}$$

18-56 A circuit board is cooled by a fan that blows air upwards. The average temperature on the surface of the circuit board is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.



Properties The properties of air at 1 atm and 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (60+35)/2 = 47.5°C = 320.5$ K are (Table A-18)

$$k = 0.0275 \text{ W/m.°C}$$

$$\upsilon = 1.77 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.710$$

$$\beta = \frac{1}{T_f} = \frac{1}{320.5 \text{ K}} = 0.00312 \text{ K}^{-1}$$

Analysis We assume the surface temperature to be 60°C. We will check this assumption later on and repeat calculations with a better assumption, if necessary. The characteristic length in this case is the length of the board in the flow (vertical) direction, $\delta = 0.12$ m. Then the Reynolds number becomes

$$\text{Re} = \frac{V_\infty \delta}{\upsilon} = \frac{(0.5 \text{ m/s})(0.12 \text{ m})}{1.77 \times 10^{-5} \text{ m}^2/\text{s}} = 3390$$

which is less than critical Reynolds number (5×10^5). Therefore the flow is laminar and the forced convection Nusselt number and h are determined from

$$Nu = \frac{hL}{k} = 0.664 \text{ Re}^{0.5} \text{ Pr}^{1/3} = 0.664(3390)^{0.5}(0.710)^{1/3} = 34.5$$

$$h = \frac{k}{\delta} Nu = \frac{0.0275 \text{ W/m.°C}}{0.12 \text{ m}}(34.5) = 7.9 \text{ W/m}^2.°\text{C}$$

$$A = (0.12 \text{ m})(0.2 \text{ m}) = 0.024 \text{ m}^2$$

Then,

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 35°\text{C} + \frac{(100)(0.05 \text{ W})}{(7.9 \text{ W/m}^2.°\text{C})(0.024 \text{ m}^2)} = \mathbf{61.4°C}$$

which is sufficiently close to the assumed value in the evaluation of properties. Therefore, there is no need to repeat calculations.

(b) The Rayleigh number is

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2} \text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00312 \text{ K}^{-1})(60 - 35 \text{ K})(0.12 \text{ m})^3}{(1.77 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.710) = 3 \times 10^6$$

$$Nu = 0.59 Ra^{1/4} = 0.59(3 \times 10^6)^{1/4} = 24.5$$

This is an assisting flow and the combined Nusselt number is determined from

$$Nu_{combined} = (Nu_{forced}{}^n + Nu_{natural}{}^n)^{1/n} = (34.5^3 + 24.5^3)^{1/3} = 38.2$$

Then, $$h = \frac{k}{\delta} Nu_{combined} = \frac{0.0275 \text{ W/m.°C}}{0.12 \text{ m}}(38.2) = 8.8 \text{ W/m}^2.°\text{C}$$

and $$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 35°\text{C} + \frac{(100)(0.05 \text{ W})}{(8.8 \text{ W/m}^2.°\text{C})(0.024 \text{ m}^2)} = \mathbf{58.8°C}$$

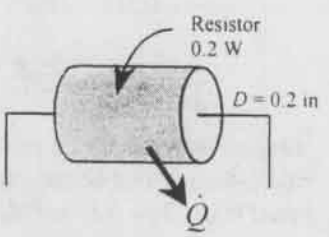
Therefore, natural convection lowers the surface temperature in this case by about 2°C.

Review Problems

18-57E A small cylindrical resistor mounted on the lower part of a vertical circuit board. The approximate surface temperature of the resistor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** Heat transfer through the connecting wires is negligible.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (220+120)/2 = 170°F$ are (Table A-18E)



$$k = 0.0168 \text{ Btu/h.ft.}°\text{F}$$

$$\upsilon = 0.221\times10^{-3} \text{ ft}^2/\text{s}$$

$$\text{Pr} = 0.72$$

$$\beta = \frac{1}{T_f} = \frac{1}{(170+460)\text{R}} = 0.001587 \text{ R}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 220°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the diameter of resistor, $\delta = D = 0.2$ in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001587 \text{ R}^{-1})(220-120 \text{ R})(0.2/12 \text{ ft})^3}{(0.221\times10^{-3} \text{ ft}^2/\text{s})^2}(0.72) = 348.8$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(348.8)^{1/6}}{\left[1+(0.559/0.72)^{9/16}\right]^{8/27}}\right\}^2 = 2.11$$

$$h = \frac{k}{\delta}Nu = \frac{0.0168 \text{ Btu/h.ft.}°\text{F}}{0.2/12 \text{ ft}}(2.11) = 2.13 \text{ Btu/h.ft}^2\text{.}°\text{F}$$

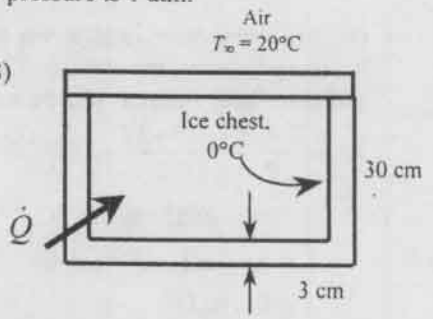
$$A = \pi DL + 2D^2/4 = \pi(0.2/12 \text{ ft})(0.3/12 \text{ ft}) + 2\pi(0.2/12 \text{ ft})^2/4 = 0.00175 \text{ ft}^2$$

and $$\dot{Q} = hA(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 120°\text{F} + \frac{(0.1\times3.412) \text{ Btu/h}}{(2.13 \text{ Btu/h.ft}^2\text{.}°\text{F})(0.00175 \text{ ft}^2)} = \mathbf{212°F}$$

which is sufficiently close to the assumed temperature for the evaluation of properties. Therefore, there is no need to repeat calculations.

18-58 An ice chest filled with ice at 0°C is exposed to ambient air. The time it will take for the ice in the chest to melt completely is to be determined for natural and forced convection cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the base of the ice chest is disregarded. **4** Radiation effects are negligible. **5** Heat transfer coefficient is the same for all surfaces considered. **6** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (20+15)/2 = 17.5°C = 290.5$ K are (Table A-18)

$$k = 0.0254 \text{ W/m.°C}$$

$$\upsilon = 1.49 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.714$$

$$\beta = \frac{1}{T_f} = \frac{1}{290.5 \text{ K}} = 0.00344 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 15°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length for the side surfaces is the height of the chest, $\delta = 0.3$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00344 \text{ K}^{-1})(20-15 \text{ K})(0.3 \text{ m})^3}{(1.49\times10^{-5} \text{ m}^2/\text{s})^2}(0.714) = 1.46\times10^7$$

$$Nu = 0.59Ra^{1/4} = 0.59(1.46\times10^7)^{1/4} = 36.5$$

$$h = \frac{k}{\delta}Nu = \frac{0.0254 \text{ W/m.°C}}{0.3 \text{ m}}(36.5) = 3.1 \text{ W/m}^2\text{.°C}$$

The heat transfer coefficient at the top surface can be determined similarly. However, the top surface constitutes only about one-fourth of the heat transfer area, and thus we can use the heat transfer coefficient for the side surfaces for the top surface also for simplicity. The heat transfer surface area is

$$A_{side} = 4(0.3 \text{ m})(0.4 \text{ m}) + (0.4 \text{ m})(0.4 \text{ m}) = 0.64 \text{ m}^2$$

Then the rate of heat transfer becomes

$$\dot{Q} = \frac{T_\infty - T_{s,i}}{R_{wall} + R_{conv,o}} = \frac{T_\infty - T_{s,i}}{\frac{L}{kA} + \frac{1}{hA}}$$

$$= \frac{(20-0)°\text{C}}{\frac{0.03 \text{ m}}{(0.033 \text{ W/m.°C})(0.64 \text{ m}^2)} + \frac{1}{(3.1 \text{ W/m}^2\text{.°C})(0.64 \text{ m}^2)}} = 10.4 \text{ W}$$

The outer surface temperature of the ice chest is determined from Newton's law of cooling to be

$$\dot{Q} = hA(T_\infty - T_s) \rightarrow T_s = T_\infty - \frac{\dot{Q}}{hA} = 20°\text{C} - \frac{10.4 \text{ W}}{(3.1 \text{ W/m}^2\text{.C})(0.64 \text{ m}^2)} = 14.8°\text{C}$$

which is almost identical to the assumed value of 15°C used in the evaluation of properties and h. Therefore, there is no need to repeat the calculations. Then the rate at which the ice will melt becomes

$$\dot{Q} = \dot{m}h_{if} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{10.4\times10^{-3} \text{ kJ/s}}{333.7 \text{ kJ/kg}} = 3.12\times10^{-5} \text{ kg/s}$$

Prob. 18-58 (*continued*)

Therefore, the melting of the ice in the chest completely will take

$$m = \dot{m}\Delta t \longrightarrow \Delta t = \frac{m}{\dot{m}} = \frac{40}{3.12\times10^{-5}} = 12.8\times10^{5}\ \text{s} = \mathbf{356\ h = 14.8\ days}$$

(b) The temperature drop across the styrofoam will be much greater in this case than that across thermal boundary layer on the surface. Thus we assume outer surface temperature of the styrofoam to be 19 °C. Radiation heat transfer will be neglected. The properties of air at 1 atm and the film temperature of

$$T_f = \frac{T_s + T_\infty}{2} = \frac{19+20}{2} = 19.5°\text{C} = 292.5\ \text{K}\ \text{ are (Table A-18)}$$

$$k = 0.0255\ \text{W/m.°C}$$

$$\upsilon = 1.51\times10^{-5}\ \text{m}^2/\text{s}$$

$$\text{Pr} = 0.714$$

$$\beta = \frac{1}{T_f} = \frac{1}{292.5\ \text{K}} = 0.00342\ \text{K}^{-1}$$

The characteristic length in this case is the length of the chest, $\delta = 0.4$ m. Then,

$$\text{Re} = \frac{V_\infty \delta}{\upsilon} = \frac{(50\times1000/3600\ \text{m/s})(0.4\ \text{m})}{1.51\times10^{-5}\ \text{m}^2/\text{s}} = 367{,}918$$

which is less than critical Reynolds number (5×10^5). Therefore the flow is laminar, and the Nusselt number is determined from

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}^{0.5}\,\text{Pr}^{1/3} = 0.664(367{,}918)^{0.5}(0.714)^{1/3} = 360$$

$$h = \frac{k}{\delta}Nu = \frac{0.0255\ \text{W/m.°C}}{0.4\ \text{m}}(360) = 23.0\ \text{W/m}^2\text{.°C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \frac{T_\infty - T_{s,i}}{R_{wall} + R_{conv,o}} = \frac{T_\infty - T_{s,i}}{\frac{L}{kA} + \frac{1}{hA}}$$

$$= \frac{(20-0)°\text{C}}{\frac{0.03\ \text{m}}{(0.033\ \text{W/m.°C})(0.64\ \text{m}^2)} + \frac{1}{(23\ \text{W/m}^2\text{.°C})(0.64\ \text{m}^2)}} = 13.5\ \text{W}$$

The outer surface temperature of the ice chest is determined from Newton's law of cooling to be

$$\dot{Q} = hA(T_\infty - T_s) \ \rightarrow\ T_s = T_\infty - \frac{\dot{Q}}{hA} = 20°\text{C} - \frac{13.5\ \text{W}}{(23\ \text{W/m}^2\text{.C})(0.64\ \text{m}^2)} = 19.1°\text{C}$$

which is almost identical to the assumed value of 19°C used in the evaluation of properties and h. Therefore, there is no need to repeat the calculations. Then the rate at which the ice will melt becomes

$$\dot{Q} = \dot{m}h_{if} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{13.5\times10^{-3}\ \text{kJ/s}}{333.7\ \text{kJ/kg}} = 4.046\times10^{-5}\ \text{kg/s}$$

Therefore, the melting of the ice in the chest completely will take

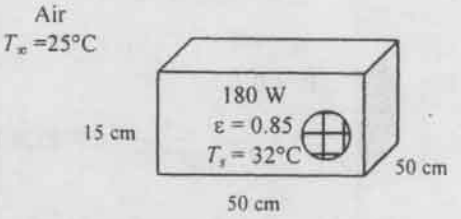
$$m = \dot{m}\Delta t \longrightarrow \Delta t = \frac{m}{\dot{m}} = \frac{40}{4.046\times10^{-5}} = 9.89\times10^{5}\ \text{s} = \mathbf{275\ h = 11.4\ days}$$

18-59 An electronic box is cooled internally by a fan blowing air into the enclosure. The fraction of the heat lost from the outer surfaces of the electronic box is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the base surface is disregarded. **4** The pressure of air inside the enclosure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (32+15)/2 = 28.5°C = 301.5$ K are (Table A-18)

$$k = 0.0261 \text{ W/m.°C}$$
$$\upsilon = 1.57\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.712$$
$$\beta = \frac{1}{T_f} = \frac{1}{301.5 \text{ K}} = 0.003317 \text{ K}^{-1}$$



Analysis Heat loss from the horizontal top surface:

The characteristic length in this case is $\delta = \frac{A}{P} = \frac{(0.5 \text{ m})^2}{2[(0.5 \text{ m})+(0.5 \text{ m})]} = 0.125$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003317 \text{ K}^{-1})(32-25 \text{ K})(0.125 \text{ m})^3}{(1.57\times10^{-5} \text{ m}^2/\text{s})^2}(0.712) = 1.28\times10^6$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.28\times10^6)^{1/4} = 18.2$$

$$h = \frac{k}{\delta}Nu = \frac{0.0261 \text{ W/m.°C}}{0.125 \text{ m}}(18.2) = 3.8 \text{ W/m}^2\text{.°C}$$

$$A_{top} = (0.5 \text{ m})^2 = 0.25 \text{ m}^2$$

and $$\dot{Q}_{top} = hA_{top}(T_s - T_\infty) = (3.8 \text{ W/m}^2\text{.°C})(0.25 \text{ m}^2)(32-25)°C = 6.6 \text{ W}$$

Heat loss from vertical side surfaces:

The characteristic length in this case is the height of the box $\delta = 0.15$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003317 \text{ K}^{-1})(32-25 \text{ K})(0.15 \text{ m})^3}{(1.57\times10^{-5} \text{ m}^2/\text{s})^2}(0.712) = 2.22\times10^6$$

$$Nu = 0.59Ra^{1/4} = 0.59(2.22\times10^6)^{1/4} = 22.8$$

$$h = \frac{k}{\delta}Nu = \frac{0.0261 \text{ W/m.°C}}{0.15 \text{ m}}(22.8) = 3.96 \text{ W/m}^2\text{.°C}$$

$$A_{side} = 4(0.15 \text{ m})(0.5 \text{ m}) = 0.3 \text{ m}^2$$

and

$$\dot{Q}_{side} = hA_{side}(T_s - T_\infty) = (3.96 \text{ W/m}^2\text{.°C})(0.3 \text{ m}^2)(32-25)°C = 8.3 \text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.85)(0.25+0.3 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(32+273 \text{ K})^4 - (25+273 \text{ K})^4] = 20.3 \text{ W}$$

Then the fraction of the heat loss from the outer surfaces of the box is determined to be

$$f = \frac{(6.6+8.3+20.3)\text{W}}{180 \text{ W}} = 0.196 = \mathbf{19.6\ \%}$$

18-60 A spherical tank made of stainless steel is used to store iced water. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Thermal resistance of the tank is negligible. **4** The local atmospheric pressure is 1 atm.

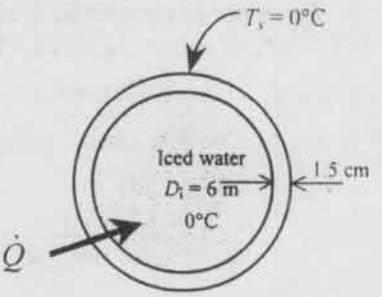
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (0+20)/2 = 10°C = 283$ K are (Table A-18)

$$k = 0.0248 \text{ W/m.°C}$$

$$\upsilon = 1.42\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.716$$

$$\beta = \frac{1}{T_f} = \frac{1}{283 \text{ K}} = 0.003534 \text{ K}^{-1}$$



Analysis (*a*) The characteristic length in this case is $\delta = 6.03$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.003534 \text{ K}^{-1})(20-0 \text{ K})(6.039.47 \text{ m})^3}{(1.42\times10^{-5} \text{ m}^2/\text{s})^2}(0.716) = 0.540\times10^{12}$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+(0.469/\Pr)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(0.540\times10^{12})^{1/4}}{\left[1+(0.469/0.712)^{9/16}\right]^{4/9}} = 392$$

$$h = \frac{k}{\delta}Nu = \frac{0.0248 \text{ W/m.°C}}{6.03 \text{ m}}(392) = 1.61 \text{ W/m}^2.°C$$

$$A = \pi D^2 = \pi(6.03 \text{ m})^2 = 114.2 \text{ m}^2$$

and

$$\dot{Q} = hA(T_\infty - T_s) = (1.61 \text{ W/m}^2.°C)(114.2 \text{ m}^2)(20-0)°C = 3682 \text{ W}$$

Heat transfer by radiation and the total rate of heat transfer are

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$= (1)(114.2 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(20+273 \text{ K})^4 - (0+273 \text{ K})^4] = 11755 \text{ W}$$

$$\dot{Q}_{total} = 3682 + 11{,}755 = 15{,}437 \text{ W} \cong \mathbf{15.4 \text{ kW}}$$

(*b*) The total amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (15.4 \text{ kJ/s})(24 \text{ h/day}\times3600 \text{ s/h}) = 1.33\times10^6 \text{ kJ/day}$$

Then the amount of ice that melts during this period becomes

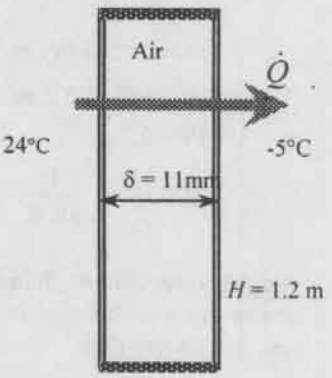
$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{1.33\times10^6 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{3986 \text{ kg}}$$

18-61 A double-pane window consisting of two layers of glass separated by an air space is considered. The rate of heat transfer through the window and the temperature of its inner surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Radiation effects are negligible. **4** The pressure of air inside the enclosure is 1 atm.

Properties We expect the average temperature of the air gap to be roughly the average of the indoor and outdoor temperatures, and evaluate The properties of air at 1 atm and the average temperature of $(T_s+T_\infty)/2 = (-5+24)/2 = 9.5°C = 282.5$ K are (Table A-18)

$$k = 0.0248 \text{ W/m.°C}$$
$$\upsilon = 1.42 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.716$$
$$\beta = \frac{1}{T_f} = \frac{1}{282.5 \text{ K}} = 0.00354 \text{ K}^{-1}$$



Analysis We "guess" the temperature difference across the air gap to be 20°C for use in the Ra relation. The characteristic length in this case is the air gap, $\delta = 0.011$ m. Then,

$$Ra = \frac{g\beta(T_1 - T_2)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.00354 \text{ K}^{-1})(20 \text{ K})(0.011 \text{ m})^3}{(1.42 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.716) = 3270$$

Then the Nusselt number and the heat transfer coefficient are determined to be

$$Nu = 0.197Ra^{1/4}\left(\frac{H}{\delta}\right)^{-1/9} = 0.197(3270)^{1/4}\left(\frac{1.2}{0.011}\right)^{-1/9} = 0.88$$

which is less than 1. But we take $Nu = 1$, which indicates that the heat transfer through the air layer between the glasses is by pure conduction, and the Nusselt number cannot be less than 1.(The Nu relation above is valid for $H/\delta < 42$ but in our case it is $H/\delta = 109$, which explains the unrealistic result obtained). Then the rate of heat transfer through this double pane window is determined to be

$$A = HL = (1.2 \text{ m})(2 \text{ m}) = 2.4 \text{ m}^2$$

$$\dot{Q} = \frac{T_{\infty,i} - T_{\infty,o}}{R_{conv,i} + R_{cond,glasses} + R_{cond,air} + R_{conv,o}}$$

$$= \frac{T_\infty - T_{s,i}}{\frac{1}{h_i A} + \frac{2L}{k_{glass} A} + \frac{L}{k_{air} A} + \frac{1}{h_o A}}$$

$$= \frac{[24-(-5)]}{\frac{1}{(10)(2.4)} + \frac{2(0.003)}{(0.78)(2.4)} + \frac{0.011}{(0.0248)(2.4)} + \frac{1}{(25)(2.4)}}$$

$$= \mathbf{118 \text{ W}}$$

Check: The temperature drop across the air gap is determined from

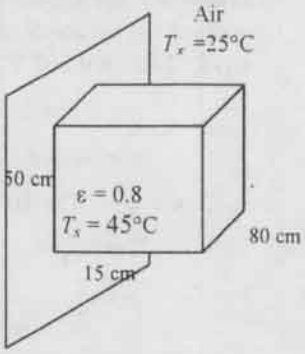
$$\dot{Q} = kA\frac{\Delta T}{L} \rightarrow \Delta T = \frac{\dot{Q}L}{kA} = \frac{(118 \text{ W})(0.011 \text{ m})}{(0.0248 \text{ W/m.°C})(2.4 \text{ m}^2)} = 21.8°C$$

which is very close to the assumed value of 20°C used in the evaluation of the Ra number.

18-62 An electric resistance space heater filled with oil is placed against a wall. The power rating of the heater and the time it will take for the heater to reach steady operation when it is first turned on are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the back, bottom, and top surfaces are disregarded. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (45+25)/2 = 35°C = 308$ K are (Table A-18)



$$k = 0.0267 \text{ W/m.°C}$$
$$\upsilon = 1.65\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.711$$
$$\beta = \frac{1}{T_f} = \frac{1}{308 \text{ K}} = 0.003247 \text{ K}^{-1}$$

Analysis Heat transfer from the top and bottom surfaces are said to be negligible, and thus the heat transfer area in this case consists of the three exposed side surfaces. The characteristic length is the height of the box, $\delta = 0.5$ m . Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.003247 \text{ K}^{-1})(45-25 \text{ K})(0.5 \text{ m})^3}{(1.65\times10^{-5} \text{ m}^2/\text{s})^2}(0.711) = 2.08\times10^8$$

$$Nu = 0.59Ra^{1/4} = 0.59(2.08\times10^8)^{1/4} = 70.8$$

$$h = \frac{k}{\delta}Nu = \frac{0.0267 \text{ W/m.°C}}{0.5 \text{ m}}(70.8) = 3.78 \text{ W/m}^2\text{.°C}$$

$$A = (0.5 \text{ m})(0.8 \text{ m}) + 2(0.15 \text{ m})(0.5 \text{ m}) = 0.55 \text{ m}^2$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (3.78 \text{ W/m}^2\text{.°C})(0.55 \text{ m}^2)(45-25)°C = 41.6 \text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.8)(0.55 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(45+273 \text{ K})^4 - (25+273 \text{ K})^4] = 58.4 \text{ W}$$

Then the total rate of heat transfer, thus the power rating of the heater becomes

$$\dot{Q}_{total} = 41.6 + 58.4 = \mathbf{100 \text{ W}}$$

The specific heat of the oil at the average temperature of the oil is 1943 kJ/kg.°C. Then the amount of heat transfer needed to raise the temperature of the oil to the steady operating temperature and the time it takes become

$$Q = mC_p(T_2 - T_1) = (45 \text{ kg})(1.943 \text{ kJ/kg.°C})(45-25)°C = 1749 \text{ kJ}$$

$$Q = \dot{Q}\Delta t \longrightarrow \Delta t = \frac{Q}{\dot{Q}} = \frac{1749 \text{ kJ}}{0.100 \text{ kJ/s}} = 17{,}490 \text{ s} = \mathbf{4.85 \text{ h}}$$

which is not practical. Therefore, the surface temperature of the heater must be allowed to be higher than 45°C.

Chapter 19
RADIATION HEAT TRANSFER

Electromagnetic and Thermal Radiation

19-1C Electromagnetic waves are caused by accelerated charges or changing electric currents giving rise to electric and magnetic fields. Sound waves are caused by disturbances. Electromagnetic waves can travel in vacuum, sound waves cannot.

19-2C Electromagnetic waves are characterized by their frequency ν and wavelength λ. These two properties in a medium are related by $\lambda = c/\nu$ where c is the speed of light in that medium.

19-3C Visible light is a kind of electromagnetic wave whose wavelength is between 0.40 and 0.76 μm. It differs from the other forms of electromagnetic radiation in that it triggers the sensation of seeing in the human eye.

19-4C Infrared radiation lies between 0.76 and 100 μm whereas ultraviolet radiation lies between the wavelengths 0.01 and 0.40 μm. The human body does not emit any radiation in the ultraviolet region since bodies at room temperature emit radiation in the infrared region only.

19-5C Thermal radiation is the radiation emitted as a result of vibrational and rotational motions of molecules, atoms and electrons of a substance, and it extends from about 0.1 to 100 μm in wavelength. Unlike the other forms of electromagnetic radiation, thermal radiation is emitted by bodies because of their temperature.

19-6C Light (or visible) radiation consists of narrow bands of colors from violet to red. The color of a surface depends on its ability to reflect certain wavelength. For example, a surface that reflects radiation in the wavelength range 0.63-0.76 μm while absorbing the rest appears red to the eye. A surface that reflects all the light appears white while a surface that absorbs the entire light incident on it appears black. The color of a surface at room temperature is not related to the radiation it emits.

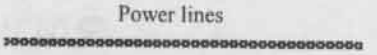
19-7C Radiation in opaque solids is considered surface phenomena since only radiation emitted by the molecules in a very thin layer of a body at the surface can escape the solid.

19-8C Because the snow reflects almost all of the visible and ultraviolet radiation, and the skin is exposed to radiation both from the sun and from the snow.

19-9C Microwaves in the range of 10^2 to 10^5 μm are very suitable for use in cooking as they are reflected by metals, transmitted by glass and plastics and absorbed by food (especially water) molecules. Thus the electric energy converted to radiation in a microwave oven eventually becomes part of the internal energy of the food with no conduction and convection thermal resistances involved. In conventional cooking, on the other hand, conduction and convection thermal resistances slow down the heat transfer, and thus the heating process.

19-10 Electricity is generated and transmitted in power lines at a frequency of 60 Hz. The wavelength of the electromagnetic waves is to be determined.

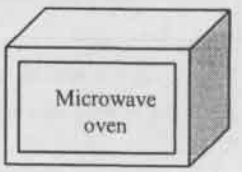
Analysis The wavelength of the electromagnetic waves is



$$\lambda = \frac{c}{\nu} = \frac{2.998\times10^{8}\ \text{m/s}}{60\ \text{Hz}(1/\text{s})} = 4.997\times10^{6}\ \text{m}$$

19-11 A microwave oven operates at a frequency of 2.8×10^{9} Hz. The wavelength of these microwaves and the energy of each microwave are to be determined.

Analysis The wavelength of these microwaves is



$$\lambda = \frac{c}{\nu} = \frac{2.998\times10^{8}\ \text{m/s}}{2.8\times10^{9}\ \text{Hz}(1/\text{s})} = 0.107\ \text{m} = 107\ \text{mm}$$

Then the energy of each microwave becomes

$$e = h\nu = \frac{hc}{\lambda} = \frac{(6.625\times10^{-34}\ \text{Js})(2.998\times10^{8}\ \text{m/s})}{0.107\ \text{m}} = 1.86\times10^{-24}\ \text{J}$$

19-12 A radio station is broadcasting radiowaves at a wavelength of 300 m. The frequency of these waves is to be determined.

Analysis The frequency of the waves is determined from



$$\lambda = \frac{c}{\nu} \longrightarrow \nu = \frac{c}{\lambda} = \frac{2.998\times10^{8}\ \text{m/s}}{300\text{m}} = 1.0\times10^{6}\ \text{Hz}$$

19-13 A cordless telephone operates at a frequency of 8.5×10^{8} Hz. The wavelength of these telephone waves is to be determined.

Analysis The wavelength of the telephone waves is



$$\lambda = \frac{c}{\nu} = \frac{2.998\times10^{8}\ \text{m/s}}{8.5\times10^{8}\ \text{Hz}(1/\text{s})} = 0.35\ \text{m} = 350\ \text{mm}$$

Blackbody Radiation

19-14C A blackbody is a perfect emitter and absorber of radiation. A blackbody does not actually exist. It is an idealized body that emits the maximum amount of radiation that can be emitted by a surface at a given temperature.

19-15C *Spectral blackbody emissive power* is the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area and per unit wavelength about wavelength λ. The integration of the spectral blackbody emissive power over the entire wavelength spectrum gives the *total blackbody emissive power*,

$$E_b(T) = \int_0^\infty E_{b\lambda}(T)d\lambda = \sigma T^4$$

The spectral blackbody emissive power varies with wavelength, the total blackbody emissive power does not.

19-16C We defined the blackbody radiation function f_λ because the integration $\int_0^\infty E_{b\lambda}(T)d\lambda$ cannot be performed. The blackbody radiation function f_λ represents the fraction of radiation emitted from a blackbody at temperature T in the wavelength range from $\lambda = 0$ to λ. This function is used to determine the fraction of radiation in a wavelength range between λ_1 and λ_2.

19-17C The larger the temperature of a body, the larger the fraction of the radiation emitted in shorter wavelengths. Therefore, the body at 1500 K will emit more radiation in the shorter wavelength region. The body at 1000 K emits more radiation at $20\ \mu m$ than the body at 1500 K since $\lambda T = \text{constant}$.

19-18 An isothermal cubical body is suspended in the air. The rate at which the cube emits radiation energy and the spectral blackbody emissive power are to be determined. √

Assumptions The body behaves as a black body.

Analysis (a) The total blackbody emissive power is determined from Stefan-Boltzman Law to be

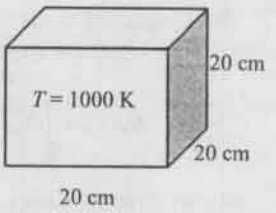
$$A = 6a^2 = 6(0.2^2) = 0.24 \text{ m}^2$$

$$E_b(T) = \sigma T^4 A = (5.67\times10^{-8} \text{ W/m}^2\text{K}^4)(1000\text{K})^4(0.24\text{m}^2) = 1.36\times10^4 \text{ W}$$

(b) The spectral blackbody emissive power at a wavelength of 4 μm is determined from Plank's distribution law,

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]} = \frac{3.743\times10^8 \text{ W}\cdot\mu\text{m}^4/\text{m}^2}{(4\,\mu\text{m})^5\left[\exp\left(\frac{1.4387\times10^4\ \mu\text{m}\cdot\text{K}}{(4\,\mu\text{m})(1000\text{K})}\right)-1\right]}$$

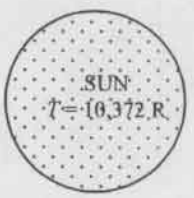
$$= 10.3 \text{ kW/m}^2\cdot\mu\text{m}$$



19-19E The sun is at an effective surface temperature of 10,372 R. The rate of infrared radiation energy emitted by the sun is to be determined.

Assumptions The sun behaves as a black body.

Analysis Noting that T = 10,372 R = 5762 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 19-2 to be



$$\lambda_1 T = (0.76\ \mu\text{m})(5762\text{ K}) = 4379.12\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.548796$$

$$\lambda_2 T = (100\ \mu\text{m})(5762\text{ K}) = 576{,}200\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.999$$

Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.999 - 0.548 = 0.451 \quad \text{(or 45.1\%)}$$

The total blackbody emissive power of the sun is determined from Stefan-Boltzman Law to be

$$E_b = \sigma T^4 = (0.1714\times10^{-8} \text{ Btu/h.ft}^2\text{R}^4)(10{,}372\text{R})^4 = 1.984\times10^7 \text{ Btu/h.ft}^2$$

Then,

$$E_{\text{infrared}} = (0.451)E_b = (0.451)(1.984\times10^7 \text{ Btu/h.ft}^2) = 8.95\times10^6 \text{ Btu/h.ft}^2$$

19-20 The temperature of the filament of an incandescent light bulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a black body.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu m$ to $\lambda_2 = 0.76\ \mu m$. Noting that $T = 3200$ K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 19-2 to be

$$\lambda_1 T = (0.40\ \mu m)(3200\ K) = 1280\ \mu mK \longrightarrow f_{\lambda_1} = 0.0043964$$

$$\lambda_2 T = (0.76\ \mu m)(3200\ K) = 2432\ \mu mK \longrightarrow f_{\lambda_2} = 0.147114$$



Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.14711424 - 0.0043964 = \mathbf{0.142718} \quad \text{(or 14.3\%)}$$

The wavelength at which the emission of radiation from the filament is maximum is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu m \cdot K \longrightarrow \lambda_{\text{max power}} = \frac{2897.8\ \mu m \cdot K}{3200\ K} = \mathbf{0.905\ mm}$$

19-21 An incandescent light bulb emits 20% of its energy at wavelengths shorter than 1 μm. The temperature of the filament is to be determined.

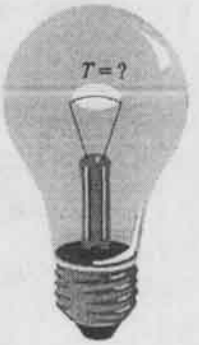
Assumptions The filament behaves as a black body.

Analysis From the Table 19-2 for the fraction of the radiation, we read

$$f_\lambda = 0.2 \longrightarrow \lambda T = 2700\ \mu mK$$

For the wavelength range of $\lambda_1 = 0.0\ \mu m$ to $\lambda_2 = 1.0\ \mu m$

$$\lambda = 1\ \mu m \longrightarrow \lambda T = 2700\ \mu mK \longrightarrow T = \mathbf{2700\ K}$$



19-22 Radiation emitted by a light source is maximum in the blue range. The temperature of this light source and the fraction of radiation it emits in the visible range are to be determined.

Assumptions The light source behaves as a black body.

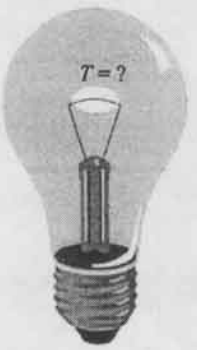
Analysis The temperature of this light source is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu m \cdot K \longrightarrow T = \frac{2897.8\ \mu m \cdot K}{0.47\ \mu m} = \mathbf{6166\ K}$$

The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu m$ to $\lambda_2 = 0.76\ \mu m$. Noting that $T = 6166$ K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 19-2 to be

$$\lambda_1 T = (0.40\ \mu m)(6166\ K) = 2466\ \mu mK \longrightarrow f_{\lambda_1} = 0.15444$$

$$\lambda_2 T = (0.76\ \mu m)(6166\ K) = 4686\ \mu mK \longrightarrow f_{\lambda_2} = 0.59141$$



Then the fraction of radiation emitted between these two wavelengths becomes

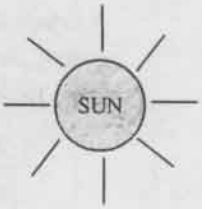
$$f_{\lambda_2} - f_{\lambda_1} = 0.59141 - 0.15444 = \mathbf{0.437} \quad \text{(or 43.7\%)}$$

19-23 A glass window transmits 90% of the radiation in a specified wavelength range and is opaque for radiation at other wavelengths. The rate of radiation transmitted through this window is to be determined for two cases. √

Assumptions The sources behave as a black body.

Analysis The surface area of the glass window is

$$A = 4\ \text{m}^2$$



(*a*) For a blackbody source at 5800 K, the total blackbody radiation emission is

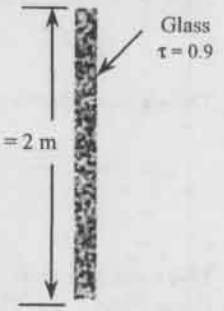
$$E_b(T) = \sigma T^4 A = (5.67\times10^{-8}\ \text{kW/m}^2\text{K})^4 (5800\text{K})^4 (4\ \text{m}^2) = 2.567\times10^5\ \text{kW}$$

The fraction of radiation in the range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\ \mu\text{m})(5800\text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.03345$$

$$\lambda_2 T = (3.0\ \mu\text{m})(5800\text{K}) = 17{,}40\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.97875$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.97875 - 0.03345 = 0.9453$$



Noting that 90% of the total radiation is transmitted through the window,

$$E_{\text{transmit}} = 0.90\,\Delta f E_b(T) = (0.90)(0.9453)(2.567\times10^5\ \text{kW}) = \mathbf{2.184\times10^5\ kW}$$

(*b*) For a blackbody source at 1000 K, the total blackbody emissive power is

$$E_b(T) = \sigma T^4 A = (5.67\times10^{-8}\ \text{W/m}^2\text{K}^4)(1000\text{K})^4 (4\ \text{m}^2) = 226.8\ \text{kW}$$

The fraction of radiation in the visible range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\ \mu\text{m})(1000\text{K}) = 300\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0000$$

$$\lambda_2 T = (3.0\ \mu\text{m})(1000\text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.27323$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.27323 - 0$$

and

$$E_{\text{transmit}} = 0.90\,\Delta f E_b(T) = (0.90)(0.27323)(226.8\ \text{kW}) = \mathbf{55.8\ kW}$$

Radiation Properties

19-24C The emissivity ε is the ratio of the radiation emitted by the surface to the radiation emitted by a blackbody at the same temperature. The fraction of radiation absorbed by the surface is called the absorptivity α,

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \quad \text{and} \quad \alpha = \frac{\text{absorbed radiation}}{\text{incident radiation}} = \frac{G_{abs}}{G}$$

When the surface temperature is equal to the temperature of the source of radiation, the total hemispherical emissivity of a surface at temperature T is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the same temperature $\varepsilon_\lambda(T) = \alpha_\lambda(T)$.

19-25C The fraction of irradiation reflected by the surface is called reflectivity ρ and the fraction transmitted is called the transmissivity τ

$$\rho = \frac{G_{ref}}{G} \quad \text{and} \quad \tau = \frac{G_{tr}}{G}$$

Surfaces are assumed to reflect in a perfectly spectral or diffuse manner for simplicity. In spectral (or mirror like) reflection, the angle of reflection equals the angle of incidence of the radiation beam. In diffuse reflection, radiation is reflected equally in all directions.

19-26C A body whose surface properties are independent of wavelength is said to be a gray body. The emissivity of a blackbody is one for all wavelengths, the emissivity of a graybody is between zero and one.

19-27C The heating effect which is due to the non-gray characteristic of glass, clear plastic, or atmospheric gases is known as the greenhouse effect since this effect is utilized primarily in greenhouses. The combustion gases such as CO_2 and water vapor in the atmosphere transmit the bulk of the solar radiation but absorb the infrared radiation emitted by the surface of the earth, acting like a heat trap. There is a concern that the energy trapped on earth will eventually cause global warming and thus drastic changes in weather patterns.

19-28C Glass has a transparent window in the wavelength range 0.3 to 3 μm and it is not transparent to the radiation which has wavelength range greater than 3 μm. Therefore, because the microwaves are in the range of 10^2 to 10^5 μm, the harmful microwave radiation can not escape from the glass door.

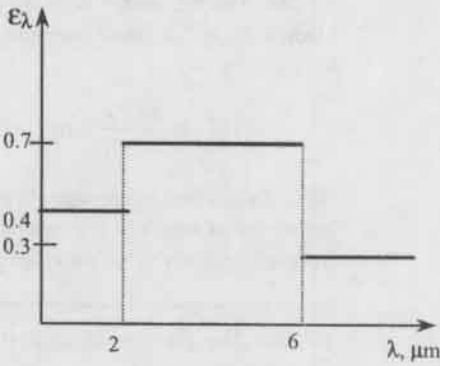
19-29 The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b_\lambda}(T)d\lambda}{\sigma T^4}$$

$$= \varepsilon_1 f_{0-\lambda_1} + \varepsilon_2 f_{\lambda_1-\lambda_2} + \varepsilon_3 f_{\lambda_2-\infty}$$

$$= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$



where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (2\ \mu\text{m})(1000\ \text{K}) = 2000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.066728$$

$$\lambda_2 T = (6\ \mu\text{m})(1000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.737818$$

$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1}$ since $f_0 = 0$ and $f_{\lambda_2-\infty} = f_\infty - f_{\lambda_2}$ since $f_\infty = 1$.

and,

$$\varepsilon = (0.4)0.066728 + (0.7)(0.737818 - 0.066728) + (0.3)(1 - 0.737818) = \mathbf{0.575}$$

Then the emissive power of the surface becomes

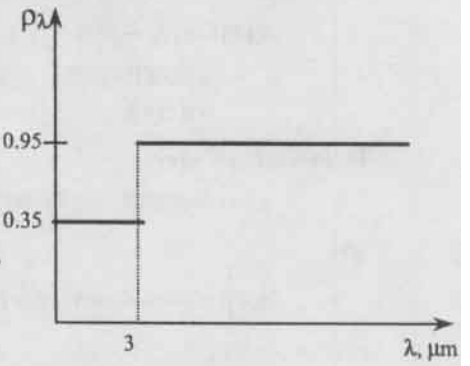
$$E = \varepsilon\sigma T^4 = 0.575(5.67\times10^{-8}\ \text{W/m}^2\text{K}^4)(1000\text{K})^4 = \mathbf{32.6\ kW/m^2}$$

19-30 The variation of reflectivity of a surface with wavelength is given. The average reflectivity, emissivity, and absorptivity of the surface are to be determined for two source temperatures.

Analysis The average reflectivity of this surface for solar radiation ($T = 5800$ K) is determined to be

$$\lambda T = (3\ \mu\text{m})(5800\ \text{K}) = 17400\ \mu\text{mK} \rightarrow f_\lambda = 0.978746$$

$$\begin{aligned}\rho(T) &= \rho_1 f_{0-\lambda_1}(T) + \rho_2 f_{\lambda_1-\infty}(T) \\ &= \rho_1 f_{\lambda_1} + \rho_2 (1 - f_{\lambda_1}) \\ &= (0.35)(0.978746) + (0.95)(1 - 0.978746) \\ &= \mathbf{0.362}\end{aligned}$$



Noting that this is an opaque surface, $\tau = 0$

At $T = 5800$ K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.362 = \mathbf{0.638}$

Repeating calculations for radiation coming from surfaces at $T = 300$ K,

$$\lambda T = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0001685$$

$$\rho(T) = (0.35)(0.0001685) + (0.95)(1 - 0.0001685) = \mathbf{0.95}$$

At $T = 300$ K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.95 = \mathbf{0.05}$

and $\varepsilon = \alpha = \mathbf{0.05}$

The temperature of the aluminum plate is close to room temperature, and thus emissivity of the plate will be equal to its absorptivity at room temperature. That is,

$$\varepsilon = \varepsilon_{room} = 0.05$$
$$\alpha = \alpha_s = 0.638$$

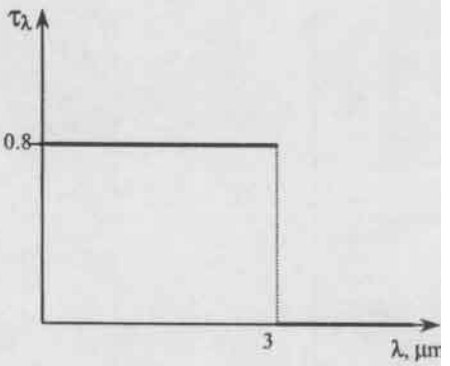
which makes it suitable as a solar collector. ($\alpha_s = 1$ and $\varepsilon_{room} = 0$ for an ideal solar collector)

19-31 The variation of transmissivity of the glass window of a furnace at a specified temperature with wavelength is given. The fraction and the rate of radiation coming from the furnace and transmitted through the window are to be determined.

Assumptions The window glass behaves as a black body.

Analysis The fraction of radiation at wavelengths smaller than 3 μm is

$$\lambda T = (3\ \mu\text{m})(1200\ \text{K}) = 3600\ \mu\text{mK} \longrightarrow f_\lambda = 0.403607$$



The fraction of radiation coming from the furnace and transmitted through the window is

$$\begin{aligned}\tau(T) &= \tau_1 f_\lambda + \tau_2 (1 - f_\lambda) \\ &= (0.8)(0.403607) + (0)(1 - 0.403607) \\ &= \mathbf{0.323}\end{aligned}$$

Then the rate of radiation coming from the furnace and transmitted through the window becomes

$$G_{tr} = \tau A \sigma T^4 = 0.323(0.04\ \text{m}^2)(5.67 \times 10^{-8}\ \text{W/m}^2.\text{K}^4)(1200\ \text{K})^4 = \mathbf{1519\ W}$$

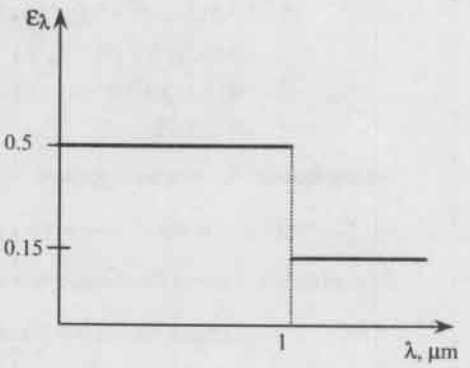
19-32 The variation of emissivity of a tungsten filament with wavelength is given. The average emissivity, absorptivity, and reflectivity of the filament are to be determined for two temperatures.

Analysis (a) $T = 1500$ K

$$\lambda_1 T = (1\ \mu\text{m})(1500\ \text{K}) = 1500\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.013754$$

The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.5)(0.013754) + (0.15)(1 - 0.013754) \\ &= \mathbf{0.155}\end{aligned}$$



From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.155} \quad \text{(at 1500 K)}$$

and

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.155 = \mathbf{0.845}$$

(b) $T = 3000$ K

$$\lambda_1 T = (1\ \mu\text{m})(3000\ \text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.273232$$

Then

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.5)(0.273232) + (0.15)(1 - 0.273232) \\ &= \mathbf{0.246}\end{aligned}$$

From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.246} \quad \text{(at 1500 K)}$$

and

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.246 = \mathbf{0.754}$$

19-33 The variations of emissivity of two surfaces are given. The average emissivity, absorptivity, and reflectivity of each surface are to be determined at the given temperature.

Analysis For the first surface:

$$\lambda_1 T = (3\ \mu\text{m})(3000\ \text{K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$

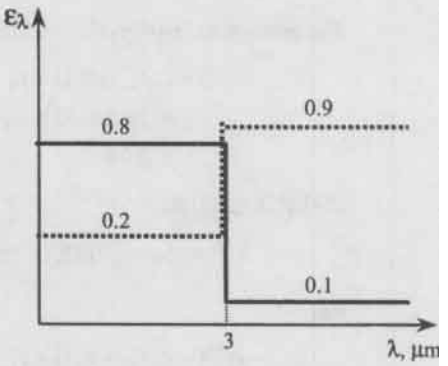
The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.890029) + (0.1)(1 - 0.890029) \\ &= \mathbf{0.19}\end{aligned}$$

The absorptivity and reflectivity are determined from Kirchhoff's law

$$\varepsilon = \alpha = \mathbf{0.19} \quad \text{(at 3000 K)}$$

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.19 = \mathbf{0.81}$$



For the second surface:

$$\lambda_1 T = (3\ \mu\text{m})(3000\ \text{K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$

The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.8)(0.890029) + (0.9)(1 - 0.890029) \\ &= 0.81\end{aligned}$$

Then,

$$\varepsilon = \alpha = \mathbf{0.81} \quad \text{(at 3000 K)}$$

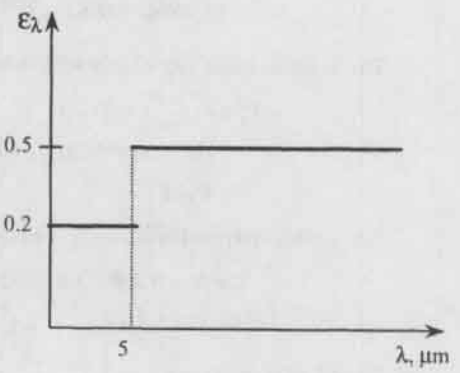
$$\alpha + \rho = 1 \rightarrow \rho = 1 - \alpha = 1 - 0.81 = \mathbf{0.19}$$

Discussion The second surface is more suitable to serve as a solar absorber since its absorptivity is greater.

19-34 The variation of emissivity of a surface with wavelength is given. The average emissivity and absorptivity of the surface are to be determined for two temperatures.

Analysis (a) For $T = 5800$ K:

$$\lambda_1 T = (5\ \mu\text{m})(5800\ \text{K}) = 29{,}000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.99534$$



The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.99534) + (0.9)(1 - 0.99534) \\ &= \mathbf{0.203}\end{aligned}$$

(b) For $T = 300$ K:

$$\lambda_1 T = (5\mu\text{m})(300\ \text{K}) = 1500\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.013754$$

and

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.013754) + (0.9)(1 - 0.013754) \\ &= \mathbf{0.89}\end{aligned}$$

The absorptivities of this surface for radiation coming from sources at 5800 K and 300 K are, from Kirchhoff's law,

$$\alpha = \varepsilon = \mathbf{0.203} \quad \text{(at 5800 K)}$$

$$\alpha = \varepsilon = \mathbf{0.89} \quad \text{(at 300 K)}$$

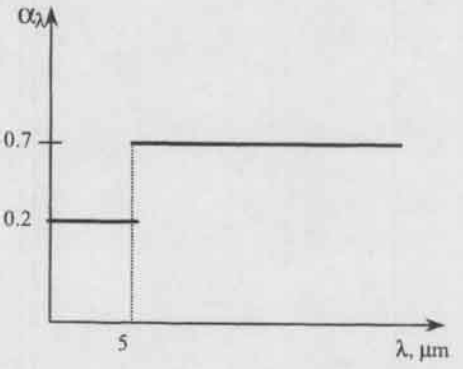
19-35 The variation of absorptivity of a surface with wavelength is given. The average absorptivity, reflectivity, and emissivity of the surface are to be determined at given temperatures.

Analysis For $T = 2500$ K:

$$\lambda_1 T = (2\ \mu\text{m})(2500\ \text{K}) = 5000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.633747$$

The average absorptivity of this surface is

$$\begin{aligned}\alpha(T) &= \alpha_1 f_{\lambda_1} + \alpha_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.633747) + (0.7)(1 - 0.633747) \\ &= \mathbf{0.38}\end{aligned}$$



Then the reflectivity of this surface becomes

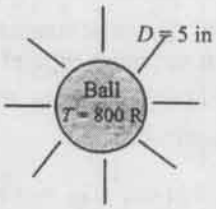
$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.38 = \mathbf{0.62}$$

Using Kirchhoff's law, $\alpha = \varepsilon$, the average emissivity of this surface at $T = 3000$ K is determined to be

$$\lambda T = (2\mu\text{m})(3000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda} = 0.737818$$

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.737818) + (0.7)(1 - 0.737818) \\ &= \mathbf{0.33}\end{aligned}$$

19-36E A spherical ball emits radiation at a certain rate. The average emissivity of the ball is to be determined at the given temperature.



Analysis The surface area of the ball is

$$A = \pi D^2 = \pi(5/12 \text{ ft})^2 = 0.5454 \text{ ft}^2$$

Then the average emissivity of the ball at this temperature is determined to be

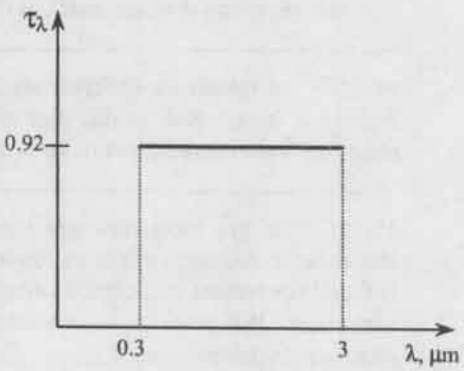
$$E = \varepsilon A \sigma T^4 \longrightarrow \varepsilon = \frac{E}{A\sigma T^4} = \frac{120 \text{ Btu/h}}{(0.5454 \text{ ft}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2\cdot\text{K})(800\text{R})^4} = \mathbf{0.31}$$

19-37 The variation of transmissivity of a glass is given. The average transmissivity of the pane at two temperatures and the amount of solar radiation transmitted through the pane are to be determined.

Analysis For T=5800 K:

$$\lambda_1 T_1 = (0.3 \text{ μm})(5800 \text{ K}) = 1740 \text{ μmK} \longrightarrow f_{\lambda_1} = 0.035$$

$$\lambda_2 T_1 = (3 \text{ μm})(5800 \text{ K}) = 17{,}400 \text{ μmK} \longrightarrow f_{\lambda_2} = 0.977$$



The average transmissivity of this surface is

$$\tau(T) = \tau_1(f_{\lambda_2} - f_{\lambda_1}) = (0.9)(0.977 - 0.035) = \mathbf{0.848}$$

For T=300 K:

$$\lambda_1 T_2 = (0.3 \text{ μm})(300 \text{ K}) = 90 \text{ μmK} \longrightarrow f_{\lambda_1} = 0.0$$

$$\lambda_2 T_2 = (3 \text{ μm})(300 \text{ K}) = 900 \text{ μmK} \longrightarrow f_{\lambda_2} = 0.0001685$$

Then,

$$\tau(T) = \tau_1(f_{\lambda_2} - f_{\lambda_1}) = (0.9)(0.0001685 - 0.0) = \mathbf{0.00015} \approx \mathbf{0}$$

The amount of solar radiation transmitted through this glass is

$$G_{tr} = \tau G_{incident} = 0.848(650 \text{ W/m}^2) = \mathbf{551 \text{ W/m}^2}$$

Atmospheric and Solar Radiation

19-38C The solar constant represents the rate at which solar energy is incident on a surface normal to sun's rays at the outer edge of the atmosphere when the earth is at its mean distance from the sun. It value is $G_s = 1353\ \text{W/m}^2$. The solar constant is used to estimate the effective surface temperature of the sun from the requirement that

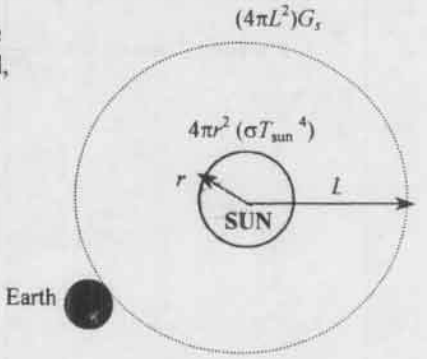
$$(4\pi L^2)G_{s1} = (4\pi r^2)\sigma T_{sun}{}^4$$

where L is the mean distance between the sun and the earth and r is the radius of the sun. If the distance between the earth and the sun doubled, the value of G_s drops to one-fourth since

$$4\pi(2L)^2 G_{s2} = (4\pi r^2)\sigma T_{sun}{}^4$$

$$16\pi L^2 G_{s2} = (4\pi r^2)\sigma T_{sun}{}^4$$

$$16\pi L^2 G_{s2} = 4\pi L^2 G_{s1} \longrightarrow \mathbf{G_{s2} = \frac{G_{s1}}{4}}$$



19-39C The amount of solar radiation incident on earth will decrease by a factor of

$$\text{Reduction factor} = \frac{\sigma T_{sun}{}^4}{\sigma T_{sun,new}{}^4} = \frac{5762^4}{2000^4} = 68.9$$

(or to 1.5% of what it was). Also, the fraction of radiation in the visible range would be much smaller.

19-40C Air molecules scatter blue light much more than they do red light. This molecular scattering in all directions is what gives the sky its bluish color. At sunset, the light travels through a thicker layer of atmosphere, which removes much of the blue from the natural light, letting the red dominate.

19-41C The reason for different seasons is the tilt of the earth which causes the solar radiation to travel through a longer path in the atmosphere in winter, and a shorter path in summer. Therefore, the solar radiation is attenuated much more strongly in winter.

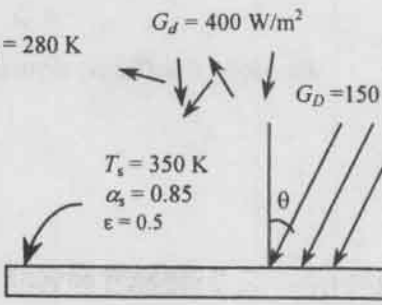
19-42C The gas molecules and the suspended particles in the atmosphere emit radiation as well as absorbing it. Although this emission is far from resembling the distribution of radiation from a blackbody, it is found convenient in radiation calculations to treat the atmosphere as a blackbody at some lower fictitious temperature that emits an equivalent amount of radiation energy. This fictitious temperature is called the effective sky temperature T_{sky}.

19-43C There is heat loss from both sides of the bridge (top and bottom surfaces of the bridge) which reduces temperature of the bridge surface to very low values. The relatively warm earth under a highway supply heat to the surface continuously, making the water on it less likely to freeze.

19-44C Due to its nearly horizontal orientation, windshield exchanges heat with the sky that is at very low temperature. Side windows on the other hand exchange heat with surrounding surfaces that are at relatively high temperature.

19-45C Because of different wavelengths of solar radiation and radiation originating from surrounding bodies, the surfaces usually have quite different absorptivities. Solar radiation is concentrated in the short wavelength region and the surfaces in the infrared region.

19-46 A surface is exposed to solar and sky radiation. The net rate of radiation heat transfer is to be determined.



Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.85$ and $\varepsilon = 0.5$.

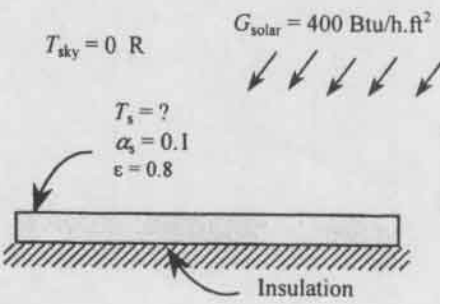
Analysis The total solar energy incident on the surface is

$$G_{solar} = G_D \cos\theta + G_d = (350 \text{ W/m}^2)\cos 30° + (400 \text{ W/m}^2) = 703.1 \text{ W/m}^2$$

Then the net rate of radiation heat transfer in this case becomes

$$\begin{aligned}\dot{q}_{net,rad} &= \alpha_s G_{solar} - \varepsilon\sigma(T_s^4 - T_{sky}^4) \\ &= 0.85(703.1 \text{ W/m}^2) - 0.5(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)\left[(350 \text{ K})^4 - (280 \text{ K})^4\right] \\ &= \mathbf{347 \text{ W/m}^2} \text{ (to the surface)}\end{aligned}$$

19-47E A surface is exposed to solar and sky radiation. The equilibrium temperature of the surface is to be determined.



Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.10$ and $\varepsilon = 0.8$.

Analysis The equilibrium temperature of the surface in this case is

$$\dot{q}_{net,rad} = \alpha_s G_{solar} - \varepsilon\sigma(T_s^4 - T_{sky}^4) = 0$$

$$\alpha_s G_{solar} = \varepsilon\sigma(T_s^4 - T_{sky}^4)$$

$$0.1(400 \text{ Btu/h.ft}^2) = 0.8(0.1714\times10^{-8} \text{ Btu/h.ft}^2\cdot\text{R}^4)\left[T_s^4 - (0 \text{ R})^4\right]$$

$$T_s = \mathbf{413 \text{ R}}$$

19-48 Water is observed to have frozen one night while the air temperature is above freezing temperature. The effective sky temperature is to be determined.

Properties The emissivity of water is $\varepsilon = 0.95$ (Table A-24).

Analysis Assuming the water temperature to be 0°C, the value of the effective sky temperature is determined from an energy balance on water to be

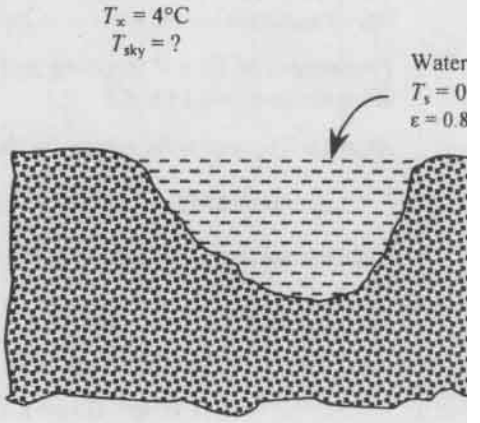
$$h(T_{air} - T_{surface}) = \varepsilon\sigma(T_s^4 - T_{sky}^4)$$

and

$$(10\ \text{W/m}^2\cdot°\text{C})(4°\text{C} - 0°\text{C}) = 0.95(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(273\ \text{K})^4 - T_{sky}^4\right]$$

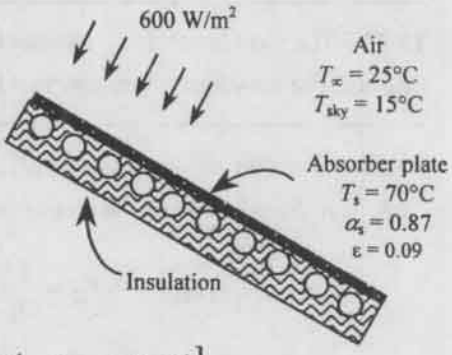
$$\longrightarrow T_{sky} = \mathbf{263\ K}$$

Therefore, the effective sky temperature must have been below 263 K.



19-49 The absorber plate of a solar collector is exposed to solar and sky radiation. The net rate of solar energy absorbed by the absorber plate is to be determined. √

Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.87$ and $\varepsilon = 0.09$.



Analysis The net rate of solar energy delivered by the absorber plate to the water circulating behind it can be determined from an energy balance to be

$$\dot{q}_{net} = \dot{q}_{gain} - \dot{q}_{loss}$$

$$\dot{q}_{net} = \alpha_s G_{solar} - \left[\varepsilon\sigma(T_s^4 - T_{sky}^4) + h(T_s - T_{air})\right]$$

Then,

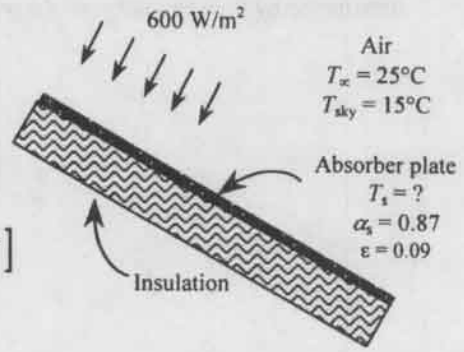
$$\dot{q}_{net} = 0.87(600\ \text{W/m}^2) - 0.09(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(70+273\ \text{K})^4 - (15+273\ \text{K})^4\right]$$
$$-(10\ \text{W/m}^2\cdot\text{K})(70°\text{C} - 25°\text{C})$$
$$= \mathbf{36.5\ W/m^2}$$

Therefore, heat is gained by the plate and transferred to water at a rate of 36.5 W per m^2 surface area.

19-50 The absorber surface of a solar collector is exposed to solar and sky radiation. The equilibrium temperature of the absorber surface is to be determined if the backside of the plate is insulated.

Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.87$ and $\varepsilon = 0.09$.

Analysis The backside of the absorbing plate is insulated (instead of being attached to water tubes), and thus



$$\dot{q}_{net} = 0$$

$$\alpha_s G_{solar} = \varepsilon\sigma(T_s^4 - T_{sky}^4) + h(T_s - T_{air})$$

$$(0.87)(600\ \text{W/m}^2) = (0.09)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(T_s)^4 - (288\ \text{K})^4\right]$$
$$+(10\ \text{W/m}^2\cdot\text{K})(T_s - 298\ \text{K})$$

$$T_s = \mathbf{346\ K}$$

View Factors

19-51C The view factor $F_{i\to j}$ represents the fraction of the radiation leaving surface i that strikes surface j directly. The view factor from a surface to itself is non-zero for concave surfaces.

19-52C The pair of view factors $F_{i\to j}$ and $F_{j\to i}$ are related to each other by the reciprocity rule $A_i F_{ij} = A_j F_{ji}$ where A_i is the area of the surface i and A_j is the area of the surface j. Therefore,

$$A_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \frac{A_2}{A_1} F_{21}$$

19-53C The summation rule for an enclosure and is expressed as $\sum_{j=1}^{N} F_{i\to j} = 1$ where N is the number of surfaces of the enclosure. It states that the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself must be equal to unity.

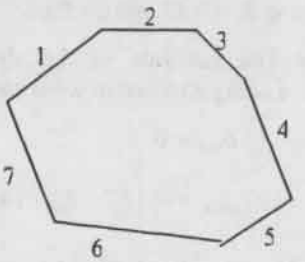
The superposition rule is stated as the view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j, $F_{1\to(2,3)} = F_{1\to 2} + F_{1\to 3}$.

19-54C The cross-string method is applicable to geometries which are very long in one direction relative to the other directions. By attaching strings between corners the Crossed-Strings Method is expressed as

$$F_{i\to j} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{string on surface } i}$$

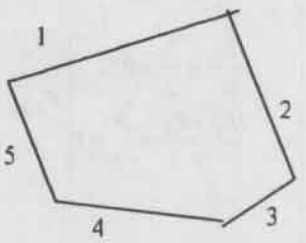
19-55 An enclosure consisting of seven surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.

Analysis A seven surface enclosure (N=7) involves $N^2 = 7^2 = 49$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{7(7-1)}{2} = 21$ view factors directly. The remaining 419-21 = 28 of the view factors can be determined by the application of the reciprocity and summation rules.



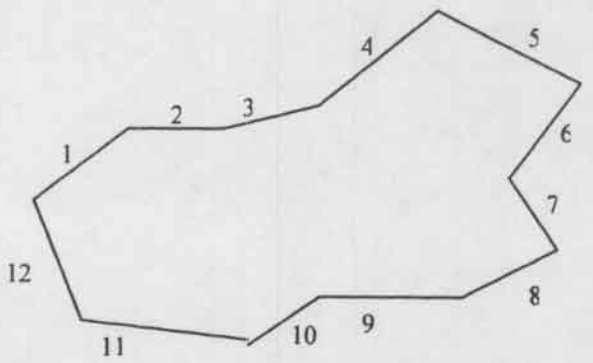
19-56 An enclosure consisting of five surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.

Analysis A five surface enclosure (N=5) involves $N^2 = 5^2 = 25$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{5(5-1)}{2} = 10$ view factors directly. The remaining 25-10 = 15 of the view factors can be determined by the application of the reciprocity and summation rules.



19-57 An enclosure consisting of twelve surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.

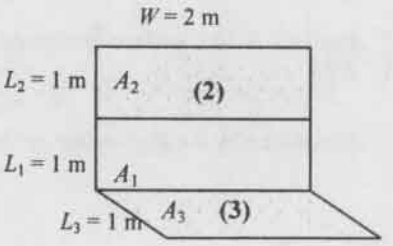
Analysis A twelve surface enclosure (N=12) involves $N^2 = 12^2 = \mathbf{144}$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{12(12-1)}{2} = 66$ view factors directly. The remaining 144-66 = **78** of the view factors can be determined by the application of the reciprocity and summation rules.



19-58 The view factors between the rectangular surfaces shown in the figure are to be determined. √

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis From Fig. 19-42,



$$\left.\begin{aligned}\frac{L_3}{W}&=\frac{1}{2}=0.5\\ \frac{L1}{W}&=\frac{1}{2}=0.5\end{aligned}\right\}F_{31}=0.24$$

and

$$\left.\begin{aligned}\frac{L_3}{W}&=\frac{1}{2}=0.5\\ \frac{L_1+L_2}{W}&=\frac{2}{2}=1\end{aligned}\right\}F_{3\to(1+2)}=0.29$$

We note that $A_1 = A_3$. Then the reciprocity and superposition rules gives

$$A_1F_{13}=A_3F_{31}\longrightarrow F_{13}=F_{31}=\mathbf{0.24}$$

$$F_{3\to(1+2)}=F_{31}+F_{32}\longrightarrow\quad 0.29=0.24+F_{32}\longrightarrow F_{32}=0.05$$

Finally,

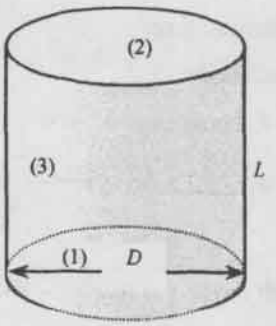
$$A_2=A_3\longrightarrow F_{23}=F_{32}=\mathbf{0.05}$$

19-59 A cylindrical enclosure is considered. The view factor from the side surface of this cylindrical enclosure to its base surface is to be determined. √

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We designate the surfaces as follows:

Base surface by (1),
top surface by (2), and
side surface by (3).



Then from Fig. 19-43 (or Table 19-4 for better accuracy)

$$\left.\begin{aligned}\frac{L}{r_1}&=\frac{4r_1}{r_1}=4\\ \frac{r_2}{L}&=\frac{r_2}{4r_2}=0.25\end{aligned}\right\}F_{12}=F_{21}=0.05$$

summation rule: $F_{11}+F_{12}+F_{13}=1$

$$0+0.05+F_{13}=1\longrightarrow F_{13}=0.95$$

reciprocity rule: $A_1F_{13}=A_3F_{31}\longrightarrow F_{31}=\dfrac{A_1}{A_3}F_{13}=\dfrac{\pi r_1^2}{2\pi r_1 L}F_{13}=\dfrac{\pi r_1^2}{2\pi r_1(4r_1)}F_{13}=\dfrac{1}{8}(0.95)=\mathbf{0.119}$

Discussion This problem can be solved more accurately by using the view factor relation from Table 19-4 to be

$$R_1=\frac{r_1}{L}=\frac{r_1}{4r_1}=0.25,$$

$$R_2=\frac{r_2}{L}=\frac{r_2}{4r_2}=0.25$$

$$S=1+\frac{1+R_2^2}{R_1^2}=1+\frac{1+0.25^2}{0.25^2}=18$$

$$F_{12}=\frac{1}{2}\left\{S-\left[S^2-4\left(\frac{R_2}{R_1}\right)^2\right]^{0.5}\right\}=\frac{1}{2}\left\{18-\left[18^2-4\left(\frac{0.25}{0.25}\right)^2\right]^{0.5}\right\}=0.0557$$

$$F_{13}=1-F_{12}=1-0.0557=0.9443$$

$$A_1F_{13}=A_3F_{31}\longrightarrow F_{31}=\frac{A_1}{A_3}F_{13}=\frac{\pi r_1^2}{2\pi r_1 L}F_{13}=\frac{\pi r_1^2}{2\pi r_1(4r_1)}F_{13}=\frac{1}{8}(0.9443)=\mathbf{0.118}$$

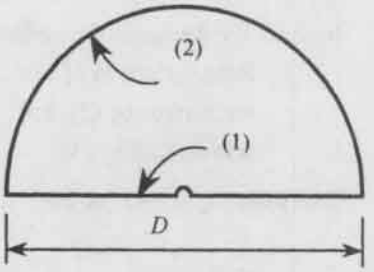
19-60 A semispherical furnace is considered. The view factor from the dome of this furnace to its flat base is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We number the surfaces as follows:

(1): circular base surface

(2): dome surface



Surface (1) is flat, and thus $F_{11} = 0$.

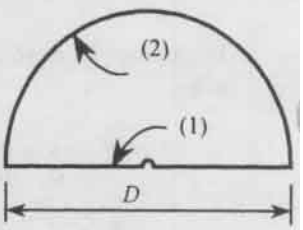
Summation rule: $F_{11} + F_{12} = 1 \rightarrow F_{12} = 1$

reciprocity rule: $A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{A_1}{A_2}(1) = \dfrac{\frac{\pi D^2}{4}}{\frac{\pi D^2}{2}} = \dfrac{1}{2} = \mathbf{0.5}$

19-61 Two view factors associated with three very long ducts with different geometries are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

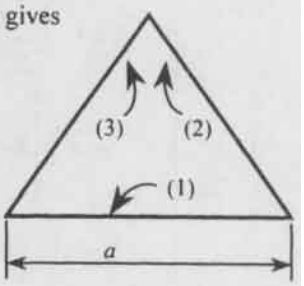
Analysis (*a*) Surface (1) is flat, and thus $F_{11} = 0$.



summation rule: $F_{11} + F_{12} = 1 \rightarrow F_{12} = \mathbf{1}$

reciprocity rule: $A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{Ds}{\left(\frac{\pi D}{2}\right)s}(1) = \dfrac{2}{\pi} = \mathbf{0.64}$

(*b*) Noting that surfaces 2 and 3 are symmetrical and thus $F_{12} = F_{13}$, the summation rule gives



$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow 0 + F_{12} + F_{13} = 1 \longrightarrow F_{12} = \mathbf{0.5}$$

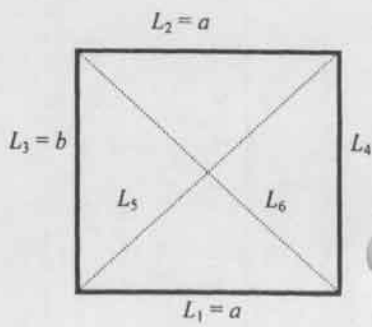
Also by using the equation obtained in Example 19-8,

$$F_{12} = \frac{L_1 + L_2 - L_3}{2L_1} = \frac{a + b - b}{2a} = \frac{a}{2a} = \frac{1}{2} = \mathbf{0.5}$$

reciprocity rule: $A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{a}{b}\left(\dfrac{1}{2}\right) = \mathbf{\dfrac{a}{2b}}$

(*c*) Applying the crossed-string method gives

$$F_{12} = F_{21} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1} = \frac{2\sqrt{a^2 + b^2} - 2b}{2a} = \mathbf{\frac{\sqrt{a^2 + b^2} - b}{a}}$$

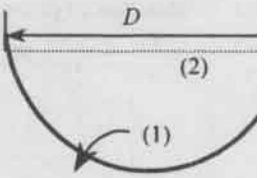


19-62 View factors from the very long grooves shown in the figure to the surroundings are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis (*a*) We designate the circular dome surface by (1) and the imaginary flat top surface by (2). Noting that (2) is flat,

$$F_{22} = 0$$

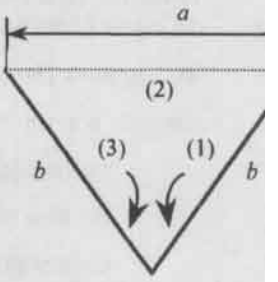


summation rule: $F_{21} + F_{22} = 1 \longrightarrow F_{21} = 1$

reciprocity rule: $A_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \dfrac{A_2}{A_1} F_{21} = \dfrac{D}{\dfrac{\pi D}{2}}(1) = \dfrac{2}{\pi} = \mathbf{0.64}$

(*b*) We designate the two identical surfaces of length *b* by (1) and (3), and the imaginary flat top surface by (2). Noting that (2) is flat,

$$F_{22} = 0$$



summation rule: $F_{21} + F_{22} + F_{23} = 1 \longrightarrow F_{21} = F_{23} = 0.5$ (symmetry)

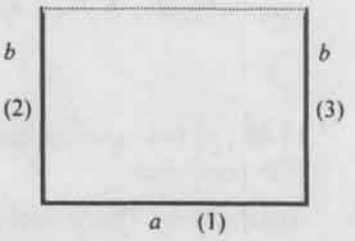
summation rule: $F_{22} + F_{2\to(1+3)} = 1 \longrightarrow F_{2\to(1+3)} = 1$

reciprocity rule: $A_2 F_{2\to(1+3)} = A_{(1+3)} F_{(1+3)\to 2}$

$$\longrightarrow F_{(1+3)\to 2} = F_{(1+3)\to surr} = \frac{A_2}{A_{(1+3)}}(1) = \mathbf{\frac{a}{2b}}$$

(*c*) We designate the bottom surface by (1), the side surfaces by (2) and (3), and the imaginary top surface by (4). Surface 4 is flat and is completely surrounded by other surfaces. Therefore, $F_{44} = 0$ and $F_{4\to(1+2+3)} = 1$.

(4)



reciprocity rule: $A_4 F_{4\to(1+2+3)} = A_{(1+2+3)} F_{(1+2+3)\to 4}$

$$\longrightarrow F_{(1+2+3)\to 4} = F_{(1+2+3)\to surr} = \frac{A_4}{A_{(1+2+3)}}(1) = \mathbf{\frac{a}{a+2b}}$$

19-63 The view factors from the base of a cube to each of the other five surfaces are to be determined.

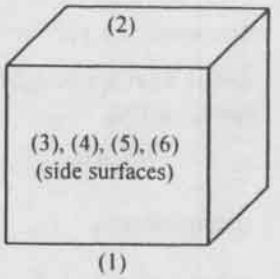
Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Noting that $L_1 / w = L_2 / w = 1$, from Fig. 19-42 we read

$$F_{12} = 0.2$$

Because of symmetry, we have

$$F_{12} = F_{13} = F_{14} = F_{15} = F_{16} = \mathbf{0.2}$$



19-64 The view factor from the conical side surface to a hole located at the center of the base of a conical enclosure is to be determined.

Assumptions The conical side surface is diffuse emitter and reflector.

Analysis We number different surfaces as

the hole located at the center of the base (1)

the base of conical enclosure (2)

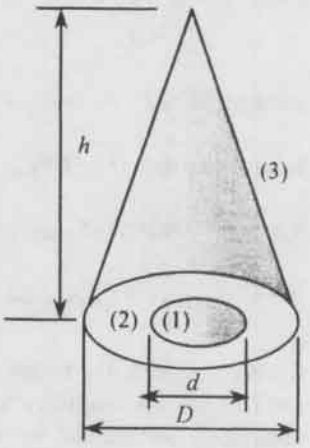
conical side surface (3)

Surfaces 1 and 2 are flat , and they have no direct view of each other. Therefore,

$$F_{11} = F_{22} = F_{12} = F_{21} = 0$$

summation rule : $F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1$

reciprocity rule : $A_1 F_{13} = A_3 F_{31} \longrightarrow \dfrac{\pi d^2}{4}(1) = \dfrac{\pi D h}{2} F_{31} \longrightarrow F_{31} = \mathbf{\dfrac{d^2}{2Dh}}$



19-65 The four view factors associated with an enclosure formed by two very long concentric cylinders are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis We number different surfaces as

the outer surface of the inner cylinder (1)

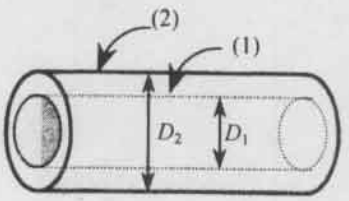
the inner surface of the outer cylinder (2)

No radiation leaving surface 1 strikes itself and thus $F_{11} = \mathbf{0}$

All radiation leaving surface 1 strikes surface 2 and thus $F_{12} = 1$

reciprocity rule : $A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{\pi D_1 h}{\pi D_2 h}(1) = \mathbf{\dfrac{D_1}{D_2}}$

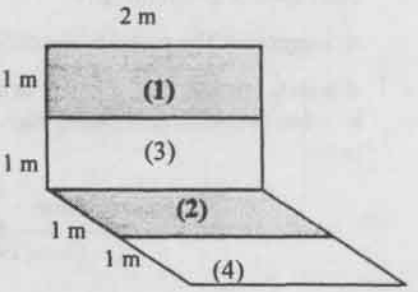
summation rule : $F_{21} + F_{22} = 1 \longrightarrow F_{22} = 1 - F_{21} = \mathbf{1 - \dfrac{D_1}{D_2}}$



19-66 The view factors between the rectangular surfaces shown in the figure are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We designate the different surfaces as follows:
shaded part of perpendicular surface by (1),
bottom part of perpendicular surface by (3),
shaded part of horizontal surface by (2), and
front part of horizontal surface by (4).



(*a*) From Fig.19-42

$$\left.\begin{matrix}\frac{Y}{X}=\frac{1}{2}\\ \frac{Z}{X}=\frac{1}{2}\end{matrix}\right\}F_{23}=0.24 \quad \text{and} \quad \left.\begin{matrix}\frac{Y}{X}=\frac{1}{2}\\ \frac{Z}{X}=\frac{2}{2}\end{matrix}\right\}F_{2\to(1+3)}=0.29$$

superposition rule: $F_{2\to(1+3)}=F_{21}+F_{23}\longrightarrow F_{21}=F_{2\to(1+3)}-F_{23}=0.29-0.24=0.05$

reciprocity rule: $A_1=A_2\longrightarrow A_1F_{12}=A_2F_{21}\longrightarrow F_{12}=F_{21}=\mathbf{0.05}$

(*b*) From Fig.19-42,

$$\left.\begin{matrix}\frac{Y}{X}=\frac{2}{2} \text{ and } \frac{Z}{X}=\frac{1}{2}\end{matrix}\right\}F_{(4+2)\to3}=0.15 \quad \text{and} \quad \left.\begin{matrix}\frac{Y}{X}=\frac{2}{2} \text{ and } \frac{Z}{X}=\frac{2}{2}\end{matrix}\right\}F_{(4+2)\to(1+3)}=0.20$$

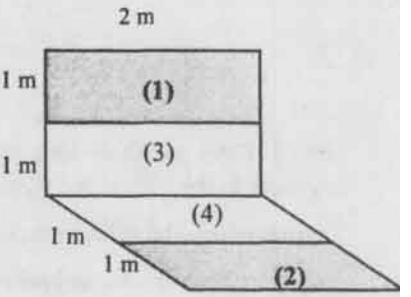
superposition rule: $F_{(4+2)\to(1+3)}=F_{(4+2)\to1}+F_{(4+2)\to3}\longrightarrow F_{(4+2)\to1}=0.20-0.15=0.05$

reciprocity rule: $A_{(4+2)}F_{(4+2)\to1}=A_1F_{1\to(4+2)}$

$$\longrightarrow F_{1\to(4+2)}=\frac{A_{(4+2)}}{A_1}F_{(4+2)\to1}=\frac{4}{2}(0.05)=0.10$$

superposition rule: $F_{1\to(4+2)}=F_{14}+F_{12}\longrightarrow F_{14}=0.10-0.05=\mathbf{0.05}$

since $F_{12}=0.05$ (from part *a*).. Note that F_{14} in part (*b*) is equivalent to F_{12} in part (*a*).



(*c*) We designate
shaded part of top surface by (1),
remaining part of top surface by (3),
remaining part of bottom surface by (4), and
shaded part of bottom surface by (2).

From Fig.19-42,

$$\left.\begin{matrix}\frac{X}{L}=\frac{2}{2}\\ \frac{Y}{L}=\frac{2}{2}\end{matrix}\right\}F_{(2+4)\to(1+3)}=0.20 \quad \text{and} \quad \left.\begin{matrix}\frac{X}{L}=\frac{2}{2}\\ \frac{Y}{L}=\frac{1}{2}\end{matrix}\right\}F_{14}=0.12$$

superposition rule: $F_{(2+4)\to(1+3)}=F_{(2+4)\to1}+F_{(2+4)\to3}$

symmetry rule: $F_{(2+4)\to1}=F_{(2+4)\to3}$

Substituting symmetry rule gives

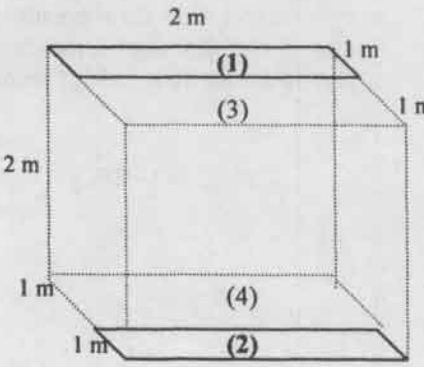
$$F_{(2+4)\to1}=F_{(2+4)\to3}=\frac{F_{(2+4)\to(1+3)}}{2}=\frac{0.20}{2}=0.10$$

reciprocity rule: $A_1F_{1\to(2+4)}=A_{(2+4)}F_{(2+4)\to1}$

$(2)F_{1\to(2+4)}=(4)(0.10)\longrightarrow F_{1\to(2+4)}=0.20$

superposition rule: $F_{1\to(2+4)}=F_{12}+F_{14}$

$0.20=F_{12}+0.12\longrightarrow F_{12}=0.20-0.12=\mathbf{0.08}$



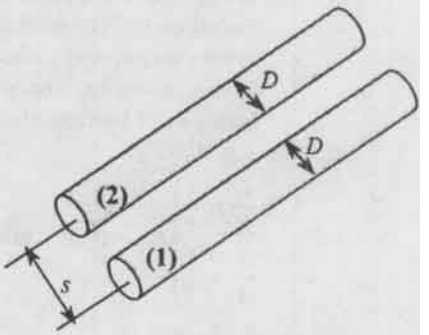
19-67 The view factor between the two infinitely long parallel cylinders located a distance s apart from each other is to be determined. √

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Using the crossed-strings method, the view factor between two cylinders facing each other for $s/D > 3$ is determined to be

$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{String on surface 1}} = \frac{2\sqrt{s^2 + D^2} - 2s}{2(\pi D/2)}$$

or $$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$



19-68 Three infinitely long cylinders are located parallel to each other. The view factor between the cylinder in the middle and the surroundings is to be determined. √

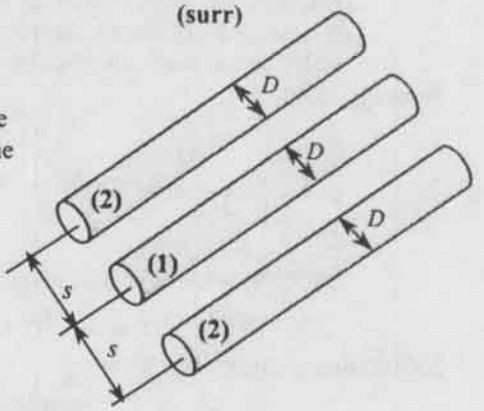
Assumptions The cylinder surfaces are diffuse emitters and reflectors.

Analysis The view factor between two cylinder facing each other is, from Prob. 19-67,

$$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$

Noting that the radiation leaving cylinder 1 that does not strike the cylinder will strike the surroundings, and this is also the case for the other half of the cylinder, the view factor between the cylinder in the middle and the surroundings becomes

$$F_{1-surr} = 1 - 2F_{1-2} = 1 - \frac{4\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$



Radiation Heat Transfer Between Surfaces

19-69C The analysis of radiation exchange between black surfaces is relatively easy because of the absence of reflection.

19-70C Radiosity is the total radiation energy leaving a surface per unit time and per unit area. Radiosity includes the emitted radiation energy as well as reflected energy. Radiosity and emitted energy are equal for blackbodies since a blackbody does not reflect any radiation.

19-71C Radiation surface resistance is given as $R_i = \dfrac{1-\varepsilon_i}{A_i \varepsilon_i}$ and it represents the resistance of a surface to the emission of radiation. It is zero for black surfaces. The space resistance is the radiation resistance between two surfaces and is expressed as $R_i = \dfrac{1-\varepsilon_i}{A_i \varepsilon_i}$

19-72C The two methods used in radiation analysis are the matrix and network methods. In matrix method, equations 19-46 and 19-47 give N linear algebraic equations for the determination of the N unknown radiosities for an N -surface enclosure. Once the radiosities are available, the unknown surface temperatures and heat transfer rates can be determined from these equations respectively. This method involves the use of matrices especially when there are a large number of surfaces. Therefore this method requires some knowledge of linear algebra.

The network method involves drawing a surface resistance associated with each surface of an enclosure and connecting them with space resistances. Then the radiation problem is solved by treating it as an electrical network problem where the radiation heat transfer replaces the current and the radiosity replaces the potential. The network method is not practical for enclosures with more than three or four surfaces due to the increased complexity of the network.

19-73C Some surfaces encountered in numerous practical heat transfer applications are modeled as being adiabatic as the back sides of these surfaces are well insulated and net heat transfer through these surfaces is zero. When the convection effects on the front (heat transfer) side of such a surface is negligible and steady-state conditions are reached, the surface must lose as much radiation energy as it receives. Such a surface is called reradiating surface. In radiation analysis, the surface resistance of a reradiating surface is taken to be zero since there is no heat transfer through it.

19-74E The top and side surfaces of a cubical furnace are black, and are maintained at uniform temperatures. The net radiation heat transfer rate to the base surface from the top and side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities are given to be $\varepsilon = 0.7$ for the bottom surface and 1 for other surfaces.

Analysis We consider the base surface to be surface 1, the top surface to be surface 2 and the side surfaces to be surface 3. The cubical furnace can be considered to be three-surface enclosure with a radiation network shown in the figure. The areas and blackbody emissive powers of surfaces are

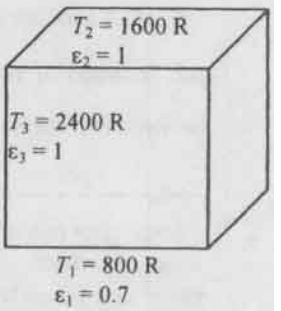
$$A_1 = A_2 = (10\ \text{ft})^2 = 100\ \text{ft}^2$$

$$A_3 = 4(10\ \text{ft})^2 = 400\ \text{ft}^2$$

$$E_{b1} = \sigma T_1^4 = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(800\ \text{R})^4 = 702\ \text{Btu/h.ft}^2$$

$$E_{b2} = \sigma T_2^4 = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(1600\ \text{R})^4 = 11{,}233\ \text{Btu/h.ft}^2$$

$$E_{b3} = \sigma T_3^4 = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(2400\ \text{R})^4 = 56{,}866\ \text{Btu/h.ft}^2$$



The view factor from the base to the top surface of the cube is $F_{12} = 0.2$.

The view factor from the base or the top to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Then the radiation resistances become

$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.7}{(100\ \text{ft}^2)(0.7)} = 0.0043\ \text{ft}^{-2}$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(100\ \text{ft}^2)(0.2)} = 0.0500\ \text{ft}^{-2}$$

$$R_{13} = \frac{1}{A_1 F_{13}} = \frac{1}{(100\ \text{ft}^2)(0.8)} = 0.0125\ \text{ft}^{-2}$$

Note that the side and the top surfaces are black, and thus their radiosities are equal to their emissive powers. The radiosity of the base surface is determined

$$\frac{E_{b1} - J_1}{R_1} + \frac{E_{b2} - J_1}{R_{12}} + \frac{E_{b3} - J_1}{R_{13}} = 0$$

Substituting,

$$\frac{702 - J_1}{0.0043} + \frac{11{,}233 - J_1}{0.500} + \frac{56{,}866 - J_1}{0.0125} = 0 \longrightarrow J_1 = 15{,}054\ \text{W/m}^2$$

(*a*) The net rate of radiation heat transfer between the base and the side surfaces is

$$\dot{Q}_{31} = \frac{E_{b3} - J_1}{R_{13}} = \frac{(56{,}866 - 15{,}054)\ \text{Btu/h.ft}^2}{0.0125\ \text{ft}^{-2}} = \mathbf{3.345\times10^6\ Btu/h}$$

(*b*) The net rate of radiation heat transfer between the base and the top surfaces is

$$\dot{Q}_{12} = \frac{J_1 - E_{b2}}{R_{12}} = \frac{(15{,}054 - 11{,}233)\ \text{Btu/h.ft}^2}{0.05\ \text{ft}^{-2}} = \mathbf{7.642\times10^4\ Btu/h}$$

The net rate of radiation heat transfer to the base surface is finally determined from

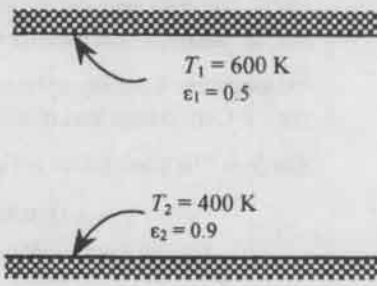
$$\dot{Q}_1 = \dot{Q}_{21} + \dot{Q}_{31} = -76{,}420 + 3{,}344{,}960 = \mathbf{3.269\times10^6\ Btu/h}$$

Discussion The same result can be found form

$$\dot{Q}_1 = \frac{J_1 - E_{b1}}{R_1} = \frac{(15{,}054 - 702)\ \text{Btu/h.ft}^2}{0.0043\ \text{ft}^{-2}} = 3.338\times10^6\ \text{Btu/h}$$

The small difference is due to round-off error.

19-75 Two very large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities ε of the plates are given to be 0.5 and 0.9.

Analysis The net rate of radiation heat transfer between the two surfaces per unit area of the plates is determined directly from

$$\frac{\dot{Q}_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (400\ \text{K})^4]}{\frac{1}{0.5}+\frac{1}{0.9}-1} = \mathbf{2795\ W/m^2}$$

19-76 The base, top, and side surfaces of a furnace of cylindrical shape are black, and are maintained at uniform temperatures. The net rate of radiation heat transfer to or from the top surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Properties The emissivity of all surfaces are $\varepsilon = 1$ since they are black.

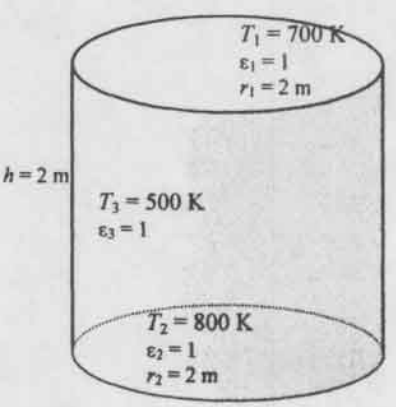
Analysis We consider the top surface to be surface 1, the base surface to be surface 2 and the side surfaces to be surface 3. The cylindrical furnace can be considered to be three-surface enclosure. We assume that steady-state conditions exist. Since all surfaces are black, the radiosities are equal to the emissive power of surfaces, and the net rate of radiation heat transfer from the top surface can be determined from

$$\dot{Q} = A_1 F_{12}\sigma(T_1^4 - T_2^4) + A_1 F_{13}\sigma(T_1^4 - T_3^4)$$

and

$$A_1 = \pi r^2 = \pi(2\ \text{m})^2 = 12.57\ \text{m}^2$$

The view factor from the base to the top surface of the cylinder is $F_{12} = 0.38$ (From Figure 19-44). The view factor from the base to the side surfaces is determined by applying the summation rule to be



$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.38 = 0.62$$

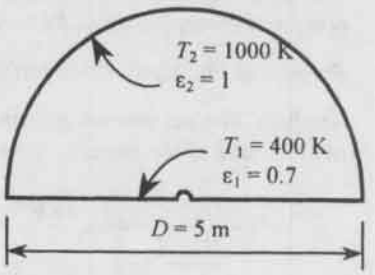
Substituting,

$$\begin{aligned}\dot{Q} &= A_1 F_{12}\sigma(T_1^4 - T_2^4) + A_1 F_{13}\sigma(T_1^4 - T_3^4)\\ &= (12.57\ \text{m}^2)(0.38)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(700\ \text{K}^4 - 500\ \text{K}^4)\\ &\quad + (12.57\ \text{m}^2)(0.62)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(700\ \text{K}^4 - 800\ \text{K}^4)\\ &= -2.68\times10^4\ \text{W} = \mathbf{-26.8\ kW}\end{aligned}$$

Discussion The negative sign indicates that net heat transfer is to the top surface.

19-77 The base and the dome of a hemispherical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Analysis The view factor is first determined from

$$F_{11} = 0 \text{ (flat surface)}$$
$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \text{ (summation rule)}$$

Noting that the dome is black, net rate of radiation heat transfer from dome to the base surface can be determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\varepsilon A_1 F_{12} \sigma (T_1^4 - T_2^4)$$
$$= -(0.7)[\pi (5\,\text{m})^2/4\,](1)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(400\,\text{K})^4 - (1000\,\text{K})^4]$$
$$= 7.594\times 10^5\ \text{W}$$
$$= \mathbf{759.4\ kW}$$

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

19-78 Two very long concentric cylinders are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined. √

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.7$.

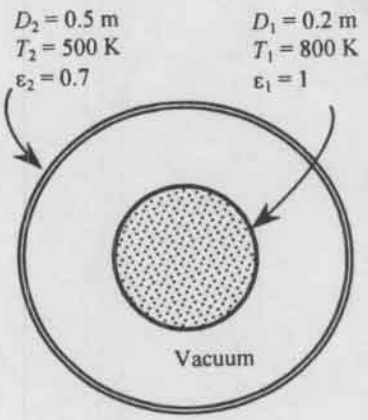
Analysis The net rate of radiation heat transfer between the two cylinders per unit length of the cylinders is determined from

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)}$$

$$= \frac{[\pi(0.2\text{ m})(1\text{ m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(800\text{ K})^4 - (500\text{ K})^4]}{\frac{1}{1} + \frac{1-0.7}{0.7}\left(\frac{2}{5}\right)}$$

$$= 1.06\times10^4\text{ W}$$

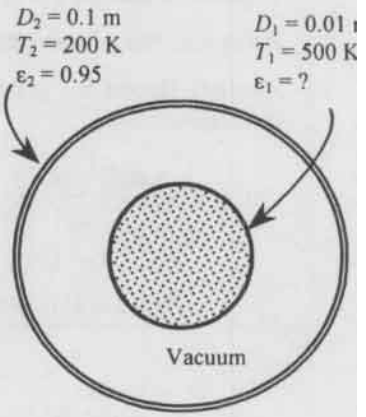
$$= \mathbf{10.6\text{ kW}}$$



19-79 A long cylindrical rod coated with a new material is placed in an evacuated long cylindrical enclosure which is maintained at a uniform temperature. The emissivity of the coating on the rod is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivity of the enclosure is given to be $\varepsilon_2 = 0.95$.



Analysis The emissivity of the coating on the rod is determined from

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

$$8\ \text{W} = \frac{[\pi(0.01\,\text{m})(1\,\text{m})](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(500\,\text{K})^4 - (200\,\text{K})^4]}{\dfrac{1}{\varepsilon_1} + \dfrac{1-0.95}{0.95}\left(\dfrac{1}{10}\right)}$$

which gives

$$\varepsilon_1 = \mathbf{0.074}$$

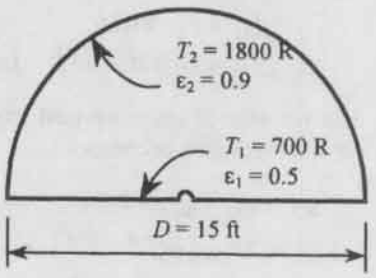
19-80E The base and the dome of a long semicylindrical duct are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.9$.

Analysis The view factor from the base to the dome is first determined from

$$F_{11} = 0 \text{ (flat surface)}$$
$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \text{ (summation rule)}$$



The net rate of radiation heat transfer from dome to the base surface can be determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$= -\frac{(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)[(700\ \text{R})^4 - (1800\ \text{R})^4]}{\frac{1-0.5}{(15\ \text{ft}^2)(0.5)} + \frac{1}{(15\ \text{ft}^2)(1)} + \frac{1-0.9}{\left[\frac{\pi(15\ \text{ft})(1\ \text{ft})}{2}\right](0.9)}}$$

$$= \mathbf{1.27\times10^5\ Btu/h}$$

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

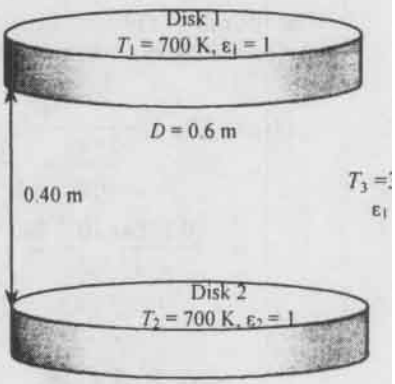
19-81 Two parallel disks whose back sides are insulated are black, and are maintained at a uniform temperature. The net rate of radiation heat transfer from the disks to the environment is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are $\varepsilon = 1$ since they are black.

Analysis Both disks possess same properties and they are black. Noting that environment can also be considered to be blackbody, we can treat this geometry as a three surface enclosure. We consider the two disks to be surfaces 1 and 2 and the environment to be surface 3. Then from Figure 19-44, we read

$$F_{12} = F_{21} = 0.26$$
$$F_{13} = 1 - 0.26 = 0.74 \quad \text{(summation rule)}$$



The net rate of radiation heat transfer from the disks into the environment then becomes

$$\dot{Q}_3 = \dot{Q}_{13} + \dot{Q}_{23} = 2\dot{Q}_{13}$$
$$\dot{Q}_3 = 2F_{13}A_1\sigma(T_1^4 - T_3^4)$$
$$= 2(0.74)[\pi(0.3\text{ m})^2](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(700\text{ K})^4 - (300\text{ K})^4]$$
$$= \mathbf{5505\ W}$$

19-82 A furnace shaped like a long equilateral-triangular duct is considered. The temperature of the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** End effects are neglected.

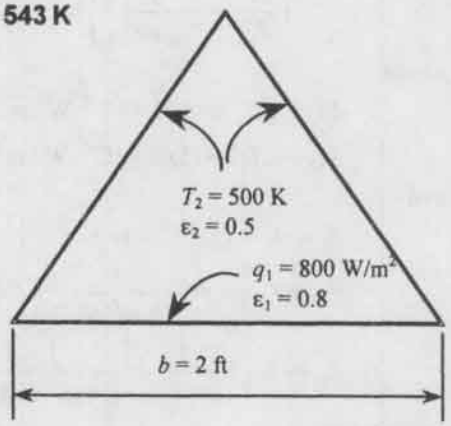
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.5$.

Analysis This geometry can be treated as a two surface enclosure since two surfaces have identical properties. We consider base surface to be surface 1 and other two surface to be surface 2. Then the view factor between the two becomes $F_{12} = 1$. The temperature of the base surface is determined from

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$800\ \text{W} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(T_1)^4 - (500\ \text{K})^4]}{\dfrac{1-0.8}{(1\ \text{m}^2)(0.8)} + \dfrac{1}{(1\ \text{m}^2)(1)} + \dfrac{1-0.5}{(2\ \text{m}^2)(0.5)}} \longrightarrow \mathbf{T_1 = 543\ K}$$

Note that $A_1 = 1\,\text{m}^2$ and $A_2 = 2\,\text{m}^2$.



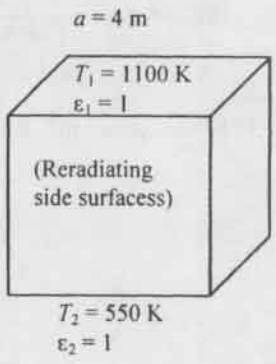
19-83 The floor and the ceiling of a cubical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer between the floor and the ceiling is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are $\varepsilon = 1$ since they are black or reradiating.

Analysis We consider the ceiling to be surface 1, the floor to be surface 2 and the side surfaces to be surface 3. The furnace can be considered to be three-surface enclosure with a radiation network shown in the figure. We assume that steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. The view factor from the ceiling to the floor of the furnace is $F_{12} = 0.2$. Then the rate of heat loss from the ceiling can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}}$$



where

$$E_{b1} = \sigma T_1^4 = (5.67 \times 10^{-8}\ \text{W/m}^2.\text{K}^4)(1100\ \text{K})^4 = 83{,}015\ \text{W/m}^2$$

$$E_{b2} = \sigma T_2^4 = (5.67 \times 10^{-8}\ \text{W/m}^2.\text{K}^4)(550\ \text{K})^4 = 5188\ \text{W/m}^2$$

and

$$A_1 = A_2 = (4\ \text{m})^2 = 16\ \text{m}^2$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(16\ \text{m}^2)(0.2)} = 0.3125\ \text{m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(16\ \text{m}^2)(0.8)} = 0.078125\ \text{m}^{-2}$$

Substituting,

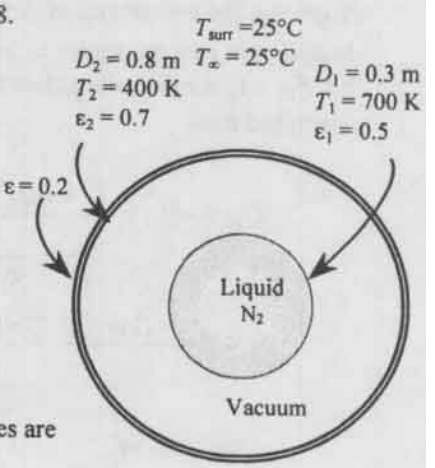
$$\dot{Q}_{12} = \frac{(83{,}015 - 5188)\ \text{W/m}^2}{\left(\dfrac{1}{0.3125\ \text{m}^{-2}} + \dfrac{1}{2(0.078125\ \text{m}^{-2})}\right)^{-1}} = 7.47 \times 10^5\ \text{W} = \mathbf{747\ kW}$$

19-84 Two concentric spheres are maintained at uniform temperatures. The net rate of radiation heat transfer between the two spheres and the convection heat transfer coefficient at the outer surface are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis The net rate of radiation heat transfer between the two spheres is



$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1^2}{r_2^2}\right)}$$

$$= \frac{[\pi(0.3\,\text{m})^2](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(700\,\text{K})^4 - (400\,\text{K})^4]}{\frac{1}{0.5} + \frac{1-0.7}{0.7}\left(\frac{0.15\,\text{m}}{0.4\,\text{m}}\right)^2}$$

$$= \mathbf{1669\ W}$$

Radiation heat transfer rate from the outer sphere to the surrounding surfaces are

$$\dot{Q}_{rad} = \varepsilon F A_2 \sigma(T_2^4 - T_{surr}^4)$$

$$= (0.2)(1)[\pi(0.8\,\text{m})^2](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(400\,\text{K})^4 - (25+273\,\text{K})^4] = 404\ \text{W}$$

The convection heat transfer rate at the outer surface of the cylinder is determined from requirement that heat transferred from the inner sphere to the outer sphere must be equal to the heat transfer from the outer surface of the outer sphere to the environment by convection and radiation. That is,

$$\dot{Q}_{conv} = \dot{Q}_{12} - \dot{Q}_{rad} = 1669 - 404 = 1265\ \text{W}$$

Then the convection heat transfer coefficient becomes

$$\dot{Q}_{conv} = hA_2(T_2 - T_\infty)$$

$$1265\ \text{W} = h[\pi(0.8\,\text{m})^2](400\,\text{K} - 298\,\text{K}) \longrightarrow h = \mathbf{6.17\ W/m^2\cdot{}^\circ C}$$

19-85 A spherical tank filled with liquid nitrogen is kept in an evacuated cubic enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

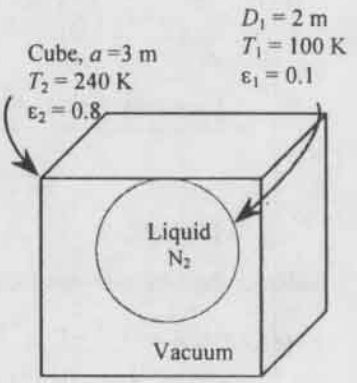
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis We take the sphere to be surface 1 and the surrounding cubic enclosure to be surface 2. Noting that $F_{12} = 1$, for this two-surface enclosure, the net rate of radiation heat transfer to liquid nitrogen can be determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{A_1}{A_2}\right)}$$

$$= -\frac{\left[\pi(2\,\text{m})^2\right]\left[5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right]\left[(100\,\text{K})^4 - (240\,\text{K})^4\right]}{\dfrac{1}{0.1} + \dfrac{1-0.8}{0.8}\left[\dfrac{\pi(2\,\text{m})^2}{6(3\,\text{m})^2}\right]}$$

$$= \mathbf{228\ W}$$



19-86 A spherical tank filled with liquid nitrogen is kept in an evacuated spherical enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

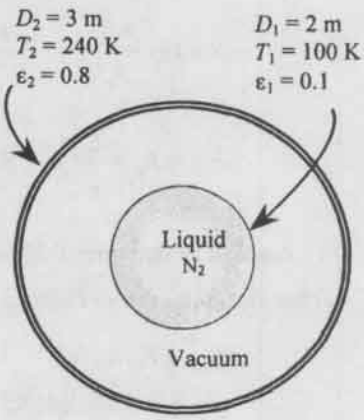
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis The net rate of radiation heat transfer to liquid nitrogen can be determined from

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1^2}{r_2^2}\right)}$$

$$= \frac{[\pi(2\text{ m})^2](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(240\text{ K})^4 - (100\text{ K})^4]}{\frac{1}{0.1} + \frac{1-0.8}{0.8}\left(\frac{(1\text{ m})^2}{(1.5\text{ m})^2}\right)}$$

$$= \mathbf{227\text{ W}}$$



19-87 A circular grill is considered. The bottom of the grill is covered with hot coal bricks, while the wire mesh on top of the grill is covered with steaks. The initial rate of radiation heat transfer from coal bricks to the steaks is to be determined for two cases. √

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities are $\varepsilon = 1$ for all surfaces since they are black or reradiating.

Analysis We consider the coal bricks to be surface 1, the steaks to be surface 2 and the side surfaces to be surface 3. First we determine the view factor between the bricks and the steaks (Table 19-4),

$$R_i = R_j = \frac{r_i}{L} = \frac{0.15\,\text{m}}{0.20\,\text{m}} = 0.75$$

$$S = 1 + \frac{1 + R_j^{\,2}}{R_i^{\,2}} = \frac{1 + 0.75^2}{0.75^2} = 3.7778$$

$$F_{12} = F_{ij} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{R_j}{R_i}\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{3.7778 - \left[3.7778^2 - 4\left(\frac{0.75}{0.75}\right)^2\right]^{1/2}\right\} = 0.2864$$

(It can also be determined from Fig. 19-44).

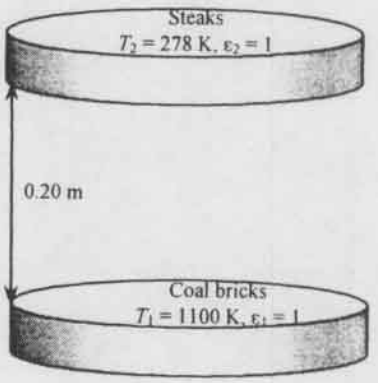
Then the initial rate of radiation heat transfer from the coal bricks to the stakes becomes

$$\begin{aligned}\dot{Q}_{12} &= F_{12} A_1 \sigma (T_1^{\,4} - T_2^{\,4}) \\ &= (0.2864)[\pi(0.3\,\text{m})^2/4](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1100\,\text{K})^4 - (278\,\text{K})^4] \\ &= \mathbf{1674\ W}\end{aligned}$$

When the side opening is closed with aluminum foil, the entire heat lost by the coal bricks must be gained by the stakes since there will be no heat transfer through a reradiating surface. The grill can be considered to be three-surface enclosure. Then the rate of heat loss from the room can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}}$$

where



$$E_{b1} = \sigma T_1^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1100\ \text{K})^4 = 83{,}015\ \text{W/m}^2$$

$$E_{b2} = \sigma T_2^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(18+273\ \text{K})^4 = 407\ \text{W/m}^2$$

and

$$A_1 = A_2 = \frac{\pi(0.3\,\text{m})^2}{4} = 0.07069\ \text{m}^2$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(0.07069\,\text{m}^2)(0.2864)} = 49.39\ \text{m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(0.07069\,\text{m}^2)(1-0.2864)} = 19.82\ \text{m}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(83{,}015 - 407)\ \text{W/m}^2}{\left(\dfrac{1}{49.39\ \text{m}^{-2}} + \dfrac{1}{2(19.82\ \text{m}^{-2})}\right)^{-1}} = \mathbf{3757\ W}$$

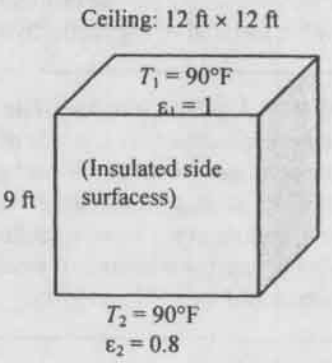
19-88E A room is heated by lectric resistance heaters placed on the ceiling which is maintained at a uniform temperature. The rate of heat loss from the room through the floor is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** There is no heat loss through the side surfaces.

Properties The emissivities are $\varepsilon = 1$ for the ceiling and $\varepsilon = 0.8$ for the floor. The emissivity of insulated (or reradiating) surfaces is also 1.

Analysis The room can be considered to be three-surface enclosure with the ceiling surface 1, the floor surface 2 and the side surfaces surface 3. We assume steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. Then the rate of heat loss from the room through its floor can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1} + R_2}$$



where

$$E_{b1} = \sigma T_1^4 = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(90+460\ \text{R})^4$$
$$= 157\ \text{Btu/h.ft}^2$$
$$E_{b2} = \sigma T_2^4 = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(65+460\ \text{R})^4$$
$$= 130\ \text{Btu/h.ft}^2$$

and

$$A_1 = A_2 = (12\ \text{ft})^2 = 144\ \text{ft}^2$$

The view factor from the floor to the ceiling of the room is $F_{12} = 0.27$ (From Figure 19-42). The view factor from the ceiling or the floor to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.27 = 0.73$$

since the ceiling is flat and thus $F_{11} = 0$. Then the radiation resistances which appear in the equation above become

$$R_2 = \frac{1-\varepsilon_2}{A_2\varepsilon_2} = \frac{1-0.8}{(144\ \text{ft}^2)(0.8)} = 0.00174\ \text{ft}^{-2}$$
$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(144\ \text{ft}^2)(0.27)} = 0.02572\ \text{ft}^{-2}$$
$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(144\ \text{ft}^2)(0.73)} = 0.009513\ \text{ft}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(157-130)\ \text{Btu/h.ft}^2}{\left(\dfrac{1}{0.02572\ \text{ft}^{-2}} + \dfrac{1}{2(0.009513\ \text{ft}^{-2})}\right)^{-1} + 0.00174\ \text{ft}^{-2}} = \mathbf{2130\ Btu/h}$$

Radiation Shields and The Radiation Effect

19-89C Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high reflectivity(low emissivity) sheet of material between the two surfaces. Such highly reflective thin plates or shells are known as radiation shields. Multilayer radiation shields constructed of about 20 shields per cm. thickness separated by evacuated space are commonly used in cryogenic and space applications to minimize heat transfer. Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect.

19-90C The influence of radiation on heat transfer or temperature of a surface is called the radiation effect. The radiation exchange between the sensor and the surroundings may cause the thermometer to indicate a different reading for the medium temperature. To minimize the radiation effect, the sensor should be coated with a material of high reflectivity (low emissivity).

19-91C A person who feels fine in a room at a specified temperature may feel chilly in an other room at the same temperature as a result of radiation effect if the walls of second room are at a considerably lower temperature. For example most people feel comfortable in a room at 22 °C if the walls of the room are also roughly at that temperature. When the wall temperature drops to 5 °C for some reason, the interior temperature of the room must be raised to at least 27 °C to maintain the same level of comfort. Also, people sitting near the windows of a room in winter will feel colder because of the radiation exchange between the person and the cold windows.

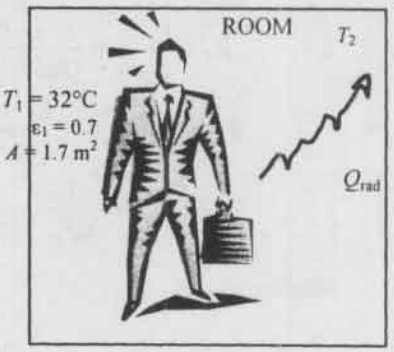
19-92 The rate of heat loss from a person by radiation in a large room whose walls are maintained at a uniform temperature is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the person is given to be $\varepsilon_1 = 0.7$.

Analysis (a) Noting that the view factor from the person to the walls $F_{12} = 1$, the rate of heat loss from that person to the walls at a large room which are at a temperature of 300 K is

$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1^4 - T_2^4) \\ &= (0.7)(1)(1.7\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(305\,\text{K})^4 - (300\,\text{K})^4] \\ &= \mathbf{37.4\ W}\end{aligned}$$



(b) When the walls are at a temperature of 280 K,

$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1^4 - T_2^4) \\ &= (0.7)(1)(1.7\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(305\,\text{K})^4 - (280\,\text{K})^4] \\ &= \mathbf{169\ W}\end{aligned}$$

19-93 A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined for the cases of with and without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

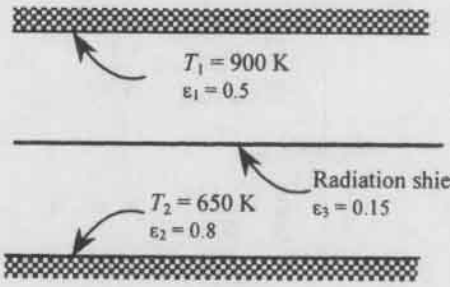
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.8$, and $\varepsilon_3 = 0.15$.

Analysis The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is

$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_{3,1}}+\frac{1}{\varepsilon_{3,2}}-1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4 - (650\ \text{K})^4]}{\left(\frac{1}{0.5}+\frac{1}{0.8}-1\right)+\left(\frac{1}{0.15}+\frac{1}{0.15}-1\right)}$$

$$= \mathbf{1857\ W/m^2}$$



The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\ \text{shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4 - (650\ \text{K})^4]}{\left(\frac{1}{0.5}+\frac{1}{0.8}-1\right)} = 12{,}035\ \text{W/m}^2$$

Then the ratio of radiation heat transfer for the two cases becomes

$$\frac{\dot{Q}_{12,\text{one shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{1857\ \text{W}}{12{,}035\ \text{W}} \cong \frac{1}{6}$$

19-94 Two very large plates are maintained at uniform temperatures. The number of thin aluminum sheets that will reduce the net rate of radiation heat transfer between the two plates to one-fifth is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

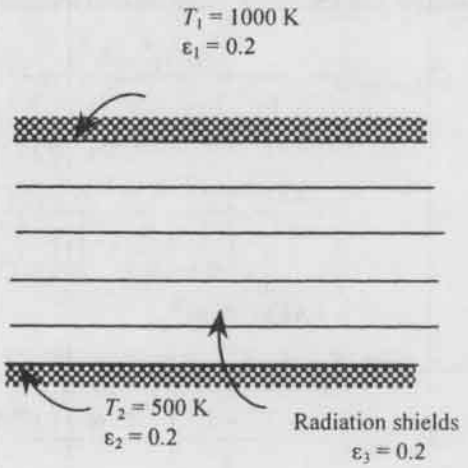
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$, and $\varepsilon_3 = 0.2$.

Analysis Since the plates and the sheets have the same emissivity value, the number of sheets that need to be inserted can be determined from

$$\frac{\dot{Q}_{12,\text{N shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{1}{N+1}$$

Substituting,

$$\frac{1}{5} = \frac{1}{N+1} \longrightarrow N = 4$$

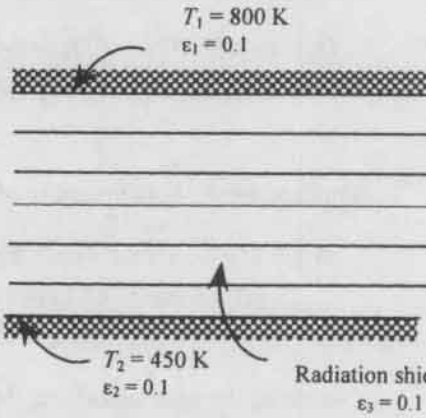


19-95 Five identical thin aluminum sheets are placed between two very large parallel plates which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.1$ and $\varepsilon_3 = 0.1$.

Analysis Since the plates and the sheets have the same emissivity value, the net rate of radiation heat transfer with 5 thin aluminum shield can be determined from



$$\dot{Q}_{12,5\,\text{shield}} = \frac{1}{N+1}\dot{Q}_{12,\,\text{no shield}} = \frac{1}{N+1}\frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)}$$

$$= \frac{1}{5+1}\frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\,\text{K})^4 - (450\,\text{K})^4]}{\left(\frac{1}{0.1}+\frac{1}{0.1}-1\right)}$$

$$= \mathbf{183\ W/m^2}$$

The net rate of radiation heat transfer without the shield is

$$\dot{Q}_{12,5\,\text{shield}} = \frac{1}{N+1}\dot{Q}_{12,\,\text{no shield}} \longrightarrow \dot{Q}_{12,\,\text{no shield}} = (N+1)\dot{Q}_{12,5\,\text{shield}} = 6\times183\ \text{W} = \mathbf{1098\ W}$$

19-96E A radiation shield is placed between two parallel disks which are maintained at uniform temperatures. The net rate of radiation heat transfer through the shields is to be determined. √

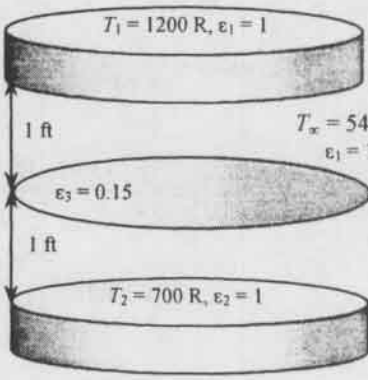
Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 1$ and $\varepsilon_3 = 0.15$.

Analysis From Fig. 19-44 we have $F_{32} = F_{13} = 0.52$. Then $F_{34} = 1 - 0.52 = 0.48$. The disk in the middle is surrounded by black surfaces on both sides. Therefore, heat transfer between the top surface of the middle disk and its black surroundings can expressed as

$$\dot{Q}_3 = \varepsilon A_3 \sigma[F_{31}(T_3^4 - T_1^4)] + \varepsilon A_3 \sigma[F_{32}(T_3^4 - T_2^4)]$$

$$= 0.15(7.069\,\text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\{0.52[(T_3^4 - (1200\,\text{R})^4] + 0.48[T_3^4 - (540\,\text{K})^4]\}$$



Similarly, for the bottom surface of the middle disk, we have

$$-\dot{Q}_3 = \varepsilon A_3 \sigma[F_{34}(T_3^4 - T_4^4)] + \varepsilon A_3 \sigma[F_{35}(T_3^4 - T_5^4)]$$

$$= 0.15(7.069\,\text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\{0.48[(T_3^4 - (700\,\text{R})^4] + 0.52[T_3^4 - (540\,\text{K})^4]\}$$

Combining the equations above, the rate of heat transfer between the disks through the radiation shield (the middle disk) is determined to be

$$\dot{Q} = \mathbf{866\ Btu/h} \quad \text{and} \quad T_3 = 895\ \text{K}$$

19-97 A radiation shield is placed between two large parallel plates which are maintained at uniform temperatures. The emissivity of the radiation shield is to be determined if the radiation heat transfer between the plates is reduced to 15% of that without the radiation shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.6$ and $\varepsilon_2 = 0.9$.

Analysis First, the net rate of radiation heat transfer between the two large parallel plates per unit area without a shield is

$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\ \text{K})^4 - (400\ \text{K})^4]}{\frac{1}{0.6}+\frac{1}{0.9}-1} = 4877\ \text{W/m}^2$$

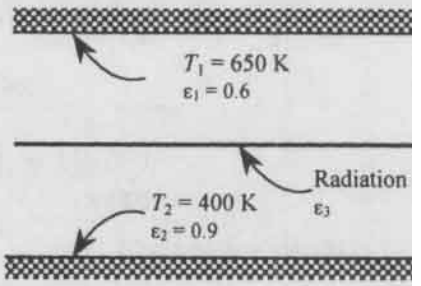
The radiation heat transfer in the case of one shield is

$$\dot{Q}_{12,\text{one shield}} = 0.15\times\dot{Q}_{12,\text{no shield}} = 0.15\times 4877\ \text{W/m}^2 = 731.6\ \text{W/m}^2$$

Then the emissivity of the radiation shield becomes

$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_{3,1}}+\frac{1}{\varepsilon_{3,2}}-1\right)}$$

$$731.6\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\ \text{K})^4 - (400\ \text{K})^4]}{\left(\frac{1}{0.6}+\frac{1}{0.9}-1\right)+\left(\frac{2}{\varepsilon_3}-1\right)}$$



which gives $\varepsilon_3 = \mathbf{0.18}$

19-98 A coaxial radiation shield is placed between two coaxial cylinders which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.4$. and $\varepsilon_3 = 0.2$.

Analysis The surface areas of the cylinders and the shield per unit length are

$$A_{pipe,inner} = A_1 = \pi D_1 L = \pi(0.2\text{ m})(1\text{ m}) = 0.628\text{ m}^2$$

$$A_{pipe,outer} = A_2 = \pi D_2 L = \pi(0.1\text{ m})(1\text{ m}) = 0.314\text{ m}^2$$

$$A_{shield} = A_3 = \pi D_3 L = \pi(0.3\text{ m})(1\text{ m}) = 0.942\text{ m}^2$$

The net rate of radiation heat transfer between the two cylinders with a shield per unit length is

$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{13}} + \dfrac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \dfrac{1}{A_3F_{3,2}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(750\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1-0.7}{(0.314)(0.7)} + \dfrac{1}{(0.314)(1)} + 2\dfrac{1-0.2}{(0.628)(0.2)} + \dfrac{1}{(0.628)(1)} + \dfrac{1-0.4}{(0.942)(0.4)}}$$

$$= \mathbf{703\text{ W}}$$

If there was no shield,

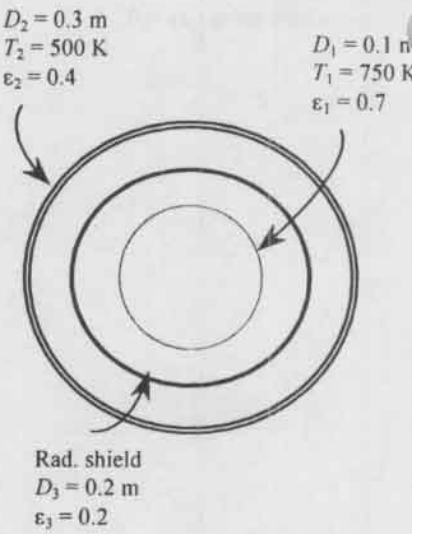
$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(750\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1}{0.7} + \dfrac{1-0.4}{0.4}\left(\dfrac{0.1}{0.3}\right)}$$

$$= \mathbf{7465\text{ W}}$$

Then their ratio becomes

$$\frac{\dot{Q}_{12,\text{one shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{703\text{ W}}{7465\text{ W}} = \mathbf{0.094}$$



Review Problems

19-99 The temperature of air in a duct is measured by a thermocouple. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.

Assumptions The surfaces are opaque, diffuse, and gray.

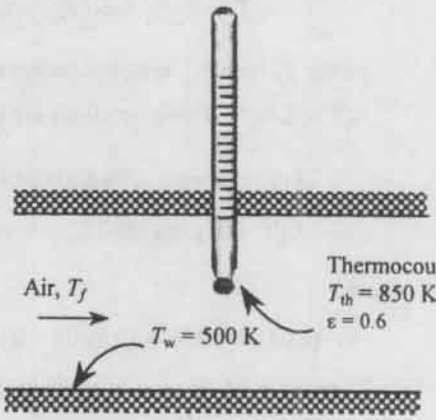
Properties The emissivity of thermocouple is given to be $\varepsilon = 0.6$.

Analysis The actual temperature of the air can be determined from

$$T_f = T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}$$

$$= 850\,\text{K} + \frac{(0.6)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(850\,\text{K})^4 - (500\,\text{K})^4]}{60\ \text{W/m}^2\cdot{}^\circ\text{C}}$$

$$= \mathbf{1111\,K}$$



19-100 The temperature of hot gases in a duct is measured by a thermocouple. The actual temperature of the gas is to be determined, and compared with that without a radiation shield.

Assumptions The surfaces are opaque, diffuse, and gray.

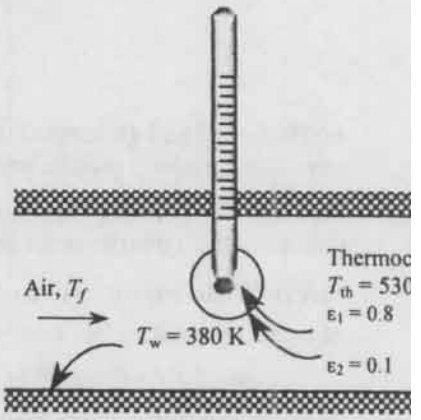
Properties The emissivity of the thermocouple is given to be $\varepsilon = 0.8$.

Analysis Assuming the area of the shield to be very close to the sensor of the thermometer, the radiation heat transfer from the sensor is determined from

$$\dot{Q}_{\text{rad, from sensor}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} - 1\right) + \left(2\frac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(530\,\text{K})^4 - (380\,\text{K})^4]}{\left(\frac{1}{0.8} - 1\right) + \left(2\frac{1}{0.1} - 1\right)}$$

$$= 171\ \text{W/m}^2$$



Then the actual temperature of the gas can be determined from a heat transfer balance to be

Without the shield the temperature of the gas would be

$$T_f = T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}$$

$$= 530\,\text{K} + \frac{(0.8)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(530\,\text{K})^4 - (380\,\text{K})^4]}{120\ \text{W/m}^2\cdot{}^\circ\text{C}}$$

$$= \mathbf{552\,K}$$

19-101 The variation of emissivity of an opaque surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from

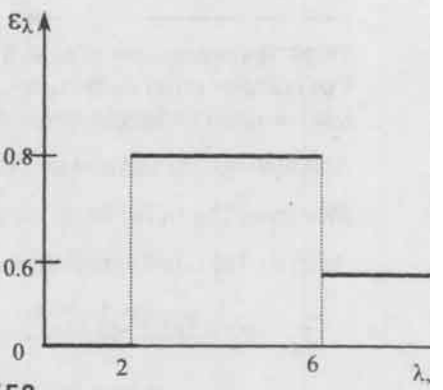
$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 19-1 to be

$$\lambda_1 T = (2\ \mu\text{m})(1200\ \text{K}) = 2400\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.140256$$

$$\lambda_2 T = (6\ \mu\text{m})(1200\ \text{K}) = 7200\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.819217$$

and



$$\varepsilon = (0.0)(0.140256) + (0.8)(0.819217 - 0.140256) + (0.6)(1 - 0.819217) = \mathbf{0.652}$$

Then the emissive power of the surface becomes

$$E = \varepsilon\sigma T^4 = (0.652)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1200\ \text{K})^4 = \mathbf{7.67\times10^4\ W/m^2}$$

19-102E A sealed electronic box is placed in a vacuum chamber. The highest temperature at which the surrounding surfaces must be kept if this box is cooled by radiation alone is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** Heat transfer from the bottom surface of the box is negligible.

Properties The emissivity of the outer surface of the box is $\varepsilon = 0.95$.

Analysis The total surface area is

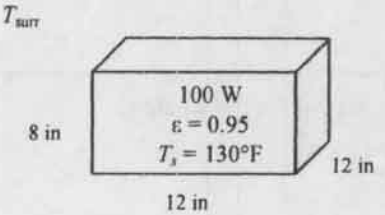
$$A_{total} = 4\times(8\times1/12) + (1\times1) = 3.67\ \text{ft}^2$$

Then the temperature of the surrounding surfaces is determined to be

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$(100\times3.41214)\ \text{Btu/h} = (0.95)(3.67\ \text{m}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)[(590\ \text{R})^4 - T_{surr}^4]$$

$$\longrightarrow T_{surr} = 503\ \text{R} = \mathbf{43°F}$$

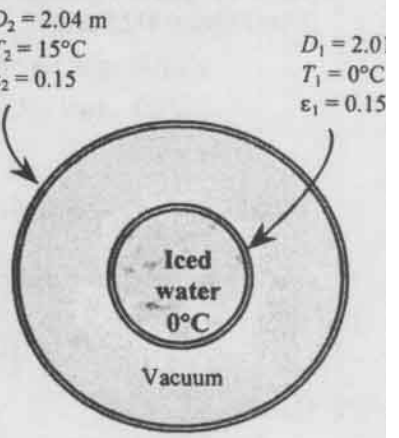


19-103 A double-walled spherical tank is used to store iced water. The air space between the two walls is evacuated. The rate of heat transfer to the iced water and the amount of ice that melts a 24-h period are to be determined. √

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivities of both surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.15$.

Analysis (*a*) Assuming the conduction resistance s of the walls to be negligible, the rate of heat transfer to the iced water in the tank is determined to be



$$A_1 = \pi D_1^2 = \pi(2.01\text{ m})^2 = 12.69\text{ m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)^2}$$

$$= \frac{(12.69\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(15+273\text{ K})^4 - (0+273\text{ K})^4]}{\frac{1}{0.15} + \frac{1-0.15}{0.15}\left(\frac{2.01}{2.04}\right)^2}$$

$$= \mathbf{78.4\text{ W}}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.0784\text{ kJ/s})(24\times3600\text{ s}) = 6774\text{ kJ}$$

The amount of ice that melts during this period then becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{6774\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{20.3\text{ kg}}$$

19-104 Two concentric spheres which are maintained at uniform temperatures are separated by air at 1 atm pressure. The rate of heat transfer between the two spheres by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

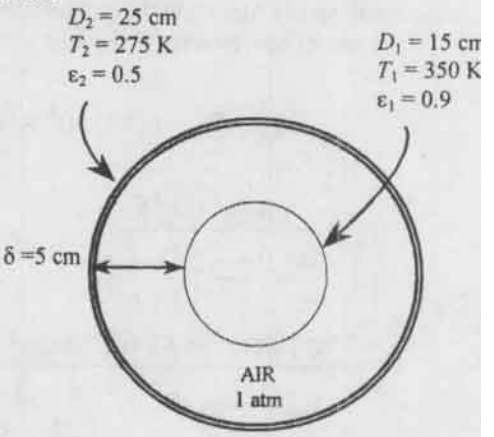
Properties The emissivities of the surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.5$. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (350+275)/2 = 312.5$ K are

$$k = 0.0270 \text{ W/m.°C}$$

$$\upsilon = 1.69 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.711$$

$$\beta = \frac{1}{312.5 \text{ K}} = 0.0032 \text{ K}^{-1}$$



Analysis (a) The characteristic length in this case is

$$\delta = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.25 \text{ m} - 0.15 \text{ m}) = 0.05 \text{ m}.$$

Then

$$Ra = \frac{g\beta(T_1 - T_2)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(0.0032 \text{ K}^{-1})(350-275)\text{K}(0.05 \text{ m})^3}{(1.69 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.711) = 731{,}886$$

$$Nu = 0.228Ra^{0.226} = 0.228(731{,}886)^{0.226} = 4.82$$

$$A = \pi D_1 D_2 = \pi(0.15 \text{ m})(0.25 \text{ m}) = 0.1178 \text{ m}^2$$

and

$$\dot{Q}_{conv} = kNuA\frac{T_1 - T_2}{\delta} = (0.0270 \text{ W/m.°C})(4.82)(0.1178 \text{ m}^2)\frac{(350-275)°\text{C}}{0.05 \text{ m}} = \mathbf{23.0 \text{ W}}$$

(b) The rate of heat transfer by radiation is determined from Eq. 19-53 to be

$$A_1 = \pi D_1^2 = \pi(0.15 \text{ m})^2 = 0.0707 \text{ m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)^2} = \frac{(0.0707 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(350 \text{ K})^4 - (275 \text{ K})^4]}{\frac{1}{0.9} + \frac{1-0.9}{0.9}\left(\frac{0.15}{0.25}\right)^2} = \mathbf{32.3 \text{ W}}$$

19-105 A solar collector is considered. The absorber plate and the glass cover are maintained at uniform temperatures, and are separated by air. The rate of heat loss from the absorber plate by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

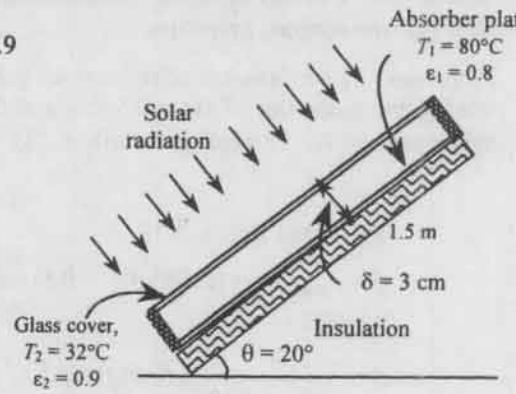
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.9$ for glass and $\varepsilon_2 = 0.8$ for the absorber plate. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (80+32)/2 = 56°C = 329$ K are

$$k = 0.0283 \text{ W/m.°C}$$

$$\upsilon = 1.86\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.708$$

$$\beta = \frac{1}{329 \text{ K}} = 0.00304 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the distance between the glasses, $\delta = 0.03$ m. Then

$$Ra = \frac{g\cos\theta\beta(T_1 - T_2)\delta^3}{\upsilon^2}\text{Pr} = \frac{(9.8 \text{ m/s}^2)(\cos 20°)(0.00304 \text{ K}^{-1})(80-32)\text{K}(0.03 \text{ m})^3}{(1.86\times10^{-5} \text{ m}^2/\text{s})^2}(0.708) = 74{,}250$$

$$Nu = 0.212Ra^{1/4} = 0.212(74{,}250)^{1/4} = 3.50$$

$$A = (1.5 \text{ m})(3 \text{ m}) = 4.5 \text{ m}^2$$

and

$$\dot{Q}_{conv} = kNuA\frac{T_1 - T_2}{\delta} = (0.0283 \text{ W/m.°C})(3.50)(4.5 \text{ m}^2)\frac{(80-32)°\text{C}}{0.03 \text{ m}} = \mathbf{713.2 \text{ W}}$$

Neglecting the end effects, the rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} = \frac{(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(80+273 \text{ K})^4 - (32+273 \text{ K})^4]}{\frac{1}{0.8}+\frac{1}{0.9}-1} = \mathbf{1289 \text{ W}}$$

19-106E A solar collector consists of a horizontal aluminum tube enclosed in a concentric thin glass tube. The annular space between the aluminum and the glass tubes is filled with air at 0.5 atm pressure. The pump circulating the water fails during a clear day. The equilibrium temperature of the aluminum tube is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

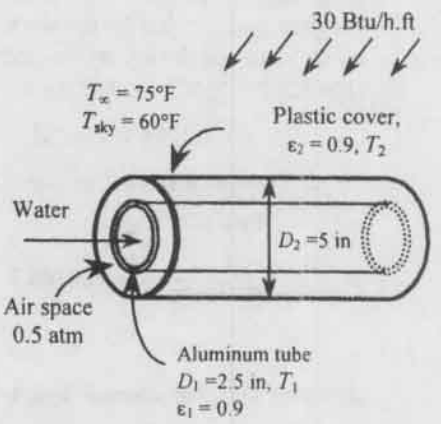
Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. We use the properties of air at 0.5 atm and the anticipated average temperature of 100°F, which are (Table A-18)

$$k = 0.0154 \text{ Btu/h.ft.°F}$$

$$\upsilon = \upsilon_{1atm} / P = (0.180 \times 10^{-3} / 0.5) = 0.36 \times 10^{-3} \text{ ft}^2/\text{s}$$

$$\Pr = 0.72$$

$$\beta = \frac{1}{(100+460)\text{R}} = 0.001786 \text{ R}^{-1}$$



Analysis We assume the glass temperature is 85°F. The characteristic length is $\delta = D_2 = 5$ in . Then,

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\Pr = \frac{(32.2 \text{ ft/s}^2)(0.001786 \text{ R}^{-1})(85-75)\text{R}(5/12 \text{ ft})^3}{(0.36\times10^{-3} \text{ ft}^2/\text{s})^2}(0.72) = 231{,}117$$

The natural convection Nusselt number for this geometry and orientation can be determined from

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(231{,}117)^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = 9.67$$

and

$$h_2 = \frac{k}{\delta}Nu = \frac{0.0154 \text{ Btu/h.ft.°F}}{(5/12) \text{ ft}}(9.67) = 0.357 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_{glass} = A_2 = \pi D_2 L = \pi(5/12 \text{ ft})(1 \text{ ft}) = 1.309 \text{ ft}^2$$

and

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$$

$$= h_2 A_2 (T_2 - T_\infty) + \varepsilon_2 A_2 (T_2^4 - T_{sky}^4)$$

$$30 = (0.357)(1.309)(T_2 - 75) + (0.9)(1.309)(0.1714\times10^{-8})[(T_2+460)^4 - (60+460)^4]$$

$$T_2 = 83°\text{F}$$

The temperature of aluminum tube is determined in a similar manner using natural convection, radiation relations for two horizontal concentric cylinders. The characteristic length in this case is

$$\delta = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(5 - 2.5) = 1.25 \text{ inch}$$

Also

$$A_{tube} = A_1 = \pi D_1 L = \pi(2.5/12 \text{ ft})(1 \text{ ft}) = 0.654 \text{ ft}^2$$

$$A = \frac{\pi L(D_2 - D_1)}{\ln(D_2/D_1)} = \frac{\pi(1 \text{ ft})(5/12 - 2.5/12)\text{ft}}{\ln(8/5)} = 1.393 \text{ ft}^2$$

We start calculations by assuming the tube temperature to be 130°F. It gives

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2}\,\mathrm{Pr} = \frac{(32.2\ \mathrm{ft/s^2})(0.001786\ \mathrm{R^{-1}})(130-83)\mathrm{R}(1.25/12\ \mathrm{ft})^3}{(0.36\times10^{-3}\ \mathrm{ft^2/s})^2}(0.72) = 16{,}973$$

$$Nu = 0.11Ra^{0.29} = 0.11(16{,}973)^{0.29} = 1.854$$

The convection heat transfer rate is

$$\dot{Q}_{1,conv} = kNuA\frac{T_1 - T_2}{\delta} = (0.0154\ \mathrm{Btu/h.ft.^\circ F})(1.854)(1.393\ \mathrm{ft^2})\frac{(130-83)^\circ\mathrm{F}}{(1.25/12)\ \mathrm{ft}} = 17.9\ \mathrm{Btu/h}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{1,\mathrm{rad}} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)}$$

$$= \frac{(0.654\ \mathrm{ft^2})(T_1^4 - T_2^4)[(130+460\,\mathrm{R})^4 - (83+460\,\mathrm{R})^4]}{\frac{1}{0.9} + \frac{1-0.9}{0.9}\left(\frac{2.5}{5}\right)}$$

$$= 32.9\ \mathrm{Btu/h}$$

and

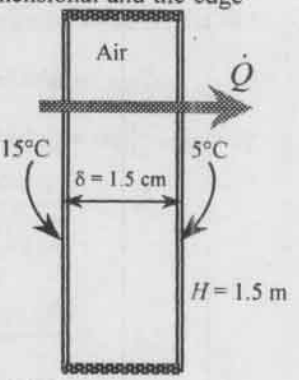
$$\dot{Q}_{1,total} = \dot{Q}_{1,conv} + \dot{Q}_{1,rad} = 17.9 + 32.9 = 50.8\ \mathrm{Btu/h}$$

which is greater than 30 Btu/h. Repeating calculations at different temperature values yields the tube temperature to be about 112°F. That is,

$$T_{tube} = 112^\circ\mathrm{F}$$

19-107 A double-pane window consists of two sheets of glass separated by an air space. The rates of heat transfer through the window by natural convection and radiation are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats. **4** Heat transfer through the window is one-dimensional and the edge effects are negligible.



Properties The emissivities of glass surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air at 0.3 atm and the average temperature of $(T_1+T_2)/2 = (15+5)/2 = 10°C = 283$ K are

$$k = 0.0248 \text{ W/m.°C}$$

$$\upsilon = \upsilon_{1atm} / 0.3 = 1.42 \times 10^{-5} / 0.3 = 4.75 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.716$$

$$\beta = \frac{1}{283 \text{ K}} = 0.003534 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the distance between the glasses, $\delta = 0.015$ m

$$Ra = \frac{g\beta(T_1 - T_2)\delta^3}{\upsilon^2} \Pr = \frac{(9.8 \text{ m/s}^2)(0.003534 \text{ K}^{-1})(15-5)\text{K}(0.015 \text{ m})^3}{(4.75 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.716) = 371$$

$$Nu = 0.197 Ra^{1/4}\left(\frac{H}{\delta}\right)^{-1/9} = 0.197(371)^{1/4}\left(\frac{15}{0.015}\right)^{-1/9} = 0.40$$

$$A = (1.5 \text{ m})(3 \text{ m}) = 4.5 \text{ m}^2$$

Then the rate of heat transfer by natural convection becomes

$$\dot{Q}_{conv} = kNuA\frac{T_1 - T_2}{\delta} = (0.0248 \text{ W/m.°C})(0.40)(4.5 \text{ m}^2)\frac{(15-5)°\text{C}}{0.015 \text{ m}} = \mathbf{29.8 \text{ W}}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{(4.5 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(15+273 \text{ K})^4 - (5+273 \text{ K})^4]}{\frac{1}{0.9} + \frac{1}{0.9} - 1}$$

$$= \mathbf{189 \text{ W}}$$

Then the rate of total heat transfer becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 29.8 + 189 = \mathbf{218.9 \text{ W}}$$

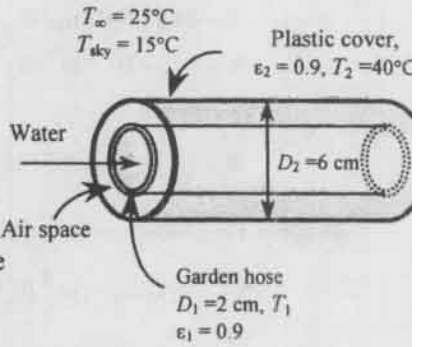
Discussion Note that heat transfer through the window is overwhelmingly by radiation.

19-108 A simple solar collector is built by placing a clear plastic tube around a garden hose. The rate of heat loss from the water in the hose by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air are at 1 atm and the film temperature of $T_f = \dfrac{T_s + T_\infty}{2} = \dfrac{40+25}{2} = 32.5°C = 305.5\ K$ are (Table A-18)

$$k = 0.0265\ W/m.°C$$
$$\upsilon = 1.62 \times 10^{-5}\ m^2/s$$
$$Pr = 0.711$$
$$\beta = \frac{1}{305.5\ K} = 0.003273\ K^{-1}$$



Analysis Under steady conditions, the heat transfer rate from the water in the hose equals to the rate of heat loss from the clear plastic tube to the surroundings by natural convection and radiation. The characteristic length in this case is the diameter of the plastic tube, $\delta = D_{plastic} = 0.06\ m$.

$$Ra = \frac{g\beta(T_s - T_\infty)\delta^3}{\upsilon^2} Pr = \frac{(9.81\ m/s^2)(0.003273\ K^{-1})(40-25)K(0.06\ m)^3}{(1.62\times10^{-5}\ m^2/s)^2}(0.711) = 4.45\times10^5$$

$$Nu = 0.11Ra^{0.29} = 0.11(4.45\times10^5)^{0.29} = 4.78$$

$$h = \frac{k}{\delta} Nu = \frac{0.0265\ W/m.°C}{0.06\ m}(4.78) = 2.11\ W/m^2.°C$$

$$A_{plastic} = A = \pi D_2 L = \pi(0.06\ m)(1\ m) = 0.1885\ m^2$$

Then the rate of heat transfer by natural convection becomes

$$\dot{Q}_{conv} = hA(T_s - T_\infty) = (2.11\ W/m^2.°C)(0.1885\ m^2)(40-25)°C = \mathbf{6.0\ W}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{sky}^4)$$
$$= (0.90)(0.1885\ m^2)(5.67\times10^{-8}\ W/m^2\cdot K^4)[(40+273\ K)^4 - (15+273\ K)^4]$$
$$= \mathbf{26.2\ W}$$

Finally,

$$\dot{Q}_{total,loss} = 6.0 + 26.2 = 32.2\ W$$

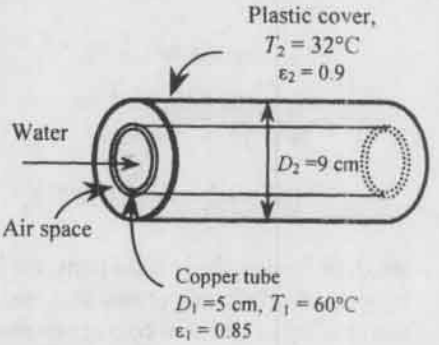
Discussion Note that heat transfer through the window is overwhelmingly by radiation.

19-109 A solar collector consists of a horizontal copper tube enclosed in a concentric thin glass tube. The annular space between the copper and the glass tubes is filled with air at 1 atm. The rate of heat loss from the collector by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.85$ for the tube surface and $\varepsilon_2 = 0.9$ for glass cover. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (60+32)/2 = 46°C = 319$ K are (Table A-18)

$$k = 0.0275 \text{ W/m.°C}$$
$$\upsilon = 1.76\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.710$$
$$\beta = \frac{1}{319 \text{ K}} = 0.003135 \text{ K}^{-1}$$



Analysis The characteristic length in this case is

$$\delta = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.09 \text{ m} - 0.05 \text{ m}) = 0.02 \text{ m}$$

$$Ra = \frac{g\beta(T_1 - T_2)\delta^3}{\upsilon^2}\Pr = \frac{(9.8 \text{ m/s}^2)(0.003135 \text{ K}^{-1})(60-32)\text{K}(0.02 \text{ m})^3}{(1.76\times10^{-5} \text{ m}^2/\text{s})^2}(0.710) = 15{,}774$$

$$Nu = 0.11Ra^{0.29} = 0.11(15{,}774)^{0.29} = 1.81$$

$$A_{tube} = A_1 = \pi D_1 L = \pi(0.05 \text{ m})(1 \text{ m}) = 0.1571 \text{ m}^2$$

$$A = \frac{\pi L(D_2 - D_1)}{\ln(D_2/D_1)} = \frac{\pi(1 \text{ m})(0.09 - 0.05)\text{m}}{\ln(9/5)} = 0.2138 \text{ m}^2$$

Then the rate of heat transfer by natural convection becomes

$$\dot{Q}_{conv} = kNuA\frac{T_1 - T_2}{\delta} = (0.0275 \text{ W/m.°C})(1.81)(0.2138 \text{ m}^2)\frac{(60-32)°\text{C}}{0.02 \text{ m}} = \mathbf{14.9 \text{ W}}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)}$$

$$= \frac{(0.1571\text{m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(60+273\text{K})^4 - (32+273\text{K})^4]}{\frac{1}{0.85} + \frac{1-0.9}{0.9}\left(\frac{5}{9}\right)}$$

$$= \mathbf{26.2 \text{ W}}$$

Finally,

$$\dot{Q}_{total,loss} = 14.9 + 26.2 = 41.1 \text{ W} \quad \text{(per m length)}$$

Chapter 20
HEAT EXCHANGERS

Types of Heat Exchangers

20-1C Heat exchangers are classified according to the flow type as parallel flow, counter flow, and cross-flow arrangement. In parallel flow, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction. In counter-flow, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite direction. In cross-flow, the hot and cold fluid streams move perpendicular to each other.

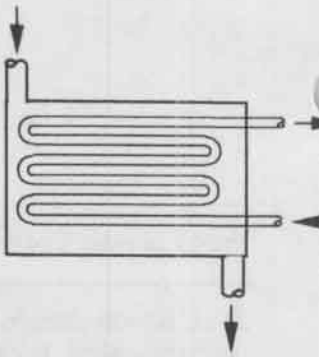
20-2C In terms of construction type, heat exchangers are classified as compact, shell and tube and regenerative heat exchangers. Compact heat exchangers are specifically designed to obtain large heat transfer surface areas per unit volume. The large surface area in compact heat exchangers is obtained by attaching closely spaced thin plate or corrugated fins to the walls separating the two fluids. Shell and tube heat exchangers contain a large number of tubes packed in a shell with their axes parallel to that of the shell. Regenerative heat exchangers involve the alternate passage of the hot and cold fluid streams through the same flow area. In compact heat exchangers, the two fluids usually move perpendicular to each other.

20-3C A heat exchanger is classified as being compact if $\beta > 700\ m^2/m^3$ or $(200\ ft^2/ft^3)$ where β is the ratio of the heat transfer surface area to its volume which is called the area density. The area density for double-pipe heat exchanger can not be in the order of 700. Therefore, it can not be classified as a compact heat exchanger.

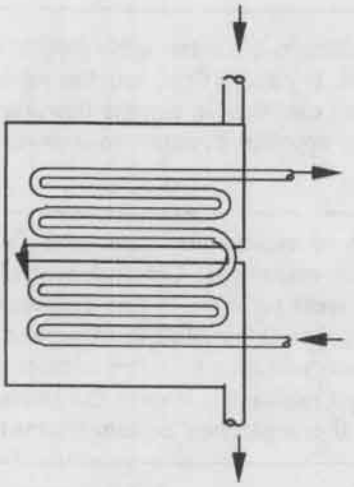
20-4C In counter-flow heat exchangers, the hot and the cold fluids move parallel to each other but both enter the heat exchanger at opposite ends and flow in opposite direction. In cross-flow heat exchangers, the two fluids usually move perpendicular to each other. The cross-flow is said to be unmixed when the plate fins force the fluid to flow through a particular interfin spacing and prevent it from moving in the transverse direction. When the fluid is free to move in the transverse direction, the cross-flow is said to be mixed.

20-5C In the shell and tube exchangers, baffles are commonly placed in the shell to force the shell side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes. Baffles disrupt the flow of fluid, and an increased pumping power will be needed to maintain flow. On the other hand, baffles eliminate dead spots and increase heat transfer rate.

20-6C Using six-tube passes in a shell and tube heat exchanger increases the heat transfer surface area, and the rate of heat transfer increases. But it also increases the manufacturing costs.



20-7C Using so many tubes increases the heat transfer surface area which in turn increases the rate of heat transfer.



20-8C Regenerative heat exchanger involves the alternate passage of the hot and cold fluid streams through the same flow area. The static type regenerative heat exchanger is basically a porous mass which has a large heat storage capacity, such as a ceramic wire mash. Hot and cold fluids flow through this porous mass alternately. Heat is transferred from the hot fluid to the matrix of the regenerator during the flow of the hot fluid and from the matrix to the cold fluid. Thus the matrix serves as a temporary heat storage medium. The dynamic type regenerator involves a rotating drum and continuous flow of the hot and cold fluid through different portions of the drum so that any portion of the drum passes periodically through the hot stream, storing heat and then through the cold stream, rejecting this stored heat. Again the drum serves as the medium to transport the heat from the hot to the cold fluid stream.

The Overall Heat Transfer Coefficient

20-9C Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction and from the wall to the cold fluid again by convection.

20-10C When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, which is usually the case, the thermal resistance of the tube is negligible.

20-11C The heat transfer surface areas are $A_i = \pi D_1 L$ and $A_o = \pi D_2 L$. When the thickness of inner tube is small, it is reasonable to assume $A_i \cong A_o \cong A$.

20-12C No, it is not reasonable to say $h_i \approx h_o \approx h$

20-13C When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, the thermal resistance of the tube is negligible and the inner and the outer surfaces of the tube are almost identical ($A_i \cong A_o \cong A$). Then the overall heat transfer coefficient of a heat exchanger can be determined to from $U = (1/h_i + 1/h_o)^{-1}$

20-14C None.

20-15C When one of the convection coefficients is much smaller than the other $h_i << h_o$, and $A_i \approx A_0 \approx A$. Then we have $(1/h_i >> 1/h_o)$ and thus $U_i = U_0 = U \cong h_i$.

20-16C The most common type of fouling is the precipitation of solid deposits in a fluid on the heat transfer surfaces. Another form of fouling is corrosion and other chemical fouling. Heat exchangers may also be fouled by the growth of algae in warm fluids. This type of fouling is called the biological fouling. Fouling represents additional resistance to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease, and the pressure drop to increase.

20-17C The effect of fouling on a heat transfer is represented by a fouling factor R_f. Its effect on the heat transfer coefficient is accounted for by introducing a thermal resistance R_f/A. The fouling increases with increasing temperature and decreasing velocity.

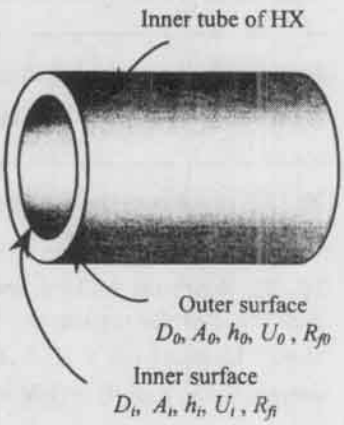
20-18 The heat transfer coefficients and the fouling factors on tube and shell side of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions **1** The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis (a) The total thermal resistance of the heat exchanger per unit length is

$$R = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln(D_o / D_i)}{2\pi kL} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o}$$

$$R = \frac{1}{(700\ \text{W/m}^2.^\circ\text{C})[\pi(0.012\ \text{m})(1\ \text{m})]} + \frac{(0.0005\ \text{m}^2.^\circ\text{C/W})}{[\pi(0.012\ \text{m})(1\ \text{m})]}$$
$$+ \frac{\ln(1.6/1.2)}{2\pi(380\ \text{W/m.}^\circ\text{C})(1\ \text{m})} + \frac{(0.0002\ \text{m}^2.^\circ\text{C/W})}{[\pi(0.016\ \text{m})(1\ \text{m})]}$$
$$+ \frac{1}{(700\ \text{W/m}^2.^\circ\text{C})[\pi(0.016\ \text{m})(1\ \text{m})]}$$
$$= \mathbf{0.0837^\circ C/W}$$



(b) The overall heat transfer coefficient based on the inner and the outer surface areas of the tube per length are

$$R = \frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.0837\,^\circ\text{C/W})[\pi(0.012\,\text{m})(1\,\text{m})]} = \mathbf{317\ W/m^2.^\circ C}$$

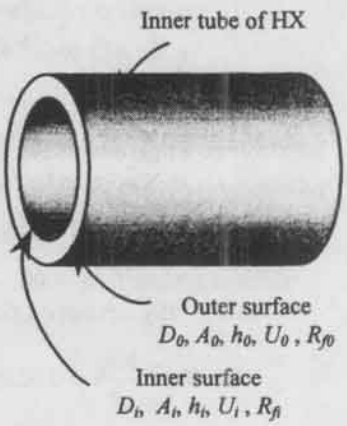
$$U_o = \frac{1}{RA_o} = \frac{1}{(0.0837\,^\circ\text{C/W})[\pi(0.016\,\text{m})(1\,\text{m})]} = \mathbf{238\ W/m^2.^\circ C}$$

20-19 Water flows through the tubes in a boiler. The overall heat transfer coefficient of this boiler based on the inner surface area is to be determined.

Assumptions **1** Water flow is fully developed. **2** Properties of the water are constant.

Properties The properties water at 107°C ≈ 110°C are (Table A-15)

$$\upsilon = \mu/\rho = 0.268\times10^{-6}\ \text{m}^2/\text{s}$$
$$k = 0.682\ \text{W/m}^2.\text{K}$$
$$\text{Pr} = 1.58$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(3.5\ \text{m/s})(0.01\ \text{m})}{0.268\times10^{-6}\ \text{m}^2/\text{s}} = 130{,}600$$

which is greater than 4000. Therefore, the flow is turbulent.
Assuming fully developed flow,

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(130{,}600)^{0.8}(1.58)^{0.4} = 342$$

and
$$h = \frac{k}{D_h}Nu = \frac{0.682\ \text{W/m.°C}}{0.01\ \text{m}}(342) = 23{,}324\ \text{W/m}^2.\text{°C}$$

The total resistance of this heat exchanger is then determined from

$$R = R_{total} = R_i + R_{wall} + R_o = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o}$$
$$= \frac{1}{(23{,}324\ \text{W/m}^2.\text{°C})[\pi(0.01\ \text{m})(5\ \text{m})]} + \frac{\ln(1.4/1)}{[2\pi(14.2\ \text{W/m.°C})(5\ \text{m})]}$$
$$+ \frac{1}{(8400\ \text{W/m}^2.\text{°C})[\pi(0.014\ \text{m})(5\ \text{m})]}$$
$$= 0.00157\text{°C/W}$$

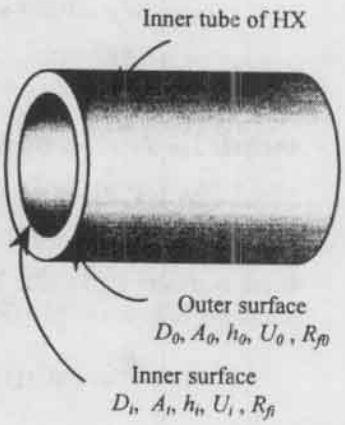
and
$$R = \frac{1}{U_i A_i} \longrightarrow U_i = \frac{1}{RA_i} = \frac{1}{(0.00157\text{°C/W})[\pi(0.01\ \text{m})(5\ \text{m})]} = \mathbf{4055\ W/m^2.°C}$$

20-20 Water is flowing through the tubes in a boiler. The overall heat transfer coefficient of this boiler based on the inner surface area is to be determined.

Assumptions **1** Water flow is fully developed. **2** Properties of water are constant. **3** The heat transfer coefficient and the fouling factor are constant and uniform.

Properties The properties water at 107°C ≈ 110°C are (Table A-15)

$$\upsilon = \mu/\rho = 0.268\times10^{-6}\ \text{m}^2/\text{s}$$
$$k = 0.682\ \text{W/m}^2.\text{K}$$
$$\text{Pr} = 1.58$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(3.5\ \text{m/s})(0.01\ \text{m})}{0.268\times10^{-6}\ \text{m}^2/\text{s}} = 130{,}600$$

which is greater than 4000. Therefore, the flow is turbulent. Assuming fully developed flow,

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(130{,}600)^{0.8}(1.58)^{0.4} = 342$$

and $$h = \frac{k}{D_h}Nu = \frac{0.682\ \text{W/m.°C}}{0.01\ \text{m}}(342) = 23{,}324\ \text{W/m}^2.\text{°C}$$

The thermal resistance of heat exchanger with a fouling factor of $R_{f,i} = 0.0005\ \text{m}^2.\text{°C/W}$ is determined from

$$R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o}$$

$$R = \frac{1}{(23{,}324\ \text{W/m}^2.\text{°C})[\pi(0.01\ \text{m})(5\ \text{m})]} + \frac{0.0005\ \text{m}^2.\text{°C/W}}{[\pi(0.01\ \text{m})(5\ \text{m})]}$$
$$+ \frac{\ln(1.4/1)}{2\pi(14.2\ \text{W/m.°C})(5\ \text{m})} + \frac{1}{(8400\ \text{W/m}^2.\text{°C})[\pi(0.014\ \text{m})(5\ \text{m})]}$$
$$= 0.00476\text{°C/W}$$

Then,

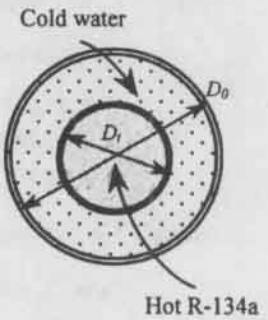
$$R = \frac{1}{U_i A_i} \longrightarrow U_i = \frac{1}{RA_i} = \frac{1}{(0.00476\text{°C/W})[\pi(0.01\ \text{m})(5\ \text{m})]} = 1337\ \text{W/m}^2.\text{°C}$$

20-21 Refrigerant-134a is cooled by water in a double-pipe heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the water and refrigerant-134a flow are fully developed. **3** Properties of the water and refrigerant-134a are constant.

Properties The properties water at 20°C are (Table A-15)

$$\rho = 998\ \text{kg/m}^3$$
$$\upsilon = \mu/\rho = 1.004\times10^{-6}\ \text{m}^2/\text{s}$$
$$k = 0.598\ \text{W/m.°C}$$
$$\text{Pr} = 7.01$$



Analysis The hydraulic diameter for annular space is

$$D_h = D_o - D_i = 0.025 - 0.01 = 0.015\ \text{m}$$

The average velocity of water in the tube and the Reynolds number are

$$V_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho\left(\pi\dfrac{D_o^{\,2} - D_i^{\,2}}{4}\right)} = \frac{0.3\ \text{kg/s}}{(998\ \text{kg/m}^3)\left(\pi\dfrac{(0.025\ \text{m})^2 - (0.01\ \text{m})^2}{4}\right)} = 0.729\ \text{m/s}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(0.729\ \text{m/s})(0.015\ \text{m})}{1.004\times10^{-6}\ \text{m}^2/\text{s}} = 10{,}890$$

which is greater than 4000. Therefore flow is turbulent. Assuming fully developed flow,

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(10{,}890)^{0.8}(7.01)^{0.4} = 85.0$$

and

$$h_o = \frac{k}{D_h}Nu = \frac{0.598\ \text{W/m.°C}}{0.015\ \text{m}}(85.0) = 3390\ \text{W/m}^2\text{.°C}$$

Then the overall heat transfer coefficient becomes

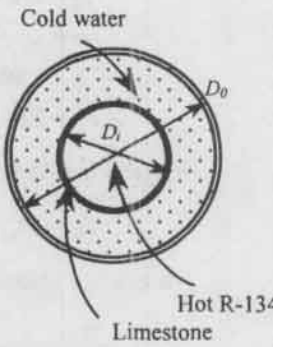
$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{5000\ \text{W/m}^2\text{.°C}} + \dfrac{1}{3390\ \text{W/m}^2\text{.°C}}} = \mathbf{2020\ W/m^2.°C}$$

20-22 Refrigerant-134a is cooled by water in a double-pipe heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the water and refrigerant-134a flows are fully developed. **3** Properties of the water and refrigerant-134a are constant. **4** The limestone layer can be treated as a plain layer since its thickness is very small relative to its diameter.

Properties The properties water at 20°C are (Table A-15)

$$\rho = 998\ \text{kg/m}^3$$
$$\upsilon = \mu/\rho = 1.004\times10^{-6}\ \text{m}^2/\text{s}$$
$$k = 0.598\ \text{W/m.°C}$$
$$\text{Pr} = 7.01$$



Analysis The hydraulic diameter for annular space is

$$D_h = D_o - D_i = 0.025 - 0.01 = 0.015\ \text{m}$$

The average velocity of water in the tube and the Reynolds number are

$$V_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho\left(\pi\dfrac{D_o^{\,2} - D_i^{\,2}}{4}\right)} = \frac{0.3\ \text{kg/s}}{(998\ \text{kg/m}^3)\left(\pi\dfrac{(0.025\ \text{m})^2 - (0.01\ \text{m})^2}{4}\right)} = 0.729\ \text{m/s}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(0.729\ \text{m/s})(0.015\ \text{m})}{1.004\times10^{-6}\ \text{m}^2/\text{s}} = 10{,}890$$

which is greater than 4000. Therefore flow is turbulent. Assuming fully developed flow,

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(10{,}890)^{0.8}(7.01)^{0.4} = 85.0$$

and

$$h_o = \frac{k}{D_h}Nu = \frac{0.598\ \text{W/m.°C}}{0.015\ \text{m}}(85.0) = 3390\ \text{W/m}^2\text{.°C}$$

Disregarding the curvature effects, the overall heat transfer coefficient is determined to be

$$U = \frac{1}{\dfrac{1}{h_i} + \left(\dfrac{L}{k}\right)_{\text{limeston}} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{5000\ \text{W/m}^2\text{.°C}} + \dfrac{0.002\ \text{m}}{1.3\ \text{W/m.°C}} + \dfrac{1}{3390\ \text{W/m}^2\text{.°C}}} = \mathbf{493\ W/m^2.°C}$$

20-23E Water is cooled by air in a cross-flow heat exchanger. The overall heat transfer coefficient is to be determined. √

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the water and air flow are fully developed. **3** Properties of the water and air are constant.

Properties The properties water at 140°F are (Table A-15E)

$$k = 0.378 \text{ Btu/h.ft.°F}$$

$$\upsilon = 5.11\times10^{-6} \text{ ft}^2/\text{s}$$

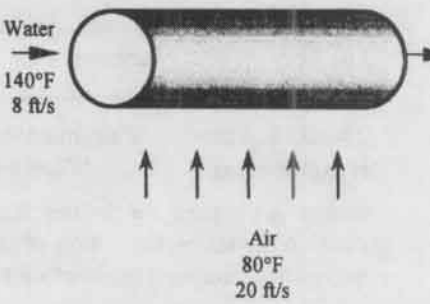
$$\text{Pr} = 2.98$$

The properties of air at 80°F are (Table A-18E)

$$k = 0.0150 \text{ Btu/h.ft.°F}$$

$$\upsilon = 0.17\times10^{-3} \text{ ft}^2/\text{s}$$

$$\text{Pr} = 0.72$$



Analysis The overall heat transfer coefficient can be determined from

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

The Reynolds number of water is

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(8 \text{ ft/s})[0.75/12 \text{ ft}]}{5.11\times10^{-6} \text{ ft}^2/\text{s}} = 97{,}850$$

which is greater than 4000. Therefore the flow of water is turbulent. Assuming the flow to be fully developed, the Nusselt number is determined from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(97{,}850)^{0.8}(2.98)^{0.4} = 350$$

and

$$h_i = \frac{k}{D_h} Nu = \frac{0.378 \text{ Btu/h.ft.°F}}{0.75/12 \text{ ft}}(350) = 2117 \text{ Btu/h.ft}^2\text{.°F}$$

The Reynolds number of air is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{(20 \text{ ft/s})[3/(4\times12) \text{ ft}]}{0.17\times10^{-3} \text{ ft}^2/\text{s}} = 7353$$

The flow of air is across the cylinder. The proper relation for Nusselt number in this case is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(7353)^{0.5}(0.72)^{1/3}}{\left[1+(0.4/0.72)^{2/3}\right]^{1/4}}\left[1+\left(\frac{7353}{282{,}000}\right)^{5/8}\right]^{4/5} = 45.6$$

and

$$h_o = \frac{k}{D} Nu = \frac{0.0150 \text{ Btu/h.ft.°F}}{0.75/12 \text{ ft}}(45.6) = 10.9 \text{ Btu/h.ft}^2\text{.°F}$$

Then the overall heat transfer coefficient becomes

$$U = \frac{1}{\frac{1}{h_i}+\frac{1}{h_o}} = \frac{1}{\frac{1}{2117 \text{ Btu/h.ft}^2\text{.°F}} + \frac{1}{10.9 \text{ Btu/h.ft}^2\text{.°F}}} = \mathbf{10.9 \text{ Btu/h.ft}^2\text{.°F}}$$

Analysis of Heat Exchangers

20-24C The heat exchangers usually operate for long periods of time with no change in their operating conditions, and then they can be modeled as steady-flow devices. As such , the mass flow rate of each fluid remains constant and the fluid properties such as temperature and velocity at any inlet and outlet remain constant. The kinetic and potential energy changes are negligible. The specific heat of a fluid can be treated as constant in a specified temperature range. Axial heat conduction along the tube is negligible. Finally, the outer surface of the heat exchanger is assumed to be perfectly insulated so that there is no heat loss to the surrounding medium and any heat transfer thus occurs is between the two fluids only.

20-25C That relation is valid under steady operating conditions, constant specific heats, and negligible heat loss from the heat exchanger.

20-26C The product of the mass flow rate and the specific heat of a fluid is called the heat capacity rate and is expressed as $C = \dot{m}C_p$. When the heat capacity rates of the cold and hot fluids are equal, the temperature change is the same for the two fluids in a heat exchanger. That is, the temperature rise of the cold fluid is equal to the temperature drop of the hot fluid. A heat capacity of infinity for a fluid in a heat exchanger is experienced during a phase-change process in a condenser or boiler.

20-27C The mass flow rate of the cooling water can be determined from $\dot{Q} = (\dot{m}C_P\Delta T)_{\text{cooling water}}$. The rate of condensation of the steam is determined from $\dot{Q} = (\dot{m}h_{fg})_{\text{steam}}$, and the total thermal resistance of the condenser is determined from $R = \dot{Q}/\Delta T$.

20-28C When the heat capacity rates of the cold and hot fluids are identical, the temperature rise of the cold fluid will be equal to the temperature drop of the hot fluid.

The Log Mean Temperature Difference Method

20-29C ΔT_{lm} is called the log mean temperature difference, and is expressed as

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

where

$$\Delta T_1 = T_{h,in} - T_{c,in} \qquad \Delta T_2 = T_{h,out} - T_{c,out}$$

for parallel-flow heat exchangers and

$$\Delta T = T_{h,in} - T_{c,out} \qquad \Delta T_2 = T_{h,out} - T_{c,in}$$

for counter-flow heat exchangers

20-30C The temperature difference between the two fluids decreases from ΔT_1 at the inlet to ΔT_2 at the outlet, and arithmetic mean temperature difference is defined as $\Delta T_m = \frac{\Delta T_1 + \Delta T_2}{2}$. The logarithmic mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluids along the heat exchanger, and is an exact representation of the average temperature difference between the hot and cold fluids. It truly reflects the exponential decay of the local temperature difference. The logarithmic mean temperature difference is always less than the arithmetic mean temperature.

20-31C ΔT_{lm} can not be greater than both ΔT_1 and ΔT_2 because ΔT_{ln} is always less than or equal to ΔT_m (arithmetic mean) which can not be greater than both ΔT_1 and ΔT_2.

20-32C No, it cannot. When ΔT_1 is less than ΔT_2 the ratio of them must be less than one and the natural logarithms of the numbers which are less than 1 are negative. But the numerator is also negative in this case. When ΔT_1 is greater than ΔT_2, we obtain positive numbers at the both numerator and denominator.

20-33C In the parallel-flow heat exchangers the hot and cold fluids enter the heat exchanger at the same end, and the temperature of the hot fluid decreases and the temperature of the cold fluid increases along the heat exchanger. But the temperature of the cold fluid can never exceed that of the hot fluid. In case of the counter-flow heat exchangers the hot and cold fluids enter the heat exchanger from the opposite ends and the outlet temperature of the cold fluid may exceed the outlet temperature of the hot fluid.

20-34C The ΔT_{lm} will be greatest for double-pipe counter-flow heat exchangers.

20-35C The factor F is called as correction factor which depends on the geometry of the heat exchanger and the inlet and the outlet temperatures of the hot and cold fluid streams. It represents how closely a heat exchanger approximates a counter-flow heat exchanger in terms of its logarithmic mean temperature difference. F can not be greater than unity.

20-36C In this case it is not practical to use the LMTD method because it requires tedious iterations. Instead, the effectiveness-NTU method should be used.

20-37C First heat transfer rate is determined from $\dot{Q} = \dot{m}C_p[T_{in} - T_{out}]$, ΔT_{ln} from $\Delta T_{lm} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$, correction factor from the figures, and finally the surface area of the heat exchanger from $\dot{Q} = UAFDT_{lm,cf}$

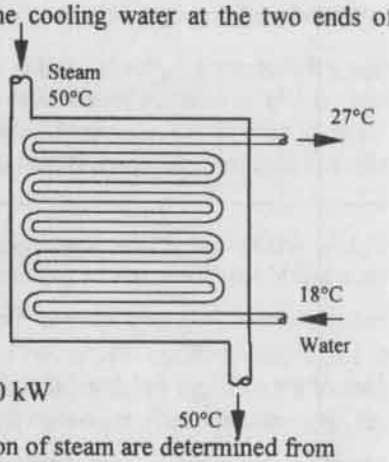
20-38 Steam is condensed by cooling water in the condenser of a power plant. The mass flow rate of the cooling water and the rate of condensation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The heat of vaporization of water at 50°C is given to be h_{fg} = 2305 kJ/kg and specific heat of cold water at the average temperature of 22.5°C is given to be C_p = 4180 J/kg.°C.

Analysis The temperature differences between the steam and the cooling water at the two ends of the condenser are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 50°C - 27°C = 23°C$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 50°C - 18°C = 32°C$$



and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{23 - 32}{\ln(23/32)} = 27.3°C$$

Then the heat transfer rate in the condenser becomes

$$\dot{Q} = UA\Delta T_{lm} = (2400\ \text{W/m}^2.°\text{C})(58\ \text{m}^2)(27.3°\text{C}) = 3800\ \text{kW}$$

The mass flow rate of the cooling water and the rate of condensation of steam are determined from

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{cooling water}} \longrightarrow \dot{m}_{\substack{\text{cooling}\\\text{water}}} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})}$$
$$= \frac{3800\ \text{kJ/s}}{(4.18\ \text{kJ/kg.°C})(27°\text{C} - 18°\text{C})} = \mathbf{101\ kg/s}$$

$$\dot{Q} = (\dot{m}h_{fg})_{steam} \longrightarrow \dot{m}_{steam} = \frac{\dot{Q}}{h_{fg}} = \frac{3800\ \text{kJ/s}}{2305\ \text{kJ/kg}} = \mathbf{1.65\ kg/s}$$

20-39 Water is heated in a double-pipe parallel-flow heat exchanger by geothermal water. The required length of tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and geothermal fluid are given to be 4.18 and 4.31 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in the heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} = (0.2\ \text{kg/s})(4.18\ \text{kJ/kg.°C})(60°\text{C} - 25°\text{C}) = 29.26\ \text{kW}$$

Then the outlet temperature of the geothermal water is determined from

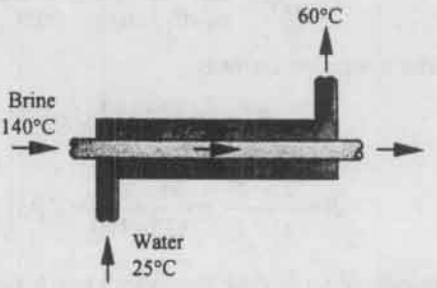
$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{geot.water}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 140°\text{C} - \frac{29.26\ \text{kW}}{(0.3\ \text{kg/s})(4.31\ \text{kJ/kg.°C})} = 117.4°\text{C}$$

The logarithmic mean temperature difference is

$$\Delta T_1 = T_{h,in} - T_{c,in} = 140°\text{C} - 25°\text{C} = 115°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,out} = 117.4°\text{C} - 60°\text{C} = 57.4°\text{C}$$



and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{115 - 57.4}{\ln(115/57.4)} = 82.9°\text{C}$$

The surface area of the heat exchanger is determined from

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{29.26\ \text{kW}}{(0.55\ \text{kW/m}^2)(82.9°\text{C})} = 0.642\ \text{m}^2$$

Then the length of the tube required becomes

$$A = \pi DL \longrightarrow L = \frac{A}{\pi D} = \frac{0.642\ \text{m}^2}{\pi(0.008\ \text{m})} = \mathbf{25.5\ m}$$

20-40E Glycerin is heated by hot water in a 1-shell pass and 8-tube passes heat exchanger. The rate of heat transfer for the cases of fouling and no fouling are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

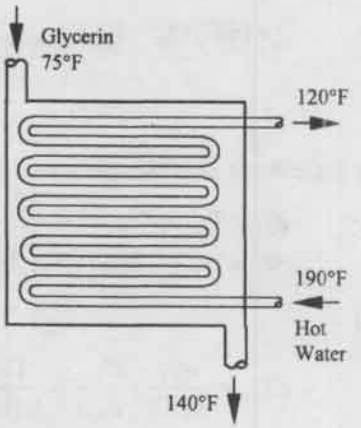
Properties The specific heats of glycerin and water are given to be 0.60 and 1.0 Btu/lbm.°F, respectively.

Analysis (a) The tubes are thin walled and thus we assume the inner surface area of the tube to be equal to the outer surface area. Then the heat transfer surface area of this heat exchanger becomes

$$A = n\pi DL = 8\pi(0.5/12\ \text{ft})(500\ \text{ft}) = 523.6\ \text{ft}^2$$

The temperature differences at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 190°\text{F} - 140°\text{F} = 50°\text{F}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 120°\text{F} - 75°\text{F} = 45°\text{F}$$



and

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{50-45}{\ln(50/45)} = 47.5°\text{F}$$

The correction factor is

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{120-190}{75-190} = 0.61 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{75-140}{120-190} = 0.93 \end{aligned}\right\} F = 0.55$$

In case of no fouling, the overall heat transfer coefficient is determined from

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{50\ \text{Btu/h.ft}^2.°\text{F}} + \frac{1}{4\ \text{Btu/h.ft}^2.°\text{F}}} = 3.7\ \text{Btu/h.ft}^2.°\text{F}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UAF\Delta T_{lm,CF} = (3.7\ \text{Btu/h.ft}^2.°\text{F})(523.6\ \text{ft}^2)(0.55)(47.5°\text{F}) = \mathbf{5.06 \times 10^4\ Btu/h}$$

(b) The thermal resistance of the heat exchanger with a fouling factor is

$$R = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{1}{h_o A_o}$$
$$= \frac{1}{(50\ \text{Btu/h.ft}^2.°\text{F})(523.6\ \text{ft}^2)} + \frac{0.002\ \text{h.ft}^2.°\text{F/Btu}}{523.6\ \text{ft}^2} + \frac{1}{(4\ \text{Btu/h.ft}^2.°\text{F})(523.6\ \text{ft}^2)}$$
$$= 0.0005195\ \text{h.°F/Btu}$$

The overall heat transfer coefficient in this case is

$$R = \frac{1}{UA} \longrightarrow U = \frac{1}{RA} = \frac{1}{(0.0005195\ \text{h.°F/Btu})(523.6\ \text{ft}^2)} = 3.68\ \text{Btu/h.ft}^2.°\text{F}$$

Then rate of heat transfer becomes

$$\dot{Q} = UAF\Delta T_{lm,CF} = (3.68\ \text{Btu/h.ft}^2.°\text{F})(523.6\ \text{ft}^2)(0.55)(47.5°\text{F}) = \mathbf{5.03 \times 10^4\ Btu/h}$$

20-41 During an experiment, the inlet and exit temperatures of water and oil and the mass flow rate of water are measured. The overall heat transfer coefficient based on the inner surface area is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of water and oil are given to be 4180 and 2150 J/kg.°C, respectively.

Analysis The rate of heat transfer from the oil to the water is

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{water} = (5\ \text{kg/s})(4.18\ \text{kJ/kg.°C})(55°\text{C} - 20°\text{C}) = 731.5\ \text{kW}$$

The heat transfer area on the tube side is

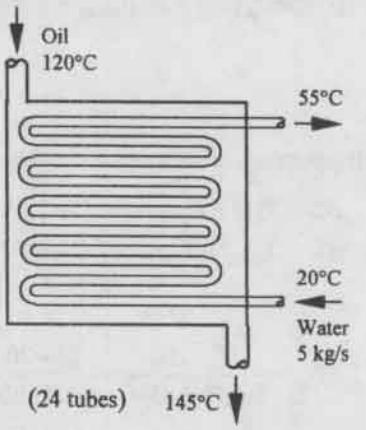
$$A_i = n\pi D_i L = 24\pi(0.012\ \text{m})(2\ \text{m}) = 1.8\ \text{m}^2$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 120°\text{C} - 55°\text{C} = 65°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 45°\text{C} - 20°\text{C} = 25°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{65 - 25}{\ln(65/25)} = 41.9°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{55 - 20}{120 - 20} = 0.35 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{120 - 45}{55 - 20} = 2.14 \end{aligned}\right\} F = 0.70$$



Then the overall heat transfer coefficient becomes

$$\dot{Q} = U_i A_i F \Delta T_{lm,CF} \longrightarrow U_i = \frac{\dot{Q}}{A_i F \Delta T_{lm,CF}} = \frac{731.5\ \text{kW}}{(1.8\ \text{m}^2)(0.70)(41.9°\text{C})} = \mathbf{13.9\ kW/m^2.°C}$$

20-42 Ethylene glycol is cooled by water in a double-pipe counter-flow heat exchanger. The rate of heat transfer, the mass flow rate of water, and the heat transfer surface area on the inner side of the tubes are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and ethylene glycol are given to be 4.18 and 2.56 kJ/kg.°C, respectively.

Analysis (a) The rate of heat transfer is

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{glycol}} = (2\ \text{kg/s})(2.56\ \text{kJ/kg.°C})(80°\text{C} - 40°\text{C}) = \mathbf{204.8\ kW}$$

(b) The rate of heat transfer from water must be equal to the rate of heat transfer to the glycol. Then,

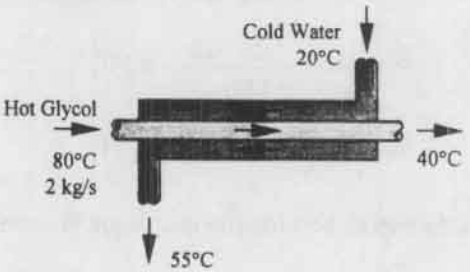
$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} \longrightarrow \dot{m}_{\text{water}} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})}$$

$$= \frac{204.8\ \text{kJ/s}}{(4.18\ \text{kJ/kg.°C})(55°\text{C} - 20°\text{C})} = \mathbf{1.4\ kg/s}$$

(c) The temperature differences at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 80°\text{C} - 55°\text{C} = 25°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 40°\text{C} - 20°\text{C} = 20°\text{C}$$

and



$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{25 - 20}{\ln(25/20)} = 22.4°\text{C}$$

Then the heat transfer surface area becomes

$$\dot{Q} = U_i A_i \Delta T_{lm} \longrightarrow A_i = \frac{\dot{Q}}{U_i \Delta T_{lm}} = \frac{204.8\ \text{kW}}{(0.25\ \text{kW/m}^2\text{.°C})(22.4°\text{C})} = \mathbf{36.6\ m^2}$$

20-43 Water is heated by steam in a double-pipe counter-flow heat exchanger. The required length of the tubes is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

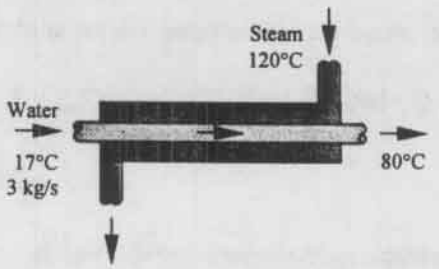
Properties The specific heat of water is given to be 4.18 kJ/kg.°C. The heat of condensation of steam at 120°C is given to be 2203 kJ/kg.

Analysis The rate of heat transfer is

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{water} = (3\ \text{kg/s})(4.18\ \text{kJ/kg.°C})(80°\text{C} - 17°\text{C}) = 790.02\ \text{kW}$$

The logarithmic mean temperature difference is

$$\Delta T_1 = T_{h,in} - T_{c,out} = 120°\text{C} - 80°\text{C} = 40°\text{C}$$
$$\Delta T_2 = T_{h,in} - T_{c,in} = 120°\text{C} - 17°\text{C} = 103°\text{C}$$
$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{40 - 103}{\ln(40/103)} = 66.6°\text{C}$$



The heat transfer surface area is

$$\dot{Q} = U_i A_i \Delta T_{lm} \longrightarrow A_i = \frac{\dot{Q}}{U_i \Delta T_{lm}} = \frac{790.02\ \text{kW}}{(1.5\ \text{kW/m}^2.°\text{C})(66.6°\text{C})} = 7.9\ \text{m}^2$$

Then the length of tube required becomes

$$A_i = \pi D_i L \longrightarrow L = \frac{A_i}{\pi D_i} = \frac{7.9\ \text{m}^2}{\pi(0.025\ \text{m})} = \mathbf{100.6\ m}$$

20-44 Oil is cooled by water in a thin-walled double-pipe counter-flow heat exchanger. The overall heat transfer coefficient of the heat exchanger is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heats of water and oil are given to be 4.18 and 2.20 kJ/kg.°C, respectively.

Analysis The rate of heat transfer from the water to the oil is

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{oil}} = (2\ \text{kg/s})(2.2\ \text{kJ/kg.°C})(150°\text{C} - 40°\text{C}) = 484\ \text{kW}$$

The outlet temperature of the water is determined from

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} \longrightarrow T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m}C_p}$$

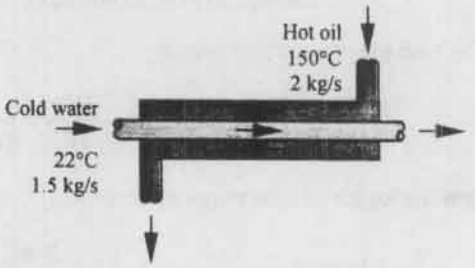
$$= 22°\text{C} + \frac{484\ \text{kW}}{(1.5\ \text{kg/s})(4.18\ \text{kJ/kg.°C})} = 99.2°\text{C}$$

The logarithmic mean temperature difference is

$$\Delta T_1 = T_{h,in} - T_{c,out} = 150°\text{C} - 99.2°\text{C} = 50.8°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 40°\text{C} - 22°\text{C} = 18°\text{C}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{50.8 - 18}{\ln(50.8/18)} = 31.6°\text{C}$$



Then the overall heat transfer coefficient becomes

$$U = \frac{\dot{Q}}{A\Delta T_{lm}} = \frac{484\ \text{kW}}{\pi(0.025\ \text{m})(6\ \text{m})(31.6°\text{C})} = \mathbf{32.5\ kW/m^2.°C}$$

20-45 The inlet and outlet temperatures of the cold and hot fluids in a double-pipe heat exchanger are given. It is to be determined whether this is a parallel-flow or counter-flow heat exchanger.

Analysis In parallel-flow heat exchangers, the temperature of the cold water can never exceed that of the hot fluid. In this case $T_{\text{cold out}} = 50°\text{C}$ which is greater than $T_{\text{hot out}} = 45°\text{C}$. Therefore this must be a counter-flow heat exchanger.

20-46 Cold water is heated by hot water in a double-pipe counter-flow heat exchanger. The rate of heat transfer and the heat transfer surface area of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heats of cold and hot water are given to be 4.18 and 4.19 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{cold water}} = (0.25\ \text{kg/s})(4.18\ \text{kJ/kg.°C})(45°\text{C} - 15°\text{C}) = \mathbf{31.35\ kW}$$

The outlet temperature of the hot water is determined from

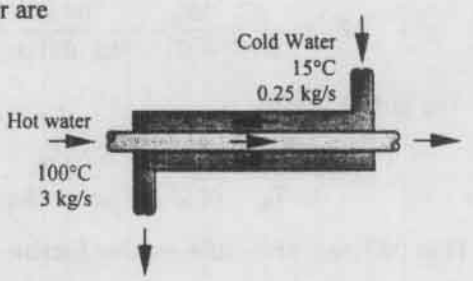
$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{hot water}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 100°\text{C} - \frac{31.35\ \text{kW}}{(3\ \text{kg/s})(4.19\ \text{kJ/kg.°C})} = 97.5°\text{C}$$

The temperature differences at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 100°\text{C} - 45°\text{C} = 55°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 97.5°\text{C} - 15°\text{C} = 82.5°\text{C}$$



and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{55 - 82.5}{\ln(55/82.5)} = 67.8°\text{C}$$

Then the surface area of this heat exchanger becomes

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow A_i = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{31.35\ \text{kW}}{(0.95\ \text{kW/m}^2.°\text{C})(67.8°\text{C})} = \mathbf{0.49\ m^2}$$

20-47 Engine oil is heated by condensing steam in a condenser. The rate of heat transfer and the length of the tube required are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

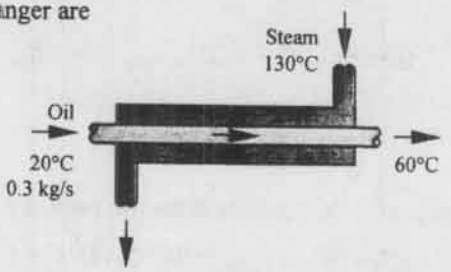
Properties The specific heat of engine oil is given to be 2.1 kJ/kg.°C. The heat of condensation of steam at 130°C is given to be 2174 kJ/kg.

Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{oil}} = (0.3\ \text{kg/s})(2.1\ \text{kJ/kg.°C})(60°\text{C} - 20°\text{C}) = \mathbf{25.2\ kW}$$

The temperature differences at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 130°\text{C} - 60°\text{C} = 70°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 130°\text{C} - 20°\text{C} = 110°\text{C}$$



and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{70 - 110}{\ln(70/110)} = 88.5°\text{C}$$

The surface area is

$$A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{25.2\ \text{kW}}{(0.65\ \text{kW/m}^2.°\text{C})(88.5°\text{C})} = 0.44\ \text{m}^2$$

Then the length of the tube required becomes

$$A = \pi DL \longrightarrow L = \frac{A}{\pi D} = \frac{0.44\ \text{m}^2}{\pi(0.02\ \text{m})} = \mathbf{7.0\ m}$$

20-48E Water is heated by geothermal water in a double-pipe counter-flow heat exchanger. The mass flow rate of each fluid and the total thermal resistance of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and geothermal fluid are given to be 1.0 and 1.03 Btu/lbm.°F, respectively.

Analysis The mass flow rate of each fluid are determined from

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} \longrightarrow \dot{m}_{\text{water}} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})}$$

$$= \frac{30 \text{ Btu/s}}{(1.0 \text{ Btu/lbm.°F})(200°\text{F} - 140°\text{F})}$$

$$= \mathbf{0.5 \text{ lbm/s}}$$

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{geo. water}} \longrightarrow \dot{m}_{\text{geo. water}} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})}$$

$$= \frac{30 \text{ Btu/s}}{(1.03 \text{ Btu/lbm.°F})(250°\text{F} - 180°\text{F})}$$

$$= \mathbf{0.42 \text{ lbm/s}}$$

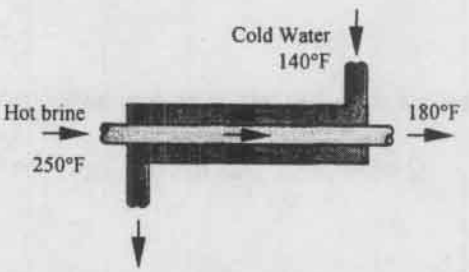
The temperature differences at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 250°\text{F} - 200°\text{F} = 50°\text{F}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 180°\text{F} - 140°\text{F} = 40°\text{F}$$

and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{50 - 40}{\ln(50/40)} = 44.8°\text{F}$$



Then

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow UA = \frac{\dot{Q}}{\Delta T_{lm}} = \frac{30 \text{ Btu/s}}{44.8°\text{F}} = 0.669 \text{ Btu/s.°F}$$

$$U = \frac{1}{RA} \longrightarrow R = \frac{1}{UA} = \frac{1}{0.669 \text{ Btu/s.°F}} = \mathbf{1.49 \text{ s.°F/Btu}}$$

20-49 Glycerin is heated by ethylene glycol in a thin-walled double-pipe parallel-flow heat exchanger. The rate of heat transfer, the outlet temperature of the glycerin, and the mass flow rate of the ethylene glycol are to be determined.

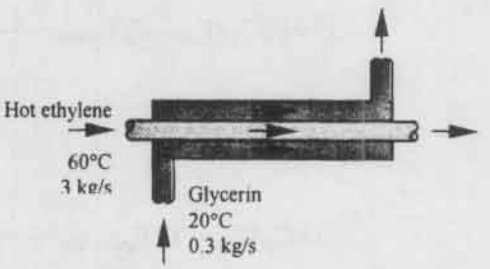
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heats of glycerin and ethylene glycol are given to be 2.4 and 2.5 kJ/kg.°C, respectively.

Analysis (a) The temperature differences at the two ends are

$$\Delta T_1 = T_{h,in} - T_{c,in} = 60°\text{C} - 20°\text{C} = 40°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,out} = T_{h,out} - (T_{h,out} - 15°\text{C}) = 15°\text{C}$$



and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{40 - 15}{\ln(40/15)} = 25.5°\text{C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA\Delta T_{lm} = (240\ \text{W/m}^2.°\text{C})(7.6\ \text{m}^2)(25.5°\text{C}) = 4.65\times10^4\ \text{W} = \mathbf{46.5\ kW}$$

(*b*) The outlet temperature of the glycerin is determined from

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{glycerin}} \longrightarrow T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 20°\text{C} + \frac{46.5\ \text{kW}}{(0.3\ \text{kg/s})(2.4\ \text{kJ/kg.°C})} = \mathbf{84.6°C}$$

(*c*) Then the mass flow rate of ethylene glycol becomes

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{ethylene glycol}} \longrightarrow \dot{m}_{\text{ethylene glycol}} = \frac{\dot{Q}}{C_p(T_{in} - T_{out})}$$

$$= \frac{46.5\ \text{kJ/s}}{(2.5\ \text{kJ/kg.°C})[(84.6+15)°\text{C} - 60°\text{C}]}$$

$$= \mathbf{0.47\ kg/s}$$

20-50 Air is preheated by hot exhaust gases in a cross-flow heat exchanger. The rate of heat transfer and the outlet temperature of the air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

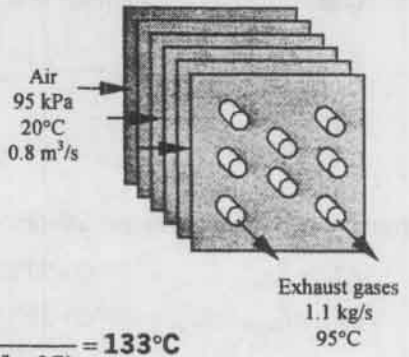
Properties The specific heats of air and combustion gases are given to be 1005 and 1100 J/kg.°C, respectively.

Analysis The rate of heat transfer is

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{gas}} = (1.1\,\text{kg/s})(1.1\,\text{kJ/kg.°C})(180°\text{C} - 95°\text{C}) = \mathbf{103\,kW}$$

The mass flow rate of air is

$$\dot{m} = \frac{P\dot{V}}{RT} = \frac{(95\,\text{kPa})(0.8\,\text{m}^3/\text{s})}{(0.287\,\text{kPa.m}^3/\text{kg.K}) \times 293\,\text{K}} = 0.904\,\text{kg/s}$$



Then the outlet temperature of the air becomes

$$\dot{Q} = \dot{m}C_p(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 20°\text{C} + \frac{103 \times 10^3\,\text{W}}{(0.904\,\text{kg/s})(1005\,\text{J/kg.°C})} = \mathbf{133°C}$$

20-51 Water is heated by hot oil in a 2-shell passes and 12-tube passes heat exchanger. The heat transfer surface area on the tube side is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and oil are given to be 4.18 and 2.3 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} = (4.5\ \text{kg/s})(4.18\ \text{kJ/kg.°C})(70°\text{C} - 20°\text{C}) = 940.5\ \text{kW}$$

The outlet temperature of the hot water is determined from

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{oil}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p}$$

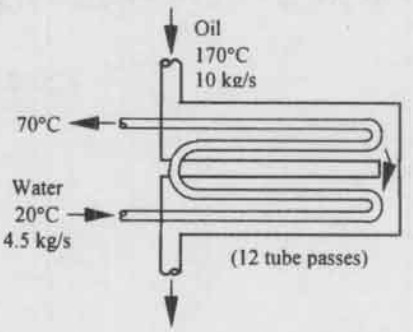
$$= 170°\text{C} - \frac{940.5\ \text{kW}}{(10\ \text{kg/s})(2.3\ \text{kJ/kg.°C})} = 129°\text{C}$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 170°\text{C} - 70°\text{C} = 100°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 129°\text{C} - 20°\text{C} = 109°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{100 - 109}{\ln(100/109)} = 105°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{129 - 170}{20 - 170} = 0.27 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 70}{129 - 170} = 1.2 \end{aligned}\right\} F = 1.0$$



Then the heat transfer surface area on the tube side becomes

$$\dot{Q} = UAF\Delta T_{lm,CF} \longrightarrow A = \frac{\dot{Q}}{UF\Delta T_{lm,CF}} = \frac{940.5\ \text{kW}}{(0.6\ \text{kW/m}^2.°\text{C})(1.0)(105°\text{C})} = \mathbf{15\ m^2}$$

20-52 Water is heated by hot oil in a 2-shell passes and 12-tube passes heat exchanger. The heat transfer surface area on the tube side is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and oil are given to be 4.18 and 2.3 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{water}} = (2\ \text{kg/s})(4.18\ \text{kJ/kg.°C})(70°\text{C} - 20°\text{C}) = 418\ \text{kW}$$

The outlet temperature of the oil is determined from

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{oil}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 170°\text{C} - \frac{418\ \text{kW}}{(10\ \text{kg/s})(2.3\ \text{kJ/kg.°C})} = 151.8°\text{C}$$

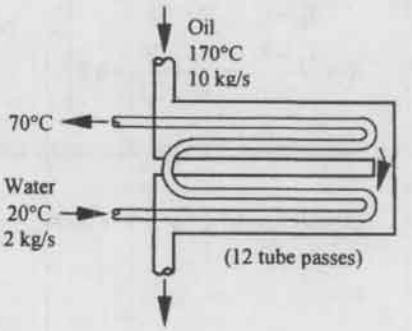
The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 170°\text{C} - 70°\text{C} = 100°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 151.8°\text{C} - 20°\text{C} = 131.8°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{100 - 131.8}{\ln(100/131.8)} = 115.2°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{151.8 - 170}{20 - 170} = 0.12 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 70}{151.8 - 170} = 2.7 \end{aligned}\right\} F = 1.0$$



Then the heat transfer surface area on the tube side becomes

$$\dot{Q} = U_i A_i F \Delta T_{lm,CF} \longrightarrow A_i = \frac{\dot{Q}}{U_i F \Delta T_{lm,CF}} = \frac{418\ \text{kW}}{(0.6\ \text{kW/m}^2.°\text{C})(1.0)(115.2°\text{C})} = \mathbf{6.05\ m^2}$$

20-53 Ethyl alcohol is heated by water in a 2-shell passes and 8-tube passes heat exchanger. The heat transfer surface area of the heat exchanger is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and ethyl alcohol are given to be 4.19 and 2.67 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is

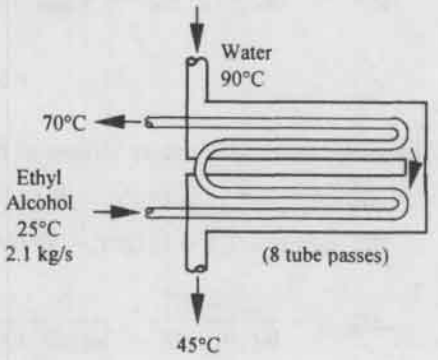
$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{ethyl alcohol}} = (2.1\,\text{kg/s})(2.67\,\text{kJ/kg.°C})(70°\text{C} - 25°\text{C}) = 252.3\,\text{kW}$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 95°\text{C} - 70°\text{C} = 25°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 45°\text{C} - 25°\text{C} = 20°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{25 - 20}{\ln(25/20)} = 22.4°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{45 - 95}{25 - 95} = 0.7 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{25 - 70}{45 - 95} = 0.9 \end{aligned}\right\} F = 0.77$$



Then the heat transfer surface area on the tube side becomes

$$\dot{Q} = U_i A_i F \Delta T_{lm,CF} \longrightarrow A_i = \frac{\dot{Q}}{U_i F \Delta T_{lm,CF}} = \frac{252.3\,\text{kW}}{(0.8\,\text{kW/m}^2.°\text{C})(0.77)(22.4°\text{C})} = \mathbf{18.3\,m^2}$$

20-54 Water is heated by ethylene glycol in a 2-shell passes and 12-tube passes heat exchanger. The rate of heat transfer and the heat transfer surface area on the tube side are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and ethylene glycol are given to be 4.18 and 2.68 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is :

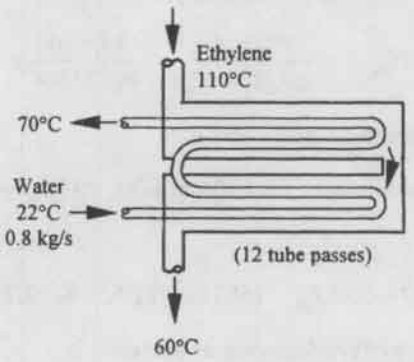
$$\dot{Q} = [\dot{m} C_p (T_{out} - T_{in})]_{\text{water}} = (0.8\,\text{kg/s})(4.18\,\text{kJ/kg.°C})(70°\text{C} - 22°\text{C}) = \mathbf{160.5\,kW}$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 110°\text{C} - 70°\text{C} = 40°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 60°\text{C} - 22°\text{C} = 38°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{40 - 38}{\ln(40/38)} = 39°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{60 - 110}{22 - 110} = 0.57 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{22 - 70}{60 - 110} = 0.96 \end{aligned}\right\} F = 0.94$$



Then the heat transfer surface area on the tube side becomes

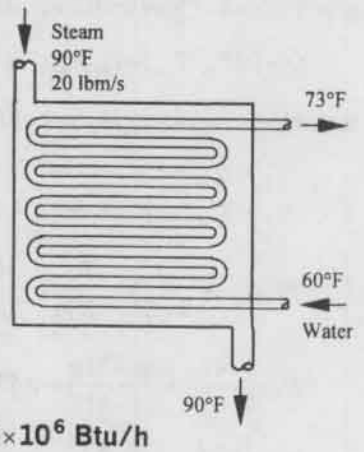
$$\dot{Q} = U_i A_i F \Delta T_{lm,CF} \longrightarrow A_i = \frac{\dot{Q}}{U_i F \Delta T_{lm,CF}} = \frac{160.5\,\text{kW}}{(0.28\,\text{kW/m}^2.°\text{C})(0.94)(39°\text{C})} = \mathbf{15.6\,m^2}$$

20-55E Steam is condensed by cooling water in a condenser. The rate of heat transfer, the rate of condensation of steam, and the mass flow rate of cold water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties We take specific heat of water are given to be 1.0 Btu/lbm.°F. The heat of condensation of steam at 90°F is 1043 Btu/lbm.

Analysis (a) The log mean temperature difference is determined from



$$\Delta T_1 = T_{h,in} - T_{c,out} = 90°\text{F} - 73°\text{F} = 17°\text{F}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 90°\text{F} - 60°\text{F} = 30°\text{F}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{17 - 30}{\ln(17/30)} = 22.9°\text{F}$$

The heat transfer surface area is

$$A = 8n\pi DL = 8 \times 50 \times \pi(3/48\ \text{ft})(5\ \text{ft}) = 392.7\ \text{ft}^2$$

and

$$\dot{Q} = UA\Delta T_{lm} = (600\ \text{Btu/h.ft}^2.°\text{F})(392.7\ \text{ft}^2)(22.9°\text{F}) = \mathbf{5.396 \times 10^6\ Btu/h}$$

(b) The rate of condensation of the steam is

$$\dot{Q} = (\dot{m}h_{fg})_{steam} \longrightarrow \dot{m}_{steam} = \frac{\dot{Q}}{h_{fg}} = \frac{5.396 \times 10^6\ \text{Btu/h}}{1043\ \text{Btu/lbm}} = \mathbf{5173\ lbm/h = 1.44\ lbm/s}$$

(c) Then the mass flow rate of cold water becomes

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{cold water}}$$

$$\dot{m}_{\text{cold water}} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})} = \frac{5.396 \times 10^6\ \text{Btu/h}}{(1.0\ \text{Btu/lbm.°F})(73°\text{F} - 60°\text{F}]}$$
$$= \mathbf{4.15 \times 10^5\ lbm/h = 115\ lbm/s}$$

20-56 Glycerin is heated by hot water in a 1-shell pass and 20-tube passes heat exchanger. The mass flow rate of glycerin and the overall heat transfer coefficient of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and glycerin are given to be 4.18 and 2.48 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{water}} = (5\text{ kg/s})(4.18\text{ kJ/kg.°C})(100°\text{C} - 55°\text{C}) = 940.5\text{ kW}$$

The mass flow rate of the glycerin is determined from

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{\text{glycerin}}$$

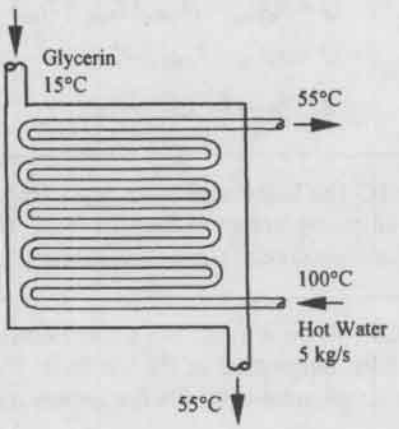
$$\dot{m}_{\text{glycerin}} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})} = \frac{940.5\text{ kJ/s}}{(2.48\text{ kJ/kg.°C})[(55°\text{C} - 15°\text{C}]} = \mathbf{9.5\text{ kg/s}}$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 100°\text{C} - 55°\text{C} = 45°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 55°\text{C} - 15°\text{C} = 40°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{45 - 40}{\ln(45/40)} = 42.5°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{55 - 100}{15 - 100} = 0.53 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{15 - 55}{55 - 100} = 0.89 \end{aligned}\right\} F = 0.77$$



The heat transfer surface area is

$$A = n\pi DL = 10\pi(0.015\text{ m})(2\text{ m}) = 0.94\text{ m}^2$$

Then the overall heat transfer coefficient of the heat exchanger is determined to be

$$\dot{Q} = UAF\Delta T_{lm,CF} \longrightarrow U = \frac{\dot{Q}}{AF\Delta T_{lm,CF}} = \frac{940.5\text{ kW}}{(0.94\text{ m}^2)(0.77)(42.5°\text{C})} = \mathbf{30.6\text{ kW/m}^2.°\text{C}}$$

The Effectiveness-NTU Method

20-57C When the heat transfer surface area A of the heat exchanger is known, but the outlet temperatures are not, the effectiveness-NTU method is definitely preferred.

20-58C The effectiveness of a heat exchanger is defined as the ratio of the actual heat transfer rate to the maximum possible heat transfer rate and represents how closely the heat transfer in the heat exchanger approaches to maximum possible heat transfer. Since the actual heat transfer rate can not be greater than maximum possible heat transfer rate, the effectiveness can not be greater than one. The effectiveness of a heat exchanger depends on the geometry of the heat exchanger as well as the flow arrangement.

20-59C For a specified fluid pair, inlet temperatures and mass flow rates, the counter-flow heat exchanger will have the highest effectiveness.

20-60C Once the effectiveness ε is known, the rate of heat transfer and the outlet temperatures of cold and hot fluids in a heat exchanger are determined from

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,in} - T_{c,in})$$

$$\dot{Q} = \dot{m}_c C_{p,c}(T_{c,out} - T_{c,in})$$

$$\dot{Q} = \dot{m}_h C_{p,h}(T_{h,in} - T_{h,out})$$

20-61C The heat transfer in a heat exchanger will reach its maximum value when the hot fluid is cooled to the inlet temperature of the cold fluid. Therefore, the temperature of the hot fluid can not drop below the inlet temperature of the cold fluid at any location in a heat exchanger.

20-62C The heat transfer in a heat exchanger will reach its maximum value when the cold fluid is heated to the inlet temperature of the hot fluid. Therefore, the temperature of the cold fluid can not rise above the inlet temperature of the hot fluid at any location in a heat exchanger.

20-63C The fluid with the lower mass flow rate will experience a larger temperature change. This is clear from the relation

$$\dot{Q} = \dot{m}_c C_p \Delta T_{cold} = \dot{m}_h C_p \Delta T_{hot}$$

20-64C The maximum possible heat transfer rate is in a heat exchanger is determined from

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in})$$

where $C_{\min}$ is the smaller heat capacity rate. The value of $\dot{Q}_{\max}$ does not depend on the type of heat exchanger.

20-65C The longer heat exchanger is more likely to have a higher effectiveness.

20-66C The increase of effectiveness with NTU is not linear. The effectiveness increases rapidly with NTU for small values (up to about NTU = 1.5), but rather slowly for larger values. Therefore, the effectiveness will not double when the length of heat exchanger is doubled.

20-67C A heat exchanger has the smallest effectiveness value when the heat capacity rates of two fluids are identical. Therefore, reducing the mass flow rate of cold fluid by half will increase its effectiveness.

20-68C When the capacity ratio is equal to zero and the number of transfer units value is greater than 5, a counter-flow heat exchanger has an effectiveness of one. In this case the exit temperature of the fluid with smaller capacity rate will equal to inlet temperature of the other fluid. For a parallel-flow heat exchanger the answer would be the same.

20-69C The NTU of a heat exchanger is defined as $NTU = \dfrac{UA}{C_{\min}} = \dfrac{UA}{(\dot{m}C_p)_{\min}}$ where U is the overall heat transfer coefficient and A is the heat transfer surface area of the heat exchanger. For specified values of U and $C_{\min}$, the value of NTU is a measure of the heat exchanger surface area A. Because the effectiveness increases slowly for larger values of NTU, a large heat exchanger cannot be justified economically. Therefore, a heat exchanger with a very large NTU is not necessarily a good one to buy.

20-70C The value of effectiveness increases slowly with a large values of NTU (usually larger than 3). Therefore, doubling the size of the heat exchanger will not save much energy in this case since the increase in the effectiveness will be very small.

20-71C The value of effectiveness increases rapidly with a small values of NTU (up to about 1.5). Therefore, tripling the NTU will cause a rapid increase in the effectiveness of the heat exchanger, and thus saves energy. I would support this proposal.

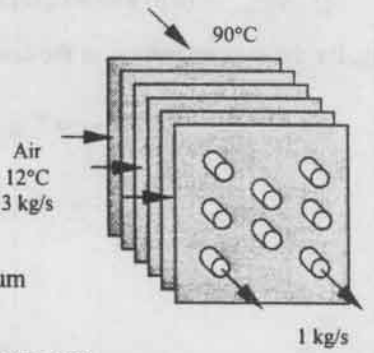
20-72 Air is heated by a hot water stream in a cross-flow heat exchanger. The maximum heat transfer rate and the outlet temperatures of the cold and hot fluid streams are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of water and air are given to be 4.19 and 1.005 kJ/kg.°C.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h C_{ph} = (1\ \text{kg/s})(4190\ \text{J/kg.°C}) = 4190\ \text{W/°C}$$
$$C_c = \dot{m}_c C_{pc} = (3\ \text{kg/s})(1005\ \text{J/kg.°C}) = 3015\ \text{W/°C}$$



Therefore

$$C_{\min} = C_c = 3015\ \text{W/°C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (3015\ \text{W/°C})(90°\text{C} - 12°\text{C}) = \mathbf{235.2\ kW}$$

The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 12°\text{C} + \frac{235.2\ \text{kW}}{3.015\ \text{kW/°C}} = \mathbf{90°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} + \frac{\dot{Q}}{C_h} = 90°\text{C} + \frac{235.2\ \text{kW}}{4.19\ \text{kW/°C}} = \mathbf{33.9°C}$$

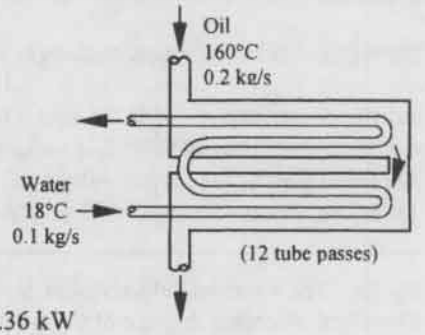
20-73 Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The thickness of the tube is negligible since it is thin-walled. **5** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and oil are given to be 4.18 and 2.2 kJ/kg.°C, respectively.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h C_{ph} = (0.2 \text{ kg/s})(2200 \text{ J/kg.°C}) = 440 \text{ W/°C}$$
$$C_c = \dot{m}_c C_{pc} = (0.1 \text{ kg/s})(4180 \text{ J/kg.°C}) = 418 \text{ W/°C}$$



Therefore, $C_{\min} = C_c = 418 \text{ W/°C}$ and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{418}{440} = 0.95$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (418 \text{ W/°C})(160°\text{C} - 18°\text{C}) = 59.36 \text{ kW}$$

The heat transfer surface area is

$$A = n(\pi DL) = (12)(\pi)(0.018 \text{ m})(3 \text{ m}) = 2.04 \text{ m}^2$$

The NTU of this heat exchanger is

$$NTU = \frac{UA}{C_{\min}} = \frac{(340 \text{ W/m}^2\text{.°C})(2.04 \text{ m}^2)}{418 \text{ W/°C}} = 1.659$$

Then the effectiveness of this heat exchanger corresponding to $C = 0.95$ and NTU = 1.659 is determined from fig. 20-26d to be

$$\varepsilon = 0.61$$

Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.61)(59.36 \text{ kW}) = \mathbf{36.2 \text{ kW}}$$

Finally, the outlet temperatures of the cold and hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 18°\text{C} + \frac{36.2 \text{ kW}}{0.418 \text{ kW/°C}} = \mathbf{104.6°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 160°\text{C} - \frac{36.2 \text{ kW}}{0.44 \text{ kW/°C}} = \mathbf{77.7°C}$$

20-74 Inlet and outlet temperatures of the hot and cold fluids in a double-pipe heat exchanger are given. It is to be determined whether this is a parallel-flow or counter-flow heat exchanger and the effectiveness of it.

Analysis This is a counter-flow heat exchanger because in the parallel-flow heat exchangers the outlet temperature of the cold fluid (55°C in this case) can not exceed the outlet temperature of the hot fluid which is (45°C in this case). Noting that the mass flow rates of both hot and cold oil streams are the same, we have $C_{\min} = C_{\max} = C$. Then the effectiveness of this heat exchanger is determined from

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{C_h(T_{h,in} - T_{h,out})}{C_h(T_{h,in} - T_{c,in})} = \frac{80°\text{C} - 45°\text{C}}{80°\text{C} - 20°\text{C}} = \mathbf{0.583}$$

20-75E Inlet and outlet temperatures of the hot and cold fluids in a double-pipe heat exchanger are given. It is to be determined the fluid which has the smaller heat capacity rate and the effectiveness of the heat exchanger.

Analysis Hot water has the smaller heat capacity rate since it experiences a greater temperature change. The effectiveness of this heat exchanger is determined from

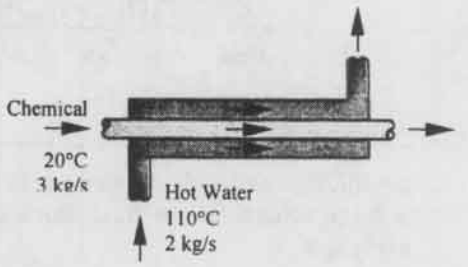
$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{C_h(T_{h,in} - T_{h,out})}{C_h(T_{h,in} - T_{c,in})} = \frac{200°\text{F} - 100°\text{F}}{200°\text{F} - 70°\text{F}} = \mathbf{0.769}$$

20-76 A chemical is heated by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The outlet temperatures of both fluids are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The thickness of the tube is negligible since tube is thin-walled. **5** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and chemical are given to be 4.18 and 1.8 kJ/kg.°C, respectively.

Analysis The heat capacity rates of the hot and cold fluids are



$$C_h = \dot{m}_h C_{ph} = (2 \text{ kg/s})(4.18 \text{ kJ/kg.°C}) = 8.36 \text{ kW/°C}$$
$$C_c = \dot{m}_c C_{pc} = (3 \text{ kg/s})(1.8 \text{ kJ/kg.°C}) = 5.40 \text{ kW/°C}$$

Therefore, $C_{\min} = C_c = 5.4 \text{ kW/°C}$ and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{5.40}{8.36} = 0.646$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (5.4 \text{ kW/°C})(110°\text{C} - 20°\text{C}) = 486 \text{ kW}$$

The NTU of this heat exchanger is

$$NTU = \frac{UA}{C_{\min}} = \frac{(1.2 \text{ kW/m}^2.°\text{C})(7 \text{ m}^2)}{5.4 \text{ kW/°C}} = 1.556$$

Then the effectiveness of this parallel-flow heat exchanger corresponding to $C = 0.646$ and NTU=1.556 is determined from

$$\varepsilon = \frac{1 - \exp[-NTU(1+C)]}{1+C} = \frac{1 - \exp[-1.556(1+0.646)]}{1+0.646} = 0.56$$

Then the actual rate of heat transfer rate becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.56)(486 \text{ kW}) = 272.2 \text{ kW}$$

Finally, the outlet temperatures of the cold and hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 20°\text{C} + \frac{272.2 \text{ kW}}{5.4 \text{ kW/°C}} = \mathbf{70.4°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 110°\text{C} - \frac{272.2 \text{ kW}}{8.36 \text{ kW/°C}} = \mathbf{77.4°C}$$

20-77 Water is heated by hot air in a heat exchanger. The mass flow rates and the inlet temperatures are given. The heat transfer surface area of the heat exchanger on the water side is to be determined.

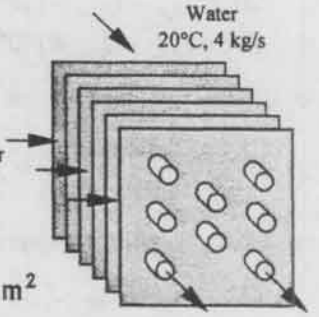
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and air are given to be 4.18 and 1.01kJ/kg.°C, respectively.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h C_{ph} = (4\ \text{kg/s})(4.18\ \text{kJ/kg.°C}) = 16.72\ \text{kW/°C}$$
$$C_c = \dot{m}_c C_{pc} = (9\ \text{kg/s})(1.01\ \text{kJ/kg.°C}) = 9.09\ \text{kW/°C}$$

Therefore, $C_{\min} = C_c = 9.09\ \text{kW/°C}$ and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{9.09}{16.72} = 0.544$



Then the NTU of this heat exchanger corresponding to $C = 0.544$ and $\varepsilon = 0.65$ is determined from Fig. 20-26 to be

$$\text{NTU} = 1.5$$

Then the surface area of this heat exchanger becomes

$$\text{NTU} = \frac{UA}{C_{\min}} \longrightarrow A = \frac{\text{NTU}\, C_{\min}}{U} = \frac{(1.5)(9.09\ \text{kW/°C})}{0.260\ \text{kW/m}^2\text{.°C}} = \mathbf{52.4\ m^2}$$

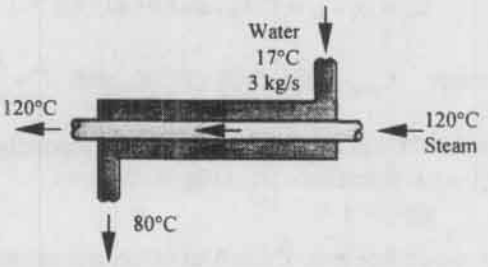
20-78 Water is heated by steam condensing in a condenser. The required length of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heat of the water is given to be 4.18 kJ/kg.°C. The heat of vaporization of water at 120°C is given to be 2203 kJ/kg.

Analysis (a) The temperature differences between the steam and the water at the two ends of the condenser are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 120°C - 80°C = 40°C$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 120°C - 17°C = 103°C$$



The logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{40 - 103}{\ln(40/103)} = 66.6°C$$

The rate of heat transfer is determined from

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) = (3\ \text{kg/s})(4.18\ \text{kJ/kg.°C})(80°C - 17°C) = 790.02\ \text{kW}$$

The surface area of heat transfer is

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{790.02\ \text{kW}}{0.9\ \text{kW/m}^2.°C)(66.6°C)} = 13.18\ \text{m}^2$$

The length of tube required then becomes

$$A = \pi DL \longrightarrow L = \frac{A}{\pi D} = \frac{13.18\ \text{m}^2}{\pi(0.025\ \text{m})} = \mathbf{167.8\ m}$$

(b) The rate of heat transfer is

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) = (3\ \text{kg/s})(4.18\ \text{kJ/kg.°C})(80°C - 17°C) = 790.02\ \text{kW}$$

and the maximum rate of heat transfer rate is

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (12.54\ \text{W/°C})(120°C - 17°C) = 1291.62\ \text{kW}$$

Then the effectiveness of this heat exchanger becomes

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{790.02\ \text{kW}}{1291.62\ \text{kW}} = 0.61$$

The NTU of this heat exchanger is determined using the relation in Table 20-5 to be

$$\text{NTU} = -\ln(1 - \varepsilon) = -\ln(1 - 0.61) = 0.942$$

The surface area is

$$NTU = \frac{UA}{C_{\min}} \longrightarrow A = \frac{NTU\, C_{\min}}{U} = \frac{(0.942)(12.54\ \text{kW/°C})}{0.9\ \text{kW/m}^2.°C} = 13.12\ \text{m}^2$$

Finally, the length of tube required is

$$A = \pi DL \longrightarrow L = \frac{A}{\pi D} = \frac{13.12\ \text{m}^2}{\pi(0.025\ \text{m})} = \mathbf{167\ m}$$

20-79 Ethanol is vaporized by hot oil in a double-pipe parallel-flow heat exchanger. The outlet temperature and the mass flow rate of oil are to be determined using the LMTD and NTU methods.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heat of oil is given to be 2.2 kJ/kg.°C. The heat of vaporization of ethanol at 78°C is given to be 846 kJ/kg.

Analysis (a) The rate of heat transfer is

$$\dot{Q} = \dot{m}h_{fg} = (0.03 \text{ kg/s})(846 \text{ kJ/kg}) = 25.38 \text{ kW}$$

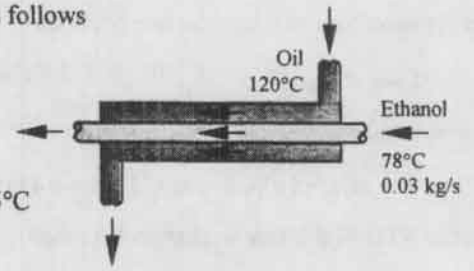
The log mean temperature difference is

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow \Delta T_{lm} = \frac{\dot{Q}}{UA} = \frac{25{,}380 \text{ W}}{(210 \text{ W/m}^2.^\circ\text{C})(7.8 \text{ m}^2)} = 15.5^\circ\text{C}$$

The outlet temperature of the hot fluid can be determined as follows

$$\Delta T_1 = T_{h,in} - T_{c,in} = 120^\circ\text{C} - 78^\circ\text{C} = 42^\circ\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,out} = T_{h,out} - 78^\circ\text{C}$$



and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{42 - (T_{h,out} - 78)}{\ln[42/(T_{h,out} - 78)]} = 15.5^\circ\text{C}$$

by trial-error

$$T_{h,out} = \mathbf{81.5^\circ C}$$

Then the mass flow rate of the hot oil becomes

$$\dot{Q} = \dot{m}C_p(T_{h,in} - T_{h,out}) \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p(T_{h,in} - T_{h,out})} = \frac{25{,}380 \text{ W}}{(2200 \text{ J/kg.}^\circ\text{C})(120^\circ\text{C} - 81.5^\circ\text{C})} = \mathbf{0.3 \text{ kg/s}}$$

(b) The heat capacity rate $C = \dot{m}C_p$ of a fluid condensing or evaporating in a heat exchanger is infinity, and thus $C = C_{\min}/C_{\max} = 0$.

The efficiency in this case is determined from

$$\varepsilon = 1 - e^{-NTU}$$

where $NTU = \dfrac{UA}{C_{\min}} = \dfrac{(210 \text{ W/m}^2.^\circ\text{C})(7.8 \text{ m}^2)}{(\dot{m}, \text{ kg/s})(2200 \text{ J/kg.}^\circ\text{C})}$

and $\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in})$

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{C_{\min}(T_{h,in} - T_{c,in})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{120 - T_{h,out}}{120 - 78}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) = 25.38 \text{ kW}$$

$$\dot{Q} = \dot{m} \times 2200(120 - T_{h,out}) = 25.38 \text{ kW} \quad (1)$$

Also

$$\frac{120 - T_{h,out}}{120 - 78} = 1 - e^{-\frac{7.8 \times 210}{\dot{m} \times 2200}} \quad (2)$$

Solving (1) and (2) simultaneously gives

$$\dot{m}_h = \mathbf{0.3 \text{ kg/s}} \text{ and } T_{h,out} = \mathbf{81.5^\circ C}$$

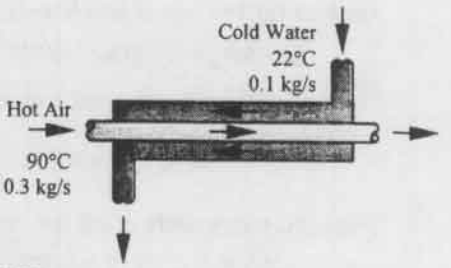
20-80 Water is heated by solar-heated hot air in a heat exchanger. The mass flow rates and the inlet temperatures are given. The outlet temperatures of the water and the air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and air are given to be 4.18 and 1.01 kJ/kg.°C, respectively.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h C_{ph} = (0.3\text{kg/s})(1010\text{ J/kg.°C}) = 303\text{ W/°C}$$
$$C_c = \dot{m}_c C_{pc} = (0.1\text{ kg/s})(4180\text{ J/kg.°C}) = 418\text{ W/°C}$$



Therefore, $C_{\min} = C_c = 303\text{ W/°C}$ and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{303}{418} = 0.725$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (303\text{ W/°C})(90°\text{C} - 22°\text{C}) = 20{,}604\text{ kW}$$

The heat transfer surface area is

$$A = \pi DL = (\pi)(0.012\text{ m})(12\text{ m}) = 0.45\text{ m}^2$$

Then the NTU of this heat exchanger becomes

$$NTU = \frac{UA}{C_{\min}} = \frac{(80\text{ W/m}^2\text{.°C})(0.45\text{ m}^2)}{303\text{ W/°C}} = 0.119$$

The effectiveness of this counter-flow heat exchanger corresponding to C = 0.725 and NTU = 0.119 is determined using the relation in Table 20-5 to be

$$\varepsilon = \frac{1-\exp[-NTU(1-C)]}{1-C\exp[-NTU(1-C)]} = \frac{1-\exp[-0.119(1-0.725)]}{1-0.725\exp[-0.119(1-0.725)]} = 0.108$$

Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = (0.108)(20{,}604\text{ W}) = 2225.2\text{ W}$$

Finally, the outlet temperatures of the cold and hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 22°\text{C} + \frac{2225.2\text{ W}}{418\text{ W/°C}} = \mathbf{27.3°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 90°\text{C} - \frac{2225.2\text{ W}}{303\text{ W/°C}} = \mathbf{82.7°C}$$

20-81E Oil is cooled by water in a double-pipe heat exchanger. The overall heat transfer coefficient of this heat exchanger is to be determined using both the LMTD and NTU methods.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The thickness of the tube is negligible since it is thin-walled.

Properties The specific heats of the water and oil are given to be 1.0 and 0.525 Btu/lbm.°F, respectively.

Analysis (a) The rate of heat transfer is

$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,in} - T_{h,out}) = (5\ \text{lbm/s})(0.525\ \text{Btu/lbm.°F})(300 - 105°\text{F}) = 511.9\ \text{Btu/s}$$

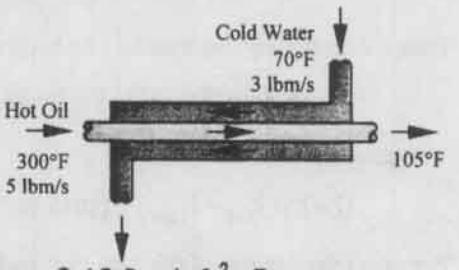
The outlet temperature of the cold fluid is

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}_c C_{pc}} = 70°\text{F} + \frac{511.9\ \text{Btu/s}}{(3\ \text{lbm/s})(1.0\ \text{Btu/lbm.°F})} = 240.6°\text{F}$$

The temperature differences between the two fluids at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 300°\text{F} - 240.6°\text{F} = 59.4°\text{F}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 105°\text{F} - 70°\text{F} = 35°\text{F}$$



The logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{59.4 - 35}{\ln(59.4/35)} = 46.1°\text{F}$$

Then the overall heat transfer coefficient becomes

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow U = \frac{\dot{Q}}{A\Delta T_{lm}} = \frac{511.9\ \text{Btu/s}}{\pi(1/12\ \text{m})(20\ \text{ft})(46.1°\text{F})} = \mathbf{2.12\ Btu/s.ft^2.°F}$$

(b) The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h C_{ph} = (5\ \text{lbm/s})(0.525\ \text{Btu/lbm.°F}) = 2.625\ \text{Btu/s.°F}$$

$$C_c = \dot{m}_c C_{pc} = (3\ \text{lbm/s})(1.0\ \text{Btu/lbm.°F}) = 3.0\ \text{Btu/s.°F}$$

Therefore, $C_{\min} = C_h = 2.625\ \text{Btu/s.°F}$ and $C = \frac{C_{\min}}{C_{\max}} = \frac{2.625}{3.0} = 0.875$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (2.625\ \text{Btu/s.°F})(300°\text{F} - 70°\text{F}) = 603.75\ \text{Btu/s}$$

The actual rate of heat transfer and the effectiveness are

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) = (2.625\ \text{Btu/s.°F})(300°\text{F} - 105°\text{F}) = 511.9\ \text{Btu/s}$$

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{511.9}{603.75} = 0.85$$

The NTU of this heat exchanger is determined using the relation in Table 20-3 to be

$$NTU = \frac{1}{C-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C - 1}\right) = \frac{1}{0.875 - 1}\ln\left(\frac{0.85 - 1}{0.85 \times 0.875 - 1}\right) = 4.28$$

The heat transfer surface area of the heat exchanger is

$$A = \pi DL = \pi(1/12\ \text{ft})(20\ \text{ft}) = 5.24\ \text{ft}^2$$

and

$$NTU = \frac{UA}{C_{\min}} \longrightarrow U = \frac{NTU\ C_{\min}}{A} = \frac{(4.28)(2.625\ \text{Btu/s.°F})}{5.24\ \text{ft}^2} = \mathbf{2.14\ Btu/s.ft^2.°F}$$

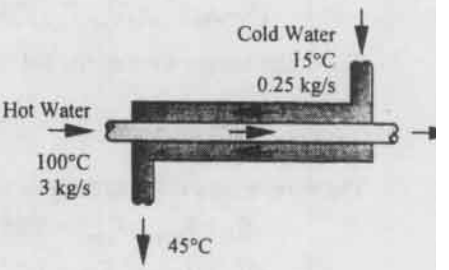
20-82 Cold water is heated by hot water in a heat exchanger. The net rate of heat transfer and the heat transfer surface area of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform. **5** The thickness of the tube is negligible.

Properties The specific heats of the cold and hot water are given to be 4.18 and 4.19 kJ/kg.°C, respectively.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h C_{ph} = (0.25 \text{ kg/s})(4180 \text{ J/kg.°C}) = 1045 \text{ W/°C}$$
$$C_c = \dot{m}_c C_{pc} = (3 \text{ kg/s})(4190 \text{ J/kg.°C}) = 12{,}570 \text{ W/°C}$$



Therefore, $C_{\min} = C_c = 1045 \text{ W/°C}$ and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{1045}{12{,}570} = 0.083$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (1045 \text{ W/°C})(100°\text{C} - 15°\text{C}) = 88{,}825 \text{ W}$$

The actual rate of heat transfer is

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) = (1045 \text{ W/°C})(45°\text{C} - 15°\text{C}) = \mathbf{31{,}350 \text{ W}}$$

Then the effectiveness of this heat exchanger becomes

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{31{,}350}{88{,}825} = 0.35$$

The NTU of this heat exchanger is determined using the relation in Table 20-5 to be

$$NTU = \frac{1}{C-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C - 1}\right) = \frac{1}{0.083 - 1}\ln\left(\frac{0.35 - 1}{0.35 \times 0.083 - 1}\right) = 0.438$$

Then the surface area of the heat exchanger is determined from

$$NTU = \frac{UA}{C_{\min}} \longrightarrow A = \frac{NTU\, C_{\min}}{U} = \frac{(0.438)(1045 \text{ W/°C})}{950 \text{ W/m}^2\text{.°C}} = \mathbf{0.482 \text{ m}^2}$$

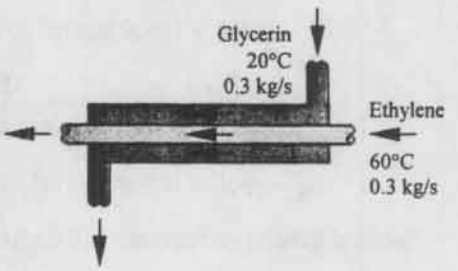
20-83 Glycerin is heated by ethylene glycol in a heat exchanger. Mass flow rates and inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform. **5** The thickness of the tube is negligible.

Properties The specific heats of the glycerin and ethylene glycol are given to be 2.4 and 2.5 kJ/kg.°C, respectively.

Analysis (a) The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h C_{ph} = (0.3 \text{ kg/s})(2400 \text{ J/kg.°C}) = 720 \text{ W/°C}$$
$$C_c = \dot{m}_c C_{pc} = (0.3 \text{ kg/s})(2500 \text{ J/kg.°C}) = 750 \text{ W/°C}$$



Therefore, $C_{\min} = C_h = 720 \text{ W/°C}$ and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{720}{750} = 0.96$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (720 \text{ W/°C})(60°\text{C} - 20°\text{C}) = 28.8 \text{ kW}$$

The NTU of this heat exchanger is

$$NTU = \frac{UA}{C_{\min}} = \frac{(240 \text{ W/m}^2.°\text{C})\,(7.6 \text{ m}^2)}{720 \text{ W/°C}} = 2.53$$

Effectiveness of this heat exchanger corresponding to C = 0.96 and NTU = 2.53 is determined using the proper relation in Table 20-4

$$\varepsilon = \frac{1 - \exp[-NTU(1+C)]}{1+C} = \frac{1 - \exp[-2.53(1+0.96)]}{1+0.96} = 0.5$$

Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.5)(28.8 \text{ kW}) = \mathbf{14.4 \text{ kW}}$$

(b) Finally, the outlet temperatures of the cold and the hot fluid streams are determined from

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 20°\text{C} + \frac{14.4 \text{ kW}}{0.72 \text{ kW/°C}} = \mathbf{40°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 60°\text{C} - \frac{14.4 \text{ kW}}{0.75 \text{ kW/°C}} = \mathbf{40.8°C}$$

20-84 Water is heated by hot air in a cross-flow heat exchanger. Mass flow rates and inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform. **5** The thickness of the tube is negligible.

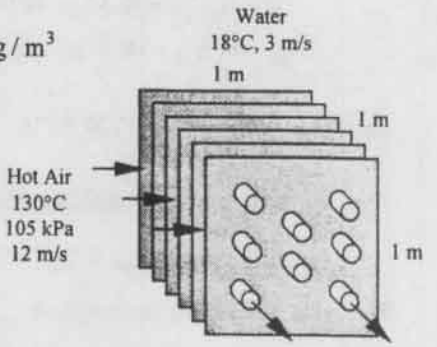
Properties The specific heats of the water and air are given to be 4.18 and 1.01 kJ/kg.°C, respectively.

Analysis The mass flow rates of the hot and the cold fluids are

$$\dot{m}_c = \rho V A_c = (1000 \text{ kg/m}^3)(3 \text{ m/s})[40\pi(0.01 \text{ m})^2/4] = 9.425 \text{ kg/s}$$

$$\rho_{air} = \frac{P}{RT} = \frac{105 \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg.K}) \times (130+273 \text{ K})} = 0.908 \text{ kg/m}^3$$

$$\dot{m}_h = \rho V A_c = (0.908 \text{ kg/m}^3)(12 \text{ m/s})(1 \text{ m})^2 = 10.90 \text{ kg/s}$$



The heat transfer surface area and the heat capacity rates are

$$A = n\pi DL = 80\pi(0.01 \text{ m})(1 \text{ m}) = 2.513 \text{ m}^2$$

$$C_h = \dot{m}_h C_{ph} = (9.425 \text{ kg/s})(4.18 \text{ kJ/kg.°C}) = 39.4 \text{ kW/°C}$$

$$C_c = \dot{m}_c C_{pc} = (10.9 \text{ kg/s})(1.010 \text{ kJ/kg.°C}) = 11.01 \text{ kW/°C}$$

Therefore, $C_{\min} = C_c = 11.01 \text{ kW/°C}$ and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{11.01}{39.40} = 0.2794$

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (11.01 \text{ kW/°C})(30°\text{C} - 18°\text{C}) = 1233 \text{ kW}$$

The NTU of this heat exchanger is

$$NTU = \frac{UA}{C_{\min}} = \frac{(80 \text{ W/m}^2.°\text{C})(2.513 \text{ m}^2)}{11{,}010 \text{ W/°C}} = 0.01826$$

Noting that this heat exchanger involves mixed cross-flow, the fluid with $C_{\min}$ is mixed, $C_{\max}$ unmixed, effectiveness of this heat exchanger corresponding to C = 0.2794 and NTU =0.01826 is determined using the proper relation in Table 20-4 to be

$$\varepsilon = 1 - \exp\left[-\frac{1}{C}(1 - e^{-CNTU})\right] = 1 - \exp\left[-\frac{1}{0.2794}(1 - e^{-0.2794 \times 0.01826})\right] = 0.01805$$

Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.01805)(1233 \text{ kW}) = \mathbf{22.26 \text{ kW}}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined from

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 18°\text{C} + \frac{22.26 \text{ kW}}{39.40 \text{ kW/°C}} = \mathbf{18.6°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 130°\text{C} - \frac{22.26 \text{ kW}}{11.01 \text{ kW/°C}} = \mathbf{128.0°C}$$

20-85 Ethyl alcohol is heated by water in a shell-and-tube heat exchanger. The heat transfer surface area of the heat exchanger is to be determined using both the LMTD and NTU methods.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the ethyl alcohol and water are given to be 2.67 and 4.19 kJ/kg.°C, respectively.

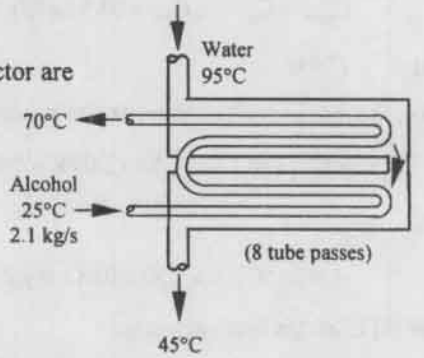
Analysis (a) The temperature differences between the two fluids at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 95°C - 70°C = 25°C$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 45°C - 25°C = 20°C$$

The logarithmic mean temperature difference and the correction factor are

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{25-20}{\ln(25/20)} = 22.4°C$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{70-25}{95-25} = 0.64 \\ R &= \frac{T_2 - T_1}{t_1 - t_1} = \frac{95-45}{70-25} = 1.11 \end{aligned}\right\} F = 0.81$$



The rate of heat transfer is determined from

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) = (2.1\ \text{kg/s})(2.67\ \text{kJ/kg.°C})(70°C - 25°C) = 252.3\ \text{kW}$$

The surface area of heat transfer is

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow A = \frac{\dot{Q}}{UF\Delta T_{lm}} = \frac{252.3\ \text{kW}}{0.8\ \text{kW/m}^2.°C)(0.81)(22.4°C)} = \mathbf{17.4\ m^2}$$

(b) The rate of heat transfer is

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) = (2.1\ \text{kg/s})(2.67\ \text{kJ/kg.°C})(70°C - 25°C) = 252.3\ \text{kW}$$

The mass flow rate of the hot fluid is

$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,in} - T_{h,out}) \rightarrow \dot{m}_h = \frac{\dot{Q}}{C_{ph}(T_{h,in} - T_{h,out})} = \frac{252.3\ \text{kW}}{(4.19\ \text{kJ/kg.°C})(95°C - 45°C)} = 1.2\ \text{kg/s}$$

The heat capacity rates of the hot and the cold fluids are

$$C_h = \dot{m}_h C_{ph} = (1.2\ \text{kg/s})(4.19\ \text{kJ/kg.°C}) = 5.03\ \text{kW/°C}$$
$$C_c = \dot{m}_c C_{pc} = (2.1\ \text{kg/s})(2.67\ \text{kJ/kg.°C}) = 5.6\ \text{kW/°C}$$

Therefore, $C_{\min} = C_h = 5.03$ W/°C and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{5.03}{5.6} = 0.9$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (5.03\ \text{W/°C})(95°C - 25°C) = 352.1\ \text{kW}$$

The effectiveness of this heat exchanger is

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{252.3}{352.1} = 0.72$$

The NTU of this heat exchanger is determined from Fig. 20-26d to be NTU = 7. Then the surface area of heat exchanger is determined to be

$$NTU = \frac{UA}{C_{\min}} \longrightarrow A = \frac{NTU\, C_{\min}}{U} = \frac{(7)(5.03\ \text{kW/°C})}{0.8\ \text{kW/m}^2.°C} = \mathbf{44.0\ m^2}$$

The small difference between the two results is due to the reading error of the chart.

20-86 Steam is condensed by cooling water in a shell-and-tube heat exchanger. The rate of heat transfer and the rate of condensation of steam are to be determined.

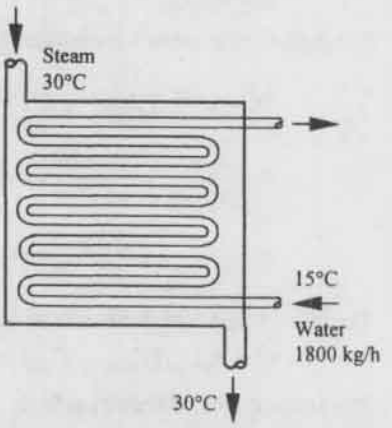
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform. **5** The thickness of the tube is negligible.

Properties The specific heat of the water is given to be 4.18 kJ/kg.°C. The heat of condensation of steam at 30°C is given to be 2430 kJ/kg.

Analysis (a) The heat capacity rate of a fluid condensing in a heat exchanger is infinity. Therefore,

$$C_{\min} = C_c = \dot{m}_c C_{pc} = (0.5 \text{ kg/s})(4.18 \text{ kJ/kg.°C}) = 2.09 \text{ kW/°C}$$

and $C = 0$



Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (2.09 \text{ kW/°C})(30°\text{C} - 15°\text{C}) = 31.35 \text{ kW}$$

and

$$A = 8n\pi DL = 8 \times 50\pi(0.015 \text{ m})(2 \text{ m}) = 37.7 \text{ m}^2$$

The NTU of this heat exchanger

$$NTU = \frac{UA}{C_{\min}} = \frac{(3 \text{ kW/m}^2.\text{°C})(37.7 \text{ m}^2)}{2.09 \text{ kW/°C}} = 54.11$$

Then the effectiveness of this heat exchanger corresponding to C = 0 and NTU = 6.76 is determined using the proper relation in Table 20-5

$$\varepsilon = 1 - \exp(-\text{NTU}) = 1 - \exp(-6.76) = 1$$

Then the actual heat transfer rate becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (1)(31.35 \text{ kW}) = \mathbf{31.35 \text{ kW}}$$

(b) Finally, the rate of condensation of the steam is determined from

$$\dot{Q} = \dot{m} h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{31.4 \text{ kJ/s}}{2430 \text{ kJ/kg}} = \mathbf{0.0129 \text{ kg/s}}$$

20-87 Cold water is heated by hot oil in a shell-and-tube heat exchanger. The rate of heat transfer is to be determined using both the LMTD and NTU methods.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and oil are given to be 4.18 and 2.2 kJ/kg.°C, respectively.

Analysis (a) The LMTD method in this case involves iterations, which involves the following steps:

1) Choose $T_{h,out}$

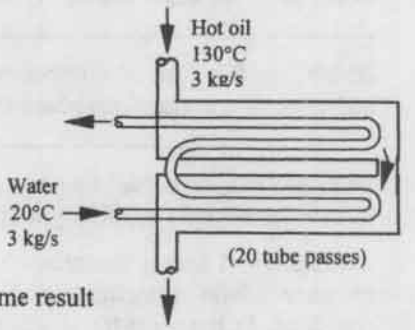
2) Calculate $\dot{Q}$ from $\dot{Q} = \dot{m}_h C_p (T_{h,out} - T_{h,in})$

3) Calculate $T_{h,out}$ from $\dot{Q} = \dot{m}_h C_p (T_{h,out} - T_{h,in})$

4) Calculate $\Delta T_{\ln,CF}$

5) Calculate $\dot{Q}$ from $\dot{Q} UAF \Delta T_{\ln,CF}$

6) Compare to the $\dot{Q}$ calculated at step 2, and repeat until reaching the same result



Result: **385 kW**

(b) The heat capacity rates of the hot and the cold fluids are

$$C_h = \dot{m}_h C_{ph} = (3 \text{ kg/s})(2.2 \text{ kJ/kg.°C}) = 6.6 \text{ kW/°C}$$
$$C_c = \dot{m}_c C_{pc} = (3 \text{ kg/s})(4.18 \text{ kJ/kg.°C}) = 12.54 \text{ kW/°C}$$

Therefore, $C_{\min} = C_h = 6.6 \text{ kW/°C}$ and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{6.6}{12.54} = 0.53$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (6.6 \text{ kW/°C})(130°\text{C} - 20°\text{C}) = 726 \text{ kW}$$

The NTU of this heat exchanger is

$$NTU = \frac{UA}{C_{\min}} = \frac{(0.3 \text{ kW/m}^2.°\text{C})\,(20 \text{ m}^2)}{6.6 \text{ kW/°C}} = 0.91$$

Then the effectiveness of this heat exchanger corresponding to C = 0.53 and NTU = 0.91 is determined from Fig. 20-26d to be

$$\varepsilon = 0.53$$

The actual rate of heat transfer then becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.53)(726 \text{ kW}) = \mathbf{385 \text{ kW}}$$

Selection Of The Heat Exchangers

20-88C 1) Calculate heat transfer rate, 2) select a suitable type of heat exchanger, 3) select a suitable type of cooling fluid, and its temperature range, 4) calculate or select U, and 5) calculate the size (surface area) of heat exchanger

20-89C The first thing we need to do is determine the life expectancy of the system. Then we need to evaluate how much the larger will save in pumping cost, and compare it to the initial cost difference of the two units. If the larger system saves more than the cost difference in its lifetime, it should be preferred.

20-90C In the case of automotive and aerospace industry, where weight and size considerations are important, and in situations where the space availability is limited, we choose the smaller heat exchanger.

20-91 Oil is to be cooled by water in a heat exchanger. The heat transfer rating of the heat exchanger is to be determined and a suitable type is to be proposed.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the oil is given to be 2.2 kJ/kg.°C.

Analysis The heat transfer rate of this heat exchanger is

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) = (20\ \text{kg/s})(2.2\ \text{kJ/kg.°C})(120°\text{C} - 50°\text{C}) = \mathbf{3080\ kW}$$

We propose a compact heat exchanger (like the car radiator) if air cooling is to be used., or a tube-and-shell or plate heat exchanger if water cooling is to be used.

20-92 Water is to be heated by steam in a shell-and-tube process heater. The number of tube passes need to be used is to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the water is given to be 4.19 kJ/kg.°C.

Analysis The mass flow rate of the water is

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) \longrightarrow \dot{m} = \frac{\dot{Q}}{C_{pc}(T_{c,out} - T_{c,in})} = \frac{600\ \text{kW}}{(4.19\ \text{kJ/kg.°C})(90°\text{C} - 20°\text{C})} = 2.046\ \text{kg/s}$$

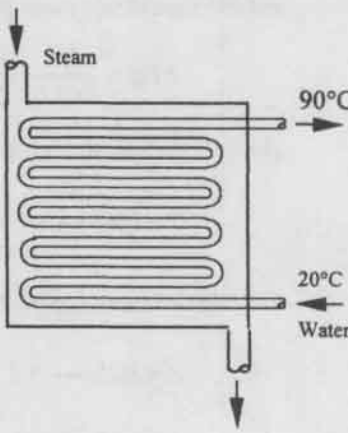
The total cross-section area of the tubes corresponding to this mass flow rate is

$$\dot{m} = \rho V A_c \rightarrow A_c = \frac{\dot{m}}{\rho V} = \frac{2.046\ \text{kg/s}}{(1000\ \text{kg/m}^3)(3\ \text{m/s})} = 6.82 \times 10^{-4}\ \text{m}^2$$

Then the number of tubes that need to be used becomes

$$A = n\frac{\pi D^2}{4} \longrightarrow n = \frac{4A}{\pi D^2} = \frac{4(6.82 \times 10^{-4}\ \text{m}^2)}{\pi(0.01\ \text{m})^2} = 8.68 \cong \mathbf{9}$$

Therefore, we need to use at least 9 tubes entering the heat exchanger.



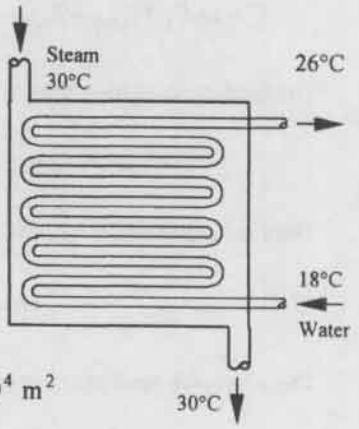
20-93 Cooling water is used to condense the steam in a power plant. The total length of the tubes required in the condenser is to be determined and a suitable HX type is to be proposed.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heat of the water is given to be 4.18 kJ/kg.°C. The heat of condensation of steam at 30°C is given to be 2430 kJ/kg.

Analysis The temperature differences between the steam and the water at the two ends of condenser are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 30°C - 26°C = 4°C$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 30°C - 18°C = 12°C$$



and the logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{4-12}{\ln(4/12)} = 7.28°C$$

The heat transfer surface area is

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{500\times10^6\ \text{W}}{(3500\ \text{W/m}^2.°C)(7.28°C)} = 1.96\times10^4\ \text{m}^2$$

The total length of the tubes required in this condenser then becomes

$$A = \pi DL \longrightarrow L = \frac{A}{\pi D} = \frac{1.96\times10^4\ \text{m}^2}{\pi(0.02\ \text{m})} = 3.12\times10^5\ m = \mathbf{312\ km}$$

A multi-pass shell-and-tube heat exchanger is suitable in this case.

20-94 Cold water is heated by hot water in a heat exchanger. The net rate of heat transfer and the heat transfer surface area of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the cold and hot water are given to be 4.18 and 4.19 kJ/kg.°C, respectively.

Analysis The temperature differences between the steam and the water at the two ends of condenser are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 30°C - 26°C = 4°C$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 30°C - 18°C = 12°C$$

and the logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{4-12}{\ln(4/12)} = 7.28°C$$

The heat transfer surface area is

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{300\times10^6\ \text{W}}{(3500\ \text{W/m}^2.°\text{C})(7.28°\text{C})} = 1.177\times10^4\ \text{m}^2$$

The total length of the tubes required in this condenser then becomes

$$A = \pi DL \longrightarrow L = \frac{A}{\pi D} = \frac{1.177\times10^3\ \text{m}^2}{\pi(0.02\ \text{m})} = 1.87\times10^5\ m = \mathbf{187\ km}$$

A multi-pass shell-and-tube heat exchanger is suitable in this case.

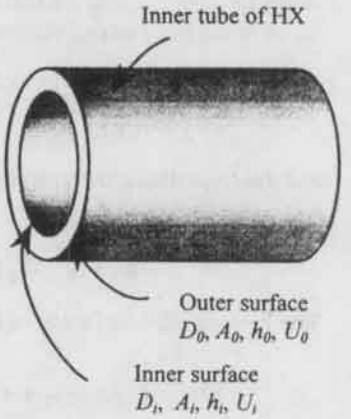
Review Problems

20-95 Hot oil is cooled by water in a multi-pass shell-and-tube heat exchanger. The overall heat transfer coefficient based on the inner surface is to be determined.

Assumptions **1** Water flow is fully developed. **2** Properties of the water are constant.

Properties The properties of water at 300 K ≈25°C are (Table A-15)

$$k = 0.607 \text{ W/m.°C}$$
$$\upsilon = \mu/\rho = 0.894\times10^{-6} \text{ m}^2/\text{s}$$
$$\text{Pr} = 6.14$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_m D}{\upsilon} = \frac{(3 \text{ m/s})(0.013 \text{ m})}{0.894\times10^{-6} \text{ m}^2/\text{s}} = 43{,}771$$

which is greater than 2300. Therefore, we assume fully developed turbulent flow, and determine Nusselt number from

$$Nu = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4}\;0.023(43{,}771)^{0.8}(6.14)^{0.4} = 245$$

and

$$h_i = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.013 \text{ m}}(245) = 11{,}440 \text{ W/m}^2\text{.°C}$$

The inner and the outer surface areas of the tube are

$$A_i = \pi D_i L = \pi(0.013 \text{ m})(1 \text{ m}) = 0.04084 \text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.015 \text{ m})(1 \text{ m}) = 0.04712 \text{ m}^2$$

The total thermal resistance of this heat exchanger per unit length is

$$R = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o}$$
$$= \frac{1}{(11{,}440 \text{ W/m}^2\text{.°C})(0.04084 \text{ m}^2)} + \frac{\ln(1.5/1.3)}{2\pi(110 \text{ W/m.°C})(1 \text{ m})} + \frac{1}{(35 \text{ W/m}^2\text{.°C})(0.04712 \text{ m}^2)}$$
$$= 0.609\text{°C/W}$$

Then the overall heat transfer coefficient of this heat exchanger based on the inner surface becomes

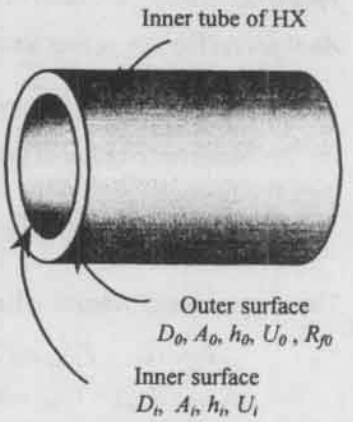
$$R = \frac{1}{U_i A_i} \longrightarrow U_i = \frac{1}{RA_i} = \frac{1}{(0.609\text{°C/W})(0.04084 \text{ m}^2)} = \mathbf{40.2 \text{ W/m}^2\text{.°C}}$$

20-96 Hot oil is cooled by water in a multi-pass shell-and-tube heat exchanger. The overall heat transfer coefficient based on the inner surface is to be determined.

Assumptions **1** Water flow is fully developed. **2** Properties of the water are constant.

Properties The properties of water at 300 K ≈25°C are (Table A-15)

$$k = 0.607 \text{ W/m.°C}$$
$$\upsilon = \mu/\rho = 0.894 \times 10^{-6} \text{ m}^2/\text{s}$$
$$\text{Pr} = 6.14$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_m D}{\upsilon} = \frac{(3 \text{ m/s})(0.013 \text{ m})}{0.894 \times 10^{-6} \text{ m}^2/\text{s}} = 43{,}771$$

which is greater than 2300. Therefore, we assume fully developed turbulent flow, and determine Nusselt number from

$$Nu = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} \; 0.023(43{,}771)^{0.8}(6.14)^{0.4} = 245$$

and

$$h_i = \frac{k}{D} Nu = \frac{0.607 \text{ W/m.°C}}{0.013 \text{ m}}(245) = 11{,}440 \text{ W/m}^2\text{.°C}$$

The inner and the outer surface areas of the tube are

$$A_i = \pi D_i L = \pi(0.013 \text{ m})(1 \text{ m}) = 0.04084 \text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.015 \text{ m})(1 \text{ m}) = 0.04712 \text{ m}^2$$

The total thermal resistance of this heat exchanger per unit length of it with a fouling factor is

$$R = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$
$$= \frac{1}{(11{,}440 \text{ W/m}^2\text{.°C})(0.04084 \text{ m}^2)} + \frac{\ln(15/13)}{2\pi(110 \text{ W/m.°C})(1 \text{ m})}$$
$$+ \frac{0.0004 \text{ m}^2\text{.°C/W}}{0.04712 \text{ m}^2} + \frac{1}{(35 \text{ W/m}^2\text{.°C})(0.04712 \text{ m}^2)}$$
$$= 0.617°\text{C/W}$$

Then the overall heat transfer coefficient of this heat exchanger based on the inner surface becomes

$$R = \frac{1}{U_i A_i} \longrightarrow U_i = \frac{1}{R A_i} = \frac{1}{(0.617°\text{C/W})(0.04084 \text{ m}^2)} = \mathbf{39.7 \text{ W/m}^2\text{.°C}}$$

20-97 Water is heated by hot oil in a multi-pass shell-and-tube heat exchanger. The rate of heat transfer and the heat transfer surface area on the outer side of the tube are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and oil are given to be 4.18 and 2.2 kJ/kg.°C, respectively.

Analysis (a)The rate of heat transfer in this heat exchanger is

$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,in} - T_{h,out}) = (3\ \text{kg/s})(2.2\ \text{kJ/kg.°C})(130°\text{C} - 60°\text{C}) = \mathbf{462\ kW}$$

(b) The outlet temperature of the cold water is

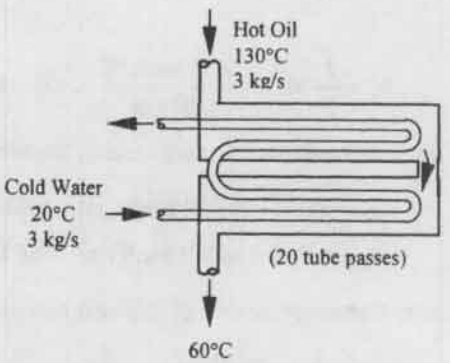
$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}_c C_{pc}} = 20°\text{C} + \frac{462\ \text{kW}}{(3\ \text{kg/s})(4.18\ \text{kJ/kg.°C})} = 56.8°\text{C}$$

The temperature difference at the two ends are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 130°\text{C} - 56.8°\text{C} = 73.2°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 60°\text{C} - 20°\text{C} = 40°\text{C}$$

The logarithmic mean temperature difference is

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{73.2 - 40}{\ln(73.2/40)} = 54.9°\text{C}$$



and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{56.8 - 20}{130 - 20} = 0.335 \\ R &= \frac{T_2 - T_1}{t_2 - t_1} = \frac{130 - 60}{56.8 - 20} = 1.90 \end{aligned}\right\} F = 0.96$$

The heat transfer surface area on the outer side of the tube is then determined from

$$\dot{Q} = UAF\Delta T_{lm} \longrightarrow A = \frac{\dot{Q}}{UF\Delta T_{lm}} = \frac{462\ \text{kW}}{(0.3\ \text{kW/m}^2.°\text{C})(0.96)(54.9°\text{C})} = \mathbf{29.2\ m^2}$$

20-98E Water is heated by solar-heated hot air in a double-pipe counter-flow heat exchanger. The required length of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and air are given to be 1.0 and 0.24 Btu/lbm.°F, respectively.

Analysis The rate of heat transfer in this heat exchanger is

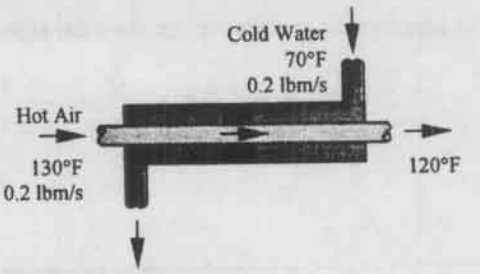
$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,in} - T_{h,out}) = (0.7 \text{ lbm/s})(0.24 \text{ Btu/lbm.°F})(190°\text{F} - 120°\text{F}) = 11.76 \text{ Btu/s}$$

The outlet temperature of the cold water is

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}_c C_{pc}} = 70°\text{F} + \frac{11.76 \text{ Btu/s}}{(0.2 \text{ lbm/s})(1.0 \text{ Btu/lbm.°F})} = 128.8°\text{F}$$

The temperature difference at the two ends are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 190°\text{F} - 128.8°\text{F} = 61.2°\text{F}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 120°\text{F} - 70°\text{F} = 50°\text{F}$$



The logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{61.2 - 50}{\ln(61.2/50)} = 55.4°\text{F}$$

The heat transfer surface area on the outer side of the tube is determined from

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{11.76 \text{ Btu/s}}{(20/3600 \text{ Btu/s.ft}^2\text{.°F})(55.4°\text{F})} = 38.21 \text{ ft}^2$$

Then the length of the tube required becomes

$$A = \pi DL \longrightarrow L = \frac{A}{\pi D} = \frac{38.21 \text{ ft}^2}{\pi(0.5/12 \text{ ft})} = \mathbf{291.9 \text{ ft}}$$

20-99 It is to be shown that when $\Delta T_1 = \Delta T_2$ for a heat exchanger, the ΔT_{lm} relation reduces to $\Delta T_{lm} = \Delta T_1 = \Delta T_2$.

Analysis When $\Delta T_1 = \Delta T_2$ we obtain

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{0}{0}$$

This case can be handled by applying L'Hospital's rule (taking derivatives of nominator and denominator separately with respect to ΔT_1 or ΔT_2).

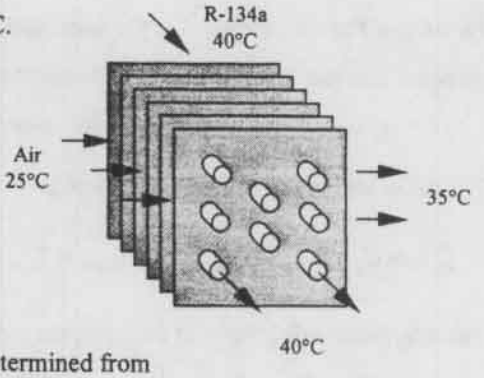
That is,

$$\Delta T_{lm} = \frac{d(\Delta T_1 - \Delta T_2)/d\Delta T_1}{d[\ln(\Delta T_1 / \Delta T_2)]/d\Delta T_1} = \frac{1}{1/\Delta T_1} = \Delta T_1 = \Delta T_2$$

20-100 Refrigerant-134a is condensed by air in the condenser of a room air conditioner. The heat transfer area on the refrigerant side is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heat of air is given to be 1.005 kJ/kg.°C.



Analysis The temperature difference at the two ends are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 40°C - 35°C = 5°C$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 40°C - 25°C = 15°C$$

The logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{5-15}{\ln(5/15)} = 9.1°C$$

The heat transfer surface area on the outer side of the tube is determined from

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{(15{,}000/3600)\ \text{kW}}{(0.150\ \text{kW/m}^2.°C)(9.1°C)} = \mathbf{3.05\ m^2}$$

20-101 Air is preheated by hot exhaust gases in a cross-flow heat exchanger. The rate of heat transfer is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of air and combustion gases are given to be 1.005 and 1.1 kJ/kg.°C, respectively.

Analysis The rate of heat transfer is simply

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{\text{gas}} = (1.1\ \text{kg/s})(1.1\ \text{kJ/kg.°C})(180°C - 95°C) = \mathbf{102.9\ kW}$$

20-102 A water-to-water heat exchanger is proposed to preheat the incoming cold water by the drained hot water in a plant to save energy. The heat transfer rating of the heat exchanger and the amount of money this heat exchanger will save are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the hot water is given to be 4.18 kJ/kg.°C.

Analysis The maximum rate of heat transfer is

$$\dot{Q}_{\max} = \dot{m}_h C_{ph}(T_{h,in} - T_{c,in}) = (8/60\ \text{kg/s})(4.18\ \text{kJ/kg.°C})(60°\text{C} - 14°\text{C}) = 25.6\ \text{kW}$$

Noting that the heat exchanger will recover 72% of it, the actual heat transfer rate becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.72)(25.6\ \text{kJ/s}) = 18.43\ \text{kW}$$

which is the heat transfer rating. The operating hours per year are

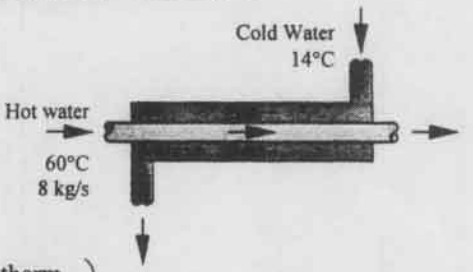
The annual operating hours = (8 h/day)(5 days/week)(52 week/year) = 2080 h/year

The energy saved during the entire year will be

Energy saved = (heat transfer rate)(operating time)

= (18.43 kJ/s)(2080 h/year)(3600 s/h)

= 1.38×10^8 kJ/year



Then amount of fuel and money saved will be

$$\text{Fuel saved} = \frac{\text{Energy saved}}{\text{Furnace efficiency}} = \frac{1.38\times10^8\ \text{kJ/year}}{0.78}\left(\frac{1\ \text{therm}}{105{,}500\ \text{kJ}}\right)$$

$$= 1677\ \text{therms/year}$$

Money saved = (fuel saved)(the price of fuel)

= (1677 therms/year)($ 0.54/therm) = **$906/year**

Chapter 21
HEATING AND COOLING OF BUILDINGS

A Brief History

21-1C Ice can be made by evacuating the air in a water tank. During evacuation, vapor is also thrown out, and thus the vapor pressure in the tank drops, causing a difference between the vapor pressures at the water surface and in the tank. This pressure difference is the driving force of vaporization, and forces the liquid to evaporate. But the liquid must absorb the heat of vaporization before it can vaporize, and it absorbs it from the liquid and the air in the neighborhood, causing the temperature in the tank to drop. The process continues until water starts freezing. The process can be made more efficient by insulating the tank well so that the entire heat of vaporization comes essentially from the water.

21-2C The first ammonia absorption refrigeration system was developed in 1851 by Ferdinand Carre. The formulas related to dry-bulb, wet-bulb, and dew-point temperatures were developed by Willis Carrier in 1911.

21-3C The concept of *heat pump* was conceived by Sadi Carnot in 1824. The first heat pump was built by T. G. N. Haldane in 1930, and the heat pumps were mass produced in 1952.

Human Body and Thermal Comfort

21-4C The metabolism refers to the burning of foods such as carbohydrates, fat, and protein in order to perform the necessary bodily functions. The metabolic rate for an average man ranges from 108 W while reading, writing, typing, or listening to a lecture in a classroom in a seated position to 1250 W at age 20 (730 at age 70) during strenuous exercise. The corresponding rates for women are about 30 percent lower. Maximum metabolic rates of trained athletes can exceed 2000 W. We are interested in metabolic rate of the occupants of a building when we deal with heating and air conditioning because the metabolic rate represents the rate at which a body generates heat and dissipates it to the room. This body heat contributes to the heating in winter, but it adds to the cooling load of the building in summer.

21-5C The metabolic rate is proportional to the size of the body, and the metabolic rate of women, in general, is lower than that of men because of their smaller size. Clothing serves as insulation, and the thicker the clothing, the lower the environmental temperature that feels comfortable.

21-6C Asymmetric thermal radiation is caused by the *cold surfaces* of large windows, uninsulated walls, or cold products on one side, and the *warm surfaces* of gas or electric radiant heating panels on the walls or ceiling, solar heated masonry walls or ceilings on the other. Asymmetric radiation causes discomfort by exposing different sides of the body to surfaces at different temperatures and thus to different rates of heat loss or gain by radiation. A person whose left side is exposed to a cold window, for example, will feel like heat is being drained from that side of his or her body.

21-7C (*a*) Draft causes undesired local cooling of the human body by exposing parts of the body to high heat transfer coefficients. (*b*) Direct contact with *cold floor surfaces* causes localized discomfort in the feet by excessive heat loss by conduction, dropping the temperature of the bottom of the feet to uncomfortable levels.

21-8C Stratification is the formation of vertical still air layers in a room at difference temperatures, with highest temperatures occurring near the ceiling. It is likely to occur at places with high ceilings. It causes discomfort by exposing the head and the feet to different temperatures. This effect can be prevented or minimized by using destratification fans (ceiling fans running in reverse).

21-9C It is necessary to ventilate buildings to provide adequate fresh air and to get rid of excess carbon dioxide, contaminants, odors, and humidity. Ventilation increases the energy consumption for heating in winter by replacing the warm indoors air by the colder outdoors air. Ventilation also increases the energy consumption for cooling in summer by replacing the cold indoors air by the warm outdoors air. It is not a good idea to keep the bathroom fans on all the time since they will waste energy by expelling conditioned air (warm in winter and cool in summer) by the unconditioned outdoor air.

Heat Transfer from the Human Body

21-10C Yes, roughly one-third of the metabolic heat generated by a person who is resting or doing light work is dissipated to the environment by convection, one-third by evaporation, and the remaining one-third by radiation.

21-11C Sensible heat is the energy associated with a temperature change. The sensible heat loss from a human body increases as (*a*) the skin temperature increases, (*b*) the environment temperature decreases, and (*c*) the air motion (and thus the convection heat transfer coefficient) increases.

21-12C Latent heat is the energy released as water vapor condenses on cold surfaces, or the energy absorbed from a warm surface as liquid water evaporates. The latent heat loss from a human body increases as (*a*) the skin wettedness increases and (*b*) the relative humidity of the environment decreases. The rate of evaporation from the body is related to the rate of latent heat loss by $\dot{Q}_{\text{latent}} = \dot{m}_{\text{vapor}} h_{fg}$ where h_{fg} is the latent heat of vaporization of water at the skin temperature.

21-13C The insulating effect of clothing is expressed in the unit **clo** with 1 clo = 0.155 $\text{m}^2.^\circ\text{C/W}$ = 0.880 $\text{ft}^2.^\circ\text{F.h/Btu}$. Clothing serves as insulation, and thus reduces heat loss from the body by convection, radiation, and evaporation by serving as a resistance against heat flow and vapor flow. Clothing decreases heat gain from the sun by serving as a radiation shield.

21-14C (*a*) Heat is lost through the skin by convection, radiation, and evaporation. (*b*) The body loses both sensible heat by convection and latent heat by evaporation from the lungs, but there is no heat transfer in the lungs by radiation.

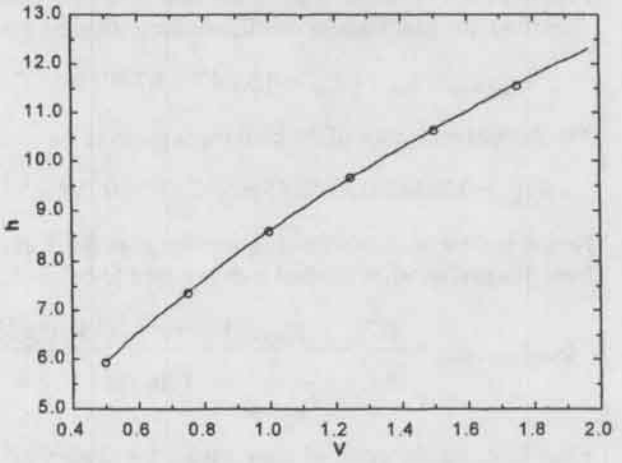
21-15C The *operative temperature* $T_{\text{operative}}$ is the average of the mean radiant and ambient temperatures weighed by their respective convection and radiation heat transfer coefficients, and is expressed as

$$T_{\text{operative}} = \frac{h_{\text{conv}} T_{\text{ambient}} + h_{\text{rad}} T_{\text{surr}}}{h_{\text{conv}} + h_{\text{rad}}} \cong \frac{T_{\text{ambient}} + T_{\text{surr}}}{2}$$

When the convection and radiation heat transfer coefficients are equal to each other, the operative temperature becomes the arithmetic average of the ambient and surrounding surface temperatures.

21-16 The convection heat transfer coefficient for a clothed person while walking in still air at a velocity of 0.5 to 2 m/s is given by $h = 8.6V^{0.53}$ where V is in m/s and h is in W/m^2.°C. The convection coefficients in that range vary from 5.96 W/m^2.°C at 0.5 m/s to 12.4 W/m^2.°C at 2 m/s. Therefore, at low velocities, the radiation and convection heat transfer coefficients are comparable in magnitude. But at high velocities, the convection coefficient is much larger than the radiation heat transfer coefficient.

Velocity, m/s	$h = 8.6V^{0.53}$ W/m^2.°C
0.50	5.96
0.75	7.38
1.00	8.60
1.25	9.68
1.50	10.66
1.75	11.57
2.00	12.40



21-17 A man wearing summer clothes feels comfortable in a room at 22°C. The room temperature at which this man would feel thermally comfortable when unclothed is to be determined.

Assumptions **1** Steady conditions exist. **2** The latent heat loss from the person remains the same. **3** The heat transfer coefficients remain the same. **4** The air in the room is still (there are no winds or running fans). **5** The surface areas of the clothed and unclothed person are the same.

Analysis At low air velocities, the convection heat transfer coefficient for a standing man is given in Table 21-3 to be 4.0 W/m^2.°C. The radiation heat transfer coefficient at typical indoor conditions is 4.7 W/m^2.°C. Therefore, the heat transfer coefficient for a standing person for combined convection and radiation is

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} = 4.0 + 4.7 = 8.7 \text{ W/m}^2.^\circ\text{C}$$

The thermal resistance of the clothing is given to be

$$R_{\text{cloth}} = 0.7 \text{ clo} = 0.7 \times 0.155 \text{ m}^2.^\circ\text{C/W} = 0.109 \text{ m}^2.^\circ\text{C/W}$$

Noting that the surface area of an average man is 1.8 m^2, the sensible heat loss from this person when clothed is determined to be

$$\dot{Q}_{\text{sensible,clothed}} = \frac{A(T_{\text{skin}} - T_{\text{ambient}})}{R_{\text{cloth}} + \dfrac{1}{h_{\text{combined}}}} = \frac{(1.8 \text{ m}^2)(33-20)^\circ\text{C}}{0.109 \text{ m}^2.^\circ\text{C/W} + \dfrac{1}{8.7 \text{ W/m}^2.^\circ\text{C}}} = 104 \text{ W}$$

From heat transfer point of view, taking the clothes off is equivalent to removing the clothing insulation or setting $R_{\text{cloth}} = 0$. The heat transfer in this case can be expressed as

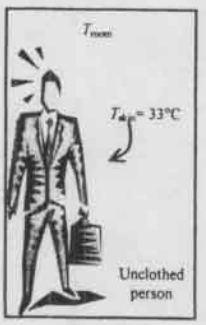
$$\dot{Q}_{\text{sensible,unclothed}} = \frac{A(T_{\text{skin}} - T_{\text{ambient}})}{\dfrac{1}{h_{\text{combined}}}} = \frac{(1.8 \text{ m}^2)(33 - T_{\text{ambient}})^\circ\text{C}}{\dfrac{1}{8.7 \text{ W/m}^2.^\circ\text{C}}}$$

To maintain thermal comfort after taking the clothes off, the skin temperature of the person and the rate of heat transfer from him must remain the same. Then setting the equation above equal to 104 W gives

$$T_{\text{ambient}} = \mathbf{26.4^\circ C}$$

Therefore, the air temperature needs to be raised from 22 to 26.4°C to ensure that the person will feel comfortable in the room after he takes his clothes off. Note that the effect of clothing on latent heat is assumed to be negligible in the solution above. We also assumed the surface area of the clothed and unclothed person to be the same for simplicity, and these two effects should counteract each other.





21-18E An average person produces 0.50 lbm of moisture while taking a shower. The contribution of showers of a family of four to the latent heat load of the air-conditioner per day is to be determined.

Assumptions All the water vapor from the shower is condensed by the air-conditioning system.

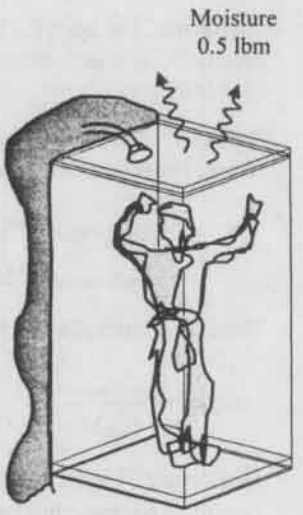
Properties The latent heat of vaporization of water is given to be 1050 Btu/lbm.

Analysis The amount of moisture produced per day is

$$\dot{m}_{\text{vapor}} = (\text{Moisture produced per person})(\text{No. of persons})$$
$$= (0.5 \text{ lbm / person})(4 \text{ persons / day}) = 2 \text{ lbm / day}$$

Then the latent heat load due to showers becomes

$$\dot{Q}_{\text{latent}} = \dot{m}_{\text{vapor}} h_{fg} = (2 \text{ lbm / day})(1050 \text{ Btu / lbm}) = \mathbf{2100 \text{ Btu / day}}$$



21-19 There are 100 chickens in a breeding room. The rate of total heat generation and the rate of moisture production in the room are to be determined.

Assumptions All the moisture from the chickens is condensed by the air-conditioning system.

Properties The latent heat of vaporization of water is given to be 2430 kJ/kg. The average metabolic rate of chicken during normal activity is 10.2 W (3.78 W sensible and 6.42 W latent).

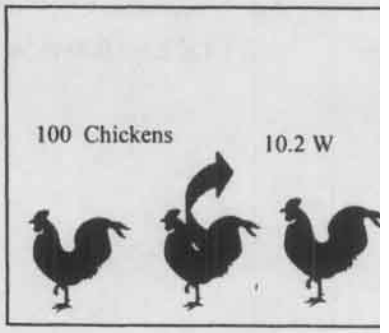
Analysis The total rate of heat generation of the chickens in the breeding room is

$$\dot{Q}_{\text{gen, total}} = \dot{q}_{\text{gen, total}} (\text{No. of chickens}) = (10.2 \text{ W / chicken})(100 \text{ chickens}) = \mathbf{1020 \text{ W}}$$

The latent heat generated by the chicken and the rate of moisture production are

$$\dot{Q}_{\text{gen, latent}} = \dot{q}_{\text{gen, latent}} (\text{No. of chickens})$$
$$= (6.42 \text{ W/chicken})(100 \text{ chickens}) = 642 \text{ W}$$
$$= 0.642 \text{ kW}$$

$$\dot{m}_{\text{moisture}} = \frac{\dot{Q}_{\text{gen, latent}}}{h_{fg}} = \frac{0.642 \text{ kJ / s}}{2430 \text{ kJ / kg}} = 0.000264 \text{ kg / s} = \mathbf{0.264 \text{ g / s}}$$



21-20 Chilled air is to cool a room by removing the heat generated in a large insulated classroom by lights and students. The required flow rate of air that needs to be supplied to the room is to be determined.

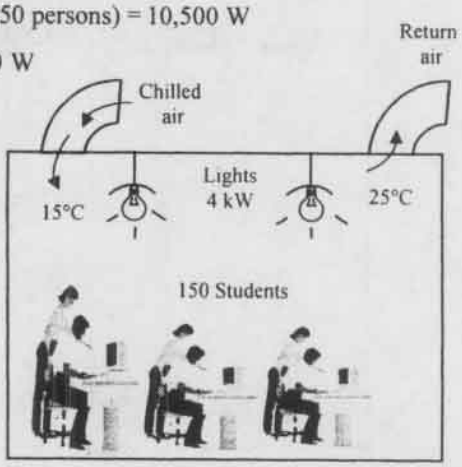
Assumptions **1** The moisture produced by the bodies leave the room as vapor without any condensing, and thus the classroom has no latent heat load. **2** Heat gain through the walls and the roof is negligible.

Properties The specific heat of air at room temperature is 1.00 kJ/kg·°C (Table A-18). The average rate of metabolic heat generation by a person sitting or doing light work is 115 W (70 W sensible, and 45 W latent) (Table 21-8).

Analysis The rate of sensible heat generation by the people in the room and the total rate of sensible internal heat generation are

$$\dot{Q}_{\text{gen, sensible}} = \dot{q}_{\text{gen, sensible}}(\text{No. of people}) = (70\ \text{W / person})(150\ \text{persons}) = 10{,}500\ \text{W}$$

$$\dot{Q}_{\text{total, sensible}} = \dot{Q}_{\text{gen, sensible}} + \dot{Q}_{\text{lighting}} = 10{,}500 + 4000 = 14{,}500\ \text{W}$$



Then the required mass flow rate of chilled air becomes

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{total, sensible}}}{C_p \Delta T} = \frac{14.5\ \text{kJ / s}}{(1.0\ \text{kJ / kg·°C})(25-15)°\text{C}} = \mathbf{1.45\ kg / s}$$

Discussion The latent heat will be removed by the air-conditioning system as the moisture condenses outside the cooling coils.

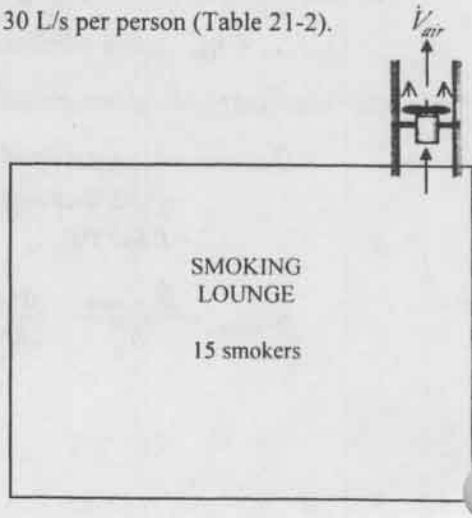
21-21 A smoking lounge that can accommodate 15 smokers is considered. The required minimum flow rate of air that needs to be supplied to the lounge is to be determined.

Assumptions Infiltration of air into the smoking lounge is negligible.

Properties The minimum fresh air requirements for a smoking lounge is 30 L/s per person (Table 21-2).

Analysis The required minimum flow rate of air that needs to be supplied to the lounge is determined directly from

$$\dot{V}_{\text{air}} = \dot{V}_{\text{air per person}}(\text{No. of persons}) = (30\ \text{L/s} \cdot \text{person})(15\ \text{persons})$$
$$= 450\ \text{L/s} = \mathbf{0.45\ m^3/s}$$



21-22 The average mean radiation temperature during a cold day drops to 18°C. The required rise in the indoor air temperature to maintain the same level of comfort in the same clothing is to be determined.

Assumptions **1** Air motion in the room is negligible. **2** The average clothing and exposed skin temperature remains the same. **3** The latent heat loss from the body remains constant. **4** Heat transfer through the lungs remain constant.

Properties The emissivity of the person is 0.95 (Table A-25). The convection heat transfer coefficient from the body in still air or air moving with a velocity under 0.2 m/s is $h_{conv} = 3.1$ W/m^2·°C (Table 21-3).

Analysis The total rate of heat transfer from the body is the sum of the rates of heat loss by convection, radiation, and evaporation,

$$\dot{Q}_{\text{body, total}} = \dot{Q}_{\text{sensible}} + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}} = (\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}) + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}}$$

Noting that heat transfer from the skin by evaporation and from the lungs remains constant, the sum of the convection and radiation heat transfer from the person must remain constant.

$$\dot{Q}_{\text{sensible,old}} = hA(T_s - T_{\text{air, old}}) + \varepsilon A\sigma(T_s^4 - T_{\text{surr,old}}^4) = hA(T_s - 22) + 0.95A\sigma[(T_s + 273)^4 - (22 + 273)^4]$$

$$\dot{Q}_{\text{sensible,new}} = hA(T_s - T_{\text{air, new}}) + \varepsilon A\sigma(T_s^4 - T_{\text{surr,new}}^4) = hA(T_s - T_{\text{air, new}}) + 0.95A\sigma[(T_s + 273)^4 - (18 + 273)^4]$$

Setting the two relations above equal to each other, canceling the surface area A, and simplifying gives

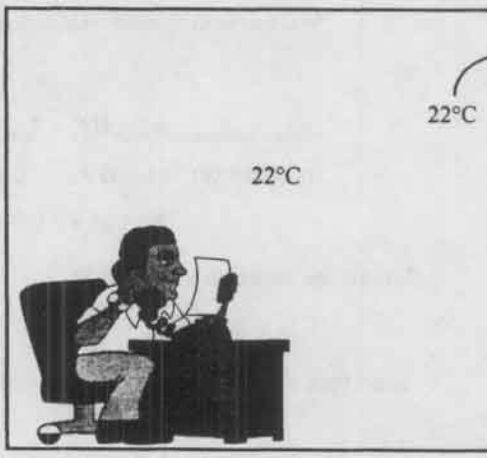
$$-22h - 0.95\sigma(22 + 273)^4 = -hT_{\text{air, new}} - 0.95\sigma(18 + 273)^4$$

$$3.1(T_{\text{air, new}} - 22) + 0.95 \times 5.67 \times 10^{-8}(291^4 - 295^4) = 0$$

Solving for the new air temperature gives

$$T_{\text{air, new}} = \mathbf{29.0°C}$$

Therefore, the air temperature must be raised to 29°C to counteract the increase in heat transfer by radiation.



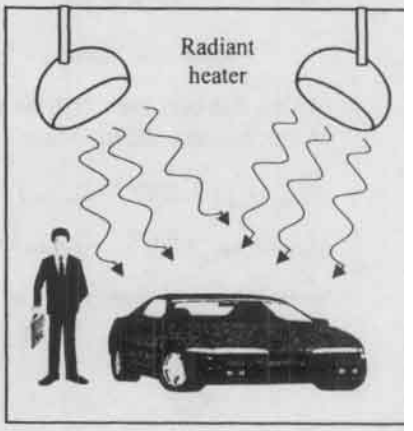
21-23 A car mechanic is working in a shop heated by radiant heaters in winter. The lowest ambient temperature the worker can work in comfortably is to be determined.

Assumptions **1** The air motion in the room is negligible, and the mechanic is standing. **2** The average clothing and exposed skin temperature of the mechanic is 33°C.

Properties The emissivity and absorptivity of the person is given to be 0.95. The convection heat transfer coefficient from a standing body in still air or air moving with a velocity under 0.2 m/s is h_{conv} = 4.0 W/m^2·°C (Table 21-3).

Analysis The equivalent thermal resistance of clothing is

$$R_{cloth} = 0.7 \text{ clo} = 0.7 \times 0.155 \text{ m}^2.^\circ\text{C/W} = 0.1085 \text{ m}^2.^\circ\text{C/W}$$



Radiation from the heaters incident on the person and the rate of sensible heat generation by the person are

$$\dot{Q}_{\text{rad, incident}} = 0.05 \times \dot{Q}_{\text{rad, total}} = 0.05(10 \text{ kW}) = 0.5 \text{ kW} = 500 \text{ W}$$
$$\dot{Q}_{\text{gen, sensible}} = 0.5 \times \dot{Q}_{\text{gen, total}} = 0.5(350 \text{ W}) = 175 \text{ W}$$

Under steady conditions, and energy balance on the body can be expressed as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$
$$\dot{Q}_{\text{rad from heater}} - \dot{Q}_{\text{conv+rad from body}} + \dot{Q}_{\text{gen, sensible}} = 0$$

or

$$\alpha\dot{Q}_{\text{rad, incident}} - h_{conv}A(T_s - T_{surr}) - \varepsilon A\sigma(T_s^4 - T_{surr}^4) + \dot{Q}_{\text{gen, sensible}} = 0$$
$$0.95(500 \text{ W}) - (4.0 \text{ W/m}^2 \cdot \text{K})(1.8 \text{ m}^2)(306 - T_{surr})$$
$$- 0.95(1.8 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(306 \text{ K})^4 - T_{surr}^4) + 175 \text{ W} = 0$$

Solving the equation above gives

$$T_{surr} = 266.2 \text{ K} = \mathbf{-7.0^\circ C}$$

Therefore, the mechanic can work comfortably at temperatures as low as -7°C.

Design conditions for Heating and Cooling

21-24C The extreme outdoor temperature under which a heating or cooling system must be able to maintain a building at the indoor design conditions is called the *outdoor design temperature.* It differs from the average winter temperature in that the average temperature represents the arithmetic average of the hourly outdoors temperatures. The 97.5% winter design temperature ensures that the heating system will provide thermal comfort 97.5 percent of the time, but may fail to do so during 2.5 percent of the time. The 99% winter design temperature, on the other hand, ensures that the heating system will provide thermal comfort 99 percent of the time, but may fail to do so during 1 percent of the time in an average year.

21-25C Yes, it is possible for a city A to have a lower winter design temperature but a higher average winter temperature than another city B. In that case, a house in city A will require a larger heating system, but it will use less energy during a heating season.

21-26C The solar radiation has no effect on the design heating load in winter since the coldest outdoor temperatures occur before sunrise, but it may reduce the annual energy consumption for heating considerably. Similarly, the heat generated by people, lights, and appliances has no effect on the design heating load in winter since the heating system should be able to meet the heating load of a house even when there is no internal heat generation, but it will reduce the annual energy consumption for heating.

21-27C The solar radiation constitutes a major part of the cooling load, and thus it increases both the design cooling load in summer and the annual energy consumption for cooling. Similarly, the heat generated by people, lights, and appliances constitute a significant part of the cooling load, and thus it increases both the design cooling load in summer and the annual energy consumption for cooling.

21-28C The moisture level of the outdoor air contributes to the latent heat load, and it affects the cooling load in summer. This is because the humidity ratio of the outdoor air is higher than that of the indoor air in summer, and the outdoor air that infiltrates into the building increases the amount of moisture inside. This excess moisture must be removed by the air-conditioning system. The moisture level of the outdoor air, in general, does affect the heating load in winter since the humidity ratio of the outdoor air is much lower than that of the indoor air in winter, and the moisture production in the building is sufficient to keep the air moist. However, in some cases, it may be necessary to add moisture to the indoor air. The heating load in this case will increase because of the energy needed to vaporize the water.

21-29C The reason for different values of recommended design heat transfer coefficients for combined convection and radiation on the outer surface of a building in summer and in winter is the wind velocity. In winter, the wind velocity and thus the heat transfer coefficient is higher.

21-30C The *sol-air temperature* is defined as the equivalent outdoor air temperature that gives the same rate of heat flow to a surface as would the combination of incident solar radiation, convection with the ambient air, and radiation exchange with the sky and the surrounding surfaces. It is used to account for the effect of solar radiation by considering the outside temperature to be higher by an amount equivalent to the effect of solar radiation. The higher the solar absorptivity of the outer surface of a wall, the higher is the amount of solar radiation absorption and thus the sol-air temperature.

21-31C Most of the solar energy absorbed by the walls of a brick house will be transferred to the outdoors since the thermal resistance between the outer surface and the indoor air (the wall resistance + the convection resistance on the inner surface) is much larger than the thermal resistance between the outer surface and the outdoor air (just the convection resistance).

21-32 The climatic conditions for major cities in the U.S. are listed in Table 21-4, and for the indicated design levels we read

Winter: $T_{outdoor} = -19°C$ (97.5 percent level)

Summer: $T_{outdoor} = 35°C$
$T_{wet\text{-}bulb} = 23°C$ (2.5 percent level)

Therefore, the heating and cooling systems in Lincoln, Nebraska for common applications should be sized for these outdoor conditions. Note that when the wet-bulb and ambient temperatures are available, the relative humidity and the humidity ratio of air can be determined from the psychrometric chart.

21-33 The climatic conditions for major cities in the U.S. are listed in Table 21-4, and for the indicated design levels we read

Winter: $T_{outdoor} = -16°C$ (99 percent level)

Summer: $T_{outdoor} = 37°C$
$T_{wet\text{-}bulb} = 23°C$ (2.5 percent level)

Therefore, the heating and cooling systems in Wichita, Kansas for common applications should be sized for these outdoor conditions. Note that when the wet-bulb and ambient temperatures are available, the relative humidity and the humidity ratio of air can be determined from the psychrometric chart.

21-34 The south wall of a house is subjected to solar radiation at summer design conditions. The design heat gain, the fraction of heat gain due to solar heating, and the fraction of solar radiation that is transferred to the house are to be determined.

Assumptions **1** Steady conditions exist. **2** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The overall heat transfer coefficient of the wall is given to be 1.6 W/m^2·°C.

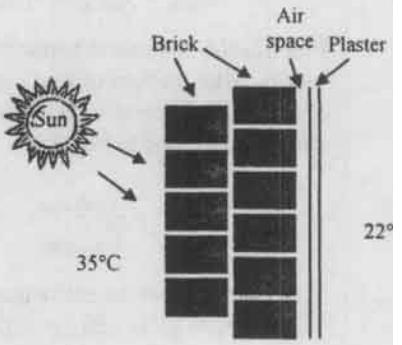
Analysis (*a*) The house is located at 40° N latitude, and thus we can use the sol-air temperature data directly from Table 21-7. At 15:00 the tabulated air temperature is 35°C, which is identical to the air temperature given in the problem. Therefore, the sol-air temperature on the south wall in this case is 43.6°C, and the heat gain through the wall is determined to be

$$\dot{Q}_{\text{wall}} = UA(T_{\text{sol-air}} - T_{\text{inside}}) = (1.6\ \text{W/m}^2\cdot^\circ\text{C})(20\ \text{m}^2)(43.6-22)^\circ\text{C} = \mathbf{691.2\ W}$$

(*b*) Heat transfer is proportional to the temperature difference, and the overall temperature difference in this case is 43.6 - 22 = 21.6°C. Also, the difference between the sol-air temperature and the ambient air temperature is

$$\Delta T_{\text{solar}} = T_{\text{sol-air}} - T_{\text{ambient}} = 43.6 - 35 = 8.6^\circ\text{C}$$

which is the equivalent temperature rise of the ambient air due to solar heating. The fraction of heat gain due to solar heating is equal to the fraction of the solar temperature difference to the overall temperature difference, and is determined to be



$$\text{Solar fraction} = \frac{\dot{Q}_{\text{wall,solar}}}{\dot{Q}_{\text{wall,total}}} = \frac{UA\Delta T_{\text{solar}}}{UA\Delta T_{\text{total}}} = \frac{\Delta T_{\text{solar}}}{\Delta T_{\text{total}}} = \frac{8.6^\circ\text{C}}{21.6^\circ\text{C}} = \mathbf{0.40}\ (\text{or } 40\%)$$

Therefore, almost half of the heat gain through the west wall in this case is due to solar heating of the wall.

(*c*) The outer layer of the wall is made of red brick which is dark colored. Therefore, the value of α_s / h_o is 0.052 m^2.°C/W. Then the fraction of incident solar energy transferred to the interior of the house is determined directly from Eq. 21-20 to be

$$\text{Solar fraction tranferred} = U\frac{\alpha_s}{h_o} = (1.6\ \text{W/m}^2\cdot^\circ\text{C})(0.052\ \text{m}^2\cdot^\circ\text{C/W}) = \mathbf{0.0832}$$

Discussion Less than 10 percent of the solar energy incident on the surface will be transferred to the house in this case. Note that a glass wall would transmit about 10 times more energy into the house.

21-35E The west wall of a house is subjected to solar radiation at summer design conditions. The design heat gain and the fraction of heat gain due to solar heating are to be determined.

Assumptions **1** Steady conditions exist. **2** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The overall heat transfer coefficient of the wall is given to be 0.14 Btu/h·ft²·°F.

Analysis (*a*) The house is located at 40°N latitude, and thus we can use the sol-air temperature data directly from Table 21-7. At 15:00 the tabulated air temperature is 94°F, which is 8°F higher than the air temperature given in the problem. But we can still use the data in that table provided that we subtract 8°F from all temperatures. Therefore, the sol-air temperature on the west wall in this case is 159 - 8 = 151°F, and the heat gain through the wall is determined to be

$$\dot{Q}_{\text{wall}} = UA(T_{\text{sol-air}} - T_{\text{inside}}) = (0.14\ \text{Btu/h}\cdot\text{ft}^2\cdot^\circ\text{F})(150\times 12\ \text{ft}^2)(151-72)^\circ\text{F} = \mathbf{19{,}908\ Btu/h}$$

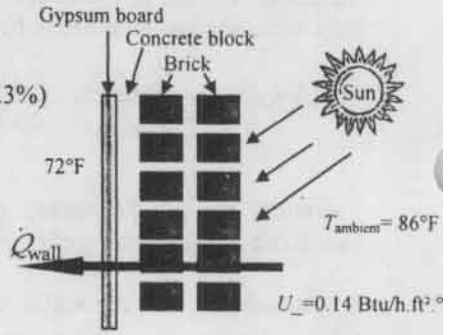
(*b*) Heat transfer is proportional to the temperature difference, and the overall temperature difference in this case is 151 - 72 = 79°F. Also, the difference between the sol-air temperature and the ambient air temperature is

$$\Delta T_{\text{solar}} = T_{\text{sol-air}} - T_{\text{ambient}} = 151 - 86 = 65^\circ\text{F}$$

which is the equivalent temperature rise of the ambient air due to solar heating. The fraction of heat gain due to solar heating is equal to the ratio of the solar temperature difference to the overall temperature difference, and is determined to be

$$\text{Solar fraction} = \frac{\dot{Q}_{\text{wall,solar}}}{\dot{Q}_{\text{wall,total}}} = \frac{UA\Delta T_{\text{solar}}}{UA\Delta T_{\text{total}}} = \frac{\Delta T_{\text{solar}}}{\Delta T_{\text{total}}} = \frac{65^\circ\text{F}}{79^\circ\text{F}} = \mathbf{0.823}\ (\text{or } 82.3\%)$$

Therefore, almost the entire heat gain through the west wall in this case is due to solar heating of the wall.



21-36 The roof of a house is subjected to solar radiation at summer design conditions. The design heat gain and the fraction of heat gain due to solar heating are to be determined.

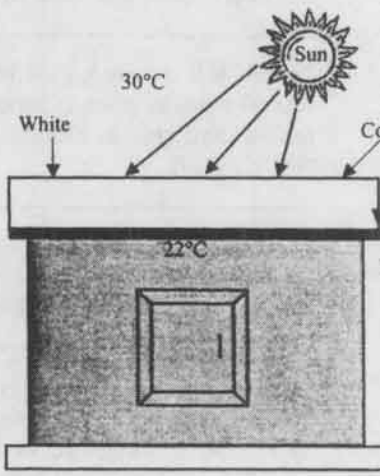
Assumptions **1** Steady conditions exist. **2** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The overall heat transfer coefficient of the roof is given to be 1.8 W/m^2·°C.

Analysis (*a*) The house is located at 40°N latitude, and thus we can use the sol-air temperature data directly from Table 21-7. At 16:00 the tabulated air temperature is 34.7°C, which is 4.7°C higher than the air temperature given in the problem. But we can still use the data in that table provided that we subtract 4.7°C from all temperatures. Therefore, the sol-air temperature on the roof in this case is 42.7 - 4.7 = 38.0°C, and the heat gain through the roof is determined to be

$$\dot{Q}_{\text{roof}} = UA(T_{\text{sol-air}} - T_{\text{inside}}) = (1.8\ \text{W/m}^2 \cdot °\text{C})(150\ \text{m}^2)(38-22)°\text{C} = \mathbf{4320\ W}$$

(*b*) Heat transfer is proportional to the temperature difference, and the overall temperature difference in this case is 38 - 22 = 16°C. Also, the difference between the sol-air temperature and the ambient air temperature is



$$\Delta T_{\text{solar}} = T_{\text{sol-air}} - T_{\text{ambient}} = 38 - 30 = 8°\text{C}$$

which is the equivalent temperature rise of the ambient air due to solar heating. The fraction of heat gain due to solar heating is equal to the ratio of the solar temperature difference to the overall temperature difference, and is determined to be

$$\text{Solar fraction} = \frac{\dot{Q}_{\text{wall,solar}}}{\dot{Q}_{\text{wall,total}}} = \frac{UA\Delta T_{\text{solar}}}{UA\Delta T_{\text{total}}} = \frac{\Delta T_{\text{solar}}}{\Delta T_{\text{total}}} = \frac{8°\text{C}}{16°\text{C}} = \mathbf{0.50}\ (\text{or } 50\%)$$

Therefore, half of the heat gain through the roof in this case is due to solar heating of the roof.

Heat Gain from People, Lights, and Appliances

21-37C The heat given off by people in a concert hall is an important consideration in the sizing of the air-conditioning system for that building because of the high density of people in the hall, and the heat generation from the people contributes a significant amount to the cooling load.

21-38C By replacing the incandescent lamps of a building by high-efficiency fluorescent lamps, (a) the design cooling load will decrease, (*b*) annual energy consumption for cooling will also decrease, and (*c*) annual energy consumption for heating for the building will increase since fluorescent lamps generate much less heat than incandescent lamps for the same light output.

21-39C It is usually a good idea to replace incandescent light bulbs by compact fluorescent bulbs that may cost 40 times as much to purchase since incandescent lights waste energy by (1) consuming more electricity for the same amount of lighting, and (2) making the cooling system work harder and longer to remove the heat given off.

21-40C The motors and appliances in a building generate heat, and thus (*a*) they increase the design cooling load, (*b*) they increase the annual energy consumption for cooling, and (*c*) they reduce the annual energy consumption for heating of the building.

21-41C The motor efficiency η_{motor} is defined as the ratio of the shaft power delivered to the electrical power consumed by the motor. The higher the motor efficiency, the lower is the amount of heat generated by the motor. Therefore, high efficiency motors decrease the design cooling load of a building and the annual energy consumption for cooling.

21-42C The heat generated by a hooded range in a kitchen with a powerful fan that exhausts all the air heated and humidified by the range still needs to be considered in the determination of the cooling load of the kitchen although all the heated air is exhausted since part of the energy (32% of it) is radiated to the surroundings from the hot surfaces.

21-43 A hooded electric open burner and a gas burner are considered. The amount of the electrical energy used directly for cooking, the cost of energy per "utilized" kWh, and the contribution of this burner to the design cooling load are to be determined.



Analysis The efficiency of the electric heater is given to be 78 percent. Therefore, a burner that consumes 3-kW of electrical energy will supply

$$\dot{Q}_{\text{utilized}} = (\text{Energy input}) \times (\text{Efficiency}) = (3\ \text{kW})(0.73) = \mathbf{2.19\ kW}$$

of useful energy. The unit cost of utilized energy is inversely proportional to the efficiency, and is determined from

$$\text{Cost of utilized energy} = \frac{\text{Cost of energy input}}{\text{Efficiency}} = \frac{\$0.09/\text{kWh}}{0.73} = \mathbf{\$0.123/kWh}$$

The design heat gain from a hooded appliance is taken to be 32% of the half of its rated energy consumption, and is determined to be

$$\dot{Q}_{\text{hooded appliance}} = 0.5 \times 0.32 \dot{Q}_{\text{appliance, input}} = 0.5 \times 0.32 \times (3\ \text{kW}) = \mathbf{0.48\ kW} \quad \text{(electric burner)}$$

Noting that the efficiency of a gas burner is 38 percent, the energy input to a gas burner that supplies utilized energy at the same rate (2.19 kW) is

$$\dot{Q}_{\text{input, gas}} = \frac{\dot{Q}_{\text{utilized}}}{\text{Efficiency}} = \frac{2.19\ \text{kW}}{0.38} = \mathbf{5.76\ kW}\ \ (= 19{,}660\ \text{Btu/h})$$

since 1 kW = 3412 Btu/h. Therefore, a gas burner should have a rating of at least 19,660 Btu/h to perform as well as the electric unit.

Noting that 1 therm = 29.3 kWh, the unit cost of utilized energy in the case of gas burner is determined the same way to be

$$\text{Cost of utilized energy} = \frac{\text{Cost of energy input}}{\text{Efficiency}} = \frac{\$0.55/(29.3\ \text{kWh})}{0.38} = \mathbf{\$0.049/kWh}$$

which is about one-quarter of the unit cost of utilized electricity. The design heat gain from this hooded gas burner is determined similarly to be

$$\dot{Q}_{\text{unhooded appliance}} = 0.5 \times 0.32 \dot{Q}_{\text{appliance, input}} = 0.5 \times 0.32 \times (5.76\ \text{kW}) = \mathbf{0.922\ kW} \quad \text{(gas burner)}$$

which is almost twice as much as that of the electric burner. Therefore, a hooded gas appliance will contribute more to the heat gain than a comparable electric appliance.

21-44 Several people are working out in an exercise room. The rate of heat gain from people and the equipment, and the fraction of that heat in the latent form are to be determined.

Analysis The 8 weight lifting machines do not have any motors, and thus they do not contribute to the internal heat gain directly. The usage factors of the motors of the treadmills are taken to be unity since they are used constantly during peak periods. Noting that 1 hp = 746 W, the total heat generated by the motors is

$$\dot{Q}_{\text{motors}} = (\text{No. of motors}) \times \dot{W}_{\text{motor}} \times f_{\text{load}} \times f_{\text{usage}} / \eta_{\text{motor}} = 4 \times (2.5 \times 746\ \text{W}) \times 0.70 \times 1.0 / 0.77 = 6782\ \text{W}$$

The average rate of heat dissipated by people in an exercise room is given in Table 21-8 to be 525 W, of which 315 W is in latent form. Therefore, the heat gain from 14 people is

$$\dot{Q}_{\text{people}} = (\text{No. of people}) \times \dot{Q}_{\text{person}} = 14 \times (525\ \text{W}) = 7350\ \text{W}$$



Then the total rate of heat gain (or the internal heat load) of the exercise room during peak period becomes

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{motors}} + \dot{Q}_{\text{people}} = 6782 + 7350 = \mathbf{14{,}132\ W}$$

The entire heat given off by the motors is in sensible form. Therefore, the latent heat gain is due to people only, which is determined to be

$$\dot{Q}_{\text{latent}} = (\text{No. of people}) \times \dot{Q}_{\text{latent, per person}} = 14 \times (315\ \text{W}) = \mathbf{4410\ W}$$

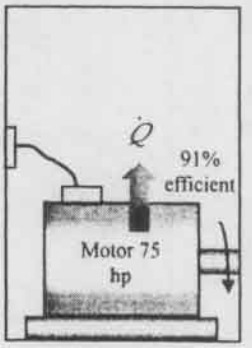
The remaining 14,132 - 4410 = 9722 W of heat gain is in the sensible form.

21-45 A worn out standard motor is replaced by a high efficiency one. The reduction in the internal heat gain due to higher efficiency under full load conditions is to be determined.

Assumptions **1** The motor and the equipment driven by the motor are in the same room. **2** The motor operates at full load so that $f_{\text{load}} = 1$.

Analysis The heat generated by a motor is due to its inefficiency, and the difference between the heat generated by two motors that deliver the same shaft power is simply the difference between the electric power drawn by the motors,

$$\dot{W}_{\text{in, electric, standard}} = \dot{W}_{\text{shaft}} / \eta_{\text{motor}} = (75 \times 746\ \text{W})/0.91 = 61{,}484\ \text{W}$$
$$\dot{W}_{\text{in, electric, efficient}} = \dot{W}_{\text{shaft}} / \eta_{\text{motor}} = (75 \times 746\ \text{W})/0.954 = 58{,}648\ \text{W}$$



Then the reduction in heat generation becomes

$$\dot{Q}_{\text{reduction}} = \dot{W}_{\text{in, electric, standard}} - \dot{W}_{\text{in, electric, efficient}} = 61{,}484 - 58{,}648 = \mathbf{2836\ W}$$

21-46 An electric hot plate and a gas hot plate are considered. For the same amount of "utilized" energy, the ratio of internal heat generated by gas hot plates to that by electric ones is to be determined.

Assumptions Hot plates are not hooded and thus the entire energy they consume is dissipated to the room they are in.

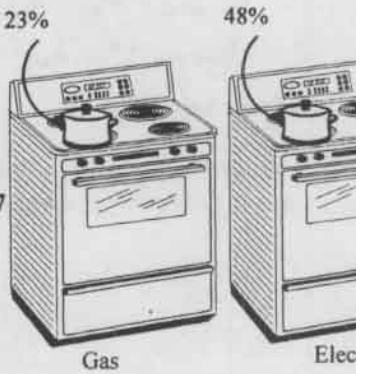
Analysis The utilized energy can be expressed in terms of the energy input and the efficiency as

$$\dot{E}_{\text{utilized}} = (\text{Energy input}) \times (\text{Efficiency}) = \dot{E}_{\text{in}} \times \eta \quad \rightarrow \quad \dot{E}_{\text{in}} = \dot{E}_{\text{utilized}} / \eta$$

Noting that the utilized energy is the same for both the electric and gas hot plates, the ratio of internal heat generated by gas hot plates to that by electric ones is determined to be

$$\text{Ratio of heat generation} = \frac{\dot{E}_{\text{in, gas}}}{\dot{E}_{\text{in, electric}}} = \frac{\dot{E}_{\text{utilized}} / \eta_{\text{gas}}}{\dot{E}_{\text{utilized}} / \eta_{\text{electric}}} = \frac{\eta_{\text{electric}}}{\eta_{\text{gas}}} = \frac{0.48}{0.23} = \mathbf{2.07}$$

Therefore, the gas hot plate will contribute twice as much to the internal heat gain of the room.



21-47 A classroom has 40 students, one instructor, and 18 fluorescent light bulbs. The rate of internal heat generation in this classroom is to be determined.

Assumptions **1** There is a mix of men, women, and children in the classroom. **2** The amount of light (and thus energy) leaving the room through the windows is negligible.

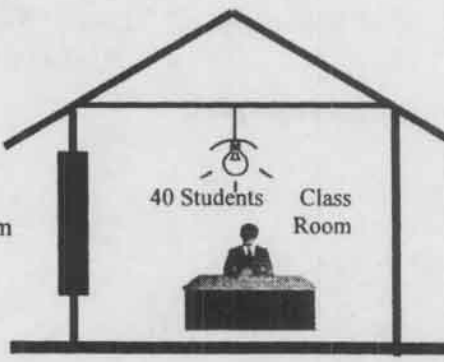
Properties The average rate of heat generation from people seated in a room/office is 115 W (Table 21-8).

Analysis The amount of heat dissipated by the lamps is equal to the amount of electrical energy consumed by the lamps, including the 10% additional electricity consumed by the ballasts. Therefore,

$$\dot{Q}_{\text{lighting}} = (\text{Energy consumed per lamp}) \times (\text{No. of lamps})$$
$$= (40\ \text{W})(1.1)(18) = 792\ \text{W}$$
$$\dot{Q}_{\text{people}} = (\text{No. of people}) \times \dot{Q}_{\text{person}} = 41 \times (115\ \text{W}) = 4715\ \text{W}$$

Then the total rate of heat gain (or the internal heat load) of the classroom from the lights and people become

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{lighting}} + \dot{Q}_{\text{people}} = 792 + 4715 = \mathbf{5507\ W}$$



21-48 An electric car is powered by an electric motor mounted in the engine compartment. The rate of heat supply by the motor to the engine compartment at full load conditions is to be determined.

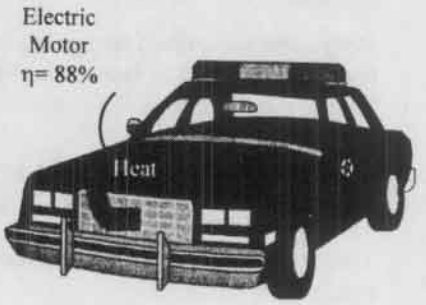
Assumptions The motor operates at full load so that $f_{load} = 1$.

Analysis The heat generated by a motor is due to its inefficiency, and the heat generated by a motor is equal to the difference between the electrical energy it consumes and the shaft power it delivers,

$$\dot{W}_{\text{in, electric}} = \dot{W}_{\text{shaft}} / \eta_{\text{motor}} = (60\text{ hp})/0.88 = 68.18\text{ hp}$$

$$\dot{Q}_{\text{generation}} = \dot{W}_{\text{in, electric}} - \dot{W}_{\text{shaft out}} = 68.18 - 60 = 8.18\text{ hp} = \mathbf{6.10\ kW}$$

since 1 hp = 0.746 kW.



Discussion The motor will supply as much heat to the compartment as a 6.1 kW resistance heater.

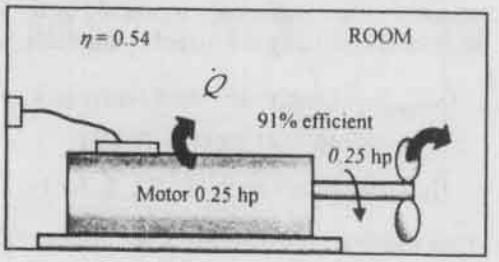
21-49 A room is cooled by circulating chilled water, and the air is circulated through the heat exchanger by a fan. The contribution of the fan-motor assembly to the cooling load of the room is to be determined.

Assumptions The fan motor operates at full load so that $f_{load} = 1$.

Analysis The entire electrical energy consumed by the motor, including the shaft power delivered to the fan, is eventually dissipated as heat. Therefore, the contribution of the fan-motor assembly to the cooling load of the room is equal to the electrical energy it consumes,

$$\dot{Q}_{\text{internal generation}} = \dot{W}_{\text{in, electric}} = \dot{W}_{\text{shaft}} / \eta_{\text{motor}}$$
$$= (0.25\text{ hp})/0.54 = 0.463\text{ hp} = \mathbf{345\ W}$$

since 1 hp = 746 W.



21-50 An office that is being cooled adequately by a 12,000 Btu/h window air-conditioner is converted to a computer room. The number of additional air-conditioners that need to be installed is to be determined.

Assumptions **1** The computer are operated by 4 adult men. **2** The computers consume 40 percent of their rated power at any given time.

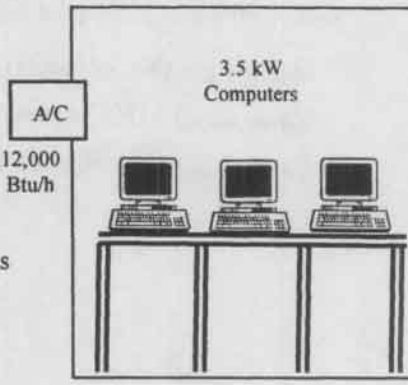
Properties The average rate of heat generation from a man seated in a room/office is 130 W (Table 21-8).

Analysis The amount of heat dissipated by the computers is equal to the amount of electrical energy they consume. Therefore,

$$\dot{Q}_{\text{computers}} = (\text{Rated power}) \times (\text{Usage factor}) = (3.5\ \text{kW})(0.4) = 1.4\ \text{kW}$$

$$\dot{Q}_{\text{people}} = (\text{No. of people}) \times \dot{Q}_{\text{person}} = 4 \times (130\ \text{W}) = 520\ \text{W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{computers}} + \dot{Q}_{\text{people}} = 1400 + 520 = 1920\ \text{W} = 6551\ \text{Btu/h}$$



since 1 W = 3.412 Btu/h. Then noting that each available air conditioner provides 4,000 Btu/h cooling, the number of air-conditioners needed becomes

$$\text{No. of air conditioners} = \frac{\text{Cooling load}}{\text{Cooling capacity of A/C}} = \frac{6551\ \text{Btu/h}}{4000\ \text{Btu/h}}$$

$$= 1.6 \approx \mathbf{2\ Air\ conditioners}$$

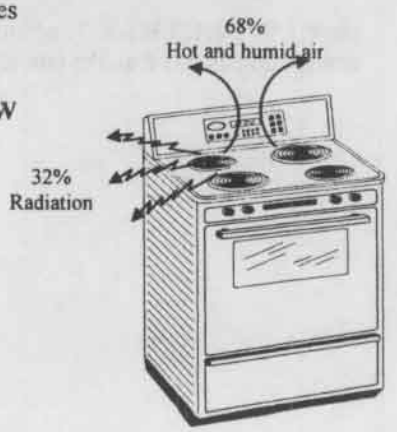
21-51 A restaurant purchases a new 8-kW electric range for its kitchen. The increase in the design cooling load is to be determined for the cases of hooded and unhooded range.

Assumptions **1** The contribution of an appliance to the design cooling is half of its rated power. **2** The hood of an appliance removes all the heated air and moisture generated, except the radiation heat that constitutes 38 percent of the heat generated.

Analysis The design cooling load due to an unhooded range is half of its rated power, and the design cooling of a hooded range is half of the radiation component of heat dissipation, which is taken to be 38 percent. Then the increase in the design cooling load for both cases becomes

$$\dot{Q}_{\text{unhooded appliance}} = 0.5 \times \dot{Q}_{\text{appliance, input}} = 0.5 \times (8\ \text{kW}) = \mathbf{4\ kW}$$

$$\dot{Q}_{\text{hooded appliance}} = 0.5 \times 0.32 \dot{Q}_{\text{appliance, input}} = 0.5 \times 0.32 \times (8\ \text{kW}) = \mathbf{1.28\ kW}$$



21-52 A department store expects to have 95 people at peak times in summer. The contribution of people to the sensible, latent, and total cooling load of the store is to be determined.

Assumptions There is a mix of men, women, and children in the classroom.

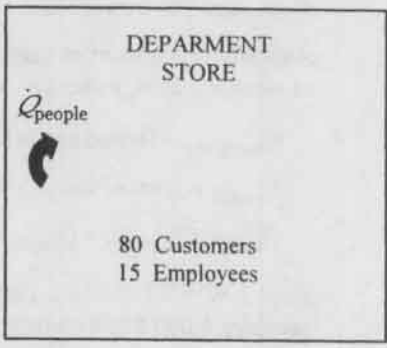
Properties The average rate of heat generation from people in a shopping center is 130 W, and 75 W of it is in sensible form and 55 W in latent form (Table 21-8).

Analysis The contribution of people to the sensible, latent, and total cooling load of the store are

$$\dot{Q}_{\text{people, total}} = (\text{No. of people}) \times \dot{Q}_{\text{person, total}} = 95 \times (130\ \text{W}) = \mathbf{12{,}350\ W}$$

$$\dot{Q}_{\text{people, sensible}} = (\text{No. of people}) \times \dot{Q}_{\text{person, sensible}} = 95 \times (70\ \text{W}) = \mathbf{7125\ W}$$

$$\dot{Q}_{\text{people, latent}} = (\text{No. of people}) \times \dot{Q}_{\text{person, latent}} = 95 \times (35\ \text{W}) = \mathbf{5225\ W}$$



21-53E There are 500 people in a movie theater in winter. It is to be determined if the theater needs to be heated or cooled.

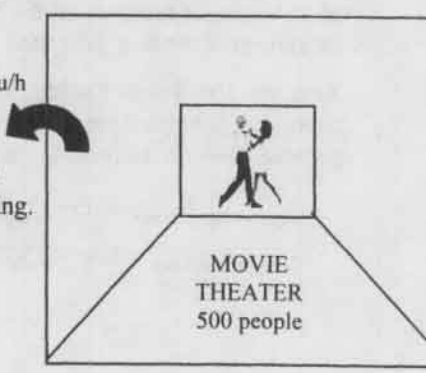
Assumptions There is a mix of men, women, and children in the classroom.

Properties The average rate of heat generation from people in a movie theater is 105 W, and 70 W of it is in sensible form and 35 W in latent form (Table 21-8).

Analysis Noting that only the sensible heat from a person contributes to the heating load of a building, the contribution of people to the heating of the building is

$$\begin{aligned}\dot{Q}_{\text{people, sensible}} &= (\text{No. of people}) \times \dot{Q}_{\text{person, sensible}} \\ &= 500 \times (70\ \text{W}) = 35{,}000\ \text{W} \\ &= \mathbf{119{,}420\ Btu/h}\end{aligned}$$

since 1 W = 3.412 Btu/h. The building needs to be heated since the heat gain from people is less than the rate of heat loss of 150,000 Btu/h from the building.



Heat Transfer Through the Walls and Roofs

21-54C The *R*-value of a wall is the thermal resistance of the wall per unit surface area. It is the same as the unit thermal resistance of the wall. It is the inverse of the *U*-factor of the wall, $R = 1/U$.

21-55C The effective emissivity for a plane-parallel air space is the "equivalent" emissivity of one surface for use in the relation $\dot{Q}_{rad} = \varepsilon_{effective}\sigma A(T_2^4 - T_1^4)$ that results in the same rate of radiation heat transfer between the two surfaces across the air space. It is determined from

$$\frac{1}{\varepsilon_{effective}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1$$

where ε_1 and ε_2 are the emissivities of the surfaces of the air space. When the effective emissivity is known, the radiation heat transfer through the air space is determined from the $\dot{Q}_{rad}$ relation above.

21-56C The unit thermal resistances (*R*-value) of both 40-mm and 90-mm vertical air spaces are given to be the same, which implies that more than doubling the thickness of air space in a wall has no effect on heat transfer through the wall. This is not surprising since the convection currents that set in in the thicker air space offset any additional resistance due to a thicker air space.

21-57C Radiant barriers are highly reflective materials that minimize the radiation heat transfer between surfaces. Highly reflective materials such as aluminum foil or aluminum coated paper are suitable for use as radiant barriers. Yes, it is worthwhile to use radiant barriers in the attics of homes by covering at least one side of the attic (the roof or the ceiling side) since they reduce radiation heat transfer between the ceiling and the roof considerably.

21-58C The roof of a house whose attic space is ventilated effectively so that the air temperature in the attic is the same as the ambient air temperature at all times will still have an effect on heat transfer through the ceiling since the roof in this case will act as a radiation shield, and reduce heat transfer by radiation.

21-59 The *R*-value and the *U*-factor of a wood frame wall are to be determined.

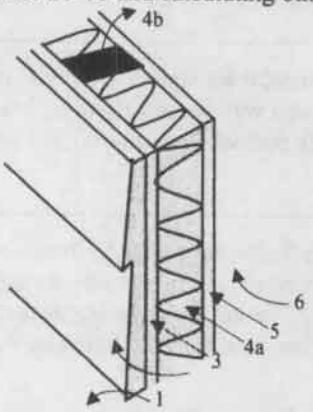
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 21-10.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{overall} = 1/U_{overall} \quad \text{where} \quad U_{overall} = (Uf_{area})_{insulation} + (Uf_{area})_{stud}$$

and the value of the area fraction f_{area} is 0.80 for insulation section and 0.20 for stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 21-10 and calculating others, the total *R*-values for each section is determined in the table below.



Construction	*R*-value, m².°C/W	
	Between studs	At studs
1. Outside surface, 12 km/h wind	0.044	0.044
2. Wood bevel lapped siding	0.14	0.14
3. Fiberboard sheathing, 25 mm	0.23	0.23
4a. Mineral fiber insulation, 140 mm	3.696	--
4b. Wood stud, 38 mm by 140 mm	--	0.98
5. Gypsum wallboard, 13 mm	0.079	0.079
6. Inside surface, still air	0.12	0.12

Total unit thermal resistance of each section, R (in m².°C/W)	4.309	1.593
The U-factor of each section, $U = 1/R$, in W/m².°C	0.232	0.628
Area fraction of each section, f_{area}	0.80	0.20
Overall U-factor, $U = \Sigma f_{area,i} U_i = 0.80 \times 0.232 + 0.20 \times 0.628$	**0.311 W/m².°C**	
Overall unit thermal resistance, $R = 1/U$	**3.213 m².°C/W**	

Therefore, the *R*-value and *U*-factor of the wall are $R = 3.213$ m².°C/W and $U = 0.311$ W/m².°C.

21-60 The change in the *R*-value of a wood frame wall due to replacing fiberwood sheathing in the wall by rigid foam sheathing is to be determined.

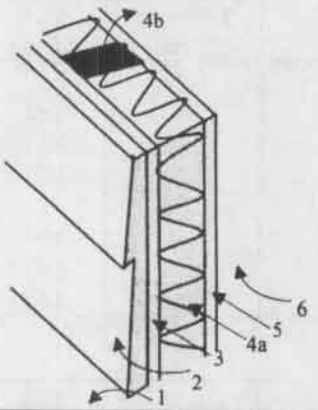
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 21-10.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{insulation}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.80 for insulation section and 0.20 for stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 21-10 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.



Construction	*R*-value, m².°C/W	
	Between studs	At studs
1. Outside surface, 12 km/h wind	0.044	0.044
2. Wood bevel lapped siding	0.14	0.14
3. Fiberboard sheathing, 25 mm	0.23	0.23
4a. Mineral fiber insulation, 140 mm	3.696	--
4b. Wood stud, 38 mm by 140 mm	--	0.98
5. Gypsum wallboard, 13 mm	0.079	0.079
6. Inside surface, still air	0.12	0.12

Total unit thermal resistance of each section, *R* (in m².°C/W)	4.309	1.593
The *U*-factor of each section, $U = 1/R$, in W/m².°C	0.232	0.628
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{\text{area},i} U_i = 0.80\times0.232+0.20\times0.628$	0.311 W/m².°C	
Overall unit thermal resistance, $R = 1/U$	3.213 m².°C/W	

Therefore, the *R*-value of the existing wall is $R = 3.213$ m².°C/W.

Noting that the *R*-values of the wood fiberboard and the rigid foam insulation are 0.23 m².°C/W and 0.98 m².°C/W, respectively, and the added and removed thermal resistances are in series, the overall *R*-value of the wall after modification becomes

$$R_{\text{new}} = R_{\text{old}} - R_{\text{removed}} + R_{\text{added}} = 3.213 - 0.23 + 0.98 = 3.963 \text{ m}^2.^\circ\text{C/W}$$

Then the change in the *R*-value becomes

$$\%\text{Change} = \frac{\Delta R - \text{value}}{R - \text{value, old}} = \frac{3.963 - 3.213}{3.963} = 0.233 \quad (\text{or } \mathbf{23.3\%})$$

21-61E The *R*-value and the *U*-factor of a masonry cavity wall are to be determined.

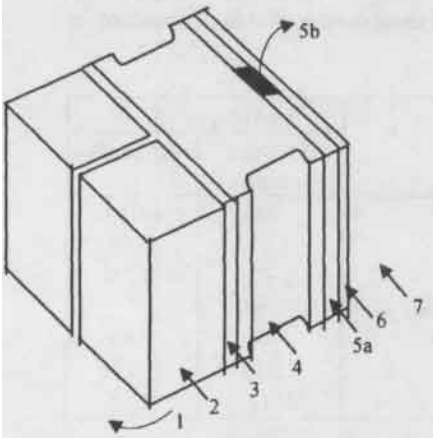
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 21-10.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{overall} = 1/U_{overall} \quad \text{where} \quad U_{overall} = (Uf_{area})_{air\ space} + \quad (Uf_{area})_{stud}$$

and the value of the area fraction f_{area} is 0.80 for air space and 0.20 for the ferrings and similar structures. Using the available *R*-values from Table 21-10 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.



Construction	*R*-value, h.ft².°F/Btu	
	Between furring	At furring
1. Outside surface, 15 mph wind	0.17	0.17
2. Face brick, 4 in	0.43	0.43
3. Cement mortar, 0.5 in	0.10	0.10
4. Concrete block, 4-in	1.51	1.51
5a. Air space, 3/4-in, nonreflective	2.91	--
5b. Nominal 1 × 3 vertical ferring	--	0.94
6. Gypsum wallboard, 0.5 in	0.45	0.45
7. Inside surface, still air	0.68	0.68

Total unit thermal resistance of each section, *R*	6.25	4.28
The *U*-factor of each section, $U = 1/R$, in Btu/h.ft².°F	0.160	0.234
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{area,i} U_i = 0.80 \times 0.160 + 0.20 \times 0.234$	**0.175 Btu/h.ft².°F**	
Overall unit thermal resistance, $R = 1/U$	**5.72 h.ft².°F/Btu**	

Therefore, the overall unit thermal resistance of the wall is $R = 5.72$ h.ft².°F/Btu and the overall U-factor is $U = 0.118$ Btu/h.ft².°F. These values account for the effects of the vertical ferring.

21-62 The winter *R*-value and the *U*-factor of a flat ceiling with an air space are to be determined for the cases of air space with reflective and nonreflective surfaces.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the ceiling is one-dimensional. **3** Thermal properties of the ceiling and the heat transfer coefficients are constant.

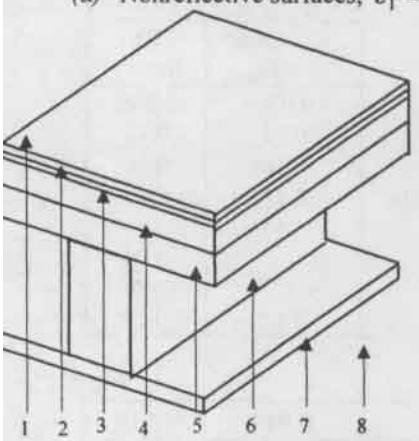
Properties The R-values of different materials are given in Table 21-10. The R-values of different air layers are given in Table 21-13.

Analysis The schematic of the ceiling as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.82 for air space and 0.18 for stud section since the headers which constitute a small part of the wall are to be treated as studs.

(a) Nonreflective surfaces, $\varepsilon_1 = \varepsilon_2 = 0.9$ and thus $\varepsilon_{\text{effective}} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.9 + 1/0.9 - 1} = 0.82$.



Construction	*R*-value, m².°C/W	
	Between studs	At studs
1. Still air above ceiling	0.12	0.044
2. Linoleum (R = 0.009 m².°C/W)	0.009	0.14
3. Felt (R = 0.011 m².°C/W)	0.011	0.23
4. Plywood, 13 mm	0.11	
5. Wood subfloor (R = 0.166 m².°C/W)	0.166	
6a. Air space, 90 mm, nonreflective	0.16	---
6b. Wood stud, 38 mm by 90 mm	---	0.63
7. Gypsum wallboard, 13 mm	0.079	0.079
8. Still air below ceiling	0.12	0.12

Total unit thermal resistance of each section, R (in m².°C/W)	0.655	1.125
The U-factor of each section, $U = 1/R$, in W/m².°C	1.527	0.889
Area fraction of each section, f_{area}	0.82	0.18
Overall U-factor, $U = \Sigma f_{\text{area},i} U_i = 0.82\times1.527+0.18\times0.889$	**1.412 W/m².°C**	
Overall unit thermal resistance, $R = 1/U$	**0.708 m².°C/W**	

(b) One-reflective surface, $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0.9 \rightarrow \varepsilon_{\text{effective}} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.05 + 1/0.9 - 1} = 0.05$

In this case we replace item 6a from 0.16 to 0.47 m².°C/W. It gives R = 0.965 m².°C/W and U = 1.036 W/m².°C for the air space. Then,

Overall U-factor, $U = \Sigma f_{\text{area},i} U_i = 0.82\times1.036+0.18\times0.889$	**1.01 W/m².°C**
Overall unit thermal resistance, $R = 1/U$	**0.99 m².°C/W**

(c) Two-reflective surface, $\varepsilon_1 = \varepsilon_2 = 0.05 \rightarrow \varepsilon_{\text{effective}} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.05 + 1/0.05 - 1} = 0.03$

In this case we replace item 6a from 0.16 to 0.49 m².°C/W. It gives R = 0.985 m².°C/W and U = 1.015 W/m².°C for the air space. Then,

Overall U-factor, $U = \Sigma f_{\text{area},i} U_i = 0.82\times1.015+0.18\times0.889$	**0.992 W/m².°C**
Overall unit thermal resistance, $R = 1/U$	**1.01 m².°C/W**

21-63 The winter *R*-value and the *U*-factor of a masonry cavity wall are to be determined.

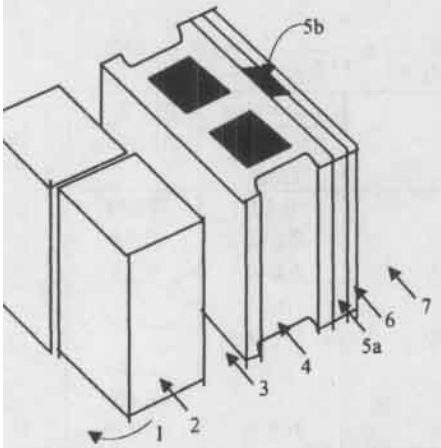
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 21-10.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{overall} = 1/U_{overall} \quad \text{where} \quad U_{overall} = (Uf_{area})_{air\ space} + (Uf_{area})_{stud}$$

and the value of the area fraction f_{area} is 0.84 for air space and 0.16 for the ferrings and similar structures. Using the available *R*-values from Tables 21-10 and 21-13 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.



Construction	*R*-value, m².°C/W	
	Between furring	At furring
1. Outside surface, 24 km/h	0.030	0.030
2. Face brick, 100 mm	0.12	0.12
3. Air space, 90-mm, nonreflective	0.16	0.16
4. Concrete block, lightweight, 100-mm	0.27	0.27
5a. Air space, 20 mm, nonreflective	0.17	---
5b. Vertical ferring, 20 mm thick	---	0.94
6. Gypsum wallboard, 13	0.079	0.079
7. Inside surface, still air	0.12	0.12

Total unit thermal resistance of each section, *R*	0.949	1.719
The *U*-factor of each section, $U = 1/R$, in W/m².°C	1.05	0.582
Area fraction of each section, f_{area}	0.84	0.16
Overall *U*-factor, $U = \Sigma f_{area,i} U_i = 0.84 \times 1.05 + 0.16 \times 0.582$	**0.978 W/m².°C**	
Overall unit thermal resistance, $R = 1/U$	**1.02 m².°C/W**	

Therefore, the overall unit thermal resistance of the wall is $R = 1.02$ m².°C/W and the overall U-factor is $U = 0.978$ W/m².°C. These values account for the effects of the vertical ferring.

21-64 The winter *R*-value and the *U*-factor of a masonry cavity wall with a reflective surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 21-10. The *R*-values of air spaces are given in Table 21-13.

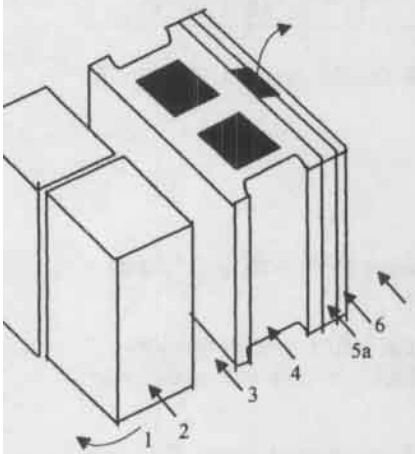
Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + \quad (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.84 for air space and 0.16 for the ferrings and similar structures. For an air space with one-reflective surface, we have $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0.9$, and thus

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{1}{1/0.05 + 1/0.9 - 1} = 0.05$$

Using the available *R*-values from Tables 21-10 and 21-13 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.



Construction	*R*-value, m².°C/W	
	Between furring	At furring
1. Outside surface, 24 km/h	0.030	0.030
2. Face brick, 100 mm	0.12	0.12
3. Air space, 90-mm, reflective with ε = 0.05	0.45	0.45
4. Concrete block, lightweight, 100-mm	0.27	0.27
5a. Air space, 20 mm, reflective with ε =0.05	0.49	---
5b. Vertical ferring, 20 mm thick	---	0.94
6. Gypsum wallboard, 13	0.079	0.079
7. Inside surface, still air	0.12	0.12

Total unit thermal resistance of each section, *R*	1.559	2.009
The *U*-factor of each section, $U = 1/R$, in W/m².°C	0.641	0.498
Area fraction of each section, f_{area}	0.84	0.16
Overall *U*-factor, $U = \Sigma f_{\text{area},i} U_i = 0.84 \times 1.05 + 0.16 \times 0.582$	**0.618 W/m².°C**	
Overall unit thermal resistance, $R = 1/U$	**1.62 m².°C/W**	

Therefore, the overall unit thermal resistance of the wall is $R = 1.62$ m².°C/W and the overall U-factor is $U = 0.618$ W/m².°C. These values account for the effects of the vertical ferring.

Discussion The change in the *U*-value as a result of adding reflective surfaces is

$$\text{Change} = \frac{\Delta U - \text{value}}{U - \text{value, nonreflective}} = \frac{0.978 - 0.618}{0.978} = 0.368$$

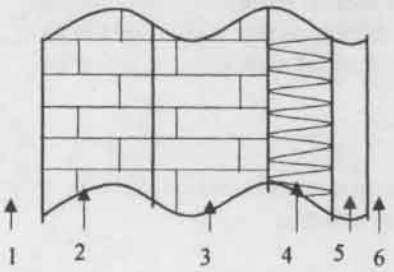
Therefore, the rate of heat transfer through the wall will decrease by 36.8% as a result of adding a reflective surface.

21-65 The winter *R*-value and the *U*-factor of a masonry wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 21-10.

Analysis Using the available *R*-values from Tables 21-10, the total *R*-value of the wall is determined in the table below.



Construction	*R*-value, $m^2.°C/W$
1. Outside surface, 24 km/h	0.030
2. Face brick, 100 mm	0.075
3. Common brick, 100 mm	0.12
4. Urethane foam insulation, 25-mm	0.98
5. Gypsum wallboard, 13 mm	0.079
6. Inside surface, still air	0.12

Total unit thermal resistance of each section, *R*	**1.404 $m^2.°C/W$**
The *U*-factor of each section, $U = 1/R$	**0.712 $W/m^2.°C$**

Therefore, the overall unit thermal resistance of the wall is $R = 1.404\ m^2.°C/W$ and the overall U-factor is $U = 0.712\ W/m^2.°C$.

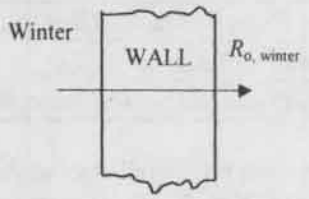
21-66 The *U*-value of a wall under winter design conditions is given. The *U*-value of the wall under summer design conditions is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant except the one at the outer surface.

Properties The R-values at the outer surface of a wall for summer (12 km/h winds) and winter (24 km/h winds) conditions are given in Table 21-10 to be $R_{o,\,summer} = 0.044\ m^2.°C/W$ and $R_{o,\,winter} = 0.030\ m^2.°C/W$.

Analysis The *R*-value of the existing wall is

$$R_{winter} = 1/U_{winter} = 1/1.55 = 0.645\ m^2 \cdot °C/W$$

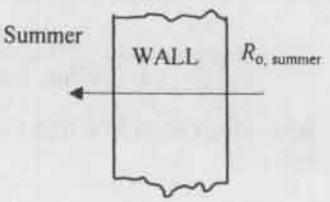


Noting that the added and removed thermal resistances are in series, the overall *R*-value of the wall under summer conditions becomes

$$\begin{aligned} R_{summer} &= R_{winter} - R_{o,winter} + R_{o,summer} \\ &= 0.645 - 0.030 + 0.044 \\ &= 0.659\ m^2 \cdot °C/W \end{aligned}$$

Then the summer *U*-value of the wall becomes

$$R_{summer} = 1/U_{summer} = 1/0.659 = \mathbf{1.52\ m^2 \cdot °C/W}$$



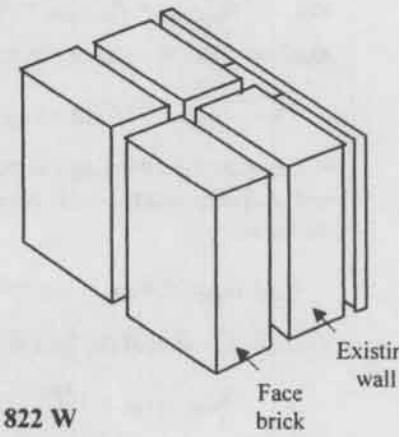
21-67 The U-value of a wall is given. A layer of face brick is added to the outside of a wall, leaving a 20-mm air space between the wall and the bricks. The new U-value of the wall and the rate of heat transfer through the wall is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The U-value of a wall is given to be $U = 2.25$ W/m^2.°C. The R - values of 100-mm face brick and a 20-mm air space between the wall and the bricks various layers are 0.075 and 0.170 m^2.°C/W, respectively.

Analysis The R-value of the existing wall for the winter conditions is

$$R_{\text{existing wall}} = 1/U_{\text{existing wall}} = 1/2.25 = 0.444 \text{ m}^2\cdot\text{°C/W}$$



Noting that the added thermal resistances are in series, the overall R-value of the wall becomes

$$R_{\text{modified wall}} = R_{\text{existing wall}} + R_{\text{brick}} + R_{\text{air layer}}$$
$$= 0.44 + 0.075 + 0.170 = 0.689 \text{ m}^2\cdot\text{°C/W}$$

Then the U-value of the wall after modification becomes

$$R_{\text{modified wall}} = 1/U_{\text{modified wall}} = 1/0.689 = \mathbf{1.45 \text{ m}^2\cdot\text{°C/W}}$$

The rate of heat transfer through the modified wall is

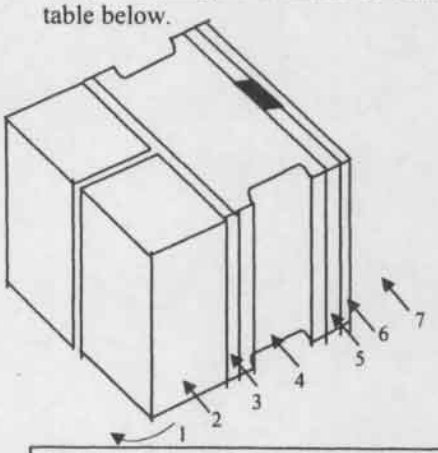
$$\dot{Q}_{\text{wall}} = (UA)_{\text{wall}}(T_i - T_o) = (1.45 \text{ W/m}^2\cdot\text{°C})(21 \text{ m}^2)[22 - (-5)\text{°C}] = \mathbf{822 \text{ W}}$$

21-68 The summer and winter R-values of a masonry wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant. **4** The air cavity does not have any reflecting surfaces.

Properties The R-values of different materials are given in Table 21-10.

Analysis Using the available R-values from Tables 21-10, the total R-value of the wall is determined in the table below.



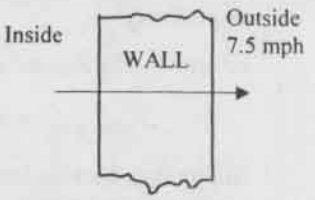
Construction	R-value, m^2.°C/W	
	Summer	Winter
1a. Outside surface, 24 km/h (winter)	---	0.030
1b. Outside surface, 12 km/h (summer)	0.044	---
2. Face brick, 100 mm	0.075	0.075
3. Cement mortar, 13 mm	0.12	0.12
4. Concrete block, lightweight, 100 mm	0.27	0.27
5. Air space, nonreflecting, 40-mm	0.16	0.16
5. Plaster board, 20 mm	0.122	0.122
6. Inside surface, still air	0.12	0.12
Total unit thermal resistance of each section (the R-value), m^2.°C/W	**0.809**	**0.795**

Therefore, the overall unit thermal resistance of the wall is $R = 0.809$ m^2.°C/W in summer and $R = 0.795$ m^2.°C/W in winter.

21-69E The *U*-value of a wall for 7.5 mph winds outside are given. The *U*-value of the wall for the case of 15 mph winds outside is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant except the one at the outer surface.

Properties The R-values at the outer surface of a wall for summer (7.5 mph winds) and winter (15 mph winds) conditions are given in Table 21-10 to be



$$R_{o,\ 7.5\ mph} = R_{o,\ summer} = 0.25\ h.ft^2.°F/Btu$$

and $$R_{o,\ 15\ mph} = R_{o,\ winter} = 0.17\ h.ft^2.°F/Btu$$

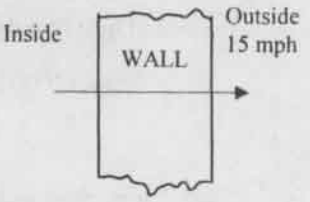
Analysis The *R*-value of the wall at 7.5 mph winds (summer) is

$$R_{wall,\ 7.5\ mph} = 1/U_{wall,\ 7.5\ mph} = 1/0.09 = 11.11\ h.ft^2.°F/Btu$$

Noting that the added and removed thermal resistances are in series, the overall *R*-value of the wall at 15 mph (winter) conditions is obtained by replacing the summer value of outer convection resistance by the winter value,

$$R_{wall,\ 15\ mph} = R_{wall,\ 7.5\ mph} - R_{o,\ 7.5\ mph} + R_{o,\ 15\ mph} = 11.11 - 0.25 + 0.17 = 11.03\ h.ft^2.°F/Btu$$

Then the U-value of the wall at 15 mph winds becomes



$$R_{wall,\ 15\ mph} = 1/U_{wal,\ 15\ mph} = 1/11.03 = \mathbf{0.0907\ h.ft^2.°F/Btu}$$

Discussion Note that the effect of doubling the wind velocity on the *U*-value of the wall is less than 1 percent since

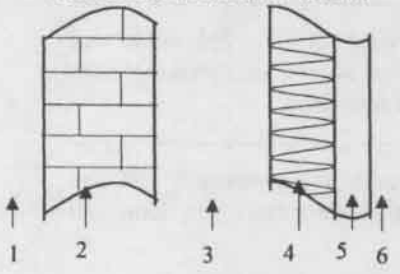
$$\text{Change} = \frac{\Delta U - \text{value}}{U - \text{value}} = \frac{0.0907 - 0.09}{0.09} = 0.0078 \quad (\text{or } 0.78\%)$$

21-70 Two homes are identical, except that their walls are constructed differently. The house that is more energy efficient is to be determined.

Assumptions **1** The homes are identical, except that their walls are constructed differently. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 21-10.

Analysis Using the available *R*-values from Tables 21-10, the total *R*-value of the masonry wall is determined in the table below.



Construction	*R*-value, $m^2.°C/W$
1. Outside surface, 24 km/h (winter)	0.030
2. Concrete block, light weight, 200 mm	2×0.27=0.54
3. Air space, nonreflecting, 20 mm	0.17
4. Urethane foam insulation, 25-mm	0.98
5. Plasterboard, 20 mm	0.12
6. Inside surface, still air	0.12

Total unit thermal resistance (the *R*-value)	**0.98 $m^2.°C/W$**

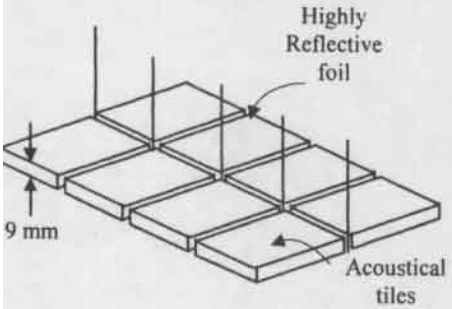
which is less than 2.4 $m^2.°C/W$. Therefore, the standard *R*-2.4 $m^2.°C/W$ wall is better insulated and thus it is more energy efficient.

21-71 A ceiling consists of a layer of reflective acoustical tiles. The *R*-value of the ceiling is to be determined for winter conditions.

Assumptions **1** Heat transfer through the ceiling is one-dimensional. **3** Thermal properties of the ceiling and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Tables 21-10 and 21-11.

Analysis Using the available *R*-values, the total *R*-value of the ceiling is determined in the table below.



Construction	*R*-value, $m^2.°C/W$
1. Still air, reflective horizontal surface facing up	$R = 1/h = 1/4.32$ = 0.23
2. Acoustic tile, 19 mm	2×0.27 = 0.54
3. Still air, horizontal surface, facing down	$R = 1/h = 1/9.26$ = 0.11

Total unit thermal resistance (the *R*-value)	**0.76 $m^2.°C/W$**

Therefore, the R-value of the hanging ceiling is 0.76 $m^2.°C/W$.

Heat Loss from Basement Walls and Floors

21-72C The mechanism of heat transfer from the basement walls and floors to the ground is *conduction heat transfer* because of the direct contact between the walls and the floor. The rate of heat loss through the ground depends on the thermal conductivity of the soil, which depends on the composition and moisture content of the soil. The higher the moisture content, the higher the thermal conductivity, and the higher the rate of heat transfer.

21-73C For a basement wall that is completely below grade, the heat loss through the upper half of the wall will be greater than the heat loss through the lower half since the heat at a lower section must pass thorough a longer path to reach the ground surface, and thus overcome a larger thermal resistance.

21-74C A building loses more heat to the ground through the below grade section of the basement wall than it does through the floor of a basement per unit surface area. This is because the floor has a very long path for heat transfer to the ground surface compared to the wall.

21-75C Heat transfer from a floor on grade at ground level is proportional to the perimeter of the floor, not the surface area.

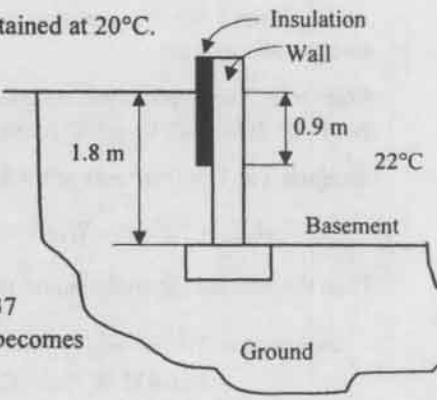
21-76C Venting a crawl space increases heat loss through the floor since it will expose the bottom of the floor to a lower temperature in winter. Venting a crawl space in summer will increase heat gain through the floor since it will expose the bottom of the floor to a higher temperature.

21-77C The cold water pipes in an unheated crawl space in winter does not need to be insulated to avoid the danger of freezing in winter if the vents of the crawl space are tightly closed since, in this case, the temperature in the crawl space will be somewhere between the house temperature and the ambient temperature that will normally be above freezing.

21-78 The peak heat loss from a below grade basement in Anchorage, Alaska to the ground through its walls and the floor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The basement is maintained at 20°C.

Properties The heat transfer coefficients are given in Table 21-14a, and the amplitudes in Fig. 21-37.



Solution The floor and wall areas of the basement are

$$A_{\text{wall}} = \text{Height} \times \text{Perimeter} = 2 \times (1.8\ \text{m})(7 + 10\ \text{m}) = 61.2\ \text{m}^2$$

$$A_{\text{floor}} = \text{Length} \times \text{Width} = (7\ \text{m})(10\ \text{m}) = 70\ \text{m}^2$$

The amplitude of the annual soil temperature is determined from Fig. 21-37 to be 10°C. Then the ground surface temperature for the design heat loss becomes

$$T_{\text{ground surface}} = T_{\text{winter, mean}} - A = -5 - 10 = -15°\text{C}$$

The top 0.9-m section of the wall below the grade is insulated with *R*-2.2, and the heat transfer coefficients through that section are given in Table 21-14a to be 1.27, 1.20, and 1.00 W/m^2.°C through the 1st, 2nd, and 3rd 0.3-m wide depth increments, respectively. The heat transfer coefficients through the uninsulated section of the wall which extends from 0.9 m to 1.8 m level is determined from the same table to be 2.23, 1.80, and 1.50 W/m^2.°C for each of the remaining 0.3-m wide depth increments. The average overall heat transfer coefficient is

$$U_{\text{wall, ave}} = \frac{\sum U_{\text{wall}}}{\text{No. of increments}} = \frac{1.27 + 1.2 + 1.0 + 2.23 + 1.8 + 1.5}{6} = 1.50\ \text{W/m}^2.°\text{C}$$

Then the heat loss through the basement wall becomes

$$\dot{Q}_{\text{basement walls}} = U_{\text{wall, ave}} A_{\text{wall}} (T_{\text{basement}} - T_{\text{ground surface}})$$

$$= (1.50\ \text{W/m}^2.°\text{C})(61.2\ \text{m}^2)[20 - (-15)°\text{C}] = 3213\ \text{W}$$

The shortest width of the house is 7 m, and the depth of the foundation below grade is 1.8 m. The floor heat transfer coefficient is given in Table 21-14b to be 0.15 W/m^2.°C. Then the heat loss through the floor of the basement becomes

$$\dot{Q}_{\text{basement floor}} = U_{\text{floor}} A_{\text{floor}} (T_{\text{basement}} - T_{\text{ground surface}})$$

$$= (0.15\ \text{W/m}^2.°\text{C})(70\ \text{m}^2)[20 - (-15)]°\text{C} = 368\ \text{W}$$

which is considerably less than the heat loss through the wall. The total heat loss from the basement is then determined to be

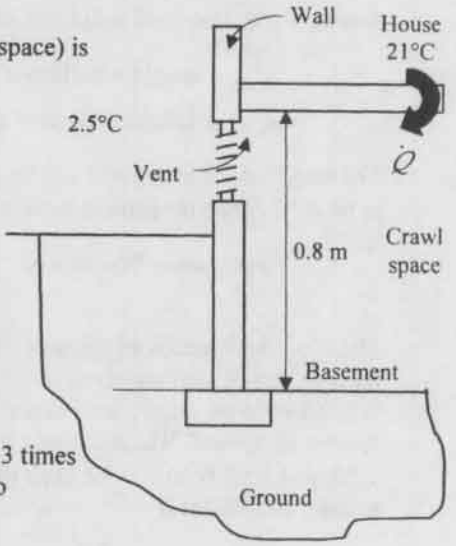
$$\dot{Q}_{\text{basement}} = \dot{Q}_{\text{basement wall}} + \dot{Q}_{\text{basement floor}} = 3213 + 368 = \mathbf{3581\ W}$$

Discussion This is the *design* or *peak* rate of heat transfer from below-grade section of the basement, and this is the value to be used when sizing the heating system. The actual heat loss from the basement will be much less than that most of the time.

21-79 The vent of the crawl space is kept open. The rate of heat loss to the crawl space through insulated and uninsulated floors is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The vented crawl space temperature is the same as the ambient temperature.

Properties The overall heat transfer coefficient for the insulated floor is given in Table 21-15 to be 0.432 W/m^2.°C. It is 1.42 W/m^2.°C for the uninsulated floor.



Analysis (*a*) The floor area of the house (or the ceiling area of the crawl space) is

$$A_{\text{floor}} = \text{Length} \times \text{Width} = (8\ \text{m})(12\ \text{m}) = 96\ \text{m}^2$$

Then the heat loss from the house to the crawl space becomes

$$\dot{Q}_{\text{insulated floor}} = U_{\text{insulated floor}} A_{\text{floor}} (T_{\text{indoor}} - T_{\text{crawl}})$$
$$= (0.432\ \text{W/m}^2\cdot°\text{C})(180\ \text{m}^2)(21-2.5)°\text{C} = \mathbf{1439\ W}$$

(*b*) The heat loss for the uninsulated case is determined similarly to be

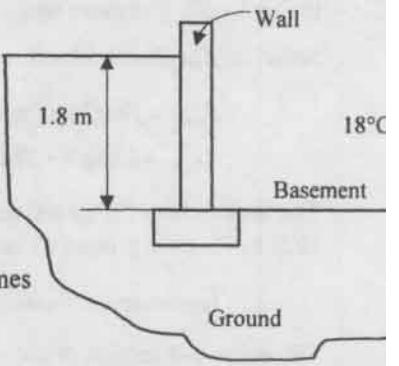
$$\dot{Q}_{\text{uninsulated floor}} = U_{\text{uninsulated floor}} A_{\text{floor}} (T_{\text{indoor}} - T_{\text{crawl}})$$
$$= (1.42\ \text{W/m}^2\cdot°\text{C})(180\ \text{m}^2)(21-2.5)°\text{C} = \mathbf{4729\ W}$$

Discussion Note that heat loss through the uninsulated floor is more than 3 times the heat loss through the insulated floor. Therefore, it is a good practice to insulate floors when the crawl space is ventilated to conserve energy and enhance comfort.

21-80 The peak heat loss from a below grade basement in Boise, Idaho to the ground through its walls and the floor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The basement is maintained at 18°C.

Properties The heat transfer coefficients are given in Table 21-14a, and the amplitudes in Fig. 21-37. The mean winter temperature in Boise is 4.6°C (Table 21-5).



Solution The floor and wall areas of the basement are

$$A_{\text{wall}} = \text{Height} \times \text{Perimeter} = 2 \times (1.8\ \text{m})(8+12\ \text{m}) = 108\ \text{m}^2$$

$$A_{\text{floor}} = \text{Length} \times \text{Width} = (8\ \text{m})(12\ \text{m}) = 96\ \text{m}^2$$

The amplitude of the annual soil temperature is determined from Fig. 21-37 to be 11°C. Then the ground surface temperature for the design heat loss becomes

$$T_{\text{ground surface}} = T_{\text{winter, mean}} - A = 4.6 - 11 = -6.4^\circ\text{C}$$

The entire 1.8-m section of the wall below the grade is uninsulated, and the heat transfer coefficients through that section are given in Table 21-14a to be 7.77, 4.20, 2.93, 2.23, 1.80, and 1.50 W/m^2.°C for each 0.3-m wide depth increments. The average overall heat transfer coefficient is

$$U_{\text{wall, ave}} = \frac{\sum U_{\text{wall}}}{\text{No. of increments}} = \frac{7.77+4.20+2.93+2.23+1.8+1.5}{6} = 3.405\ \text{W/m}^2\cdot^\circ\text{C}$$

Then the heat loss through the basement wall becomes

$$\dot{Q}_{\text{basement walls}} = U_{\text{wall, ave}} A_{\text{wall}} (T_{\text{basement}} - T_{\text{ground surface}})$$
$$= (3.405\ \text{W/m}^2\cdot^\circ\text{C})(108\ \text{m}^2)[18-(-6.4)^\circ\text{C}] = 8973\ \text{W}$$

The shortest width of the house is 12 m, and the depth of the foundation below grade is 1.8 m. The floor heat transfer coefficient is given in Table 21-14b to be 0.10 W/m^2.°C. Then the heat loss through the floor of the basement becomes

$$\dot{Q}_{\text{basement floor}} = U_{\text{floor}} A_{\text{floor}} (T_{\text{basement}} - T_{\text{ground surface}})$$
$$= (0.10\ \text{W/m}^2\cdot^\circ\text{C})(96\ \text{m}^2)[18-(-6.4)]^\circ\text{C} = 234\ \text{W}$$

which is considerably less than the heat loss through the wall. The total heat loss from the basement is then determined to be

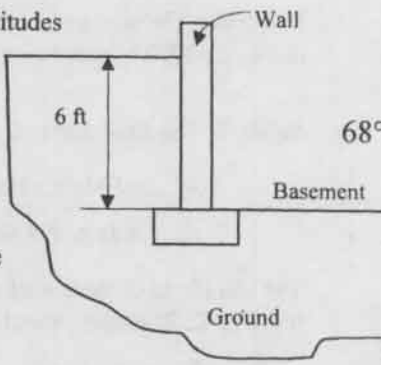
$$\dot{Q}_{\text{basement}} = \dot{Q}_{\text{basement wall}} + \dot{Q}_{\text{basement floor}} = 8973 + 234 = \mathbf{9207\ W}$$

Discussion This is the *design* or *peak* rate of heat transfer from below-grade section of the basement, and this is the value to be used when sizing the heating system. The actual heat loss from the basement will be much less than that most of the time.

21-81E The peak heat loss from a below grade basement in Boise, Idaho to the ground through its walls and the floor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The basement is maintained at 68°F.

Properties The heat transfer coefficients are given in Table 21-14a, and the amplitudes in Fig. 21-37. The mean winter temperature in Boise is 39.7°F (Table 21-5).



Solution The floor and wall areas of the basement are

$$A_{\text{wall}} = \text{Height} \times \text{Perimeter} = 2 \times (6\ \text{ft})(32 + 60\ \text{ft}) = 1104\ \text{ft}^2$$

$$A_{\text{floor}} = \text{Length} \times \text{Width} = (32\ \text{fr})(60\ \text{ft}) = 1920\ \text{ft}^2$$

The amplitude of the annual soil temperature is determined from Fig. 21-37 to be 19.8°F. Then the ground surface temperature for the design heat loss becomes

$$T_{\text{ground surface}} = T_{\text{winter, mean}} - A = 39.7 - 19.8 = 19.9°\text{F}$$

The entire 6-ft section of the wall below the grade is uninsulated, and the heat transfer coefficients through that section are given in Table 21-14a to be 0.410, 0.222, 0.155, 0.119, 0.096, 0.079 Btu/h.ft^2.°F for each 1-ft wide depth increments. The average overall heat transfer coefficient is

$$U_{\text{wall, ave}} = \frac{\sum U_{\text{wall}}}{\text{No. of increments}} = \frac{0.410 + 0.222 + 0.155 + 0.119 + 0.096 + 0.079}{6} = 0.180\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}$$

Then the heat loss through the basement wall becomes

$$\dot{Q}_{\text{basement walls}} = U_{\text{wall, ave}} A_{\text{wall}} (T_{\text{basement}} - T_{\text{ground surface}})$$

$$= (0.180\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F})(1104\ \text{ft}^2)(68 - 19.9)°\text{F} = 9558\ \text{Btu/h}$$

The shortest width of the house is 32 ft, and the depth of the foundation below grade is 6 ft. The floor heat transfer coefficient is given in Table 21-14b to be 0.02 Btu/h.ft^2.°F. Then the heat loss through the floor of the basement becomes

$$\dot{Q}_{\text{basement floor}} = U_{\text{floor}} A_{\text{floor}} (T_{\text{basement}} - T_{\text{ground surface}})$$

$$= (0.022\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F})(1920\ \text{ft}^2)(68 - 19.9)°\text{F} = 2032\ \text{Btu/h}$$

which is considerably less than the heat loss through the wall. The total heat loss from the basement is then determined to be

$$\dot{Q}_{\text{basement}} = \dot{Q}_{\text{basement wall}} + \dot{Q}_{\text{basement floor}} = 9558 + 2032 = \mathbf{11{,}590\ Btu/h}$$

Discussion This is the *design* or *peak* rate of heat transfer from below-grade section of the basement, and this is the value to be used when sizing the heating system. The actual heat loss from the basement will be much less than that most of the time.

21-82 A house with a concrete slab floor sits directly on the ground at grade level, and the wall below grade is insulated. The heat loss from the floor at winter design conditions is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The house is maintained at 22°C. **3** The weather in Baltimore is moderate.

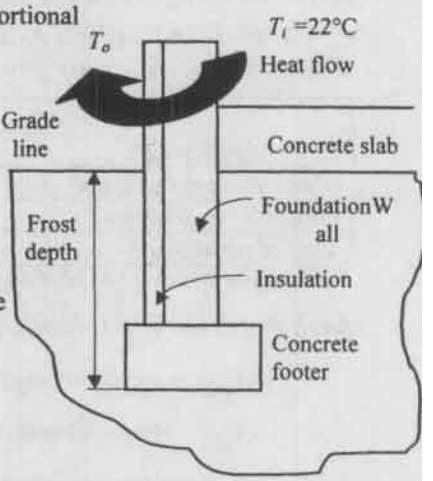
Properties The 97.5% winter design conditions in Baltimore is -11°C (Table 21-4). The heat transfer coefficient for the insulated wall below grade is $U = 0.86$ W/m.°C (Table 21-14c).

Solution Heat transfer from a floor on the ground at the grade level is proportional to the perimeter of the floor, and the perimeter in this case is

$$p_{floor} = 2 \times (\text{Length} + \text{Width}) = 2(15+18)\ \text{m} = 66\ \text{m}$$

Then the heat loss from the floor becomes

$$\dot{Q}_{floor} = U_{floor}\, p_{floor}\, (T_{indoor} - T_{outdoor})$$
$$= (0.86\ \text{W/m}\cdot°\text{C})(66\ \text{m})[22-(-11)]°\text{C} = \mathbf{1873\ W}$$



Discussion This is the *design* or *peak* rate of heat transfer from below-grade section of the basement, and this is the value to be used when sizing the heating system. The actual heat loss from the basement will be much less than that most of the time.

21-83 A house with a concrete slab floor sits directly on the ground at grade level, and the wall below grade is uninsulated. The heat loss from the floor at winter design conditions is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The house is maintained at 22°C. **3** The weather in Baltimore is moderate.

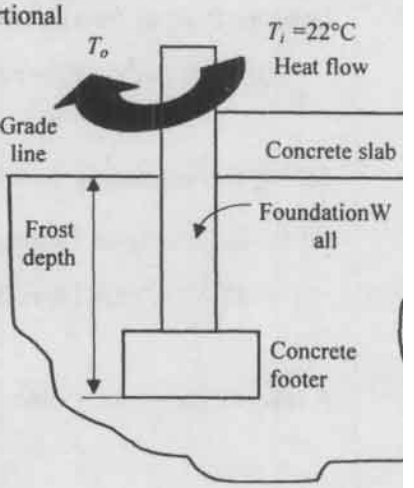
Properties The 97.5% winter design conditions in Baltimore is -11°C (Table 21-4). The heat transfer coefficient for the uninsulated wall below grade is $U = 1.17$ W/m.°C (Table 21-14c).

Solution Heat transfer from a floor on the ground at the grade level is proportional to the perimeter of the floor, and the perimeter in this case is

$$p_{floor} = 2 \times (\text{Length} + \text{Width}) = 2(15+18)\ \text{m} = 66\ \text{m}$$

Then the heat loss from the floor becomes

$$\dot{Q}_{floor} = U_{floor}\, p_{floor}\, (T_{indoor} - T_{outdoor})$$
$$= (1.17\ \text{W/m}\cdot°\text{C})(66\ \text{m})[22-(-11)]°\text{C} = \mathbf{2548\ W}$$



Discussion This is the *design* or *peak* rate of heat transfer from below-grade section of the basement, and this is the value to be used when sizing the heating system. The actual heat loss from the basement will be much less than that most of the time.

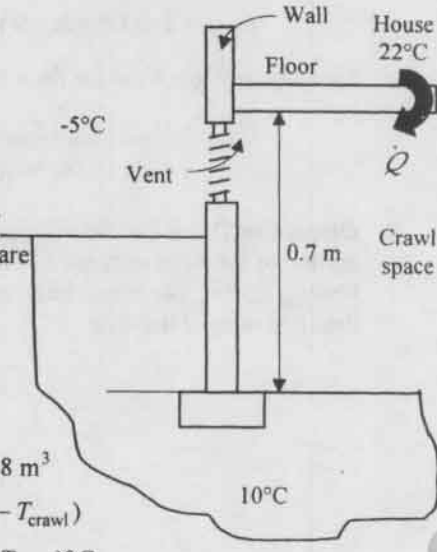
21-84 The vents of the crawl space of a house are kept closed, but air still infiltrates The heat loss from the house to the crawl space and the crawl space temperature are to be determined for the cases of insulated and uninsulated walls, floor, and ceiling of the crawl space.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties and heat transfer coefficients remain constant. **3** The atmospheric pressure is 1 atm.

Properties The indoor and outdoor design temperatures are given to be 22°C and -5°C, respectively, and the deep-down ground temperature is 10°C. The properties of air at -5°C and 1 atm are $\rho = 1.328\ \text{kg/m}^3$ and $C_p = 1.004$ kJ/kg.°C (Table A-18). The overall heat transfer coefficients (the *U*-values) for insulated and uninsulated crawl spaces are given in Table 21-15 to be

Application	*U*, W/m².°C	
	uninsulated	insulated*
Floor above crawl space	1.42	0.432
Ground of crawl space	0.437	0.437
Wall of crawl space	2.77	1.07

*An insulation *R*-value of 1.94 m².°C/W is used on the floor, and 0.95 m².°C/W on the walls.



Analysis (*a*) The floor (ceiling), ground, and wall areas of the crawl space are

$$A_{\text{floor}} = A_{\text{ground}} = \text{Length} \times \text{Width} = (12\ \text{m})(20\ \text{m}) = 240\ \text{m}^2$$

$$A_{\text{wall}} = \text{Height} \times \text{Perimeter} = (0.7\ \text{m})[2 \times (12 + 20\ \text{m})] = 44.8\ \text{m}^2$$

The volume of the crawl space and the infiltration heat loss is

$$V_{\text{crawl}} = (\text{Height})(\text{Width})(\text{Length}) = (0.7\ \text{m})(12\ \text{m})(20\ \text{m}) = 168\ \text{m}^3$$

$$\dot{Q}_{\text{infiltration}} = \rho \dot{V} C_p (T_{\text{ambient}} - T_{\text{crawl}}) = \rho (V_{\text{crawl}} \times ACH) C_p (T_{\text{ambient}} - T_{\text{crawl}})$$

$$= (1.328\ \text{kg/m}^3)(168\ \text{m}^3)(1.2/\text{h})(1004\ \text{J/kg}\cdot\text{°C})(-5 - T_{\text{crawl}})\ \text{°C}$$

$$= 268{,}800(-5 - T_{\text{crawl}})\ \text{J/h} = 74.7(-5 - T_{\text{crawl}})\ \text{W} \quad (\text{since } 1\ \text{h} = 3600\ \text{s})$$

Noting that under steady conditions the net heat transfer to the crawl space is zero, the energy balance for the crawl space can be written as

$$\dot{Q}_{\text{floor}} + \dot{Q}_{\text{wall}} + \dot{Q}_{\text{ground}} + \dot{Q}_{\text{infiltration}} = 0$$

or

$$[UA(T_{\text{indoor}} - T_{\text{crawl}})]_{\text{floor}} + [UA(T_{\text{ambient}} - T_{\text{crawl}})]_{\text{wall}} + [UA(T_{\text{ground}} - T_{\text{crawl}})]_{\text{ground}} + \rho \dot{V} C_p (T_{\text{ambient}} - T_{\text{crawl}}) = 0$$

Using the *U*-values from the table above for the **insulated case** and substituting,

$$(0.432\ \text{W/m}^2\cdot\text{°C})(240\ \text{m}^2)(22 - T_{\text{crawl}})\ \text{°C} + (1.07\ \text{W/m}^2\cdot\text{°C})(44.8\ \text{m}^2)(-5 - T_{\text{crawl}})\ \text{°C}$$
$$+ (0.437\ \text{W/m}^2\cdot\text{°C})(240\ \text{m}^2)(10 - T_{\text{crawl}})\ \text{°C} + 74.7(-5 - T_{\text{crawl}})\ \text{W} = 0$$

Solving for the equation above for the crawl space temperature gives $T_{\text{crawl}} = \mathbf{1.3°C}$.

(*b*) Similarly, using the *U*-values from the table above for the **uninsulated case** and substituting,

$$(1.42\ \text{W/m}^2\cdot\text{°C})(240\ \text{m}^2)(22 - T_{\text{crawl}})\ \text{°C} + (2.77\ \text{W/m}^2\cdot\text{°C})(44.8\ \text{m}^2)(-5 - T_{\text{crawl}})\ \text{°C}$$
$$+ (0.437\ \text{W/m}^2\cdot\text{°C})(240\ \text{m}^2)(10 - T_{\text{crawl}})\ \text{°C} + 74.7(-5 - T_{\text{crawl}})\ \text{W} = 0$$

Solving for the equation above for the crawl space temperature gives $T_{\text{crawl}} = \mathbf{11.7°C}$.

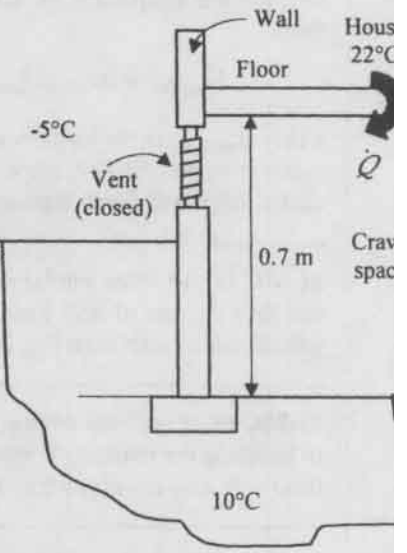
21-85 The vents of the crawl space of a house are tightly sealed, and no air infiltrates. The heat loss from the house to the crawl space and the crawl space temperature are to be determined for the cases of insulated and uninsulated walls, floor, and ceiling of the crawl space.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties and heat transfer coefficients remain constant. **3** The atmospheric pressure is 1 atm. **4** Air infiltration is negligible.

Properties The indoor and outdoor design temperatures are given to be 22°C and -5°C, respectively, and the deep-down ground temperature is 10°C. The properties of air at -5°C and 1 atm are $\rho = 1.328\ \text{kg/m}^3$ and $C_p = 1.004$ kJ/kg.°C (Table A-18). The overall heat transfer coefficients (the *U*-values) for insulated and uninsulated crawl spaces are given in Table 21-15 to be

Application	U, W/m².°C	
	uninsulated	insulated*
Floor above crawl space	1.42	0.432
Ground of crawl space	0.437	0.437
Wall of crawl space	2.77	1.07

*An insulation *R*-value of 1.94 m².°C/W is used on the floor, and 0.95 m².°C/W on the walls.



Analysis (*a*) The floor (ceiling), ground, and wall areas of the crawl space are

$$A_{\text{floor}} = A_{\text{ground}} = \text{Length} \times \text{Width} = (12\ \text{m})(20\ \text{m}) = 240\ \text{m}^2$$

$$A_{\text{wall}} = \text{Height} \times \text{Perimeter} = (0.7\ \text{m})[2 \times (12 + 20\ \text{m})] = 44.8\ \text{m}^2$$

Noting that under steady conditions the net heat transfer to the crawl space is zero, the energy balance for the crawl space can be written as

$$\dot{Q}_{\text{floor}} + \dot{Q}_{\text{wall}} + \dot{Q}_{\text{ground}} = 0$$

or

$$[UA(T_{\text{indoor}} - T_{\text{crawl}})]_{\text{floor}} + [UA(T_{\text{ambient}} - T_{\text{crawl}})]_{\text{wall}} + [UA(T_{\text{ground}} - T_{\text{crawl}})]_{\text{ground}} = 0$$

Using the *U*-values from the table above for the **insulated case** and substituting,

$$(0.432\ \text{W/m}^2\cdot°\text{C})(240\ \text{m}^2)(22 - T_{\text{crawl}})°\text{C} + (1.07\ \text{W/m}^2\cdot°\text{C})(44.8\ \text{m}^2)(-5 - T_{\text{crawl}})°\text{C} + (0.437\ \text{W/m}^2\cdot°\text{C})(240\ \text{m}^2)(10 - T_{\text{crawl}})°\text{C} = 0$$

Solving for the equation above for the crawl space temperature gives $T_{\text{crawl}} = \mathbf{6.5°C}$.

(*b*) Similarly, using the *U*-values from the table above for the **uninsulated case** and substituting,

$$(1.42\ \text{W/m}^2\cdot°\text{C})(240\ \text{m}^2)(22 - T_{\text{crawl}})°\text{C} + (2.77\ \text{W/m}^2\cdot°\text{C})(44.8\ \text{m}^2)(-5 - T_{\text{crawl}})°\text{C} + (0.437\ \text{W/m}^2\cdot°\text{C})(240\ \text{m}^2)(10 - T_{\text{crawl}})°\text{C} = 0$$

Solving for the equation above for the crawl space temperature gives $T_{\text{crawl}} = \mathbf{13.9°C}$.

Heat Transfer Through Windows

21-86C Windows are considered in three regions when analyzing heat transfer through them because the structure and properties of the frame are quite different than those of the glazing. As a result, heat transfer through the frame and the edge section of the glazing adjacent to the frame is two-dimensional. Even in the absence of solar radiation and air infiltration, heat transfer through the windows is more complicated than it appears to be. Therefore, it is customary to consider the windows in three regions when analyzing heat transfer through them: (1) the *center-of-glass*, (2) the *edge-of-glass*, and (3) the *frame* regions. When the heat transfer coefficient for all three regions are known, the overall U-value of the window is determined from

$$U_{\text{window}} = (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}}) / A_{\text{window}}$$

where A_{window} is the window area, and A_{center}, A_{edge}, and A_{frame} are the areas of the center, edge, and frame sections of the window, respectively, and U_{center}, U_{edge}, and U_{frame} are the heat transfer coefficients for the center, edge, and frame sections of the window.

21-87C Of the three similar double pane windows with air gab widths of 5, 10, and 20 mm, the U-factor and thus the rate of heat transfer through the window will be a minimum for the window with 10-mm air gab, as can be seen from Fig. 21-44.

21-88C In an ordinary double pane window, about half of the heat transfer is by radiation. A practical way of reducing the radiation component of heat transfer is to reduce the emissivity of glass surfaces by coating them with low-emissivity (or "low-e") material.

21-89C When a thin polyester film is used to divide the 20-mm wide air of a double pane window space into two 10-mm wide layers, both (*a*) convection and (*b*) radiation heat transfer through the window will be reduced.

21-90C When a double pane window whose air space is flashed and filled with argon gas, (*a*) convection heat transfer will be reduced but (*b*) radiation heat transfer through the window will remain the same.

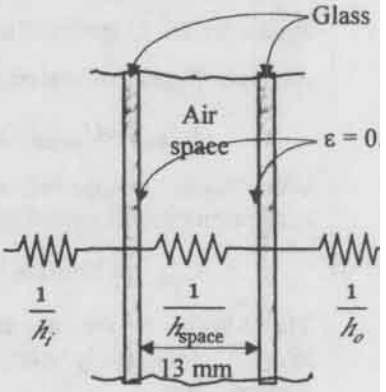
21-91C The heat transfer rate through the glazing of a double pane window is higher at the edge section than it is at the center section because of the two-dimensional effects due to heat transfer through the frame.

21-92C The *U*-factors of windows with aluminum frames will be highest because of the higher conductivity of aluminum. The U-factors of wood and vinyl frames are comparable in magnitude.

21-93 The U-factor for the center-of-glass section of a double pane window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** The thermal resistance of glass sheets is negligible.

Properties The emissivity of clear glass is given to be 0.84. The values of h_i and h_o for winter design conditions are $h_i = 8.29$ W/m^2.°C and $h_o = 34.0$ W/m^2.°C (Table 21-11).



Analysis Disregarding the thermal resistance of glass sheets, which are small, the U-factor for the center region of a double pane window is determined from

$$\frac{1}{U_{\text{center}}} \cong \frac{1}{h_i} + \frac{1}{h_{\text{space}}} + \frac{1}{h_o}$$

where h_i, h_{space}, and h_o are the heat transfer coefficients at the inner surface of window, the air space between the glass layers, and the outer surface of the window, respectively. The effective emissivity of the air space of the double pane window is

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{1}{1/0.84 + 1/0.84 - 1} = 0.72$$

For this value of emissivity and an average air space temperature of 10°C with a temperature difference across the air space to be 15°C, we read $h_{\text{space}} = 5.7$ W/m^2.°C from Table 21-16 for 13-mm thick air space. Therefore,

$$\frac{1}{U_{\text{center}}} = \frac{1}{8.29} + \frac{1}{5.7} + \frac{1}{34.0} \rightarrow U_{\text{center}} = \mathbf{3.07\ W/m^2.°C}$$

Discussion The overall U-factor of the window will be higher because of the edge effects of the frame.

21-94 The rate of heat loss through an double-door wood framed window and the inner surface temperature are to be determined for the cases of single pane, double pane, and low-e triple pane windows.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factors of the windows are given in Table 21-19.

Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}} (T_i - T_o)$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area which is determined to be

$$A_{\text{window}} = \text{Height} \times \text{Width} = (4\ \text{ft})(6\ \text{ft}) = 24\ \text{ft}^2$$

The U-factors for the three cases can be determined directly from Table 21-19 to be 5.57, 2.86, and 1.46 W/m^2.°C, respectively. Also, the inner surface temperature of the window glass can be determined from Newton's law,

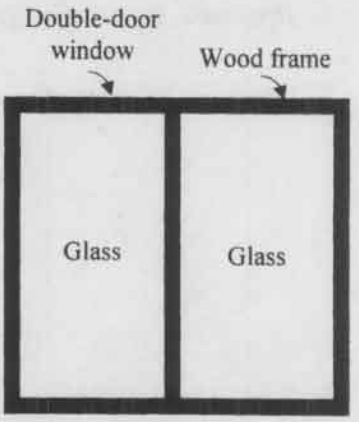
$$\dot{Q}_{\text{window}} = h_i A_{\text{window}} (T_i - T_{\text{glass}}) \quad \rightarrow \quad T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}}$$

where h_i is the heat transfer coefficient on the inner surface of the window which is determined from Table 21-18 to be h_i = 8.3 W/m^2.°C. Then the rate of heat loss and the interior glass temperature for each case are determined as follows:

(*a*) Single glazing:

$$\dot{Q}_{\text{window}} = (5.57\ \text{W/m}^2.^\circ\text{C})(2.16\ \text{m}^2)[20-(-8)]^\circ\text{C} = \mathbf{337\ W}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 20^\circ\text{C} - \frac{337\ \text{W}}{(8.29\ \text{W/m}^2.^\circ\text{C})(2.16\ \text{m}^2)} = \mathbf{1.2^\circ C}$$



(*b*) Double glazing (13 mm air space):

$$\dot{Q}_{\text{window}} = (2.86\ \text{W/m}^2.^\circ\text{C})(2.16\ \text{m}^2)[20-(-8)]^\circ\text{C} = \mathbf{173\ W}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 20^\circ\text{C} - \frac{173\ \text{W}}{(8.29\ \text{W/m}^2.^\circ\text{C})(2.16\ \text{m}^2)} = \mathbf{10.3^\circ C}$$

(*c*) Triple glazing (13 mm air space, low-e coated):

$$\dot{Q}_{\text{window}} = (1.46\ \text{W/m}^2.^\circ\text{C})(2.16\ \text{m}^2)[20-(-8)]^\circ\text{C} = \mathbf{88.3\ W}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 20^\circ\text{C} - \frac{80.3\ \text{W}}{(8.29\ \text{W/m}^2.^\circ\text{C})(2.16\ \text{m}^2)} = \mathbf{15.1^\circ C}$$

Discussion Note that heat loss through the window will be reduced by 49 percent in the case of double glazing and by 74 percent in the case of triple glazing relative to the single glazing case. Also, in the case of single glazing, the low inner glass surface temperature will cause considerable discomfort in the occupants because of the excessive heat loss from the body by radiation. It is raised from 1.2°C to 10.3°C in the case of double glazing and to 15.1°C in the case of triple glazing.

21-95 The overall U-factor for a double-door type window is to be determined, and the result is to be compared to the value listed in Table 21-19.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional.

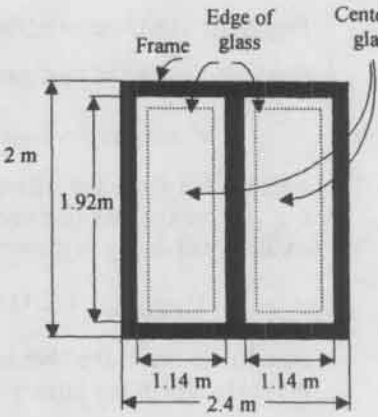
Properties The U-factors for the various sections of windows are given in Tables 21-11 and 21-19.

Analysis The areas of the window, the glazing, and the frame are

$$A_{\text{window}} = \text{Height} \times \text{width} = (2\text{ m})(2.4\text{ m}) = 4.80\text{ m}^2$$

$$A_{\text{glazing}} = 2 \times (\text{Height} \times \text{width}) = 2(1.92\text{ m})(1.14\text{ m}) = 4.38\text{ m}^2$$

$$A_{\text{frame}} = A_{\text{window}} - A_{\text{glazing}} = 4.80 - 4.38 = 0.42\text{ m}^2$$



The edge-of-glass region consists of a 6.5-cm wide band around the perimeter of the glazings, and the areas of the center and edge sections of the glazing are determined to be

$$A_{\text{center}} = 2 \times (\text{Height} \times \text{width}) = 2(1.92 - 0.13\text{ m})(1.14 - 0.13\text{ m}) = 362\text{ m}^2$$

$$A_{\text{edge}} = A_{\text{glazing}} - A_{\text{center}} = 4.38 - 23.62 = 0.76\text{ m}^2$$

The U-factor for the frame section is determined from Table 21-17 to be $U_{\text{frame}} = 2.8$ W/m^2.°C. The U-factor for the center and edge sections are determined from Table 21-19 to be $U_{\text{center}} = 2.78$ W/m^2.°C and $U_{\text{edge}} = 3.40$ W/m^2.°C. Then the overall U-factor of the entire window becomes

$$U_{\text{window}} = (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}}) / A_{\text{window}}$$

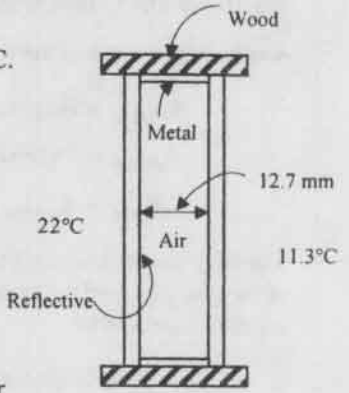
$$= (2.78 \times 3.62 + 3.40 \times 0.76 + 2.8 \times 0.42) / 4.80$$

$$= \mathbf{2.88\ W/m^2.°C}$$

Discussion The overall U-factor listed in Table 21-19 for the specified type of window is 2.86 W/m^2.°C, which is sufficiently close to the value obtained above.

21-96 The windows of a house in Atlanta are of double door type with wood frames and metal spacers. The average rate of heat loss through the windows in winter is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.



Properties The U-factor of the window is given in Table 21-19 to be 2.13 W/m².°C.

Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window, ave}} = U_{\text{overall}} A_{\text{window}} (T_i - T_{o,\text{ave}})$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Substituting,

$$\dot{Q}_{\text{window, ave}} = (2.13\ \text{W/m}^2 \cdot ^\circ\text{C})(20\ \text{m}^2)(22\text{-}11.3)^\circ\text{C} = \mathbf{456\ W}$$

Discussion This is the "average" rate of heat transfer through the window in winter in the absence of any infiltration.

21-97E The *R*-value of the common double door windows that are double pane with 1/4-in of air space and have aluminum frames is to be compared to the *R*-value of *R*-13 wall. It is also to be determined if more heat is transferred through the windows or the walls.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factor of the window is given in Table 21-19 to be 4.66×0.176 = 0.801 Btu/h.ft².°F.

Analysis The *R*-value of the windows is simply the inverse of its *U*-factor, and is determined to be

$$R_{\text{window}} = \frac{1}{U} = \frac{1}{0.801\ \text{Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}} = 1.25\ \text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F/Btu}$$

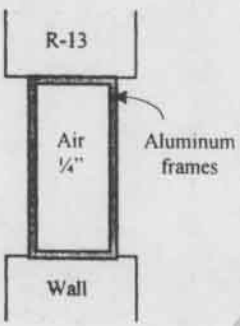
which is less than 13. Therefore, the *R*-value of a double pane window is **much less** than the *R*-value of an *R*-13 wall.

Now consider a 1-ft² section of a wall. The solid wall and the window areas of this section are $A_{\text{wall}} = 0.8\ \text{ft}^2$ and $A_{\text{window}} = 0.2\ \text{ft}^2$. Then the rates of heat transfer through the two sections are determined to be

$$\dot{Q}_{\text{wall}} = U_{\text{wall}} A_{\text{wall}} (T_i - T_o) = A_{\text{wall}} \frac{T_i - T_o}{R\text{-value, wall}} = (0.8\ \text{ft}^2) \frac{\Delta T(^\circ\text{F})}{13\ \text{h.ft}^2.^\circ\text{F/Btu}} = 0.615 \Delta T\ \text{Btu/h}$$

$$\dot{Q}_{\text{window}} = U_{\text{window}} A_{\text{window}} (T_i - T_o) = A_{\text{window}} \frac{T_i - T_o}{R\text{-value}} = (0.2\ \text{ft}^2) \frac{\Delta T(^\circ\text{F})}{1.25\ \text{h.ft}^2.^\circ\text{F/Btu}} = 0.160 \Delta T\ \text{Btu/h}$$

Therefore, the rate of heat transfer through a double pane window is **much more** than the rate of heat transfer through an *R*-13 wall.



Discussion The ratio of heat transfer through the wall and through the window is

$$\frac{\dot{Q}_{\text{wall}}}{\dot{Q}_{\text{window}}} = \frac{0.615 \Delta T\ \text{Btu/h}}{0.160 \Delta T\ \text{Btu/h}} = 2.60$$

Therefore, 2.6 times more heat is lost through the windows than through the walls although the windows occupy only 20% of the wall area.

21-98 The overall U-factor of a window is given to be $U = 2.76$ W/m^2.°C for 12 km/h winds outside. The new U-factor when the wind velocity outside is doubled is to be determined.

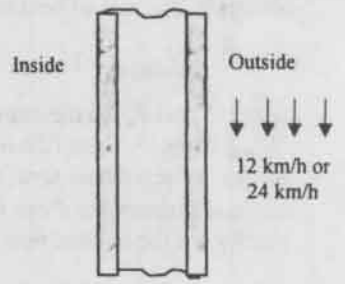
Assumptions Thermal properties of the windows and the heat transfer coefficients are constant.

Properties The heat transfer coefficients at the outer surface of the window are $h_o = 22.7$ W/m^2.°C for 12 km/h winds, and $h_o = 22.7$ W/m^2.°C for 24 km/h winds (Table 21-11).

Analysis The corresponding convection resistances for the outer surfaces of the window are

$$R_{o,12\ km/h} = \frac{1}{h_{o,12\ km/h}} = \frac{1}{22.7\ W/m^2 \cdot °C} = 0.044\ m^2 \cdot °C/W$$

$$R_{o,24\ km/h} = \frac{1}{h_{o,24\ km/h}} = \frac{1}{34.0\ W/m^2 \cdot °C} = 0.029\ m^2 \cdot °C/W$$



Also, the R-value of the window at 12 km/h winds is

$$R_{window,12\ km/h} = \frac{1}{U_{window,12\ km/h}} = \frac{1}{2.76\ W/m^2 \cdot °C} = 0.362\ m^2 \cdot °C/W$$

Noting that all thermal resistances are in series, the thermal resistance of the window for 24 km/h winds is determined by replacing the convection resistance for 12 km/h winds by the one for 24 km/h:

$$R_{window,24\ km/h} = R_{window,12\ km/h} - R_{o,12\ km/h} + R_{o,24\ km/h} = 0.362 - 0.044 + 0.029 = 0.347\ m^2 \cdot °C/W$$

Then the U-factor for the case of 24 km/h winds becomes

$$U_{window,24\ km/h} = \frac{1}{R_{window,24\ km/h}} = \frac{1}{0.347\ m^2 \cdot °C/W} = \mathbf{2.88\ W/m^2 \cdot °C}$$

Discussion Note that doubling of the wind velocity increases the U-factor only slightly (about 4%) from 2.76 to 2.88 W/m^2.°C.

21-99 The existing wood framed single pane windows of an older house in Wichita are to be replaced by double-door type vinyl framed double pane windows with an air space of 6.4 mm. The amount of money the new windows will save the home owner per month is to be determined.

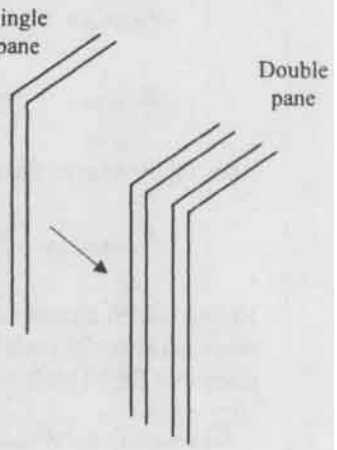
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factors of the windows are 5.57 W/m^2.°C for the old single pane windows, and 3.20 W/m^2.°C. for the new double pane windows (Table 21-19).

Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}} (T_i - T_o)$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Noting that the heaters will turn on only when the outdoor temperature drops below 18°C, the rates of heat transfer due to electric heating for the old and new windows are determined to be



$$\dot{Q}_{\text{window, old}} = (557 \text{ W/m}^2 \cdot \text{°C})(12 \text{ m}^2)(18 - 7.1)\text{°C} = 729 \text{ W}$$

$$\dot{Q}_{\text{window, new}} = (3.20 \text{ W/m}^2 \cdot \text{°C})(12 \text{ m}^2)(18 - 7.1)\text{°C} = 419 \text{ W}$$

$$\dot{Q}_{\text{saved}} = \dot{Q}_{\text{window, old}} - \dot{Q}_{\text{window, new}} = 729 - 419 = 310 \text{ W} = 0.310 \text{ kW}$$

Then the electrical energy and cost savings per month becomes

Energy savings = $\dot{Q}_{\text{saved}} \Delta t$ = (0.310 kW)(30 × 24 h/month) = 223 kWh/month

Cost savings = (Energy savings)(Unit cost of energy) = (223 kWh/month)($0.07/kWh) = **$15.62/month**

Discussion We would obtain the same result if we used the actual indoor temperature (probably 22°C) for T_i instead of the balance point temperature of 18°C.

Solar Heat Gain Through Windows

21-100C (*a*) The spectral distribution of solar radiation beyond the earth's atmosphere resembles the energy emitted by a black body at 5982°C, with about 39 percent in the visible region (0.4 to 0.7 μm), and the 52 percent in the near infrared region (0.7 to 3.5 μm). (*b*) At a solar altitude of 41.8°, the total energy of direct solar radiation incident at sea level on a clear day consists of about 3 percent ultraviolet, 38 percent visible, and 59 percent infrared radiation.

21-101C A window that transmits visible part of the spectrum while absorbing the infrared portion is ideally suited for minimizing the air-conditioning load since such windows provide maximum daylighting and minimum solar heat gain. The ordinary window glass approximates this behavior remarkably well.

21-102C The **solar heat gain coefficient** (SHGC) is defined as the fraction of incident solar radiation that enters through the glazing. The solar heat gain of a glazing relative to the solar heat gain of a reference glazing, typically that of a standard 3 mm (1/8 in) thick double-strength clear window glass sheet whose SHGC is 0.87, is called the **shading coefficient**. They are related to each other by

$$SC = \frac{\text{Solar heat gain of product}}{\text{Solar heat gain of reference glazing}} = \frac{SHGC}{SHGC_{ref}} = \frac{SHGC}{0.87} = 1.15 \times SHGC$$

For single pane clear glass window, SHGC = 0.87 and SC = 1.0.

21-103C The SC (shading coefficient) of a device represents the solar heat gain relative to the solar heat gain of a reference glazing, typically that of a standard 3 mm (1/8 in) thick double-strength clear window glass sheet whose SHGC is 0.87. The shading coefficient of a 3-mm thick *clear glass* is SC = 1.0 whereas SC = 0.88 for 3-mm thick *heat absorbing glass*.

21-104C A device that blocks solar radiation and thus reduces the solar heat gain is called a shading device. External shading devices are more effective in reducing the solar heat gain since they intercept sun's rays before they reach the glazing. The solar heat gain through a window can be reduced by as much as 80 percent by exterior shading. *Light colored* shading devices maximize the back reflection and thus minimize the solar gain. *Dark colored* shades, on the other hand, minimize the back refection and thus maximize the solar heat gain.

21-105C A low-e coating on the inner surface of a window glass reduces both the (*a*) heat loss in winter and (*b*) heat gain in summer. This is because the radiation heat transfer to or from the window is proportional to the emissivity of the inner surface of the window. In winter, the window is colder and thus radiation heat loss from the room to the window is low. In summer, the window is hotter and the radiation transfer from the window to the room is low.

21-106C Glasses coated with reflective films on the outer surface of a window glass reduces solar heat both in summer and in winter.

21-107 The net annual cost savings due to installing reflective coating on the West windows of a building and the simple payback period are to be determined.

Assumptions **1** The calculations given below are for an average year. **2** The unit costs of electricity and natural gas remain constant.

Analysis Using the daily averages for each month and noting the number of days of each month, the total solar heat flux incident on the glazing during summer and winter months are determined to be

$$Q_{\text{solar, summer}} = 4.24\times30 + 4.16\times31 + 3.93\times31 + 3.48\times30 = 482 \text{ kWh/year}$$

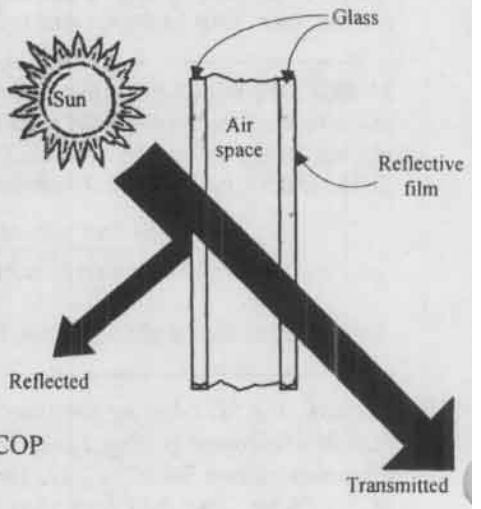
$$Q_{\text{solar, winter}} = 2.94\times31 + 2.33\times30 + 2.07\times31 + 2.35\times31 + 3.03\times28 + 3.62\times31 + 4.00\times30 = 615 \text{ kWh/year}$$

Then the decrease in the annual cooling load and the increase in the annual heating load due to reflective film become

$$\text{Cooling load decrease} = Q_{\text{solar, summer}} A_{\text{glazing}} (\text{SHGC}_{\text{without film}} - \text{SHGC}_{\text{with film}})$$
$$= (482 \text{ kWh/year})(60 \text{ m}^2)(0.766-0.261)$$
$$= 14{,}605 \text{ kWh/year}$$

$$\text{Heating load increase} = Q_{\text{solar, winter}} A_{\text{glazing}} (\text{SHGC}_{\text{without film}} - \text{SHGC}_{\text{with film}})$$
$$= (615 \text{ kWh/year})(60 \text{ m}^2)(0.766-0.261)$$
$$= 18{,}635 \text{ kWh/year} = 635.8 \text{ therms/year}$$

since 1 therm = 29.31 kWh. The corresponding decrease in cooling costs and the increase in heating costs are



$$\text{Decrease in cooling costs} = (\text{Cooling load decrease})(\text{Unit cost of electricity})/\text{COP}$$
$$= (14{,}605 \text{ kWh/year})(\$0.09/\text{kWh})/3.2 = \$411/\text{year}$$

$$\text{Increase in heating costs} = (\text{Heating load increase})(\text{Unit cost of fuel})/\text{Efficiency}$$
$$= (635.8 \text{ therms/year})(\$0.45/\text{therm})/0.80 = \$358/\text{year}$$

Then the net annual cost savings due to reflective films become

$$\text{Cost Savings} = \text{Decrease in cooling costs} - \text{Increase in heating costs}$$
$$= \$411 - 358 = \$53/\text{year}$$

The implementation cost of installing films is

$$\text{Implementation Cost} = (\$20/\text{m}^2)(60 \text{ m}^2) = \$1200$$

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$1200}{\$53/\text{year}} = 23 \text{ years}$$

Discussion The reflective films will pay for themselves in this case in about 23 years, which is unacceptable to most manufacturers since they are not usually interested in any energy conservation measure which does not pay for itself within 3 years.

21-108 A house located at 40° N latitude has ordinary double pane windows. The total solar heat gain of the house at 9:00, 12:00, and 15:00 solar time in July and the total amount of solar heat gain per day for an average day in January are to be determined.

Assumptions The calculations are performed for an average day in a given month.

Properties The shading coefficient of a double pane window with 6-mm thick glasses is SC = 0.82 (Table 21-21). The incident radiation at different windows at different times are given as (Table 21-20)

Month	Time	Solar radiation incident on the surface, W/m^2			
		North	East	South	West
July	9:00	117	701	190	114
July	12:00	138	149	395	149
July	15:00	117	114	190	701
January	Daily total	446	1863	5897	1863

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq.21-40 to be

$$SHGC = 0.87 \times SC = 0.87 \times 0.82 = 0.7134$$

The rate of solar heat gain is determined from

$$\dot{Q}_{\text{solar gain}} = SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}} = 0.7134 \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}}$$

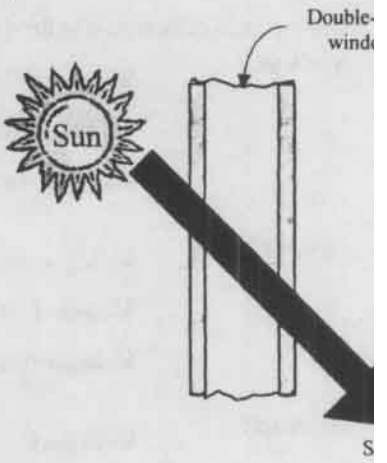
Then the rates of heat gain at the 4 walls at 3 different times in July become

North wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.7134 \times (4\,\text{m}^2) \times (117\,\text{W/m}^2) = \mathbf{334\ W}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134 \times (4\,\text{m}^2) \times (138\,\text{W/m}^2) = \mathbf{394\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.7134 \times (4\,\text{m}^2) \times (117\,\text{W/m}^2) = \mathbf{334\ W}$$



East wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.7134 \times (6\,\text{m}^2) \times (701\,\text{W/m}^2) = \mathbf{3001\ W}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134 \times (6\,\text{m}^2) \times (149\,\text{W/m}^2) = \mathbf{638\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.7134 \times (6\,\text{m}^2) \times (114\,\text{W/m}^2) = \mathbf{488\ W}$$

South wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.7134 \times (8\,\text{m}^2) \times (190\,\text{W/m}^2) = \mathbf{1084\ W}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134 \times (8\,\text{m}^2) \times (395\,\text{W/m}^2) = \mathbf{2254\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.7134 \times (8\,\text{m}^2) \times (190\,\text{W/m}^2) = \mathbf{1084\ W}$$

West wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.7134 \times (6\,\text{m}^2) \times (114\,\text{W/m}^2) = \mathbf{488\ W}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134 \times (6\,\text{m}^2) \times (149\,\text{W/m}^2) = \mathbf{638\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.7134 \times (6\,\text{m}^2) \times (701\,\text{W/m}^2) = \mathbf{3001\ W}$$

Similarly, the solar heat gain of the house through all of the windows in January is determined to be

January:

$$\dot{Q}_{\text{solar gain, North}} = 0.7134 \times (4\,\text{m}^2) \times (446\,\text{Wh/m}^2 \cdot \text{day}) = 1273\ \text{Wh/day}$$

$$\dot{Q}_{\text{solar gain, East}} = 0.7134 \times (6\,\text{m}^2) \times (1863\,\text{Wh/m}^2 \cdot \text{day}) = 7974\ \text{Wh/day}$$

$$\dot{Q}_{\text{solar gain, South}} = 0.7134 \times (8\,\text{m}^2) \times (5897\,\text{Wh/m}^2 \cdot \text{day}) = 33{,}655\ \text{Wh/day}$$

$$\dot{Q}_{\text{solar gain, West}} = 0.7134 \times (6\,\text{m}^2) \times (1863\,\text{Wh/m}^2 \cdot \text{day}) = 7974\ \text{Wh/day}$$

Therefore, for an average day in January,

$$\dot{Q}_{\text{solar gain per day}} = 446 + 1863 + 5897 + 1863 = 58{,}876\ \text{Wh/day} \cong \mathbf{58.9\ kWh/day}$$

21-109 A house located at 40° N latitude has gray-tinted double pane windows. The total solar heat gain of the house at 9:00, 12:00, and 15:00 solar time in July and the total amount of solar heat gain per day for an average day in January are to be determined.

Assumptions The calculations are performed for an average day in a given month.

Properties The shading coefficient of a gray-tinted double pane window with 6-mm thick glasses is SC = 0.58 (Table 21-21). The incident radiation at different windows at different times are given as (Table 21-20)

Month	Time	Solar radiation incident on the surface, W/m^2			
		North	East	South	West
July	9:00	117	701	190	114
July	12:00	138	149	395	149
July	15:00	117	114	190	701
January	Daily total	446	1863	5897	1863

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq.21-40 to be

$$\text{SHGC} = 0.87 \times \text{SC} = 0.87 \times 0.58 = 0.5046$$

The rate of solar heat gain is determined from

$$\dot{Q}_{\text{solar gain}} = SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}} = 0.5046 \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}}$$

Then the rates of heat gain at the 4 walls at 3 different times in July become

North wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.5046 \times (4\,\text{m}^2) \times (117\,\text{W/m}^2) = \mathbf{236\ W}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.5046 \times (4\,\text{m}^2) \times (138\,\text{W/m}^2) = \mathbf{279\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.5046 \times (4\,\text{m}^2) \times (117\,\text{W/m}^2) = \mathbf{236\ W}$$

East wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.5046 \times (6\,\text{m}^2) \times (701\,\text{W/m}^2) = \mathbf{2122\ W}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.5046 \times (6\,\text{m}^2) \times (149\,\text{W/m}^2) = \mathbf{461\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.5046 \times (6\,\text{m}^2) \times (114\,\text{W/m}^2) = \mathbf{345\ W}$$

South wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.5046 \times (8\,\text{m}^2) \times (190\,\text{W/m}^2) = \mathbf{7674\ W}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.5046 \times (8\,\text{m}^2) \times (395\,\text{W/m}^2) = \mathbf{1595\ W}$$

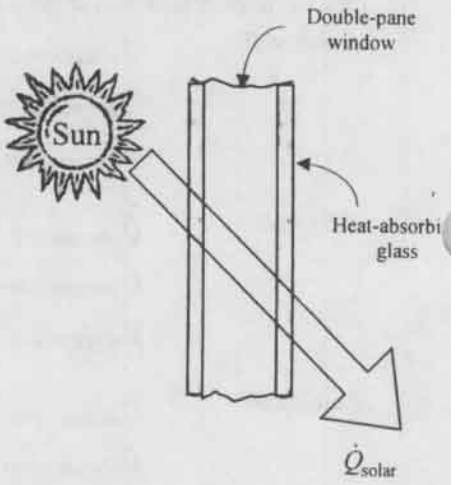
$$\dot{Q}_{\text{solar gain, 15:00}} = 0.5046 \times (8\,\text{m}^2) \times (190\,\text{W/m}^2) = \mathbf{767\ W}$$

West wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.5046 \times (6\,\text{m}^2) \times (114\,\text{W/m}^2) = \mathbf{345\ W}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.5046 \times (6\,\text{m}^2) \times (149\,\text{W/m}^2) = \mathbf{451\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.5046 \times (6\,\text{m}^2) \times (701\,\text{W/m}^2) = \mathbf{2122\ W}$$



Similarly, the solar heat gain of the house through all of the windows in January is determined to be

January:

$$\dot{Q}_{\text{solar gain, North}} = 0.5046 \times (4\,\text{m}^2) \times (446\,\text{Wh/m}^2 \cdot \text{day}) = 900\ \text{Wh/day}$$

$$\dot{Q}_{\text{solar gain, East}} = 0.5046 \times (6\,\text{m}^2) \times (1863\,\text{Wh/m}^2 \cdot \text{day}) = 5640\ \text{Wh/day}$$

$$\dot{Q}_{\text{solar gain, South}} = 0.5046 \times (8\,\text{m}^2) \times (5897\,\text{Wh/m}^2 \cdot \text{day}) = 23{,}805\ \text{Wh/day}$$

$$\dot{Q}_{\text{solar gain, West}} = 0.5046 \times (6\,\text{m}^2) \times (1863\,\text{Wh/m}^2 \cdot \text{day}) = 5640\ \text{Wh/day}$$

Therefore, for an average day in January,

$$\dot{Q}_{\text{solar gain per day}} = 900 + 5640 + 23{,}805 + 5640 = 35{,}985\ \text{Wh/day} = \mathbf{35.895\ kWh/day}$$

21-110 A building at 40° N latitude has double pane heat absorbing type windows that are equipped with light colored venetian blinds. The total solar heat gains of the building through the south windows at solar noon in April for the cases of with and without the blinds are to be determined.

Assumptions The calculations are performed for an "average" day in April, and may vary from location to location.

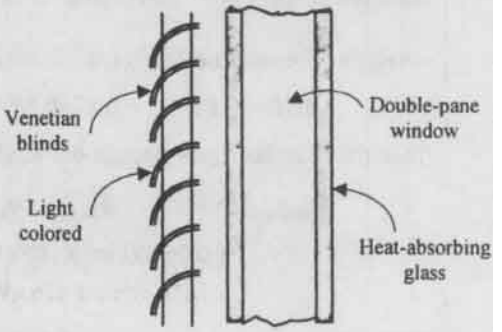
Properties The shading coefficient of a double pane heat absorbing type windows is SC = 0.58 (Table 21-21). It is given to be SC = 0.30 in the case of blinds. The solar radiation incident at a South-facing surface at 12:00 noon in April is 559 W/m^2 (Table 21-20).

Analysis The solar heat gain coefficient (SHGC) of the windows without the blinds is determined from Eq.21-40 to be

$$\text{SHGC} = 0.87 \times \text{SC} = 0.87 \times 0.58 = 0.5046$$

Then the rate of solar heat gain through the window becomes

$$\begin{aligned} \dot{Q}_{\text{solar gain, no blinds}} &= SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}} \\ &= 0.5046(200\ \text{m}^2)(559\ \text{W/m}^2) \\ &= \mathbf{56{,}414\ W} \end{aligned}$$



In the case of windows equipped with venetian blinds, the SHGC and the rate of solar heat gain become

$$\text{SHGC} = 0.87 \times \text{SC} = 0.87 \times 0.30 = 0.261$$

Then the rate of solar heat gain through the window becomes

$$\begin{aligned} \dot{Q}_{\text{solar gain, no blinds}} &= SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}} \\ &= 0.261(200\ \text{m}^2)(559\ \text{W/m}^2) \\ &= \mathbf{29{,}180\ W} \end{aligned}$$

Discussion Note that light colored venetian blinds significantly reduce the solar heat, and thus air-conditioning load in summers.

21-111 A house has double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers. It is to be determined if the house is losing more or less heat than it is gaining from the sun through an east window in a typical day in January.

Assumptions **1** The calculations are performed for an "average" day in January. **2** Solar data at 40° latitude can also be used for a location at 39° latitude.

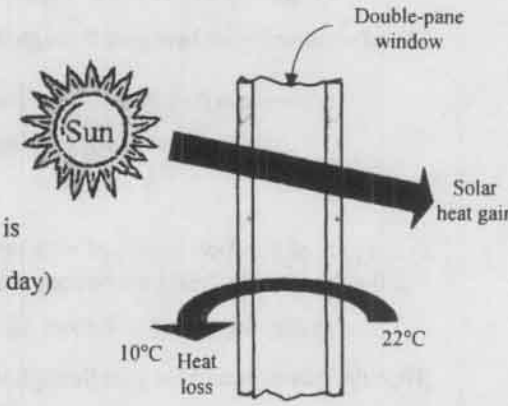
Properties The shading coefficient of a double pane window with 3-mm thick clear glass is SC = 0.88 (Table 21-21). The overall heat transfer coefficient for double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers is 4.55 W/m^2.°C (Table 21-19). The total solar radiation incident at an East-facing surface in January during a typical day is 1863 Wh/m^2 (Table 21-20).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq.21-40 to be

$$\text{SHGC} = 0.87 \times \text{SC} = 0.87 \times 0.88 = 0.7656$$

Then the solar heat gain through the window per unit area becomes

$$\begin{aligned} Q_{\text{solar gain}} &= SHGC \times A_{\text{glazing}} \times q_{\text{solar, daily total}} \\ &= 0.7656(1\,\text{m}^2)(1863\,\text{Wh/m}^2) \\ &= \mathbf{1426\,Wh = 1.426\,kWh} \end{aligned}$$



The heat loss through a unit area of the window during a 24-h period is

$$\begin{aligned} Q_{\text{loss, window}} &= \dot{Q}_{\text{loss, window}}\,\Delta t = U_{\text{window}} A_{\text{window}} (T_i - T_{0,\text{ave}})(1\,\text{day}) \\ &= (4.55\,\text{W/m}^2 \cdot °\text{C})(1\,\text{m}^2)(22-10)°\text{C}(24\,\text{h}) \\ &= \mathbf{1310\,Wh = 1.31\,kWh} \end{aligned}$$

Therefore, the house is **gaining** more heat than it is loosing through the East windows during a typical day in January.

21-112 A house has double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers. It is to be determined if the house is losing more or less heat than it is gaining from the sun through a South window in a typical day in January.

Assumptions **1** The calculations are performed for an "average" day in January. **2** Solar data at 40° latitude can also be used for a location at 39° latitude.

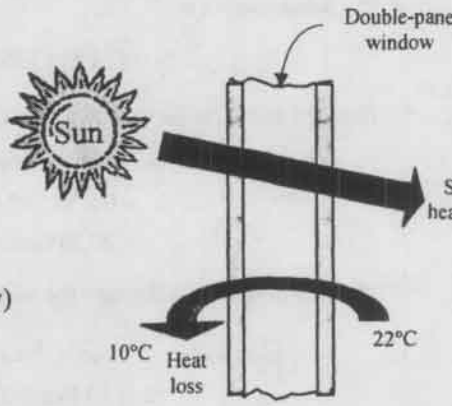
Properties The shading coefficient of a double pane window with 3-mm thick clear glass is SC = 0.88 (Table 21-21). The overall heat transfer coefficient for double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers is 4.55 W/m².°C (Table 21-19). The total solar radiation incident at a South-facing surface in January during a typical day is 5897 Wh/m² (Table 21-20).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq.21-40 to be

$$\text{SHGC} = 0.87 \times \text{SC} = 0.87 \times 0.88 = 0.7656$$

Then the solar heat gain through the window per unit area becomes

$$\begin{aligned} Q_{\text{solar gain}} &= SHGC \times A_{\text{glazing}} \times q_{\text{solar, daily total}} \\ &= 0.7656(1\,\text{m}^2)(5897\,\text{Wh/m}^2) \\ &= \mathbf{4515\,Wh = 4.515\,kWh} \end{aligned}$$



The heat loss through a unit area of the window during a 24-h period is

$$\begin{aligned} Q_{\text{loss, window}} &= \dot{Q}_{\text{loss, window}}\,\Delta t = U_{\text{window}}\,A_{\text{window}}\,(T_i - T_{0,\text{ave}})(1\,\text{day}) \\ &= (4.55\,\text{W/m}^2\cdot\text{°C})(1\,\text{m}^2)(22-10)\text{°C}(24\,\text{h}) \\ &= \mathbf{1310\,Wh = 1.31\,kWh} \end{aligned}$$

Therefore, the house is **gaining** much more heat than it is loosing through the South windows during a typical day in January.

21-113E A house has 1/8-in thick single pane windows with aluminum frames on a West wall. The rate of net heat gain (or loss) through the window at 3 PM during a typical day in January is to be determined.

Assumptions **1** The calculations are performed for an "average" day in January. **2** The frame area relative to glazing area is small so that the glazing area can be taken to be the same as the window area.

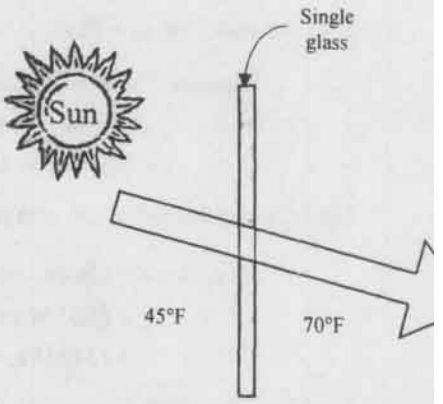
Properties The shading coefficient of a 1/8-in thick single pane window is SC = 1.0 (Table 21-21). The overall heat transfer coefficient for 1/8-in thick single pane windows with aluminum frames is 6.63 $W/m^2.°C$ = 1.17 $Btu/h.ft^2.°F$ (Table 21-19). The total solar radiation incident at a West-facing surface at 3 PM in January during a typical day is 557 W/m^2 = 177 $Btu/h.ft^2$ (Table 21-20).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq.21-40 to be

$$SHGC = 0.87 \times SC = 0.87 \times 1.0 = 0.87$$

The window area is

$$A_{window} = (9\text{ ft})(15\text{ ft}) = 135\text{ ft}^2$$



Then the rate of solar heat gain through the window at 3 PM becomes

$$\dot{Q}_{\text{solar gain, 3 PM}} = SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, 3 PM}}$$
$$= 0.87(135\text{ ft}^2)(177\text{ Btu/h.ft}^2)$$
$$= 20{,}789\text{ Btu/h}$$

The rate of heat loss through the window at 3 PM is

$$\dot{Q}_{\text{loss, window}} = U_{\text{window}} A_{\text{window}} (T_i - T_0)$$
$$= (1.17\text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F})(135\text{ ft}^2)(70-45)°\text{F}$$
$$= 3949\text{ Btu/h}$$

The house will be gaining heat at 3 PM since the solar heat gain is larger than the heat loss. The rate of net heat gain through the window is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{solar gain, 3 PM}} - \dot{Q}_{\text{loss, window}} = 20{,}789 - 394 = \mathbf{16{,}840\text{ Btu/h}}$$

Discussion The actual heat gain will be less because of the area occupied by the window frame.

21-114 A building located near 40° N latitude has equal window areas on all four sides. The side of the building with the highest solar heat gain in summer is to be determined.

Assumptions The shading coefficients of windows on all sides of the building are identical.

Analysis The reflective films should be installed on the side that receives the most incident solar radiation in summer since the window areas and the shading coefficients on all four sides are identical. The incident solar radiation at different windows in July are given to be (Table 21-20)

Month	Time	The daily total solar radiation incident on the surface, Wh/m^2			
		North	East	South	West
July	Daily total	1621	4313	2552	4313

Therefore, the reflective film should be installed on the **East** or **West** windows (instead of the South windows) in order to minimize the solar heat gain and thus the cooling load of the building.

Infiltration Heat Load and Weatherizing

21-115C The uncontrolled entry of outside air into a building through unintentional openings is called *infiltration*. It differs from ventilation in that ventilation is intentional and controlled whereas infiltration is unintentional and uncontrolled. Infiltration increases the heating loss in winter and the cooling load in summer since the air entering must be heated in winter and cooled in summer. Therefore, the warm air leaving the house represents *energy loss*. This is also the case for cool air leaving in summer since some electricity is used to cool that air.

21-116C Air infiltration rate of a building can be determined by direct measurements by (1) *injecting a tracer gas* into a building and observing the decline of its concentration with time, or (2) *pressurizing the building* to 10 to 75 Pa gage pressure by a large fan mounted on a door or window, and measuring the air flow required to maintain a specified indoor-outdoor pressure difference. The larger the airflow to maintain a pressure difference, the more the building may leak. Sulfur hexafluoride (SF_6) is commonly used as a tracer gas because it is inert, nontoxic, and easily detectable at concentrations as low as 1 part per billion. Pressurization testing is easier to conduct, and thus preferable to tracer gas testing.

The *design infiltration rate* determined at design conditions is used to size a heating or cooling equipment where as the *seasonal average infiltration rate* is used to properly estimate the seasonal energy consumption for heating or cooling.

21-117C The infiltration unit ACH (air changes per hour) is defined as

$$\text{ACH} = \frac{\text{Flow rate of outdoor air into the building (per hour)}}{\text{Internal volume of the building}} = \frac{\dot{V}\,(\text{m}^3/\text{h})}{V\,(\text{m}^3)}$$

Therefore, the quantity ACH represents the number of building volumes of outdoor air that infiltrates (and eventually exfiltrates) per hour.

Too low and too high values of ACH should be avoided since too little fresh air will cause health and comfort problems such as the sick-building syndrome, which is experienced in super airtight buildings, and too much of it will waste energy. Therefore, the rate of fresh air supply should be just enough to maintain the indoor air quality at an acceptable level.

21-118C The energy of the air vented out from the kitchens and bathrooms can be saved by installing an *air-to-air heat exchanger* (also called "economizer" or "heat recuperator") that transfers the heat from the exhausted stale air to the incoming fresh air without any mixing. Such heat exchangers are commonly used in superinsulated houses, but the benefits of such heat exchangers must be weighed against the cost and complexity of their installation.

21-119C The latent heat load of infiltration **is not** necessarily zero when the relative humidity of the hot outside air in summer is the same as that of inside air since $\dot{Q}_{\text{infiltration, latent}} = \rho_o h_{fg} (ACH)(V_{\text{building}})(w_i - w_o)$ where $w_i - w_o$ is the humidity ratio difference between the indoor and outdoor air, and w is higher at higher temperatures for the same relative humidity.

21-120C The latent heat load of infiltration **is** necessarily zero when the humidity ratio w of the hot outside air in summer is the same as that of inside air since $\dot{Q}_{\text{infiltration, latent}} = \rho_o h_{fg} (ACH)(V_{\text{building}})(w_i - w_o) = 0$ when $w_i = w_o$.

21-121C Some practical ways of preventing infiltration in homes are (1) *caulking* that can be applied with a caulking gun inside and outside where two stationary surfaces such as a wall and a window frame meet and (2) *weather-stripping* with a narrow piece of metal, vinyl, rubber, felt or foam that seals the contact area between the fixed and movable sections of a joint.

21-122C Yes, it is true that the infiltration rate and infiltration losses can be reduced by using radiant panel heaters since the air temperature can be lowered without sacrificing comfort, and the lower the temperature difference between the indoors and the outdoors the lower the infiltration loss. It is also true that radiant panels will increase the heat losses through the wall and the roof by conduction as a result of increased surface temperature if this turns out to be the case since heat conduction through the wall is proportional to the temperature difference across the wall.

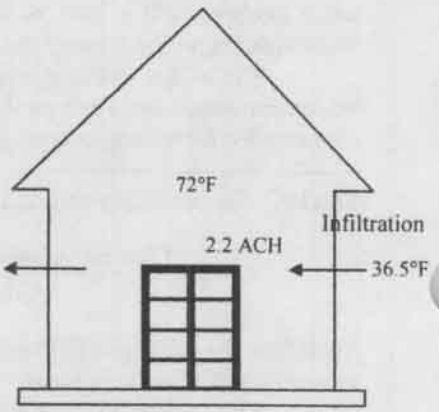
21-123E A winterizing project is to reduce the infiltration rate of a house from 2.2 ACH to 1.1 ACH. The resulting cost savings are to be determined.

Assumptions **1** The house is maintained at 72°F at all times. **2** The latent heat load during the heating season is negligible. **3** The infiltrating air is heated to 72°F before it exfiltrates.

Properties The gas constant of air is 0.3704 psia.ft^3/lbm·R (Table A-1E). The specific heat of air at room temperature is 0.24 Btu/lbm·°F (Table A-3E).

Analysis The density of air at the outdoor conditions is

$$\rho_o = \frac{P_o}{RT_o} = \frac{13.5 \text{ psia}}{(0.3704 \text{ psia.ft}^3/\text{lbm.R})(496.5 \text{ R})} = 0.0734 \text{ lbm/ft}^3$$



The volume of the building is

$$V_{\text{building}} = (\text{Floor area})(\text{Height}) = (3000 \text{ ft}^2)(9 \text{ ft}) = 27{,}000 \text{ ft}^3$$

The reduction in the infiltration rate is 2.2 – 1.1 = 1.1 ACH.
The sensible infiltration heat load corresponding to it is

$$\dot{Q}_{\text{infiltration, saved}} = \rho_o C_p (ACH_{\text{saved}})(V_{\text{building}})(T_i - T_o)$$
$$= (0.0734 \text{ lbm/ft}^3)(0.24 \text{ Btu/lbm.°F})(1.1/\text{h})(27{,}000 \text{ ft}^3)(72 - 36.5)°\text{F}$$
$$= 18{,}573 \text{ Btu/h} = 0.18573 \text{ therm/h}$$

since 1 therm = 100,000 Btu. The number of hours during a six month period is 6×30×24 = 4320 h. Noting that the furnace efficiency is 0.65 and the unit cost of natural gas is $0.62/therm, the energy and money saved during the 6-month period are

$$\text{Energy savings} = (\dot{Q}_{\text{infiltration, saved}})(\text{No. of hours per year})/\text{Efficiency}$$
$$= (0.18573 \text{ therm/h})(4320 \text{ h/year})/0.65$$
$$= 1234 \text{ therms/year}$$

$$\text{Cost savings} = (\text{Energy savings})(\text{Unit cost of energy})$$
$$= (1234 \text{ therms/year})(\$0.62/\text{therm})$$
$$= \mathbf{\$765/year}$$

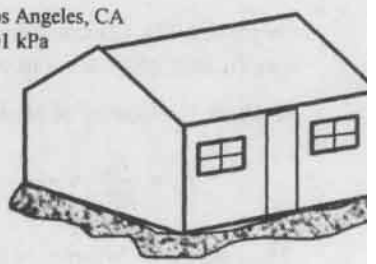
Therefore, reducing the infiltration rate by one-half will reduce the heating costs of this homeowner by $765 per year.

21-124 Two identical buildings in Los Angeles and Denver have the same infiltration rate. The ratio of the heat losses by infiltration at the two cities under identical conditions is to be determined.

Assumptions Both homes are identical and both are subjected to the same conditions except the atmospheric conditions.

Analysis The sensible infiltration heat loss is given as

$$\dot{Q}_{\text{infiltration}} = \dot{m}_{\text{air}} C_p (T_i - T_o) = \rho_{o,\text{air}} (ACH)(V_{\text{building}}) C_p (T_i - T_o)$$

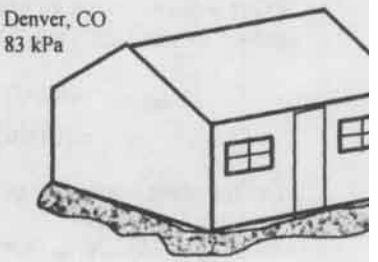


Therefore, the infiltration heat loss is proportional to the density of air, and thus the ratio of infiltration heat losses at the two cities is simply the densities of outdoor air at those cities,

$$\text{Infiltration heat loss ratio} = \frac{\dot{Q}_{\text{infiltration, Los Angeles}}}{\dot{Q}_{\text{infiltration, Denver}}} = \frac{\rho_{o,\text{air, Los Angeles}}}{\rho_{o,\text{air, Denver}}}$$

$$= \frac{(P_0 / RT_0)_{\text{Los Angeles}}}{(P_0 / RT_0)_{\text{Denver}}} = \frac{P_{o,\text{Los Angeles}}}{P_{0,\text{Denver}}}$$

$$= \frac{101\,\text{kPa}}{83\,\text{kPa}} = \mathbf{1.22}$$



Therefore, the infiltration heat loss in Los Angeles will be 22% higher than that in Denver under identical conditions.

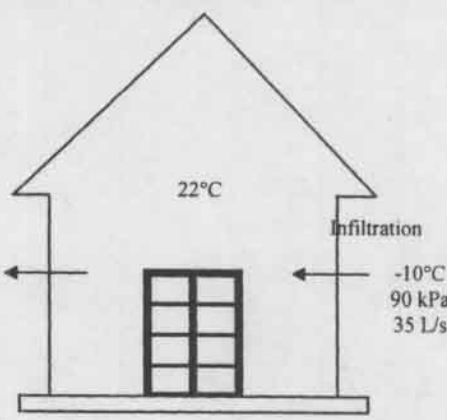
21-125 Outdoor air at -10°C and 90 kPa enters the building at a rate of 35 L/s when the indoors is maintained at 22°C. The rate of sensible heat loss from the building due to infiltration is to be determined.

Assumptions **1** The house is maintained at 22°C at all times. **2** The latent heat load is negligible. **3** The infiltrating air is heated to 22°C before it exfiltrates.

Properties The gas constant of air is R = 0.287 kPa.m^3/kg·K (Table A-1). The specific heat of air at room temperature is C_p = 1.0 kJ/kg·°C (Table A-2).

Analysis The density of air at the outdoor conditions is

$$\rho_o = \frac{P_o}{RT_o} = \frac{90\,\text{kPa}}{(0.287\,\text{kPa.m}^3/\text{kg.K})(-10+273\,\text{K})} = 1.19\,\text{kg/m}^3$$



Then the sensible infiltration heat load corresponding to an infiltration rate of 35 L/s becomes

$$\dot{Q}_{\text{infiltration}} = \rho_o \dot{V}_{air} C_p (T_i - T_o)$$

$$= (1.19\,\text{kg/m}^3)(0.035\,\text{m}^3/\text{s})(1.0\,\text{kJ/kg.°C})[22-(-10)]\text{°C}$$

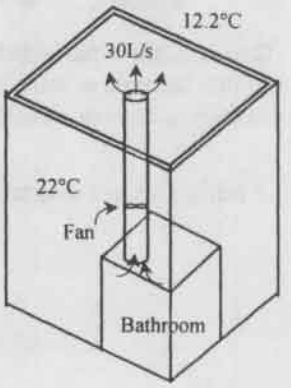
$$= \mathbf{1.335\,kW}$$

Therefore, sensible heat will be lost at a rate of 1.335 kJ/s due to infiltration.

21-126 The ventilating fan of the bathroom of an electrically heated building in San Francisco runs continuously. The amount and cost of the heat "vented out" per month in winter are to be determined.

Assumptions **1** We take the atmospheric pressure to be 1 atm = 101.3 kPa since San Francisco is at sea level. **2** The house is maintained at 22°C at all times. **3** The infiltrating air is heated to 22°C before it exfiltrates.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg·K (Table A-1). The specific heat of air at room temperature is $C_p = 1.0$ kJ/kg·°C (Table A-2).



Analysis The density of air at the indoor conditions of 1 atm and 22°C is

$$\rho_o = \frac{P_o}{RT_o} = \frac{(101.3\,\text{kPa})}{(0.287\,\text{kPa.m}^3/\text{kg.K})(22+273\,\text{K})} = 1.20\,\text{kg/m}^3$$

Then the mass flow rate of air vented out becomes

$$\dot{m}_{\text{air}} = \rho \dot{V}_{\text{air}} = (1.20\,\text{kg/m}^3)(0.030\,\text{m}^3/\text{s}) = 0.036\,\text{kg/s}$$

Noting that the indoor air vented out at 22°C is replaced by infiltrating outdoor air at 12.2°C, this corresponds to energy loss at a rate of

$$\dot{Q}_{\text{loss by fan}} = \dot{m}_{\text{air}} C_p (T_{\text{indoors}} - T_{\text{outdoors}})$$
$$= (0.036\,\text{kg/s})(1.005\,\text{kJ/kg.°C})(22-12.2)°\text{C} = 0.355\,\text{kJ/s} = 0.355\,\text{kW}$$

Then the amount and cost of the heat "vented out" per month (1 month = 30×24 = 720 h) becomes

Energy loss = $\dot{Q}_{\text{loss by fan}}\,\Delta t = (0.355\,\text{kW})(720\,\text{h/month}) =$ **256 kWh/month**

Money loss = (Energy loss)(Unit cost of energy) = (256 kWh/month)($0.09/kWh) = **$23.0/month**

Discussion Note that the energy and money loss associated with ventilating fans can be very significant. Therefore, ventilating fans should be used with care.

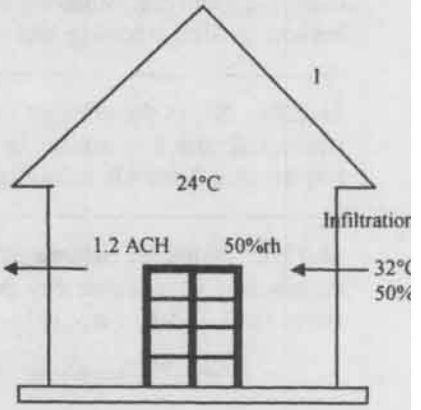
21-127 The infiltration rate of a building is estimated to be 1.2 ACH. The sensible, latent, and total infiltration heat loads of the building at sea level are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air infiltrates at the outdoor conditions, and exfiltrates at the indoor conditions. **3** Excess moisture condenses at 5°C. **4** The effect of water vapor on air density is negligible.

Properties The gas constant and the specific heat of air are $R = 0.287$ kPa.m^3/kg.K and $C_p = 1.0$ kJ/kg·°C (Tables A-1 and A-2). The heat of vaporization of water at 5°C is $h_{fg} = h_{fg@5°C} = 2490$ kJ/kg (Table A-15). The properties of the ambient and room air are determined from the psychrometric chart to be

$$\left.\begin{matrix} T_{\text{ambient}} = 32°\text{C} \\ \phi_{\text{ambient}} = 50\% \end{matrix}\right\} w_{\text{ambient}} = 0.0150 \text{ kg/kg dryair}$$

$$\left.\begin{matrix} T_{\text{room}} = 24°\text{C} \\ \phi_{\text{room}} = 50\% \end{matrix}\right\} w_{\text{room}} = 0.0093 \text{ kg/kg dryair}$$



Analysis Noting that the infiltration of ambient air will cause the air in the cold storage room to be changed 0.8 times every hour, the air will enter the room at a mass flow rate of

$$\rho_{\text{ambient}} = \frac{P_o}{RT_o} = \frac{101.325 \text{ kPa}}{(0.287 \text{ kPa.m}^3\text{/kg.K})(32+273 \text{ K})} = 1.16 \text{ kg/m}^3$$

$$\dot{m}_{\text{air}} = \rho_{\text{ambient}} V_{\text{room}} ACH = (1.16 \text{ kg/m}^3)(20\times13\times3 \text{ m}^3)(1.2 \text{ h}^{-1}) = 1085 \text{ kg/s} = 0.3016 \text{ kg/s}$$

Then the sensible, latent, and total infiltration heat loads of the room are determined to be

$$\dot{Q}_{\text{infiltration,sensible}} = \dot{m}_{\text{air}} C_p (T_{\text{ambient}} - T_{\text{room}}) = (0.3016 \text{ kg/s})(1.0 \text{ kJ/kg.°C})(32-24)°\text{C} = \mathbf{2.41 \text{ kW}}$$

$$\dot{Q}_{\text{infiltration,latent}} = \dot{m}_{\text{air}} (w_{\text{ambient}} - w_{\text{room}}) h_{fg} = (0.3016 \text{ kg/s})(0.0150-0.0093)(2490 \text{ kJ/kg}) = \mathbf{4.28 \text{ kW}}$$

$$\dot{Q}_{\text{infiltration, total}} = \dot{Q}_{\text{infiltration,sensible}} + \dot{Q}_{\text{infiltration,latent}} = 2.41+4.28 = \mathbf{6.69 \text{ kW}}$$

Discussion The specific volume of the dry air at the ambient conditions could also be determined from the psychrometric chart at ambient conditions.

Annual Energy Consumption

21-128C Yes, it is possible for a building in a city to have a higher peak heating load but a lower energy consumption for heating in winter than an identical building in another city. This will be the case for a city with severe but relatively short heating seasons.

21-129C No, we cannot determine the annual energy consumption of a building for heating by simply multiplying the design heating load of the building by the number of hours in the heating season. This is because the design heating load represents the heat loss under extreme conditions, not average conditions.

21-130C No, as the manager of a large commercial building, I would not lower the thermostat setting in winter and raise it in summer by a few degrees. Although this practice will save energy and thus money, it may cost much more in reduced productivity.

21-131C Taking the balance point temperature to be 18°C, as is commonly done, the number of heating degree-days for a winter day during which the average outdoor temperature was 10°C, and never went above 18°C, is determined to be

$$DD_{1\ \text{day}} = (T_{\text{balance point}} - T_{\text{daily average}})(1\ \text{day}) = (18 - 10)°\text{C}(1\ \text{day}) = \mathbf{8°C\text{-}day}$$

21-132C Taking the balance point temperature to be 18°C, as is commonly done, the number of heating degree-days for a winter month during which the average outdoor temperature was 12°C, and never rose above 18°C, is determined to be

$$DD_{1\ \text{mont}} = (T_{\text{balance point}} - T_{\text{monthly average}})(1\ \text{month}) = (18 - 12)°\text{C}(30\ \text{days}) = \mathbf{180°C\text{-}day}$$

21-133C The °C-days are based on temperature differences, and $\Delta T(°\text{F}) = 1.8\Delta T(°\text{C})$ for temperature differences. Therefore, we should not add 32 to the result.

21-134C The outdoor temperature above which no heating is required is called the *balance point temperature* T_{balance}. The balance-point temperature is used in the determination of degree-days instead of the actual thermostat setting of a building since the internal heat generated by people, lights, and appliances in occupied buildings as well as the heat gain from the sun during the day, $\dot{Q}_{\text{gain}}$, will be sufficient to compensate for the heat losses from the building until the outdoor temperature drops below T_{balance}.

21-135C It is proper to use the degree-day method to determine the annual energy consumption of a building under relatively *steady* conditions. The method is based on constant indoor conditions during the heating or cooling season, and it assumes the efficiency of the heating or cooling equipment is not affected by the variation of outdoor temperature. These conditions will be closely approximated if all the thermostats in a building are set at the same temperature at the beginning of a heating or cooling season, and are never changed, and a seasonal average efficiency is used (rather than the full-load or design efficiency) for the furnaces or coolers.

21-136 A person offers to his roommate in Syracuse, New York, to pay the heating bills during the upcoming year (starting January 1st) if he pays the heating bills for the current calendar year until Dec. 31. It is to be determined if this is a good offer.

Assumptions The calculations are performed for an "average" year. **2** The time value of money is not considered.

Properties The annual heating degree-days of Syracuse, NY, is 6756°F-days (Table 21-5). The monthly distribution of degree-days are 6, 28, 132, 415, 744, 1153, 1271, 1140, 1004, 570, 248, and 45°F-days for July through June, respectively.

Analysis It makes sense to accept this offer if the cost of heating before December 31st is less than the cost of heating after December 31st. The amount and cost of energy consumption of a building for heating is proportional to the heating degree days. For Syracuse, we have

$$DD_{\text{heating, before Dec. 31}} = 6 + 28 + 132 + 415 + 744 + 1153 = 2478\text{°F-days} \quad (2478/6756 = 0.367)$$

$$DD_{\text{heating, after Dec. 31}} = 1271 + 1140 + 1004 + 570 + 248 + 45 = 4278\text{°F-days} \quad (4278/6756 = 0.633)$$

This is clearly a **good offer** for the roommate since 63.3% of the heating load occurs after December 31st, and the proposer is offering to pay for it. Therefore, the offer should be accepted.

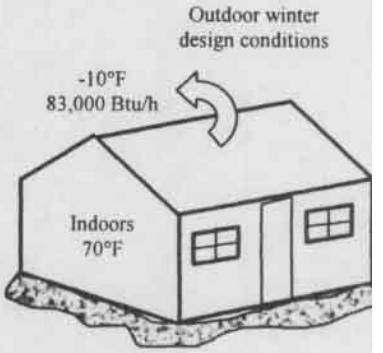
21-137E A house whose design heat load is 83,000 Btu/h is heated by a high-efficiency natural gas furnace. The annual gas consumption of this house and its cost are to be determined.

Assumptions **1** The house is maintained at 70°F at all times during the heating season. **2** The calculations are performed for an "average" year.

Properties The annual heating degree-days of Billing, Montana, is 7049°F-days (Table 21-5). The winter design temperature of Billing is given to be -10°F.

Analysis The fuel (natural gas) consumption rate of the house for heating at design conditions is

$$\dot{Q}_{\text{design}} = \frac{\dot{Q}_{\text{design, load}}}{\eta_{\text{heating}}} = \frac{83{,}000\ \text{Btu/h}}{0.95} = 87{,}368\ \text{Btu/h} = 0.874\ \text{therm/h}$$



Then the annual natural gas usage of the house and its cost can be determined from Eq. 21-48 to be

$$\text{Annual fuel consumption} = Q_{\text{heating, year}} = \frac{DD_{\text{heating}}}{(T_i - T_o)_{\text{design}}}\dot{Q}_{\text{design}}$$

$$= \frac{7049°\text{F-day}}{[70-(-10)]°\text{F}}\left(\frac{24\ \text{h}}{1\ \text{day}}\right)(0.874\ \text{therm/h})$$

$$= \mathbf{1848\ therms/year}$$

and

$$\text{Annual fuel cost} = (\text{Annual fuel consumption})(\text{Unit cost of fuel})$$

$$= (1848\ \text{therms/year})(\$0.65/\text{therm}) = \mathbf{\$1201/year}$$

Therefore, it will cost $1201 per year to heat this house during an average year.

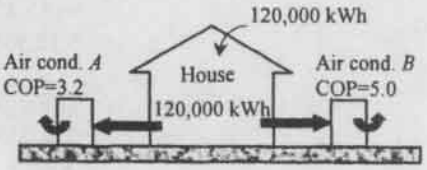
21-138 A decision is to be made between a cheaper but inefficient and an expensive but efficient air-conditioner for a building.

Assumptions The two air conditioners are comparable in all aspects other than the initial cost and the efficiency.

Analysis The unit that will cost less during its lifetime is a better buy. The total cost of a system during its lifetime (the initial, operation, maintenance, etc.) can be determined by performing a life cycle cost analysis. A simpler alternative is to determine the simple payback period. The energy and cost savings of the more efficient air conditioner in this case is

$$\begin{aligned}\text{Energy savings} &= (\text{Annual energy usage of A}) - (\text{Annual energy usage of B})\\ &= (\text{Annual cooling load})(1/\text{COP}_A - 1/\text{COP}_B)\\ &= (120{,}000\ \text{kWh/year})(1/3.2 - 1/5.0)\\ &= 13{,}500\ \text{kWh/year}\end{aligned}$$

$$\begin{aligned}\text{Cost savings} &= (\text{Energy savings})(\text{Unit cost of energy})\\ &= (13{,}500\ \text{kWh/year})(\$0.10/\text{kWh}) = \mathbf{\$1350/year}\end{aligned}$$



The installation cost difference between the two air-conditioners is

$$\text{Cost difference} = \text{Cost of B} - \text{cost of A} = 7000 - 5500 = \$1500$$

Therefore, the more efficient air-conditioner B will pay for the $1500 cost differential in this case in about 1 year.

Discussion A cost conscious consumer will have no difficulty in deciding that the more expensive but more efficient air-conditioner B is clearly a better buy in this case since air conditioners last at least 15 years. But the decision would not be so easy if the unit cost of electricity at that location was much less than $0.10/kWh, or if the annual air-conditioning load of the house was much less than 120,000 kWh.

21-139 An industrial facility is to replace its 40-W standard fluorescent lamps by their 34-W high efficiency counterparts. The amount of energy and money that will be saved a year as a result of switching to the high efficiency fluorescent lamps as well as the simple payback period are to be determined.

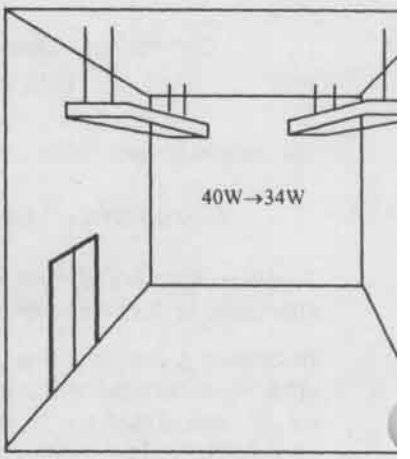
Analysis The reduction in the total electric power consumed by the lighting as a result of switching to the high efficiency fluorescent is

$$\begin{aligned}\text{Wattage reduction} &= (\text{Wattage reduction per lamp})(\text{Number of lamps})\\ &= (40 - 34\ \text{W/lamp})(700\ \text{lamps})\\ &= 4200\ \text{W}\end{aligned}$$

Then using the relations given earlier, the energy and cost savings associated with the replacement of the high efficiency fluorescent lamps are determined to be

$$\begin{aligned}\text{Energy Savings} &= (\text{Total wattage reduction})(\text{Ballast factor})(\text{Operating hours})\\ &= (4.2\ \text{kW})(1.1)(2800\ \text{h/year})\\ &= \mathbf{12{,}936\ kWh/year}\end{aligned}$$

$$\begin{aligned}\text{Cost Savings} &= (\text{Energy savings})(\text{Unit electricity cost})\\ &= (12{,}936\ \text{kWh/year})(\$0.08/\text{kWh})\\ &= \mathbf{\$1035}\end{aligned}$$



The implementation cost of this measure is simply the extra cost of the energy efficient fluorescent bulbs relative to standard ones, and is determined to be

$$\begin{aligned}\text{Implementation Cost} &= (\text{Cost difference of lamps})(\text{Number of lamps})\\ &= [(\$2.26-\$1.77)/\text{lamp}](700\ \text{lamps})\\ &= \$343\end{aligned}$$

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$343}{\$1035/\text{year}} = \mathbf{0.33\ year}\ (4.0\ \text{months})$$

Discussion Note that if all the lamps were burnt out today and are replaced by high-efficiency lamps instead of the conventional ones, the savings from electricity cost would pay for the cost differential in about 4 months. The electricity saved will also help the environment by reducing the amount of CO_2, CO, NO_x, etc. associated with the generation of electricity in a power plant.

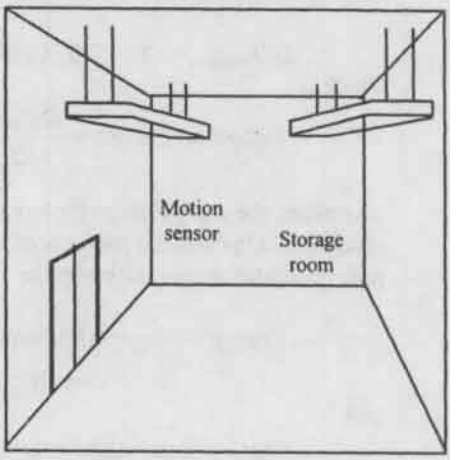
21-140 The lighting energy consumption of a storage room is to be reduced by installing motion sensors. The amount of energy and money that will be saved as a result of installing motion sensor as well as the simple payback period are to be determined.

Assumptions The electrical energy consumed by the ballasts is negligible.

Analysis The plant operates 12 hours a day, and thus currently the lights are on for the entire 12 hour period. The motion sensors installed will keep the lights on for 3 hours, and off for the remaining 9 hours every day. This corresponds to a total of 9×365 = 3285 off hours per year. Disregarding the ballast factor, the annual energy and cost savings become

$$\begin{aligned}\text{Energy Savings} &= (\text{Number of lamps})(\text{Lamp wattage})(\text{Reduction of annual operating hours})\\ &= (24\ \text{lamps})(60\ \text{W/lamp})(3285\ \text{hours/year})\\ &= \mathbf{4730\ kWh/year}\end{aligned}$$

$$\begin{aligned}\text{Cost Savings} &= (\text{Energy Savings})(\text{Unit cost of energy})\\ &= (5{,}203\ \text{kWh/year})(\$0.08/\text{kWh})\\ &= \mathbf{\$378/year}\end{aligned}$$



The implementation cost of this measure is the sum of the purchase price of the sensor plus the labor,

$$\text{Implementation Cost} = \text{Material} + \text{Labor} = \$32 + \$40 = \$72$$

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$72}{\$378/\text{year}}$$
$$= \mathbf{0.19\ year}\ (2.3\ \text{months})$$

Therefore, the motion sensor will pay for itself in about 2 months.

21-141 The existing manual thermostats of an office building are to be replaced by programmable ones to reduce the heating costs by lowering the temperature setting shortly before closing at 6 PM and raising it shortly before opening at 8 AM. The annual energy and cost savings as result of installing programmable thermostats as well as the simple payback period are to be determined.

Assumptions **1** The ambient temperature remains below 7.2°C during the entire heating season (Nov.-Apr). This assumption will most likely be violated some days, and thus the result is optimistic. **2** The balance point temperature is 18°C so that no heating is required at temperatures above 18°C.

Properties The annual heating degree-days for Reno, Nevada is given to be 3346°C-days.

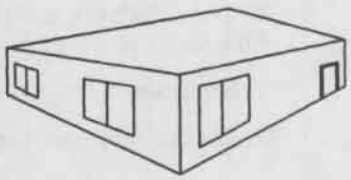
Analysis The energy usage for heating is proportional to degree days, and the reduction in the degree days due to lowering the thermostat setting to 7.2°C from about 6 PM to 8 AM for 10 h for everyday during the heating season for 180 days is

$$DD_{\text{reduction}} = (22 - 7.2)°\text{C}(10\ \text{h/day})(180\ \text{days}) = 32{,}614°\text{C-h} = 1359°\text{C-day}$$

which is

$$\text{Reduction fraction} = \frac{DD_{\text{reduction}}}{DD_{\text{annual}}} = \frac{1359°\text{C-days}}{3346°\text{C-days}} = 0.406 \qquad (\text{or } 40.6\%)$$

Reno, NV



Therefore, the energy usage for heating will be reduced by 40.6%. Then the reduction in the amount and cost of heating energy as a result of installing programmable thermostats become

$$\text{Energy Savings} = (\text{Reduction fraction})(\text{Annual heating energy usage})$$

$$= 0.406(3530\ \text{therms/year}) = \mathbf{1433\ therms/year}$$

and

$$\text{Cost Savings} = (\text{Reduction fraction})(\text{Annual heating bill}) = 0.406(\$2{,}047/\text{year}) = \mathbf{\$831/year}$$

The total implementation cost of installation of 5 programmable thermostats is

$$\text{Implementation Cost} = 5 \times \$190 = \$950$$

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$950}{\$831/\text{year}} = \mathbf{1.14\ year}$$

Therefore, the programmable thermostats will almost pay for themselves during the first heating season.

21-142 A worn out standard motor is to be replaced by a high efficiency one. The amount of electrical energy and money savings as a result of installing the high efficiency motor instead of the standard one as well as the simple payback period are to be determined.

Assumptions The load factor of the motor remains constant at 0.75.

Analysis The electric power drawn by each motor and their difference can be expressed as

$$\dot{W}_{\text{electric in, standard}} = \dot{W}_{\text{shaft}} / \eta_{\text{standar}} = (\text{Power rating})(\text{Load factor}) / \eta_{\text{standard}}$$

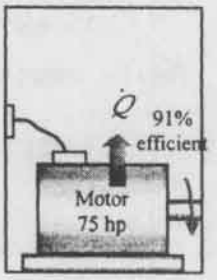
$$\dot{W}_{\text{electric in, efficient}} = \dot{W}_{\text{shaft}} / \eta_{\text{efficien}} = (\text{Power rating})(\text{Load factor}) / \eta_{\text{efficient}}$$

$$\text{Power savings} = \dot{W}_{\text{electric in, standard}} - \dot{W}_{\text{electric in, efficient}}$$

$$= (\text{Power rating})(\text{Load factor})[1/\eta_{\text{standard}} - 1/\eta_{\text{efficient}}]$$

where η_{standard} is the efficiency of the standard motor, and $\eta_{\text{efficient}}$ is the efficiency of the comparable high efficiency motor. Then the annual energy and cost savings associated with the installation of the high efficiency motor are determined to be

Energy Savings = (Power savings)(Operating Hours)
= (Power Rating)(Operating Hours)(Load Factor)($1/\eta_{\text{standard}}$- $1/\eta_{\text{efficient}}$)
= (75 hp)(0.746 kW/hp)(4,368 hours/year)(0.75)(1/0.91 - 1/0.954)
= **9,290 kWh/year**



Cost Savings = (Energy savings)(Unit cost of energy)
= (9,290 kWh/year)($0.08/kWh)
= **$743/year**

The implementation cost of this measure consists of the excess cost the high efficiency motor over the standard one. That is,

Implementation Cost = Cost differential = $5,520 - $5,449 = $71

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$71}{\$743/\text{year}} = \mathbf{0.096\ year}\ (\text{or } 1.1 \text{ months})$$

Therefore, the high-efficiency motor will pay for its cost differential in about one month.

21-143 The combustion efficiency of a furnace is raised from 0.7 to 0.8 by tuning it up. The annual energy and cost savings as a result of tuning up the boiler are to be determined.

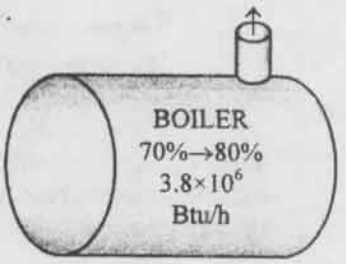
Assumptions The boiler operates at full load while operating.

Analysis The heat output of boiler is related to the fuel energy input to the boiler by

$$\text{Boiler output} = (\text{Boiler input})(\text{Combustion efficiency}) \quad \text{or} \quad \dot{Q}_{\text{out}} = \dot{Q}_{\text{in}}\,\eta_{\text{furnace}}$$

The current rate of heat input to the boiler is given to be $\dot{Q}_{\text{in, current}} = 3.8\times10^6$ Btu/h.
Then the rate of useful heat output of the boiler becomes

$$\dot{Q}_{\text{out}} = (\dot{Q}_{\text{in}}\,\eta_{\text{furnace}})_{\text{current}} = (3.8\times10^6\ \text{Btu/h})(0.7) = 2.66\times10^6\ \text{Btu/h}$$



The boiler must supply useful heat at the same rate after the tune up. Therefore, the rate of heat input to the boiler after the tune up and the rate of energy savings become

$$\dot{Q}_{\text{in, new}} = \dot{Q}_{\text{out}} / \eta_{\text{furnace, new}} = (2.66\times10^6\ \text{Btu/h})/0.8 = 3.325\times10^6\ \text{Btu/h}$$

$$\dot{Q}_{\text{in, saved}} = \dot{Q}_{\text{in, current}} - \dot{Q}_{\text{in, new}} = 3.8\times10^6 - 3.325\times10^6 = 0.475\times10^6\ \text{Btu/h}$$

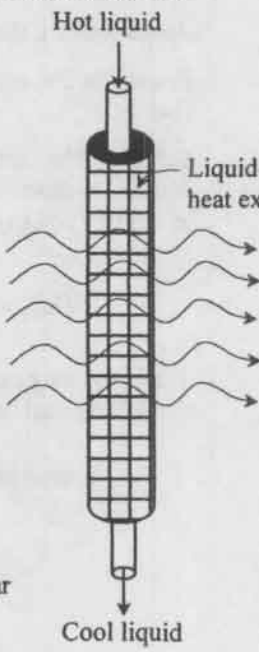
Then the annual energy and cost savings associated with tuning up the boiler become

$$\text{Energy Savings} = \dot{Q}_{\text{in, saved}}\,(\text{Operation hours})$$
$$= (0.475\times10^6\ \text{Btu/h})(1500\ \text{h/year}) = \mathbf{712.5\times10^6\ Btu/yr}$$

$$\text{Cost Savings} = (\text{Energy Savings})(\text{Unit cost of energy})$$
$$= (712.5\times10^6\ \text{Btu/yr})(\$4.35\ \text{per}\ 10^6\ \text{Btu}) = \mathbf{\$3099/year}$$

Discussion Notice that tuning up the boiler will save $3099 a year, which is a significant amount. The implementation cost of this measure is negligible if the adjustment can be made by in-house personnel. Otherwise it is worthwhile to have an authorized representative of the boiler manufacturer to service the boiler twice a year.

21-144 The gas space heating of a facility is to be supplemented by air heated in a liquid-to-air heat exchanger of a compressor. The amount of money that will be saved by diverting the compressor waste heat into the facility during the heating season is to be determined.



Assumptions The atmospheric pressure at that location is 1 atm.

Analysis The mass flow rate of air through the liquid-to-air heat exchanger is

$$\begin{aligned}\text{Mass flow rate of air} &= (\text{Density of air})(\text{Average velocity})(\text{Flow area})\\ &= (1.21\ \text{kg/m}^3)(3\ \text{m/s})(1.0\ \text{m}^2)\\ &= 3.63\ \text{kg/s} = 13{,}068\ \text{kg/h}\end{aligned}$$

Noting that the exit temperature of air is 52°C, the rate at which heat can be recovered (or the rate at which heat is transferred to air) is

$$\begin{aligned}\text{Rate of Heat Recovery} &= (\text{Mass flow rate of air})(\text{Specific heat of air})(\text{Temperature rise})\\ &= (13{,}068\ \text{kg/h})(1.0\ \text{kJ/kg.°C})(52-20)\text{°C}\\ &= 418{,}176\ \text{kJ/h}\end{aligned}$$

The number of operating hours of this compressor during the heating season is

$$\text{Operating hours} = (20\ \text{hours/day})(5\ \text{days/week})(26\ \text{weeks/year}) = 2600\ \text{hours/year}$$

Then the annual energy and cost savings become

$$\begin{aligned}\text{Energy Savings} &= (\text{Rate of Heat Recovery})(\text{Annual Operating Hours})/\text{Efficiency}\\ &= (418{,}176\ \text{kJ/h})(2600\ \text{h/year})/0.8\\ &= 1{,}359{,}100{,}000\ \text{kJ/year}\\ &= 12{,}882\ \text{therms/year}\end{aligned}$$

$$\begin{aligned}\text{Cost Savings} &= (\text{Energy savings})(\text{Unit cost of energy saved})\\ &= (12{,}882\ \text{therms/year})(\$0.50/\text{therm})\\ &= \$6441/\text{year}\end{aligned}$$

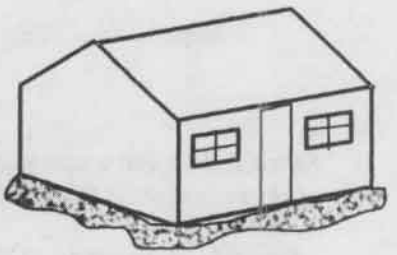
Discussion Notice that utilizing the waste heat from the compressor will save $6441 per year from the heating costs. The implementation of this measure requires the installation of an ordinary sheet metal duct from the outlet of the heat exchanger into the building. The installation cost associated with this measure is relatively low. Several manufacturing facilities already have this conservation system in place. A damper is used to direct the air into the building in winter and to the ambient in summer. Combined compressor/heat-recovery systems are available in the market for both air-cooled (greater than 50 hp) water cooled (greater than 125 hp) systems.

21-145 An Atlanta family has moved to an identical house in Denver, CO where the fuel and electricity prices are the same. The annual heating cost of this family in their new house is to be determined.

Assumptions Calculations are performed for an average year.

Properties The annual heating degree-days are 2961°F-days for Atlanta, and 6283°F-days for Denver (Table 21-5).

Solution The heating cost is proportional to the energy consumption, which is proportional to the degree-days. The ratio of the degree-days in the two cities is



$$DD_{\text{ratio}} = \frac{\text{°C - days for Atlanta}}{\text{°C - day for Denver}} = \frac{6283\text{°F - days}}{2961\text{°F - days}} = 2.12$$

Therefore, the heating load will increase by factor of 2.12 in Denver. Then the annual heating cost of this new house in Denver becomes

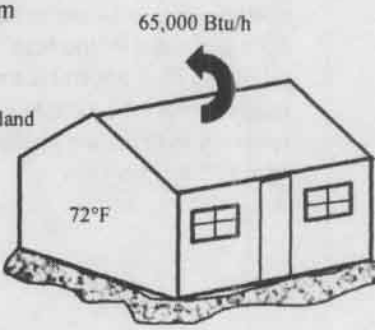
Annual heating cost in Denver = 2.12×(Annual heating cost in Atlanta) = 2.12($600/yr) = **$1272**

21-146E The annual gas consumption and its cost for a house in Cleveland, Ohio with a design heat load of 65,000 Btu/h and a furnace efficiency of 90% are to be determined.

Assumptions The house is maintained at 72°F at all times during the heating season.

Properties The annual heating degree-days of Cleveland, Ohio is 6351°F-days (Table 21-5). The 97.5% winter design temperature of Cleveland is 5°F.

Analysis The overall heat loss coefficient K_{overall} of the building is determined from



$$\dot{Q}_{\text{design}} = UA\Delta T_{\text{design}} = UA(T_i - T_o)_{\text{design}} = K_{\text{overall}}(T_i - T_o)_{\text{design}}$$

Substituting,

$$K_{\text{overall}} = \frac{\dot{Q}_{\text{design}}}{(T_i - T_o)_{\text{design}}} = \frac{65{,}000\text{ Btu/h}}{(72-5)\text{°F}} = 970\text{ Btu/h.°F}$$

Then the annual gas consumption of the house for heating is determined to be

$$\text{Annual Gas Consumption} = \frac{K_{\text{overall}}}{\eta_{\text{heating}}} DD_{\text{heating}}$$

$$= \frac{970\text{ Btu/h.°F}}{0.90}(6351\text{°F - days})\left(\frac{24\text{ h}}{1\text{day}}\right)\left(\frac{1\text{ therm}}{100{,}000\text{ Btu}}\right)$$

$$= 1643\text{ therms}$$

Therefore, the house will consume 1643 therms of natural gas for heating.

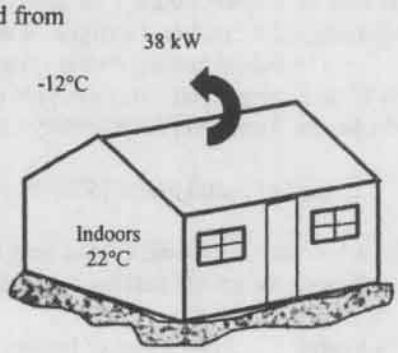
21-147 A house in Boise, Idaho is heated by electric resistance heaters. The amount of money the home owner will save if she lowers the thermostat from 22°C to 14°C every night in December is to be determined.

Assumptions **1** The house is maintained at 22°C during the day, and 14°C for 8 hours at night. **2** The efficiency of electric resistance heating system is 100%.

Properties The annual heating degree-days of Cleveland, Ohio is 6351°F-days (Table 21-5). The 97.5% winter design temperature of Boise is given to be -12°C.

Analysis The overall heat loss coefficient K_{overall} of the building is determined from

$$\dot{Q}_{\text{design}} = UA\Delta T_{\text{design}} = UA(T_{\text{i}} - T_o)_{\text{design}} = K_{\text{overall}}(T_{\text{i}} - T_o)_{\text{design}}$$



Substituting,

$$K_{\text{overall}} = \frac{\dot{Q}_{\text{design}}}{(T_{\text{i}} - T_o)_{\text{design}}} = \frac{38\,\text{kW}}{[72-(-12)]^\circ\text{C}} = 1.12\,\text{kW/°C}$$

The rate at which energy is saved at night is

$$\dot{Q}_{\text{saved}} = UA\Delta T_{\text{reduction}} = K_{\text{overall}}\Delta T_{\text{reduction}}$$
$$= (1.12\,\text{kW/°C})(22-14)^\circ\text{C} = 8.96\,\text{kW}$$

Then the energy and cost savings in December due to lowering the thermostat becomes

$$\text{Energy savings} = \dot{Q}_{\text{saved}}\Delta t = (8.96\,\text{kW})(31\times 8\,\text{h}) = \mathbf{2222\,kWh}$$

$$\text{Cost savings} = (\text{Energy savings})(\text{Unit cost of eergy}) = (2222\,\text{kWh})(\$0.06/\text{kWh}) = \mathbf{\$133}$$

Discussion Note that thermostat setback results in considerable savings in winter, and is commonly used in practice.

eview Problems

-148 A decision is to be made between a cheaper but inefficient and an expensive but efficient natural gas ater for a house.

sumptions The two heater are comparable in all aspects other than the initial cost and efficiency.

alysis Other things being equal, the logical choice is the heater that will cost less during its lifetime. The al cost of a system during its lifetime (the initial, operation, maintenance, etc.) can be determined by rforming a life cycle cost analysis. A simpler alternative is to determine the simple payback period.

The annual heating cost is given to be $1100. Noting that the existing heater is 60% efficient, only % of that energy (and thus money) is delivered to the house, and the rest is wasted due to the inefficiency the heater. Therefore, the monetary value of the heating load of the house is

Cost of useful heat = (60%)(Current annual heating cost) = 0.60×($1100/yr)=$660/yr

is is how much it would cost to heat this house with a heater that is 100% efficient. For heaters that are s efficient, the annual heating cost is determined by dividing $660 by the efficiency:

% heater: Annual cost of heating = (Cost of useful heat)/Efficiency = ($660/yr)/0.82 = $805/yr

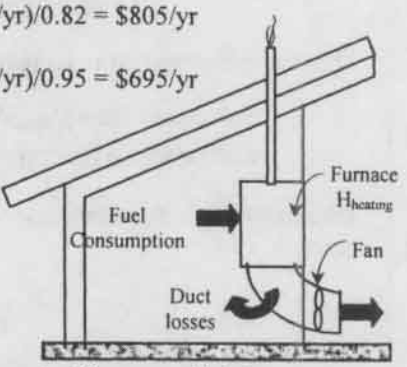
% heater: Annual cost of heating = (Cost of useful heat)/Efficiency = ($660/yr)/0.95 = $695/yr

nual cost savings with the efficient heater = 805 - 695 = $110

cess initial cost of the efficient heater = 2700 - 1600 = $1100

e simple payback period becomes

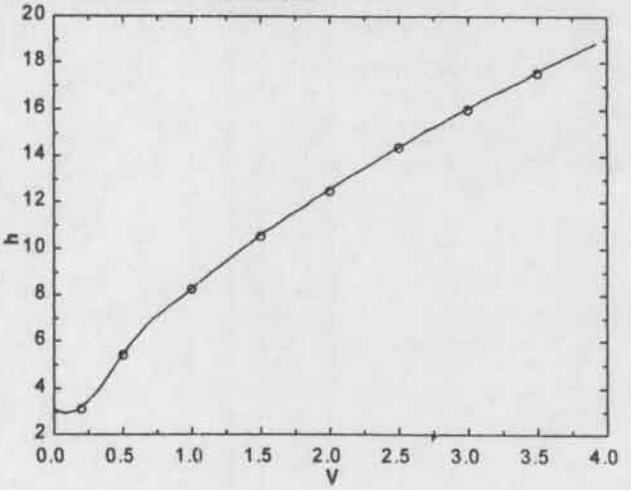
$$\text{Simple payback period} = \frac{\text{Excess initial cost}}{\text{Annaul cost savings}} = \frac{\$1100}{\$110/\text{yr}} = \mathbf{10\ years}$$



erefore, the more efficient heater will pay for the $1100 cost differential in this case in 10 years, which is re than the 8-year limit. Therefore, the purchase of the cheater and less efficient heater is indicated in this se.

-149 The convection heat transfer coefficient for a clothed person seated in moving air at a velocity of to 4 m/s is given by $h = 8.3V^{0.6}$ where V is in m/s and h is in W/m^2.°C. The convection coefficients in t range vary from 3.16 W/m^2.°C at 0.2 m/s to 19.07 W/m^2.°C at 4.0 m/s. Therefore, at low velocities, the liation and convection heat transfer coefficients are comparable in magnitude. But at high velocities, the nvection coefficient is much larger than the radiation heat transfer coefficient.

Velocity, m/s	$h = 8.3V^{0.6}$ W/m^2.°C
0.0	3.10
0.20	3.16
0.5	5.48
1.00	8.30
1.5	10.59
2.0	12.58
2.5	14.40
3.0	16.05
3.5	17.60
4.0	19.07



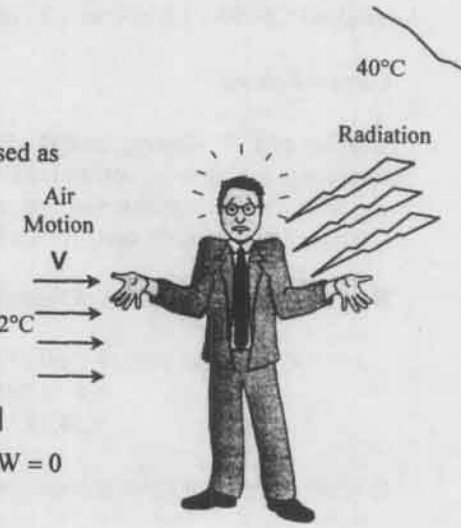
21-150 Workers in a casting facility are surrounded with hot surfaces. The velocity of air needed to provide comfort for the workers is to be determined.

Assumptions **1** The average clothing and exposed skin temperature of the workers is 30°C. **2** The workers are standing in moving air.

Properties The emissivity of the person is 0.95 (Table A-25). The convection heat transfer coefficient for a standing man in air moving with a velocity V is given by $h = 14.8V^{0.69}$ where V is in m/s and h is in W/m².°C (Table 21-3).

Analysis The rate of sensible heat transfer from the person is

$$\dot{Q}_{\text{gen, sensible}} = 0.5 \times \dot{Q}_{\text{gen, total}} = 0.5(300\ \text{W}) = 150\ \text{W}$$



Under steady conditions, and energy balance on the body can be expressed as

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_{\text{gen}} = 0$$

$$\dot{Q}_{\text{in, radiation}} - \dot{Q}_{\text{out, convection}} + \dot{Q}_{\text{gen, sensible}} = 0$$

or

$$\varepsilon A\sigma(T_{\text{surr}}^4 - T_{\text{person}}^4) - h_{\text{conv}} A(T_{\text{person}} - T_{\text{ambient}}) + \dot{Q}_{\text{gen, sensible}} = 0$$

$$0.95(1.8\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(40+273\,\text{K})^4 - (30+273\,\text{K})^4]$$
$$- h(1.8\,\text{m}^2)(30-22)°\text{C} + 150\ \text{W} = 0$$

Solving the equation above for h gives

$$h = 18.3\ \text{W/m}^2\cdot°\text{C}$$

Then the velocity of air needed to provide comfort for the workers is determined from $h = 14.8V^{0.69} = 18.3$ to be

$$V = \mathbf{1.36\ m/s}$$

Therefore, the velocity of air must be at least 1.36 m/s.

21-151 Switching to energy efficient lighting reduces the electricity consumed for lighting as well as the cooling load in summer, but increases the heating load in winter. It is to be determined if switching to efficient lighting will increase or decrease the total heating and cooling cost of the building whose annual heating load is roughly equal to the annual cooling load.

Assumptions **1** The annual heating load of the building is roughly equal to the annual cooling load. **2** The light escaping through the windows is negligible so that the entire lighting energy becomes part of the internal heat generation.

Analysis Consider 1 h of operation of lighting in summer and 1 h of operation winter.

Current lighting:

Lighting cost: (Energy used)(Unit cost)= (2 kWh)($0.08/kWh) = $0.16
Increase in the air conditioning load: = 1 kWh
Increase in the air conditioning cost: (Increase in load/COP)(unit cost)=(1 kWh/3.5)($0.08/kWh) = $0.0229
Decrease in the heating cost = (1/29.3 therm)($0.70/therm) = 0.0239

Net cost of 1 h of operation of lighting in summer and 1 h of operation in winter is

Current net cost = Cost of lighting - Cost of heating + Cost of Air-conditioning.
= 0.16-0.0229+0.0239
= **$0.16**



15W



60W

(*a*) *Energy-efficient lighting* (consumes 1/4th of the electricity for the same lighting)

Lighting cost: (Energy used)(Unit cost)= (0.5 kWh)($0.08/kWh) = $0.04

Increase in the air conditioning load: = 0.25 kWh
Increase in air conditioning cost: (Increase in load/COP)(unit cost)=(0.25 kWh/3.5)($0.08/kWh) = $0.0057
Decrease in the heating cost = (0.25/29.3 therm)($0.70/therm) = $0.0060

Net cost of 1 h of operation of lighting in summer and 1 h of operation in winter is

Current net cost = Cost of lighting - Cost of heating + Cost of Air-conditioning.
= 0.04-0.0057+0.0060
= **$0.04**

Therefore, the energy efficient lighting will reduce the total energy usage and cost of this family considerably.

21-152 The outer surfaces of the walls of a brick farmhouse are exposed to 24 km/h winds. The rate of heat transfer through a 20-m^2 section of the wall is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

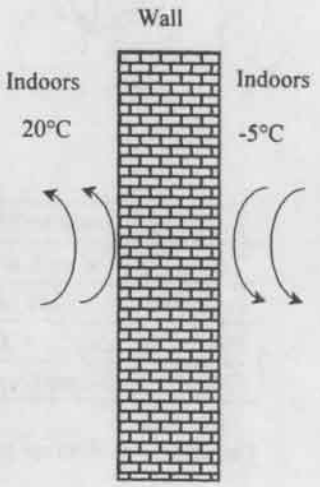
Properties The R-values of different materials are given in Table 21-10.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{insulation}} + \quad (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.80 for insulation section and 0.20 for stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 21-10 and calculating others, the total *R*-values for each section is determined in the table below.

Construction	*R*-value, $m^2 \cdot °C/W$
1. Outside surface, 24 km/h winds	0.030
2. Common brick, 200 mm	0.12×2 =0.24
3. Inside surface, still air	0.12
TOTAL	0.39



Then the *U*-factor of the wall after the rate of heat transfer through the wall become

$$U_{\text{wall}} = 1/R_{\text{wall}} = 1/0.39 = 2.56\ \text{m}^2 \cdot °\text{C/W}$$

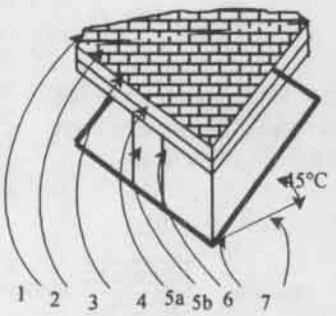
$$\dot{Q}_{\text{wall}} = UA(T_i - T_o) = (2.56\ \text{W/m}^2 \cdot °\text{C})(20\ \text{m}^2)[20-(-5)°\text{C}] = \mathbf{1280\ W}$$

21-153E The *R*-value and the *U*-factor of a 45° pitched roof are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the roof is one-dimensional. **3** Thermal properties of the roof and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 21-10.

Analysis The schematic of the pitched roof as well as the different elements used in its construction are shown below. Using the available *R*-values from Table 21-10, the overall *R*-value of the roof can be determined as shown in the table below.



Construction	*R*-value, h.ft^2.°F/Btu	
	Between studs	At studs
1. Outside surface, 15 mph wind	0.17	0.17
2. Asphalt shingle roofing	0.44	0.44
3. Building paper	0.10	0.10
4. Plywood deck, 5/8 in	0.78	0.78
5a. Reflective air space, 3.5-in	2.17	----
5b. Wood stud, 2 in by 4 in	----	3.58
6. Gypsum wallboard, 0.5 in	0.45	0.45
7. Inside surface, still air	0.63	0.63

Total unit thermal resistance of each section, *R*	4.74	6.15
The *U*-factor of each section, $U = 1/R$, in Btu/h.ft^2.°F	0.211	0.163
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{area,i} U_i = 0.80 \times 0.211 + 0.20 \times 0.163$	**0.201 Btu/h.ft^2.°F**	
Overall unit thermal resistance, $R = 1/U$	**4.97 h.ft^2.°F/Btu**	

Therefore, the *R*-value and *U*-factor of the roof are $R = 4.97$ h.ft^2.°F/Btu and $U = 0.201$ Btu/h.ft^2.°F

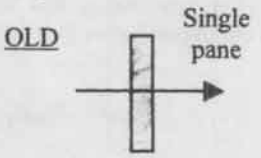
21-154 Heat losses through the windows of a house with aluminum frame single pane windows account for 26 percent of the total. The reduction in the heat load of the house as a result of switching to vinyl frame double pane windows is to be determined.

Properties The *U*-factors of the aluminum frame single pane and vinyl frame double pane windows are 7.16 and 2.74 W/m^2.°C, respectively.

Analysis The rate of heat transfer through the windows for the existing house is

$$\dot{Q}_{\text{window, old}} = (\text{Fraction of heat loss through windows})\dot{Q}_{\text{house, total}} = 0.26 \times (32\,\text{kW}) = 8.32\,\text{kW}$$

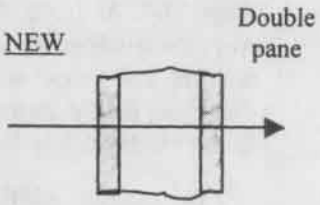
Then the rate of heat transfer through the new windows becomes



$$\frac{\dot{Q}_{\text{window, new}}}{\dot{Q}_{\text{window, old}}} = \frac{U_{\text{new}} A \Delta T}{U_{\text{old}} A \Delta T}$$

which gives

$$\dot{Q}_{\text{window, new}} = \frac{U_{\text{new}}}{U_{\text{old}}} \dot{Q}_{\text{window, old}} = \frac{2.74}{7.14}(8.32\,\text{kW}) = 3.19\,\text{kW}$$



Therefore, the reduction in the heat load of the house is

$$\dot{Q}_{\text{reducdtion}} = \dot{Q}_{\text{window, old}} - \dot{Q}_{\text{window, new}} = 8.32 - 3.19 = \mathbf{5.13\,kW}$$

Discussion Note that the heat load from the house will go down by 16% since 5.13/32 = 0.16.

21-155 The attic of a house in Thessaloniki, Greece is not vented in summer. The rate of heat gain through the roof in late July is to be determined assuming the roof is (*a*) light colored and (*b*) dark colored.

Assumptions **1** Steady operating conditions exist. **2** Thermal properties of the roof and the heat transfer coefficients are constant. **3** The effect of air infiltration on the attic temperature is negligible (this will result in a higher rate of heat transfer than actual). **4** The sol-air temperature for a horizontal surface can be used for the tilted roof.

Properties The *R*-values of the roof and the ceiling are given to be 1.4 m^2.°C/W and 0.50 m^2.°C/W, respectively.

Analysis Noting that the roof-to-ceiling area ratio is 1.4, the thermal resistance of the roof-attic-ceiling combination per unit area of the ceiling is

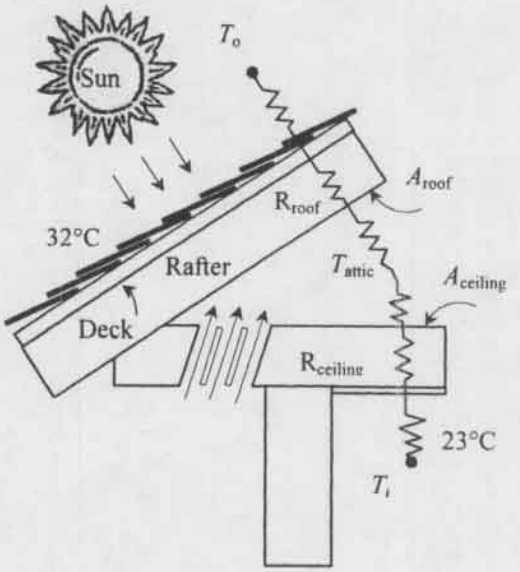
$$R_{\text{total}} = R_{\text{ceiling}} + \left(\frac{A_{\text{ceiling}}}{A_{\text{roof}}}\right) R_{\text{roof}} = 0.50 + \tfrac{1}{4}1.40 = 1.50\,\text{m}^2 \cdot \text{°C/W}$$

(*a*) The house is located at 40° N latitude, and thus we can use the sol-air temperature data directly from Table 21-7. At 16:00, the tabulated sol-air temperatures for a light-colored horizontal surface is 42.7°C. Also, the tabulated air temperature is 34.7°C, which is 2.7°C higher than the air temperature given in the problem. But we can still use the data in that table provided that we subtract 2.7°C from all temperatures. Therefore, the sol-air temperature on the roof in this case is 42.7 - 2.7 = 40.0°C, and the heat gain through the roof is determined to be

$$\begin{aligned}\dot{Q}_{\text{roof, light color}} &= UA(T_{\text{sol-air, light color}} - T_{\text{inside}}) \\ &= A\frac{T_{\text{sol-air, light color}} - T_{\text{inside}}}{R} \\ &= (150\,\text{m}^2)\frac{(40-23)\text{°C}}{1.50\,\text{m}^2 \cdot \text{°C/W}} = \mathbf{1700\ W}\end{aligned}$$

(*b*) The tabulated sol-air temperatures for a dark-colored horizontal surface is 54.7°C. Therefore, the sol-air temperature on the roof in this case is 54.7 - 2.7 = 52.0°C, and the heat gain through the roof is determined to be

$$\begin{aligned}\dot{Q}_{\text{roof, dark color}} &= UA(T_{\text{sol-air, dark color}} - T_{\text{inside}}) \\ &= A\frac{T_{\text{sol-air, darh color}} - T_{\text{inside}}}{R} \\ &= (150\,\text{m}^2)\frac{(52-23)\text{°C}}{1.50\,\text{m}^2 \cdot \text{°C/W}} = \mathbf{2900\ W}\end{aligned}$$

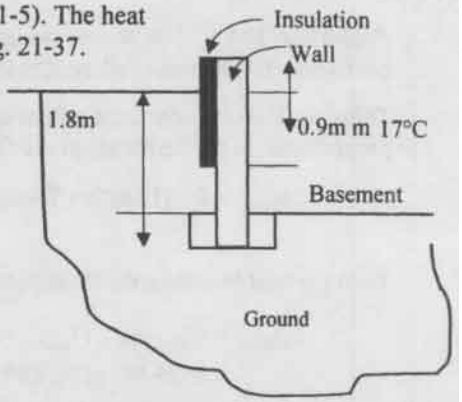


Discussion Note that the color of the exposed surface of the roof has a major effect on the rate of heat gain through the roof.

21-156 The peak heat loss from a below grade basement in Norfolk, Virginia to the ground through its walls and floor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The basement is maintained at 17°C.

Properties The winter average temperature of Norfolk is 9.9°C (Table 21-5). The heat transfer coefficients are given in Table 21-14a, and the amplitudes in Fig. 21-37.



Solution The floor and wall areas of the basement are

$$A_{\text{wall}} = \text{Height} \times \text{Perimeter} = 2 \times (1.8\ \text{m})(10 + 19\ \text{m}) = 104.4\ \text{m}^2$$

$$A_{\text{floor}} = \text{Length} \times \text{Width} = (10\ \text{m})(19\ \text{m}) = 190\ \text{m}^2$$

The amplitude of the annual soil temperature is determined from Fig. 21-37 to be 10°C. Then the ground surface temperature for the design heat loss becomes

$$T_{\text{ground surface}} = T_{\text{winter, mean}} - A = 9.9 - 10 = -0.1°\text{C}$$

The top 0.9-m section of the wall below the grade is insulated with *R*-0.73, and the heat transfer coefficients through that section are given in Table 21-14a to be 2.87and 2.20 W/m^2.°C through the 1st and 2nd 0.3-m wide depth increments, respectively. The heat transfer coefficients through the uninsulated section of the wall which extends from 0.6 m to 1.8 m level is determined from the same table to be 2.93, 2.23, 1.80, and 1.50 W/m^2.°C for each of the remaining 0.3-m wide depth increments. The average overall heat transfer coefficient is

$$U_{\text{wall, ave}} = \frac{\sum U_{\text{wall}}}{\text{No. of increments}} = \frac{2.87 + 2.20 + 2.93 + 2.23 + 1.8 + 1.5}{6} = 2.255\ \text{W/m}^2.°\text{C}$$

Then the heat loss through the basement wall becomes

$$\dot{Q}_{\text{basement walls}} = U_{\text{wall, ave}} A_{\text{wall}} (T_{\text{basement}} - T_{\text{ground surface}})$$

$$= (2.255\ \text{W/m}^2.°\text{C})(104.4\ \text{m}^2)[17 - (-0.1)°\text{C}] = 4026\ \text{W}$$

The shortest width of the house is 10 m, and the depth of the foundation below grade is 1.8 m. The floor heat transfer coefficient is determined from Table 21-14b to be 0.12 W/m^2.°C. Then the heat loss through the floor of the basement becomes

$$\dot{Q}_{\text{basement floor}} = U_{\text{floor}} A_{\text{floor}} (T_{\text{basement}} - T_{\text{ground surface}})$$

$$= (0.12\ \text{W/m}^2.°\text{C})(104.4\ \text{m}^2)[17 - (-0.1)]°\text{C} = 390\ \text{W}$$

which is considerably less than the heat loss through the wall. The total heat loss from the basement is then determined to be

$$\dot{Q}_{\text{basement}} = \dot{Q}_{\text{basement wall}} + \dot{Q}_{\text{basement floor}} = 4026 + 390 = \mathbf{4416\ W}$$

Discussion This is the *design* or *peak* rate of heat transfer from below-grade section of the basement, and this is the value to be used when sizing the heating system. The actual heat loss from the basement will be much less than that most of the time.

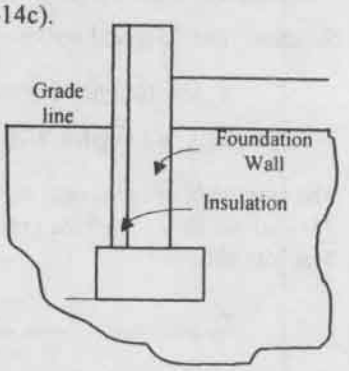
21-157 A house with a concrete slab floor sits directly on the ground at grade level, and the wall below grade is insulated. The heat loss from the floor at winter design conditions is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The house is maintained at 22°C. **3** The weather in Anchorage is severe.

Properties The 97.5% winter design conditions in Anchorage is -28°C (Table 21-4). The heat transfer coefficient for the insulated wall below grade is $U = 0.86$ W/m.°C (Table 21-14c).

Solution Heat transfer from a floor on the ground at the grade level is proportional to the perimeter of the floor, and the perimeter in this case is

$$p_{floor} = 2 \times (\text{Length} + \text{Width}) = 2(15 + 20)\ \text{m} = 70\ \text{m}$$



Then the heat loss from the floor becomes

$$\dot{Q}_{floor} = U_{floor} p_{floor} (T_{indoor} - T_{outdoor})$$
$$= (0.86\ \text{W/m·°C})(70\ \text{m})[22 - (-28)]\text{°C} = \mathbf{2890\ W}$$

Discussion This is the *design* or *peak* rate of heat transfer from below-grade section of the basement, and this is the value to be used when sizing the heating system. The actual heat loss from the basement will be much less than that most of the time.

21-158 A house with a concrete slab floor sits directly on the ground at grade level, and the wall below grade is uninsulated. The heat loss from the floor at winter design conditions is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The house is maintained at 22°C. **3** The weather in Anchorage is severe.

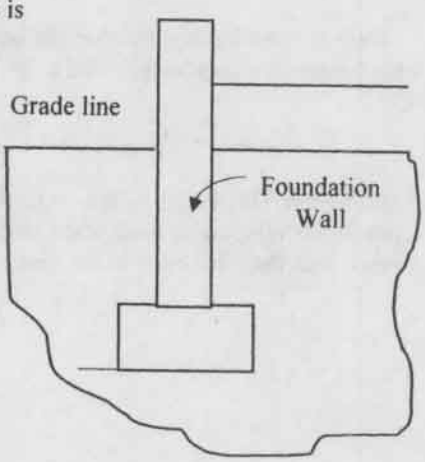
Properties The 97.5% winter design conditions in Baltimore is -28°C (Table 21-4). The heat transfer coefficient for the uninsulated wall below grade is $U = 1.17$ W/m.°C (Table 21-14c).

Analysis Heat transfer from a floor on the ground at the grade level is proportional to the perimeter of the floor, and the perimeter in this case is

$$p_{floor} = 2 \times (\text{Length} + \text{Width}) = 2(15 + 20)\ \text{m} = 70\ \text{m}$$

Then the heat loss from the floor becomes

$$\dot{Q}_{floor} = U_{floor} p_{floor} (T_{indoor} - T_{outdoor})$$
$$= (1.17\ \text{W/m·°C})(70\ \text{m})[22 - (-28)]\text{°C} = \mathbf{3931\ W}$$



Discussion This is the *design* or *peak* rate of heat transfer from below-grade section of the basement, and this is the value to be used when sizing the heating system. The actual heat loss from the basement will be much less than that most of the time.

21-159 The classrooms and faculty offices of a university campus are not occupied an average of 4 hours a day, but the lights are kept on. The amounts of electricity and money the campus will save a year if the lights are turned off during unoccupied periods are to be determined.

Analysis The total electric power consumed by the lights in the classrooms and faculty offices is

$$\dot{E}_{\text{lighting, classroom}} = (\text{Power consumed per lamp}) \times (\text{No. of lamps}) = (200 \times 12 \times 110\ \text{W}) = 264{,}000 = 264\ \text{kW}$$

$$\dot{E}_{\text{lighting, offices}} = (\text{Power consumed per lamp}) \times (\text{No. of lamps}) = (400 \times 6 \times 110\ \text{W}) = 264{,}000 = 264\ \text{kW}$$

$$\dot{E}_{\text{lighting, total}} = \dot{E}_{\text{lighting, classroom}} + \dot{E}_{\text{lighting, offices}} = 264 + 264 = 528\ \text{kW}$$

Noting that the campus is open 240 days a year, the total number of unoccupied work hours per year is

$$\text{Unoccupied hours} = (4\ \text{hours/day})(240\ \text{days/year}) = 960\ \text{h/yr}$$

Then the amount of electrical energy consumed per year during unoccupied work period and its cost are

$$\text{Energy savings} = (\dot{E}_{\text{lighting, classroom}})(\text{Unoccupied hours}) = (528\ \text{kW})(960\ \text{h/yr}) = \mathbf{506{,}880\ kWh}$$

$$\text{Cost savings} = (\text{Energy savings})(\text{Unit cost of energy}) = (506{,}880\ \text{kWh})(\$0.075/\text{kWh}) = \mathbf{\$38{,}016/yr}$$

Discussion Note that simple conservation measures can result in significant energy and cost savings.

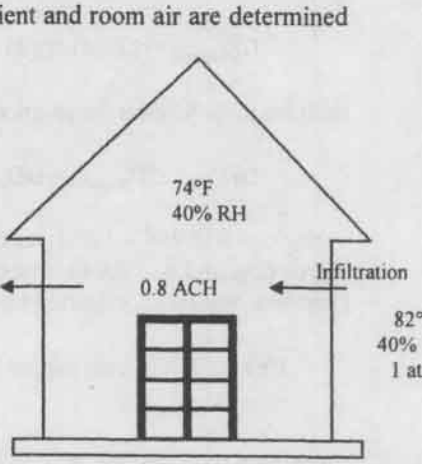
21-160E The infiltration rate of a building is estimated to be 0.8 ACH. The sensible, latent, and total infiltration heat loads of the building at sea level are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air infiltrates at the outdoor conditions, and exfiltrates at the indoor conditions. **3** Excess moisture condenses at 40°F. **4** The effect of water vapor on air density is negligible.

Properties The gas constant and the specific heat of air are $R = 0.3704$ psia.ft^3/lbm.R and $C_p = 0.24$ Btu/lbm·°F (Table A-18E). The heat of vaporization of water at 40°F is $h_{fg} = h_{fg@40°F} = 1071$ Btu/lbm (Table A-15E). The properties of the ambient and room air are determined from the psychrometric chart to be

$$\left.\begin{aligned} T_{\text{ambient}} &= 82°\text{F} \\ \phi_{\text{ambient}} &= 40\% \end{aligned}\right\} \begin{aligned} w_{\text{ambient}} &= 0.094\ \text{lbm/lbm dryair} \\ v &= 13.7\ \text{ft}^3/\text{lbm dry air} \end{aligned}$$

$$\left.\begin{aligned} T_{\text{room}} &= 74°\text{F} \\ \phi_{\text{room}} &= 40\% \end{aligned}\right\} w_{\text{room}} = 0.073\ \text{lbm/lbm dryair}$$



Analysis Noting that the infiltration of ambient air will cause the air in the cold storage room to be changed 0.8 times every hour, the air will enter the room at a mass flow rate of

$$\dot{m}_{\text{air}} = \frac{V_{\text{room}}}{v_{\text{ambient}}} ACH = \frac{9 \times 50 \times 60\ \text{ft}^3}{13.7\ \text{ft}^3/\text{lbm dry air}} (0.8\ \text{h}^{-1}) = 176\ \text{lbm/h}$$

Then the sensible, latent, and total infiltration heat loads of the room are

$$\dot{Q}_{\text{infiltration,sensible}} = \dot{m}_{\text{air}} C_p (T_{\text{ambient}} - T_{\text{room}}) = (176\ \text{lbm/h})(0.24\ \text{Btu/lbm.°F})(82 - 74)°\text{F} = \mathbf{337\ Btu/h}$$

$$\dot{Q}_{\text{infiltration,latent}} = \dot{m}_{\text{air}} (w_{\text{ambient}} - w_{\text{room}}) h_{fg} = (176\ \text{lbm/h})(0.094 - 0.073)(1071\ \text{Btu/lbm}) = \mathbf{3952\ Btu/h}$$

$$\dot{Q}_{\text{infiltration}} = \dot{Q}_{\text{infiltration,sensible}} + \dot{Q}_{\text{infiltration,latent}} = 3952 + 337 = \mathbf{4289\ Btu/h}$$

Discussion The specific volume of the dry air at the ambient conditions could also be determined from the ideal gas relation, $v = RT / P = (0.3704\ \text{psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(82 + 460\ \text{R})/14.7\ \text{psia} = 13.7\ \text{ft}^3/\text{lbm}$.

21-161 It is believed that January is the coldest month in the Northern hemisphere. On the basis of Table 21-5, it is to be determined if this is true for all locations.

Analysis Degree-days is a measure of coldness of a location. We notice from Table 21-5 that in Atlanta, Georgia the heating degree-days are 648°F-days in December and 636°F-days in January. Therefore, January is **not** necessarily the coldest month in Northern Hemisphere.

21-162 The December space heating bill of a fully occupied house is given. The heating bill of this house is to be determined if it were not occupied.

Assumptions The outdoors temperature never rises above 18°C in January.

Properties The heating degree-days for Louisville, Kentucky for the month of January is 890°F-days (Table 21-5). It is equivalent to 890/1.8 = 494°C-days.

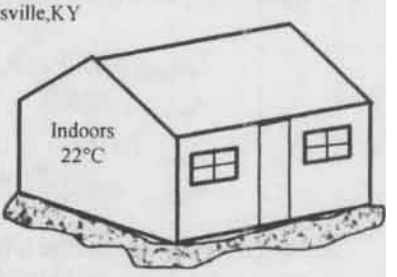
Analysis The degree-days method is based on the assumption that the internal heat gain can meet the heating needs of a house when the outdoors temperature is above 18°C. Therefore, for an indoor temperature of 22°C, the number of degree-days that correspond to the temperature range of 18 to 22°C is

$$DD_{internal} = (22-18)\ °C(31\ days) = 124°C\text{-days}$$

Then the total °C-days for an unoccupied building becomes

$$DD_{total} = DD_{standard} + DD_{internal} = 494+124 = 618°C\text{-days}$$

Noting that 618/494 = 1.251, we conclude that the number of heating degree-days and thus the heating bill in January will increase by 25.1%. Therefore, the January heating bill for the unoccupied house will be



$$\text{Heating bill of unoccupied house} = 1.251 \times (\text{Heating bill of occupied house})$$
$$= 1.251 \times \$110 = \mathbf{\$138}$$

Discussion Note that the heating bill of this house will increase by 25.1% in the absence of any internal heat gain.

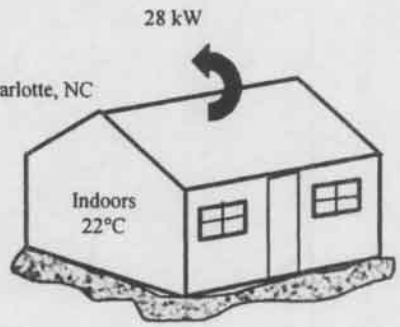
21-163 The annual gas consumption and its cost for a house in Charlotte, NC with a design heat load of 28 kW and a furnace efficiency of 80% are to be determined.

Assumptions The house is maintained at 22°C at all times during the heating season.

Properties The annual heating degree-days of Charlotte, NC is 3191°F-days (Table 21-5). It is equivalent to 3191/1.8 = 1773°C-days. The 97.5% winter design temperature of Charlotte is -6°C.

Analysis (*a*) The overall heat loss coefficient K_{overall} of the building is determined from

$$\dot{Q}_{\text{design}} = UA\Delta T_{\text{design}} = UA(T_i - T_o)_{\text{design}} = K_{\text{overall}}(T_i - T_o)_{\text{design}}$$



Substituting,

$$K_{\text{overall}} = \frac{\dot{Q}_{\text{design}}}{(T_i - T_o)_{\text{design}}} = \frac{28{,}000\ \text{W}}{[22-(-6)]^\circ\text{C}} = 1000\text{W} = \mathbf{1\ kW/^\circ C}$$

(*b*) The annual gas consumption of the house for heating and its cost are determined to be

$$\text{Annual Gas Consumption} = \frac{K_{\text{overall}}}{\eta_{\text{heating}}} DD_{\text{heating}}$$

$$= \frac{1000\ \text{W/°C}}{0.80}(1773^\circ\text{C-days})\left(\frac{24\times 3600\ \text{s}}{1\text{day}}\right)$$

$$= 1.915\times 10^8 = 1814\ \text{therms} \qquad (\text{since 1 therm} = 105{,}500\ \text{kJ})$$

Then the annual heating cost becomes

Annual heating cost = (Annual energy consumption)(Unit cost of energy)

= (1815 therms/year)($0.70/therm) = **$1271/year**

Therefore, it will cost $1281 per year to heat this house.

Chapter 22
COOLING OF ELECTRONIC EQUIPMENT

Introduction and History

22-1C The invention of vacuum diode started the electronic age. The invention of the transistor marked the beginning of a revolution in that age since the transistors performed the functions of the vacuum tubes with greater reliability while occupying negligible space and consuming negligible power compared to the vacuum tubes.

22-2C Integrated circuits are semiconductor devices in which several components such as diodes, transistors, resistors and capacitors are housed together. The initials MSI, LSI, and VLSI stand for medium scale integration, large scale integration, and very large scale integration, respectively.

22-3C The electrical resistance R is a measure of resistance against current flow, and the friction between the electrons and the material causes heating. The amount of the heat generated can be determined from Ohm's law, $W = I^2 R$.

22-4C The electrical energy consumed by the TV is eventually converted to heat, and the blanket wrapped around the TV prevents the heat from escaping. Then the temperature of the TV set will have to start rising as a result of heat build up. The TV set will have to burn up if operated this way for a long time. However, for short time periods, the temperature rise will not reach destructive levels.

22-5C Since the heat generated in the incandescent light bulb which is completely wrapped can not escape, the temperature of the light bulb will increase, and will possibly start a fire by igniting the towel.

22-6C When the air flow to the radiator is blocked, the hot water coming off the engine cannot be cooled, and thus the engine will overheat and fail, and possible catch fire.

22-7C A car is much more likely to break since it has more moving parts than a TV.

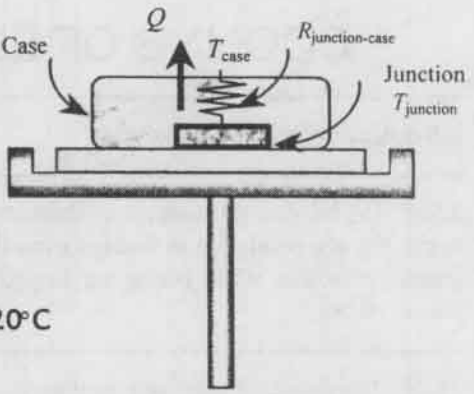
22-8C Diffusion in semi-conductor materials, chemical reactions and creep in the bending materials cause electronic components to fail under prolonged use at high temperatures.

22-9 The case temperature of a power transistor and the junction-to-case resistance are given. The junction temperature is to be determined.

Assumptions Steady operating conditions exist.

Analysis The rate of heat transfer between the junction and the case in steady operation is

$$\dot{Q} = \left(\frac{\Delta T}{R}\right)_{junction-case} = \frac{T_{junction} - T_{case}}{R_{junction-case}}$$



Then the junction temperature is determined to be

$$T_{junction} = T_{case} + \dot{Q}R_{junction-case} = 60°C + (12\ W)(5°C/W) = \mathbf{120°C}$$

22-10 The power dissipated by an electronic component as well as the junction and case temperatures are measured. The junction-to-case resistance is to be determined.

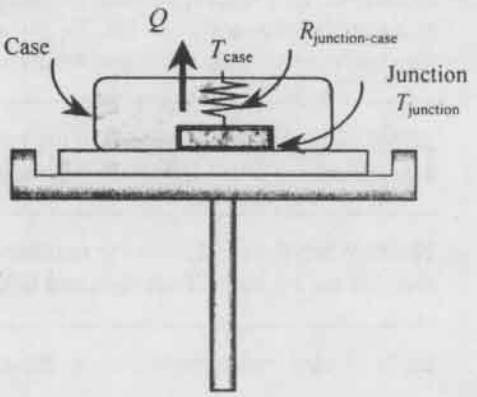
Assumptions Steady operating conditions exist.

Analysis The rate of heat transfer from the component is

$$\dot{W}_e = \dot{Q} = VI = (12\ V)(0.15\ A) = 1.8\ W$$

Then the junction-to-case thermal resistance of this component becomes

$$R_{junction-case} = \frac{T_{junction} - T_{case}}{\dot{Q}} = \frac{(90-65)°C}{1.8\ W} = \mathbf{13.9°C/W}$$



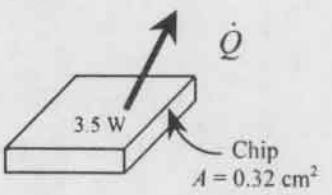
22-11 A logic chip dissipates 3.5 W power. The amount of heat this chip dissipates during a 10-h period and the heat flux on the surface of the chip are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer from the surface is uniform.

Analysis (*a*) The amount of heat this chip dissipates during an eight-hour work day is

$$Q = \dot{Q}\Delta t = (0.0035\ kW)(8\ h) = \mathbf{0.028\ kWh}$$

(*b*) The heat flux on the surface of the chip is



$$\dot{q} = \frac{\dot{Q}}{A} = \frac{3.5\ W}{0.32\ cm^2} = \mathbf{10.9\ W/cm^2}$$

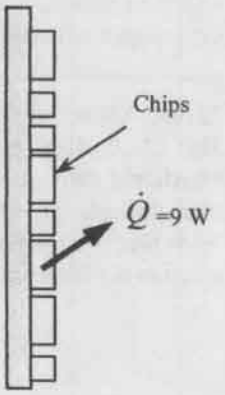
22-12 A circuit board houses 90 closely spaced logic chips, each dissipating 0.1 W. The amount of heat this chip dissipates in 10 h and the heat flux on the surface of the circuit board are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer from the back surface of the board is negligible.

Analysis (a) The rate of heat transfer and the amount of heat this circuit board dissipates during a ten-hour period are

$$\dot{Q}_{total} = (90)(0.1\text{ W}) = 9\text{ W}$$

$$Q_{total} = \dot{Q}_{total}\Delta t = (0.009\text{ kW})(10\text{ h}) = \mathbf{0.09\ kWh}$$



(b) The average heat flux on the surface of the circuit board is

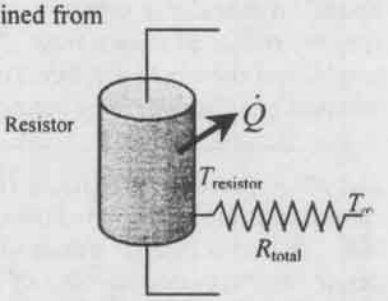
$$\dot{q} = \frac{\dot{Q}_{total}}{A} = \frac{9\text{ W}}{(15\text{ cm})(20\text{ cm})} = \mathbf{0.03\ W/cm^2}$$

22-13E The total thermal resistance and the temperature of a resistor are given. The power at which it can operate safely in a particular environment is to be determined.

Assumptions Steady operating conditions exist.

Analysis The power at which this resistor can be operate safely is determined from

$$\dot{Q} = \frac{T_{resistor} - T_{ambient}}{R_{total}} = \frac{(360 - 120)^\circ\text{F}}{130^\circ\text{F/W}} = \mathbf{1.85\ W}$$



22-14 The surface-to-ambient thermal resistance and the surface temperature of a resistor are given. The power at which it can operate safely in a particular environment is to be determined.

Assumptions Steady operating conditions exist.

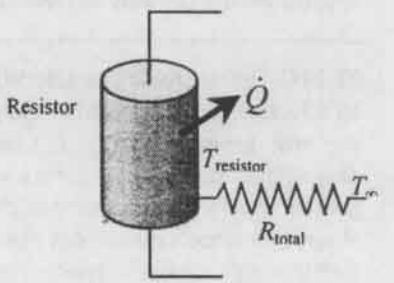
Analysis The power at which this resistor can operate safely is determined from

$$\dot{Q} = \frac{T_{resistor} - T_{ambient}}{R_{total}} = \frac{(150 - 30)^\circ\text{C}}{300^\circ\text{C/W}} = \mathbf{0.4\ W}$$

At specified conditions, the resistor dissipates

$$\dot{Q} = \frac{V^2}{R} = \frac{(7.5\text{ V})^2}{(100\ \Omega)} = 0.5625\text{ W}$$

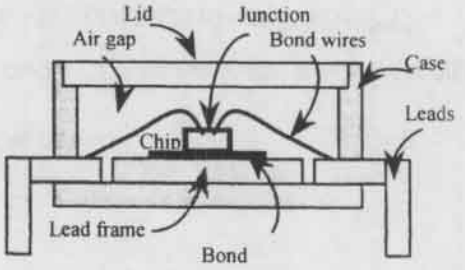
of power. Therefore, the current operation is not safe.



Manufacturing of Electronic Equipment

22-15C The thermal expansion coefficient of the plastic is about 20 times that of silicon. Therefore, bonding the silicon directly to the plastic case will result in such large thermal stresses that the reliability would be seriously jeopardized. To avoid this problem, a lead frame made of a copper alloy with a thermal expansion coefficient close to that of silicon is used as the bonding surface.

22-16C The schematic of chip carrier is given in Fig. 22-20. Heat generated at the junction is transferred through the chip to the led frame, then through the case to the leads. From the leads heat is transferred to the ambient or to the medium the leads are connected to.



22-17C The cavity of the chip carrier is filled with a gas which is a poor conductor of heat. Also, the case is often made of materials which are also poor conductors of heat. This results in a relatively large thermal resistance between the chip and the case, called the junction-to-case thermal resistance. It depends on the geometry and the size of the chip carrier as well as the material properties of the bonding material and the case.

22-18C A hybrid chip carrier houses several chips, individual electronic components, and ordinary circuit elements connected to each other. The result is improved performance due to the shortening of the wiring lengths, and enhanced reliability. Lower cost would be an added benefit of multi-chip packages if they are produced in sufficiently large quantities.

22-19C A printed circuit board (PCB) is a properly wired plane board on which various electronic components such as the ICs, diodes, transistors, resistors, and capacitors are mounted to perform a certain task. The board of a PCB is made of polymers and glass epoxy materials. The thermal resistance between a device on the board and edge of the board is called as device-to-PCB edge thermal resistance. This resistance is usually high (about 20 to 60 °C/W) because of the low thickness of the board and the low thermal conductivity of the board material.

22-20C The three types of circuit boards are the single-sided, double-sided, and multi-layer boards. The single-sided PCBs have circuitry lines on one side of the board only, and are suitable for low density electronic devices (10-20 components). The double-sided PCBs have circuitry on both sides, and are best suited for intermediate density devices. Multi-layer PCBs contain several layers of circuitry, and they are suitable for high density devices. They are equivalent to several PCBs sandwiched together.

22-21C The desirable characteristics of the materials used in the fabrication of circuit boards are: (1) being an effective electrical insulator to prevent electrical breakdown, (2) being a good heat conductor to conduct the heat generated away, (3) having high material strength to withstand the forces and to maintain dimensional stability, (4) having a thermal expansion coefficient which closely matches to that of copper to prevent cracking in the copper cladding during thermal cycling, (5) having a high resistance to moisture absorption since moisture can effect both mechanical and electrical properties and degrade performance, (6) stability in properties at temperature levels encountered in electronic applications, (7) ready availability and manufacturability, and, of course (8) low cost.

22-22C An electronic enclosure (a case or a cabinet) house the circuit boards and the necessary peripheral equipment and connectors. It protects them from the detrimental effects of the environment, and may provide a cooling path. An electronic enclosure can simply be made of sheet metals such as thin gauge aluminum or steel.

Cooling Load of Electronic Equipment and Thermal Environment

22-23C The heating load of an electronic box which consumes 120 W of power is simply 120 W because of the conservation of energy principle.

22-24C Superconductor materials will generate hardly any heat and as a result, more components can be packed into a smaller volume, resulting in enhanced speed and reliability without having to resort to some exotic cooling techniques.

22-25C The actual power dissipated by a device can be considerably less than its rated power, depending on its duty cycle (the fraction of time it is on). A 5 W power transistor, for example, will dissipate an average of 2 W of power if it is active only 40 percent of the time. Then we can treat this transistor as a 2-W device when designing a cooling system. This may allow the selection of a simpler and cheaper cooling mechanism.

22-26C The cyclic variation of temperature of an electronic device during operation is called the temperature cycling. The thermal stresses caused by temperature cycling undermines the reliability of electronic devices. The failure rate of electronic devices subjected to deliberate temperature cycling of more than 20 °C is observed to increase by eight-fold.

22-27C The ultimate heat sink for a TV is the room air with a temperature range of about 10 to 30°C. For an airplane it is the ambient air with a temperature range of about −50 °C to 50 °C. The ultimate heat sink for a ship is the sea water with a temperature range of 0 °C to 30 °C.

22-28C The ultimate heat sink for a VCR is the room air with a temperature range of about 10 to 30°C. For a spacecraft it is the ambient air or space with a temperature range of about −273°C to 50 °C. The ultimate heat sink for a communication system on top of a mountain is the ambient air with a temperature range of about −20 °C to 50 °C.

Electronics Cooling in Different Applications

22-29C The electronics of short-range missiles do not need any cooling because of their short cruising times. The missiles reach their destinations before the electronics reach unsafe temperatures. The long-range missiles must be cooled because of their long cruise times (several hours). The electronics in this case are cooled by passing the liquid fuel they carry through the cold plate of the electronics enclosure as it flows towards the combustion chamber.

22-30C Dynamic temperature is the rise in the temperature of a fluid as a result of the ramming effect or the stagnation process. This is due to the conversion of kinetic energy to internal energy which is significant at high velocities. It is determined from $T_{dynamic} = V^2 / (2C_p)$ where V is the velocity and C_p is the specific heat of the fluid. It is significant at velocities above 100 m/s.

22-31C The electronic equipment in ships and submarines are usually housed in rugged cabinets to protect them from vibrations and shock during stormy weather. Because of easy access to water, water cooled heat exchangers are commonly used to cool sea-born electronics. Often air in a closed or open loop is cooled in an air-to-water heat exchanger, and is forced to the electronic cabinet by a fan.

22-32C The electronics of communication systems operate for long periods of time under adverse conditions such as rain, snow, high winds, solar radiation, high altitude, high humidity, and too high or too low temperatures. Large communication systems are housed in specially built shelters. Sometimes it is necessary to air-condition these shelters to safely dissipate the large quantities of heat generated by the electronics of communication systems.

22-33C The electronic components used in the high power microwave equipment such as radars generate enormous amounts of heat because of the low conversion efficiency of electrical energy to microwave energy. The klystron tubes of high power radar systems where radio frequency (RF) energy is generated can yield local heat fluxes as high as 2000 W/cm^2. The safe and reliable dissipation of such high heat fluxes usually require the immersion of such equipment into a suitable dielectric fluid which can remove large quantities of heat by boiling.

22-34C The electronic equipment in space vehicles are usually cooled by a liquid circulated through the components where heat is picked up, and then through a space radiator where the waste heat is radiated into deep space at 0 K. In such systems it may be necessary to run a fan in the box to circulate the air since there is no natural convection currents in space because of the absence of a gravity field.

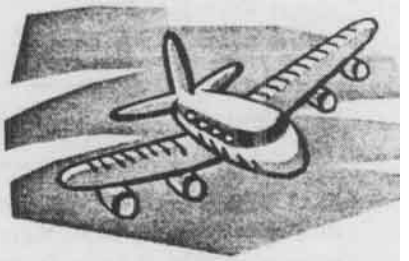
22-35 An airplane cruising in the air at a temperature of -10°C at a velocity of 850 km/h is considered. The temperature rise of air is to be determined.

Assumptions Steady operating conditions exist.

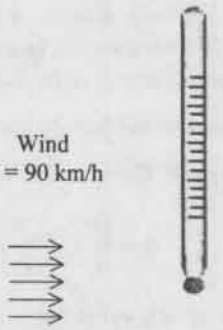
Analysis The temperature rise of air (dynamic temperature) at this speed is

$$T_{dynamic} = \frac{V^2}{2C_p} = \frac{(850 \times 1000/3600\ \text{m/s})^2}{(2)(1003\ \text{J/kg.°C})}\left(\frac{1\ \text{J/kg}}{1\ \text{m}^2/\text{s}^2}\right) = \mathbf{27.8°C}$$

V = 850 km/h



22-36 The temperature of air in the wind at a wind velocity of 90 km/h is measured to be 12°C. The true temperature of air is to be determined.



Assumptions Steady operating conditions exist.

Analysis The temperature rise of air (dynamic temperature) at this speed is

$$T_{dynamic} = \frac{\mathbf{V}^2}{2C_p} = \frac{(90\times 1000/3600\ \text{m/s})^2}{(2)(1005\ \text{J/kg.°C})}\left(\frac{1\ \text{J/kg}}{1\ \text{m}^2/\text{s}^2}\right) = \mathbf{0.3°C}$$

Therefore, the true temperature of air is

$$T_{true} = T_{measured} - T_{dynamic} = (12-0.3)°\text{C} = \mathbf{11.7°C}$$

22-37 Air at 25°C is flowing in a channel. The temperature a stationary probe inserted into the channel will read is to be determined for different air velocities.

Assumptions Steady operating conditions exist.

Analysis (*a*) The temperature rise of air (dynamic temperature) for an air velocity of 1 m/s is

$$T_{dynamic} = \frac{\mathbf{V}^2}{2C_p} = \frac{(1\ \text{m/s})^2}{(2)(1005\ \text{J/kg.°C})}\left(\frac{1\ \text{J/kg}}{1\ \text{m}^2/\text{s}^2}\right) = 0.0005°\text{C}$$

Then the temperature which a stationary probe will read becomes

$$T_{measured} = T_{true} + T_{dynamic} = 25 + 0.0005 = \mathbf{25.0005°C}$$

(*b*) For an air velocity of 10 m/s the temperature rise is

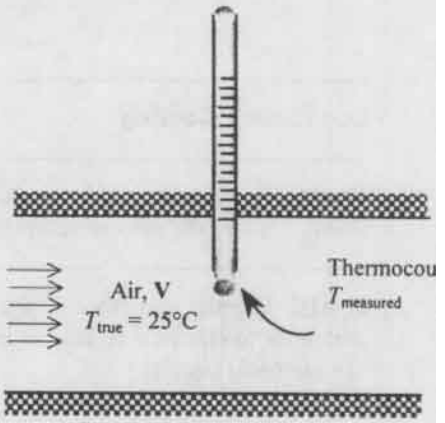
$$T_{dynamic} = \frac{V^2}{2C_p} = \frac{(10\ \text{m/s})^2}{(2)(1005\ \text{J/kg.°C})}\left(\frac{1\ \text{J/kg}}{1\ \text{m}^2/\text{s}^2}\right) = 0.05°\text{C}$$

Then, $T_{measured} = T_{true} + T_{dynamic} = 25 + 0.05 = \mathbf{25.05°C}$

(c) For an air velocity of 100 m/s the temperature rise is

$$T_{dynamic} = \frac{V^2}{2C_p} = \frac{(100\ \text{m/s})^2}{(2)(1005\ \text{J/kg.°C})}\left(\frac{1\ \text{J/kg}}{1\ \text{m}^2/\text{s}^2}\right) = 4.98°\text{C}$$

Then, $T_{measured} = T_{true} + T_{dynamic} = 25 + 4.98 = \mathbf{29.98°C}$

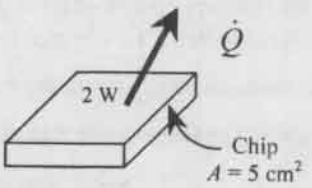


(*d*) For an air velocity of 1000 m/s the temperature rise is

$$T_{dynamic} = \frac{V^2}{2C_p} = \frac{(1000\ \text{m/s})^2}{(2)(1005\ \text{J/kg.°C})}\left(\frac{1\ \text{J/kg}}{1\ \text{m}^2/\text{s}^2}\right) = 497.5°\text{C}$$

Then, $T_{measured} = T_{true} + T_{dynamic} = 25 + 497.5 = \mathbf{522.5°C}$

22-38 Power dissipated by an electronic device as well as its surface area and surface temperature are given. A suitable cooling technique for this device is to be determined.



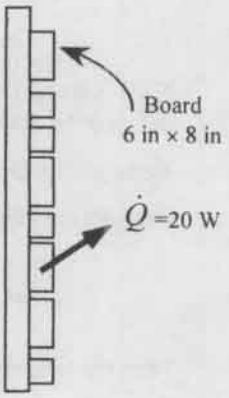
Assumptions Steady operating conditions exist.

Analysis The heat flux on the surface of this electronic device is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{2\ \text{W}}{5\ \text{cm}^2} = 0.4\ \text{W/cm}^2$$

For an allowable temperature rise of 50°C, the suitable cooling technique for this device is determined from Fig. 22-17 to be **forced convection** with direct air.

22-39E Power dissipated by a circuit board as well as its surface area and surface temperature are given. A suitable cooling mechanism is to be selected.



Assumptions Steady operating conditions exist.

Analysis The heat flux on the surface of this electronic device is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{20\ \text{W}}{(6\ \text{in} \times 2.54\ \text{cm/in})(8\ \text{in} \times 2.54\ \text{cm/in})} = 0.065\ \text{W/cm}^2$$

For an allowable temperature rise of 85°F, the suitable cooling technique for this device is determined from Fig. 22-17 to be **natural convection** with direct air.

Conduction Cooling

22-40C The major considerations in the selection of a cooling technique are the magnitude of the heat generated, the reliability requirements, the environmental conditions, and the cost.

22-41C Thermal resistance is the resistance of a materiel or device against heat flow through it. It is analogous to electrical resistance in electrical circuits, and the thermal resistance networks can be analyzed like electrical circuits.

22-42C If the rate of heat conduction through a medium $\dot{Q}$, and the thermal resistance R of the medium are known, then the temperature difference across the medium can be determined from $\Delta T = \dot{Q}R$.

22-43C The voltage drop across the wire is determined from $\Delta V = IR$. The length of the wire is proportional to the electrical resistance [$R = L/(\rho A)$], which is proportional to the voltage drop. Therefore, doubling the wire length while the current I is held constant will double the voltage drop.

The temperature drop across the wire is determined from $\Delta T = \dot{Q}R$. The length of the wire is proportional to the thermal resistance [$R = L/(kA)$], which is proportional to the temperature drop. Therefore, doubling the wire length while the heat flow Q is held constant will double the temperature drop.

22-44C A heat frame is a thick metal plate attached to a circuit board. It enhances heat transfer by providing a low resistance path for the heat flow from the circuit board to the heat sink. The thicker the heat frame, the lower the thermal resistance and thus the smaller the temperature difference between the center and the ends of the heat frame. The electronic components at the middle of a PCB operate at the highest temperature since they are furthest away from the heat sink.

22-45C Heat flow from the junction to the body of a chip is three-dimensional, but can be approximated as being one-dimensional by adding a constriction thermal resistance to the thermal resistance network. For a small heat generation area of diameter a on a considerably larger body, the constriction resistance is given by $R_{constriction} = 1/(2\sqrt{\pi}ak)$ where k is the thermal conductivity of the larger body. The constriction resistance is analogous to a partially closed valve in fluid flow, and a sudden drop in the cross-sectional area of an wire in electric flow.

22-46C The junction-to-case thermal resistance of an electronic component is the overall thermal resistance of all parts of the electronic component between the junction and case. In practice, this value is determined experimentally. When the junction-to-case resistance, the power dissipation, and the case temperature are known, the junction temperature of a component is determined from

$$T_{junction} = T_{case} + \dot{Q}R_{junction-case}$$

22-47C The case-to-ambient thermal resistance of an electronic device is the total thermal resistance of all parts of the electronic device between its outer surface and the ambient. In practice, this value is determined experimentally. Usually, manufacturers list the total resistance between the junction and the ambient for devices they manufacture for various configurations and ambient conditions likely to be encountered. When the case-to-ambient resistance, the power dissipation, and the ambient temperature are known, the junction temperature of the device is determined from $T_{junction} = T_{ambient} + \dot{Q}R_{junction-ambient}$

22-48C The junction temperature in this case is determined from

$$T_{junction} = T_{ambient} + \dot{Q}\left(R_{junction-case} + R_{case-ambient}\right).$$

When $R_{junction-case} > R_{case-ambient}$, the case temperature will be closer to the ambient temperature.

22-49C The PCBs are made of electrically insulating materials such as glass-epoxy laminates which are poor conductors of heat. Therefore, the rate of heat conduction along a PCB is very low. Heat conduction from the mid parts of a PCB to its outer edges can be improved by attaching heat frames or clamping cold plates to it. Heat conduction across the thickness of the PCB can be improved by planting copper or aluminum pins across the thickness of the PCB to serve as thermal bridges.

22-50C The thermal expansion coefficients of aluminum and copper are about twice as large as that of the epoxy-glass. This large difference in the thermal expansion coefficients can cause warping on the PCBs if the epoxy and the metal are not bonded properly. Warping is a major concern because it decreases reliability. One way of avoiding warping is to use PCBs with components on both sides.

22-51C The thermal conduction module received a lot of attention from thermal designers because the thermal design was incorporated at the initial stages of electrical design. The TCM was different from previous chip designs in that it incorporated both electrical and thermal considerations in early stages of design. The cavity in the TCM is filled with helium (instead of air) because of its very high thermal conductivity (about six times that of air).

22-52 The dimensions and power dissipation of a chip are given. The junction temperature of the chip is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through various components is one-dimensional. **3** Heat transfer through the air gap and the lid on top of the chip is negligible because of the very large thermal resistance involved along this path.

Analysis The various thermal resistances on the path of primary heat flow are

$$R_{constriction} = \frac{1}{2\sqrt{\pi}ak} = \frac{1}{2\sqrt{\pi}(0.5\times10^{-3}\ \text{m})(120\ \text{W/m.°C})} = 4.7°\text{C/W}$$

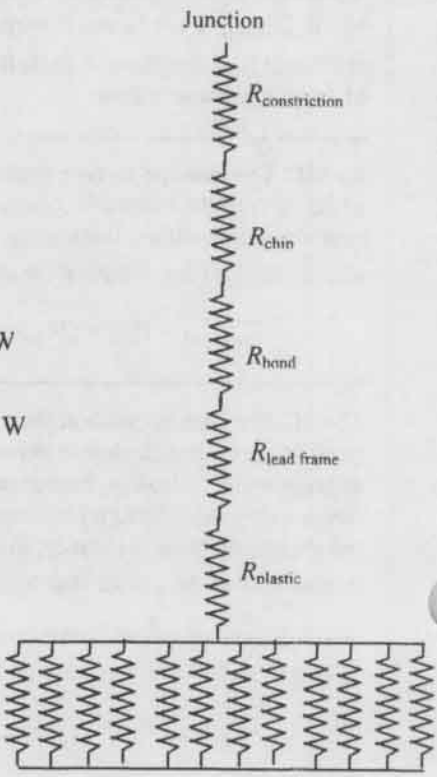
$$R_{chip} = \frac{L}{kA} = \frac{0.5\times10^{-3}\ \text{m}}{(120\ \text{W/m.°C})(0.004\times0.004)\text{m}^2} = 0.26°\text{C/W}$$

$$R_{bond} = \frac{L}{kA} = \frac{0.05\times10^{-3}\ \text{m}}{(296\ \text{W/m.°C})(0.004\times0.004)\text{m}^2} = 0.011°\text{C/W}$$

$$R_{\substack{lead\\frame}} = \frac{L}{kA} = \frac{0.25\times10^{-3}\ \text{m}}{(386\ \text{W/m.°C})(0.004\times0.004)\text{m}^2} = 0.04°\text{C/W}$$

$$R_{plastic} = \frac{L}{kA} = \frac{0.3\times10^{-3}\ \text{m}}{(1\ \text{W/m.°C})(18\times0.001\times0.00025)\text{m}^2} = 66.67°\text{C/W}$$

$$R_{leads} = \frac{L}{kA} = \frac{6\times10^{-3}\ \text{m}}{(386\ \text{W/m.°C})(18\times0.001\times0.00025)\text{m}^2} = 3.45°\text{C/W}$$



Since all resistances are in series, the total thermal resistance between the junction and the leads is determined by simply adding them up

$$R_{total} = R_{junction-lead} = R_{constriction} + R_{chip} + R_{bond} + R_{\substack{lead\\frame}} + R_{plastic} + R_{leads}$$

$$= 4.7 + 0.26 + 0.011 + 0.04 + 66.67 + 3.45 = 75.13°\text{C/W}$$

Knowing the junction-to-leads thermal resistance, the junction temperature is determined from

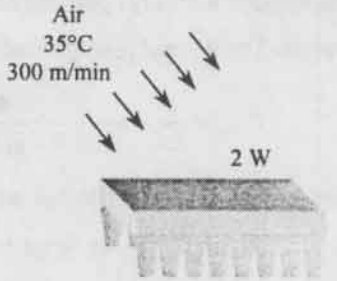
$$\dot{Q} = \frac{T_{junction} - T_{leads}}{R_{junction-case}}$$

$$T_{junction} = T_{leads} + \dot{Q}R_{junction-case} = 50°\text{C} + (0.8\ \text{W})(75.13°\text{C/W}) = \mathbf{110.1°C}$$

22-53 A plastic DIP with 16 leads is cooled by forced air. Using data supplied by the manufacturer, the junction temperature is to be determined.

Assumptions Steady operating conditions exist.

Analysis The junction-to-ambient thermal resistance of the device with 16 leads corresponding to an air velocity of 300 m/min is determined from Fig.22-23 to be



$$R_{junction-ambient} = 50°C/W$$

Then the junction temperature becomes

$$\dot{Q} = \frac{T_{junction} - T_{ambient}}{R_{junction-ambient}}$$

$$T_{junction} = T_{ambient} + \dot{Q}R_{junction-ambient} = 35°C + (2\ W)(50°C/W) = \mathbf{135°C}$$

When the fan fails the total thermal resistance is determined from Fig.22-23 by reading the value for zero air velocity (the intersection point of the curve with the vertical axis) to be

$$R_{junction-ambient} = 70°C/W$$

which yields

$$\dot{Q} = \frac{T_{junction} - T_{ambient}}{R_{junction-ambient}}$$

$$T_{junction} = T_{ambient} + \dot{Q}R_{junction-ambient} = 35°C + (2\ W)(70°C/W) = \mathbf{175°C}$$

22-54 A PCB with copper cladding is given. The percentages of heat conduction along the copper and epoxy layers as well as the effective thermal conductivity of the PCB are to be determined.

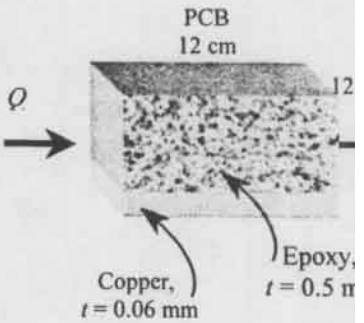
Assumptions **1** Steady operating conditions exist. **2** Heat conduction along the PCB is one-dimensional since heat transfer from side surfaces is negligible. **3** The thermal properties of epoxy and copper layers are constant.

Analysis Heat conduction along a layer is proportional to the thermal conductivity-thickness product (kt) which is determined for each layer and the entire PCB to be

$$(kt)_{copper} = (386\ W/m.°C)(0.06\times10^{-3}\ m) = 0.02316\ W/°C$$

$$(kt)_{epoxy} = (0.26\ W/m.°C)(0.5\times10^{-3}\ m) = 0.00013\ W/°C$$

$$(kt)_{PCB} = (kt)_{copper} + (kt)_{epoxy} = 0.02316 + 0.00013 = 0.02329\ W/°C$$



Therefore the percentages of heat conduction along the epoxy board are

$$f_{epoxy} = \frac{(kt)_{epoxy}}{(kt)_{PCB}} = \frac{0.00013\ W/°C}{0.02316\ W/°C} = 0.0056 \cong \mathbf{0.6\%}$$

and $f_{copper} = (100 - 0.6)\% = \mathbf{99.4\%}$

Then the effective thermal conductivity becomes

$$k_{eff} = \frac{(kt)_{epoxy} + (kt)_{copper}}{t_{epoxy} + t_{copper}} = \frac{(0.02316 + 0.00013)\ W/°C}{(0.06 + 0.5)\times10^{-3}\ m} = \mathbf{41.6\ W/m.°C}$$

22-55 The heat generated in a silicon chip is conducted to a ceramic substrate to which it is attached. The temperature difference between the front and back surfaces of the chip is to be determined.

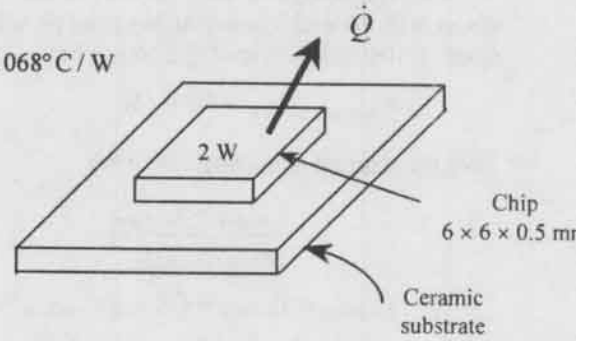
Assumptions **1** Steady operating conditions exist. **2** Heat conduction along the chip is one-dimensional.

Analysis The thermal resistance of silicon chip is

$$R_{chip} = \frac{L}{kA} = \frac{0.5 \times 10^{-3}\ \text{m}}{(130\ \text{W/m.°C})(0.006 \times 0.006)\text{m}^2} = 0.1068°\text{C/W}$$

Then the temperature difference across the chip becomes

$$\Delta T = \dot{Q} R_{chip} = (2\ \text{W})(0.1068°\text{C/W}) = \mathbf{0.21°C}$$



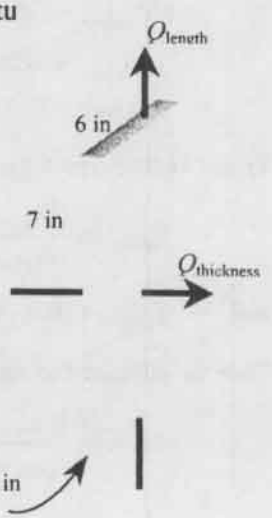
22-56E The dimensions of an epoxy glass laminate are given. The thermal resistances for heat flow along the layers and across the thickness are to be determined.

Assumptions **1** Heat conduction in the laminate is one-dimensional in either case. **2** Thermal properties of the laminate are constant.

Analysis The thermal resistances of the PCB along the 7 in long side and across its thickness are

(*a*) $$R_{\substack{along\\length}} = \frac{L}{kA} = \frac{(7/12)\ \text{ft}}{(0.15\ \text{Btu/h.ft.°F})(6/12\ \text{ft})(0.05/12\ \text{ft})} = \mathbf{1867\ h.°F/Btu}$$

(*b*) $$R_{\substack{across\\thickness}} = \frac{L}{kA} = \frac{(0.05/12)\ \text{ft}}{(0.15\ \text{Btu/h.ft.°F})(7/12\ \text{ft})(6/12\ \text{ft})} = \mathbf{0.095\ h.°F/Btu}$$



22-57 Cylindrical copper fillings are planted throughout an epoxy glass board. The thermal resistance of the board across its thickness is to be determined.

Assumptions **1** Heat conduction along the board is one-dimensional. **2** Thermal properties of the board are constant.

Analysis The number of copper fillings on the board is

$$n = \frac{\text{Area of board}}{\text{Area of one square}} = \frac{(150\text{ mm})(180\text{ mm})}{(3\text{ mm})(3\text{ mm})} = 3000$$

The surface areas of the copper fillings and the remaining part of the epoxy layer are

$$A_{copper} = n\frac{\pi D^2}{4} = (3000)\frac{\pi(0.001\text{ m})^2}{4} = 0.002356\text{ m}^2$$

$$A_{total} = (length)(width) = (0.15\text{ m})(0.18\text{ m}) = 0.027\text{ m}^2$$

$$A_{epoxy} = A_{total} - A_{copper} = 0.027 - 0.002356 = 0.024644\text{ m}^2$$

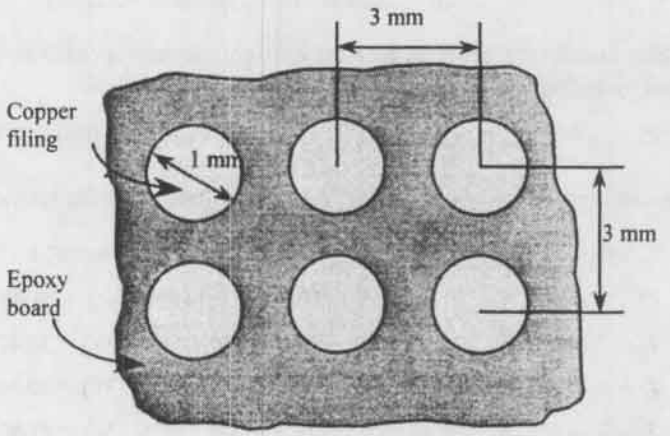
The thermal resistance of each material is

$$R_{copper} = \frac{L}{kA} = \frac{0.0014\text{ m}}{(386\text{ W/m.°C})(0.002356\text{ m}^2)} = 0.00154°\text{C/W}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.0014\text{ m}}{(0.26\text{ W/m.°C})(0.024644\text{ m}^2)} = 0.2185°\text{C/W}$$

Since these two resistances are in parallel, the equivalent thermal resistance of the entire board is

$$\frac{1}{R_{board}} = \frac{1}{R_{epoxy}} + \frac{1}{R_{copper}} = \frac{1}{0.2185°\text{C/W}} + \frac{1}{0.00154°\text{C/W}} \longrightarrow R_{board} = 0.00153°\text{C/W}$$



22-58 A circuit board with uniform heat generation is to be conduction cooled by a copper heat frame. Temperature distribution along the heat frame and the maximum temperature in the PCB are to be determined.

Assumptions **1** Steady operating conditions exist **2** Thermal properties are constant. **3** There is no direct heat dissipation from the surface of the PCB, and thus all the heat generated is conducted by the heat frame to the heat sink.

Analysis The properties and dimensions of various section of the PCB are summarized below as

Section and material	Thermal conductivity	Thickness	Heat transfer surface area
Epoxy board	0.26 W/m.°C	2 mm	10 mm × 120 mm
Epoxy adhesive	1.8 W/m.°C	0.12 mm	10 mm × 120 mm
Copper heat frame (normal to frame)	386 W/m.°C	1.5 mm	10 mm × 120 mm
Copper heat frame (along the frame)	386 W/m.°C	10 mm	15 mm × 120 mm

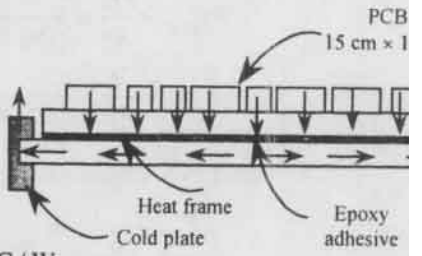
Using the values in the table, the various thermal resistances are determined to be

$$R_{epoxy} = \frac{L}{kA} = \frac{0.002\ \text{m}}{(0.26\ \text{W/m.°C})(0.01\ \text{m} \times 0.12\ \text{m})} = 6.41°\text{C/W}$$

$$R_{adhesive} = \frac{L}{kA} = \frac{0.00012\ \text{m}}{(1.8\ \text{W/m.°C})(0.01\ \text{m} \times 0.12\ \text{m})} = 0.056°\text{C/W}$$

$$R_{copper,\perp} = \frac{L}{kA} = \frac{0.0015\ \text{m}}{(386\ \text{W/m.°C})(0.01\ \text{m} \times 0.12\ \text{m})} = 0.0032°\text{C/W}$$

$$R_{frame} = R_{copper,parallel} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(386\ \text{W/m.°C})(0.0015 \times 0.12\ \text{m})} = 0.144°\text{C/W}$$



The combined resistance between the electronic components on each strip and the heat frame can be determined by adding the three thermal resistances in series to be

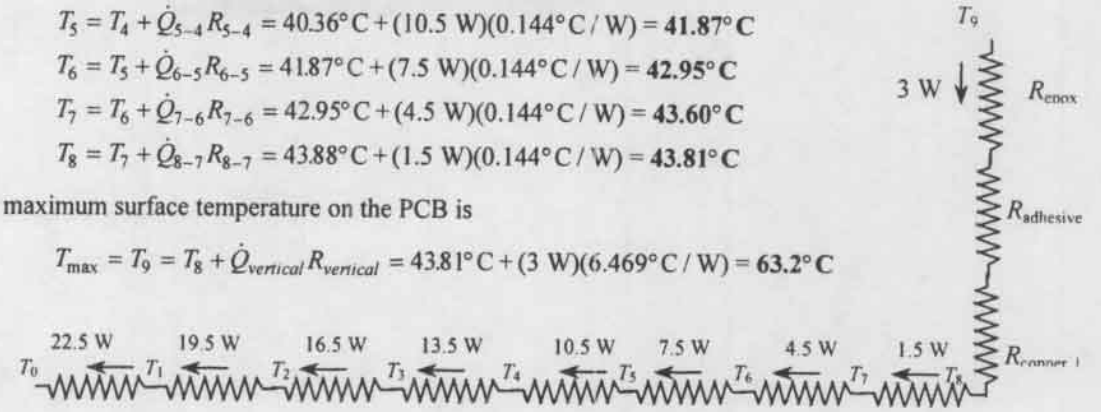
$$R_{vertical} = R_{epoxy} + R_{adhesive} + R_{copper,\perp} = 6.41 + 0.056 + 0.0032 = 6.469°\text{C/W}$$

The temperatures along the heat frame can be determined from the relation $\Delta T = T_{high} - T_{low} = \dot{Q}R$. Then,

$$T_1 = T_0 + \dot{Q}_{1-0}R_{1-0} = 30°\text{C} + (22.5\ \text{W})(0.144°\text{C/W}) = \mathbf{33.24°C}$$
$$T_2 = T_1 + \dot{Q}_{2-1}R_{2-1} = 33.24°\text{C} + (19.5\ \text{W})(0.144°\text{C/W}) = \mathbf{36.05°C}$$
$$T_3 = T_2 + \dot{Q}_{3-2}R_{3-2} = 36.05°\text{C} + (16.5\ \text{W})(0.144°\text{C/W}) = \mathbf{38.42°C}$$
$$T_4 = T_3 + \dot{Q}_{4-3}R_{4-3} = 38.42°\text{C} + (13.5\ \text{W})(0.144°\text{C/W}) = \mathbf{40.36°C}$$
$$T_5 = T_4 + \dot{Q}_{5-4}R_{5-4} = 40.36°\text{C} + (10.5\ \text{W})(0.144°\text{C/W}) = \mathbf{41.87°C}$$
$$T_6 = T_5 + \dot{Q}_{6-5}R_{6-5} = 41.87°\text{C} + (7.5\ \text{W})(0.144°\text{C/W}) = \mathbf{42.95°C}$$
$$T_7 = T_6 + \dot{Q}_{7-6}R_{7-6} = 42.95°\text{C} + (4.5\ \text{W})(0.144°\text{C/W}) = \mathbf{43.60°C}$$
$$T_8 = T_7 + \dot{Q}_{8-7}R_{8-7} = 43.88°\text{C} + (1.5\ \text{W})(0.144°\text{C/W}) = \mathbf{43.81°C}$$

The maximum surface temperature on the PCB is

$$T_{max} = T_9 = T_8 + \dot{Q}_{vertical}R_{vertical} = 43.81°\text{C} + (3\ \text{W})(6.469°\text{C/W}) = \mathbf{63.2°C}$$

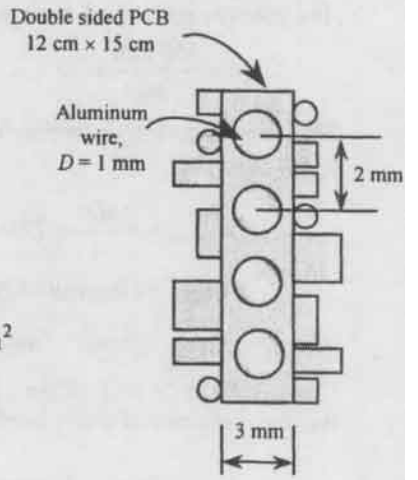


22-59 A circuit board with uniform heat generation is to be conduction cooled by aluminum wires inserted into it. The magnitude and location of the maximum temperature in the PCB is to be determined.

Assumptions **1** Steady operating conditions exist **2** Thermal properties are constant. **3** There is no direct heat dissipation from the surface of the PCB.

Analysis The number of wires in the board is

$$n = \frac{150 \text{ mm}}{2 \text{ mm}} = 75$$



The surface areas of the aluminum wires and the remaining part of the epoxy layer are

$$A_{aluminum} = n\frac{\pi D^2}{4} = (75)\frac{\pi(0.001 \text{ m})^2}{4} = 0.0000589 \text{ m}^2$$

$$A_{total} = (length)(width) = (0.003 \text{ m})(0.15 \text{ m}) = 0.00045 \text{ m}^2$$

$$A_{epoxy} = A_{total} - A_{aluminum} = 0.00045 - 0.0000589 = 0.0003911 \text{ m}^2$$

Considering only half of the circuit board because of symmetry, the thermal resistance of each material per 1-cm length is determined to be

$$R_{aluminum} = \frac{L}{kA} = \frac{0.01 \text{ m}}{(237 \text{ W/m.°C})(0.0000589 \text{ m}^2)} = 0.716°\text{C/W}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.01 \text{ m}}{(0.26 \text{ W/m.°C})(0.0003911 \text{ m}^2)} = 98.34°\text{C/W}$$

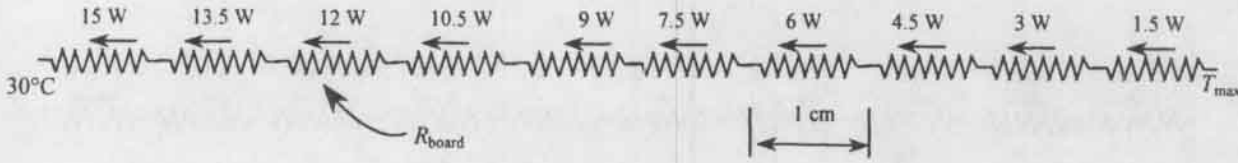
Since these two resistances are in parallel, the equivalent thermal resistance per cm is determined from

$$\frac{1}{R_{board}} = \frac{1}{R_{epoxy}} + \frac{1}{R_{aluminum}} = \frac{1}{0.716°\text{C/W}} + \frac{1}{98.34°\text{C/W}} \longrightarrow R_{board} = 0.711°\text{C/W}$$

Maximum temperature occurs in the middle of the plate along the 20 cm length which is determined to be

$$T_{max} = T_{end} + \Delta T_{board,total} = T_{end} + \sum \dot{Q}_i R_{board,1-cm} = T_{end} + R_{board,1-cm} \sum \dot{Q}_i$$

$$= 30°\text{C} + (0.711°\text{C/W})(15 + 13.5 + 12 + 10.5 + 9 + 7.5 + 6 + 4.5 + 3 + 1.5)\text{W} = \mathbf{88.7°C}$$

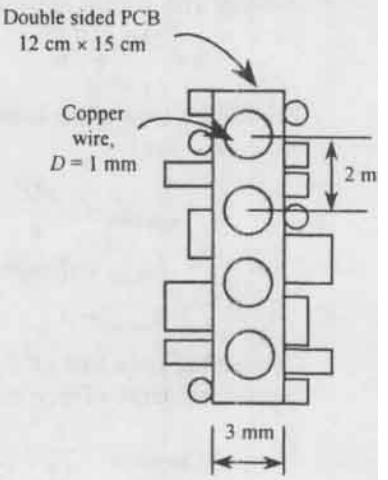


22-60 A circuit board with uniform heat generation is to be conduction cooled by copper wires inserted in it. The magnitude and location of the maximum temperature in the PCB is to be determined.

Assumptions **1** Steady operating conditions exist **2** Thermal properties are constant. **3** There is no direct heat dissipation from the surface of the PCB.

Analysis The number of wires in the circuit board is

$$n = \frac{150 \text{ mm}}{2 \text{ mm}} = 75$$



The surface areas of the copper wires and the remaining part of the epoxy layer are

$$A_{copper} = n\frac{\pi D^2}{4} = (75)\frac{\pi(0.001 \text{ m})^2}{4} = 0.0000589 \text{ m}^2$$

$$A_{total} = (length)(width) = (0.003 \text{ m})(0.15 \text{ m}) = 0.00045 \text{ m}^2$$

$$A_{epoxy} = A_{total} - A_{copper} = 0.00045 - 0.0000589 = 0.0003911 \text{ m}^2$$

Considering only half of the circuit board because of symmetry, the thermal resistance of each material per 1-cm length is determined to be

$$R_{copper} = \frac{L}{kA} = \frac{0.01 \text{ m}}{(386 \text{ W/m.°C})(0.0000589 \text{ m}^2)} = 0.440°\text{C/W}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.01 \text{ m}}{(0.26 \text{ W/m.°C})(0.0003911 \text{ m}^2)} = 98.34°\text{C/W}$$

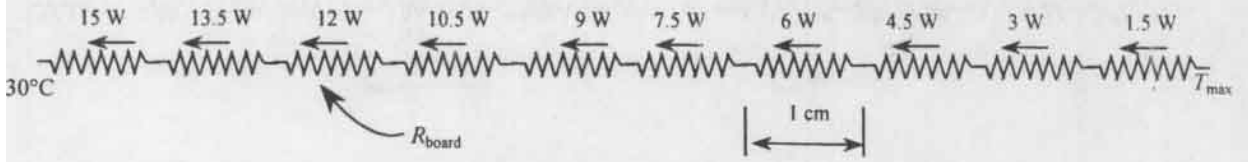
Since these two resistances are in parallel, the equivalent thermal resistance is determined from

$$\frac{1}{R_{board}} = \frac{1}{R_{epoxy}} + \frac{1}{R_{copper}} = \frac{1}{0.440°\text{C/W}} + \frac{1}{98.34°\text{C/W}} \longrightarrow R_{board} = 0.438°\text{C/W}$$

Maximum temperature occurs in the middle of the plate along the 20 cm length which is determined to be

$$T_{max} = T_{end} + \Delta T_{board,total} = T_{end} + \sum \dot{Q}_i R_{board,1-cm} = T_{end} + R_{board,1-cm} \sum \dot{Q}_i$$

$$= 30°\text{C} + (0.438°\text{C/W})(15+13.5+12+10.5+9+7.5+6+4.5+3+1.5)\text{W} = \mathbf{66.1°C}$$

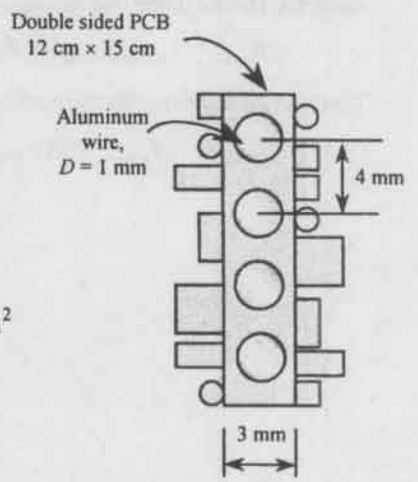


22-61 A circuit board with uniform heat generation is to be conduction cooled by aluminum wires inserted into it. The magnitude and location of the maximum temperature in the PCB is to be determined.

Assumptions **1** Steady operating conditions exist **2** Thermal properties are constant. **3** There is no direct heat dissipation from the surface of the PCB.

Analysis The number of wires in the board is

$$n = \frac{150 \text{ mm}}{4 \text{ mm}} = 37$$



The surface areas of the aluminum wires and the remaining part of the epoxy layer are

$$A_{aluminum} = n\frac{\pi D^2}{4} = (37)\frac{\pi(0.001 \text{ m})^2}{4} = 0.000029 \text{ m}^2$$

$$A_{total} = (length)(width) = (0.003 \text{ m})(0.15 \text{ m}) = 0.00045 \text{ m}^2$$

$$A_{epoxy} = A_{total} - A_{aluminum} = 0.00045 - 0.000029 = 0.000421 \text{ m}^2$$

Considering only half of the circuit board because of symmetry, the thermal resistance of each material per 1-cm length is determined to be

$$R_{aluminum} = \frac{L}{kA} = \frac{0.01 \text{ m}}{(237 \text{ W/m.°C})(0.000029 \text{ m}^2)} = 1.455°\text{C/W}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.01 \text{ m}}{(0.26 \text{ W/m.°C})(0.000421 \text{ m}^2)} = 91.36°\text{C/W}$$

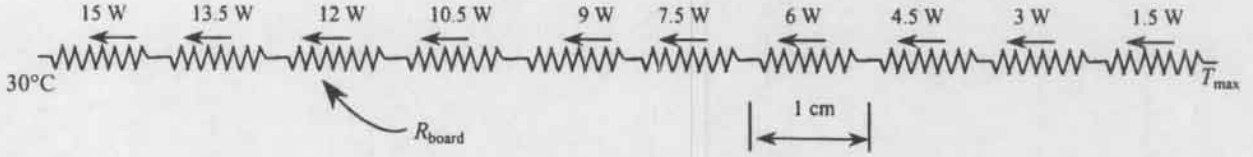
Since these two resistances are in parallel, the equivalent thermal resistance is determined from

$$\frac{1}{R_{board}} = \frac{1}{R_{epoxy}} + \frac{1}{R_{aluminum}} = \frac{1}{1.455°\text{C/W}} + \frac{1}{91.36°\text{C/W}} \longrightarrow R_{board} = 1.432°\text{C/W}$$

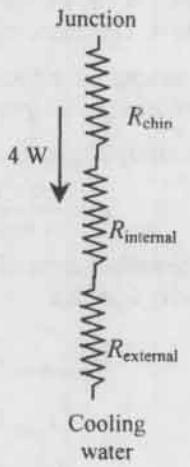
Maximum temperature occurs in the middle of the plate along the 20 cm length which is determined to be

$$T_{max} = T_{end} + \Delta T_{board,total} = T_{end} + \sum \dot{Q}_i R_{board,1-cm} = T_{end} + R_{board,1-cm} \sum \dot{Q}_i$$

$$= 30°\text{C} + (1.432°\text{C/W})(15+13.5+12+10.5+9+7.5+6+4.5+3+1.5)\text{W} = \mathbf{148.1°C}$$



22-62 A thermal conduction module with 80 chips is cooled by water. The junction temperature of the chip is to be determined.



Assumptions **1** Steady operating conditions exist **2** Heat transfer through various components is one-dimensional.

Analysis The total thermal resistance between the junction and cooling water is

$$R_{total} = R_{junction-water} = R_{chip} + R_{internal} + R_{external} = 1.2 + 9 + 7 = 17.2°C$$

Then the junction temperature becomes

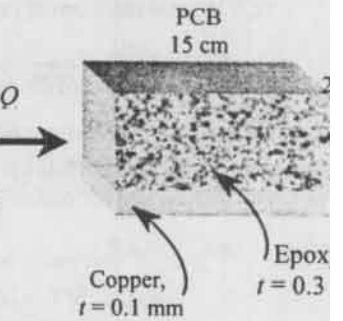
$$T_{junction} = T_{water} + \dot{Q}R_{junction-water} = 22°C + (4\ W)(17.2°C/W) = \mathbf{90.8°C}$$

22-63 A layer of copper is attached to the back surface of an epoxy board. The effective thermal conductivity of the board and the fraction of heat conducted through copper are to be determined.

Assumptions **1** Steady operating conditions exist **2** Heat transfer is one-dimensional.

Analysis Heat conduction along a layer is proportional to the thermal conductivity-thickness product (kt) which is determined for each layer and the entire PCB to be

$$(kt)_{copper} = (386\ \text{W/m.°C})(0.0001\ \text{m}) = 0.0386\ \text{W/°C}$$
$$(kt)_{epoxy} = (0.26\ \text{W/m.°C})(0.0003\ \text{m}) = 0.000078\ \text{W/°C}$$
$$(kt)_{PCB} = (kt)_{copper} + (kt)_{epoxy} = 0.0386 + 0.000078 = 0.038678\ \text{W/°C}$$



The effective thermal conductivity can be determined from

$$k_{eff} = \frac{(kt)_{epoxy} + (kt)_{copper}}{t_{epoxy} + t_{copper}} = \frac{(0.0386 + 0.000078)\ \text{W/°C}}{(0.0003\ \text{m} + 0.0001\ \text{m})} = \mathbf{96.7\ W/m.°C}$$

Then the fraction of the heat conducted along the copper becomes

$$f = \frac{(kt)_{copper}}{(kt)_{PCB}} = \frac{0.0386\ \text{W/°C}}{0.038678\ \text{W/°C}} = 0.998 = \mathbf{99.8\%}$$

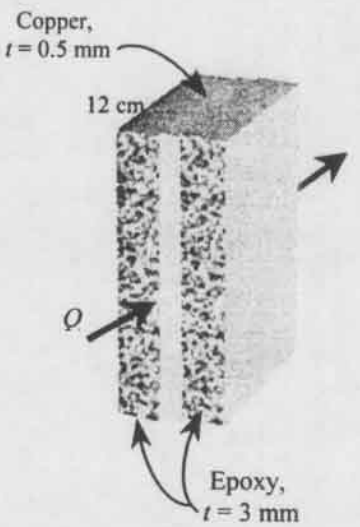
Discussion Note that heat is transferred almost entirely through the copper layer.

22-64 A copper plate is sandwiched between two epoxy boards. The effective thermal conductivity of the board and the fraction of heat conducted through copper are to be determined.

Assumptions **1** Steady operating conditions exist **2** Heat transfer is one-dimensional.

Analysis Heat conduction along a layer is proportional to the thermal conductivity-thickness product (kt) which is determined for each layer and the entire PCB to be

$$(kt)_{copper} = (386\ \text{W/m.°C})(0.0005\ \text{m}) = 0.193\ \text{W/°C}$$
$$(kt)_{epoxy} = (2)(0.26\ \text{W/m.°C})(0.003\ \text{m}) = 0.00156\ \text{W/°C}$$
$$(kt)_{PCB} = (kt)_{copper} + (kt)_{epoxy} = 0.193 + 0.00156 = 0.19456\ \text{W/°C}$$



The effective thermal conductivity can be determined from

$$k_{eff} = \frac{(kt)_{epoxy} + (kt)_{copper}}{t_{epoxy} + t_{copper}} = \frac{(0.00156 + 0.193)\ \text{W/°C}}{[(2 \times 0.003\ \text{m}) + 0.0005\ \text{m}]} = \mathbf{29.9\ W/m.°C}$$

Then the fraction of the heat conducted along the copper becomes

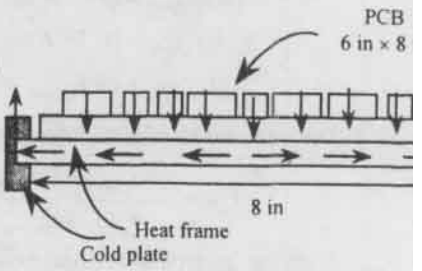
$$f = \frac{(kt)_{copper}}{(kt)_{PCB}} = \frac{0.193\ \text{W/°C}}{0.19456\ \text{W/°C}} = 0.992 = \mathbf{99.2\%}$$

22-65E A copper heat frame is used to conduct heat generated in a PCB. The temperature difference between the mid section and either end of the heat frame is to be determined.

Assumptions **1** Steady operating conditions exist **2** Heat transfer is one-dimensional.

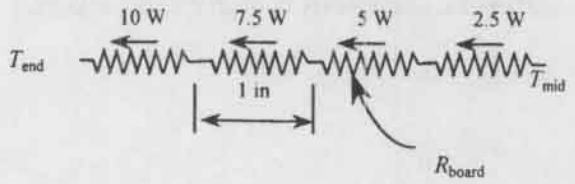
Analysis We assume heat is generated uniformly on the 6 in × 8 in board, and all the heat generated is conducted by the heat frame along the 8-in side. Noting that the rate of heat transfer along the heat frame is variable, we consider 1 in × 8 in strips of the board. The rate of heat generation in each strip is (20 W)/8 = 2.5 W, and the thermal resistance along each strip of the heat frame is

$$R_{frame} = \frac{L}{kA} = \frac{(1/12)\text{ ft}}{(223\text{ Btu/h.ft.°F})(6/12\text{ ft})(0.07/12\text{ ft})} = 0.128\text{ h.°F/Btu}$$



Maximum temperature occurs in the middle of the plate along the 20 cm length. Then the temperature difference between the mid section and either end of the heat frame becomes

$$\Delta T_{max} = \Delta T_{\text{mid section - edge of frame}} = \sum \dot{Q}_i R_{frame,1-cm} = R_{frame,1-cm} \sum \dot{Q}_i$$
$$= (0.128°\text{F.h/Btu})(10+7.5+5+2.5\text{ W})(3.4121\text{ Btu/h.W}) = \mathbf{10.9°F}$$

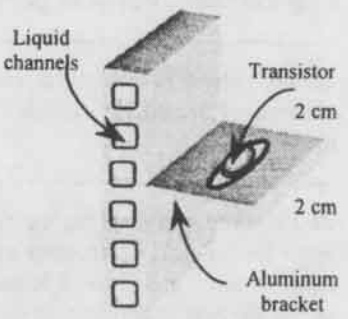


22-66 A power transistor is cooled by mounting it on an aluminum bracket that is attached to a liquid-cooled plate. The temperature of the transistor case is to be determined.

Assumptions **1** Steady operating conditions exist **2** Conduction heat transfer is one-dimensional.

Analysis The rate of heat transfer by conduction is

$$\dot{Q}_{conduction} = (0.80)(12\ \text{W}) = 9.6\ \text{W}$$



The thermal resistance of aluminum bracket and epoxy adhesive are

$$R_{aluminum} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(237\ \text{W/m.°C})(0.003\ \text{m})(0.02\ \text{m})} = 0.703°\text{C/W}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.0002\ \text{m}}{(1.8\ \text{W/m.°C})(0.003\ \text{m})(0.02\ \text{m})} = 1.852°\text{C/W}$$

The total thermal resistance between the transistor and the cold plate is

$$R_{total} = R_{case-cold\ plate} = R_{plastic} + R_{epoxy} + R_{aluminum} = 2.5 + 1.852 + 0.703 = 5.055°\text{C/W}$$

Then the temperature of the transistor case is determined from

$$T_{case} = T_{\substack{cold \\ plate}} + \dot{Q}R_{case-cold\ plate} = 50°\text{C} + (9.6\ \text{W})(5.055°\text{C/W}) = \mathbf{98.5°C}$$

Air Cooling: Natural Convection and Radiation

22-67C As the student watches the movie, the temperature of the electronic components in the VCR will keep increasing because of the blocked air passages. The VCR eventually may overheat and fail.

22-68C There is no natural convection in space because of the absence of gravity (and because of the absence of a medium outside). However, it can be cooled by radiation since radiation does not need a medium.

22-69C The openings on the side surfaces of a TV, VCR or other electronic enclosures provide passage ways for the cold air to enter and warm air to leave. If a TV or VCR is enclosed in a cabinet with no free space around, and if there is no other cooling process involved, the temperature of device will keep rising due to the heat generation in device, which may cause the device to fail eventually.

22-70C The magnitude of radiation, in general, is comparable to the magnitude of natural convection. Therefore, radiation heat transfer should be always considered in the analysis of natural convection cooled electronic equipment.

22-71C The effect of atmospheric pressure to heat transfer coefficient can be written as $h_{conv,P\ atm} = h_{conv,1\ atm}\sqrt{P}$ $(W/m^2.^\circ C)$ where P is the air pressure in atmosphere. Therefore, the greater the air pressure, the greater the heat transfer coefficient. The best and the worst orientation for heat transfer from a square surface are vertical and horizontal, respectively, since the former maximizes and the latter minimizes natural convection.

4-72C The view factor from surface 1 to surface 2 is the fraction of radiation which leaves surface 1 and strikes surface 2 directly. The magnitude of radiation heat transfer between two surfaces is proportional to the view factor. The larger the view factor, the larger the radiation exchange between the two surfaces.

22-73C Emissivity of a surface is the ratio of the radiation emitted by a surface at a specified temperature to the radiation emitted by a blackbody (which is the maximum amount) at the same temperature. The magnitude of radiation heat transfer between a surfaces and it surrounding surfaces is proportional to the emissivity. The larger the emissivity, the larger the radiation heat exchange between the two surfaces.

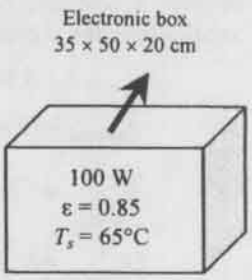
22-74C For most effective natural convection cooling of a PCB array, the PCB should be placed vertically to take advantage of natural convection currents which tend to rise naturally, and to minimize trapped air pockets. Placing the PCBs too close to each other tends to choke the flow because of the increased resistance. Therefore, the PCBs should be placed far from each other for effective heat transfer (A distance of about 2 cm between the PCBs turns out to be adequate for effective natural convection cooling.)

22-75C Radiation heat transfer from the components on the PCBs in an enclosure is negligible since the view of the components is largely blocked by other heat generating components at about the same temperature, and hot components face other hot surfaces instead of cooler surfaces.

22-76 The surface temperature of a sealed electronic box placed on top of a stand is not to exceed 65°C. It is to be determined if this box can be cooled by natural convection and radiation alone.

Assumptions **1** Steady operating conditions exist. **2** The local atmospheric pressure is 1 atm.

Analysis Using Table 22-1, the heat transfer coefficient and the natural convection heat transfer from side surfaces are determined to be



$$L = 0.2\ \text{m}$$

$$A_{side} = (2)(0.5\ \text{m} + 0.35\ \text{m})(0.2\ \text{m}) = 0.34\ \text{m}^2$$

$$h_{conv,side} = 1.42\left(\frac{\Delta T}{L}\right)^{0.25} = 1.42\left(\frac{65-30}{0.2}\right)^{0.25} = 5.16\ \text{W/m.°C}$$

$$\dot{Q}_{conv,side} = h_{conv,side} A_{side} (T_s - T_{fluid})$$

$$= (5.16\ \text{W/m.°C})(0.34\ \text{m}^2)(65-30)°\text{C} = 61.5\ \text{W}$$

The heat transfer from the horizontal top surface by natural convection is

$$L = \frac{4A}{p} = \frac{4(0.5\ \text{m})(0.35\ \text{m})}{(2)(0.5\ \text{m} + 0.35\ \text{m})} = 0.41\ \text{m}$$

$$A_{top} = (0.5\ \text{m})(0.35\ \text{m}) = 0.175\ \text{m}^2$$

$$h_{conv,top} = 1.32\left(\frac{\Delta T}{L}\right)^{0.25} = 1.32\left(\frac{65-30}{0.41}\right)^{0.25} = 4.01\ \text{W/m.°C}$$

$$\dot{Q}_{conv,top} = h_{conv,top} A_{top} (T_s - T_{fluid}) = (4.01\ \text{W/m.°C})(0.175\ \text{m}^2)(65-30)°\text{C} = 24.6\ \text{W}$$

The rate of heat transfer from the box by radiation is determined from

$$\dot{Q}_{rad} = \varepsilon A \sigma (T_s^{\,4} - T_{surr}^{\,4})$$

$$= (0.85)(0.34\ \text{m}^2 + 0.175\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(65+273\ \text{K})^4 - (30+273\ \text{K})^4] = 114.7\ \text{W}$$

Then the total rate of heat transfer from the box becomes

$$\dot{Q}_{total} = \dot{Q}_{conv,side} + \dot{Q}_{conv,top} + \dot{Q}_{rad} = 61.5 + 24.6 + 114.7 = \mathbf{200.8\ W}$$

which is greater than 100 W. Therefore, this box can be cooled by combined natural convection and radiation.

22-77 The surface temperature of a sealed electronic box placed on top of a stand is not to exceed 65°C. It is to be determined if this box can be cooled by natural convection and radiation alone.

Assumptions **1** Steady operating conditions exist. **2** The local atmospheric pressure is 1 atm.

Analysis In given orientation, two side surfaces and the top surface will be vertical and other two side surfaces will be horizontal. Using Table 22-1, the heat transfer coefficient and the natural convection heat transfer from the vertical surfaces are determined to be

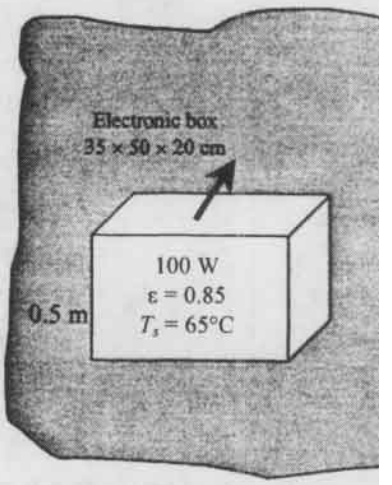
$$L = 0.5\ \text{m}$$

$$A_{vertical} = (2\times 0.2\times 0.5 + 0.5\times 0.35) = 0.375\ \text{m}^2$$

$$h_{\substack{conv\\vertical}} = 1.42\left(\frac{\Delta T}{L}\right)^{0.25} = 1.42\left(\frac{65-30}{0.5}\right)^{0.25} = 4.107\ \text{W/m.°C}$$

$$\dot{Q}_{\substack{conv\\vertical}} = h_{\substack{conv\\vertical}} A_{vertical}(T_s - T_{fluid})$$

$$= (4.107\ \text{W/m.°C})(0.375\ \text{m}^2)(65-30)°\text{C} = 53.9\ \text{W}$$



The heat transfer from the horizontal top surface by natural convection is

$$A_{top} = (0.2\ \text{m})(0.35\ \text{m}) = 0.07\ \text{m}^2$$

$$L = \frac{4A_s}{p} = \frac{(4)(0.07\ \text{m}^2)}{(4)(0.2\ \text{m}+0.35\ \text{m})} = 0.1273\ \text{m}$$

$$h_{\substack{conv\\top}} = 1.32\left(\frac{\Delta T}{L}\right)^{0.25} = 1.32\left(\frac{65-30}{0.1273}\right)^{0.25} = 5.4\ \text{W/m.°C}$$

$$\dot{Q}_{\substack{conv\\top}} = h_{\substack{conv\\top}} A_{top}(T_s - T_{fluid})$$

$$= (5.4\ \text{W/m.°C})(0.07\ \text{m}^2)(65-30)°\text{C} = 13.2\ \text{W}$$

The heat transfer from the horizontal top surface by natural convection is

$$h_{\substack{conv\\bottom}} = 0.59\left(\frac{\Delta T}{L}\right)^{0.25} = 0.59\left(\frac{65-30}{0.1273}\right)^{0.25} = 2.4\ \text{W/m.°C}$$

$$\dot{Q}_{\substack{conv\\bottom}} = h_{\substack{conv\\bottom}} A_{bottom}(T_s - T_{fluid}) = (2.4\ \text{W/m.°C})(0.07\ \text{m}^2)(65-30)°\text{C} = 5.9\ \text{W}$$

The rate of heat transfer from the box by radiation is determined from

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$= (0.85)(0.34\ \text{m}^2 + 0.175\ \text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2.\text{K}^4)[(65+273\ \text{K})^4 - (30+273\ \text{K})^4] = 114.7\ \text{W}$$

Then the total rate of heat transfer from the box becomes

$$\dot{Q}_{total} = \dot{Q}_{\substack{conv\\vertical}} + \dot{Q}_{\substack{conv\\top}} + \dot{Q}_{\substack{conv\\bottom}} + \dot{Q}_{rad} = 53.9 + 13.2 + 5.9 + 114.7 = \mathbf{187.7\ W}$$

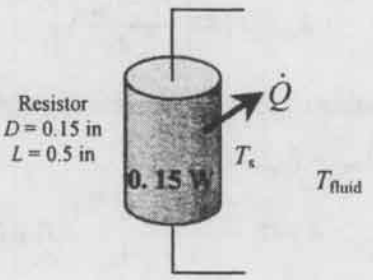
which is greater than 100 W. Therefore, this box can be cooled by combined natural convection and radiation.

22-78E A small cylindrical resistor mounted on a PCB is being cooled by natural convection and radiation. The surface temperature of the resistor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The local atmospheric pressure is 1 atm. **3** Radiation is negligible in this case since the resistor is surrounded by surfaces which are at about the same temperature, and the radiation heat transfer between two surfaces at the same temperature is zero. This leaves natural convection as the only mechanism of heat transfer from the resistor.

Analysis For components on a circuit board, the heat transfer coefficient relation from Table 22-1 is

$$h_{conv} = 0.50\left(\frac{T_s - T_{fluid}}{D}\right)^{0.25} \qquad (L = D)$$



Substituting it into the heat transfer relation to get

$$\dot{Q}_{conv} = h_{conv} A(T_s - T_{fluid})$$

$$= 0.50\left(\frac{T_s - T_{fluid}}{D}\right)^{0.25} A(T_s - T_{fluid})$$

$$= 0.50A\frac{(T_s - T_{fluid})^{1.25}}{D^{0.25}}$$

Calculating surface area and substituting it into above equation for the surface temperature yields

$$A = 2\left(\frac{\pi D^2}{4}\right) + \pi DL = 2\left[\frac{\pi(0.15/12\,\text{ft})^2}{4}\right] + \pi(0.15/12\,\text{ft})(0.5/12\,\text{ft}) = 0.00188\,\text{ft}^2$$

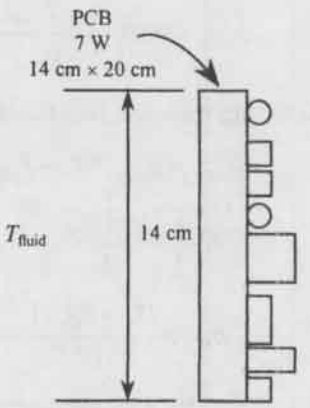
$$(0.15\,\text{W} \times 3.41214\,\text{Btu/h.W}) = (0.50)(0.00188\,\text{ft}^2)\frac{(T_s - 130)^{1.25}}{(0.15/12\,\text{ft})^{0.25}} \longrightarrow T_s = \mathbf{194°F}$$

22-79 The surface temperature of a PCB is not to exceed 90°C. The maximum environment temperatures for safe operation at sea level and at 3,000 m altitude are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation heat transfer is negligible since the PCB is surrounded by other PCBs at about the same temperature. **3** Heat transfer from the back surface of the PCB will be very small and thus negligible.

Analysis Using the simplified relation for a vertical orientation from Table 22-1, the natural convection heat transfer coefficient is determined to be

$$h_{conv} = 1.42\left(\frac{T_s - T_{fluid}}{L}\right)^{0.25}$$



Substituting it into the heat transfer relation to get

$$\dot{Q}_{conv} = h_{conv} A(T_s - T_{fluid})$$

$$= 1.42\left(\frac{T_s - T_{fluid}}{L}\right)^{0.25} A(T_s - T_{fluid})$$

$$= 1.42A\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

Calculating surface area and characteristic length and substituting them into above equation for the surface temperature yields

$$L = 0.14 \text{ m}$$

$$A = (0.14 \text{ m})(0.2 \text{ m}) = 0.028 \text{ m}^2$$

$$7 \text{ W} = (1.42)(0.028 \text{ m}^2)\frac{(90 - T_{fluid})^{1.25}}{(0.14 \text{ m})^{0.25}} \longrightarrow T_{fluid} = \mathbf{47.8°C}$$

At an altitude of 3000 m, the atmospheric pressure is 70.12 kPa which is equivalent to

$$P = (70.12 \text{ kPa})\frac{1 \text{ atm}}{101.325 \text{ kPa}} = 0.692 \text{ atm}$$

Modifying the heat transfer relation for this pressure (by multiplying by the square root of it) yields

$$7 \text{ W} = (1.42)(0.028 \text{ m}^2)\frac{(90 - T_{fluid})^{1.25}}{(0.14 \text{ m})^{0.25}}\sqrt{0.692} \longrightarrow T_{fluid} = \mathbf{41.1°C}$$

22-80 A cylindrical electronic component is mounted on a board with its axis in the vertical direction. The average surface temperature of the component is to be determined.

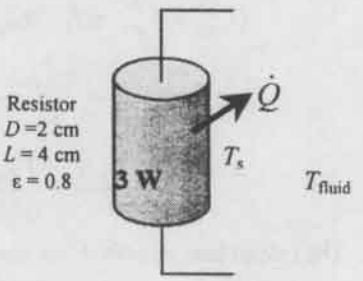
Assumptions **1** Steady operating conditions exist. **2** The local atmospheric pressure is 1 atm.

Analysis The natural convection heat transfer coefficient for vertical orientation using Table 22-1 can be determined from

$$h_{conv} = 1.42\left(\frac{T_s - T_{fluid}}{L}\right)^{0.25}$$

Substituting it into the heat transfer relation gives

$$\begin{aligned}\dot{Q}_{conv} &= h_{conv} A(T_s - T_{fluid}) \\ &= 1.42\left(\frac{T_s - T_{fluid}}{L}\right)^{0.25} A(T_s - T_{fluid}) \\ &= 1.42A\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}\end{aligned}$$



The rate of heat transfer from the cylinder by radiation is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

Then the total rate of heat transfer can be written as

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 1.42A\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}} + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

We will calculate total surface area of the cylindrical component including top and bottom surfaces, and assume the natural heat transfer coefficient to be the same throughout all surfaces of the component

$$A = 2\left(\frac{\pi D^2}{4}\right) + \pi DL = 2\left[\frac{\pi(0.02\text{ m})^2}{4}\right] + \pi(0.02\text{ m})(0.04\text{ m}) = 0.00314\text{ m}^2$$

Substituting

$$\begin{aligned}3\text{ W} = (1.42)(0.00314\text{ m}^2)&\frac{[T_s - (30+273\text{ K})]^{1.25}}{(0.04\text{ m})^{0.25}} \\ &+ (0.8)(0.00314\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{K}^4)[T_s^4 - (20+273\text{ K})^4]\end{aligned}$$

Solving for the surface temperature gives

$$T_s = 363\text{ K} = \mathbf{90°C}$$

22-81 A cylindrical electronic component is mounted on a board with its axis in horizontal direction. The average surface temperature of the component is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The local atmospheric pressure is 1 atm.

Analysis Since atmospheric pressure is not given, we assume it to be 1 atm. The natural convection heat transfer coefficient for horizontal orientation using Table 22-1 can be determined from

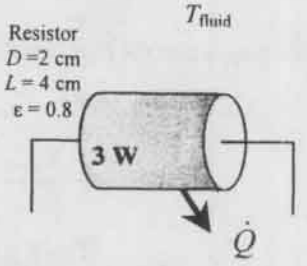
$$h_{conv} = 1.32\left(\frac{T_s - T_{fluid}}{D}\right)^{0.25}$$

Substituting it into the heat transfer relation to get

$$\dot{Q}_{conv} = h_{conv} A(T_s - T_{fluid})$$

$$= 1.32\left(\frac{T_s - T_{fluid}}{D}\right)^{0.25} A(T_s - T_{fluid})$$

$$= 1.32A\frac{(T_s - T_{fluid})^{1.25}}{D^{0.25}}$$



The rate of heat transfer from the cylinder by radiation is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

Then the total heat transfer can be written as

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 1.32A\frac{(T_s - T_{fluid})^{1.25}}{D^{0.25}} + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

We will calculate total surface area of the cylindrical component including top and bottom surfaces, and assume the natural heat transfer coefficient to be the same throughout all surfaces of the component

$$A = 2\left(\frac{\pi D^2}{4}\right) + \pi DL = 2\left[\frac{\pi(0.02\,\text{m})^2}{4}\right] + \pi(0.02\,\text{m})(0.04\,\text{m}) = 0.00314\,\text{m}^2$$

Substituting,

$$3\,\text{W} = (1.32)(0.00314\,\text{m}^2)\frac{[T_s - (30+273)\,\text{K}]^{1.25}}{(0.02\,\text{m})^{0.25}}$$

$$+ (0.8)(0.00314\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\text{K}^4)[T_s^4 - (20+273\,\text{K})^4]$$

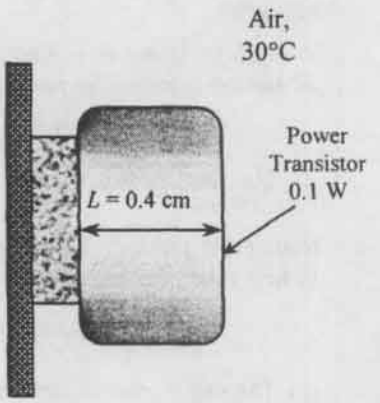
Solving for the surface temperature gives

$$T_s = 361\,\text{K} = \mathbf{88°C}$$

22-82 A power transistor dissipating 0.1 W of power is considered. The heat flux on the surface of the transistor and the surface temperature of the transistor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the transistor.

Analysis *(a)* The heat flux on the surface of the transistor is



$$A = 2\left(\frac{\pi D^2}{4}\right) + \pi DL = 2\left[\frac{\pi(0.4\text{ cm})^2}{4}\right] + \pi(0.4\text{ cm})(0.4\text{ cm}) = 0.754\text{ cm}^2$$

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{0.1\text{ W}}{0.754\text{ cm}^2} = 0.1326\text{ W/cm}^2$$

(*b*) The surface temperature of the transistor is determined from Newton's law of cooling to be

$$\dot{q} = h_{combined}(T_s - T_{fluid})$$

$$T_s = T_{fluid} + \frac{\dot{q}}{h_{combined}} = 30°\text{C} + \frac{1326\text{ W/m}^2}{12\text{ W/m}^2.°\text{C})} = \mathbf{140.5°C}$$

22-83 The components of an electronic equipment located in a horizontal duct with rectangular cross-section are cooled by forced air. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation heat transfer from the outer surfaces is negligible.

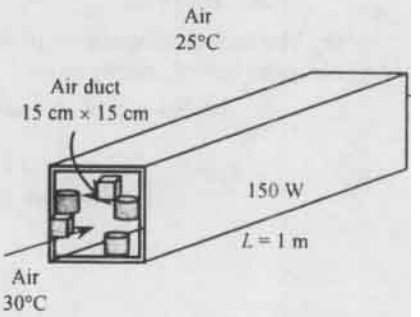
Analysis (a) Using air properties at 300 K and 1 atm, the mass flow rate of air and the heat transfer rate by forced convection are determined to be

$$\dot{m} = \rho \dot{V} = (1.177 \text{ kg/m}^3)(0.4/60 \text{ m}^3/\text{s}) = 0.00785 \text{ kg/s}$$

$$\dot{Q}_{\substack{forced \\ convection}} = \dot{m} C_p \Delta T = (0.00785 \text{ kg/s})(1005 \text{ J/kg.°C})(45-30)°\text{C} = 118.3 \text{ W}$$

Noting that radiation heat transfer is negligible, the rest of the 150 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{\substack{natural \\ convection}} = \dot{Q}_{total} - \dot{Q}_{\substack{forced \\ convection}} = 150 - 118.3 = \mathbf{31.7 \text{ W}}$$



(b) The natural convection heat transfer from the vertical side surfaces of the duct is

$$A_{side} = 2 \times (0.15 \text{ m})(1 \text{ m}) = 0.3 \text{ m}^2$$

$$h_{conv,side} = 1.42\left(\frac{\Delta T}{L}\right)^{0.25}$$

$$\dot{Q}_{conv,side} = h_{conv,side} A_{side} (T_s - T_{fluid}) = 1.42\left(\frac{(T_s - T_{fluid})}{L}\right)^{0.25} A_{side} (T_s - T_{fluid})$$

$$= 1.42 A_{side} \frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

Natural convection from the top and bottom surfaces of the duct is

$$L = \frac{4A}{p} = \frac{(4)(0.15 \text{ m})(1 \text{ m})}{(2)(0.15 \text{ m} + 1 \text{ m})} = 0.26 \text{ m}$$

$$A_{top} = (0.15 \text{ m})(1 \text{ m}) = 0.15 \text{ m}^2$$

$$h_{conv,top} = 1.32\left(\frac{\Delta T}{L}\right)^{0.25}$$

$$\dot{Q}_{conv,top} = h_{conv,top} A_{top} (T_s - T_{fluid}) = 1.32\left(\frac{(T_s - T_{fluid})}{L}\right)^{0.25} A_{top} (T_s - T_{fluid})$$

$$= 1.32 A_{top} \frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

$$h_{conv,bottom} = 0.59\left(\frac{\Delta T}{L}\right)^{0.25}$$

$$\dot{Q}_{conv,bottom} = h_{conv,bottom} A_{top} (T_s - T_{fluid}) = 0.59\left(\frac{(T_s - T_{fluid})}{L}\right)^{0.25} A_{bottom} (T_s - T_{fluid})$$

$$= 0.59 A_{bottom} \frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

Then the total heat transfer by natural convection becomes

$$\dot{Q}_{total,conv} = \dot{Q}_{conv,side} + \dot{Q}_{conv,top} + \dot{Q}_{conv,bottom}$$

$$\dot{Q}_{total,conv} = 1.42\,A_{side}\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}} + 1.32\,A_{top}\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}} + 0.59\,A_{bottom}\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

Substituting all known quantities with proper units gives the average temperature of the duct to be

$$31.7 = (1.42)(0.3)\frac{(T_s - 25)^{1.25}}{0.15^{0.25}} + (1.32)(0.15)\frac{(T_s - 25)^{1.25}}{0.26^{0.25}} + (0.59)(0.15)\frac{(T_s - 25)^{1.25}}{0.26^{0.25}}$$

$$31.7 = (1.086)(T_s - 25)^{1.25} \longrightarrow T_s = 40^{\circ}C$$

22-84 The components of an electronic equipment located in a circular horizontal duct are cooled by forced air. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation heat transfer from the outer surfaces is negligible.

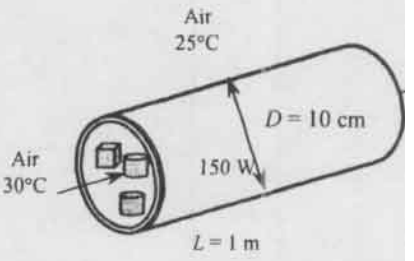
Analysis (*a*) Using air properties at 300 K and 1 atm, the mass flow rate of air and the heat transfer rate by forced convection are determined to be

$$\dot{m} = \rho \dot{V} = (1.177 \text{ kg/m}^3)(0.4/60 \text{ m}^3/\text{s}) = 0.00785 \text{ kg/s}$$

$$\dot{Q}_{\substack{forced \\ convection}} = \dot{m} C_p \Delta T = (0.00785 \text{ kg/s})(1005 \text{ J/kg.°C})(45-30)°\text{C} = 118.3 \text{ W}$$

Noting that radiation heat transfer is negligible, the rest of the 150 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{\substack{natural \\ convection}} = \dot{Q}_{total} - \dot{Q}_{\substack{forced \\ convection}} = 150 - 118.3 = \mathbf{31.7 \text{ W}}$$



(*b*) The natural convection heat transfer from the circular duct is

$$L = D = 0.1 \text{ m}$$

$$A = 2\left(\frac{\pi D^2}{4}\right) + \pi DL = 2\left[\frac{\pi (0.1 \text{ m})^2}{4}\right] + \pi (0.1 \text{ m})(1 \text{ m}) = 0.33 \text{ m}^2$$

$$h_{conv} = 1.32\left(\frac{\Delta T}{D}\right)^{0.25}$$

$$\dot{Q}_{conv} = h_{conv} A (T_s - T_{fluid}) = 1.32\left(\frac{(T_s - T_{fluid})}{D}\right)^{0.25} A_s (T_s - T_{fluid})$$

$$= 1.32 A \frac{(T_s - T_{fluid})^{1.25}}{D^{0.25}}$$

Substituting all known quantities with proper units gives the average temperature of the duct to be

$$31.7 \text{ W} = (1.32)(0.33 \text{ m}^2)\frac{(T_s - 25)^{1.25}}{(0.1 \text{ m})^{0.25}} \longrightarrow \mathbf{T_s = 44°C}$$

22-85 The components of an electronic equipment located in a horizontal duct with rectangular cross-section are cooled by forced air. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation heat transfer from the outer surfaces is negligible.

Analysis In this case the entire 150 W must be dissipated by natural convection from the outer surface of the duct. Natural convection from the vertical side surfaces of the duct can be expressed as

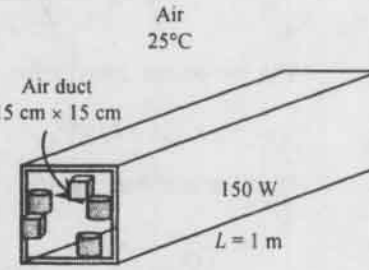
$$L = 0.15\,\text{m}$$

$$A_{side} = 2\times(0.15\,\text{m})(1\,\text{m}) = 0.3\,\text{m}^2$$

$$h_{conv,side} = 1.42\left(\frac{\Delta T}{L}\right)^{0.25}$$

$$\dot{Q}_{conv,side} = h_{conv,side}A_{side}(T_s - T_{fluid}) = 1.42\left(\frac{(T_s - T_{fluid})}{L}\right)^{0.25} A_{side}(T_s - T_{fluid})$$

$$= 1.42A_{side}\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$



Natural convection from the top surface of the duct is

$$L = \frac{4A}{p} = \frac{(4)(0.15\,\text{m})(1\,\text{m})}{(2)(0.15\,\text{m}+1\,\text{m})} = 0.26\,\text{m}$$

$$A_{top} = (0.15\,\text{m})(1\,\text{m}) = 0.15\,\text{m}^2$$

$$h_{conv,top} = 1.32\left(\frac{\Delta T}{L}\right)^{0.25}$$

$$\dot{Q}_{conv,top} = h_{conv,top}A_{top}(T_s - T_{fluid}) = 1.32\left(\frac{(T_s - T_{fluid})}{L}\right)^{0.25} A_{top}(T_s - T_{fluid})$$

$$= 1.32A_{top}\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

Natural convection from the bottom surface of the duct is

$$h_{conv,bottom} = 0.59\left(\frac{\Delta T}{L}\right)^{0.25}$$

$$\dot{Q}_{conv,bottom} = h_{conv,bottom}A_{top}(T_s - T_{fluid}) = 0.59\left(\frac{(T_s - T_{fluid})}{L}\right)^{0.25} A_{bottom}(T_s - T_{fluid})$$

$$= 0.59A_{bottom}\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

Then the total heat transfer by natural convection becomes

$$\dot{Q}_{total,conv} = \dot{Q}_{conv,side} + \dot{Q}_{conv,top} + \dot{Q}_{conv,bottom}$$

$$\dot{Q}_{total,conv} = 1.42A_{side}\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}} + 1.32A_{top}\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}} + 0.59A_{bottom}\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

Substituting all known quantities with proper units gives the average temperature of the duct to be

$$150 = (1.42)(0.3)\frac{(T_s - 25)^{1.25}}{0.15^{0.25}} + (1.32)(0.15)\frac{(T_s - 25)^{1.25}}{0.26^{0.25}} + (0.59)(0.15)\frac{(T_s - 25)^{1.25}}{0.26^{0.25}}$$

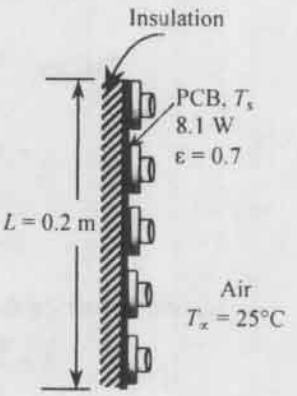
$$150 = (1.086)(T_s - 25)^{1.25} \longrightarrow \mathbf{T_s = 77°C}$$

22-86 A wall-mounted circuit board containing 81 square chips is cooled by combined natural convection and radiation. The surface temperature of the chips is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer from the back side of the circuit board is negligible. **3** Temperature of surrounding surfaces is the same as the air temperature. **3** The local atmospheric pressure is 1 atm.

Analysis The natural convection heat transfer coefficient for the vertical orientation of board can be determined from (Table 22-1)

$$h_{conv} = 1.42\left(\frac{T_s - T_{fluid}}{L}\right)^{0.25}$$



Substituting it relation into the heat transfer relation gives

$$\dot{Q}_{conv} = h_{conv} A(T_s - T_{fluid})$$

$$= 1.42\left(\frac{T_s - T_{fluid}}{L}\right)^{0.25} A(T_s - T_{fluid}) = 1.42A\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

The rate of heat transfer from the board by radiation is

$$\dot{Q}_{rad} = \varepsilon A\sigma\left(T_s^4 - T_{surr}^4\right)$$

Then the total heat transfer can be expressed as

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 1.42A\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}} + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

where $\dot{Q}_{total} = (0.1\text{ W}) \times 81 = 8.1\text{ W}$. Noting that the characteristic length is L = 0.2 m, calculating the surface area and substituting the known quantities into the above equation, the surface temperature is determined to be

$L = 0.2$ m

$A = (0.2\text{ m})(0.2\text{ m}) = 0.04\text{ m}^2$

$$8.1\text{ W} = (2.44)(0.04\text{ m}^2)\frac{[T_s - (25+273\text{ K})]^{1.25}}{(0.2\text{ m})^{0.25}} + (0.7)(0.04\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{K}^4)\left[T_s^4 - (25+273\text{ K})^4\right]$$

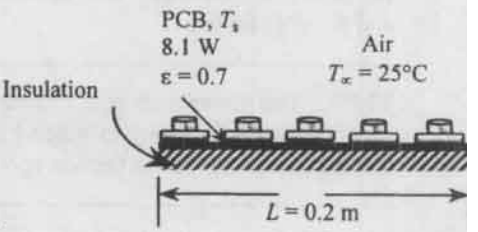
$T_s = 315\text{ K} = \mathbf{42°C}$

22-87 A horizontal circuit board containing 81 square chips is cooled by combined natural convection and radiation. The surface temperature of the chips is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer from the back side of the circuit board is negligible. **3** Temperature of surrounding surfaces is the same as the air temperature. **3** The local atmospheric pressure is 1 atm.

Analysis (*a*) The natural convection heat transfer coefficient for the horizontal orientation of board with chips facing up can be determined from (Table 22-1)

$$h_{conv} = 1.32\left(\frac{T_s - T_{fluid}}{L}\right)^{0.25}$$



Substituting it into the heat transfer relation gives

$$\dot{Q}_{conv} = h_{conv} A(T_s - T_{fluid})$$

$$= 1.32\left(\frac{T_s - T_{fluid}}{L}\right)^{0.25} A(T_s - T_{fluid}) = 1.32A\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}}$$

The rate of heat transfer from the board by radiation is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

Then the total heat transfer can be written as

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 1.32A\frac{(T_s - T_{fluid})^{1.25}}{L^{0.25}} + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

where $\dot{Q}_{total} = (0.1\text{ W}) \times 81 = 8.1\text{ W}$. Noting that the characteristic length is L = 0.2 m, calculating the surface area and substituting the known quantities into the above equation, the surface temperature is determined to be

$$L = \frac{4A}{p} = \frac{(4)(0.2\text{ m})(0.2\text{ m})}{(4)(0.2\text{ m})} = 0.2\text{ m}$$

$$A = (0.2\text{ m})(0.2\text{ m}) = 0.04\text{ m}^2$$

$$8.1\text{ W} = (1.32)(0.04\text{ m}^2)\frac{(T_s - (25+273)\text{K})^{1.25}}{(0.2\text{ m})^{0.25}} + (0.7)(0.04\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{K}^4)[T_s^4 - (25+273\text{ K})^4]$$

$$T_s = 321\text{ K} = \mathbf{48°C}$$

(*b*) The solution in this case (the chips are facing down instead of up) is identical to the one above, except we must replace the constant 1.32 in the heat transfer coefficient relation by 0.59. Then the surface temperature in this case becomes

$$8.1\text{ W} = (0.59)(0.04\text{ m}^2)\frac{(T_s - (25+273)\text{K})^{1.25}}{(0.2\text{ m})^{0.25}} + (0.7)(0.04\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{K}^4)[T_s^4 - (25+273\text{ K})^4]$$

$$T_s = 328\text{ K} = \mathbf{55°C}$$

Air Cooling: Forced Convection

22-88C Radiation heat transfer in forced air cooled systems is usually disregarded with no significant error since the forced convection heat transfer coefficient is usually much larger than the radiation heat transfer coefficient.

22-89C We would definitely prefer natural convection cooling whenever it is adequate in order to avoid all the problems associated with the fans such as cost, power consumption, noise, complexity, maintenance, and possible failure.

22-90C The convection heat transfer coefficient depends strongly on the average fluid velocity. Forced convection usually involves much higher fluid velocities, and thus much higher heat transfer coefficients. Consequently, forced convection cooling is much more effective.

22-91C Increasing the flow rate of air will increase the heat transfer coefficient. Then from Newton's law of cooling $\dot{Q}_{conv} = hA(T_s - T_{fluid})$, it becomes obvious that for a fixed amount of power, the temperature difference between the surface and the air will decrease. Therefore, the surface temperature will decrease. The exit temperature of the air will also decrease since $\dot{Q}_{conv} = \dot{m}_{air} C_p (T_{out} - T_{in})$ and the flow rate of air is increased.

22-92C Fluid flow over a body is called external flow, and flow through a confined space such as a tube or the parallel passage area between two circuit boards in an enclosure is called internal flow. A fan cooled personal computer left in windy area involves both types of flow.

22-93C For a specified power dissipation and air inlet temperature, increasing the heat transfer coefficient will decrease the surface temperature of the electronic components since, from Newton's law of cooling, $\dot{Q}_{conv} = hA(T_s - T_{fluid})$

22-94C A fan at a fixed speed (or fixed rpm) will deliver a fixed volume of air regardless of the altitude and pressure. But the mass flow rate of air will be less at high altitude as a result of the lower density of air. This may create serious reliability problems and catastrophic failures of electronic equipment if proper precautions are not taken. Variable speed fans which automatically increase the speed when the air density decreases are available to avoid such problems.

22-95C A fan placed at the inlet draws the air in and pressurizes the electronic box, and prevents air infiltration into the box through the cracks or other openings. Having only one location for air inlet makes it practical to install a filter at the inlet to catch all the dust and dirt before they enter the box. This allows the electronic system to operate in a clean environment. Also, the fan placed at the inlet handles cooler and thus denser air which results in a higher mass flow rate for the same volume flow rate or rpm. Being subjected to cool air has the added benefit that it increases the reliability and extends the life of the fan. The major disadvantage associated with the fan mounted at the inlet is that the heat generated by the fan and its motor is picked up by air on its way into the box, which adds to the heat load of the system.

When the fan is placed at the exit, the heat generated by the fan and its motor is immediately discarded to the atmosphere without getting blown first into the electronic box. However, the fan at the exit creates a vacuum inside the box, which draws air into the box through inlet vents as well as any cracks and openings. Therefore, the air is difficult to filter, and the dirt and dust which collects on the components undermine the reliability of the system.

22-96C The volume flow rate of air in a forced-air-cooled electronic system that has a constant speed fan is established at point where the fan static head curve and the system resistance curve intersects. Therefore, a fan will deliver a higher flow rate through a system which offers a lower flow resistance. A few PCBs added into an electronic box will increase the flow resistance and thus decrease the flow rate of air.

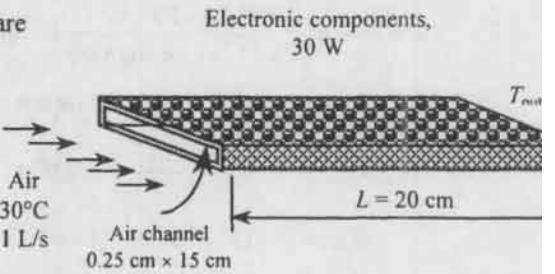
22-97C An undersized fan may cause the electronic system to overheat and fail. An oversized fan will definitely provide adequate cooling but it will needlessly be larger, noisier, more expensive, and will consume more power.

22-98 A hollow core PCB is cooled by forced air. The outlet temperature of the air and the highest surface temperature are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas. **4** The local atmospheric pressure is 1 atm. **5** The entire heat generated in electronic components is removed by the air flowing through the hollow core.

Properties The air properties at 300 K and 1 atm (Table A-18) are

$$\rho = 1.177\ \text{kg/m}^3; \qquad C_p = 1005\ \text{J/kg.°C}$$
$$\text{Pr} = 0.712; \qquad k = 0.0261\ \text{W/m.°C}$$
$$\nu = 1.57\times10^{-5}\ \text{m}^2/\text{s}$$



Analysis (*a*) The cross-sectional area of the channel and its hydraulic diameter are

$$A_c = (height)(width) = (0.15\ \text{m})(0.0025\ \text{m}) = 3.75\times10^{-4}\ \text{m}^2$$

$$D_h = \frac{4A_c}{p} = \frac{(4)(3.75\times10^{-4}\ \text{m}^2)}{(2)(0.15\ \text{m}+0.0025\ \text{m})} = 0.00492\ \text{m}$$

The average velocity and the mass flow rate of air are

$$\mathbf{V} = \frac{\dot{V}}{A_c} = \frac{1\times10^{-3}\ \text{m}^3/\text{s}}{3.75\times10^{-4}\ \text{m}^2} = 2.67\ \text{m/s}$$

$$\dot{m} = \rho\dot{V} = (1.177\ \text{kg/m}^3)(1\times10^{-3}\ \text{m}^3/\text{s}) = 1.177\times10^{-3}\ \text{kg/s}$$

Then the temperature of air at the exit of the hollow core becomes

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in})$$

$$T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m}C_p} = 30°\text{C} + \frac{30\ \text{W}}{(1.177\times10^{-3}\ \text{kg/s})(1005\ \text{J/kg.°C})} = \mathbf{55°C}$$

(*b*) The highest surface temperature in the channel will occur near the exit, and the surface temperature there can be determined from

$$\dot{q}_{conv} = h(T_s - T_{fluid})$$

To determine heat transfer coefficient, we first need to calculate the Reynolds number,

$$\text{Re} = \frac{\mathbf{V}D_h}{\nu} = \frac{(2.67\ \text{m/s})(0.00492\ \text{m})}{1.57\times10^{-5}\ \text{m}^2/\text{s}} = 837 < 2300$$

Therefore the flow is laminar. Assuming fully developed flow, the Nusselt number for the air flow in this rectangular cross-section corresponding to the aspect ratio of $a/b = height/width = 15/0.25 = 60 \approx \infty$ is determined from Table 22-3 to be $Nu = 8.24$. Then,

$$h = \frac{k}{D_h}Nu = \frac{0.0261\ \text{W/m.°C}}{0.00492\ \text{m}}(8.24) = 43.7\ \text{W/m}^2\text{.°C}$$

The surface temperature of the hollow core near the exit is determined to be

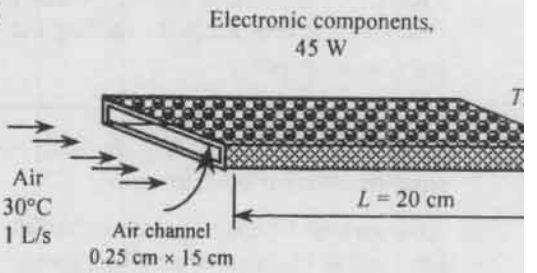
$$T_{s,\max} = T_{out} + \frac{\dot{q}}{h} = 55°\text{C} + \frac{(30\ \text{W})/(0.06\ \text{m}^2)}{(43.7\ \text{W/m}^2\text{.°C})} = \mathbf{66.4°C}$$

22-99 A hollow core PCB is cooled by forced air. The outlet temperature of air and the highest surface temperature are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas. **4** The local atmospheric pressure is 1 atm. **5** The entire heat generated in electronic components is removed by the air flowing through the hollow core.

Properties The air properties at 300 K and 1 atm are (Table A-18)

$$\rho = 1.177\ \text{kg/m}^3;\quad C_p = 1005\ \text{J/kg.°C}$$
$$\text{Pr} = 0.712;\qquad k = 0.0261\ \text{W/m.°C}$$
$$\nu = 1.57\times10^{-5}\ \text{m}^2/\text{s}$$



Analysis (a) The cross-sectional area of the channel and its hydraulic diameter are

$$A_c = (height)(width) = (0.15\ \text{m})(0.0025\ \text{m}) = 3.75\times10^{-4}\ \text{m}^2$$
$$D_h = \frac{4A_c}{p} = \frac{(4)(3.75\times10^{-4}\ \text{m}^2)}{(2)(0.15\ \text{m}+0.0025\ \text{m})} = 0.00492\ \text{m}$$

The average velocity and the mass flow rate of air are

$$\mathbf{V} = \frac{\dot{V}}{A_c} = \frac{1\times10^{-3}\ \text{m}^3/\text{s}}{3.75\times10^{-4}\ \text{m}^2} = 2.67\ \text{m/s}$$
$$\dot{m} = \rho\dot{V} = (1.177\ \text{kg/m}^3)(1\times10^{-3}\ \text{m}^3/\text{s}) = 1.177\times10^{-3}\ \text{kg/s}$$

Then the temperature of air at the exit of the hollow core becomes

$$\dot{Q} = \dot{m}C_p(T_{out}-T_{in}) \longrightarrow T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m}C_p}$$
$$= 30°\text{C} + \frac{45\ \text{W}}{(1.177\times10^{-3}\ \text{kg/s})(1005\ \text{J/kg.°C})} = \mathbf{68°C}$$

(b) The highest surface temperature in the channel will occur near the exit, and the surface temperature there can be determined from

$$\dot{q}_{conv} = h(T_s - T_{fluid})$$

where the heat transfer surface area is

$$A = 2A_{side} = (2)(height)(length) = (2)(0.15\ \text{m})(0.2\ \text{m}) = 0.06\ \text{m}^2$$

To determine heat transfer coefficient, we first need to calculate the Reynolds number

$$\text{Re} = \frac{\mathbf{V}D_h}{\nu} = \frac{(2.67\ \text{m/s})(0.00492\ \text{m})}{1.57\times10^{-5}\ \text{m}^2/\text{s}} = 837 < 2300$$

Therefore the flow is laminar. Assuming fully developed flow, the Nusselt number for the air flow in this rectangular cross-section corresponding to the aspect ratio $a/b = height/width = 15/0.25 = 60 \approx \infty$ is determined from Table 22-3 to be

$$Nu = 8.24$$

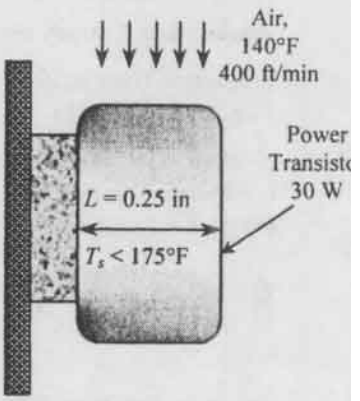
and thus

$$h = \frac{k}{D_h}Nu = \frac{0.0261\ \text{W/m.°C}}{0.00492\ \text{m}}(8.24) = 43.7\ \text{W/m}^2\text{.°C}$$

The surface temperature of the hollow core near the exit is determined to be

$$T_{s,\max} = T_{out} + \frac{\dot{q}}{hA} = 68°\text{C} + \frac{(45\ \text{W})/(0.06\ \text{m}^2)}{(43.7\ \text{W/m}^2\text{.°C})} = \mathbf{85°C}$$

22-100E A transistor mounted on a circuit board is cooled by air flowing over it. The power dissipated when its case temperature is 175°F is to be determined.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm pressure and the film temperature of $T_f = (T_s+T_{fluid})/2 = (175+140)/2 = 157.5°F$ are (Table A-18)

$$\rho = 0.066 \text{ lbm / ft}^3$$
$$C_p = 0.241 \text{ Btu / lbm.°F}$$
$$\text{Pr} = 0.72$$
$$k = 0.0166 \text{ Btu / h.ft.°F}$$
$$\nu = 0.21\times10^{-3} \text{ ft}^2 \text{ / s}$$

Analysis The transistor is cooled by forced convection through its cylindrical surface as well as its flat top surface. The characteristic length for flow over a cylinder is the diameter D=0.2 in. Then,

$$\text{Re} = \frac{VD}{\nu} = \frac{(400/60 \text{ ft / s})(0.2/12 \text{ ft})}{0.21\times10^{-3} \text{ ft}^2 \text{ / s}} = 529$$

which falls into the range of 40-4000. Using the appropriate relation from Table 22-2, the Nusselt number and the convection heat transfer coefficient are determined to be

$$Nu = 0.683\,\text{Re}^{0.466}\,\text{Pr}^{1/3} = (0.683)(529)^{0.466}(0.72)^{1/3} = 11.4$$

$$h = \frac{k}{D}Nu = \frac{0.0166 \text{ Btu / h.ft.°F}}{(0.2/12 \text{ ft})}(11.4) = 11.3 \text{ Btu / h.ft}^2\text{.°F}$$

The transistor loses heat through its cylindrical surface as well as its circular top surface. For convenience, we take the heat transfer coefficient at the top surfaces to be the same as that of the side surface. (The alternative is to treat the top surface as a flat plate, but this will double the amount of calculations without providing much improvement in accuracy since the area of the top surface is much smaller and it is circular in shape rather than being rectangular). Then,

$$A_{cyl} = \pi DL + \pi D^2/4 = \pi(0.2/12 \text{ ft})(0.25/12 \text{ ft}) + \pi(0.2/12 \text{ ft})^2/4 = 0.00131 \text{ ft}^2$$

$$\dot{Q}_{cyl} = hA_{cyl}(T_s - T_{fluid}) = (11.3 \text{ Btu / h.ft}^2\text{.°F})(0.00131 \text{ ft}^2)(175-140)°\text{F} = 0.518 \text{ Btu / h} = \mathbf{0.15\ W}$$

since 1 W = 3.4121 Btu/h. Therefore, the transistor can dissipate 0.15 W safely.

22-101 A desktop computer is to be cooled by a fan safely in hot environments and high elevations. The air flow rate of the fan and the diameter of the casing are to be determined.

Assumptions **1** Steady operation under worst conditions is considered. **2** Air is an ideal gas.

Properties The specific heat of air at the average temperature of T_{ave} = (45+60)/2 = 52.5°C is C_p = 1006 J/kg.°C (Table A-18)

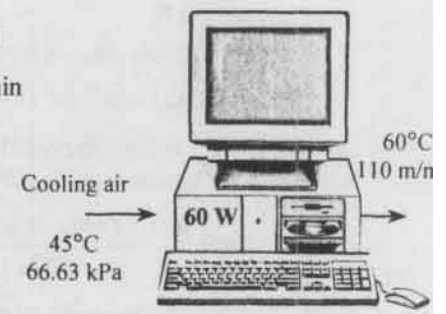
Analysis The fan selected must be able to meet the cooling requirements of the computer at worst conditions. Therefore, we assume air to enter the computer at 66.63 kPa and 45°C, and leave at 60°C. Then the required mass flow rate of air to absorb heat generated is determined to be

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in}) \rightarrow \dot{m} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})} = \frac{60\ \text{W}}{(1006\ \text{J/kg.°C})(60-45)\text{°C}}$$
$$= 0.00397\ \text{kg/s} = 0.239\ \text{kg/min}$$

The density of air entering the fan at the exit and its volume flow rate are

$$\rho = \frac{P}{RT} = \frac{66.63\ \text{kPa}}{(0.287\ \text{kPa.m}^3/\text{kg.K})(60+273)\text{K}} = 0.6972\ \text{kg/m}^3$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.239\ \text{kg/min}}{0.6972\ \text{kg/m}^3} = \mathbf{0.343\ m^3/min}$$



For an average exit velocity of 110 m/min, the diameter of the casing of the fan is determined from

$$\dot{V} = A_c V = \frac{\pi D^2}{4} V \rightarrow D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{(4)(0.343\ \text{m}^3/\text{min})}{\pi(110\ \text{m/min})}} = 0.063\ \text{m} = \mathbf{6.3\ cm}$$

22-102 A desktop computer is to be cooled by a fan safely in hot environments and high elevations. The air flow rate of the fan and the diameter of the casing are to be determined.

Assumptions **1** Steady operation under worst conditions is considered. **2** Air is an ideal gas.

Properties The specific heat of air at the average temperature of T_{ave} = (45+60)/2 = 52.5°C is C_p = 1006 J/kg.°C.

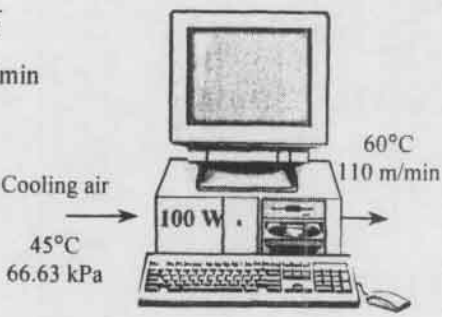
Analysis The fan selected must be able to meet the cooling requirements of the computer at worst conditions. Therefore, we assume air to enter the computer at 66.63 kPa and 45°C, and leave at 60°C. Then the required mass flow rate of air to absorb heat generated is determined to be

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in}) \rightarrow \dot{m} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})} = \frac{100\ \text{W}}{(1006\ \text{J/kg.°C})(60-45)\text{°C}}$$
$$= 0.006627\ \text{kg/s} = 0.398\ \text{kg/min}$$

The density of air entering the fan at the exit and its volume flow rate are

$$\rho = \frac{P}{RT} = \frac{66.63\ \text{kPa}}{(0.287\ \text{kPa.m}^3/\text{kg.K})(60+273)\text{K}} = 0.6972\ \text{kg/m}^3$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.398\ \text{kg/min}}{0.6972\ \text{kg/m}^3} = \mathbf{0.57\ m^3/min}$$



For an average exit velocity of 110 m/min, the diameter of the casing of the fan is determined from

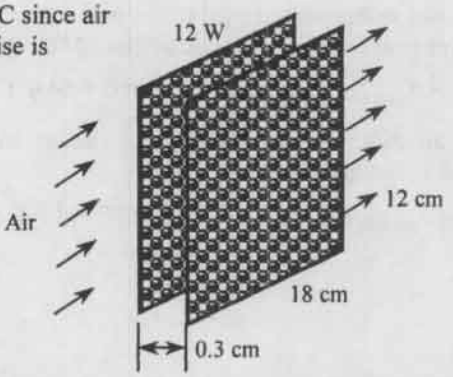
$$\dot{V} = A_c V = \frac{\pi D^2}{4} V \longrightarrow D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{(4)(0.57\ \text{m}^3/\text{min})}{\pi(110\ \text{m/min})}} = 0.081\ \text{m} = \mathbf{8.1\ cm}$$

22-103 A computer is cooled by a fan, and the temperature rise of air is limited to 15°C. The flow rate of air, the fraction of the temperature rise of air caused by the fan and its motor, and maximum allowable air inlet temperature are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** The entire heat generated in electronic components is removed by the air flowing through the opening between the PCBs. **5** The entire power consumed by the fan motor is transferred as heat to the cooling air.

Properties We use air properties at 1 atm and 27°C since air enters at room temperature, and the temperature rise is limited to 15°C (Table A-18)

$$\rho = 1.177 \text{ kg/m}^3$$
$$C_p = 1005 \text{ J/kg.°C}$$
$$\text{Pr} = 0.712$$
$$k = 0.0261 \text{ W/m.°C}$$
$$\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$$



Analysis (a) Because of symmetry, we consider the flow area between the two adjacent PCBs only. We assume the flow rate of air through all 8 channels to be identical, and to be equal to one-eighth of the total flow rate. The total mass and volume flow rates of air through the computer are determined from

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in}) \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})} = \frac{[(8 \times 12) + 25] \text{ J/s}}{(1005 \text{ J/kg.°C})(15°\text{C})} = 0.00803 \text{ kg/s}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.00803 \text{ kg/s}}{1.177 \text{ kg/m}^3} = \mathbf{0.006819 \text{ m}^3/\text{s}}$$

Noting that we have 8 PCBs and the flow area between the PCBs is 0.12 m and 0.003 m wide, the air velocity is determined to be

$$\mathbf{V} = \frac{\dot{V}}{A_c} = \frac{(0.006819 \text{ m}^3/\text{s})/8}{(0.12 \text{ m})(0.003 \text{ m})} = 2.37 \text{ m/s}$$

(b) The temperature rise of air due to the 25 W of power consumed by the fan is

$$\Delta T_{air} = \frac{\dot{Q}_{fan}}{\dot{m}C_p} = \frac{25 \text{ W}}{(0.00803 \text{ kg/s})(1005 \text{ J/kg.°C})} = 3.1°\text{C}$$

Then the fraction of temperature rise of air which is due to the heat generated by the fan becomes

$$f = \frac{3.1 \text{ °C}}{15 \text{ °C}} \times 100 = \mathbf{21\%}$$

(c) To determine the surface temperature, we need to evaluate the convection heat transfer coefficient,

$$A_c = (height)(width) = (0.12 \text{ m})(0.003 \text{ m}) = 0.00036 \text{ m}^2$$

$$D_h = \frac{4A_c}{p} = \frac{(4)(0.00036 \text{ m}^2)}{(2)(0.12 \text{ m} + 0.003 \text{ m})} = 0.00585 \text{ m}$$

$$\text{Re} = \frac{\mathbf{V}D_h}{\nu} = \frac{(2.37 \text{ m/s})(0.00585 \text{ m})}{1.57 \times 10^{-5} \text{ m}^2/\text{s}} = 884 < 2300$$

Therefore, the flow is laminar. (Actually, the components will cause the flow to be turbulent. The laminar assumption gives conservative results). Assuming fully developed flow, the Nusselt number for the air flow through this rectangular channel corresponding to the aspect ratio $a/b = 12/0.3 = 40 \approx \infty$ is determined from Table 22-3 to be $Nu = 8.24$. Then the heat transfer coefficient becomes

$$h = \frac{k}{D_h} Nu = \frac{0.0261 \text{ W/m.°C}}{0.00585 \text{ m}} (8.24) = 36.8 \text{ W/m}^2.\text{°C}$$

Disregarding the entrance effects, the temperature difference between the surface of the PCB and the air anywhere along the channel is determined to be

$$T_s - T_{fluid} = \frac{\dot{Q}}{hA} = \frac{12 \text{ W}}{(36.8 \text{ W/m}^2.\text{°C})(0.12 \times 0.18 \text{ m}^2)} = 15.1\text{°C}$$

The highest air and component temperatures will occur at the exit. Therefore, in the limiting case, the component surface temperature at the exit will be 80 °C. The air temperature at the exit in this case will be

$$T_{out,\max} = T_{s,\max} - \Delta T_{rise} = 80\text{°C} - 15.1\text{°C} = 64.9\text{°C}$$

Noting that the air experiences a temperature rise of 15 °C between the inlet and the exit, the inlet temperature of air becomes

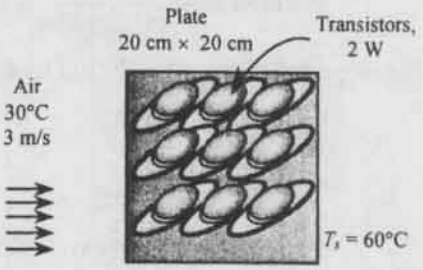
$$T_{in,\max} = T_{out,\max} - 15\text{°C} = 64.9\text{°C} - 15\text{°C} = \mathbf{49.9°C}$$

22-104 An array of power transistors is to be cooled by mounting them on a square aluminum plate and blowing air over the plate. The number of transistors that can be placed on this plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** The entire heat generated by transistors is removed by the air flowing over the plate. **5** The heat transfer from the back side of the plate is negligible.

Properties The properties of air at the free stream temperature of 30°F (303 K) are (Table A-18)

$$\rho = 1.177 \text{ kg/m}^3$$
$$C_p = 1005 \text{ J/kg.°C}$$
$$\text{Pr} = 0.712$$
$$k = 0.0261 \text{ W/m.°C}$$
$$\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$$



Analysis The plate area and the convection heat transfer coefficient are determined to be (from Table 22-2)

$$A = (0.2 \text{ m})(0.2 \text{ m}) = 0.04 \text{ m}^2$$

$$\text{Re} = \frac{VL}{\nu} = \frac{(3 \text{ m/s})(0.2 \text{ m})}{1.57 \times 10^{-5} \text{ m}^2/\text{s}} = 38{,}217$$

$$Nu = 0.664 \text{ Re}^{1/2} \text{ Pr}^{1/3} = (0.664)(38{,}217)^{1/2}(0.712)^{1/3} = 115.9$$

$$h = \frac{k}{L} Nu = \frac{0.0261 \text{ W/m.°C}}{0.2 \text{ m}}(115.9) = 15.1 \text{ W/m}^2\text{.°C}$$

The rate of heat transfer from the plate is

$$\dot{Q}_{conv} = hA(T_s - T_{fluid}) = (15.1 \text{ W/m}^2\text{.°C})(0.04 \text{ m}^2)(60 - 30)\text{°C} = 18.2 \text{ W}$$

Then the number of transistors that can be placed on this plate becomes

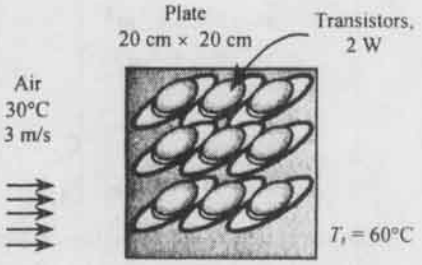
$$n = \frac{18.2 \text{ W}}{2 \text{ W}} = 9 \text{ transistors}$$

22-105 An array of power transistors is to be cooled by mounting them on a square aluminum plate and blowing air over the plate. The number of transistors that can be placed on this plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 83.4 kPa. **4** The entire heat generated by transistors is removed by the air flowing over the plate. **5** The heat transfer from the back side of the plate is negligible.

Properties At an elevation of 1610 m, the atmospheric pressure is 83.4 kPa or

$$P = (83.4 \text{ kPa})\frac{1 \text{ atm}}{101.325 \text{ kPa}} = 0.823 \text{ atm}$$



The properties of air at 30 °C (303 K) are

$$C_p = 1005 \text{ J/kg.°C}$$
$$\text{Pr} = 0.712$$
$$k = 0.0261 \text{ W/m.°C}$$
$$v = (1.57 \times 10^{-5} \text{ m}^2/\text{s})/0.823 = 1.91 \text{ m}^2/\text{s}$$

Analysis The plate area and the convection heat transfer coefficient are determined to be (from Table 22-2)

$$A = (0.2 \text{ m})(0.2 \text{ m}) = 0.04 \text{ m}^2$$

$$\dot{m} = \rho A V = \rho \frac{\pi D^2}{4} V \longrightarrow D = \sqrt{\frac{4\dot{m}}{\pi \rho V}} = \sqrt{\frac{4(0.0179 \text{ kg/s})}{\pi(1000 \text{ kg/m}^3)(0.5 \text{ m/s})}} = 0.0068 \text{ m}$$

$$= \mathbf{0.68 \text{ cm}}$$

$$\dot{Q} = \frac{T_{case} - T_{liquid}}{R_{case-liquid}} \longrightarrow T_{case} = T_{liquid} + \dot{Q}R_{case-liquid} = 25°\text{C} + (300 \text{ W})(0.04 \text{ °C/W}) = \mathbf{37°C}$$

$$h = \frac{k}{L} Nu = \frac{0.0261 \text{ W/m.°C}}{0.2 \text{ m}}(105.1) = 13.7 \text{ W/m}^2.°\text{C}$$

The rate of heat transfer from the plate is

$$\dot{Q}_{conv} = hA(T_s - T_{fluid}) = (13.7 \text{ W/m}^2.°\text{C})(0.04 \text{ m}^2)(60 - 30)°\text{C} = 16.5 \text{ W}$$

The number of transistors that can be placed on this plate for this rate of heat transfer is

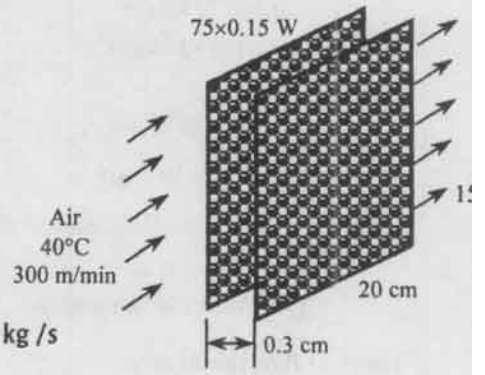
$$\dot{Q} = \frac{761.8 \text{ Btu/h}}{0.85} = \mathbf{896 \text{ Btu/h} = 263 \text{ W}}$$

22-106 An enclosure containing an array of circuit boards is cooled by forced air flowing through the clearance between the tips of the components on the PCB and the back surface of the adjacent PCB. The exit temperature of the air and the highest surface temperature of the chips are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** The entire heat generated by the PCBs is removed by the air flowing through the clearance inside the enclosure. **5** The heat transfer from the back side of the circuit board is negligible.

Properties We use the properties of air at 1 atm and 40°F (313 K):

$$\rho = 1.133 \text{ kg/m}^3$$
$$C_p = 1006 \text{ J/kg.°C}$$
$$\text{Pr} = 0.713$$
$$k = 0.027 \text{ W/m.°C}$$
$$\nu = 1.7 \times 10^{-5} \text{ m}^2/\text{s}$$



Analysis The volume and the mass flow rates of air are

$$\dot{Q} = \dot{m}C_p\Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p\Delta T} = \frac{3 \text{ kJ/s}}{(4.18 \text{ kJ/kg.°C})(4°\text{C})} = \mathbf{0.1794 \text{ kg/s}}$$

Then the exit temperature of air is determined from

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in}) \longrightarrow T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m}C_p} = 40°\text{C} + \frac{(75 \times 0.15)\text{W}}{(0.00255 \text{ kg/s})(1006 \text{ J/kg.°C})} = \mathbf{44.4°C}$$

To determine the surface temperature, we need to calculate the convection heat transfer coefficient first,

$$A = (0.15 \text{ m})(0.2 \text{ m}) = 0.03 \text{ m}^2$$

$$A_c = (0.15 \text{ m})(0.003 \text{ m}) = 0.00045 \text{ m}^2$$

$$D_h = \frac{4A_c}{p} = \frac{(4)(0.00045 \text{ m}^2)}{(2)(0.15 \text{ m} + 0.003 \text{ m})} = 0.0059 \text{ m}$$

$$\text{Re} = \frac{\mathbf{V}D_h}{\nu} = \frac{(300/60 \text{ m/s})(0.0059 \text{ m})}{1.7 \times 10^{-5} \text{ m}^2/\text{s}} = 1735 < 2300$$

Therefore, the flow is laminar. Assuming fully developed flow, the Nusselt number for the air flow in this rectangular cross-section corresponding to the aspect ratio of $a/b = height/width = 15/0.3 = 50 \approx \infty$ is determined from Table 22-3 to be $Nu = 8.24$. Then,

$$h = \frac{k}{L}Nu = \frac{0.027 \text{ W/m.°C}}{0.0059 \text{ m}}(8.24) = 37.7 \text{ W/m}^2\text{.°C}$$

The highest surface temperature of the chips then becomes

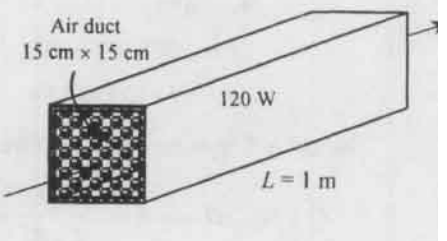
$$\dot{Q} = hA(T_{s,\max} - T_{fluid}) \longrightarrow T_{s,\max} = T_{air,out} + \frac{\dot{Q}}{hA} = 44.4°\text{C} + \frac{(75 \times 0.15) \text{ W}}{(37.7 \text{ W/m}^2\text{.°C})(0.03 \text{ m}^2)} = \mathbf{54.3°C}$$

22-107 The components of an electronic system located in a horizontal duct of rectangular cross-section are cooled by forced air flowing through the duct. The exit temperature of air and the highest component surface temperature in the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties We use the properties of air at 1 atm and 30°C (303 K) (Table A-18)

$$\rho = 1.177 \text{ kg/m}^3$$
$$C_p = 1005 \text{ J/kg.°C}$$
$$\text{Pr} = 0.712$$
$$k = 0.0261 \text{ W/m.°C}$$
$$\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$$



Analysis (*a*) The rate of heat transfer from the components to the forced air in the duct is

$$\dot{Q} = (0.80)(120 \text{ W}) = 96 \text{ W}$$

The mass flow rate of air is

$$\dot{m} = \rho \dot{V} = (1.177 \text{ kg/m}^3)(0.5/60 \text{ m}^3/\text{s}) = 0.00981 \text{ kg/s}$$

Then the exit temperature of air is determined from

$$m = \rho V = \rho AL = (8950 \text{ kg/m}^3)(0.03886 \text{ m}^2)(0.4 \text{ m}) = \mathbf{139 \text{ kg}}$$

(*b*) The highest surface temperature can be determined from

$$\dot{Q}_{conv} = hA(T_s - T_{fluid})$$

But we first need to determine convection heat transfer coefficient,

$$A = (4)(1 \text{ m})(0.15 \text{ m}) = 0.6 \text{ m}^2$$

$$\mathbf{V} = \frac{\dot{V}}{A_c} = \frac{(0.5/60 \text{ m}^3/\text{s})}{(0.15 \text{ m})^2} = 0.37 \text{ m/s}$$

$$\text{Re} = \frac{\mathbf{V} D_h}{\nu} = \frac{(0.37 \text{ m/s})(0.15 \text{ m})}{1.57 \times 10^{-5} \text{ m}^2/\text{s}} = 3539$$

From Table 22-2,

$$Nu = 0.102 \text{ Re}^{0.675} \text{ Pr}^{1/3} = (0.102)(3539)^{0.675}(0.712)^{1/3} = 22.6$$

$$h = \frac{k}{D_h} Nu = \frac{0.0261 \text{ W/m.°C}}{0.15 \text{ m}}(22.6) = 3.94 \text{ W/m}^2.\text{°C}$$

Then the highest component surface temperature in the duct becomes

$$\dot{Q} = hA(T_{s,\max} - T_{air,out}) \longrightarrow T_{s,\max} = T_{air,out} + \frac{\dot{Q}}{hA} = 39.7°\text{C} + \frac{96 \text{ W}}{(3.94 \text{ W/m}^2.\text{°C})(0.6 \text{ m}^2)} = \mathbf{80.3°\text{C}}$$

22-108 The components of an electronic system located in a circular horizontal duct are cooled by forced air flowing through the duct. The exit temperature of air and the highest component surface temperature in the duct are to be determined.

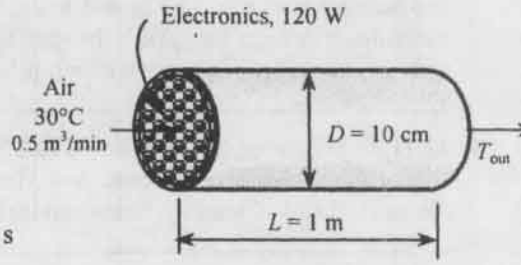
Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties We use the properties of air at 1 atm and 30°C (303 K) (Table A-18)

$$\rho = 1.177 \text{ kg/m}^3$$
$$C_p = 1005 \text{ J/kg.°C}$$
$$\text{Pr} = 0.712$$
$$k = 0.0261 \text{ W/m.°C}$$
$$\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis (*a*) The rate of heat transfer from the components to the forced air in the duct is

$$\dot{Q} = (0.80)(120 \text{ W}) = 96 \text{ W}$$



The mass flow rate of air is

$$\dot{m} = \rho \dot{V} = (1.177 \text{ kg/m}^3)(0.5/60 \text{ m}^3/\text{s}) = 0.00981 \text{ kg/s}$$

Then the exit temperature of air is determined from

$$\dot{Q} = \dot{m} C_p (T_{out} - T_{in}) \longrightarrow T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m} C_p} = 30°\text{C} + \frac{96 \text{ W}}{(0.00981 \text{ kg/s})(1005 \text{ J/kg.°C})} = \mathbf{39.7°C}$$

(*b*) The highest surface temperature can be determined from

$$\dot{Q}_{conv} = hA(T_s - T_{fluid})$$

But we first need to determine convection heat transfer coefficient,

$$A = \pi DL = \pi(0.1 \text{ m})(1 \text{ m}) = 0.314 \text{ m}^2$$

$$\mathbf{V} = \frac{\dot{V}}{A_c} = \frac{(0.5/60 \text{ m}^3/\text{s})}{\frac{\pi(0.1 \text{ m})^2}{4}} = 1.061 \text{ m/s}$$

$$\text{Re} = \frac{\mathbf{V}D}{\nu} = \frac{(1.061 \text{ m/s})(0.1 \text{ m})}{1.57 \times 10^{-5} \text{ m}^2/\text{s}} = 6758$$

From Table 22-2,

$$Nu = 0.683 \text{Re}^{0.466} \text{Pr}^{1/3} = 0.683(6758)^{0.466}(0.712)^{1/3} = 37.15$$

$$h = \frac{k}{D} Nu = \frac{0.0261 \text{ W/m.°C}}{0.1 \text{ m}}(37.15) = 9.70 \text{ W/m}^2\text{.°C}$$

Then the highest component surface temperature in the duct becomes

$$\dot{Q} = hA(T_{s,\max} - T_{air,out}) \longrightarrow T_{s,\max} = T_{air,out} + \frac{\dot{Q}}{hA} = 39.7°\text{C} + \frac{96 \text{ W}}{(9.7 \text{ W/m}^2\text{.°C})(0.314 \text{ m}^2)} = \mathbf{71.2°C}$$

Liquid Cooling

22-109C When both are adequate, we would prefer forced air cooling in order to avoid the potential risks and problems associated with water cooling such as leakage, corrosion, extra weight, and condensation.

22-110C In direct cooling systems, the electronic components are in direct contact with the liquid, and thus the heat generated in the components is transferred directly to the liquid. In indirect cooling systems, however, there is no direct contact with the components. The heat generated in this case is first transferred to a medium such as a cold plate before it is removed by the liquid.

22-111C In closed loop cooling systems the liquid is recirculated while in the open loop systems the liquid is discarded after use. The heated liquid in closed loop systems is cooled in a heat exchanger, and it is recirculated through the system. In open loop systems, liquid (usually tap water) flows through the cooling system is discarded into a drain after it is heated.

22-112C The properties of a liquid ideally suited for cooling electronic equipment include high thermal conductivity, high specific heat, low viscosity, high surface tension, high dielectric strength, chemical inertness, chemical stability, being non toxic, having low freezing and high boiling points, and low cost.

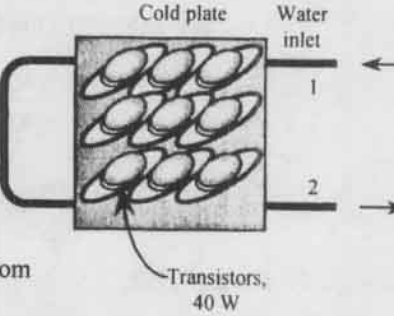
22-113 A cold plate is to be cooled by water. The mass flow rate of water, the diameter of the pipe, and the case temperature of the transistors are to be determined.

Assumptions **1** Steady operating conditions exist. **2** About 25 percent of the heat generated is dissipated from the components to the surroundings by convection and radiation.

Properties The properties of water at room temperature are $\rho = 1000$ kg/m^3 and $C_p = 4180$ J/kg.°C (Table A-15).

Analysis Noting that each of the 10 transistors dissipates 40 W of power and 75% of this power is removed by the water, the rate of heat transfer to the water is

$$\dot{Q} = (10 \text{ transistors})(40 \text{ W / transistor})(0.75) = 300 \text{ W}$$



In order to limit the temperature rise of water to 4 °C, the mass flow rate of water must be no less than

$$\dot{m} = \frac{\dot{Q}}{C_p \Delta T_{\text{rise}}} = \frac{300 \text{ W}}{(4180 \text{ J / kg.°C})(4°\text{C})} = 0.0179 \text{ kg / s} = \mathbf{1.08 \text{ kg / min}}$$

The diameter of the pipe to maintain the velocity under 0.5 m/s is determined from

$$\dot{m} = \rho A \mathrm{V} = \rho \frac{\pi D^2}{4} \mathrm{V} \longrightarrow D = \sqrt{\frac{4\dot{m}}{\pi \rho \mathrm{V}}} = \sqrt{\frac{4(0.0179 \text{ kg / s})}{\pi (1000 \text{ kg / m}^3)(0.5 \text{ m / s})}} = 0.0068 \text{ m}$$

$$= \mathbf{0.68 \text{ cm}}$$

Noting that the case-to-liquid thermal resistance is 0.04 °C/W, the case temperature of the transistors is

$$\dot{Q} = \frac{T_{case} - T_{liquid}}{R_{case-liquid}} \longrightarrow T_{case} = T_{liquid} + \dot{Q} R_{case-liquid} = 25°\text{C} + (300 \text{ W})(0.04 °\text{C / W}) = \mathbf{37°\text{C}}$$

22-114E Electronic devices mounted on a cold plate is cooled by water. The amount of heat generated by the electronic devices is to be determined.

Assumptions **1** Steady operating conditions exist. **2** About 15 percent of the heat generated is dissipated from the components to the surroundings by convection and radiation.

Properties The properties of water at room temperature are $\rho = 62.2$ lbm/ft^3 and $C_p = 0.998$ Btu/lbm.°F.

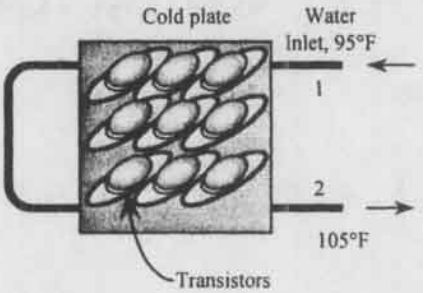
Analysis The mass flow rate of water and the rate of heat removal by the water are

$$\dot{m} = \rho A \mathbf{V} = \rho \frac{\pi D^2}{4} \mathbf{V} = (62.2 \text{ lbm/ft}^3) \frac{\pi (0.25/12 \text{ ft})^2}{4} (60 \text{ ft/min}) = 1.272 \text{ lbm/min} = 76.33 \text{ lbm/h}$$

$$\dot{Q} = \dot{m} C_p (T_{out} - T_{in}) = (76.33 \text{ lbm/h})(0.998 \text{ Btu/lbm.°F})(105-95)°\text{F} = 761.8 \text{ Btu/h}$$

which is 85 percent of the heat generated by the electronic devices. Then the total amount of heat generated by the electronic devices becomes

$$\dot{Q} = \frac{761.8 \text{ Btu/h}}{0.85} = \mathbf{896 \text{ Btu/h} = 263 \text{ W}}$$



22-115 A sealed electronic box is to be cooled by tap water flowing through channels on two of its sides. The mass flow rate of water and the amount of water used per year are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Entire heat generated is dissipated by water.

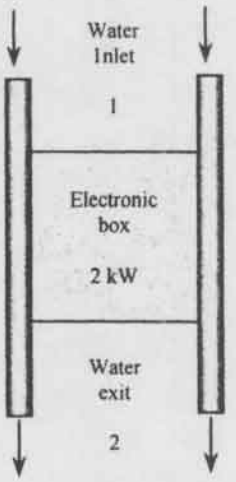
Properties The specific heat of water at room temperature is $C_p = 4180$ J/kg.°C.

Analysis The mass flow rate of tap water flowing through the electronic box is

$$\dot{Q} = \dot{m} C_p \Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p \Delta T} = \frac{2 \text{ kJ/s}}{(4.18 \text{ kJ/kg.°C})(4°\text{C})} = \mathbf{0.1196 \text{ kg/s}}$$

Therefore, 0.11962 kg water is needed per second to cool this electronic box. Then the amount of cooling water used per year becomes

$$m = \dot{m} \Delta t = (0.1196 \text{ kg/s})(365 \text{ days/yr} \times 24 \text{ h/day} \times 3600 \text{ s/h})$$
$$= 3{,}772{,}000 \text{ kg/yr} = \mathbf{3{,}772 \text{ tons/yr}}$$

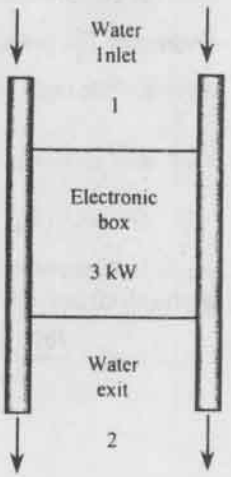


22-116 A sealed electronic box is to be cooled by tap water flowing through channels on two of its sides. The mass flow rate of water and the amount of water used per year are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Entire heat generated is dissipated by water.

Properties The specific heat of water at room temperature is $C_p = 4180$ J/kg.°C.

Analysis The mass flow rate of tap water flowing through the electronic box is



$$\dot{Q} = \dot{m}C_p\Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p\Delta T} = \frac{3\text{ kJ/s}}{(4.18\text{ kJ/kg.°C})(4\text{°C})} = \mathbf{0.1794\text{ kg/s}}$$

Therefore, 0.1794 kg water is needed per second to cool this electronic box. Then the amount of cooling water used per year becomes

$$m = \dot{m}\Delta t = (0.1794\text{ kg/s})(365\text{ days/yr} \times 24\text{ h/day} \times 3600\text{ s/h})$$
$$= 5{,}658{,}400\text{ kg/yr} = \mathbf{5{,}658\text{ tons/yr}}$$

Immersion Cooling

22-117C The desirable characteristics of a dielectric liquid used in immersion cooling of electronic devices are non-flammability, being chemically inert, compatibility with materials used in electronic equipment, and low boiling and freezing points.

22-118C An open loop immersion cooling system involves an external reservoir which supplies liquid continually to the electronic enclosure. The vapor generated inside is allowed to escape to the atmosphere. A pressure relief valve on the vapor vent line keeps the pressure and thus the temperature inside the enclosure at a preset value. In a closed loop immersion system, the vapor is condensed and returned to the electronic enclosure instead of being purged into the atmosphere.

22-119C In external immersion cooling systems, the vapor is condensed outside the enclosure whereas in internal immersion cooling systems the vapor is condensed inside the enclosure by circulating a cooling fluid through the vapor. Therefore, in condenser is built into the enclosure in internal immersion cooling systems whereas it is placed outside in external immersion cooling systems.

22-120C The heat transfer coefficient is much greater in the boiling heat transfer than it is in the forced air or liquid cooling. Therefore, in the cooling of high-power electronic devices, boiling heat transfer is used to achieve high cooling rates with minimal temperature differences.

22-121 A logic chip is to be cooled by immersion in a dielectric fluid. The minimum heat transfer coefficient and the type of cooling mechanism are to be determined.

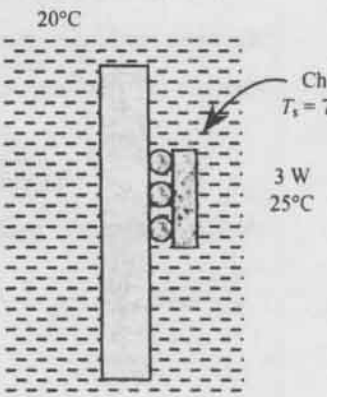
Assumptions Steady operating conditions exist.

Analysis The average heat transfer coefficient over the surface of the chip is determined from Newton's law of cooling to be

$$\dot{Q} = hA(T_{chip} - T_{fluid}) \longrightarrow h = \frac{\dot{Q}}{A(T_{chip} - T_{fluid})} = \frac{3\ \text{W}}{(0.3\times10^{-4}\ \text{m}^2)(70-20)^\circ\text{C}}$$

$$= \mathbf{2000\ W/m^2.^\circ C}$$

which is rather high. An examination of Fig. 22-62 reveals that we can obtain such heat transfer coefficients with the boiling of fluorocarbon fluids. Therefore, a suitable cooling technique in this case is immersion cooling in such a fluid.

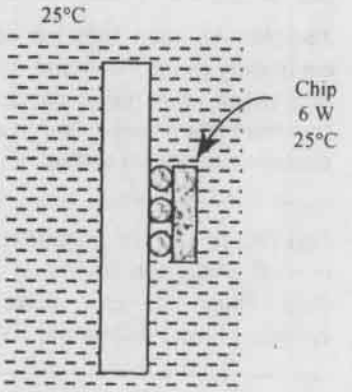


22-122 A chip is cooled by boiling in a dielectric fluid. The surface temperature of the chip is to be determined.

Assumptions The boiling curve in Fig. 22-63 is prepared for a chip having a surface area of 0.457 cm^2 being cooled in FC86 maintained at 5°C. The chart can be used for similar cases with reasonable accuracy.

Analysis The heat flux in this case is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{6\ \text{W}}{0.5\ \text{cm}^2} = 12\ \text{W/cm}^2$$



The temperature of the chip surface corresponding to this heat flux is determined from Fig. 22-63 to be

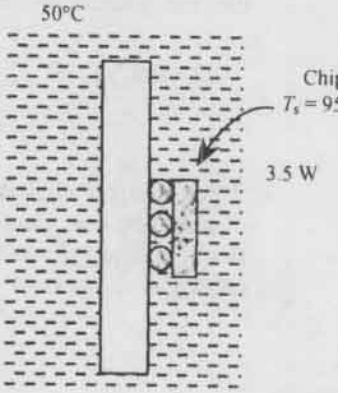
$$T_{chip} - T_{fluid} = 57°\text{C} \longrightarrow T_{chip} = (T_{fluid} + 57)°\text{C} = (25+57)°\text{C} = \mathbf{82°C}$$

22-123 A logic chip is cooled by immersion in a dielectric fluid. The heat flux and the heat transfer coefficient on the surface of the chip and the thermal resistance between the surface of the chip and the cooling medium are to be determined.

Assumptions Steady operating conditions exist.

Analysis (*a*) The heat flux on the surface of the chip is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{3.5\ \text{W}}{0.8\ \text{cm}^2} = \mathbf{4.375\ W/cm^2}$$



(*b*) The heat transfer coefficient on the surface of the chip is

$$\dot{Q} = hA(T_{chip} - T_{fluid}) \longrightarrow h = \frac{\dot{Q}}{A(T_{chip} - T_{fluid})} = \frac{3.5\ \text{W}}{(0.8\times10^{-4}\ \text{m}^2)(95-50)°\text{C}}$$
$$= \mathbf{972\ W/m^2.°C}$$

(*c*) The thermal resistance between the surface of the chip and the cooling medium is

$$\dot{Q} = \frac{T_{chip} - T_{fluid}}{R_{chip-fluid}} \longrightarrow R_{chip-fluid} = \frac{T_{chip} - T_{fluid}}{\dot{Q}} = \frac{(95-50)°\text{C}}{3.5\ \text{W}} = \mathbf{12.9°C/W}$$

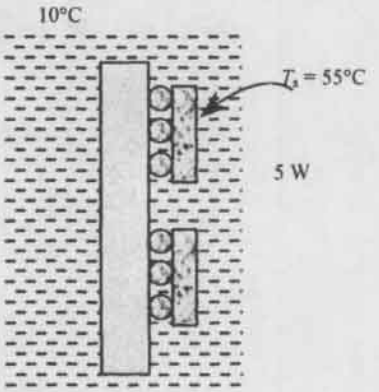
22-124 A computer chip is to be cooled by immersion in a dielectric fluid. The minimum heat transfer coefficient and the appropriate type of cooling mechanism are to be determined.

Assumptions Steady operating conditions exist.

Analysis The average heat transfer coefficient over the surface of the chip is determined from Newton's law of cooling to be

$$\dot{Q} = hA(T_{chip} - T_{fluid}) \longrightarrow h = \frac{\dot{Q}}{A(T_{chip} - T_{fluid})} = \frac{5\ \text{W}}{(0.4 \times 10^{-4}\ \text{m}^2)(55-10)^\circ\text{C}} = \mathbf{2778\ W/m^2.^\circ C}$$

which is rather high. An examination of Fig. 22-62 reveals that we can obtain such heat transfer coefficients with the boiling of fluorocarbon fluids. Therefore, a suitable cooling technique in this case is immersion cooling in such a fluid.



22-125 A chip is cooled by boiling in a dielectric fluid. The surface temperature of the chip is to be determined.

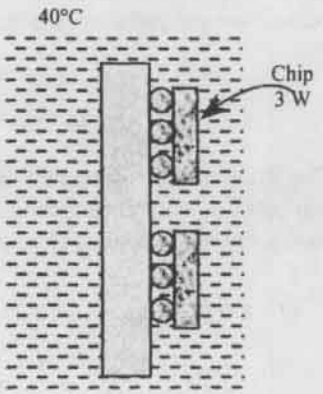
Assumptions The boiling curve in Fig. 22-63 is prepared for a chip having a surface area of 0.457 cm^2 being cooled in FC86 maintained at 5°C. The chart can be used for similar cases with reasonable accuracy.

Analysis The heat flux in this case is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{3\ \text{W}}{0.2\ \text{cm}^2} = 15\ \text{W/cm}^2$$

The temperature of the chip surface corresponding to this value is determined from Fig. 22-63 to be

$$T_{chip} - T_{fluid} = 63^\circ\text{C} \longrightarrow T_{chip} = (T_{fluid} + 63)^\circ\text{C} = (40+63)^\circ\text{C} = \mathbf{103^\circ C}$$



22-126 A chip is cooled by boiling in a dielectric fluid. The maximum power that the chip can dissipate safely is to be determined.

Assumptions The boiling curve in Fig. 22-63 is prepared for a chip having a surface area of 0.457 cm^2 being cooled in FC86 maintained at 5°C. The chart can be used for similar cases with reasonable accuracy.

Analysis The temperature difference between the chip surface and the liquid is

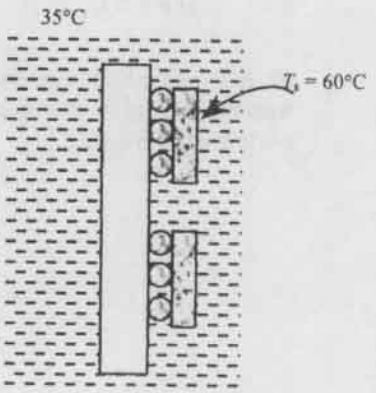
$$T_{chip} - T_{fluid} = (60 - 35)°C = 25°C$$

Using this value, the heat flux can be determined from Fig. 22-63 to be

$$\dot{q} = 3.3 \text{ W/cm}^2$$

Then the maximum power that the chip can dissipate safely becomes

$$\dot{Q} = \dot{q}A = (3.3 \text{ W/cm}^2)(0.3 \text{ cm}^2) = \mathbf{0.99 \text{ W}}$$



22-127 An electronic device is to be cooled by immersion in a dielectric fluid. It is to be determined if the heat generated inside can be dissipated to the ambient air by natural convection and radiation as well as the heat transfer coefficient at the surface of the electronic device.

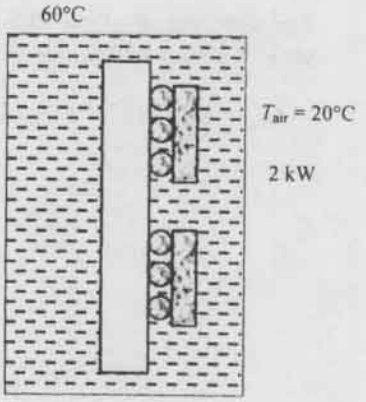
Assumptions Steady operating conditions exist.

Analysis Assuming the surfaces of the cubic enclosure to be at the temperature of the boiling dielectric fluid at 60 °C, the rate at which heat can be dissipated to the ambient air at 20 °C by combined natural convection and radiation is determined from

$$\dot{Q} = hA(T_s - T_{air}) = h(6a^2)(T_s - T_{air})$$
$$= (10 \text{ W/m}^2.°C)[6(1 \text{ m})^2](60 - 20)°C = 2400 \text{ W} = \mathbf{2.4 \text{ kW}}$$

Therefore, the heat generated inside the cubic enclosure can be dissipated by natural convection and radiation. The heat transfer coefficient at the surface of the electronic device is

$$\dot{Q} = hA(T_s - T_{fluid}) \longrightarrow h = \frac{\dot{Q}}{A(T_s - T_{fluid})} = \frac{2000 \text{ W}}{(0.012 \text{ m}^2)(80 - 60)°C}$$
$$= \mathbf{8333 \text{ W/m}^2.°C}$$



Heat Pipes

22-128C A heat pipe is a simple device with no moving parts which can transfer large quantities of heat over fairly large distances essentially at a constant temperature without requiring any power input. A heat pipe is basically a sealed slender tube containing a wick structure lined on the inner surface and a small amount of fluid such as water at the saturated state. It is composed of three sections: the evaporator section at one and where heat is absorbed and the fluid is vaporized, a condenser section at the other end where the vapor is condensed and heat is rejected, and the adiabatic section in between where the vapor and the liquid phases of the fluid flow in opposite directions through the core and the wick, respectively, to complete the cycle with no significant heat transfer between the fluid and the surrounding medium.

22-129C The boiling and condensation processes are associated with extremely high heat transfer coefficients, and thus it is natural to expect the heat pipe to be an extremely effective heat transfer device since its operation is based on alternate boiling and condensation of the working fluid.

22-130C The non-condensable gases such as air degrade the performance of the heat pipe, and can destroy it in a short time.

22-131C The heat pipes with water as the working fluid are designed to remove heat at temperatures below the boiling temperature of water at atmospheric pressure (i.e., 100°C). Therefore, the pressure inside the heat pipe must be maintained below the atmospheric pressure to provide boiling at such temperatures.

22-132C Liquid motion in the wick depends on the dynamic balance between two opposing effects: the capillary pressure which pumps the liquid through the pores, and the internal resistance to flow due to the friction between the mesh surface and the liquid. Small pores increases the capillary action, but it also increases the friction force opposing the fluid motion. At optimum core size, the difference between the capillary force and the friction force is maximum.

22-133C The orientation of the heat pipe affects its performance. The performance of a heat pipe will be best when the capillary and gravity forces act in the same direction (evaporator end down), and it will be worst when these two forces act in opposite directions (evaporator end up).

22-134C The capillary pressure which creates the suction effect to draw the liquid forces the liquid in a heat pipe to move up against the gravity without a pump. For the heat pipes which work against the gravity, it is better to have fine wicks since the capillary pressure is inversely proportional to the effective capillary radius of the mesh.

22-135C The most important consideration in the design of a heat pipe is the compatibility of the materials used for the tube, wick and the fluid.

22-136C The major cause for the premature degradation of the performance of some heat pipes is contamination which occurs during the sealing of the ends of the heat pipe tube.

22-137 A 40-cm long cylindrical heat pipe dissipates heat at a rate of 150 W. The diameter and mass of a 40-cm long copper rod that can conduct heat at the same rate are to be determined.

Assumptions Steady operating conditions exist.

Properties The properties of copper at room temperature are $\rho = 8950$ kg/m^3 and $k = 386$ W/m.°C.

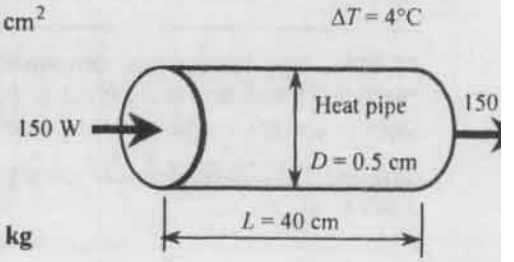
Analysis The rate of heat transfer through the copper rod can be expressed as

$$\dot{Q} = kA\frac{\Delta T}{L}$$

Solving for the cross-sectional area A and the diameter D gives

$$A = \frac{L}{k\Delta T}\dot{Q} = \frac{0.4\text{ m}}{(386\text{ W/m.°C})(4\text{°C})}(150\text{ W}) = 0.03886\text{ m}^2 = 388.6\text{ cm}^2$$

$$A = \frac{\pi D^2}{4} \longrightarrow D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(388.6\text{ cm}^2)}{\pi}} = \mathbf{22.2\text{ cm}}$$



The mass of this copper rod is

$$m = \rho V = \rho AL = (8950\text{ kg/m}^3)(0.03886\text{ m}^2)(0.4\text{ m}) = \mathbf{139\text{ kg}}$$

22-138 A 40-cm long cylindrical heat pipe dissipates heat at a rate of 150 W. The diameter and mass of a 40-cm long copper rod that can conduct heat at the same rate are to be determined.

Assumptions Steady operating conditions exist.

Properties The properties of copper at room temperature are $\rho = 2702$ kg/m^3 and $k = 237$ W/m.°C.

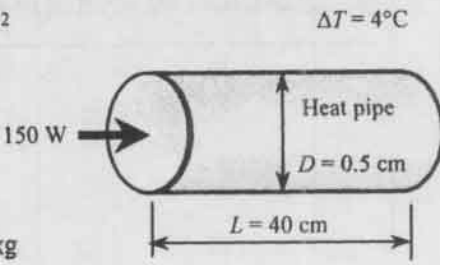
Analysis The rate of heat transfer through the aluminum rod can be expressed as

$$\dot{Q} = kA\frac{\Delta T}{L}$$

Solving for the cross-sectional area A and the diameter D gives

$$A = \frac{L}{k\Delta T}\dot{Q} = \frac{0.4\text{ m}}{(237\text{ W/m.°C})(4\text{°C})}(150\text{ W}) = 0.06329\,\text{m}^2 = 632.9\,\text{cm}^2$$

$$A = \frac{\pi D^2}{4} \longrightarrow D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(632.9\,\text{cm}^2)}{\pi}} = \mathbf{28.4\ cm}$$



The mass of this aluminum rod is

$$m = \rho V = \rho AL = (2702\text{ kg/m}^3)(0.06329\text{ m}^2)(0.4\text{ m}) = \mathbf{68.4\ kg}$$

22-139E A plate that supports 10 power transistors is to be cooled with heat pipes. The number of pipes need to be attached to this plate is to be determined.

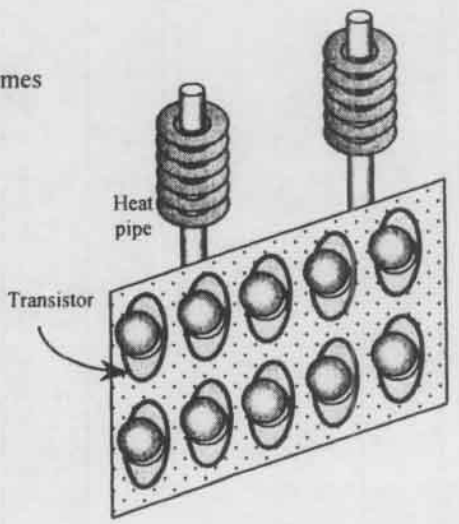
Assumptions Steady operating conditions exist.

Analysis The heat removal rate of heat pipes that have an outside diameter of 0.635 cm and length of 30.5 cm is given in Table 22-5 to be 175 W. The total rate of heat dissipated by 10 transistors each dissipating 35 W is

$$\dot{Q}_{total} = (10)(35\text{ W}) = 350\text{ W}$$

Then the number of heat pipes need to be attached to the plate becomes

$$n = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{350\text{ W}}{175\text{ W}} = \mathbf{2}$$



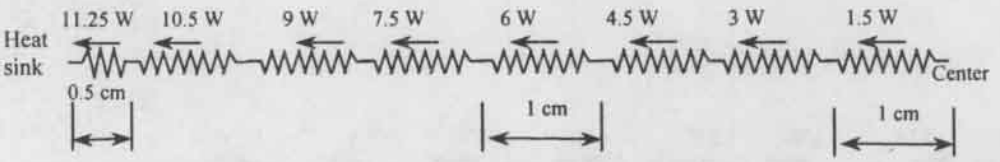
Review Problems

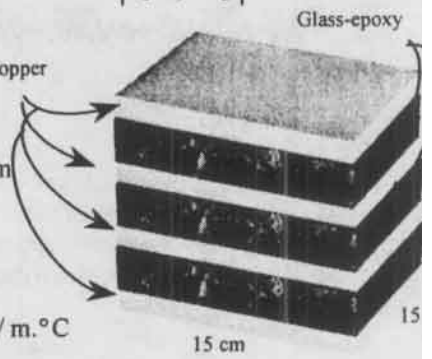
22-140C For most effective cooling, (1) the transistors must be mounted directly over the cooling lines, (2) the thermal contact resistance between the transistors and the cold plate must be minimized by attaching them tightly with a thermal grease, and (3) the thickness of the plates and the tubes should be as small as possible to minimize the thermal resistance between the transistors and the tubes.

22-141C There is no such thing as heat rising. Only heated fluid rises because of lower density due to buoyancy. Heat conduction in a solid is due to the molecular vibrations and electron movement, and gravitational force has no effect on it. Therefore, the orientation of the bar is irrelevant.

22-142 A multilayer circuit board consisting of four layers of copper and three layers of glass-epoxy sandwiched together is considered. The magnitude and location of the maximum temperature that occurs in the PCB are to be determined.

Assumptions **1** Steady operating conditions exist **2** Thermal properties are constant. **3** There is no direct heat dissipation from the surface of the PCB, and thus all the heat generated is conducted by the PCB to the heat sink.





Analysis The effective thermal conductivity of the board is determined from

$$(k_1 t_1)_{copper} = 4[(386\ \text{W/m.°C})(0.0001\ \text{m})] = 0.1544\ \text{W/°C}$$

$$(k_2 t_2)_{epoxy} = 3[(0.26\ \text{W/m.°C})(0.0005\ \text{m})] = 0.00039\ \text{W/°C}$$

$$k_{eff} = \frac{(k_1 t_1)_{copper} + (k_2 t_2)_{epoxy}}{t_1 + t_2} = \frac{(0.1544 + 0.00039)\text{W/°C}}{4(0.0001\ \text{m}) + 3(0.0005\ \text{m})} = 81.5\ \text{W/m.°C}$$

The maximum temperature will occur in the middle of the board which is farthest away from the heat sink. We consider half of the board because of symmetry, and divide the region in 1-cm thick strips, starting at the mid-plane. Then from Fourier's law, the temperature difference across a strip can be determined from

$$\dot{Q} = k_{eff} A \frac{\Delta T}{L} \quad \longrightarrow \quad \Delta T = \frac{\dot{Q} L}{k_{eff} A}$$

where L = 1 cm = 0.01 m (except it is 0.5 cm for the strip attached to the heat sink), and the heat transfer area for all the strips is

$$A = (0.15\ \text{m})[4(0.0001\ \text{m}) + 3(0.0005\ \text{m})] = 0.000285\ \text{m}^2$$

Then the temperature at the center of the board is determined by adding the temperature differences across all the strips as

$$\Delta T_{\text{center-heat sink}} = \sum \frac{\dot{Q} L}{k_{eff} A} = \frac{(\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 + \dot{Q}_4 + \dot{Q}_5 + \dot{Q}_6 + \dot{Q}_7 + \dot{Q}_8/2) L}{k_{eff} A}$$

$$= \frac{(1.5 + 3 + 4.5 + 6 + 7.5 + 9 + 10.5 + 11.25/2\ \text{W})(0.01\ \text{m})}{(81.5\ \text{W/m.°C})(0.000285\ \text{m}^2)} = 20.5°\text{C}$$

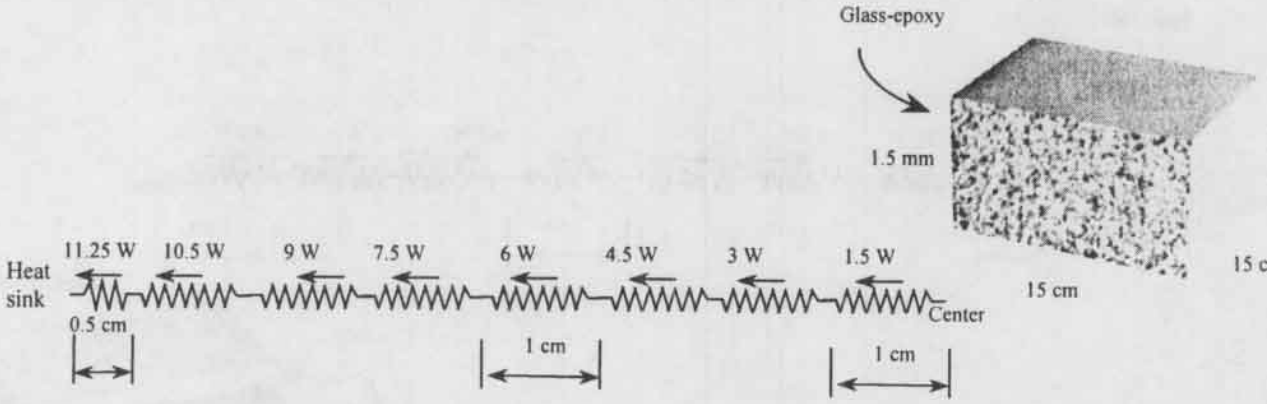
and $$T_{\text{center}} = T_{\text{heat sink}} + \Delta T_{\text{center-heat sink}} = 35°\text{C} + 20.5°\text{C} = \mathbf{55.5°C}$$

Discussion This problem can also be solved approximately by using the "average" heat transfer rate for the entire half board, and treating it as a constant. The heat transfer rate in each half changes from 0 at the center to 22.5/2 = 11.25 W at the heat sink, with an average of 11.25/2 = 5.625 W. Then the center temperature becomes

$$\dot{Q}_{ave} \cong k_{eff} A \frac{T_1 - T_2}{L} \longrightarrow T_{\text{center}} \cong T_{\text{heat sink}} + \frac{\dot{Q}_{ave} L}{k_{eff} A} = 35°\text{C} + \frac{(5.625\ \text{W})(0.075\ \text{m})}{(81.5\ \text{W/m.°C})(0.000285\ \text{m}^2)} = \mathbf{53.2°C}$$

22-143 A circuit board consisting of a single layer of glass-epoxy is considered. The magnitude and location of the maximum temperature that occurs in the PCB are to be determined.

Assumptions **1** Steady operating conditions exist **2** Thermal properties are constant. **3** There is no direct heat dissipation from the surface of the PCB, and thus all the heat generated is conducted by the PCB to the heat sink.



Analysis In this case the board consists of a 1.5-mm thick layer of epoxy. Again the maximum temperature will occur in the middle of the board which is farthest away from the heat sink. We consider half of the board because of symmetry, and divide the region in 1-cm thick strips, starting at the mid-plane. Then from Fourier's law, the temperature difference across a strip can be determined from

$$\dot{Q} = k_{eff} A \frac{\Delta T}{L} \quad \longrightarrow \quad \Delta T = \frac{\dot{Q} L}{k_{eff} A}$$

where L = 1 cm = 0.01 m (except it is 0.5 cm for the strip attached to the heat sink), and the heat transfer area for all the strips is

$$A = (0.15 \text{ m})(0.0015 \text{ m}) = 0.000225 \text{ m}^2$$

Then the temperature at the center of the board is determined by adding the temperature differences across all the strips as

$$\Delta T_{\text{center-heat sink}} = \sum \frac{\dot{Q} L}{k_{eff} A} = \frac{(\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 + \dot{Q}_4 + \dot{Q}_5 + \dot{Q}_6 + \dot{Q}_7 + \dot{Q}_8/2)L}{k_{eff} A}$$

$$= \frac{(1.5+3+4.5+6+7.5+9+10.5+11.25/2 \text{ W})(0.01 \text{ m})}{(0.26 \text{ W/m.°C})(0.000225 \text{ m}^2)} = 8141°\text{C}$$

and $$T_{\text{center}} = T_{\text{heat sink}} + \Delta T_{\text{center-heat sink}} = 35°\text{C} + 8141°\text{C} = \mathbf{8176°C}$$

Discussion This problem can also be solved approximately by using the "average" heat transfer rate for the entire half board, and treating it as a constant. The heat transfer rate in each half changes from 0 at the center to 22.5/2 = 11.25 W at the heat sink, with an average of 11.25/2 = 5.625 W. Then the center temperature becomes

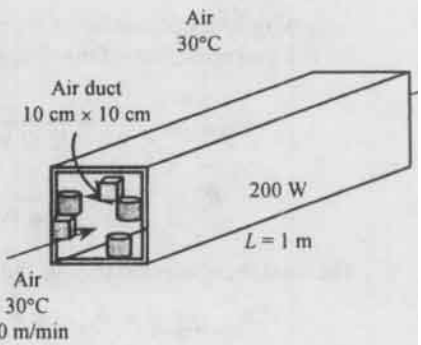
$$\dot{Q}_{ave} \cong k_{eff} A \frac{T_1 - T_2}{L} \longrightarrow T_{\text{center}} \cong T_{\text{heat sink}} + \frac{\dot{Q}_{ave} L}{k_{eff} A} = 35°\text{C} + \frac{(5.625 \text{ W})(0.075 \text{ m})}{(0.26 \text{ W/m.°C})(0.000225 \text{ m}^2)} = \mathbf{7247°C}$$

22-144 The components of an electronic system located in a horizontal duct of rectangular cross-section are cooled by forced air flowing through the duct. The heat transfer from the outer surfaces of the duct by natural convection, the average temperature of the duct and the highest component surface temperature in the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties We use the properties of air at (30+45)/2 = 37.5°C (310 K) are

$$\rho = 1.143 \text{ kg/m}^3$$
$$C_p = 1006 \text{ J/kg.°C}$$
$$\text{Pr} = 0.713$$
$$k = 0.0268 \text{ W/m.°C}$$
$$\nu = 2.05 \times 10^{-5} \text{ m}^2/\text{s}$$



Analysis (a) The volume and the mass flow rates of air are

$$\dot{V} = A_c \text{V} = (0.1 \text{ m})(0.1 \text{ m})(50/60 \text{ m/s}) = 0.008333 \text{ m}^3/\text{s}$$
$$\dot{m} = \rho \dot{V} = (1.143 \text{ kg/m}^3)(0.008333 \text{ m}^3/\text{s}) = 0.009525 \text{ kg/s}$$

The rate of heat transfer to the air flowing through the duct is

$$\dot{Q}_{forced\ conv} = \dot{m} C_p (T_{in} - T_{out}) = (0.009525 \text{ kg/s})(1006 \text{ J/kg.°C})(45-30)°\text{C} = 143.7 \text{ W}$$

Then the rate of heat loss from the outer surfaces of the duct to the ambient air by natural convection becomes

$$\dot{Q}_{conv} = \dot{Q}_{total} - \dot{Q}_{forced\ conv} = 200 \text{ W} - 143.7 \text{ W} = \mathbf{56.3\ W}$$

(b) The average surface temperature can be determined from

$$\dot{Q}_{conv} = hA(T_{s,duct} - T_{ambient\ air})$$

But we first need to determine convection heat transfer coefficient. Using the Nusselt number relation from Table 22-2,

$$A = (4)(0.1 \text{ m})(1 \text{ m}) = 0.4 \text{ m}^2$$

$$\text{Re} = \frac{\text{V} D_h}{\nu} = \frac{(50/60 \text{ m/s})(0.1 \text{ m})}{2.05 \times 10^{-5} \text{ m}^2/\text{s}} = 4065$$

$$Nu = 0.102 \text{ Re}^{0.675} \text{ Pr}^{1/3} = (0.102)(4065)^{0.675}(0.713)^{1/3} = 24.9$$

$$h = \frac{k}{D_h} Nu = \frac{0.0268 \text{ W/m.°C}}{0.1 \text{ m}}(24.9) = 6.67 \text{ W/m}^2.°\text{C}$$

Then the average surface temperature of the duct becomes

$$\dot{Q}_{conv} = hA(T_s - T_{ambient}) \longrightarrow T_s = T_{ambient} + \frac{\dot{Q}_{conv}}{hA} = 30°\text{C} + \frac{56.3 \text{ W}}{(6.67 \text{ W/m}^2.°\text{C})(0.4 \text{ m}^2)} = \mathbf{51.1°C}$$

(c) The highest component surface temperature will occur at the exit of the duct. From Newton's law relation at the exit,

$$\dot{q}_{conv} = h(T_{s,\max} - T_{air,exit}) \longrightarrow T_{s,\max} = T_{air,exit} + \frac{\dot{Q}_{conv}/A}{h} = 45°\text{C} + \frac{56.3 \text{ W}}{(6.67 \text{ W/m}^2.°\text{C})(0.4 \text{ m}^2)} = \mathbf{66.1°C}$$

22-145 Two power transistors are cooled by mounting them on the two sides of an aluminum bracket that is attached to a liquid-cooled plate. The temperature of the transistor case and the fraction of heat dissipation to the ambient air by natural convection and to the cold plate by conduction are to be determined.

Assumptions **1** Steady operating conditions exist **2** Conduction heat transfer is one-dimensional. **3** We assume the ambient temperature is 25°C.

Analysis The rate of heat transfer by conduction is

$$\dot{Q}_{conduction} = (0.80)(12\ \text{W}) = 9.6\ \text{W}$$

Assuming heat conduction in the plate to be one-dimensional for simplicity, the thermal resistance of the aluminum plate and epoxy adhesive are

$$R_{aluminum} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(237\ \text{W/m.°C})(0.003\ \text{m})(0.03\ \text{m})} = 0.938°\text{C/W}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.0002\ \text{m}}{(1.8\ \text{W/m.°C})(0.003\ \text{m})(0.03\ \text{m})} = 1.235°\text{C/W}$$

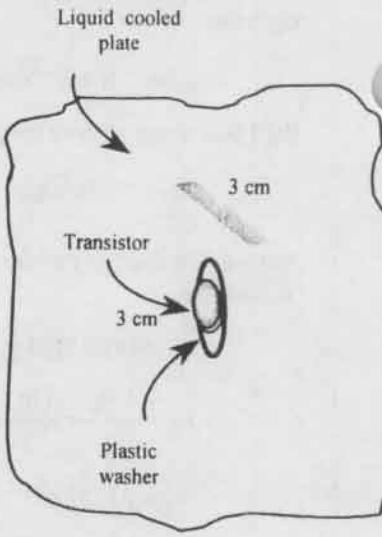
The total thermal resistance of the plate and the epoxy is

$$R_{plate+epoxy} = R_{epoxy} + R_{plate} = 1.235 + 0.938 = 2.173°\text{C/W}$$

Heat generated by the transistors is conducted to the plate, and then it is dissipated to the cold plate by conduction, and to the ambient air by convection. Denoting the plate temperature where the transistors are connected as $T_{s,max}$ and using the heat transfer coefficient relation from Table 22-1 for a vertical plate, the total heat transfer from the plate can be expressed as

$$\dot{Q}_{total} = \dot{Q}_{cond} + \dot{Q}_{conv} = \frac{\Delta T_{plate}}{R_{plate+epoxy}} + hA_{side}(T_{s,ave} - T_{air})$$

$$= \frac{T_{s,\max} - T_{cold\ plate}}{R_{plate+epoxy}} + 1.42\left(\frac{(T_{s,ave} - T_{air})}{L}\right)^{0.25} A_{side}(T_{s,\max} - T_{air})$$



where $T_{s,\,ave} = (T_{s,\,max} + T_{cold\ plate})/2$, L 0.03 m, and $A_{side} = 2(0.03\ \text{m})(0.03\ \text{m}) = 0.00018\ \text{m}^2$. Substituting the known quantities gives

$$20\ \text{W} = \frac{T_{s,\max} - 40}{2.173°\text{C/W}} + 1.42(0.00018)\frac{[(T_{s,\max} + 40)/2 - 25]^{1.25}}{(0.03)^{0.25}}$$

Solving for $T_{s,\,max}$ gives

$$T_{s,\,max} = 83.3°\text{C}$$

Then the rate of heat transfer by natural convection becomes

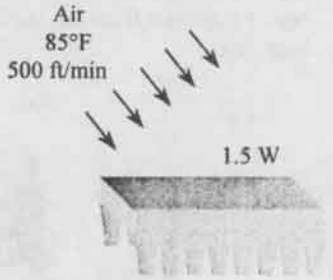
$$\dot{Q}_{conv} = 1.42(0.00018)\frac{[(83.3 + 40)/2 - 25]^{1.25}}{(0.03)^{0.25}} = 0.055\ \text{W}$$

which is 0.055/20 = 0.00275 or **0.3%** of the total heat dissipated. The remaining **99.7%** of the heat is transferred by conduction. Therefore, heat transfer by natural convection is negligible. Then the surface temperature of the transistor case becomes

$$T_{case} = T_{s,max} + \dot{Q}R_{plastic\ washer} = 83.3°\text{C} + (10\ \text{W})(2°\text{C/W}) = \mathbf{103.3°C}$$

22-146E A plastic DIP with 24 leads is cooled by forced air. Using data supplied by the manufacturer, the junction temperature is to be determined for two cases.

Assumptions Steady operating conditions exist.



Analysis The junction-to-ambient thermal resistance of the device with 24 leads corresponding to an air velocity of 500 × 0.3048 = 152.4 m/min is determined from Fig. 22-23 to be

$$R_{junction-ambient} = 50\ °\text{C/W} = (50 \times 1.8) + 32 = 122\ °\text{F/W}$$

Then the junction temperature becomes

$$\dot{Q} = \frac{T_{junction} - T_{ambient}}{R_{junction-ambient}} \longrightarrow T_{junction} = T_{ambient} + \dot{Q}R_{junction-ambient}$$

$$= 85°\text{F} + (1.5\ \text{W})(122\ °\text{F/W}) = \mathbf{268°F}$$

When the fan fails the total thermal resistance is determined from Fig. 22-23 by reading the value at the intersection of the curve at the vertical axis to be

$$R_{junction-ambient} = 66\ °\text{C/W} = (66 \times 1.8) + 32 = 150.8\ °\text{F/W}$$

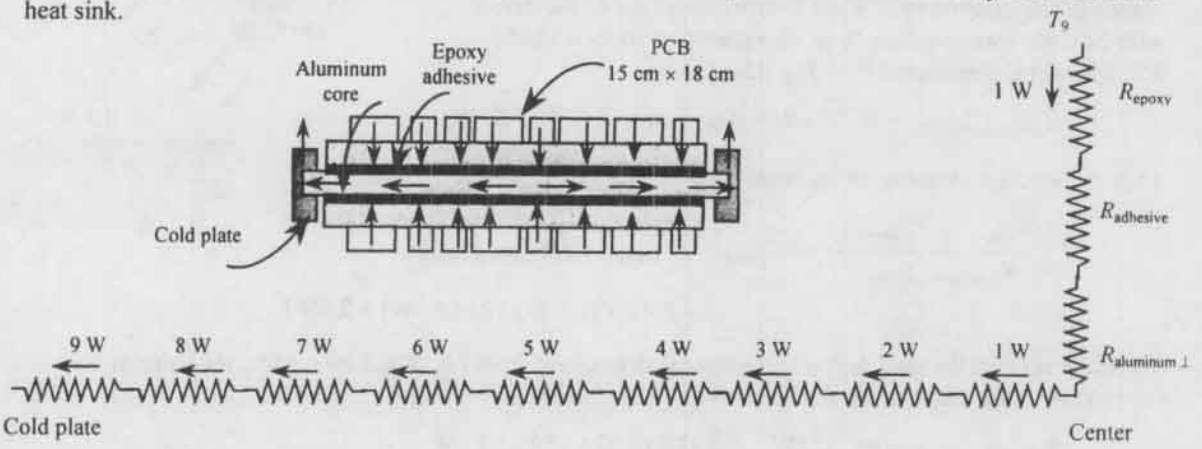
which yields

$$\dot{Q} = \frac{T_{junction} - T_{ambient}}{R_{junction-ambient}} \longrightarrow T_{junction} = T_{ambient} + \dot{Q}R_{junction-ambient}$$

$$= 85°\text{F} + (1.5\ \text{W})(150.8\ °\text{F/W}) = \mathbf{311°F}$$

22-147 A circuit board is to be conduction-cooled by aluminum core plate sandwiched between two epoxy laminates. The maximum temperature rise between the center and the sides of the PCB is to be determined.

Assumptions **1** Steady operating conditions exist **2** Thermal properties are constant. **3** There is no direct heat dissipation from the surface of the PCB., and thus all the heat generated is conducted by the PCB to the heat sink.



Analysis Using the half thickness of the aluminum frame because of symmetry, the thermal resistances against heat flow in the vertical direction for a 1-cm wide strip are

$$R_{\text{aluminum},\perp} = \frac{L}{kA} = \frac{0.0006\ \text{m}}{(237\ \text{W/m.°C})(0.01\ \text{m})(0.15\ \text{m})} = 0.00169\ °\text{C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0005\ \text{m}}{(0.26\ \text{W/m.°C})(0.01\ \text{m})(0.15\ \text{m})} = 1.28205\ °\text{C/W}$$

$$R_{\text{adhesive}} = \frac{L}{kA} = \frac{0.0001\ \text{m}}{(1.8\ \text{W/m.°C})(0.01\ \text{m})(0.15\ \text{m})} = 0.03703\ °\text{C/W}$$

$$R_{\text{vertical}} = R_{\text{aluminum},\perp} + R_{\text{epoxy}} + R_{\text{adhesive}} = 0.00169 + 1.28205 + 0.03704 = 1.321\ °\text{C/W}$$

We assume heat conduction along the epoxy and adhesive in the horizontal direction to be negligible, and heat to be conduction to the heat sink along the aluminum frame. The thermal resistance of the aluminum frame against heat conduction in the horizontal direction for a 1-cm long strip is

$$R_{\text{frame}} = R_{\text{aluminum},\parallel} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(237\ \text{W/m.°C})(0.0012\ \text{m})(0.18\ \text{m})} = 0.1953\ °\text{C/W}$$

The temperature difference across a strip is determined from

$$\Delta T = \dot{Q} R_{\text{aluminum},\parallel}$$

The maximum temperature rise across the 9 cm distance between the center and the sides of the board is determined by adding the temperature differences across all the strips as

$$\Delta T_{\text{horizontal}} = \sum \dot{Q} R_{\text{aluminum},\parallel} = (\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 + \dot{Q}_4 + \dot{Q}_5 + \dot{Q}_6 + \dot{Q}_7 + \dot{Q}_8 2) R_{\text{aluminum},\parallel}$$
$$= (1+2+3+4+5+6+7+8+9\ \text{W})(0.1953\ °\text{C/W}) = 8.8°\text{C}$$

The temperature difference between the center of the aluminum core and the outer surface of the PCB is determined similarly to be

$$\Delta T_{\text{vertical}} = \sum \dot{Q} R_{\text{vertical},\perp} = (1\ \text{W})(1.321\ °\text{C/W}) = 1.3°\text{C}$$

Then the maximum temperature rise across the 9-cm distance between the center and the sides of the PCB becomes

$$\Delta T_{\max} = \Delta T_{\text{horizontal}} + \Delta T_{\text{vertical}} = 8.8 + 1.3 = \mathbf{10.1°C}$$

22-148 Ten power transistors attached to an aluminum plate are cooled from two sides of the plate by liquid. The temperature rise between the transistors and the heat sink is to be determined.

Assumptions **1** Steady operating conditions exist **2** Thermal properties are constant.

Analysis We consider only half of the plate because of symmetry. It is stated that 85% of the heat generated is conducted through the aluminum plate, and this heat will be conducted across the 1-cm wide section between the transistors and the cooled edge of the plate. (Note that the mid section of the plate will essentially be isothermal and thus there will be no significant heat transfer towards the midsection). The rate of heat conduction to each side is of the plate is

$$\dot{Q}_{\text{cond,1-side}} = 0.85 \times (10\ \text{W}) = 8.5\ \text{W}$$

Then the temperature rise across the 1-cm wide section of the plate can be determined from

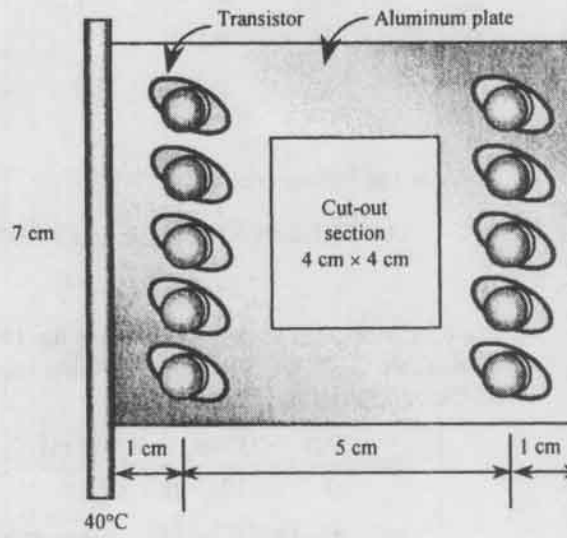
$$\dot{Q}_{\text{cond 1-side}} = kA\frac{(\Delta T)_{\text{plate}}}{L}$$

Solving for $(\Delta T)_{\text{plate}}$ and substituting gives

$$(\Delta T)_{\text{plate}} = \frac{\dot{Q}_{\text{cond 1-side}} L}{kA}$$

$$= \frac{(8.5\ \text{W})(0.01\ \text{m})}{(237\ \text{W/m.°C})(0.07 \times 0.002\ \text{m}^2)}$$

$$= \mathbf{2.6°C}$$

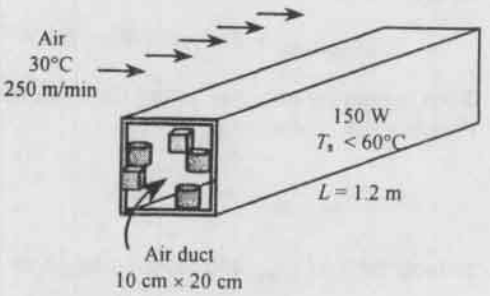


22-149 The components of an electronic system located in a horizontal duct are cooled by air flowing over the duct. The total power rating of the electronic devices that can be mounted in the duct is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** Thermal properties of air are constant. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of (30+60)/2 = 45°C = 318 K are (Table A-18)

$$\text{Pr} = 0.710$$
$$k = 0.0274 \text{ W/m.°C}$$
$$\nu = 1.74 \times 10^{-5} \text{ m}^2/\text{s}$$



Analysis The surface area of the duct is

$$A = 2\{[(1.2 \text{ m})(0.1 \text{ m})] + [(1.2 \text{ m})(0.2 \text{ m})]\} = 0.72 \text{ m}^2$$

The duct is oriented such that air strikes the 10 cm high side normally. Using the Nusselt number relation from Table 22-2 for a 10-cm by 10-cm cross-section square as an approximation, the heat transfer coefficient is determined to be

$$\text{Re} = \frac{VD}{\nu} = \frac{(250/60 \text{ m/s})(0.1 \text{ m})}{1.74 \times 10^{-5} \text{ m}^2/\text{s}} = 23{,}946$$

$$Nu = 0.102 \text{Re}^{0.675} \text{Pr}^{1/3} = (0.102)(23{,}946)^{0.675}(0.710)^{1/3} = 82.2$$

$$h = \frac{k}{D_h} Nu = \frac{0.0274 \text{ W/m.°C}}{0.1 \text{ m}}(82.2) = 22.523 \text{ W/m}^2.\text{°C}$$

Then the heat transfer rate (and thus the power rating of the components inside) in this orientation is determined from

$$\dot{Q} = hA(T_s - T_{fluid}) = (22.523 \text{ W/m}^2.\text{°C})(0.72 \text{ m}^2)(60-30)\text{°C} = \mathbf{487 \text{ W}}$$

We now consider the duct oriented such that air strikes the 20 cm high side normally. Using the Nusselt number relation from Table 22-2 for a 20-cm by 20-cm cross-section square as an approximation, the heat transfer coefficient is determined to be

$$\text{Re} = \frac{VD}{\nu} = \frac{(250/60 \text{ m/s})(0.2 \text{ m})}{1.74 \times 10^{-5} \text{ m}^2/\text{s}} = 47{,}893$$

$$Nu = 0.102 \text{Re}^{0.675} \text{Pr}^{1/3} = (0.102)(47{,}893)^{0.675}(0.712)^{1/3} = 131.3$$

$$h = \frac{k}{D_h} Nu = \frac{0.0274 \text{ W/m.°C}}{0.2 \text{ m}}(131.3) = 18.0 \text{ W/m}^2.\text{°C}$$

Then the heat transfer rate (and thus the power rating of the components inside) in this orientation is determined from

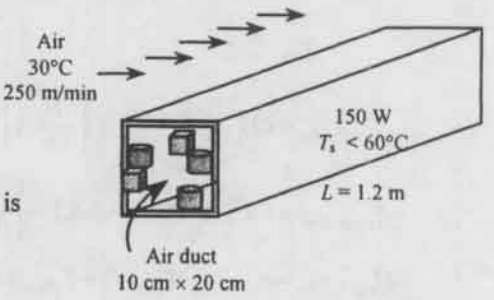
$$\dot{Q} = hA(T_s - T_{fluid}) = (18.0 \text{ W/m}^2.\text{°C})(0.72 \text{ m}^2)(60-30)\text{°C} = \mathbf{389 \text{ W}}$$

22-150 The components of an electronic system located in a horizontal duct are cooled by air flowing over the duct. The total power rating of the electronic devices that can be mounted in the duct is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** Thermal properties of air are constant.

Properties The properties of air at 1 atm and the film temperature of (30+60)/2 = 45°C = 318 K are (Table A-18)

$$\text{Pr} = 0.710$$
$$k = 0.0274 \text{ W/m.°C}$$
$$\nu = 1.74 \times 10^{-5} \text{ m}^2/\text{s}$$



Note that for ideal gases, the Prandtl number and the thermal conductivity are independent of pressure while kinematic viscosity is inversely proportional to the pressure

Analysis. The surface area of the duct is

$$A = 2\{[(1.2 \text{ m})(0.1 \text{ m})] + [(1.2 \text{ m})(0.2 \text{ m})]\} = 0.72 \text{ m}^2$$

The duct is oriented such that air strikes the 10 cm high side normally. Using the Nusselt number relation from Table 22-2 for a 10-cm by 10-cm cross-section square as an approximation, the heat transfer coefficient is determined to be

$$\text{Re} = \frac{VD}{\nu} = \frac{(250/60 \text{ m/s})(0.1 \text{ m})}{3.26 \times 10^{-5} \text{ m}^2/\text{s}} = 12{,}781$$

$$Nu = 0.102 \text{ Re}^{0.675} \text{ Pr}^{1/3} = (0.102)(12{,}781)^{0.675}(0.710)^{1/3} = 53.8$$

$$h = \frac{k}{D_h} Nu = \frac{0.0274 \text{ W/m.°C}}{0.1 \text{ m}}(53.8) = 14.7 \text{ W/m}^2\text{.°C}$$

Then the heat transfer rate (and thus the power rating of the components inside) in this orientation is determined from

$$\dot{Q} = hA(T_s - T_{fluid}) = (14.7 \text{ W/m}^2\text{.°C})(0.72 \text{ m}^2)(60-30)°\text{C} = \mathbf{319 \text{ W}}$$

We now consider the duct oriented such that air strikes the 20 cm high side normally. Using the Nusselt number relation from Table 22-2 for a 20-cm by 20-cm cross-section square as an approximation, the heat transfer coefficient is determined to be

$$\text{Re} = \frac{VD}{\nu} = \frac{(250/60 \text{ m/s})(0.2 \text{ m})}{3.26 \times 10^{-5} \text{ m}^2/\text{s}} = 25{,}571$$

$$Nu = 0.102 \text{ Re}^{0.675} \text{ Pr}^{1/3} = (0.102)(25{,}571)^{0.675}(0.710)^{1/3} = 86.4$$

$$h = \frac{k}{D_h} Nu = \frac{0.0274 \text{ W/m.°C}}{0.2 \text{ m}}(86.4) = 11.8 \text{ W/m}^2\text{.°C}$$

Then the heat transfer rate (and thus the power rating of the components inside) in this orientation is determined from

$$\dot{Q} = hA(T_s - T_{fluid}) = (11.8 \text{ W/m}^2\text{.°C})(0.72 \text{ m}^2)(60-30)°\text{C} = \mathbf{256 \text{ W}}$$

22-151E A computer is cooled by a fan blowing air into the computer enclosure. The fraction of heat lost from the outer surfaces of the computer case is to be determined.

Assumptions **1** Steady operating conditions exist **2** Thermal properties of air are constant. **3** The local atmospheric pressure is 1 atm.

Analysis Using the proper relation from Table 22-1, the heat transfer coefficient and the rate of natural convection heat transfer from the vertical side surfaces are determined to be

$$L = \frac{6}{12}\text{ ft}$$

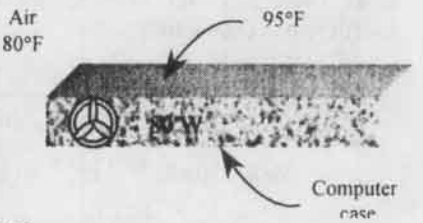
$$A_{side} = (2)\left(\frac{20}{12}\text{ ft} + \frac{24}{12}\text{ ft}\right)\left(\frac{6}{12}\text{ ft}\right) = 3.67\text{ ft}^2$$

$$h_{conv,side} = 1.42\left(\frac{\Delta T}{L}\right)^{0.25} = 1.42\left(\frac{95-80}{6/12}\right)^{0.25} = 3.32\text{ Btu/h.ft}^2\text{.°F}$$

$$\dot{Q}_{conv,side} = h_{conv,side} A_{side}(T_s - T_{fluid}) = (3.32\text{ Btu/h.ft}^2\text{.°F})(3.67\text{ ft}^2)(95-80)\text{°F} = 182.9\text{ Btu/h}$$

Similarly, the rate of heat transfer from the horizontal top surface by natural convection is determined to be

$$L = \frac{4A}{p} = \frac{4\left(\frac{20}{12}\text{ ft}\right)\left(\frac{24}{12}\text{ ft}\right)}{2\left[\left(\frac{20}{12}\text{ ft}\right) + \left(\frac{24}{12}\text{ ft}\right)\right]} = 1.82\text{ ft}$$



$$A_{top} = \left(\frac{20}{12}\text{ ft}\right)\left(\frac{24}{12}\text{ ft}\right) = 3.33\text{ ft}^2$$

$$h_{conv,top} = 1.32\left(\frac{\Delta T}{L}\right)^{0.25} = 1.32\left(\frac{95-80}{1.82}\right)^{0.25} = 2.24\text{ Btu/h.ft}^2\text{.°F}$$

$$\dot{Q}_{conv,top} = h_{conv,top} A_{top}(T_s - T_{fluid}) = (2.24\text{ Btu/h.ft}^2\text{.°F})(3.33\text{ ft}^2)(95-80)\text{°F} = 111.7\text{ Btu/h}$$

The rate of heat transfer from the outer surfaces of the computer case by radiation is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A\sigma(T_s^4 - T_{surr}^4)\\ &= (0.85)(3.67\text{ ft}^2 + 3.33\text{ ft}^2)(0.1714\text{ Btu/h.ft}^2\text{.R}^4)[(95+460\text{ R})^4 - (80+273\text{ R})^4]\\ &= 100.4\text{ Btu/h}\end{aligned}$$

Then the total rate of heat transfer from the outer surfaces of the computer case becomes

$$\dot{Q}_{total} = \dot{Q}_{conv,side} + \dot{Q}_{conv,top} + \dot{Q}_{rad} = 182.9 + 111.7 + 100.4 = 395\text{ Btu/h}$$

Therefore, the fraction of the heat loss from the outer surfaces of the computer case is

$$f = \frac{(395/3.41214)\text{ W}}{170\text{ W}} = 0.68 = \mathbf{68\%}$$